

Chapter 11

11.1

a) Firm 2 chooses its quantity to maximize:

$$\pi_2 = Q_2(1000 - 4Q_1 - 4Q_2) - 20Q_2$$

$$\frac{\partial \pi_2}{\partial Q_2} = 1000 - 4Q_1 - 8Q_2 - 20 = 0 \Rightarrow Q_2 = \frac{245 - Q_1}{2}$$

Now firm 1 chooses its quantity to maximize

$$\pi_1 = Q_1(1000 - 4Q_1 - 4Q_2) - 20Q_1 = Q_1 \left(980 - 4Q_1 - 4 \frac{245 - Q_1}{2} \right) = \frac{1}{2} Q_1(980 - 4Q_1)$$

$$\frac{\partial \pi_1}{\partial Q_1} = \frac{1}{2}(980 - 8Q_1) = 0 \Rightarrow Q_1 = \frac{980}{8} = 122.5 \Rightarrow Q_2 = 61.25$$

11.1

b)

There is no non-negative c such that the leader and the follower have the same market share. To see this, consider $c = 0$. Then the leader's quantity is 120, whereas the follower's quantity is less than 120. As c increases, the market share of the leader goes up and the market share of the follower goes down.

11.2

Let p_1 be the price charged by Ben and p_2 be the price charged by Will. Let x be the location of a consumer who is indifferent between buying from Ben and Will.

Therefore,

$$p_1 + x = p_2 + (10 - x) \Rightarrow x = \frac{1}{2}(p_2 - p_1 + 10)$$

Consequently, the demand faced by Ben is

$$D_1(p_1, p_2) = \frac{1}{2} \left(\frac{1000}{10} \right) (p_2 - p_1 + 10) = 500 + 50(p_2 - p_1)$$

11.2

Demand faced by Will is

$$D_2(p_1, p_2) = \left(\frac{1000}{10}\right) \left(10 - \frac{1}{2}(p_2 - p_1 + 10)\right) = 500 - 50(p_2 - p_1)$$

Ben's profit is given by

$$\pi_1(p_1, p_2) = (p_1 - 1)(500 + 50(p_2 - p_1)) - 250$$

Will's profit is given by

$$\pi_2(p_1, p_2) = (p_2 - 1)(500 - 50(p_2 - p_1)) - 250$$

11.2

Since Will is the follower, we first maximize π_2 with respect to p_2 , to derive Will's reaction function.

$$\frac{\partial \pi_2(p_1, p_2)}{\partial p_2} = (p_2 - 1)(-50) + (500 - 50(p_2 - p_1)) = 0$$

$$\Rightarrow p_2 = \frac{1}{100} (550 + 50p_1) = \frac{1}{2} (11 + p_1)$$

Now, substitute Will's reaction function in to Ben's profit function to get:

$$\pi_1(p_1, p_2) = (p_1 - 1) \left(500 + 50 \left(\frac{1}{2} (11 + p_1) - p_1 \right) \right) - 250 = (p_1 - 1)(775 - 25p_1) - 250$$

11.2

We now maximize π_1 with respect to p_1

$$\frac{\partial \pi_1(p_1, p_2)}{\partial p_1} = (p_2 - 1)(-25) + (775 - 25p_1) = 0$$

$$\Rightarrow p_1 = \frac{800}{50} = 16$$

Now from Will's reaction function, we get

$$\Rightarrow p_2 = \frac{1}{2}(11 + p_1) = 13.5$$

11.2

Hence, Ben will serve

$$D_1(p_1, p_2) = 500 + 50(p_2 - p_1) = 500 - 50\left(\frac{5}{2}\right) = 375$$

Will serves

$$D_2(p_1, p_2) = 500 - 50(p_2 - p_1) = 500 + 50\left(\frac{5}{2}\right) = 625$$

Ben's profit is

$$\pi_1 = 375(16 - 1) - 250 = 5375$$

Will's profit is

$$\pi_2 = 625(13.5 - 1) - 250 = 7562.5$$

Chapter 12

12.1

a) Setting marginal revenue equal to marginal cost will yield

$$\begin{aligned}MR &= 50 - 0.2q_I = 0.05q_I = MC \\&\Rightarrow 0.25q_I = 50 \\&\Rightarrow q_I = 200 \\&\Rightarrow P = 50 - 0.1q_I = 50 - 20 = 30\end{aligned}$$

The firm will have profits equal to

$$\pi_I = (30)(200) - (0,025)(200)^2 = 6000 - 1000 = 5000$$

12.1

b) The residual industry demand curve can be written as

$$P = 50 - 0.1Q = 50 - 0.1q_I - 0.1q_E = 50 - 0.1(200) - 0.1q_E = 30 - 0.1q_E$$

Marginal revenue from the entrant firm will be

$$MR_E = 30 - 0.2q_E$$

Setting marginal revenue equal to marginal cost we obtain

$$\begin{aligned} TR_E &= (30 - 0.1q_E)q_E \\ MR_E &= 30 - 0.2q_E = 10 + 0.05q_E = MC_E \\ &\Rightarrow 0.25q_E = 20 \\ &\Rightarrow q_E = 80 \end{aligned}$$

$$\Rightarrow P = 50 - 0.1(200) - 0.1(80) = 50 - 20 - 8 = 22$$

The entrant will export 80 units to the market and the price will fall from \$30 to \$22. The total quantity transacted will rise from 200 to 280. Profits for the two firms will be

$$\begin{aligned} \pi_I &= (22)(200) - (0,025)(200)^2 = 4400 - 1000 = 3400 \\ \pi_E &= (22)(80) - 10(80) - (0,025)(200)^2 = 1760 - 800 - 160 = 800 \end{aligned}$$

12.1

c) We simply need to find the level of $q_I = Q$ such that the best response of the entrant is to produce zero output. Writing the residual demand curve as a function of q_I we obtain

$$P = 50 - 0.1Q = 50 - 0.1q_I - 0.1q_E$$

Marginal revenue for the entrant firm will be

$$MR_E = 50 - 0.1q_I - 0.2q_E$$

Setting marginal revenue equal to marginal cost we obtain

$$\begin{aligned} MR_E = 50 - 0.1q_I - 0.2q_E &= 10 + 0.05q_E = MC_E \\ \Rightarrow 0.25q_E &= 40 - 0.1q_I \\ \Rightarrow q_E &= 160 - 0.4q_I \end{aligned}$$

If the incumbent chooses q_I such that the optimal $q_E = 0$, the entrant will not enter. This implies

$$q_E = 160 - 0.4q_I = 0 \Rightarrow q_I = 400 = Q$$

With this level of output price and profits of the two firms are

$$P = 10, \pi_I = 0 \text{ and } \pi_E = 0$$

12.2

Now consider a two-firm Cournot model with different cost functions for each firm. The solution is obtained by choosing q_I to maximize profit given the rival's output. For firm 1:

$$\begin{aligned}\pi_I &= [Pq_I - C(q_I)] = [(50 - 0.1q_I - 0.1q_E)q_I - 0.025q_I^2] \\ &= [50q_I - 0.1q_I^2 - 0.1q_Iq_E - 0.025q_I^2]\end{aligned}$$

$$\begin{aligned}\Rightarrow \frac{\partial \pi_I}{\partial q_I} &= 50 - 0.2q_I - 0.1q_E - 0.05q_I = 0 \\ &\Rightarrow 0.25q_I = 50 - 0.1q_E \\ &\Rightarrow q_I^* = 200 - 0.4q_E\end{aligned}$$

Similarly, the best response function for the second firm is given by

$$\begin{aligned}\pi_E &= [Pq_E - C(q_E)] = [(50 - 0.1q_I - 0.1q_E)q_E - 10q_E - 0.025q_E^2] \\ &= [50q_I - 0.1q_I^2 - 0.1q_Iq_E - 0.025q_I^2]\end{aligned}$$

$$\begin{aligned}\Rightarrow \frac{\partial \pi_E}{\partial q_E} &= 50 - 0.2q_E - 0.1q_I - 10 - 0.05q_E = 0 \\ &\Rightarrow 0.25q_E = 40 - 0.1q_I \\ &\Rightarrow q_E^* = 160 - 0.4q_I\end{aligned}$$

12.2

We solve for the optimal q_I^* and q_E^* as follows

$$\begin{aligned}q_E &= 160 - 0.4q_I = 160 - 0.4(200 - 0.4q_E) = 80 + 16q_E \\&\Rightarrow q_E^* = 95.238 \\&\Rightarrow q_I^* = 161.90476 \\&\Rightarrow P = 24.285715\end{aligned}$$

Profits are given by

$$\begin{aligned}\pi_I^* &= 3276.644 \\ \pi_E^* &= 1133.787\end{aligned}$$

The incumbent earns a profit less than if he maintains the monopoly output of 200 and the entrant produces 80. However, his output of 200 is not optimal if the entrant produces 80 units as given his reaction function he should only produce 168, which of course is not 200 so the threat is not credible in this one shot game.

12.3

a)

$$\pi_I = (100 - 2q_1 - 2q_2)q_I - 20q_I - 100 \text{ if } q_1 \leq \bar{K}_1$$

$$\pi_I = (100 - 2q_1 - 2q_2)q_I - 40q_I - 100 \text{ if } q_1 > \bar{K}_1$$

$$\pi_E = (100 - 2q_1 - 2q_2)q_E - 40q_E - 100$$

$$\frac{\partial \pi_I}{\partial q_1} = 100 - 4q_1 - 2q_2 - 20 = 0 \Rightarrow q_1 = 20 - \frac{q_2}{2} \text{ if } q_1 \leq \bar{K}_1$$

$$\frac{\partial \pi_I}{\partial q_1} = 100 - 4q_1 - 2q_2 - 40 = 0 \Rightarrow q_1 = 15 - \frac{q_2}{2} \text{ if } q_1 > \bar{K}_1$$

b)

$$\frac{\partial \pi_I}{\partial q_1} = 100 - 2q_1 - 4q_2 - 40 = 0 \Rightarrow q_2 = 15 - \frac{q_1}{2}$$

12.4

a)

$$\begin{aligned}q_2 &= 15 - \frac{q_1}{2} = 15 - \frac{15}{2} = 7.5 \\ \Rightarrow \pi_E &= (100 - 2(22.5))7.5 - 40(7.5) - 100 = 12.5 \\ \Rightarrow \pi_I &= (100 - 2(22.5))15 - 40(15) - 100 = 125\end{aligned}$$

$$\begin{aligned}q_2 &= 15 - \frac{q_1}{2} = 15 - \frac{16}{2} = 7 \\ \Rightarrow \pi_E &= (100 - 2(23))7 - 40(7) - 100 = -2\end{aligned}$$

b) The entrant is not going to enter, so:

$$\Rightarrow \pi_I = (100 - 2(16))16 - 40(16) - 100 = 348$$

Chapter 14

14.1

a)

$$\pi_{1,2} = (260 - 2Q_1 - 2Q_2)Q_{1,2} - 20Q_{1,2}$$

$$\frac{\partial \pi_{1,2}}{\partial Q_{1,2}} = 240 - 4Q_{1,2} - 2Q_{2,1} = 0$$

$$Q_1 = 60 - \frac{Q_2}{2} \quad | \quad Q_1 = Q_2 \quad \text{because of symmetry}$$

$$Q_1 = Q_2 = 60 \left(\frac{2}{3} \right) = 40$$

$$\pi_1^{Cournot} = \pi_2^{Cournot} = (100 - 20)(40) = 3200$$

14.1

b)

$$Q^{Monopoly} = \frac{260 - 20}{2(2)} = 60 \Rightarrow P^{Monopoly} = 260 - 2(60) = 140$$

Therefore, profit of each firm in a cartel is

$$\pi_1^{Cartel} = \pi_2^{Cartel} = (140 - 20)(30) = 3600$$

14.2

Without loss of generality, suppose Firm 2 cheats, but Firm 1 maintains its cartel quantity of 30.

Then, the optimal choice for Firm 2 can be found from its best response function.

$$Q_2^{Cheating} = \frac{1}{4}(260 - 20 - 2(30)) = 45$$

Therefore, the market price is $260 - 2(30 + 45) = 110$.

As a result, the profit of the cheating firm is: $\pi_2^{Cheating} = (110 - 20)(45) = 4050$.

The profit of the firm, that is cheated on, is then $\pi_1^{Cheating} = 2700$.

14.3

If Firm 2 cheats, then it earns 4050 for one period, but earns its Cournot profit; 3200, for all periods afterwards. On the other hand, if Firm 2 does not cheat, it can continue earning its cartel profit for ever. Hence, the collusive outcome can be sustained if

$$3600 + \delta(3600) + \delta^2(3600) + \dots \geq 4050 + \delta(3200) + \delta^2(3200) + \dots$$

$$\Rightarrow \frac{3600}{1 - \delta} \geq 4050 + \frac{3200\delta}{1 - \delta} \Rightarrow \delta \geq 0.53$$

, where δ is the probability adjusted discount factor.

14.4 & 14.5

14.4

a) With the Bertrand price competition $P_1 = P_2 = 20 \Rightarrow Q_1 = Q_2 = 60$ en $\pi_1 = \pi_2 = 0$

b)

$$Q^{Monopoly} = \frac{260 - 20}{2(2)} = 60 \Rightarrow P^{Monopoly} = 260 - 2(60) = 140$$

Therefore, profit of each firm in a cartel is

$$\pi_1^{Cartel} = \pi_2^{Cartel} = (140 - 20)(30) = 3600$$

14.5

Without loss of generality, let Firm 1 charge \$140, but firm 2 cheat. Firm 2 needs to undercut Firm 1 only slightly to capture almost the entire monopoly profit. At the limit, Firm 2 captures the entire monopoly profit by cheating. Therefore $\pi_2^{Cheating} = 7200$

14.6

If Firm 2 cheats, then it earns 7200 for one period, but earns its Bertrand profit, 0, for all periods afterwards. On the other hand, if Firm 2 does not cheat, it can continue earning its cartel profit forever. Hence, the collusive outcome can be sustained if

$$3600 + \delta(3600) + \delta^2(3600) + \dots \geq 7200 + \delta(0) + \delta^2(0) + \dots$$

$$\Rightarrow \frac{3600}{1 - \delta} \geq 7200 \Rightarrow \delta \geq \frac{1}{2}$$

, where δ is the probability adjusted discount factor.

14.7

Comparing the discount factors, it can be seen it is more difficult to sustain a cartel under Cournot competition, since it requires a larger discount factor, but only slightly. Cheating is more rewarding, but the punishment is harsher under Bertrand competition.