Chapter 13

13.2

In monopoly

$$\pi_I = (100 - q_I)q_I - 40q_I - 800$$

$$\frac{\partial \pi_I}{\partial q_I} = 100 - 2q_I - 40 = 0 \Rightarrow q_I = 30 \Rightarrow P = 70 \Rightarrow \pi_I = 100$$

13.2

For $P = 63$ in a monopoly situation $q_I = 37$. The profit would then be equal to $\pi_I = 51$.

Under Cournot

$$\pi_I = (100 - q_I - q_E)q_I - 40q_I - 800$$

$$\frac{\partial \pi_I}{\partial q_I} = 100 - 2q_I - q_E - 40 = 0 \Leftrightarrow q_I = \frac{60 - q_E}{2}$$

Similarly,

$$q_E = \frac{64 - q_I}{2}$$

We get $q_E = \frac{68}{3}$, $q_I = \frac{56}{3}$, $P = \frac{176}{3}$and $\pi_I = -451{,}55$ and $\pi_E = -786{,}22$

→ Natural Monopoly

Chapter 15

15.1a

First consider the 3 symmetric firms. For the first firm profit is given by:

$$\pi_1 = Pq_1 - 20q_1$$
$$\Updownarrow$$
$$\pi_1 = (100 - q_1 - q_2 - q_3 - q_4)q_1 - 20q_1$$

If we maximize profit we obtain q_1 as a function of the other firms' outputs.

$$\frac{\partial \pi_1}{\partial q_1} = 100 - 2q_1 - q_2 - q_3 - q_4 - 20 = 0$$
$$\Updownarrow$$
$$q_1 = \frac{80 - q_2 - q_3 - q_4}{2}$$

15.1a

Since the first three firms are symmetric we know $q_1 = q_2 = q_3$, so:

$$q_1 = \frac{80 - q_2 - q_3 - q_4}{2} = \frac{80 - q_1 - q_1 - q_4}{2}$$
$$\Updownarrow$$
$$q_1 = q_2 = q_3 = 20 - \frac{q_4}{4}$$

This also implies that $q_1 + q_2 + q_3 = 60 - \frac{3}{4} q_4$

15.1a

Now consider the profit maximization problem of firm 4

$$\pi_4 = Pq_4 - (20+\gamma)q_4$$
$$\Updownarrow$$
$$\pi_4 = (100 - q_1 - q_2 - q_3 - q_4)q_4 - 20q_4 - \gamma q_4$$

If we maximize profit we obtain q_4 as a function of the other firms' outputs.

$$\frac{\partial \pi_4}{\partial q_4} = 100 - q_1 - q_2 - q_3 - 2q_4 - 20 - \gamma = 0$$
$$\Updownarrow$$
$$q_4 = \frac{80 - q_1 - q_2 - q_3 - \gamma}{2}$$

15.1a

If we substitute the expression for $q_1 + q_2 + q_3$, we obtain:

$$q_4 = \frac{80 - q_1 - q_2 - q_3 - \gamma}{2} = \frac{80 - \gamma - \left(60 - \frac{3}{4} q_4\right)}{2} = \frac{20 - \gamma + \frac{3}{4} q_4}{2}$$

$$\Updownarrow$$

$$\frac{5}{4} q_4 = 20 - \gamma$$

$$\Updownarrow$$

$$q_4 = 16 - \frac{4}{5}\gamma$$

We can then find the optimal levels of the first three firms by substitutin in:

$$q_1 = q_2 = q_3 = 20 - \frac{q_4}{4} = 20 - \frac{1}{4}\left(16 - \frac{4}{5}\gamma\right) = 16 + \frac{1}{5}\gamma$$

15.1a

Price is given by:

$$P = 100 - q_1 - q_2 - q_3 - q_4 = 100 - 64 - \frac{3}{5}\gamma + \frac{4}{5}\gamma = 36 + \frac{1}{5}\gamma$$

Profit for each of the three symmetric firms is given by:

$$\pi_1 = \pi_2 = \pi_3 = Pq_1 - 20q_1 = \left(36 + \frac{1}{5}\gamma\right)\left(16 + \frac{1}{5}\gamma\right) - 20\left(16 + \frac{1}{5}\gamma\right) = \left(16 + \frac{1}{5}\gamma\right)^2 = 256 + \frac{32}{5}\gamma + \frac{1}{25}\gamma^2$$

Profit for firm 4 is given by:

$$\pi_4 = Pq_4 - 20q_4 - \gamma q_4 = \left(36 + \frac{1}{5}\gamma\right)\left(16 - \frac{4}{5}\gamma\right) - (20 + \gamma)\left(16 - \frac{4}{5}\gamma\right) = \left(16 - \frac{4}{5}\gamma\right)^2 = 256 - \frac{128}{5}\gamma + \frac{16}{25}\gamma^2$$

15.1a

The restrictions on the model are that price has to be greater than marginal cost and that quantities are nonnegative. Since marginal cost for the first three firms is equal to 20, this implies that

$$P = 36 + \frac{1}{5}\gamma \geq 20 \Rightarrow \gamma \geq -80$$

Since quantities must be positive we also have that

$$16 + \frac{1}{5}\gamma \geq 0 \Rightarrow \gamma \geq -80$$
$$16 - \frac{4}{5}\gamma \geq 0 \Rightarrow \gamma \leq 20$$

But we also require that marginal cost for the fourth firm is positive, so:

$$c_4 = 20 + \gamma \geq 0 \Rightarrow \gamma \geq -20$$

Combining the constraints we find that:

$$-20 \leq \gamma \leq 20$$

15.1b

There are now three firms, two identical and one differtent. The solution is as in part a. Denote the firms *m*, 3 and 4, where *m* is the combined firm. First consider the two symmetric firms.

$$\pi_m = Pq_m - 20q_m$$
$$\Updownarrow$$
$$\pi_m = (100 - q_m - q_3 - q_4)q_m - 20q_m$$

If we maximize profit we obtain q_m as a function of the other firms' outputs.

$$\frac{\partial \pi_m}{\partial q_m} = 100 - 2q_m - q_3 - q_4 - 20 = 0$$
$$\Updownarrow$$
$$q_m = \frac{80 - q_3 - q_4}{2}$$

15.1b

Since the first two firms are symmetric we know $q_m = q_3$, so:

$$q_m = \frac{80 - q_3 - q_4}{2} = \frac{80 - q_m - q_4}{2}$$

$$\Updownarrow$$

$$q_m = q_3 = \frac{80}{3} - \frac{q_4}{3}$$

15.1b

Now consider the profit maximization problem of firm 4

$$\pi_4 = Pq_4 - (20 + \gamma)q_4$$
$$\Updownarrow$$
$$\pi_4 = (100 - q_m - q_3 - q_4)q_4 - 20q_4 - \gamma q_4$$

If we maximize profit we obtain q_4 as a function of the other firms' outputs.

$$\frac{\partial \pi_4}{\partial q_4} = 100 - q_m - q_3 - 2q_4 - 20 - \gamma = 0$$
$$\Updownarrow$$
$$q_4 = \frac{80 - q_m - q_3 - \gamma}{2}$$

15.1b

If we substitute in for the other firms we obtain

$$q_4 = \frac{80 - q_m - q_3 - \gamma}{2} = \frac{80 - \gamma - \left(\frac{160}{3} - \frac{2}{3} q_4\right)}{2} = \frac{\frac{80}{3} - \gamma + \frac{2}{3} q_4}{2}$$

$$\Updownarrow$$

$$q_4 = 20 - \frac{3}{4}\gamma$$

We can then find the optimal levels of the first two firms by substituting in:

$$q_m = q_3 = \frac{80}{3} - \frac{1}{3} q_4 = \frac{80}{3} - \frac{1}{3}\left(20 - \frac{3}{4}\gamma\right) = 20 + \frac{1}{4}\gamma$$

15.1b

The price is given by

$$P = 100 - q_m - q_3 - q_4 = 100 - \left(40 + \frac{1}{2}\gamma\right) - \left(20 - \frac{3}{4}\gamma\right) = 40 + \frac{1}{4}\gamma$$

The profit for each symmetric firm is given by

$$\pi_m = \pi_3 = Pq_m - 20q_m = \left(40 + \frac{1}{4}\gamma\right)\left(20 + \frac{1}{4}\gamma\right) - 20\left(20 + \frac{1}{4}\gamma\right) = \left(20 + \frac{1}{4}\gamma\right)^2 = 400 + 10\gamma + \frac{1}{16}\gamma^2$$

Profit for firm 4 is given by

$$\pi_4 = Pq_4 - (20 + \gamma)q_4 = \left(40 + \frac{1}{4}\gamma\right)\left(20 - \frac{3}{4}\gamma\right) - (20 + \gamma)\left(20 - \frac{3}{4}\gamma\right) = \left(20 - \frac{3}{4}\gamma\right)^2 = 400 - 30\gamma + \frac{9}{16}\gamma^2$$

15.1b

The combined profits of firm 1 and 2 in 15.1a were

$$\pi_1 + \pi_2 = 2\left(256 + \frac{32}{5}\gamma + \frac{1}{25}\gamma^2\right) = 512 + \frac{64}{5}\gamma + \frac{2}{25}\gamma^2$$
$$= 512 + 12.8\gamma + 0{,}08\gamma^2$$

Versus $\pi_m = 400 + 10\gamma + \frac{1}{16}\gamma^2$ for the merged firm.

If γ is positive then the sum of the independent profits are higher since they are higher in every term. If γ is negative then the difference between could shrink. Consider the difference at the lower bound for $\gamma, \gamma = -20$. Then the difference is 63, at $\gamma = 0$ the difference is 112 and at $\gamma = 20$ the difference is 175.

15.1c

We assume that firms 2 and 3 are still independent. Since firm 4 has higher costs than firm 1, all production at the merged firm will take place using the facilities of firm 1. Thus we have a three-firm Cournot game where the firms are symmetric. We can proceed as in 15.1a, essentially ignoring firm 4.

For the merged firm the profit is given by

$$\pi_m = Pq_m - 20q_m = (100 - q_m - q_2 - q_3)q_m - 20q_m$$

If we maximize profit we obtain q_1 as a function of the other firms' outputs.

$$\frac{\partial \pi_m}{\partial q_m} = 100 - 2q_m - q_2 - q_3 - 20 = 0$$

$$\Updownarrow$$

$$q_m = \frac{80 - q_2 - q_3}{2}$$

Since the three firms are symmetric we know that $q_m = q_2 = q_3$. This means

$$q_m = \frac{80 - q_m - q_m}{2} \Leftrightarrow q_m = 20 \Rightarrow P = 40 \Rightarrow \pi_m = \pi_2 = \pi_3 = 400$$

15.1c

So firms 2 and 3 each have profits of 400. In 15.1a, the profits of firm 2 and 3 were $\pi_2 = \pi_3 = \left(16 + \frac{1}{5}\gamma\right)^2$. The largest possible value of γ is 20, so the maximum profit for firm 2 and 3 in the initial problem is 400. Thus, firm two and three cannot lose from this merger. We know need to compare this to the profits that occured in 15.1a to see how firms 1 and 4 do. The combined profits of firm 1 and firm 4 in part a are

$$\pi_1 + \pi_4 = \left(256 + \frac{32}{5}\gamma + \frac{1}{25}\gamma^2\right) + \left(256 - \frac{128}{5}\gamma + \frac{16}{25}\gamma^2\right) = 512 - \frac{96}{5}\gamma + \frac{17}{25}\gamma^2$$

The question is whether this is smaller than 400 for a positive γ. If it is, there is an incentive to merge. For example, if $\gamma = 10$, then the combined profits are equal to 388, so a merger would be beneficial. The difference in profits can be written as $\Delta = 512 - \frac{96}{5}\gamma + \frac{17}{25}\gamma^2 - 400 = 112 - \frac{96}{5}\gamma + \frac{17}{25}\gamma^2$. Now we know that for all the values γ between 8.235… and 20 there is an incentive to merge. You can calculate these values yourselves.

15.2a

From problem 15.1b we have the following optimal quantities and market price, which are not affected by fixed costs.

$$q_m = q_3 = 20 + \frac{1}{4}\gamma$$

$$q_4 = 20 - \frac{3}{4}\gamma$$

$$P = 40 + \frac{1}{4}\gamma$$

Profits are now given by

$$\pi_m = 400 + 10\gamma + \frac{1}{16}\gamma^2 - bF$$

$$\pi_3 = 400 + 10\gamma + \frac{1}{16}\gamma^2 - F$$

$$\pi_4 = 400 - 30\gamma + \frac{9}{16}\gamma^2 - F$$

15.2a

Combined profits for firms 1 and 2 with no mergers are

$$\pi_1 + \pi_2 = 2\left(256 + \frac{32}{5}\gamma + \frac{1}{25}\gamma^2 - F\right) = 512 + \frac{64}{5}\gamma + \frac{2}{25}\gamma^2 - 2F$$

The merger is profitable if

$$\pi_m = 400 + 10\gamma + \frac{1}{16}\gamma^2 - bF \geq 512 + \frac{64}{5}\gamma + \frac{2}{25}\gamma^2 - 2F = \pi_1 + \pi_2$$
$$\Updownarrow$$
$$0.0175\gamma^2 + 2.8\gamma + 112 \leq F(2-b)$$
$$\Updownarrow$$
$$0.0175\left(\gamma^2 + 160\gamma + 6400\right) \leq F(2-b)$$
$$\Updownarrow$$
$$0.0175(\gamma + 80)^2 \leq F(2-b)$$
$$\Downarrow$$
$$\gamma \leq \frac{\sqrt{F}(2-b)^{\frac{1}{2}}}{0,13228756} - 80$$

15.2b

From problem 15.1c, we have the following optimal quantities and equilibrium market price, which are not affected by fixed costs.

$$q_m = q_2 = q_3 = 20$$
$$P = 40$$

Profits are now given by

$$\pi_m = 400 - bF$$
$$\pi_2 = \pi_3 = 400 - F$$

Combined profits for firm 1 and 4 with no merger are

$$\pi_1 + \pi_4 = 512 - \frac{96}{5}\gamma + \frac{17}{25}\gamma^2 - 2\mathrm{F}$$

15.2b

The merger is profitable if

$$\pi_m = 400 - bF \geq 512 - \frac{96}{5}\gamma + \frac{17}{25}\gamma^2 - 2\mathrm{F} = \pi_1 + \pi_4$$

If we rearrange, we obtain

$$\frac{17}{25}\gamma^2 - \frac{96}{5}\gamma + 112 \leq F(2 - b)$$

15.2c

Mergers that create cost savings by economizing on fixed costs or by eliminating high cost firms are more likely to be profitable.

15.3a

We now have a market with three firms, two identical and one (the leader) different. Denote the firms l, 3 and 4, where l denotes the leader firm. First consider the reponse of the two follower firms.

$$\pi_3 = (100 - q_l - q_3 - q_4)q_3 - 20q_3$$

If we maximize profit and know that $q_3 = q_4$, we obtain

$$\frac{\partial \pi_3}{\partial q_3} = 100 - q_l - 2q_3 - q_4 - 20 = 0$$
$$\Updownarrow$$
$$q_3 = \frac{80 - q_l - q_4}{2}$$
$$\Updownarrow$$
$$q_3 = q_4 = \frac{80}{3} - \frac{q_l}{3}$$

15.3a

Now consider the profit maximization problem of the leader firm. This firm will take into account the best response function of the followers.

$$\pi_l = (100 - q_l - q_3 - q_4)q_l - 20q_l = 100q_l - {q_l}^2 - \left(\frac{80}{3} - \frac{q_l}{3}\right)q_l - \left(\frac{80}{3} - \frac{q_l}{3}\right)q_l - 20q_l = \frac{80}{3}q_l - \frac{1}{3}{q_l}^2$$

If we maximize profit we obtain

$$\frac{\partial \pi_l}{\partial q_l} = \frac{80}{3} - \frac{2}{3}q_l = 0$$
$$\Updownarrow$$
$$\frac{2}{3}q_l = \frac{80}{3}$$
$$\Updownarrow$$
$$q_l = 40$$

Then we know $q_l = 40 \Rightarrow q_3 = q_4 = \frac{40}{3} \Rightarrow P = \frac{100}{3} \Rightarrow \pi_3 = \pi_4 = \frac{1600}{9}, \pi_l = \frac{1600}{3}$ which is larger thant the sum of het profits from 15.1b: $512 + \frac{64}{5}\gamma + \frac{2}{25}\gamma^2$, since if $\gamma = 0$ then 533 is larger than 512.

15.3b

In this situation we will have two Cournot competitors in the leader group.

$$\pi_m = Pq_m - 20q_m = (100 - q_l - q_m)q_m - 20q_m$$

If we maximize profit, knowing $q_l = q_m$ we obtain

$$\frac{\partial \pi_m}{\partial q_m} = 100 - q_l - 2q_m - 20 = 0$$
$$\Updownarrow$$
$$q_m = \frac{80 - q_l}{2}$$
$$\Updownarrow$$
$$q_l = q_m = \frac{80}{3} \Rightarrow P = \frac{140}{3} \Rightarrow \pi_m = \pi_l = \frac{6400}{9}$$

15.4

a) The toal profit of the firms 1 and 2 after merger as Stackelberg leader is 533,33. The sum of the profits from being part of a Cournot market is 512. Thus a cost of more than 21,33333 would make the merger undesirable.

b) In the Stackelberg equilibrium, the follower firms each make 177,77 for a total profit of 355,55. In the two firm Cournot model the profit of each member of the duopoly has a profit of 711,11. If the cost of merging is more than 711,11-355,55=355,55, then the firms will not merge.

Extra Exercise

Extra Exercise - a

		Firm 2	
		Cooperate	Compete
Firm 1	Cooperate	5 , 5	1 , 7
	Compete	7 , 1	2 , 2

Extra Exercise - a

		Firm 2	
		Cooperate	Compete
Firm 1	Cooperate	5 , 5	1 , 7
	Compete	7 , 1	2 , 2

Extra Exercise - b

Sticking to the strategy has to be rational, so cooperate > Deviate

$$\frac{5}{1-R} \geq 7 + 2R + \frac{5R^2}{1-R}$$
$$\Updownarrow$$
$$5 \geq 7 - 7\mathrm{R} + 2\mathrm{R} - 2R^2 + 5R^2$$
$$\Updownarrow$$
$$0 \geq 2 - 5R + 3R^2$$
$$\Downarrow$$
$$\frac{2}{3} \leq R \leq 1$$

Trigger-1 is an equilibrium strategy if and only if $\frac{2}{3} \leq R \leq 1$.

Extra Exercise - c

Sticking to the strategy has to be rational, regardless of the situation. If in the last period both players cooperated, the same calculation as under b applies.

$$\frac{5}{1-R} \geq 7 + 2R + \frac{5R^2}{1-R}$$
$$\Updownarrow$$
$$5 \geq 7 - 7\mathrm{R} + 2\mathrm{R} - 2R^2 + 5R^2$$
$$\Updownarrow$$
$$0 \geq 2 - 5R + 3R^2$$
$$\Downarrow$$
$$\frac{2}{3} \leq R \leq 1$$

Extra Exercise - c

If in the past period one or both players competed, the strategy states that both players should compete for one period and then start cooperating again. If a player does not stick to his strategy, he cooperates in the first period instead. But that yields him only 1 instead of 2, so for the strategy to be subgame perfect, the following inequality must hold.

$$2 + \frac{5R}{1-R} \geq 1 + \frac{5R}{1-R}$$

Which is true for all values of R. Combined with the first condition we can conclude that trigger-1 is a subgame perfect equilibrium strategy if and only if $\frac{2}{3} \leq R \leq 1$.