Chapter 16

16.1a

Profits of Norman International are given by revenue minus the cost of richets and other variable costs. If a richet costs p_r per unit we obtain

$$\pi_{NI} = (50 - q_w)q_w - p_r q_w - 5q_w = (45 - q_w)q_w - p_r q_w$$

Taking the first derivative with respect to q_w and solving for the optimal level of output, yields

$$\frac{\partial \pi_{NI}}{\partial q_w} = 45 - 2q_w - p_r = 0$$
$$\Updownarrow$$
$$q_w = \frac{45 - p_r}{2}$$
$$\Downarrow$$
$$p_w = 50 - \frac{45 - p_r}{2} = \frac{55 + p_r}{2}$$

We can then write the price of richets in inverse demand or price dpendent form

$$p_r = 45 - 2q_w = 45 - 2q_r$$

16.1b

$$\pi_{PR} = (45 - 2q_r)q_r - 5q_r = 40q_r - 2{q_r}^2$$

If we maximize profit with respect to q_r we obtain

$$\frac{\partial \pi_{PR}}{\partial q_r} = 40 - 4q_r = 0$$

$$\Updownarrow$$

$$q_r = 10 = q_w$$

$$\Downarrow$$

$$p_r = 45 - 2q_r = 25 \Rightarrow p_w = 40 \Rightarrow \pi_{NI} = 100, \pi_{PR} = 200$$

16.2a

Profit of the merged firm is given by

$$\pi_{NPR} = (50 - q_w)q_w - 5q_w - 5q_w = 40q_w - q_w^2$$

Maximizing the profit with respect to q_w yields

$$\frac{\partial \pi_{NPR}}{\partial q_w} = 40 - 2q_w = 0$$
$$\Updownarrow$$
$$q_w = 20$$

The price of whatsits is then equal to

$$p_w = 50 - q_w = 50 - 20 = 30 \Rightarrow \pi_{NPR} = 400$$

16.2b

The profit of the merged firm is equal to 400, the profits of NI and PepRich as stand-alone firms, were 100 and 200 respectively. Thus, the total profits (producer surplus) has gone up by 100.

The price to consumers is now 30 instead of 40. The change in consumer surplus can be calculated as follows:

$$\Delta CS = (p_1 - p_2)q_1 + \frac{1}{2}(p_1 - p_2)(q_2 - q_1)$$
$$= (40 - 30)10 + \frac{1}{2}(40 - 30)(20 - 10)$$
$$= 150$$

Thus, consumers are also better off after the merger. Welfare, the sum of producer surplus and consumer surplus, has increased by 250.

16.2c

We can compute the net present value of the merged firm as a perpetuity where we divide the constant annual profit level by the interest rate. If the discount factor is given by

$$R = \frac{1}{1+r} \Rightarrow \mathrm{r} = \frac{1-R}{R} = \frac{0.1}{0.9} = \frac{0.1}{0.9} = 0{,}11\overline{1}$$

The net present values of the firms are then as follows:

$$NPV(NPR) = \frac{400}{0{,}11\overline{1}} = 3600$$
$$NPV(NI) = \frac{100}{0{,}11\overline{1}} = 900$$
$$NPV(PR) = \frac{200}{0{,}11\overline{1}} = 1800$$

Pepper rich would pay up to 1800 (3600-1800) for Norman International. Norman International would be expecting at least 900.

16.4a

$$p_w = 50 - q_w$$

Now let p_z denote the price of zabits and p_r the price of richets. Profits can be written as

$$\pi_{NI} = (50 - q_w)q_w - p_r q_w - p_z q_w - 5q_w$$

Maximizing, yields

$$\frac{\partial \pi_{NI}}{\partial q_w} = 50 - 2q_w - p_r - p_z - 5 = 0$$
$$\Updownarrow$$
$$q_w = \frac{45 - p_r - p_z}{2}$$
$$\Downarrow$$
$$p_w = 50 - \frac{45 - p_r - p_z}{2} = \frac{55 + p_r + p_z}{2}$$

16.4a

We can also write the price of richets and zabits in price dependent form

$$p_r = 45 - p_z - 2q_w$$
$$p_z = 45 - p_r - 2q_w$$

So in equilibrium, for PepRich and Zabcorp, and given that $q_w = q_r = q_z$ the following must hold

$$MR_{PR} = 45 - p_z - 4q_r = 5 = MC_{PR} \Rightarrow q_r = \frac{40 - p_z}{4} \Rightarrow p_z = 40 - 4q_w$$

$$MR_Z = 45 - p_r - 4q_z = 2.5 = MC_Z \Rightarrow q_z = \frac{42.5 - p_r}{4} \Rightarrow p_r = 42.5 - 4q_w$$

16.4a

Plugging this into the equation for the optimal q_w will give

$$q_w = \frac{45 - p_r - p_z}{2} = \frac{45 - (40 - 4q_w) - (42.5 - 4q_w)}{2} = \frac{-37.5 + 8q_w}{2}$$
$$\Rightarrow q_w = 6.25 \Rightarrow p_w = 43.75$$

The market clearing prices p_r and p_z are then

$$p_z = 40 - 4q_w = 40 - 25 = 15$$
$$p_r = 42.5 - 4q_w = 42.5 - 25 = 17.5$$

16.4a

Profits are then equal to

- $\pi_{NI} = (50 - q_w)q_w - p_r q_w - p_z q_w - 5q_w = 39.0625$
- $\pi_{PR} = (45 - p_z - 2q_r)q_r - 5q_r = 78.125$
- $\pi_Z = (45 - p_z - 2q_r)q_r - 2.5q_r = 78.125$

16.4b

We now need to consider the profits of each of the merged firms versus the individual profits computed in 16.4a. First consider the PepRich and Norman International merger denoted NPR. The profits of the merged firm are given by

$$\pi_{NPR} = (50 - q_w)q_w - 5q_w - p_z q_w - 5q_w = (40 - p_z - q_w)q_w$$

Maximizing the profit function gives

$$\frac{\partial \pi_{NPR}}{\partial q_w} = 40 - p_z - 2q_w = 0$$
$$\Updownarrow$$
$$q_w = \frac{40 - p_z}{2}$$
$$\Downarrow$$
$$p_w = 50 - q_w = 50 - \frac{40 - p_z}{2} = \frac{60 + p_z}{2}$$

16.4b

We can write the price of zabits in price dependent form

$$p_z = 40 - 2q_w$$

We can now calculate revenue for Zabcorp, and set marginal revenue equal to marginal cost as follows ($q_w = q_z$)

$$MR_z = 40 - 4q_z = 2.5 = MC_z$$
$$\Downarrow$$
$$q_z = \frac{37.5}{4} = 9.375$$
$$\Downarrow$$
$$p_z = 40 - 2q_z = 21.25$$

16.4b

We can then find p_w from

$$p_w = \frac{60 + p_z}{2} = \frac{60 + 21.25}{2} = 40.625$$

Profits for the comined firm NPR are

$$\pi_{NPR} = (40 - p_z - q_w)q_w = 87.890625$$

16.4b

The net present value of this is

$$NPV(NPR) = \frac{87.890625}{0{,}11\overline{1}} = 791.015625$$

The net present value of PepRich from 16.4a was

$$NPV(NPR) = \frac{78.125}{0{,}11\overline{1}} = 703.125$$

The net present value of Norman International from 16.4a was

$$NPV(NPR) = \frac{39.0625}{0{,}11\overline{1}} = 351.0625$$

16.4b

Therefore PepRich could afford to pay $(791.015625 - 703.125) =$ $\$87.890625$ for Norman International. Profits in this new market for Zabcorp are

$$\pi_z = p_z q_z - 2.5q_z = 21.25q_z - 2.5q_z = 175.78125$$

16.4b

Now consider the merger of Zabcorp and Norman International denoted ZN.

$$\pi_{ZN} = (50 - q_w)q_w - 2.5q_w - p_r q_w - 5q_w = (42.5 - p_r - q_w)q_w$$

Maximizing profits means

$$\frac{\partial \pi_{ZN}}{\partial q_w} = 42.5 - p_r - 2q_w = 0$$

$$\Updownarrow$$

$$q_w = \frac{42.5 - p_r}{2}$$

$$\Downarrow$$

$$p_w = 50 - q_w = 50 - \frac{42.5 - p_r}{2} = \frac{57.5 + p_r}{2}$$

16.4b

We can write the price of richets in inverse demand or price dependent form

$$p_r = 42.5 - 2q_w$$

We can now compute revenue for PepRich and set marginal revenue equal to marginal cost as follows ($q_w = q_r$)

$$MR_r = 42.5 - 4q_r = 5 = MC_r$$
$$\Downarrow$$
$$q_r = \frac{37.5}{4} = 9.375$$
$$\Downarrow$$
$$p_r = 42.5 - 2q_z = 23.75$$

16.4b

We can then find p_w from

$$p_w = \frac{57.5 + p_r}{2} = \frac{57.5 + 23.75}{2} = 40.625$$

Profits for the comined firm ZN are

$$\pi_{ZN} = (42.5 - p_r - q_w)q_w = 87.890625$$

The net present value of this is

$$NPV(ZN) = \frac{87.890625}{0{,}11\bar{1}} = 791.015625$$

The net present value of Zabcorp from 16.4a was

$$NPV(NPR) = \frac{78.125}{0{,}11\bar{1}} = 703.125$$

16.4b

Hence Zabcorp can afford to pay $(791.015625 - 703.125) = \$87.890625$ for Norman International.

Both firms can afford the same amount and so it is not clear who will win the bidding. However, since the price is \$40.625, which is less than the price \$43.75 in the original monopoly problem in 16.4a, consumers are better off. Quantity demanded also increases from 6.25 to 9.375.

$$\begin{aligned}\Delta CS &= (p_1 - p_2)q_1 + \frac{1}{2}(p_1 - p_2)(q_2 - q_1)\\ &= (43.75 - 40.625)6.25 + \frac{1}{2}(43.75 - 40.625)(9.375 - 6.25)\\ &= 14.6484375\end{aligned}$$

Thus consumers are better off with either merger. However, Norman International will not accept either offer of $\$87.890625$ since the present value of its profit stream before merger is

$$NPV(NI) = \frac{39.0625}{0{,}11\bar{1}} = 351.5625$$

Chapter 17

17.1a

Since demand is given by $Q = 30 - p$, inverse demand is given by $p = 30 - q$.

Profit of the Volvo dealer is given by

$$\pi^D(p, w) = pQ - wQ = (30 - Q)Q - wQ$$

Maximization yields

$$\frac{\partial \pi^D}{\partial Q} = 30 - 2Q - w = 0$$
$$\Updownarrow$$
$$Q = \frac{30 - w}{2} = 15 - \frac{w}{2}$$
$$\Downarrow$$
$$p = 15 + \frac{w}{2} \Rightarrow \pi^D = \left(15 + \frac{w}{2}\right)\left(15 - \frac{w}{2}\right) - w\left(15 - \frac{w}{2}\right) = \frac{w^2}{4} - 15w + 225$$

17.1b

Since the quantity sold is the quantity purchased, we can simply invert the equation for Q derived in 17.1a to get a demand for cars as a function of Q. This will give $w = 30 - 2Q$.

Optimization yields

$$MR^M = 30 - 4Q = 5 = MC^M$$
$$\Updownarrow$$
$$Q = 6.25$$
$$\Downarrow$$
$$w = 17.5 \Rightarrow \pi^M = 78.13$$

Based on $Q = 6.25$, the retail price $p = 30 - Q = 23.75$ and $\pi^D = 39.06$.

17.2

Now compute the integrated firm’s profit using marginal production cost as the cost of the car.

$$\pi^I = (30 - Q)Q - 5Q \Rightarrow \frac{\partial \pi^I}{\partial Q} = 30 - 2Q - 5 = 0$$

$$\Updownarrow$$

$$Q = 12.5$$

$$\Downarrow$$

$$p = 17.5 \Rightarrow \pi^I = 156.25 > \pi^M + \pi^D = 117.19$$

Chapter 22

22.1

a) Because of the network externality, the proposed merger is in the interest of the consumers.

b) Most likely Bank 1 will not agree. It has a significantly larger number of ATM machines, so it has more likely to attract new customers.

22.2a

Let $\overline{w_i}$ be reservation value of the marginal consumer who is just indifferent buying the service and not buying it. The fraction f of consumers who suscribe to the service is given by

$$f = 1 - \frac{\overline{w_i}}{50}$$

The marginal consumer's address is then given by

$$\overline{w_i} = 50(1 - f)$$

The quantity will be zero for $fw_i < p$, but will be one for each consumer who has $fw_i \geq p$. Substituting $\overline{w_i}$ in the demand equation when q_i^D is equal to one yields the inverse form as

$$p = f\overline{w} = f(50)(1 - f) = 50f(1 - f)$$

22.2b

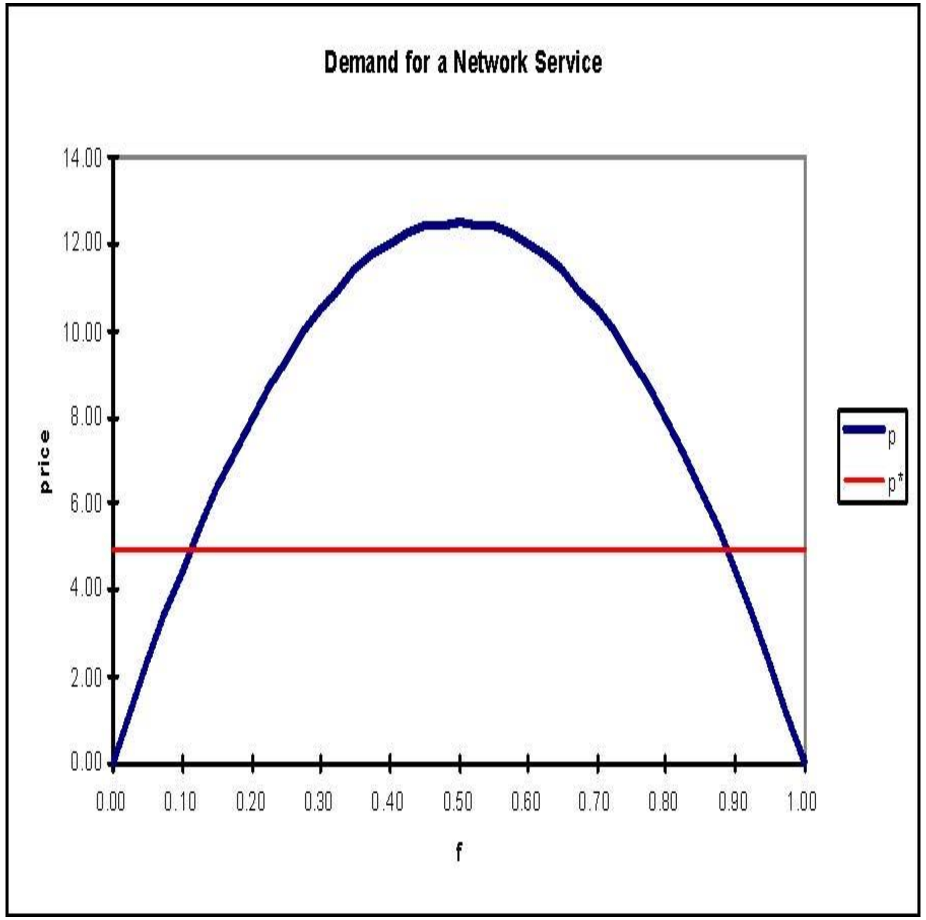
Consider the graph right.

As the proportion of suscribers rises, demand rises until $f = .5$. We can use the demand equation to solve for f low and f high by solving the aggregate demand equation for f, given $p = 5$. This is a quadratic in f. We can write it as follows

$$p = 50f(1-f) = 50f - 50f^2 \Rightarrow 5 = 50f - 50f^2$$

It is straightforward to show that $f = 0.887298$ or $f = 0.11270166$

So if f is above 11.270166% the market will move to the higher equilibrium at 88.7298% of the customers served.



22.2c

Profit as a function of f is given by

$$\pi = 50Nf^2 - 50Nf^3$$

Maximization of profit with respect to f yields $f = 0$ or $f = \frac{2}{3}$. This implies that $p = 11.11$.