Question 1 (/12)

Part 1: Discuss the model of oligopoly with competition on quantity (i.e., *a la Cournot*), focusing on the case with two firms. In particular, address the following aspects of the analysis:

a. What are the main assumptions of the model?

b. What is the strategy followed by the firms? Describe the best response functions and the equilibrium of the model.

Part 2: Is this model a good description of some oligopolistic markets? Discuss an extension of this model at your choice.

Part 3. Consider the case when one of the two firms has a first-mover advantage. What is then the optimal production level of the market leader? What is the new market equilibrium?

Solution

Part 1. All the elements needed for this solution (assumptions, best response functions, etc.) can be found in the book and in the slides. The student is required to present the model, its assumptions, derive the equilibrium (better if using both math and graphs).

Part 2. Then the goal is to discuss the relationship with the real-world competition between firms. How important is the difference between price and quantity competition, and under what circumstances the quantity competition is a good description of an oligopolistic market. Starting from the limitation of the Cournot model, the student can discuss an extension (e.g., general number of firms).

Part 3. Same as in Part 1, here the student has to discuss the Stackelberg model of oligopoly.

Question 2 (/12)

Consider the case of a monopolist firm that faces consumers of two types (in equal shares) who are not distinguishable (es. one type has lower demand for the good and the other type has higher demand). The demand of these two types are:

- $P_H = 20 - 2Q_H$
- $P_L = 14 - 2Q_L$

and the cost of production is: $C(Q) = 2Q$

a. What would be the optimal packages for the two types if the firm could distinguish them?

b. What is the firm going to offer if it wants to serve both types without the possibility to distinguish them? Find the incentive-compatible menu of packages. Which type of consumer is profiting from the limited information of the firm?

Solution

Point a. The monopolist can discriminate between the two types of consumer and it charges the corresponding two-part tariffs that extract the surplus. We consider the monopolist posting a two-part tariff with price per-unit at marginal cost, i.e., at $P_H = P_L = 2$. The quantities consumed at this price are:

$$Q_H = 10 - \frac{P_H}{2} = 10 - 1 = 9 \qquad \text{and} \qquad Q_L = 7 - \frac{P_L}{2} = 7 - 1 = 6 \tag{1}$$

The corresponding surpluses are:

$$CS_H = \frac{(20-2)9}{2} = 81 \qquad \text{and} \qquad CS_L = \frac{(14-2)6}{2} = 36 \tag{2}$$

so, the monopolist can either set entry fees of $Fee_H = 81$€ and $Fee_L = 36$€ and then price at €2 per unit, or offer the packages: (€99,9) to the high type and (€48,6) to the low type.

Point b. To find the optimal menu of contracts we need to compute the surplus for the high type pretending to be a low type. This is because, when offered the two types computed in a., the low type chooses correctly the package (€48,6) and not the package designed for the high type. However, if the high type consumes the package designed for the low type, the surplus is:

$$CS_H(48,6) = \frac{(20-8)6}{2} + 8*6 - 48 = 84 - 48 = 36$$

so, the monopolist has to give a surplus of €36 to the high type to make this consumer choosing the package designed for him/her. The packages then will be:

$$\begin{cases} Package_H: & (63,9) & \text{for the high type} \\ Package_L: & (48,6) & \text{for the low type} \end{cases} \tag{3}$$

So, the monopolist has to give a quantity discount to the high type, which pays a price of $\frac{63}{9} = 7$€ per unit, instead of the price per unit that low type is paying: $\frac{48}{6} = 8$€. The high type consumer is profiting from his/her informational advantage over the firm.

Question 3 (/12)

Consider a market with two firms selling a horizontally differentiated product. The product space can be described by the Hoteling model, where the firms are located at the two end points of the line. Consumers have a maximum valuation of $V = 20$ for the product and each consumer buys at most 1 unit of the good. Transportation costs are $t = 5$, and production cost is $C(Q) = 2Q$.
Answer to the following questions:

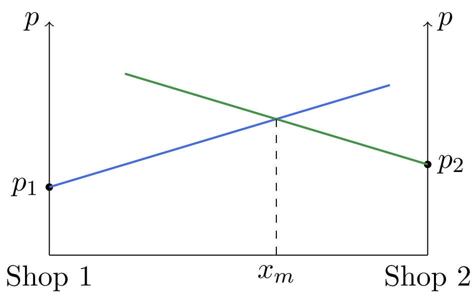
a. Compute the location of the consumer who is indifferent between the two products both in general terms, and using these parameters.

b. Derive the best response function of the two firms.

c. Compute the optimal price that is charged by the two firms.

d. What is the effect of a change in transportation costs on competition? What happens if the transportation cost goes to 0?

Solution
Point a. The consumer who is indifferent between the two products finds them of equal utility. This means that the full price (monetary price plus transportation cost) he/she gets from the product sold by firm 1 is the same as the full price he/she gets from the product sold by firm 2. Graphically:



Let's indicate the location of the indifferent consumer by x_m (measured as distance from firm 1). The indifference condition implies that:

$$p_1 + tx_m = p_2 + t(1 - x_m) \quad \Rightarrow \quad \boxed{x_m(p_1, p_2) = \frac{p_2 - p_1 + t}{2t}}$$

so, the location of the indifferent consumer increases when the price of firm 2 increases (i.e., moves to the right) and reduces when the price of firm 1 increases (moves to he left). Using the parameters of the problem:

$$\boxed{x_m(p_1, p_2) = \frac{p_2 - p_1 + 5}{10}}$$

Point b. To derive the best response function of the firms, we start from the profit function, where we use the residual demand for the firm derived in the previous point. In fact, the location of the indifferent consumer indicates the share of total consumers that buy the product from firm 1 or firm 2. The strategic variable of the firms is the price set for the own product. Taking the perspective of firm 1:

$$\pi_1 = N\frac{(p_2p_1 - p_1^2 + tp_1 + cp_1 - cp_2 - ct)}{2t}$$

Then, the FOCs imply that:

$$\frac{\partial \pi_1}{\partial p_1} = \frac{N}{2t}(p_2 - 2p_1 + t + c) = 0$$

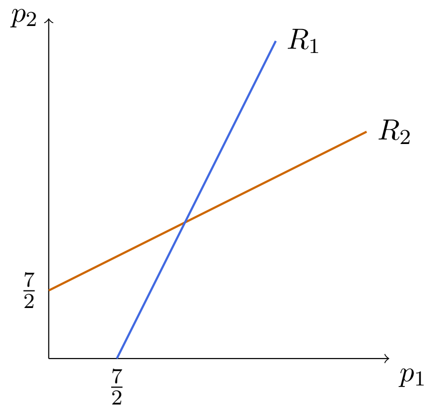
which give the following best response function for firm 1:

$$\boxed{p_1^* = \frac{p_2 + t + c}{2} = \frac{p_2 + 7}{2}}$$

and, being the game symmetric, the following best response function for firm 2:

$$\boxed{p_2^* = \frac{p_1 + t + c}{2} = \frac{p_2 + 7}{2}}$$

Graphically, the best response functions look like the in the figure below (i.e., upward sloped), indicating that price changes are strategic complements.



Point c. We can derive the equilibrium price solving the system:

$$2p_2^* = p_1 + t + c = \frac{p_2^*}{2} + 3\frac{t+c}{2} \quad \Rightarrow \quad 3p_2 = 3(t+c)$$

$$\Rightarrow \quad p_2^H = t + c = 5 + 2 = 7$$

being the game symmetric, we have that $p_1^H = 7$.

Point d. In the Hotelling model, due to the transportation cost (preference for variety), the price charged in equilibrium is larger than marginal cost. The presence of this cost soften competition with respect to the case of price competition with homogeneous products. With product differentiation firms gain some local market power over the consumers that are located close to their product and they can raise price above cost. In fact, the equilibrium price increases with the increase of the transportation cost (which, differently from the cost of production, is bore by consumers and not by the firm).

The case of zero transportation costs makes the solution of the Hotelling model coinciding with the one of the Bertrand model (in the case of the exercise, that is $p = MC = 2$. This is not surprising because a transportation cost equal to zero means that consumers do not have any preference for variety, so, that there is no product differentiation. In that case, with no capacity constraints, we know from the Bertrand model that the equilibrium price is equal to the marginal cost of production.