Chapter 1: Industrial Organization: What, How and Why?

Problem 1

Many examples imperfectly competitive markets are possible. Common ones include: (1) Automobiles, (2) Beer, (3) Telephone/Telecommunications, (4) Jet Aircraft, (5) Patented Pharmaceuticals, and (6) Computer Operating Systems, .Large entry costs, scale economies, network effects and government regulations all play a role in these examples.

Problem 2

In a perfectly competitive market, each agent is a price taker. That is, decisions of individual firm and / or consumer do not affect the market price or environment. Therefore, there is no room for strategic behavior in a perfectly competitive market.

Problem 3

In general, the Clayton Act was designed to prevent monopoly "in its incipiency" by making explicitly illegal a number of business practices. In particular, Section 2 prevents strategic manipulations of the upstream / downstream market by a firm with market power. Under Section 2 of the Clayton Act, it is illegal to "discriminate in price between different purchasers of commodities of like grade and quality". Section 7 was passed to prevent anti-competitive mergers.

Problem 4

If higher concentration leads to higher worker productivity, then industrial concentration can lower production cost, and therefore, horizontal mergers *may* improve economic efficiency.

Problem 5

Market dominance by one firm may be due to the firm's better performance, higher efficiency etc. Price fixing, however, does not indicate higher efficiencies for the participating firms. It simply hurts the consumers and reduces overall welfare.

Chapter 2: Some Basic Microeconomic Tools

Problem 1

(a) Setting inverse demand function equal to the inverse supply function, we obtain the equilibrium quantity

$$\text{Inverse Demand} = 1{,}000 - 0.025Q = 150 + 0.033Q = \text{Inverse Supply}$$
$$\Rightarrow 850 = 0.058Q$$
$$\Rightarrow \frac{850}{0.058} = Q$$
$$\rightarrow 14{,}655.172 = Q$$

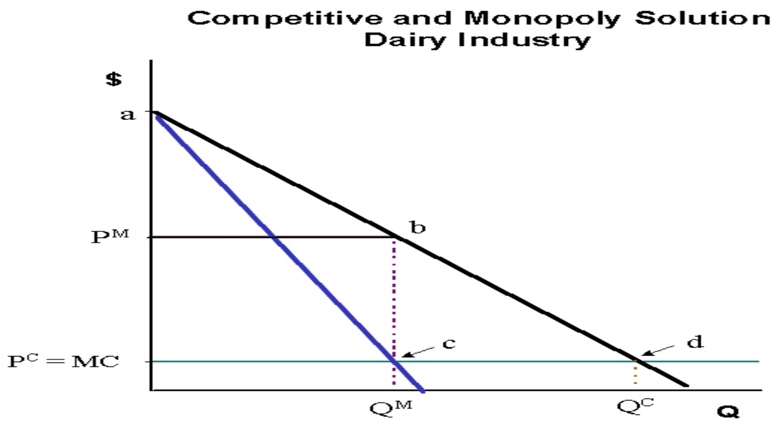
We find price by substituting Q into the inverse demand or supply equation

$$\begin{aligned} P &= 1{,}000 - 0.025Q \\ &= 1{,}000 - 0.025(14{,}655.172) \\ &= 1{,}000 - 366.3793 \\ &= 633.6207 \end{aligned}$$

(b) $CS = \frac{1}{2}(1000 - 633.6207)(14655.172) = 2684675.8$

$$PS = \frac{1}{2}(633.6207 - 150)(14655.172) = 3543772.3$$

Problem 2



Before forming the supply association, the industry price is given by P^C = MC. The quantity supplied is Q^C where price is equal to marginal cost. There are no profits and consumer surplus is equal to the area adP^C. After forming the association and restricting supply, the price rises to P^M. The quantity is Q^M. Producers now have profits equal to the area P^ccbP^M while consumer surplus falls to abP^M. The deadweight loss is equal to the area bcd.

Problem 3

(a) We set price equal to marginal cost for any one of the firms to obtain

$$MC = 4q + 2 = P$$
$$\rightarrow 4q = P - 2$$
$$\Rightarrow q = \frac{P - 2}{4}$$
$$= \frac{P}{4} - \frac{1}{2}$$

(b) Because there are 100 identical firms, we can simply multiply the supply curve in part a by 100 as follows to obtain the supply equation.

$$Q = 100q$$
$$= (100)\left(\frac{P}{4} - \frac{1}{2}\right)$$
$$= 25P - 50$$

We then solve this equation for P as a function of Q to get inverse supply

$$Q = 25P - 50$$
$$\rightarrow Q + 50 = 25P$$
$$\Rightarrow \frac{Q + 50}{25} = P$$
$$\rightarrow \frac{Q}{25} + 2 = P$$

Problem 4

(a) Find the inverse demand function by solving the demand equation for *P* as a function of *Q*

$$Q = 1{,}000 - 50P$$
$$\Rightarrow 50P = 1{,}000 - Q$$
$$\rightarrow P = 20 - \frac{Q}{50}$$

Then set this equal to marginal cost to find the competitive solution. This will give

$$P = 20 - \frac{Q}{50} = 10 = MC$$
$$\rightarrow \frac{Q}{50} = 10$$
$$\Rightarrow Q = 500$$
$$\Rightarrow P = 20 - \frac{Q}{50}$$
$$= 20 - \frac{500}{50}$$
$$= 20 - 10$$
$$= 10$$

Under monopoly we set marginal revenue equal to marginal cost. We find marginal revenue by finding total revenue first and taking the derivative with respect to Q or by applying the same intercept - twice the slope rule to the inverse demand. Using the same intercept - twice the slope rule we obtain

$$P = 20 - \frac{Q}{50}$$

$$MR = 20 - 2\left(\frac{Q}{50}\right)$$

$$= 20 - \frac{Q}{25}$$

If we derive an equation for revenue we obtain

$$R = PQ = \left(20 - \frac{Q}{50}\right)Q$$

$$= 20Q - \frac{Q^2}{50}$$

Taking the derivative we obtain

$$R = 20Q - \frac{Q^2}{50}$$

$$MR = \frac{dR}{dQ} = 20 - \frac{Q}{25}$$

Setting this equal to marginal cost we obtain

$$MR = 20 - \frac{Q}{25} = 10 = MC$$

$$\Rightarrow \frac{Q}{25} = 10$$

$$\Rightarrow Q = 250$$

$$\Rightarrow P = 20 - \frac{Q}{50}$$

$$= 20 - \frac{250}{50}$$

$$= 20 - 5$$

$$= 15$$

(b) First compute the elasticity for the competitive case where Q = 500 and P = 10.

$$\varepsilon_D(competitive) = -\frac{P}{Q}\frac{\Delta Q}{\Delta P}$$

$$= -\frac{10}{500}(-50)$$

$$= \frac{500}{500} = 1$$

Then compute the elasticity for the monopoly case where Q = 250 and P = 15.

$$\varepsilon_D(monoply) = -\frac{P}{Q}\frac{\Delta Q}{\Delta P} = -\frac{15}{250}(-50) = \frac{750}{250} = 3$$

(c) The monopoly price is P = 15. Marginal cost for this firm is MC = 10. So we obtain

$$\frac{P-MC}{P}=\frac{15-10}{15}=\frac{5}{15}=\frac{1}{3}$$

$$\varepsilon_D=3, \frac{1}{\varepsilon_D}=\frac{1}{(3)}=\frac{1}{3}$$

Problem 5

(a) To find the competitive quantity we set price equal to marginal cost and solve for Q as follows.

$$P = 3 - \frac{Q}{16{,}000} = 1 = MC$$

$$\Rightarrow \frac{Q}{16{,}000} = 2$$

$$\Rightarrow Q = 32{,}000$$

We obtain price by substituting the competitive quantity in the inverse demand function.

$$P = 3 - \frac{Q}{16{,}000}$$

$$= 3 - \frac{32{,}000}{16{,}000}$$

$$= 3 - 2 = 1$$

Or we could simply note that with P = MC, price must be equal to 1, and then substitute this in the inverse demand equation and solve for Q.

(b) With an inverse demand of P = 3 - Q/16,000, marginal revenue is given by MR = 3 - Q/8,000. Setting this equal to marginal cost will yield the monopoly value of Q.

$$MR = 3 - \frac{Q}{8{,}000} = 1 = MC$$

$$\Rightarrow \frac{Q}{8{,}000} = 2$$

$$\rightarrow Q = 16{,}000$$

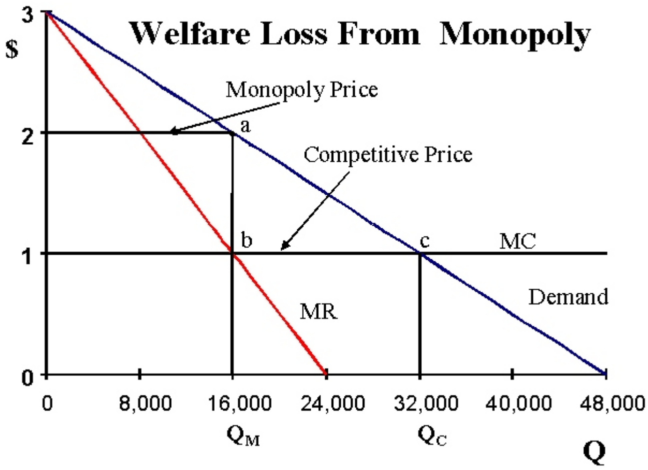
Solving for price we obtain

$$P = 3 - \frac{Q}{16{,}000}$$

$$= 3 - \frac{16{,}000}{16{,}000}$$

$$= 3 - 1 = 2$$

(c) The following diagram will be useful for this problem.



The competitive industry has no profits and so producer surplus is zero. Consumer surplus is given by the triangle that starts at 1, proceeds over to c, and then angles up to 3. The base is 32,000, the height is 2, and the area is ½(32,000)(2) = 32,000. W ith a monopoly consumer surplus is given by the triangle that starts at 2, proceeds over to a, and then angles up to 3. The base is 16,000, the height is 1, and the area is ½(16,000)(1) = 8,000. Profits or producer surplus for the monopolist are given by the rectangle beginning at 1, proceeding over to b, up to a and then back over to 2. This rectangle has dimensions 16,000x1 = 16,000. So total surplus with monopoly is 24,000. The loss from monopoly is then 32,000 - 24,000 or 8,000.

One can also compute the area of the deadweight loss triangle abc. It has base 16.,000 and height 1 for an area of $\frac{1}{2}(16{,}000)(1) = 8{,}000$.

Problem 6

(a) First find the inverse demand function as follows

$$\begin{aligned} Q &= 60 - P \\ \Rightarrow P &= 60 - Q \end{aligned}$$

Then set marginal revenue equal to marginal cost. Find marginal revenue from total revenue first as follows

$$\begin{aligned} P &= 60 - Q \\ R &= PQ = (60 - Q)Q \\ &= 60Q - Q^2 \\ MR &= 60 - 2Q \end{aligned}$$

Setting this equal to marginal cost we obtain

$$\begin{aligned} MR &= 60 - 2Q = 10 = MC \\ \rightarrow 2Q &= 50 \\ \Rightarrow Q &= 25 \\ \rightarrow P &= 60 - Q \\ &= 35 \end{aligned}$$

Profit is given by

$$\begin{aligned} \pi &= PQ - C(Q) \\ &= (35)(25) - (10)(25) \\ &= (25)(25) \\ &= 625 \end{aligned}$$

(b) First find the inverse demand function as follows

$$Q = 45 - 0.5P$$
$$\Rightarrow 0.5P = 45 - Q$$
$$\rightarrow P = 90 - 2Q$$

Then set marginal revenue equal to marginal cost.

$$P = 90 - 2Q$$
$$R = PQ = (90 - 2Q)Q$$
$$= 90Q - 2Q^2$$
$$MR = 90 - 4Q = 10 = MC$$
$$\Rightarrow 4Q = 80$$
$$\rightarrow Q = 20$$
$$\Rightarrow P = 90 - 2Q$$
$$= 50$$

Profit is given by

$$\pi = PQ - C(Q)$$
$$= (50)(20) - (10)(20)$$
$$= (40)(20)$$
$$= 800$$

(c) First find the inverse demand function as follows

$$Q = 100 - 2P$$
$$\rightarrow 2P = 100 - Q$$
$$\rightarrow P = 50 - \frac{Q}{2}$$

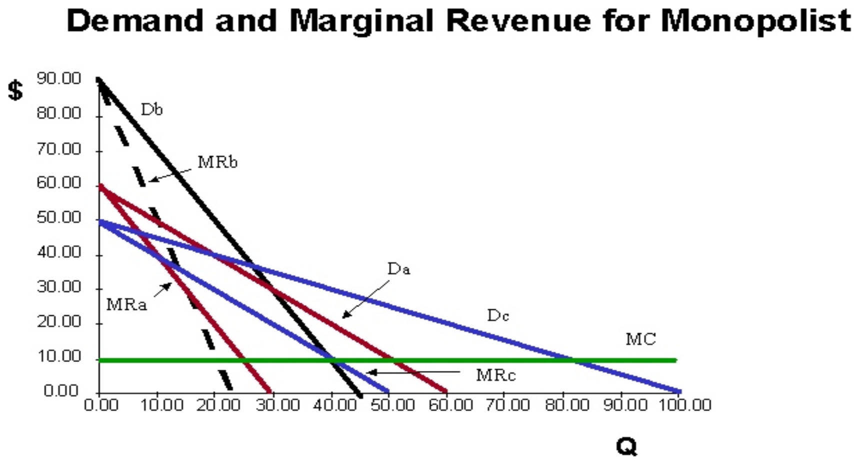
Then set marginal revenue equal to marginal cost.

$$P = 50 - \frac{Q}{2}$$
$$R = PQ = (50 - \frac{Q}{2})Q$$
$$= 50Q - \frac{Q^2}{2}$$
$$MR = 50 - Q = 10 = MC$$
$$\Rightarrow Q = 40$$
$$\rightarrow P = 50 - \frac{Q}{2}$$
$$= 30$$

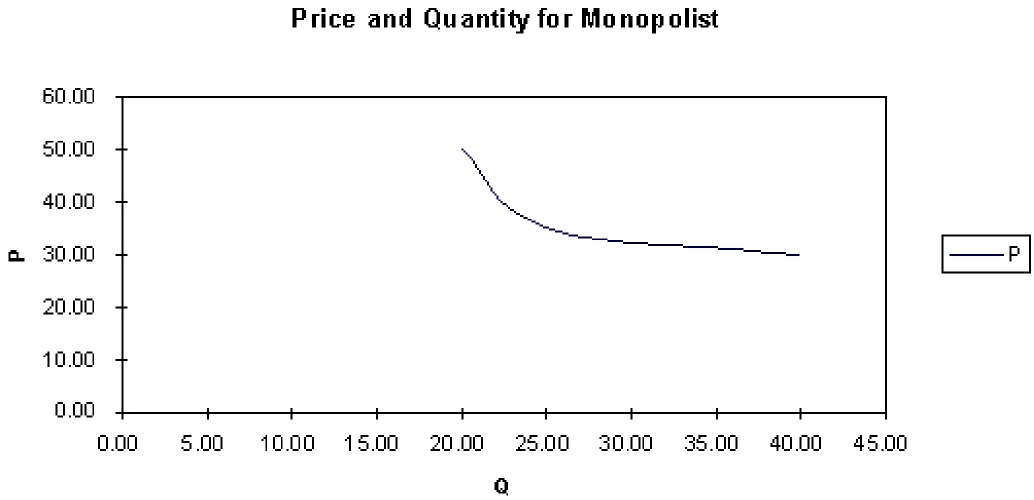
Profit is given by

$$\pi = PQ - C(Q)$$
$$= (30)(40) - (10)(40)$$
$$= (20)(40)$$
$$= 800$$

(d) The diagram below shows demand, marginal revenue, and marginal cost for parts a-c of this question.



Notice that for some values of marginal cost the firm would choose the same price but not the same quantity. And for other values of marginal cost, the firm may choose the same quantity but charge different prices. Another way to look at this is to notice that 20 is supplied with a price of $50 while at a lower price of $30, 40 units are supplied. This hardly seems like the normal notion of supply. Consider then a diagram showing price and quantity for this monopolist when the technology and marginal cost are the same.



As demand shifts, we do not trace out a supply curve as would happen in the competitive case. With a constant marginal cost in this problem, the supply curve for competitive firm would be horizontal and the shifting demand would simply show alternative quantities at the price of $10.

Chapter 3: Market Structure and Market Power

Problem 1

(a)

$$CR_4^{FT} = 0.48 + 0.30 + 0.07 + 0.06 = 0.91 = 91\%$$

$$CR_4^{TP} = 0.30 + 0.20 + 0.16 + 0.12 = 0.78 = 78\%$$

$$CR_4^{PT} = 0.37 + 0.18 + 0.12 + 0.11 = 0.78 = 78\%$$

(b)

$$\begin{aligned} H^{FT} &= .48^2 + .30^2 + .07^2 + .06^2 + .09^2 \\ &= .2304 + .0900 + .0049 + .0036 + .0081 = .3370 \\ H^{TP} &= .30^2 + .20^2 + .16^2 + .12^2 + .05^2 + .16^2 \\ &= .0900 + .0400 + .0256 + .0144 + .0025 + .0256 = .1981 \\ H^{PT} &= .37^2 + .18^2 + .12^2 + .11^2 + .04^2 + .18^2 \\ &= .1369 + .0324 + .0144 + .0121 + .0016 + .0324 = .2298 \end{aligned}$$

(c) Given the highest four-firm concentration ratio and a very high Herfindahl index, facial tissue is the most concentrated with 2 firms controlling 78% of the market.

Problem 2

a. $LI = \theta(HHI/\eta)$. If the firms collude and act as a monopoly, the Lerner Index will be $LI = 1/\eta$. Hence, in this case, $\theta = 1/HHI$.

b. Again, $LI = \theta(HHI/\eta)$. Under perfect competition, the Lerner Index is 0. Hence, in this case, $\theta = 0$.

c. Holding concentration or HHI constant, we might expect that as θ increases from 0 to 1/HHI, it indicates that the level of competition in the market is decreasing.

Problem 3

Given a downward sloping demand curve, Monopoly Air could probably fill the planes if it lowered its price. At issue here is the cost of production versus the price charged. In order to determine if this is a natural monopoly, it would be useful to have data on the demand function and the cost function for production of passenger miles. Only if one large firm can meet the market demand at cost less than two firms is there a natural monopoly.

Problem 4

We can write the Lerner index as follows

$$L = \frac{p - MC}{p} = 1 - \frac{MC}{P}.$$

First note that prices and marginal costs are always positive. Then note that a profit maximizing firm will only operate at a point where $P \geq MC$. This means that the ratio MC/P is always less than one which means than L is always less than one and greater than zero.

Given that L is ≤ 1 it is clear than $\eta \geq 1$ for a monopolist. In particular,

$$L = \frac{1}{\eta}$$

$$L \leq 1$$

$$\Rightarrow \frac{1}{\eta} \leq 1$$

$$\Rightarrow 1 \leq \eta$$

Chapter 4: Technology and Cost

Problem 1

$$AC(q) = \frac{C(q)}{q} = \frac{100 + 4q + 4q^2}{q} = \frac{100}{q} + 4 + 4q$$

$$MC(q) = 4 + 8q$$

To find the range of production characterized by scale economies, equate AC(q) with MC(q).

$$AC(q) = MC(q) \Rightarrow \frac{100}{q} + 4 + 4q = 4 + 8q \Rightarrow q = 5$$

For $q \in [0,5]$, production is characterized by scale economies. At q = 5 production level scale economies exhausted.

Problem 2

The consultant has not distinguished between fixed and variable costs. Since the fixed costs will be incurred regardless of whether the train runs or not, there is no increase in fixed cost from making a trip during off-peak hours. What matters are the variable costs of making an off-peak hour trip? As long as they are less than the revenue from the sales of 10 tickets, the train should make the trip.

> Suppose that the train makes 20 total round trips per day and the total fixed cost per day is $800. The variable cost per trip is $10. The fixed cost per trip with 20 trips is $40. Suppose the train normally makes 5 peak load trips and 15 off-peak load trips. The variable cost per passenger for an off-peak hour trip is $1. If the fare exceeds $1, then the train should make the trip since the fixed costs will accrue whether the trip is made or not. In fact, if the number of off-peak trips is reduced by 10 to 5 trips so that the total trips per day is now 10, the total cost per trip is now ((800+100)/10) $90 instead of $50.

Problem 3

(a)

$$C(q \leq 7) = 50 + 0.5q$$

$$AC = \frac{50 + 0.5q}{q} = \frac{50}{q} + 0.5$$

$$MC = 0.5$$

We can create a table with the values for various levels of q where average marginal cost is the average of the discrete changes to and away from q_i.

q	Cost	Average Cost	Discrete MC	Approx MC	MC
0	50		0.5		0.5
1	50.5	50.5000	0.5	0.5	0.5
2	51	25.5000	0.5	0.5	0.5
3	51.5	17.1667	0.5	0.5	0.5
4	52	13.0000	0.5	0.5	0.5

5	52.5	10.5000	0.5	0.5	0.5
6	53	8.8333	0.5	0.5	0.5
7	53.5	7.6429	2.5	1.5	0.5
8	56	7.0000	7	4.75	7
9	63	7.0000	7	7	7
10	70	7.0000	7	7	7
15	105	7.0000	7	7	7
20	140	7.0000			7

(b)

$$C(q > 7) = 7q$$

$$AC = \frac{7q}{q}$$

$$= 7$$

$$MC = 7$$

Problem 4

Yes, there is a minimum efficient scale of plant implied by these cost relationships. If we require integer values of q, then the minimum efficient scale is 8 units of output. Otherwise, it is any amount greater than 7.

Problem 5

Since the minimum average cost is $7.00 and this is also marginal cost we can assume that the price in market equilibrium is $7.00. Using the inverse demand curve we then obtain

$$\begin{aligned} P &= 84 - 0.5Q \\ \Rightarrow 7 &= 84 - 0.5Q \\ \Rightarrow 0.5Q &= 77 \\ \rightarrow Q &= 154 \end{aligned}$$

Since the minimum efficient scale is 8, the maximum number of firms producing 8 units is q* = 154/8 = 19.25. Each firm would produce 8 units given a total of 152. The firms would then need to allocate the remaining two units in some integer fashion among them if whole units of production are required. Otherwise we could have 21 firms each producing 7.333 units.

Problem 6

Demand has changed and so has the equilibrium quantity.

$$\begin{aligned} P &= 14 - 0.5Q \\ \Rightarrow 7 &= 14 - 0.5Q \\ \Rightarrow 0.5Q &= 7 \\ \rightarrow Q &= 14 \end{aligned}$$

Since the minimum efficient scale is 8, the maximum number of firms producing 8 units is q* = 14/8 = 1.75. One firm could produce 14 units at a total cost of $98. If there were two firms in the industry, one producing 8 units and the other one six units, the total cost of production would be $109 (56+53), which is larger than $98. If the industry price were $7, the second firm would not cover its average costs of $8.833 per unit. Thus there will be no second firm and the first firm will be a monopoly. There is not room in this industry for two firms.

If the first firm were a monopoly it would set marginal revenue equal to marginal cost and charge a price of

$$\pi = PQ - C(Q)$$
$$=(14 - 0.5Q)Q - 7Q$$
$$= 14Q - 0.5Q^2 - 7Q$$
$$= 7Q - 0.5Q^2$$
$$\frac{d\pi}{dQ} = 7 - Q = 0$$
$$\Rightarrow Q = 7$$
$$\Rightarrow P = 10.5$$

At this point a second firm will try to enter producing at least 6 units. But this will cause price to fall and the second firm will be forced out.

Problem 7

(a) It is clear that average costs start to rise once we move from 1,500 to 1,750 units of output. This can also be seen by computing the total cost at each output level and then computing a discrete measure of marginal cost as in the table below. Once we get beyond 1,500 units, marginal cost is higher than average cost.

(b) Find the MC first. Here is the answer for Q =1000. The answers for other values of Q can be found in a similar fashion.

For output level 1,000, it is computed as

$$S(1{,}000) = \frac{AC}{MC} = \frac{21.63}{18.48} = 1.17045$$

It may be more accurate here to compute the average marginal cost as opposed to the discrete one given the large changes in output.

Problem 8

If the main product is meat, then the additional costs of supplying the byproducts (offal) are quite small. For example the cost of supplying the hide is the cost of removing the hide in a fashion that preserves its usefulness for leather as opposed to a technique that might be cheaper but reduces it to a pile of scrap. These economies exist because the process of feeding and slaughtering a steer or heifer produces a whole animal (hide, horns, meat, viscera, etc.) and since these come in more or less fixed proportions, the cost of obtaining whatever is considered a byproduct is close to zero for all amounts less than that implied by the fixed proportion technology.

The supply of leather will then depend on the price of steak. If the demand for steak is very high, then the supply of cattle will be high, which will increase the supply of hides and lower the price of leather. Similarly, a very high price of gelatin may lead to a different process in removing the horns and hoofs so that more is preserved for the making of gelatin. In a similar fashion, the percentage of the animal that goes to make ribs as opposed to hamburger depends on the demand for ribs in a given area.

Chapter 7: Product Variety and Quality under Monopoly

Problem 1

(a) For z = 1, profits for this firm is given by

$$\begin{aligned}\pi &= PQ - VC(Q) - FC \\ &= (36 - 2Q)(1)Q - 0 - (65)(1^2) \\ &= 36Q - 2Q^2 - 65\end{aligned}$$

Taking the derivative of the profit with respect to Q will yield

$$\begin{aligned}\frac{d\pi}{dQ} &= 36 - 4Q = 0 \\ \Rightarrow 4Q &= 36 \\ \rightarrow Q &= 9 \\ \Rightarrow P &= 36 - 2Q = 18\end{aligned}$$

Profit is given by

$$\begin{aligned}\pi &= PQ - VC(Q) - FC \\ &= (18)(1)(9) - 65 \\ &= 162 - 65 = 97\end{aligned}$$

(b) Profit when z = 2 is given by

$$\begin{aligned}\pi &= PQ - VC(Q) - FC \\ &= (36 - 2Q)(2)Q - 0 - (65)(2^2) \\ &= 72Q - 4Q^2 - 260\end{aligned}$$

Taking the derivative of the profit with respect to Q will yield

$$\begin{aligned}\frac{d\pi}{dQ} &= 72 - 8Q = 0 \\ \rightarrow 8Q &= 72 \\ \rightarrow Q &= 9 \\ \Rightarrow P &= 36 - 2Q = 18\end{aligned}$$

Profit is given by

$$\begin{aligned}\pi &= PQ - VC(Q) - FC \\ &= (18)(2)(9) - 260 \\ &= 324 - 260 = 64\end{aligned}$$

(c) The monopolist will go with low quality.

Problem 2

This is a clear case where the individual firm incentive to increase variety is at odds with the socially optimal level of product variety. The individual firms will try to fill each niche in product space so that they can obtain some revenue from that spot as opposed to letting the revenue go to another firm. Rather than using a price instrument that would also give a lower price to consumers at other spots (where the competition may not be so fierce), they locate a brand (outlet) near the spot. If the cost of adding the brand is less than what consumers paid to travel to the old brand spot, they can offer a lower price and beat the competition. Consider, for example, the case of Wheaties, Total, Corn Total, and Raisin Bran Total, which are produced by General Mills. They compete with Corn Flakes and Raisin Bran and each other in such a way that competitors' products have a hard time finding a unique niche. Also notice that as "truly" new cereals such as Fruit and Fibre or Granola have come on the market, that there has been a rapid filling of the product space around these new competitors.

Problem 3

This is the classic location model from the chapter. The reservation price is given by V = \$5. The number of customers is N = 1,000. The length of the beach is 5 miles. The "cost" to travel from one end of the beach to the other is \$5.00. The marginal cost per crepe is c = \$0.50 and the fixed cost per stall is F = \$40. First consider one shop in the middle of the beach. Demand is given by

$$\begin{aligned} Q(p_1, 1) &= 2N\left(\frac{V - p_1}{t}\right) \\ &= (2)(1{,}000)\left(\frac{5 - p_1}{5}\right) \\ &= 2{,}000\left(\frac{5 - p_1}{5}\right) \\ &= 400(5 - p_1) \\ &= 2{,}000 - 400p_1 \end{aligned}$$

Given that there are 1,000 consumers, we can find the price that will allow this one stall to sell to all of them.

$$\begin{aligned} Q(p_1, 1) &= 2{,}000 - 400p_1 \\ 1{,}000 &= 2{,}000 - 400p_1 \\ \Rightarrow 400p_1 &= 1{,}000 \\ p_1 &= \$2.50 \end{aligned}$$

This makes sense, the customers at the ends of the beach must travel 2.5 miles (10 quarter miles) to the stall. At a cost of \$0.25 per 1/4 mile, the ten 1/4 miles gives a cost of \$2.50. This plus the price of the crepe at \$2.50 is just their reservation price. This will give profit from this one shop of

$$\begin{aligned} \pi &= (2.50)(1{,}000) - (0.50)(1{,}000) - 40 \\ &= 2{,}500 - 500 - 40 \\ &= \$1{,}960 \end{aligned}$$

If instead of supplying the whole market with this one shop, the firm were to restrict output, the optimal output level is determined by setting marginal revenue equal to marginal cost. We find marginal revenue by inverting the demand function and then using the "twice the slope" rule.

$$\begin{aligned} Q &= 2N\left(\frac{V - p}{t}\right) \\ \Rightarrow Qt &= 2NV - 2Np \\ \Rightarrow p &= \frac{2NV - Qt}{2N} \\ &= V - \frac{t}{2N}Q \\ &= 5 - \frac{5}{2{,}000}Q \\ MR &= 5 - \frac{10}{2{,}000}Q \\ &= 5 - 0.005Q \end{aligned}$$

Setting this equal to marginal cost of \$0.50 we obtain

$$MR = 5 - 0.005Q = 0.50 = MC$$
$$\rightarrow 4.50 = 0.005Q$$
$$\Rightarrow Q = 900$$

Price is then given by

$$P = V - \frac{t}{2N}Q$$
$$= 5 - \frac{5}{2{,}000}(900)$$
$$= 5 - 2.25$$
$$= 2.75$$

So with only one stall, the market is not fully served. We can see this directly using the equation in the text which says that if $V < c + t/n$, only part of the market should be served, i.e.

$$c + \frac{t}{n} < V$$
$$0.50 + \frac{5}{1} = 5.50$$
$$5.50 \nless 5.00.$$

If there are two stalls, the entire market will be served as can be seen from

$$c + \frac{t}{n} < V$$
$$0.50 + \frac{5}{2} = 3.00$$
$$3.00 < 5.00.$$

Two stalls will be located 1/4 and 3/4 of the way along the beach. Each will sell to the maximum number of customers, i.e. 500. In order to sell to 500 customers, they must charge a price of \$3.75 as can be seen below.

$$Q(p_1, 1) = 2{,}000 - 400p_1$$
$$500 = 2{,}000 - 400p_1$$
$$\Rightarrow 400p_1 = 1{,}500$$
$$p_1 = \$3.75$$

Joint profits for the two stalls can be computed as

$$\pi = (3.75)(1{,}000) - (0.50)(1{,}000) - 80$$
$$= 3{,}750 - 500 - 80$$
$$= \$3{,}170$$

Three stalls will be located 1/6, ½, and 5/6 of the way along the beach. Each will sell to the maximum number of customers, i.e. 333 1/3. In order to sell to 333 1/3 customers, they must charge a price of \$4.166 as can be seen below.

$$Q(p_1, 1) = 2{,}000 - 400p_1$$
$$333\frac{1}{3} = 2{,}000 - 400p_1$$
$$\Rightarrow 400p_1 = 1{,}666\frac{2}{3}$$
$$p_1 = \$4.16\overline{6}$$

Joint profits for the three stalls can be computed as

$$\begin{aligned}\pi &= (4.166)(1{,}000) - (0.50)(1{,}000) - 120\\ &= 4{,}166.6\overline{6} - 500 - 120\\ &= \$3{,}546.6\overline{6}\end{aligned}$$

So three stalls dominates two stalls. We can proceed in a similar fashion with four stalls each serving 250 consumers.

$$\begin{aligned}Q(p_1, 1) &= 2{,}000 - 400p_1\\ 250 &= 2{,}000 - 400p_1\\ \Rightarrow 400p_1 &= 1{,}750\\ p_1 &= \$4.375\end{aligned}$$

Joint profits for four stalls can be computed as

$$\begin{aligned}\pi &= (4.375)(1{,}000) - (0.50)(1{,}000) - 160\\ &= 4{,}375 - 500 - 160\\ &= \$3{,}715\end{aligned}$$

We could proceed in this fashion or use the equations in the text for profits with N consumers and n stalls.

$$\pi(N, n) = N\left(V - \frac{t}{2n} - c\right) - nF$$

$$\pi(N, n+1) = N\left(V - \frac{t}{2(n+1)} - c\right) - (n+1)F$$

Profit with n+1 firms will be higher than with n firms if

$$\pi(N, n+1) > \pi(N, n)$$

$$\begin{aligned}&\Rightarrow N\left(V - \frac{t}{2(n+1)} - c\right) - (n+1)F > N\left(V - \frac{t}{2n} - c\right) - nF\\ &\Rightarrow N\left(\frac{t}{2n}\right) - N\left(\frac{t}{2(n+1)}\right) + nF - (n+1)F > 0\\ &\Rightarrow N\left(\frac{t}{2n}\right) - N\left(\frac{t}{2(n+1)}\right) - F > 0\\ &\Rightarrow N\left(\frac{t}{2n} - \frac{t}{2(n+1)}\right) - F > 0\\ &\Rightarrow N\left(\frac{t(n+1)}{2n(n+1)} - \frac{tn}{2n(n+1)}\right) - F > 0\\ &\Rightarrow N\left(\frac{t}{2n(n+1)}\right) - F > 0\\ &\Rightarrow N\left(\frac{t}{2n(n+1)}\right) > F\\ &\Rightarrow \frac{tN}{F} > 2n(n+1)\\ &\Rightarrow \frac{tN}{2F} > n(n+1)\end{aligned}$$

For this problem the left-hand side of this inequality is

$$\frac{tN}{2F} = \frac{(5)(1,000)}{(2)(40)} = \frac{5,000}{80} = 62.5$$

With four stalls, n(n+1) = (4)(5) = 20. With seven stalls, n(n+1) = (7)(8) = 56, while with eight stalls, n(n+1) = (8)(9) = 72. So the firm should increase from seven to eight stalls, but not from eight to nine. So the optimal number of stalls is eight. To see this explicitly, compare profits with eight and nine stalls.

First, for eight stalls.

$$Q(p_1, 1) = 2,000 - 400p_1$$
$$125 = 2,000 - 400p_1$$
$$\rightarrow 400p_1 = 1,875$$
$$p_1 = \$4.6875$$
$$\pi = (4.6875)(1,000) - (0.50)(1,000) - 320 = 4,687.5 - 500 - 160 = \$3,867.5$$

Then, for nine stalls.

$$Q(p_1, 1) = 2,000 - 400p_1$$
$$\frac{1,000}{9} = 2,000 - 400p_1$$
$$\rightarrow 400p_1 = \frac{17,000}{9}$$
$$p_1 = \$4.72\overline{2}$$
$$\pi = (4.72\overline{2})(1,000) - (0.50)(1,000) - 360 = 4,722.2 - 500 - 360 = \$3,862.2\overline{2}$$

The table below shows the optimal price, revenue, total variable cost, total fixed cost and profit for various numbers of stalls assuming that all consumers are served in each case.

Stalls	Price	Revenue	Variable Cost	Fixed Cost	Profit	n(n+1)	tN/2F	c + t/n
1	2.5	2,500	500	40	1,960	2	62.5	5.5
2	3.75	3,750	500	80	3,170	6	62.5	3
3	4.1667	4,166.667	500	120	3,546.67	12	62.5	2.16667
4	4.375	4,375	500	160	3,715	20	62.5	1.75
5	4.5	4,500	500	200	3,800	30	62.5	1.5
6	4.5833	4,583.333	500	240	3,843.33	42	62.5	1.33333
7	4.6429	4,642.857	500	280	3,862.86	56	62.5	1.21429
8	4.6875	4,687.5	500	320	**3,867.5**	72	62.5	1.125
9	4.7222	4,722.222	500	360	3,862.22	90	62.5	1.05556
10	4.75	4,750	500	400	3,850	110	62.5	1
15	4.8333	4,833.333	500	600	3,733.33	240	62.5	0.83333
20	4.875	4,875	500	800	3,575	420	62.5	0.75

Problem 4

Given the standard assumption that the effort costs of the stall-holders are the same as those o the sunbathers, there is no change in the costs faced by consumers and profits faced by stall-holders, since the only change is a transfer from consumers' disutility cost to stall-holders' delivery cost. Therefore, the optimal number of stalls is still eight for profit maximization.

If the incurred effort costs half as much has those of the sunbathers, then in order to have

$$\pi(N,n) = N\left(V - \frac{t}{2n} - c\right) - nF < \pi(N,n+1) = N\left(V - \frac{t}{2(n+1)} - c\right) - (n+1)F \text{ require}$$

$$\frac{tN}{2F} = n(n+1) \Rightarrow \frac{(2.5)(1000)}{(2)(40)} = n(n+1) \Rightarrow n = 5$$

Then the optimal number of stalls is five.

Problem 5

Since the marginal cost of making both types of computers are identical, the profit maximizing strategy is to offer high performance laptops to both groups.

Problem 6

Profit maximization requires that Dell offers a quality – price combination (z_1, p_1) to the "normal" people and a quality – price combination (z_2, p_2) to the "techies" that works to sort the two groups and permit more surplus extraction. Therefore, to the normal people, Dell can charge a price $p_1 = 1000z_1$. from the discussion in the text, it follows that Dell should offer $z_2 = 3$, which is the highest quality. Now, Dell needs to adjust p_2 so that the techies do not buy a computer of quality z_1. Thus, the highest price it can charge the techies is p_2, where p_2 is such that $2000(z_1 - 1) - p_1 = 4000 - p_2 \Rightarrow p_2 = 6000 - 2000z_1 + p_1$

If the proportions of normal and techies are N_n and N_t, respectively, then Dell's profit is:

$$\pi = N_t(6000 - 2000z_1 + p_1) + N_n(1000z_1) - 500(N_t + N_n)$$

$$\frac{\partial \pi}{\partial z_1} = -2000N_t + 1000N_n$$

From the above equation, it follows that Dell's profit maximizing strategy depends on the proportions of techies and normal people.

Problem 7

(a) An increase in product quality affects demand positively. Observe that

$$\frac{\partial P}{\partial Z} = \frac{\partial}{\partial Z}\left[22 - \frac{Q}{100Z}\right] = \frac{Q}{100Z^2} > 0$$

(b) Find out the total profits associated with Z = 1, 2 and 3.

Note that when Z = 1, $P = 22 - \frac{Q}{100}$ and $MC(Q) = 2 + (1)^2 = 3$

Now equate MR with MC to get the optimal quantity, price and profit.

$$MR(Z=1)=22-\frac{2Q}{100}=3=MC(Z=1)\Rightarrow Q=950\Rightarrow \pi(Z=1)=9025$$

Note that when Z = 2, $P=22-\frac{Q}{200}$ and $MC(Q)=2+(2)^2=6$

Now equate MR with MC to get the optimal quantity, price and profit.

$$MR(Z=2)=22-\frac{2Q}{200}=6=MC(Z=2)\Rightarrow Q=1600\Rightarrow \pi(Z=2)=12800$$

Note that when Z = 3, $P=22-\frac{Q}{300}$ and $MC(Q)=2+(3)^2=11$

Now equate MR with MC to get the optimal quantity, price and profit.

$$MR(Z=3)=22-\frac{2Q}{300}=11=MC(Z=3)\Rightarrow Q=1650\Rightarrow \pi(Z=3)=9075$$

Therefore, Z = 2 is the profit maximizing level of quality for the monopolist.

Problem 8

To maximize the social welfare, is to maximize

$$W=[2000(z-1)-p]N_t+[1000z-p]N_n+(p-500)(N_t+N_n)$$

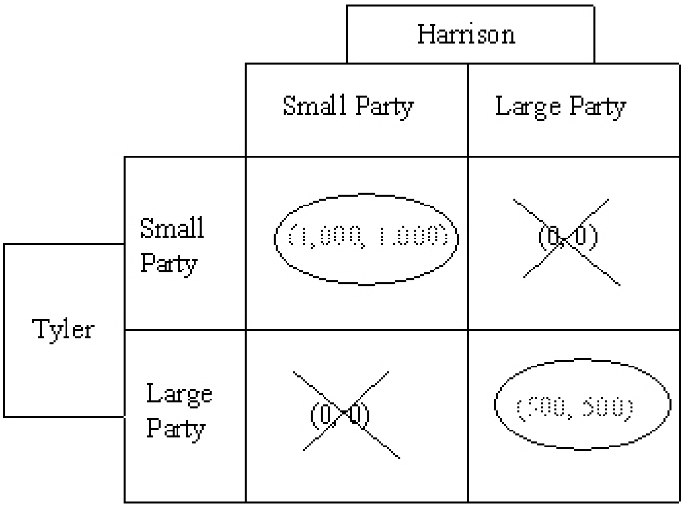
Therefore, the quality choice is the highest quality, which is z = 3.

The monopolist would maximize its profit under this social optimal quality condition. The price must be greater than its marginal cost 500, and allow both types of consumer to purchase. Therefore, the price it would charge is 3000.

Chapter 9: Static Games and Cournot Competition

Problem 1

(a) This is a classic matching problem. The easiest way to find the Nash equilibrium is to first eliminate from each row the dominated strategies for Harrison. Harrison has the second payoff in each pair. Looking at the first row, if Tyler chooses small (S), Harrison should also choose small. Thus the point (S,L) in the upper right-hand corner can be eliminated. Looking at the second row, if Tyler chooses large (L), then Harrison should also choose large. Thus the point (L,S) in the lower left-hand corner can be eliminated. Now we move to the dominated strategies for Tyler. If Harrison chooses the first column (S), then Tyler should also choose small. This is already removed and so we gain no information. Unfortunately checking the second column also yields no new information and we are left with the two Nash Equilibria (S,S) and (L,L).



(b) The Pareto optimal outcome is (1,000, 1,000) which results from the strategy pair (S,S). At this point there is no way to make either party better off.

Problem 2:

(a) Note that X is the aggregate number of people that all individuals on campus expect to show up. The intercept is twenty since 20 individuals will always show up regardless of expectations. If individuals on campus think that one person will attend, then 21 individuals will show up.

(b) One way to get the intuition here is to assume that each person on campus thinks that 100 people will attend this party. This implies that the aggregate expectation is 100 individuals at the party. Plugging this into the equation implies that attendance is given by

$$A = 20 + 0.6(100) = 80$$

Thus expectations are not correct. If each individual thought no one would attend the party then the attendance would be given by

$$A = 20 + 0.6(0) = 20$$

which again is not a correct expectation. If each individual guesses that 50 people will attend then we obtain

$$A = 20 + 0.6(50) = 50$$

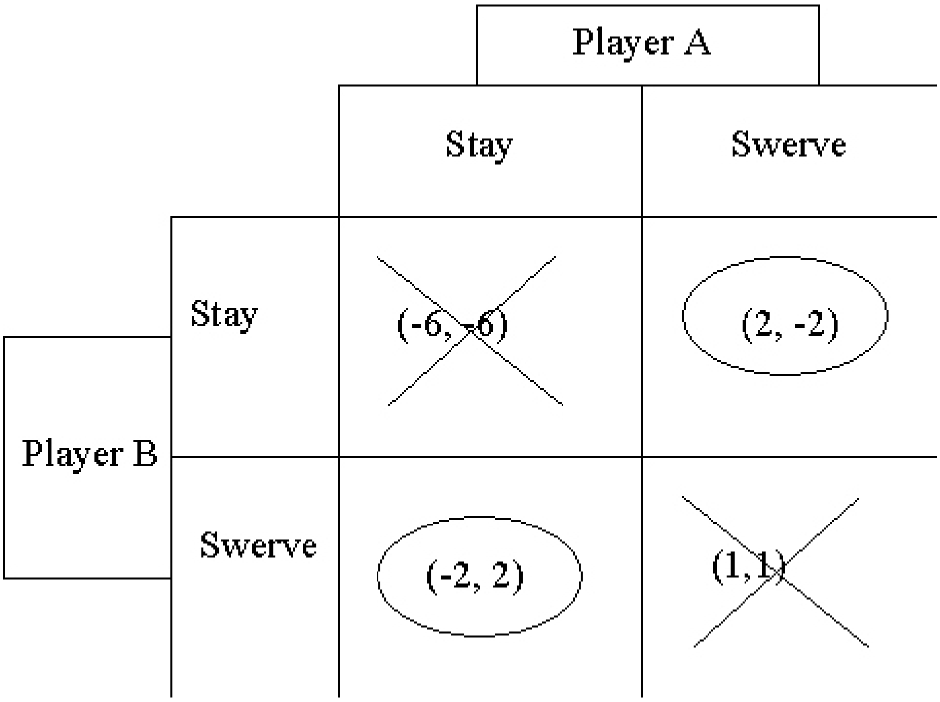
which means expectations are fulfilled. The solution procedure is to find the expected attendance (X) that makes the equation satisfied with X=A. Thus we just plug in X on the right-hand side and solve

$$\begin{aligned} X &= 20 + 0.6(X) \\ \Rightarrow 0.4X &= 20 \\ \Rightarrow X &= 50 \end{aligned}$$

It might be useful to relate this problem to the "multiplier" problem in a simple macro model of consumption where C = a + bY and Y = C+I.

Problem 3

(a)



The equilibria are the two off-diagonal elements.

(b) To solve this problem we need to use expected values. If player B chooses to Stay his expected payoff is given by the payoffs to staying weighted by the probabilities that player A will Stay or Swerve.

$$\begin{aligned} E(\pi_B|STAY) &= (0.20)(-6) + (0.8)(2) \\ &= -1.2 + 1.6 = 0.4 \end{aligned}$$

If player B chooses to Swerve his expected payoff is given by the payoffs to staying weighted by the probabilities that player A will Stay or Swerve.

$$\begin{aligned} E(\pi_B|SWERVE) &= (0.20)(-2) + (0.8)(1) \\ &= -0.4 + 0.8 = 0.4 \end{aligned}$$

(c) The easiest way to see this is to add the probabilities to the border of game matrix and then compute the joint probabilities in each cell.

		Player A	
		Stay 1/5	Swerve 4/5
Player B	Stay 1/5	(-6, -6) 1/25	(2, -2) 4/25
	Swerve 4/5	(-2, 2) 4/25	(1,1) 16/25

The probability of (Stay, Stay) is 1/25.

Problem 4

To determine my best response function, I equate my marginal revenue with my marginal cost

$$200 - 4Q_1 - 2Q_2 = 8 \Rightarrow Q_1 = \frac{1}{4}\left[200 - 2Q_2 - 8\right]$$

Since my rival and I are identical,

$$Q_1^* = Q_2^* = Q^* \Rightarrow \frac{1}{4}\left[192 - 2Q^*\right] = Q^* \Rightarrow Q^* = 32 \Rightarrow P = 72$$

My profit is

$$\pi = (72 - 8)32 - 1500 = 548$$

Note: Because of the fixed cost, there are two other asymmetric equilibria. At each, one firm produces its monopoly output and the other produces none. We assume that in this case, a symmetric equilibrium is more reasonable than an asymmetric equilibrium.

Problem 5

Assume that I am firm 1. To determine my best response function, I equate my marginal revenue with my marginal cost.

$$290 - \frac{2}{3}Q_1 - \frac{1}{3}\left(\sum_{i=2}^{14} Q_i\right) = 50 \Rightarrow Q_1 = \frac{3}{2}\left[240 - \frac{1}{3}\left(\sum_{i=2}^{14} Q_i\right)\right]$$

Since my rivals and I are identical, $Q_i^* = Q^*$ for all *i*. Therefore,

$$Q^* = \frac{3}{2}\left[240 - \frac{1}{3}\left(\sum_{i=2}^{14} Q^*\right)\right] \Rightarrow Q^* = 48 \Rightarrow P^* = 290 - \frac{1}{3}(14)(48) = 66$$

My profit is

$$\pi = (66 - 50)(48) - 200 = 568$$

Problem 6

(a) To determine firm 1's best response function, equate its marginal revenue with marginal cost

$$400 - 4Q_1 - 2Q_2 = 40 \Rightarrow Q_1 = \frac{1}{4}[360 - 2Q_2]$$

Since the firms are identical,

$$Q_1^* = Q_2^* = Q^* \Rightarrow \frac{1}{4}[360 - 2Q^*] = Q^* \Rightarrow Q^* = 60 \Rightarrow P^* = 160$$

Firm 1's profit is

$$\pi_1 = (160 - 40)60 = 7200$$

(b) The monopoly output is $Q_M = \frac{1}{4}[360] = 90$

(45, 45) is a not a solution, because if one firm produces 45, then the other produces $\frac{1}{4}[360 - 2(45)] = 67.5$ to maximize its profit.

Problem 7

To determine firm 1's best response function, equate its marginal revenue with its marginal cost

$$400 - 4Q_1 - 2Q_2 = 25 \Rightarrow Q_1 = \frac{1}{4}[375 - 2Q_2]$$

To determine firm 2's best response function, equate its marginal revenue with its marginal cost

$$400 - 4Q_2 - 2Q_1 = 40 \Rightarrow Q_2 = \frac{1}{4}[360 - 2Q_1]$$

Substitute $Q_1 = \frac{1}{4}[375 - 2Q_2]$ into $Q_2 = \frac{1}{4}[360 - 2Q_1]$ yields the equilibrium quantity for firm 2 as $Q_2^* = 57.5$. It is then easy to verify that: $Q_1^* = 65$,

Problem 8

(a) Let the long-run equilibrium number of firms be *N*. For each firm, determine its best response function by equate its marginal revenue with its marginal cost. Since the firms are identical

$$100 - 2q - (N-1)q = 20 \Rightarrow q = \frac{80}{N+1} \Rightarrow P = 100 - \frac{N}{N+1} \cdot 80$$

Each firm's profit must be zero so that no firm has incentive to leave or enter the industry

$$\pi = Pq - C = (100 - \frac{N}{N+1} \cdot 80) \cdot \frac{80}{N+1} - (256 + 20 \cdot \frac{80}{N+1}) = 0 \Rightarrow (N+1)^2 = 25 \Rightarrow N = 4$$

Thus, the long-run equilibrium number of firms is 4.

(b) At the long-run equilibrium, each firm's profit is zero and output is 16, therefore, the industry output is 64, industry price is 36 and industry profit is zero.

Chapter 10: Price Competition

Problem 1

(a) At equilibrium $p_1^* = p_2^* = 10$, assuming that if both firms charge the same price, then the firms split the market evenly.

(b) The higher cost firm makes zero profit, whereas the lower cost firm's profit is
$\left(p_1^* - c_1\right)Q_1 = (10-6)(5000-200(10)) = 12000$

(c) No, this outcome is not efficient.

Problem 2

(a) Note that the inverse demand function is $P = 30 - \frac{1}{3}Q$. Then the Cournot quantities are:

$$Q_1^* = \frac{(30-2(15)+10)}{3(1/3)} = 10, Q_2^* = \frac{((30-2(10)+15)}{3(1/3)} = 25$$

The market price is $P = 30 - \frac{1}{3}Q = 30 - \frac{1}{3}(10+25) = 18.33$

Profit of Firm 1 = $(18.33-15)(10) = 33.3$

Profit of Firm 2 = $(18.33-10)(25) = 208.25$

(b) At a Bertrand equilibrium, $p_1^* = p_2^* = 15$, assuming that if both firms charge the same price, then the consumers buy from the lower priced firm.

Total sales = 90 – 3(15) = 45. Firm 1 sells zero and earns zero profit. Firm 2 sells 45 units and earns (15 – 10) (45) = 225

Problem 3

(a) Yes, the outcome will change. The two lower cost firms will charge \$10 and share the market equally.

(b) The answer may change depending on how much premium the consumers are willing to pay for the green balls endorsed by Tiger Woods.

Problem 4

Note: Suppose that a consumer travels one mile to go to a store. Since the consumer needs to return home after purchase, it will cost her 2 (\$0.50) = \$1 to travel. Assume that V is very high.

(a) If both of them charge \$1**, e**ach will serve 500 in a day. If Ben charges \$1 and Will charges \$1.40, suppose the customer at the distance t from Ben's store is indifferent to buy fruit smoothie from each store, then since

$$2(0.5)x + 1 = 2(0.5)(10-x) + 1.4 \Rightarrow x = 5.2$$

Ben will sell 520 and Will will sell 480 per day.

(b) If Ben charges \$3, then \$8.00 will enable Will to sell 250. \$3.00 will enable him to sell 500, no positive price can enable him to sell more than 650. So, no positive price by Will permits him to reach a volume of either 750 or 1000.

(c) Suppose Ben charges p_1 and Will charges p_2. Let a consumer at a distance x from Ben is indifferent between the two firms. Therefore,

$$p_1 + x = p_2 + (10 - x) \Rightarrow 2x - 10 = p_2 - p_1 \Rightarrow x = 5 + \frac{p_2 - p_1}{2}$$

Therefore, the demand faced by Ben is $x_1(p_1) = 500 + 50(p_2 - p_1)$

Demand faced by Will is $x_2(p_2) = 1000 - x_1(p_1) = 500 - 50(p_2 - p_1)$

(d) $p_1 = 10 + p_2 - \frac{x_1}{50}$ Ben's marginal revenue function is $MR_1 = 10 + p_2 - \frac{x_1}{25}$

(e) Ben's profit is given by

$$\Pi_1 = (p_1 - 1)x_1(p_1) = (p_1 - 1)(500 + 50(p_2 - p_1))$$

Ben chooses his price to maximize his profit.

$$\frac{\partial \Pi_1}{\partial p_1} = (p_1 - 1)50(-1) + (500 + 50(p_2 - p_1)) = 0$$

Now, by symmetry, Ben and Will charge the same price in equilibrium. Therefore,

$$-p_1 + 1 + 10 = 0 \Rightarrow p_1^* = p_2^* = 11$$

Hence, the profit earned by each of them = (11 – 1) (500) - 250 = 5000 – 250 = 4750

Problem 5

George locates at the center. Let consumers at a distance of x (on both sides) are indifferent between buying from George and his rival. Consequently, George's market length is 2x.

(a) To find x, observe that a consumer located at $5 - x$ is indifferent between buying from Ben and George. Therefore, $11 + (5 - x) = p_{George} + x \Rightarrow x = \frac{(16 - p_{George})}{2}$

So, George chooses his price to maximize $\Pi_{George} = (p_{George} - 1)2x = (p_{George} - 1)(16 - p_{George})$

$$\frac{\partial \Pi_{George}}{\partial p_{George}} = ((p_{George} - 1)(-1) + (16 - p_{George})) = 0 \Rightarrow p_{George} = \frac{17}{2} = \$8.5$$

George's market length is $2x = 16 - \frac{17}{2} = \frac{15}{2} \Rightarrow \Pi_{George} = (7.5)(1000)\frac{15}{20} - 250 = 5375$

(b) Yes, Ben and Will have incentives to change their locations and prices. Otherwise, each of them makes a loss. Even after the adjustments, at the new equilibrium, both Ben and Will will make a loss and leave the market.

Problem 6

(a) Since unit costs are zero, profit is equal to revenue. And revenue is equal to price times quantity. For each firm the revenue is then given by the above expressions since the total quantity of nuts or bolts sold by the firm is equal to the market quantity.

(b) Since the choice variable in this model is quantity, it is useful to write the above profit expressions in terms of quantities. This is done by noting that from the demand equations

$$Q_B = Z - P_B - P_N \Rightarrow P_B = Z - P_N - Q_B$$
$$Q_N = Z - P_N - P_B \Rightarrow P_N = Z - P_B - Q_N$$

Revenue is then given by

$$R_B = P_B Q_B = (Z - P_N - Q_B)Q_B = ZQ_B - P_N Q_B - Q_B^2$$
$$R_N = P_N q_N = (Z - P_B - Q_N)Q_N = ZQ_N - P_B Q_N - Q_N^2$$

Marginal revenue is given by

$$MR_B = Z - P_N - 2Q_B$$
$$MR_N = Z - P_B - 2Q_N$$

Setting these equal to marginal cost (=0) and solving gives

$$MR_B = Z - P_N - 2Q_B = 0$$
$$\Rightarrow Q_B = \frac{Z - P_N}{2}$$
$$MR_N = Z - P_B - 2Q_N = 0$$
$$\Rightarrow Q_N = \frac{Z - P_B}{2}$$

Given the levels of quantity we can get the level of price from the price equations.

$$P_B = Z - P_N - \left(\frac{Z - P_N}{2}\right) = \frac{Z - P_N}{2}$$
$$P_N = Z - P_B - \left(\frac{Z - P_B}{2}\right) = \frac{Z - P_B}{2}$$

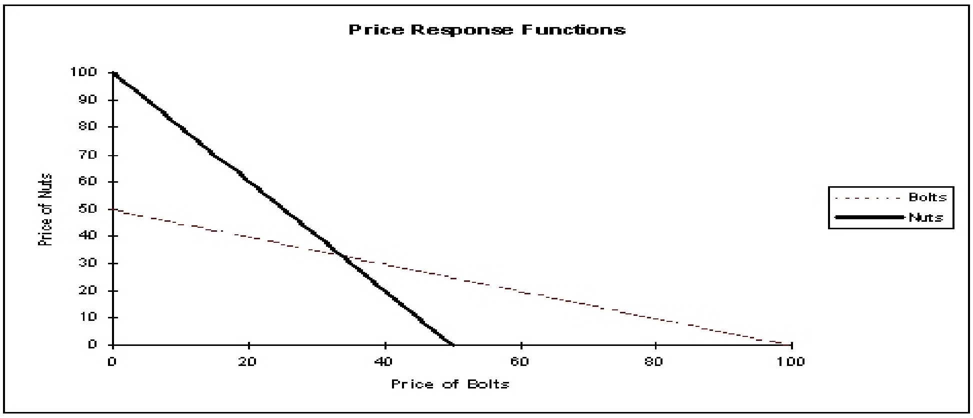
(c) Find the Nash equilibrium prices by solving the two equations simultaneously as follows

$$P_B = \frac{Z - P_N}{2} = \frac{Z - \left(\frac{Z - P_B}{2}\right)}{2} \Rightarrow P_B = \frac{1}{4}Z + \frac{1}{4}P_B \Rightarrow P_B = \frac{1}{3}Z$$

Similarly for P_N we obtain

$$P_N = \frac{Z - P_B}{2} = \frac{Z - \left(\frac{Z}{3}\right)}{2}$$
$$\rightarrow P_B = \frac{1}{2}Z - \frac{1}{6}Z$$
$$\Rightarrow P_B = \frac{1}{3}Z$$

The graphs look like this for Z = 100



(d) While this could be viewed as a coordination problem for two firms as in this problem, it could also be viewed as a joint product problem for a multiproduct monopolist. If the monopolist were to take into account the joint nature of the purchasing decision and sell "nut and bolt pairs", a higher level of production of both goods would occur. This will result in a lower price than if the firm (or two monopolists) did not coordinate the production and sales. When two monopolists sell complementary goods in separate markets, the Nash equilibrium prices for the two goods are higher than what the two monopolists would charge if they coordinated their pricing. Coordination or cooperation leads in this case to lower prices! This is because the goods are complementary so that the best response functions are downward sloping as is clear from the figure in part c.

Problem 7

(a) Without loss of generality, suppose the cost is zero, then profit for each firm is given by

$$\pi_1 = p_1 q_1 = (15 - p_1 + 0.5 p_2) p_1$$

$$\pi_2 = p_2 q_2 = (15 - p_2 + 0.5 p_1) p_2$$

Each firm chooses its price to maximize its profit

$$\frac{\partial \pi_1}{\partial p_1} = 15 - 2p_1 + 0.5 p_2 = 0 \Rightarrow p_1 = \frac{15 + 0.5 p_2}{2}$$

$$\frac{\partial \pi_2}{\partial p_2} = 15 - 2p_2 + 0.5 p_1 = 0 \Rightarrow p_2 = \frac{15 + 0.5 p_1}{2}$$

They are the best response functions. Prices are strategic complementary.

(b) From the best response functions, derive the equilibrium set of prices

$$p_1^* = \frac{1}{2}\left(15 + 0.5\left(\frac{15 + 0.5 p_1^*}{2}\right)\right) \Rightarrow p_1^* = 10$$

$$p_2^* = \frac{1}{2}(15 + 0.5 \times 10) \Rightarrow p_2^* = 10$$

The equilibrium set of prices in this market is 10 for each firm. Profits earned at these prices are

$$\pi_1^* = p_1^* q_1^* = (15 - p_1^* + 0.5 p_2^*) p_1^* = (15 - 10 + 0.5 \times 10) \times 10 = 100$$

$$\pi_2^* = p_2^* q_2^* = (15 - p_2^* + 0.5 p_1^*) p_2^* = (15 - 10 + 0.5 \times 10) \times 10 = 100$$

Chapter 11: Dynamic Games and First and Second Movers

Problem 1

(a) Firm 2 chooses its quantity to maximize

$$\pi_2 = Q_2(1000 - 4Q_1 - 4Q_2) - 20Q_2$$

$$\frac{\partial \pi_2}{\partial Q_2} = 1000 - 4Q_1 - 8Q_2 - 20 = 0 \Rightarrow Q_2 = \frac{1}{8}(980 - 4Q_1)$$

Now, Firm 1 chooses its quantity to maximize

$$\pi_1 = Q_1(1000 - 4Q_1 - 4Q_2) - 20Q_1 = Q_1\left(980 - 4Q_1 - \frac{1}{2}(980 - 4Q_1)\right) = \frac{1}{2}Q_1(980 - 4Q_1)$$

$$\frac{\partial \pi_1}{\partial Q_1} = 980 - 8Q_1 = 0 \Rightarrow Q_1 = \frac{980}{8} = 122.5 \Rightarrow Q_2 = 61.25$$

(b) There is no non-negative c such that the leader and the follower have the same market share. To see, consider $c = 0$. Then the leader's quantity is 120, whereas the follower's quantity is less than 120. As c increases, the market share of the leader goes up and the market share of the follower goes down.

Problem 2

Let p_1 be the price charged by Ben and p_2 be the price charged by Will. Let x be the location of a consumer who is indifferent between buying from Ben and Will.
Therefore,

$$p_1 + x = p_2 + (10 - x) \Rightarrow x = \frac{1}{2}(p_2 - p_1 + 10)$$

Consequently, the demand faced by Ben is

$$D_1(p_1, p_2) = \left(\frac{1000}{10}\right)\left(\frac{1}{2}\right)(p_2 - p_1 + 10) = 500 + 50(p_2 - p_1)$$

The demand faced by Will is

$$D_2(p_1, p_2) = \left(\frac{1000}{10}\right)\left[10 - \left(\frac{1}{2}\right)(p_2 - p_1 + 10)\right] = 500 - 50(p_2 - p_1)$$

Hence, Ben's profit is given by

$$\pi_1(p_1, p_2) = (p_1 - 1)(500 + 50(p_2 - p_1))$$

Will's profit is given by

$$\pi_2(p_1, p_2) = (p_2 - 1)(500 - 50(p_2 - p_1))$$

Since Will is the follower, we first maximize π_2 with respect to p_2, to derive Will's reaction function.

$$\frac{\partial \pi_2(p_1, p_2)}{\partial p_2} = (p_2 - 1)(-50) + (500 - 50(p_2 - p_1)) = 0$$

$$\Rightarrow p_2 = \frac{1}{100}[550 + 50 p_1] = \frac{1}{2}[11 + p_1]$$

Now, substitute Will's reaction function in to Ben's profit function to get

$$\pi_1(p_1, p_2) = (p_1 - 1)\left(500 + 50\left(\frac{11}{2} + \frac{p_1}{2} - p_1\right)\right) = (p_1 - 1)(775 - 25 p_1)$$

We now maximize π_1 with respect to p_1,

$$\frac{\partial \pi_1(p_1, p_2)}{\partial p_1} = (p_1 - 1)(-25) + (775 - 25 p_1) = 0$$

$$p_1 = \frac{800}{50} = 16$$

Now, from Will's reaction function, get

$$\Rightarrow p_2 = \frac{1}{2}[11 + p_1] = 13.5$$

Problem 2 (b)

$$p_2 - p_1 = 13.5 - 16 = -2.5$$

Hence, Ben will serve

$$D_1(p_1, p_2) = 500 + 50(p_2 - p_1) = 500 - 50\left(\frac{5}{2}\right) = 375$$

Will serves

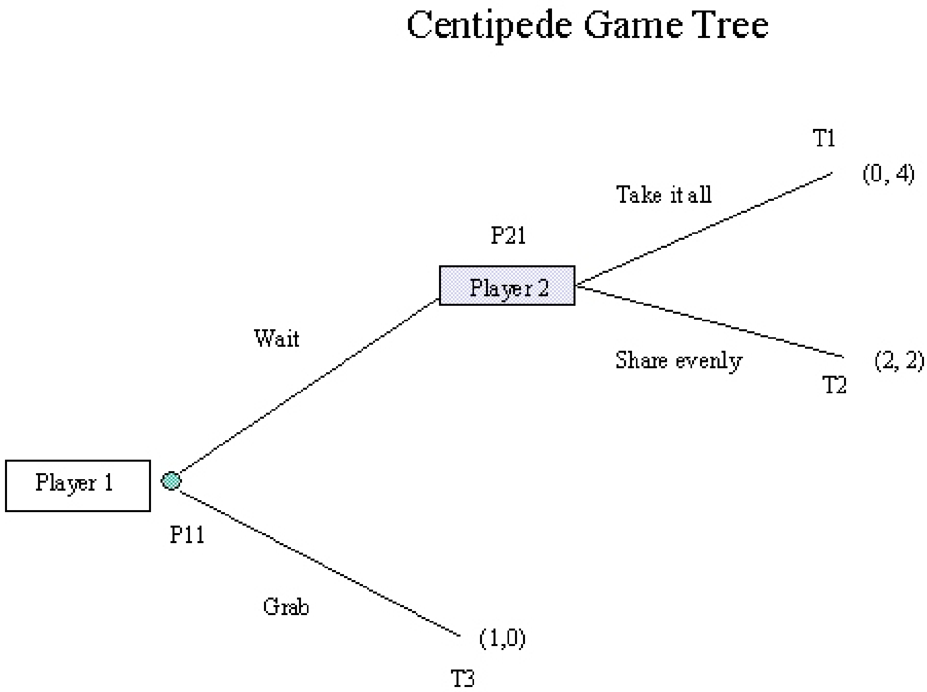
$$D_2(p_1, p_2) = 500 - 50(p_2 - p_1) = 500 + 50\left(\frac{5}{2}\right) = 625$$

Ben's profit = *375 (16 - 1) – 250 = 5375*

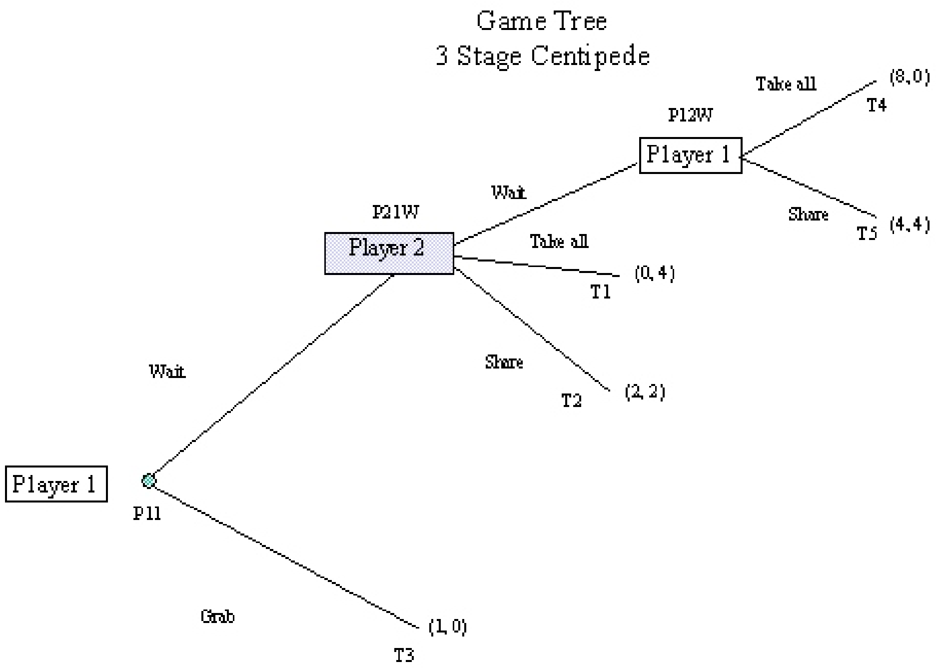
Will's profit = *625 (13.5 – 1) – 250 = 7562.5*

Problem 3

(a)

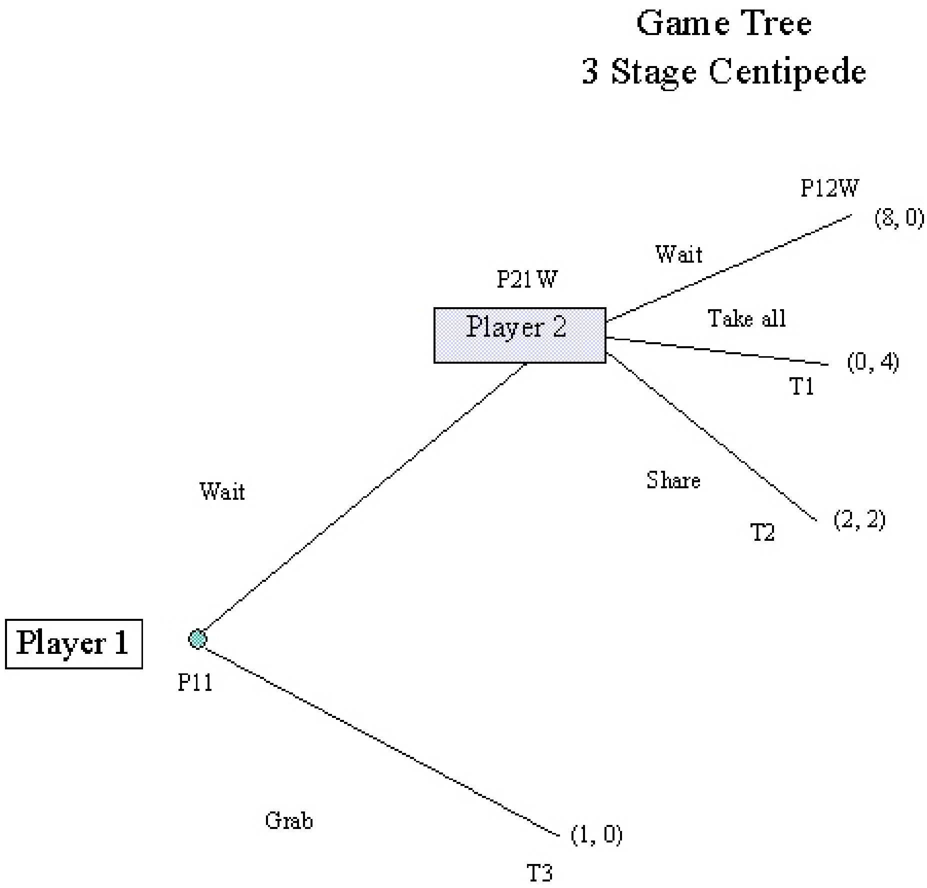


(b) The strategy of splitting the money is never an equilibrium since once the game reaches the point P21, the optimal strategy for the Player 2 is to take the entire $4. Because Player 1 knows this will be the outcome at P2, Player 1 will always choose "Grab" and the outcome will be T3 with Player 1 getting $1 and Player 2 getting nothing.



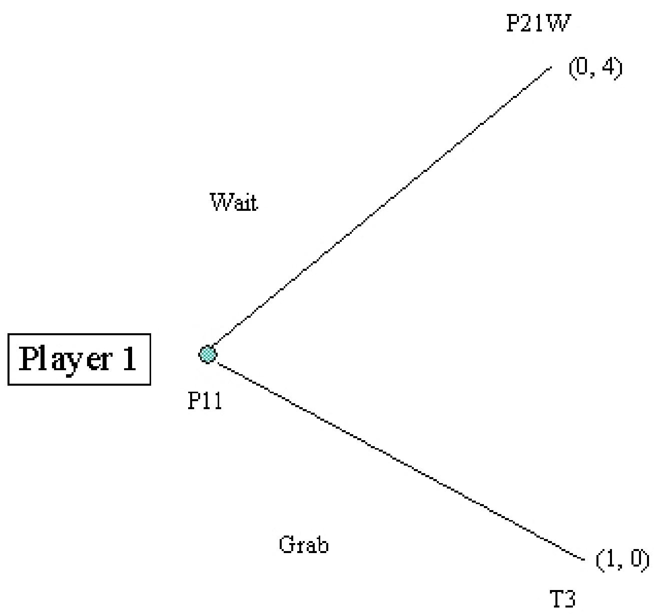
(c)

It is clear that in the third stage Player 1 will choose to keep all the money. Thus we can eliminate this choice from the tree and consider the new game with the final node removed (pruned). This will give



It is now obvious that Player 2 will choose to take it all at node P21W. Thus we can eliminate this node from the tree and replace it with the payoffs to both players when Player 2 chooses to take it all. This will give

Game Tree
3 Stage Centipede



It is now clear that Player 1 will grab the money at the initial node and the final payoff will be (1,0).

Problem 4

		Northern Springs			
		3	4	5	6
	3	(24, 24)	(30, 25)	(36, 20)	(42, 12)
Southern Pelligrino	4	(25, 30)	(32, 32)	(41, 30)	(48, 24)
	5	(20, 36)	(30, 41)	(40, 40)	(50, 36)
	6	(12, 42)	(24, 48)	(36, 50)	(48, 48)

In this case we can use Southern Pelligrino's best response function to find its optimal choice. Denote the best response of Southern Pelligrino as SP(__ NP's choice).

SP(__ 3) = 4 since 25 is the highest first element is column 1.
SP(__ 4) = 4 since 32 is the highest first element is column 2.
SP(__ 5) = 4 since 41 is the highest first element is column 3.
SP(__ 6) = 5 since 50 is the highest first element is column 4.

Now consider the best response function of Northern Springs

NS(3 __) = 4 since 25 is the highest second element is row 1.
NS(4 __) = 4 since 32 is the highest second element is row 2.
NS(5 __) = 4 since 41 is the highest second element is row 3.
NS(6 __) = 5 since 50 is the highest second element is row 4.

The Nash equilibrium is of course where NS(4 __) = 4 and SP(__ 4) = 4 and is the point (4,4). But if Northern Springs must go first and realizes that Southern Pelligrino will go second then Northern Springs has the payoff function defined by the best response function of Southern Pelligrino. The payoffs to Northern Springs are as follows

Payoff_{NS} (NP(3 __)) = Payoff_{NS} (SP(__ 3)) = Payoff_{NS} when SP chooses 4 which is 30.

Payoff_{NS} (NP(4 __)) = Payoff_{NS} (SP(__ 4)) = Payoff_{NS} when SP chooses 4 which is 32.

Payoff_{NS} (NP(5 __)) = Payoff_{NS} (SP(__ 4)) = Payoff_{NS} when SP chooses 4 which is 30.

Payoff_{NS} (NP(6 __)) = Payoff_{NS} (SP(__ 5)) = Payoff_{NS} when SP chooses 5 which is 36.

The equilibrium is now the point (5,6) where NS gets to choose the six first. Both firms are better off in this game because once NP chooses 6 and cannot deviate, the best choice for SP is to choose 5. If NP could now switch it would and go to 4, but then SP would switch and go to 4 and we would be back at the Cournot equilibrium.

(c) It is not an advantage for NP to move first. In a pricing game, the first mover is a sitting target for the firm that moves second. Both do better than in the simultaneous move game, but the second mover does best.

Problem 5

(a)

		Firm 2	
		C	Nothing
Firm 1	A	(8, 8)	(20, 8)
	B	(-3, -3)	(11, 0)
	A, B	(2, -2)	(18, 0)
	Nothing	(0, 10)	(0, 0)

There is a unique Nash equilibrium, where Firm 1 chooses A and Firm 2 chooses C.

(b) Note that A is a dominant strategy for Firm 1. Therefore, even if Firm 1 can commit before Firm 2, the answer does not change.

Problem 6

Find three examples of different ways individual firms or industries can make the strategy "This offer is good for a limited time only" a credible strategy.

i. Make the price applicable to stock on hand when there is a clear time lag in ordering additional stock or the items are one of a kind so that there can be no additional sales.

ii. Announce the price on a "special" purchase where the items are not the items normally stocked and there is a limited supply.

iii. Develop a reputation over time.

Problem 7

(a) Implied inversed demand is: $P = 3{,}000 - 0.08Q$. With $MC = 0$, implied monopoly outcome for Gizmo is: P = \$1,500; Q = 18,750; and Profit = \$28,125,000. We interpret the assumption that the metric can supply half the market to mean that it competes as a symmetric duopolist in quantities. In this case, the post-entry equilibrium is: P = \$1,000; Q = 25,000. Each firm produces q_i = 12,500 units and earns an operating profit of \$12,500,000. If the cost of entry is just \$10,000,000, then entry is profitable.

(b) Gizmo's profit falls from \$28,125,000 to \$12,500,00 or by \$15,625,000. If spending \$5 million could deter entry and prevent the \$15,625,000 loss it would surely be worth it. However, it is not clear that buying additional capacity achieves this result. The firm's current capacity of 25,000 is already more than it needs.

Chapter 12: Limit Pricing and Entry Deterrence

Problem 1

(a) Setting marginal revenue equal to marginal cost will yield

$$MR = 50 - 0.2\, q_I = 0.05 q_I = MC$$
$$\rightarrow 0.25 q_I = 50$$
$$\rightarrow q_I = 200$$
$$\Rightarrow P = 50 - 0.1 q_I$$
$$= 50 - 20$$
$$= 30$$

The firm will have profits equal to

$$\pi_I = (30)(200) - (0.025)(200)^2$$
$$= 6{,}000 - 1{,}000 = 5{,}000$$

(b) The industry demand curve can be written as

$$P = 50 - 0.1\, Q$$
$$= 50 - 0.1 q_I - 0.1 q_E$$
$$= 50 - (0.1)(200) - 0.1 q_E$$
$$= 30 - 0.1 q_E$$

Marginal revenue for the entrant firm will be

$$MR_E = 30 - 0.2 q_E$$

Setting marginal revenue equal to marginal cost we obtain

$$MR_E = 30 - 0.2 q_E = 10 + 0.05 q_E = MC_E$$
$$\Rightarrow 0.25 q_E = 20$$
$$\rightarrow q_E = 80$$
$$\rightarrow P = 50 - (0.1)(200) - (0.1)(80)$$
$$= 50 - 20 - 8$$
$$= 22$$

The entrant will export 80 units to the market and price will fall from \$30 to \$22. The total quantity transacted will rise from 200 to 280. Profits for the two firms will be

$$\pi_I = (22)(200) - (0.025)(200)^2$$
$$= 4{,}400 - 1{,}000 = 3{,}400$$
$$\pi_E = (22)(80) - (10)(80) - (0.025)(80)^2$$
$$= 1{,}760 - 800 - 160 = 800$$

(c) We simply need to find the level of $q_1 = Q$ such that the best response of the entrant is to produce zero output. Writing the residual demand curve as a function of q_I we obtain

$$P = 50 - 0.1\, Q$$
$$= 50 - 0.1 q_I - 0.1 q_E$$

Marginal revenue for the entrant firm will be

$$MR_E = 50 - 0.1 q_I - 0.2 q_E$$

Setting marginal revenue equal to marginal cost we obtain

$$MR_E = 50 - 0.1q_I - 0.2q_E = 10 + 0.05q_E = MC_E$$
$$\Rightarrow 0.25q_E = 40 - 0.1q_I$$
$$\Rightarrow q_E = 160 - 0.4q_I$$

If the incumbent chooses q_I such that the optimal $q_E = 0$, the entrant will not enter. This implies

$$q_E = 160 - 0.4q_I = 0$$
$$\rightarrow 0.4q_I = 160$$
$$\rightarrow q_I = 400$$
$$\Rightarrow \underline{Q} = 400$$

With this level of output price and profits for the two firms are

$$\begin{aligned} P &= 50 - 0.1q_I - 0.1q_E \\ &= 50 - (0.1)(400) - (0.1)(0) \\ &= 50 - 40 \\ &= 10 \\ \pi_I &= (10)(400) - (0.025)(400)^2 \\ &= 4{,}000 - 4{,}000 = 0 \\ \pi_E &= (10)(0) - (10)(0) - (0.025)(0)^2 \\ &= 0 \end{aligned}$$

If the incumbent were to produce not 400 units, but instead 350 units, then the optimal response of the entrant would be to produce 20 units. This is clear from the response equation

$$\begin{aligned} q_E &= 160 - 0.4q_I \\ &= 160 - 0.4(350) = 160 - 140 \\ &= 20 \\ \Rightarrow P &= 50 - (0.1)(350) - (0.1)(20) \\ &= 13 \\ \pi_I &= (13)(350) - (0.025)(350)^2 \\ &= 4{,}550 - 3{,}062.5 = 1{,}487.5 \\ \pi_E &= (13)(20) - (10)(20) - (0.025)(20)^2 \\ &= 260 - 200 - 10 \\ &= 50 \end{aligned}$$

Problem 2

Now consider a two-firm Cournot model with different cost functions for each firm. The solution is obtained by choosing q_i to maximize profit given the rival's output. For firm 1:

$$\begin{aligned} \pi_I &= [Pq_I - C(q_I)] \\ &= [(50 - 0.1q_I - 0.1q_E)q_I - 0.025q_I^2] \\ &= [50q_I - 0.1q_I^2 - 0.1q_Iq_E - 0.025q_I^2] \end{aligned}$$

$$\Rightarrow \frac{d\pi_I}{dq_I} = 50 - 0.2q_I - 0.1q_E - 0.05q_I = 0$$
$$\rightarrow 0.25q_I = 50 - 0.1q_E$$
$$\Rightarrow q_I^* = 200 - 0.4q_E$$

Similarly, the best response function for the second firm is given by

$$\begin{aligned}
\pi_E &= [Pq_E - C(q_E)] \\
&= [(50 - 0.1q_I - 0.1q_E)q_E - 10q_E - 0.025q_E^2] \\
&= [50q_E - 0.1q_Iq_E - 0.1q_E^2 - 10q_E - 0.025q_E^2] \\
\rightarrow \frac{d\pi_E}{dq_E} &= 50 - 0.1q_I - 0.2q_E - 10 - 0.05q_E = 0 \\
\Rightarrow 0.25q_E &= 40 - 0.1q_I \\
\rightarrow q_E^* &= 160 - 0.4q_I
\end{aligned} \tag{74}$$

We solve for the optimal q_i* simultaneously as follows

$$\begin{aligned}
q_E &= 160 - 0.4q_I \\
&= 160 - 0.4(200 - 0.4q_E) \\
&= 160 - 80 + 0.16q_E \\
&= 80 + 0.16q_E \\
\Rightarrow 0.84q_E &= 80 \\
\Rightarrow q_E &= 95.238
\end{aligned}$$

$$\begin{aligned}
q_I &= 200 - 0.4q_E \\
&= 200 - 0.4(95.238) \\
&= 200 - 38.095 \\
&= 161.90476
\end{aligned}$$

Price is given by

$$\begin{aligned}
P &= 50 - 0.1\ Q \\
&= 50 - 0.1q_I - 0.1q_E \\
&= 50 - (0.1)(161.90476) - (0.1)(95.23809) \\
&= 24.285715
\end{aligned}$$

Profits are given by

$$\begin{aligned}
\pi_I &= (24.2857)(161.9047) - (0.025)(161.9047)^2 \\
&= 3,931.973 - 655.329 = 3,276.644 \\
\pi_E &= (24.2857)(95.23809) - (10)(95.23809) - (0.025)(95.23809)^2 \\
&= 2,312.925 - 952.381 - 226.757 \\
&= 1,133.787
\end{aligned}$$

The incumbent earns a profit less than if he maintains the monopoly output and the entrant produces 80 units. However, $q_I = 200$ is not optimal if the entrant produces 80 units as

$$\begin{aligned}
q_I &= 200 - 0.4q_E \\
&= 200 - (0.4)(80) \\
&= 168
\end{aligned}$$

which of course is not 200 so the threat is not credible.

Problems 3 and 4

Similar to Practice Problem 12.2.

Problem 5

Firm 1 enters and chooses a small size. Firm 2 enters afterwards and chooses a small size as well.

Problem 6

(a) Write the market demand curve in inverse form as follows

$$q = \sum_{j=1}^{1,000} q_j = 70,000 - 2,000P$$

$$\Rightarrow 2,000P = 70,000 - \sum_{j=1}^{1,000} q_j$$

$$\Rightarrow P = 35 - (0.0005)\sum_{j=1}^{1,000} q_j$$

Now consider marginal cost for the first firm and set it equal to price (firms are price takers)

$$MC(q_1) = q_1 + 5 = 35 - (0.0005)q_1 - (0.0005)\sum_{j\neq 1}^{1,000} q_j$$

$$\Rightarrow 1.0005q_1 = 30 - (0.0005)\sum_{j\neq 1}^{1,000} q_j$$

$$\Rightarrow q_1 = 29.985007 - (0.00049975012)\sum_{j\neq 1}^{1,000} q_j$$

Since the firms are all the same we can substitute for q_j with q_1 to obtain

$$q_1 = 29.985007 - (0.00049975012)\sum_{j\neq 1}^{1,000} q_j$$

$$\Rightarrow q_1 = 29.985007 - (0.00049975012)\sum_{j\neq 1}^{1,000} q_1$$

$$\Rightarrow q_1 = 29.985007 - (0.00049975012)(999)q_1$$

$$q_1 = 29.985007 - 0.4992503q_1$$

$$\Rightarrow 1.49975012q_1 = 29.985007$$

$$\Rightarrow q_1 = 20$$

This implies that q = 20,000 and p = MC = 25.

We can also find this by horizontally adding the marginal cost functions and then setting supply equal to demand as follows

$$MC(q_i) = q_i + 5$$

$$\Rightarrow q_i = MC(q_i) - 5$$

$$\Rightarrow 1,000q_i = q = 1,000MC(q_i) - 5,000$$

$$\Rightarrow 1,000MC(q_i) = q + 5,000$$

$$\Rightarrow MC(q_i) = 0.001q + 5$$

Setting this equal to price from above we obtain

$$MC(q_i) = 0.001q + 5 = 35 - 0.0005q = P$$
$$\Rightarrow 0.0015q = 30$$
$$\rightarrow q = 20{,}000$$
$$\Rightarrow q_i = 20$$

We can also write the marginal cost relation in quantity dependent form (the normal supply curve) and then set supply equal to demand as

$$P = 0.001q + 5$$
$$\Rightarrow q = 1{,}000P - 5{,}000$$

$$q = 1{,}000P - 5{,}000 = 70{,}000 - 2{,}000P = q$$
$$\Rightarrow 3{,}000P = 75{,}000$$
$$\Rightarrow P = 25$$
$$\Rightarrow q = 20{,}000$$

(b) Denote supply by the small sellers as q_F and demand for the product of the BIG firm as q_B, with total demand given by q_T. The residual supply curve for the small sellers is given by

$$q_F = 1{,}000P - 5{,}000$$

Then we have as residual demand for the BIG firm

$$q_T = q_B + q_F = 70{,}000 - 2{,}000P$$
$$\Rightarrow q_B = 70{,}000 - q_F - 2{,}000P$$
$$= 70{,}000 - (1{,}000P - 5{,}000) - 2{,}000P$$
$$= 75{,}000 - 3{,}000P.$$

Residual inverse demand is found by inverting the residual demand function as follows

$$q_B = 75{,}000 - 3{,}000P$$
$$\Rightarrow 3{,}000P = 75{,}000 - q_B$$
$$\Rightarrow P = 25 - \frac{1}{3{,}000}q_B$$

(c) Profit for BIG is given by

$$\pi_B = Pq_B - 15q_B$$
$$= (25 - \frac{1}{3{,}000}q_B)q_B - 15q_B$$
$$= 25q_B - \frac{1}{3{,}000}q_B^2 - 15q_B$$
$$= 10q_B - \frac{1}{3{,}000}q_B^2$$

$$\frac{d\pi_B}{dq_B} = 10 - \frac{2}{3{,}000}q_B = 0$$
$$\Rightarrow \frac{2}{3{,}000}q_B = 10$$
$$\Rightarrow q_B = 15{,}000$$
$$\Rightarrow P = 25 - \frac{1}{3{,}000}(15{,}000)$$
$$= 25 - 5$$
$$= 20$$

The residual firms then supply

$$\begin{aligned} q_F &= 1{,}000P - 5{,}000 \\ &= (1{,}000)(20) - 5{,}000 \\ &= 15{,}000 \end{aligned}$$

Total quantity supplied is then q_T = 30,000. This comes from adding q_B and q_F or by plugging price in the original as opposed to the residual demand curve

$$\begin{aligned} q_T &= q_B + \sum_{j=1}^{1,000} q_j = 70{,}000 - 2{,}000(20) \\ &= 70{,}000 - 40{,}000 \\ &= 30{,}000 \end{aligned}$$

7. Throughout, we assume that $0 < r < 1$. It is easy to see that $t_1 = t_2 = 1/2$ is a Nash equilibrium. Suppose that $t_2 = 1/2$. If firm 1 choose a time $t_1 = 1/2$, its profit is simply $\pi^1 = e^{0.5(1-r)}$. If instead, it chooses $t_1 < 1/2 - U$, then its profit is $\pi^1 = e^{0.5(1-r)-(1-r)U} < e^{0.5(1-r)}$. Similarly, if it chooses $t_1 = 1/2 + U$ its profit is $\pi^1 = e^{0.5(1-r)-rU} < e^{0.5(1-r)}$. Thus, $t_1 = 1/2$ is a best response to $t_2 = 1/2$. Since the problem is symmetric, $t_1 = t_2 = 1/2$, is a pair of best responses and, hence, a Nash Equilibrium. To see that it is a unique Nash Equilibrium, first suppose t_2 is an arbitrarily small amount ε from 0. If firm 1 matches firm 2, it will then earn: $\pi^1 = e^{0.5-r\varepsilon}$. However, if it waits a short time U longer, it will then earn: $\pi^1 = e^{(1-\varepsilon)-rU}$, which is greater than $\pi^1 = e^{0.5-r\varepsilon}$ when $t_2 = \varepsilon < 1/2$. Likewise, for any value $t_2 > ½$, firm 1 always does better by going a little bit faster, i.e., by setting $t_1 < t_2$.

Chapter 14: Price Fixing and Repeated Games

Problem 1

(a) $Q_1 = Q_2 = 40 \Rightarrow P = 260 - 2(80) = 100$

$\pi_1^{Cournot} = \pi_2^{Cournot} = (100 - 20)(40) = 3200$

(b) $Q^{Monopoly} = \dfrac{260 - 20}{2(2)} = 60 \Rightarrow P^{Monopoly} = 260 - 2(60) = 140$

Therefore, profit of each firm in a cartel is

$\pi_1^{Cartel} = \pi_2^{Cartel} = (140 - 20)(30) = 3600$

Problem 2

Without loss of generality, suppose Firm 2 cheats, but Firm 1 maintains its cartel quantity of 30. Then, the optimal choice for Firm 2 can be found from its best response function.

$Q_2^{Cheating} = \dfrac{1}{4}\left(260 - 20 - 2(30)\right) = 45$

Therefore, the market price is 260 – 2 (30+45) = 110. As a result, the profit of the cheating firm is: $\pi_2^{Cheating} = (110 - 20)(45) = 4050$

Problem 3

If Firm 2 cheats, then it earns 4050 for one period, but earns its Cournot profit; 3200, for all periods afterwards. On the other hand, if Firm 2 does not cheat, it can continue earning its cartel profit for ever. Hence, the collusive outcome can be sustained if

$$3600 + \delta(3600) + \delta^2(3600) + ... \geq 4050 + \delta(3200) + \delta^2(3200) \Rightarrow \frac{3600}{1-\delta} \geq 4050 + \frac{3200\delta}{1-\delta}$$

$\Rightarrow \delta \geq 0.53$, where δ is the probability adjusted discount factor.

Problem 4

(a) With Bertrand price competition $P_1 = P_2 = 20 \Rightarrow Q_1 = Q_2 = 60, \pi_1 = \pi_2 = 0$

(b) $Q^{Monopoly} = \dfrac{260 - 20}{2(2)} = 60 \Rightarrow P^{Monopoly} = 260 - 2(60) = 140$

Therefore, profit of each firm in a cartel is

$\pi_1^{Cartel} = \pi_2^{Cartel} = (140 - 20)(30) = 3600$

Problem 5

Without loss of generality, let Firm 1 charges \$140, but Firm 2 cheat. Firm 2 needs to undercut Firm 1 only slightly to capture almost the entire monopoly profit. At the limit, Firm 2 captures the entire monopoly profit by cheating. Therefore, $\pi_2^{Cheating} = 7200$.

Problem 6

If Firm 2 cheats, then it earns 7200 for one period, but earns its Bertrand profit; 0, for all periods afterwards. On the other hand, if Firm 2 does not cheat, it can continue earning its cartel profit for ever. Hence, the collusive outcome can be sustained if

$$3600+\delta(3600)+\delta^2(3600)+...\geq 7200+\delta(0)+\delta^2(0)\Rightarrow \frac{3600}{1-\delta}\geq 7200$$

$\Rightarrow \delta \geq \frac{1}{2}$, where δ is the probability adjusted discount factor.

Problem 7

Comparing the discount factors, it can be seen that it is more difficult to sustain a cartel under Cournot competition, since it requires a larger discount factor.

Problem 8

(a) Recall that for Cournot model with *n* identical firms, with marginal cost *c*, demand intercept *a* and slope *-b*

$$Q_1 = Q_2 = ... = Q_n = \frac{(a-c)}{(n+1)b}, \pi_1 = \pi_2 = ... = \pi_n = \frac{(a-c)^2}{(n+1)^2 b}$$

Therefore, $Q_1 = Q_2 = ... = Q_4 = 24, \pi_1 = \pi_2 = ... = \pi_n = 1152, P = 68$

(b) $Q^{Monopoly} = \frac{260-20}{2(2)} = 60 \Rightarrow P^{Monopoly} = 260 - 2(60) = 140$

Therefore, $Q_1 = Q_2 = ... = Q_4 = 15$ and the profit of each firm in a cartel is

$$\pi_1^{Cartel} = \pi_2^{Cartel} = ... = \pi_4^{Cartel} = (140-20)(15) = 1800$$

Problem 9

(a) Without loss of generality, suppose Firm 4 cheats, but all other firms maintain their cartel quantities. Then, the optimal choice for Firm 4 can be found from its best response function.

$$Q_4^{Cheating} = \frac{1}{4}\left(260-20-2(15+15+15)\right) = 37.5$$

Therefore, the market price is 260 – 2 (37.5+45) = 95. As a result, the profit of the cheating firm is: $\pi_2^{Cheating} = (95-20)(37.5) = 2812.5$

Problem 10

If Firm 2 cheats, then it earns 2812.5 for one period, but earns its Cournot profit; 1152, for all periods afterwards. On the other hand, if Firm 2 does not cheat, it can continue earning its cartel profit for ever. Hence, the collusive outcome can be sustained if

$$1800+\delta(1800)+\delta^2(1800)+...\geq 2812.5+\delta(1152)+\delta^2(1152)\Rightarrow \frac{1800}{1-\delta}\geq 2812.5+\frac{1152\delta}{1-\delta}$$

$\Rightarrow \delta \geq 0.61$, where δ is the probability adjusted discount factor.

Problem 11

Comparing the discount factors, it can be seen that it is more difficult to sustain a cartel under Cournot competition, when there are more firms.

Problem 12

(a) Weighted average of the marginal cost is

$$\bar{c} = (.32)(.7)+(.32)(.7)+(.14)(.8)+(.14)(.8)+(.04)(.85)+(.04)(.85) = .74$$

The value of the Herfindahl Index is

$H = 2(.32)^2 + 2(.14)^2 + 2(0.04)^2 = 0.2472$

Therefore,

$$\frac{P^* - 0.74}{P^*} = \frac{0.2472}{1.55} = 0.16 \Rightarrow (1 - 0.16)P^* = 0.74 \Rightarrow P^* = 0.88$$

We now need to find out what would have been the total sales under Cournot equilibrium. For simplicity, assume that total sales under a Cournot equilibrium is Q^*. Then ADM's profit under a Cournot equilibrium = $(0.88 - 0.70)(0.32)Q^* = (0..0576)Q^*$

Assume a constant elasticity of demand so that η = 1.55 at all output levels. The cartel price of $1.12, reflects a 27% increase in price over the (imperfectly) competitive level. Since η = 1.55, the monopoly output of 100,000 tons should reflect a 1.55 x 27, or a 42 percent decrease in volume. In other words, the Cournot output would have been about172 thousand tons. Hence, ADM's profit would have been: (0.0576/pound)*(172 thousand tons)*(2200 pounds per ton)or roughly $21.8 million.

(b) ADM's annual profit under the cartel

$$= (1.12 - 0.70)(0.32)(2200)(100{,}000) = (0.42)(0.32)(2200)(100.000)$$
$$= (42)(32)(22)(1000) = (29568)(1000) = \$29.568 \text{ million.}$$

Problem 12

The probability-adjusted discount factor is ρ = 0.5/1.16 = 0.43. This must satisfy the condition in equation (14.7). In turn, this implies that $20.20 million ≥ $0.57*\pi^D$ or, ADM's profit from cheating on the cartel for one period exceeded $35.44 million..

Chapter 20: Advertising, Market Power, and Information

Problem 1

The information given provides point estimates for the demand elasticity and the advertising elasticity. Using the Dorfman-Steiner condition (equation 10.10) and the targeted level of sales one can find the optimal level of advertising. The Dorfman-Steiner equation is

Advertising Expenditure / Sales Revenue = $\frac{\eta_S}{\eta_P} = \frac{0.5}{2.0} = \frac{1}{4}$. Therefore, advertising expenditure = (sales revenue) / 4 = 5,000,000. So the firm should commit 5 million dollars to advertising.

Problem 2

(a) The demand elasticity is given by

$$\eta_P = -\frac{\partial Q}{\partial P}\frac{P}{Q} = \frac{1}{2}P^{-\frac{3}{2}}S^{\frac{1}{4}}\frac{P}{Q} = \frac{1}{2}$$

The advertising elasticity is given by

$$\eta_S = -\frac{\partial Q}{\partial S}\frac{S}{Q} = \frac{1}{2}P^{-\frac{1}{2}}S^{\frac{3}{4}}\frac{S}{Q} = \frac{1}{4}$$

The advertising-to-sales ratio is given by

$$\frac{\eta_S}{\eta_P} = \frac{0.25}{0.5} = \frac{1}{2}$$

(b) The answer is no. The data is in terms of expenditure. As the cost of advertising goes up, the expenditure rises but the data is in terms of expenditure. Hence, cost as a percent of sales revenue doesn’t matter.

Problem 3

(a) We get inverse demand by solving Q(P,S) as a function of P. We can then get revenue and marginal revenue in the usual manner.

$$MR_Q = 11.6 - 0.002Q + 0.02S^{\frac{1}{2}}$$

$$MR_S = 0.01S^{-\frac{1}{2}}Q$$

(b) Now set the two equations for marginal revenue equal to the respective marginal costs.

$$MR_Q = 11.6 - 0.002Q + 0.02S^{\frac{1}{2}} = 0.002Q + 4 = MC_Q$$

$$MR_S = 0.01S^{-\frac{1}{2}}Q = 1 = MC_S$$

Now solve the first equation for Q as a function of S and plug into the second equation. This will give $S = 400$, $Q = 2{,}000$

The price is given by substituting Q and S in the inverse demand function. This will give $P = 10$

(c) Profit is given by substituting the optimal levels of P, Q and S in the expression for profit. With $P = 10$, $S = 400$, and $Q = 2{,}000$, it is clear that *Profit* = 7600.

(d) We compute consumer surplus by finding the area of the triangle between the vertical intercept and the market price and the inverse demand function. The vertical intercept is 11.6. The price is 10 and the quantity is 2,000. Verify that $CS = 1600$.

Problem 4

To do this assume that S = 0 in the above model. Price and marginal revenue with respect to Q are given by $P = 11.6 - 0.001Q \Rightarrow MR_Q = 11.6 - 0.002Q$ Setting marginal revenue equals marginal cost, we obtain Q = 1900. Therefore, P = 9.7.

Profits are given by substituting the optimal levels of P and Q in the expression for profit. It is straightforward to show then that: *Profit* = 7220. *Consumer surplus* = 1805

Problem 5

The text postulates that the advertising to sales ratio will be high to convenience goods that are relatively inexpensive and frequently purchased. The text also hypothesized that the advertising-to-sales ratio for "shopping goods" that are infrequently purchased and expensive will be low since consumers will check other sources for information. Consider then each company in the above table.

Philip Morris Tobacco, beer, and food are purchased almost daily. Someone seeing a television ad or hearing a radio spot just prior to running to the store may well make an impulsive purchase of the product.

Johnson and Johnson and American Home Products These products are also purchased on a regular basis. Particularly for non-prescription drugs, consumer information is not often unclear and for some ailments, it is not obvious that any of the remedies really help. There is also quite a bit of brand competition in this area (Tums, Rolaids, Mylanta, Axid, etc.). Firms will attempt to differentiate their product by creating specific market niches (Robitussin DM, CF, etc.). These are also experience goods in the sense that consumers may keep going back once they try them and like them.

Proctor and Gamble This is in the middle as far as the benefits of advertising. A product such as soap is highly differentiated and a small part of the budget so that advertising for this convenience good makes a lot of sense. But things like paper products and some food items may see little increase in sales due to advertising. Thus, the expenditures are not as large as for beer or tobacco.

General Motors Cars and truck are a classic example of a "shopping good" where advertising is only a small part of the decision information.

Kodak There is strong brand competition in photo supplies between Kodak, Fuji, and house brands. Consumers may be able to be swayed by effective advertising, particularly since the quality of a given roll of pictures is highly variable depending on the firm, the lighting, the skill of the person shooting the pictures, etc. The consumer may not have good data on what works well for them and advertising has a good chance of success.

Pepsico This is an intermediate case like Proctor and Gamble because of the wide variety of products produced. The bottled or canned soft drinks (especially main items like Pepsi-Cola, Mountain Dew, and Sprite) are in fierce price competition and so sales will be very sensitive to advertising. But fountain drinks and other products (like Coke's Minutemaid) may be much less sensitive to advertising. In some ways one might think Pepsico would have larger advertising expenditures.

Sears, Roebuck & Co. Many of Sears' products are things like washers, dryers, and refrigerators that clearly fit the "shopping good" category. The same would be generally true to computers and lawn mowers but these will have some advertising sensitivity. Things like clothes (the softer side of Sears) would be more in the convenience good category.

Problem 6

(a) Assume that marginal cost = 0. Faced with a room full of N potential customers, drawn at random, the firm will wish to set a price that maximizes expected revenue as this is the same as maximizing profit when c = 0. Let F(p) be the cumulative distribution function of p. Hence,

$F'(p) = f(p)$ is the probability density. Expected revenue at any price p is given by: $p[1 - F(p)]N$. Maximizing this with respect to p implies:

$$1 - F(p) - pF'(p) = 0$$

Since the distribution of p is uniform between 0 and 1, $F(p) = p$, while $F'(p) = f(p) = 1$. Hence, profit maximization requires: $1 - p - p = 0$ or $2p = 1 \Rightarrow p = \$0.50$.

(b) The typical consumer will expect a price of \$0.50 when they arrive in the store. With probability 0.5, the style will not be one that they sufficiently like and they will not buy the good. However, the remaining half of the time, the typical consumer will find that she likes the product enough that she enjoys some surplus from it despite the fact that it costs \$0.50. Since her valuation of the product in these cases runs from \$0.50 to \$1, her average valuation will be \$0.75 and her average surplus in the cases in which she buys the product will be \$0.75 - \$0.50 = \$0.25. However, this surplus is realized only half the time. So, the expected surplus is 0.5 x \$0.25 = \$0.125. This is just enough of an expected surplus to induce the typical consumer to sink the \$0.125 transport cost necessary to visit the store. If the search cost were higher, the market could collapse in the absence of any credible way for the firm to commit to a price less than \$0.50 when faced with a random group of consumers in its shop. If the firm is free to set any price it wants once consumers are in the store, \$0.50 is its optimum choice. At that point, the search cost is sunk and consumers will buy or not depending on their valuation of the particular style the store has. However, if the search cost is say, \$0.15, rational consumers will foresee this outcome. Realizing that once in the store they face a "hold-up" problem in which the store can charge a high price since the transport cost is at that point sunk, they will reckon that their expected surplus net of transport cost is negative and not visit the shop.

Problem 7

If the store owner can identify potential patrons with a valuation of her style that is less than \$0.50 then, conditional on this fact, she knows that in any group of randomly selected customers now coming to the store, the conditional distribution is uniform between \$0.50 and \$1. Profit maximization again requires that $1 - F(p) - pF'(p) = 0$. Here, $F(p) = (p - 0.50)/0.50$; and $F'(p) = f(p) = 1/0.5 = 2$. Hence, profit maximization requires:

$$1 - \frac{p - 0.50}{0.50} - \frac{p}{0.5} = 0 \Rightarrow 0.50 - p + 0.50 - p = 0 \Rightarrow 1 = 2p \Rightarrow p = \$0.50$$

The profit-maximizing price remains the same at $p = \$0.50$. However, the average valuation in the group of store visitors is \$0.75. That is, those who now visit the store in response to an advertisement now know with certainty that they have an average value of the style in stock equal to \$0.75 Hence, the transport cost can now be as high as \$0.25 without deterring these consumers from visiting the store.