p 1.12 n°1

Het is mooi weer $\Rightarrow$ we gaan wandelen

ontkenning : Het is morgen mooi weer en toch gaan we niet wandelen

p 1.12 n°2

a) niet eq
b) eq
c) niet eq
d) niet eq
e) eq
f) eq
g) niet eq

p 1.13 n°3

$\forall k \in K, \exists v \in V$: k is het kind van v

$\exists v \in V, \forall k \in K$: k is het kind van v

Er is een moeder die de moeder is voor alle kinderen

p 1.13 n°4

a) $\forall p \in P, \exists d \in D$: p past op d
b) $\exists d \in D, \forall p \in P$: d past op p
c) $\forall s \in S$: (s studeert goed) $\Rightarrow$ (s zal slagen)
d) $\forall e \in E, \exists s \in S$: s slaagt voor e
e) $\exists e \in E, \forall s \in S$: s slaagt voor e
f) $\exists s \in S, \forall e \in E$: s slaagt voor e

p 1.13 n°5

a) $\exists p \in P, \forall d \in D$: p past niet op d
Er is een potje waarop geen enkel deksel past

b) $\forall d \in D, \exists p \in P$: d past niet op p
Voor elk deksel is er een potje waarop het deksel niet past

c) $\exists s \in S$: (s studeert goed) $\Rightarrow$ (s zal niet slagen)
Er is iemand die goed studeert en toch niet zal slagen

d) $\exists e \in E, \forall s \in S$: s slaagt niet voor e
Er is minstens één examen waarvoor alle studenten niet slagen.

e) $\forall e \in E, \exists s \in S$: s slaagt niet voor e
Voor alle examens is er een student die niet slaagt.

f) $\forall s \in S: \exists e \in S$: s slaagt niet voor e
Alle studenten slagen niet voor een examen.

p 1.13 n°6

a) Als het kwadraat van 2 reële getallen gelijk is, zijn ook die twee getallen gelijk.

b) $\exists x \in \mathbb{R}, \exists y \in \mathbb{R}$: $x^2 = y^2$ en $x \neq y$
Er bestaan reële getallen die verschillend zijn maar waarvan het kwadraat toch gelijk is

c) de ontkenning!
vb. $(-6)^2 = 6^2 = 36$
maar $-6 \neq 6$

p 2.20 n°1

A is eindig $\quad$ vb $\{1, 2, 3, 4, 5\} \to$ ook begrensd

$\Downarrow \quad \not\Uparrow$

A is begrensd $\quad$ vb $[1, 2] \to$ nt eindig want elementen $\in \mathbb{R}$

p 2.20 n°2

A is begrensd als er een $m, M \in \mathbb{R}$ bestaan zo dat $m \leq x \leq M$ voor alle $x \in A$. Aangezien $\mathbb{R} = \max\{|m|, |M|\}$:

$$-|m| \leq m \leq x \leq M \leq |M|$$

$$\Leftrightarrow -\mathbb{R} \leq -|m| \leq m \leq x \leq M \leq |M| \leq \mathbb{R}$$

$$\Leftrightarrow x \leq \mathbb{R}$$

p 2.20 n°3

a) Bewijs $\forall a, b, c, d \in \mathbb{R} : (a \leq b \text{ en } c \leq d) \Rightarrow (a + c \leq b + d)$

Eigenschap $\forall x, y, z \in \mathbb{R} : x \leq y \Rightarrow x + z \leq y + z$

als $a \leq b$ dan is $a + c \leq b + c$

als $c \leq d$ dan is $c + b \leq d + b$

$$\Rightarrow a + c \leq b + c \leq c + b \leq d + b$$

$$\Leftrightarrow a + c \leq d + b = b + d$$

b) $(a \leq b \text{ en } c \leq d) \Rightarrow (a - c \leq b - d)$? $\to$ Nee!

tegenvoorbeeld: $\quad 1 \leq 2$ en $-9 \leq 7$

$1 - (-9) \leq 2 - (-7)$

$10 \leq 9 \to$ klopt niet!

p 2.20 n°4

a) $-3 \leq 1$ en $-4 \leq 2$

$-3(-4) \leq 1\cdot 2 \Leftrightarrow 14 \leq 2 \rightarrow$ klopt niet !

$\forall\, a, b, c, d \in \mathbb{R}^+ \cup \forall\, a, b, c, d \in \mathbb{R}^- : (a \leq b \text{ en } c \leq d) \Rightarrow ac \leq bd$

b) $-3 \leq -2$ en $0 \leq 4 \leq 10$

$-3 \cdot 4 \leq -2 \cdot 10 \Leftrightarrow -12 \leq -20 \rightarrow$ klopt niet !

$\forall\, a, b, (c, d) \in \mathbb{R}^+ : (a \leq b \text{ en } 0 \leq c \leq d) \Rightarrow ac \leq bd$

c) $-3 \leq -2 \Rightarrow -1/3 \geq -1/2 \rightarrow$ klopt niet !

$\forall\, a, b \in \mathbb{R}^+_0 : (a \leq b) \Rightarrow 1/a \geq 1/b$

d) $-3 \leq -2 \Rightarrow (-3)^2 \leq 2^2 \Rightarrow 9 \leq 4 \rightarrow$ klopt niet !

$\forall\, a, b \in \mathbb{R}^+ : (a \leq b) \Rightarrow a^2 \leq b^2$

e) $a \leq b$ als en slechts als $\sqrt{a} \leq \sqrt{b}$

$\sqrt{a}$ kan $-a$ of $+a$ zijn !

p 2.21 n°5

a) $|x - a| < \varepsilon \Leftrightarrow a - \varepsilon < x < a + \varepsilon$

$x - a < \varepsilon$ $\qquad$ $-x + a < \varepsilon$

$x < \varepsilon + a$ $\qquad$ $x > a - \varepsilon$

$\Rightarrow a - \varepsilon < x < a + \varepsilon$

b) i) $|x - 3| \leq 5$

$-5 \leq x - 3 \leq 5$

$-2 \leq x \leq 8$

ii) $|x + 2| \geq 5$

$x + 2 \geq 5$ $\qquad$ $-x - 2 \geq 5$

$x \geq 3$ $\qquad$ $-x \geq 7$

$x \leq -7$

$]-\infty, -7] \cup [3, +\infty[$

iii) $|9 - x^2| > 9$

$*\ 9 - x^2 > 9$

$-x^2 > 0$

$*\ -9 + x^2 > 9$

$x^2 > 18$

$x > -\sqrt{18}$ of $\sqrt{18}$

$]-\infty, -\sqrt{18}[\cup]18, +\infty[$

p 2.21 n°6

a) Geg $|a| = \begin{cases} a \text{ als } a \geq 0 \\ -a \text{ als } a \leq 0 \end{cases}$

TB $-|a| \leq a \leq |a|$

Bewijs * Geval 1 : $a \geq 0$

$-a \leq a \leq a$

* Geval 2 : $a < 0$

$-a > a > a$

$|a| = -a$

$-|a| < a < |a|$

$\rightarrow -1 \leq a \leq 2$

indien $-2 \leq a \leq 0$ dan $0 \leq |a| \leq 2$

indien $0 \leq a \leq 2$ dan $0 \leq |a| \leq 2$

b) Geg $-\mathbb{R} < a \leq \mathbb{R}$

TB $|a| \leq \mathbb{R}$

Bewijs

$a \nearrow + : -\mathbb{R} < a \leq \mathbb{R} \Leftrightarrow |a| \leq \mathbb{R}$

$a \searrow - : -\mathbb{R} \leq a \leq \mathbb{R} \Leftrightarrow -\mathbb{R} \leq |a| \leq \mathbb{R}$

$\Leftrightarrow \mathbb{R} \geq |a|$

$\Leftrightarrow |a| \leq \mathbb{R}$

c) TB max waarde van a of b is $\frac{1}{2}(a+b+|a-b|)$

Bewijs * Stel $\max\{a,b\} = a$ dus $a > b$

dan $\frac{1}{2}(a+b+\underbrace{|a-b|}_{>0 \text{ want } a>b}) = \frac{1}{2}(a+b+a-b) = \frac{1}{2}\,2a = a$

* Stel $\max\{a,b\} = b$

dan $\frac{1}{2}(a+b-a+b) = \frac{1}{2}\,2b = b$

* Stel $a = b$

dan $\frac{1}{2}(a+b) \Rightarrow \frac{1}{2}a = \frac{1}{2}b$

p2.21 n°7

Propositie: ① $\forall a, b \in \mathbb{R}$ is $|a+b| \leq |a|+|b|$

② $\forall x, y, z \in \mathbb{R}$ is $|x-y| \leq |x-z| - |z-y|$

Bewijs ① Definitie abs. waarde : $-a \leq |a| \leq a$

$-b \leq |b| \leq b$

$-|a| \leq a \leq |a|$

$-|b| \leq b \leq |b|$

$-(|a|+|b|) < a+b < |a|+|b|$

$-(|a|+|b|) < |a|+|b|$ en $a+b < |a|+|b|$

Zowel $-(a+b)$ als $a+b < |a|+|b|$

$\Rightarrow |a+b| < |a|+|b|$

② Als $|a+b| \leq |a|+|b|$ dan is $|x-y| \leq |x-z| + |z-y|$

Neem $|a| = |x-z|$ en $b = |z-y|$

Dan is $|a+b| = |x-z+z-y| = |x-y|$

p2.21 n°8

a) $|ab| = |a||b| \rightarrow$ waar

b) $|a-b| \leq |a|+|b| \rightarrow$ waar

c) $|a-b| \leq |a|-|b| \rightarrow$ vals

tegenvoorbeeld $a = -5$ en $b = 3$

$|-5-3| = 8$ en $|-5| - |3| = 2$ $\quad 8 \leq 2 \rightarrow$ klopt niet!

p2.21 n°9

$$\left(\begin{array}{l} |x+5| \leq 2 \text{ en } |y+5| \leq 6 \\ \max |x-y| ? \\ \quad |(x-5)-(y+5)| \leq |x+5| + |y+5| \leq 2+6 \rightarrow \max = 8 \end{array} \right)$$

* $|x+5| \leq 2 \Leftrightarrow -2 \leq x+5 \leq 2$

$\Leftrightarrow -2-5 \leq x \leq 2-5$

$\Leftrightarrow -7 \leq x \leq -3$

* $|y+5| \geqslant 6 \Leftrightarrow -6 \leq y+5 \leq 6$

$-6-5 \leq y \leq 6-5$

$-11 \leq y \leq 1$

$\Rightarrow \max |x-y| = |-11-3| = 8$

bereikt bij $x = -3$ en $y = -11$

p 2.21 n° 10

$x \in [-3, 1]$
$y \in [-4, 2]$
$z \in [-3, -1]$
$\}$ geen enkele combinatie $\geqslant 7$:

$|-3-(-3)| = 0$

$|1-(-1)| = 2$

$|1-(-3)| = 4$

$|-3-(-1)| = 2$

$\Rightarrow$ Nee!

$|x-z| = |x-y| + |y-z|$

$\left.\begin{array}{l} |x-z| \geqslant 7 \\ |x-z| \leq 1 \end{array}\right\} \Rightarrow |x-y| \geqslant 6$

maar $|x-y|$ bereikt geen hogere waarde dan 5 voor $x = -3$ en $y = 2$

$\Rightarrow$ er bestaan geen x, y, z die hieraan voldoen!

p 2.21 n° 11

$[2, +\infty[$ is gesloten omdat het complement $\mathbb{R} \setminus [2, +\infty[$ open is

p 2.21 n° 12

$$\sum_{k=1}^{n} a_k b_k \neq \left(\sum_{k=1}^{n} a_k\right)\left(\sum_{k=1}^{n} b_k\right)$$

want RL : $(a_1 b_1) + (a_1 b_2) + (a_1 b_3) \dots + (a_2 b_1) \dots$

$\neq$ LL : $a_1 b_1 + a_2 b_2 + a_3 b_3 + \dots + a_n b_n$

p 2.22 n° 13

$$\sum_{i=1}^{n} \sum_{j=1}^{m} x_{ij} = \sum_{j=1}^{m} \sum_{i=1}^{n} x_{ij} \rightarrow \text{waar !}$$

LL : sommatie van $i = 1$ tot n van $x_{i,1}$
+ sommatie van $i = 1$ tot n van $x_{i,2}$ + ...
+ sommatie van $i = 1$ tot n van $x_{i,n}$
$= x_{1,1} + x_{2,1} + \dots + x_{n,1} + x_{1,2} + x_{2,2} + \dots + x_{n,2} + \dots + x_{n,m}$

RL . sommatie van $j = 1$ tot m van $x_{1,j}$
+ sommatie van $j = 1$ tot m van $x_{2,j}$ + ...
+ ... + sommatie van $j = 1$ tot m van $x_{n,j}$
$= x_{1,1} + x_{1,2} + \dots + x_{1,m} + x_{2,1} + x_{2,2} + \dots + x_{2,m} + \dots x_{n,m}$

LL = RL

p 2.22 n° 15

a) $1 + 2 + 3 + \dots + (n-1) + n = \frac{1}{2} n(n+1)$

Stap 1 : $1 \in S$, geldt voor $n = 1$?
$1 = \frac{1}{2} \cdot 1(1+1)$
$= 1/2 \cdot 2 = 1 \rightarrow$ ok!

Stap 2 : $m \in S$, dus $(m+1) \in S$

$$\underbrace{1 + 2 + 3 + \dots + (n-1) + n}_{m} + (n+1) = \frac{1}{2}(n+1)(n+1+1)$$

$\Leftrightarrow \frac{1}{2} n(n+1) + (n+1) = \frac{1}{2}(n^2 + 2n + n + 2)$
$\Leftrightarrow \frac{1}{2}(n^2 + n) + n + 1 = \frac{n^2}{2} + n + \frac{n}{2} + 1$
$\Leftrightarrow \frac{n^2}{2} + \frac{n}{2} + n + 1 = \frac{n^2}{2} + n + \frac{n}{2} + 1 \rightarrow$ ok!

b) $2^n \geq 1 + n$

Stap 1 : $1 \in S$, geldt voor $n = 1$?

$2^1 \geq 1 + 1 \Leftrightarrow 2 \geq 2 \rightarrow$ Ok!

Stap 2 : Als $m \in S$, geldt voor $m+1$?

$2^{n+1} \geq 1 + n + 1 \Leftrightarrow 2^n \cdot 2 \geq n + 2$

$\Leftrightarrow (n+1) \cdot 2 \geq n+2$

$\Leftrightarrow 2n + 2 \geq n + 2$

$\Leftrightarrow 2n \geq n \rightarrow$ Ok!

$2^m \geq 1 + m + 1$

$2 \cdot 2^m \geq 1 + m + 1$

$\geq 2(1+m)$

$\geq 2 + 2m$

$\geq 2 + m + m$

dus $2 + m + m \geq m + 2 = 1 + (m+1)$

c) $\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{n}{n+1}$

Stap 1 Stelling waar voor $n = 1$

zie volgend blad

c) $\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{n}{n+1}$

① Stelling waar voor $n = 1$

$$\sum_{k=1}^{1} \frac{1}{k(k+1)} = \frac{1}{2}$$

$$\frac{1}{1 \cdot 2} = \frac{1}{2} \Rightarrow \text{waar}$$

② Waar voor $n+1$?

$$TB: \sum_{k=1}^{n+1} \frac{1}{k(k+1)} = \frac{n+1}{n+2}$$

$$\text{Bewijs: } \sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{n(n+2)}{(n+1)(n+2)}$$

$$+ \frac{1}{(n+1)(n+2)} = \frac{1}{(n+1)(n+2)}$$

?

$$\sum_{k=1}^{n+1} \frac{1}{k(k+1)} = \frac{n(n+2)+1}{(n+1)(n+2)}$$

$$= \frac{n^2+2n+1}{(n+1)(n+2)}$$

$$= \frac{(n+1)^{\cancel{2}}}{(\cancel{n+1})(n+2)}$$

$$= \frac{n+1}{n+2} \Rightarrow \text{waar}$$

d) Als $r \in \mathbb{R}$ en $r \neq 1$, dan is $1 + r + r^2 + \ldots + r^n = \frac{1 - r^{n+1}}{1-r}$

① Stelling waar voor $n = 1$

$$1 + r^1 = \frac{1 - r^2}{1 - r}$$

$$1 + r = \frac{(\cancel{1-r})(1+r)}{\cancel{1-r}} \Rightarrow \text{waar}$$

$a^2 - b^2 = (a-b)(a+b)$

$1 - r^2 = (1-r)(1+r)$

② Stelling waar voor $n+1$?

TB: $1 + r + r^2 + \ldots + r^n = \dfrac{1 - r^{n+1}}{1-r}$

Bewijs: $1 + r + r^2 + \ldots + r^n = \dfrac{1 - r^{n+1}}{1-r}$

$$+ \quad r^{n+1} \qquad = r^{n+1}$$

$$1 + r + r^2 + \ldots + r^n + r^{n+1} = \frac{1 - r^{n+1} + r^{n+1} - r^{n+2}}{1-r}$$

?

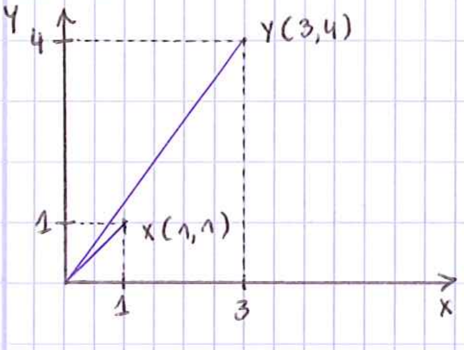
$$\frac{1-r^{n+1}}{1-r} + \frac{r^{n+1}(1-r)}{1-r}$$

$$= \frac{1 - r^{n+1} + r^{n+1} - r^{n+2}}{1-r}$$

$$= \frac{1 - r^{n+2}}{1-r}$$

$\Rightarrow$ waar

p2.31 n°1



* $\|\lambda x\| = 1$

$|\lambda| \cdot \|x\| = 1$ (Stelling ② p 2.26)

$\|x\| = \sqrt{x_1^2 + x_2^2} = \sqrt{1+1} = \sqrt{2}$

$|\lambda| \cdot \sqrt{2} = 1$

$|\lambda| = \frac{1}{\sqrt{2}}$ dus $\lambda = \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$

* $\|\mu y\| = 1$

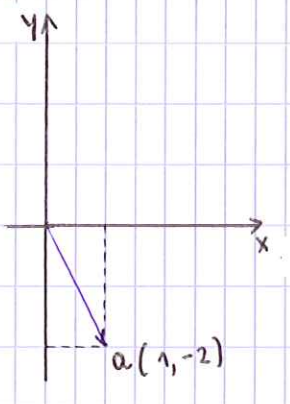
$|\mu| \cdot \|y\| = 1$

$\|y\| = \sqrt{y_1^2 + y_2^2} = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$

$|\mu| 5 = 1$

$|\mu| = 1/5$ dus $\mu = \frac{1}{5}, -\frac{1}{5}$

p2.31 n°2



scalair product van loodrechte vectoren = 0
└ orthogonaal

dus $\langle x, a \rangle = 0$

$x_1 a_1 + x_2 a_2 = 0$

$x_1 \cdot 1 + x_2(-2) = 0$

$x_1 - 2(x_2) = 0$

$? \begin{cases} x_1 = 2x_2 \\ x = (2\lambda; \lambda) \end{cases} \Rightarrow \lambda = 1 \Rightarrow 2$ en 1

vector x = 2 en 1

= -2 à -1

0-vector loodrecht op alle factoren !

p 2.31 n°4

Toon aan dat $\|x-y\|^2 = \|x+y\|^2 = \|x\|^2 + \|y\|^2$

$* \; \|x-y\|^2 = \|x\|^2 + \|y\|^2 - 2\|x\|\,\|y\|\cos\theta$ cosinusregel: $x \perp y$ dus $\theta = \pi/2$

$= \|x\|^2 + \|y\|^2$ $\cos \pi/2 = 0$

$* \; \|x+y\|^2 = \langle x+y, x+y \rangle$ $\|x+y\|^2 = \sqrt{(x_1+y_1)^2 + (x_2+y_2)^2}^2$

$= \langle x, x \rangle + \langle y + x \rangle + (x+y) + \langle y, y \rangle$

$= \|x\|^2 + \|y\|^2 + 2\langle x, y \rangle$

x en y zijn orthogonaal dus $x \perp y$

$2\langle x, y \rangle = 0$

$\|x+y\|^2 = \|x\|^2 + \|y\|^2$

p 2.31 n°5

voor alle $x, y \in \mathbb{R}^2$ geldt dat

$* \; \|x+y\|^2 + \|x-y\|^2 = 2\|x\|^2 + 2\|y\|^2$

$* \; \|x\|^2 + \|y\|^2 + \cancel{2\langle x, y \rangle} + \|x\|^2 + \|y\|^2 - \cancel{2\langle x, y \rangle} = 2\|x\|^2 + 2\|y\|^2$

$* \; 2\|x\|^2 + 2\|y\|^2 = 2\|x\|^2 + 2\|y\|^2$

of $(x_1+y_1)^2 + (x_2+y_2)^2 + (x_1-y_1)^2 + (x_2-y_2)^2$

$= 2(x_1^2 + x_2^2) + 2(y_1^2 + y_2^2)$

$\cancel{x_1^2} + \cancel{2x_1y_1} + \cancel{y_1^2} + \cancel{x_2^2} + \cancel{2x_2y_2} + \cancel{y_2^2} + \cancel{x_1^2} - \cancel{2x_1y_1} + \cancel{y_1^2} + x_2^2 - 2x_2y_2 + y_2^2$

$= 2(\cancel{x_1^2 + x_2^2}) + 2(\cancel{y_1^2 + y_2^2})$

ok!

p2.31 n°3

$\langle x, y \rangle = 0 \Leftrightarrow x_1y_1 + x_2y_2 + x_3y_3 = 0$

$$\begin{cases} x_1(1,1,0) = 0 \Leftrightarrow x_1 + x_2 = 0 \\ y_1(1,1,0) = 0 \Leftrightarrow y_1 + y_2 = 0 \end{cases}$$

$\Rightarrow x_1 + x_2 = 0 \Leftrightarrow x_1 = -x_2$

$y_1 + y_2 = 0 \Leftrightarrow y_1 = -y_2$

Bij elke keuze van x_1 en y_1 hebben x_2 en y_2 een verschillend teken

p2.31 n°6

$A = \{(x, y) \in \mathbb{R}^2 \mid |x| \leq 1 \text{ en } |y| < 1\}$

of $(x, y) \in \mathbb{R}^2$: $4 \leq x \leq 8$

en $2 < y < 4$

p 3.29 n°1

a) geen functie (verschillende y-waarden voor 1 x-waarde)
b) functie
c) functie
d) geen functie (zie a)

p 3.30 n°2 (zie eindoplossingen Toledo)

p 3.30 n°3

$f : \mathbb{R} \to \mathbb{R}$ = <u>even</u> als $f(-x) = f(x)$
<u>oneven</u> als $f(-x) = -f(x)$

De grafiek is even als ze symmetrisch tov de y-as is
oneven als ze puntsymmetrisch is tov $(0,0)$

p 3.30 n°4 (zie eindopl Toledo)

p 3.30 n°5

$g : A \to B : a \mapsto f(a)$ is <u>surjectief</u> asa voor elke $b \in B$ bestaat er een $a \in A$ zodat $g(a) = b$

$h : C \to D$: is <u>injectief</u> asa voor elke c_1 en c_2 in C geldt $h(c_1) \neq h(c_2)$

a) $\mathbb{R} \to \mathbb{R} : x \mapsto x^3$ = surjectief
= injectief

b) $\mathbb{R} \to \mathbb{R} : x \mapsto |x|$ = niet surjectief
= niet injectief

c) $\mathbb{R} \to \mathbb{R} : x \mapsto \sin^2 x$ = nt surjectief
= nt injectief

d) $\mathbb{R} \to \mathbb{R} : (x,y) \mapsto x^2 + y^2$ = nt surjectief
= nt injectief

e) $\mathbb{R}^2 \to \mathbb{R} : (x,y) \mapsto x^2 - y^2$
= surjectief
= nt injectief

p 3.31 n°6

a) $\mathbb{R} \to \mathbb{R} : x \mapsto 2x^3+5$ = injectief

$f^{-1} : \mathbb{R} \to \mathbb{R} \mapsto \sqrt[3]{\frac{x-5}{2}}$

b) $\mathbb{R} \to \mathbb{R} : x \mapsto 12x - 11$ = injectief

$f^{-1} = \mathbb{R} \to \mathbb{R} \mapsto \frac{x+11}{12}$

c) $\mathbb{R} \to \mathbb{R} : x \mapsto x^2 - 4x + 1$ = niet injectief

p 3.31 n°7

a) $g \circ h : \mathbb{R} \to \mathbb{R} : x \mapsto (x+1)^2$

$h \circ g : \mathbb{R} \to \mathbb{R} : x \mapsto x^2 + 1$

b) $g \circ h : \mathbb{R} \to \mathbb{R} : x \mapsto x+1$

$h \circ g : \mathbb{R} \to \mathbb{R} : x \mapsto x+1$

c) $g \circ h :]-\infty, -2] \cup [2, +\infty[\to \mathbb{R} : x \mapsto \sqrt{x^2-4}$

$h \circ g : \mathbb{R}^+ \to \mathbb{R} : x \mapsto x - 4$

d) $g \circ h : \mathbb{R} \setminus \{-2, 2\} \to \mathbb{R} : x \mapsto \frac{1}{x^2-4}$

$h \circ g : \mathbb{R} \setminus \{1\} \to \mathbb{R} : x \mapsto \left(\frac{1}{x-1}\right)^2 - 3$

p 3.31 n°8 (zie eindoplossingen Toledo)

p 3.31 n°9 (zie eindopl. Toledo)

p3.32 n° 10

a) Als f en g injectief zijn, dan is $f \circ g$ ook injectief

$f(g(x_1)) = f(g(x_2))$

dan is $g(x_1) = g(x_2)$ aangezien f injectief is

en $x_1 = x_2$ aangezien g injectief is

→ WAAR

b) Als $f \circ g$ injectief is, dan is g injectief.

$f \circ g$ = injectief dus $f \circ g(x_1) = f \circ g(x_2)$

$x_1 = x_2$ en dus g = injectief

→ WAAR

c) Als $f \circ g$ injectief is, dan is f injectief

stel $f : \mathbb{R} \to \mathbb{R} : x \mapsto x^2$

$g : \mathbb{R} \to \mathbb{R} : x \mapsto 2^x$

dan is $f \circ g : \mathbb{R} \to \mathbb{R} : x \mapsto 2^{2x}$ injectief maar f niet

→ VALS

p3.32 n° 11

$$f : \mathbb{R} \to \mathbb{R} : x \mapsto \frac{x}{1+x^2} + \ln(2+x^4)$$

stel $f_1 : \mathbb{R} \to \mathbb{R} : x \mapsto x$

$f_2 : \mathbb{R} \to \mathbb{R} : x \mapsto 1$

$f_3 : \mathbb{R} \to \mathbb{R} : x \mapsto x^2$

$f_4 : \mathbb{R}_0^+ \to \mathbb{R} : x \mapsto \ln x$

$f_5 : \mathbb{R} \to \mathbb{R} : x \mapsto 2$

dan is $f : \mathbb{R} \to \mathbb{R} : x \mapsto \frac{f_1}{f_2 + f_3} + f_4 \circ (f_5 + f_3 \circ f_3)$

p3.32 n°12

$f : \mathbb{R} \to \mathbb{R} : x \mapsto \frac{2(x+5)-6}{2} - 2 = \frac{2x+10-6}{2} - 2 = x + 2 - 2 = x$

p3.32 n°13

a) $c(t) = 0{,}5\,(10 + 0{,}1t^2) + 1 = 0{,}05t^2 + 6$

b) $6{,}8 = 0{,}05t^2 + 6$

$t^2 = 16$

$t = 4$

p3.32 n°14

$f : \mathbb{R} \to \mathbb{R} : t \mapsto \pi\,(r(t))^2$

p3.33 n°15

0 & 8 invullen in fü

a) Ja : $[3, 43] \to \mathbb{R} : n \mapsto \frac{n-3}{5}$

b) Neen : bij 8 zouden er plotseling geen deeltjes meer in de lucht zijn

p3.33 n°16

a) $3 \cdot 1^2(-1) - 2 \cdot 1(-1) + 1^2 - 6(-1) = 6$

$z(1,-1) = 6$

b) $\frac{1}{2} \cdot 1^{3/4} \cdot 16^{1/4} = 1$

$Q(16, 1) = 1$

c) $\frac{4 \cdot 3}{5+3} + 4 \cdot 3^2(-3) + \frac{2(-3)}{10-3} = \frac{12}{8} - 108 - \frac{6}{7} = -\frac{1503}{14}$

p 3.33 n° 17

$f_1 : \mathbb{R}^2 \to \mathbb{R} : (x, y) \mapsto x + y$
$f_2 : \mathbb{R}^2 \to \mathbb{R} : (x, y) \mapsto \sin x$
$f_3 : \mathbb{R}^2 \to \mathbb{R} : (x, y) \mapsto 4xy$

p 3.33 n° 18

$f : \mathbb{R}^2 \to \mathbb{R} : (x, y) \mapsto 5$

p 3.33 n° 20

Als bij een punt (x, y) een $f(x, y) = c_1$ en een $f(x, y) = c_2$ zou bestaan dan zou dit de definitie van een functie tegenspreken.

p 3.33 n° 21 (tekeningen : zie eindopl. Toledo)

$c = 2 : x + 2y + 1 = 2 \Leftrightarrow x + 2y = 1 \Leftrightarrow y = \frac{-x}{2} + \frac{1}{2}$

$c = 1 : x + 2y + 1 = 1 \Leftrightarrow y = -\frac{x}{2}$

$c = 0 : x + 2y + 1 = 0 \Leftrightarrow -\frac{x}{2} - \frac{1}{2}$

$c = -1 : x + 2y + 1 = -1 \Leftrightarrow y = -\frac{x}{2} - 1$

$c = -2 : x + 2y + 1 = -2 \Leftrightarrow y = -\frac{x}{2} - \frac{3}{2}$

p 3.34 n° 22 (zie eindopl. Toledo)

tot

p 3.34 n° 26 (zie eindopl. Toledo)

p3.34 n°26

$f : \mathbb{R}^2 \to \mathbb{R} : (x,y) \mapsto x^2y - 3xy$
$g : \mathbb{R} \to \mathbb{R}^2 : t \mapsto (t, t^2)$

$f \circ g : \mathbb{R} \to \mathbb{R} : t \mapsto t^4 - 3t^3$
$g \circ f : \mathbb{R}^2 \to \mathbb{R}^2 : (x,y) \mapsto ((x^2y - 3xy), (x^4y^2 - 6x^3y^2 + 9x^2y^2))$

p3.34 n°27

a) $(3, 3, 5, 9, 15, 23, \dots)$
b) $(2, 0, 2, 0, 2, 0, \dots)$

p3.35 n°28

$x_n = n^2 + 4n + 2$ = unieke formule

p3.35 n°29

a) $(5, 8, 11, 14, 17, 20, 23)$ $x_n = 3n + 5$
b) $(1, 1, 2, 6, 24, 120, 720)$ $x_n = n!$
c) $(1, 1, 2, 3, 5, 8, 13)$

p3.35 n°30

b) FOUT $\begin{cases} x_n = 1 \\ y_n = 2^n \end{cases}$ dan is $s_n = 1 + 2^n$ gn meetkundige rij

want stel $r \in \mathbb{R}_0^+$, $a \in \mathbb{R}$ zodat $x_n = ar^n$

Dan geldt $r = \frac{ar}{a} = \frac{x_1}{x_0} = \frac{1+2}{1+1} = \frac{3}{2}$

$a = x_0 = 2$ en $x_3 = 2\left(\frac{3}{2}\right)^2 = \frac{9}{2} \neq 1 + 2^3 = 9$

d) JUIST want $\begin{cases} x_n = ar^n \\ y_n = bs^n \end{cases}$ $p_n = x_n y_n = ab(rs)^n$

omdat $ab \in \mathbb{R}$, $rs \in \mathbb{R}^+$, is p_n = MR

p 3.35 n° 31

$\text{Slotwaarde} = w \cdot (1+i)^n \Rightarrow 10.000 = x \cdot 1{,}0575^5$

$x = \frac{10.000}{1{,}0575^5} = 7561{,}33$

p 3.35 n° 32

a) $x = 100.000 \cdot 1{,}07^{20} = 386.968{,}45$

b) $S = k \cdot \frac{u^n - 1}{u - 1} \Rightarrow 386.968{,}45 = k \cdot 40{,}995 \Rightarrow k = 9439{,}29$

p 3.36 n° 33

$A = k \cdot \frac{u^n - 1}{u - 1} \cdot \frac{1}{u^n} = 7000 \cdot \frac{1{,}07^{20} - 1}{0{,}07} \cdot \frac{1}{1{,}07^{20}} = 74.158{,}1$

p4.13 n°1

Dit wil zeggen dat de eerste voorwaarde voldaan is ($\lambda = 0$), maar de 2e moet nog bewezen worden. Dit is een nodige maar niet voldoende voorwaarde.

p4.13 n°2

a) $y = ax + b$ $\qquad$ $y = ax + b$

$1 = 2a + b$ $\qquad$ $-2 = 3a + b$

$$\underbrace{b = 1 - 2a \qquad -2 - 3a = b}$$

$-2 - 3a = 1 - 2a$

$-3a + 2a = 1 + 2$

$a = -3$ en $b = 1 - 2(-3) = 7$

dus $y = -3x + 7$

b) rico $= a$

$x \mapsto -3(x-1) + 2$ $\qquad$ $y = -3x + b \Leftrightarrow b = y + 3x$

$f: \mathbb{R} \to \mathbb{R}: x \mapsto -3x + 5$ $\qquad$ $b = 2 + 3 \cdot 1 = 5$

c) rico $= a$, hoogte $= b$

$f: \mathbb{R} \to \mathbb{R}: x \mapsto 2x - 8$

p4.13 n°3

$$f: \mathbb{R} \to \mathbb{R}: x \mapsto y_0 + \frac{y_1 - y_0}{x_1 - x_0}(x - x_0)$$

p4.13 n°4

$f : \mathbb{R}^2 \to \mathbb{R}^2 : (x,y) \mapsto ax + by + c$

$f(-1,0) = 2 \to a\,(x+1) + b(y-0) + 2$

invullen van $f(0,3) = -5 : a(0+1) + b(3-0) + 2 = -5$

$f(1,1) = 3 : a(1+1) + b(1-0) + 2 = 3$

stelsel: $\begin{cases} a + 3b + 2 = -5 \\ 2a + b + 2 = 3 \end{cases} \Leftrightarrow \begin{cases} a + 3b = -7 \\ 2a + b = 1 \end{cases} \quad \begin{cases} -5a = -10 \\ 2a + b = 1 \end{cases} \quad \begin{cases} a = 2 \\ b = 1 - 4 = -3 \end{cases}$

dus $2(x-1) - 3(y-0) + 2$

$= 2x + 2 - 3y + 2$

$= 2x - 3y + 4$

p4.13 n°5

a) $y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 98{,}5 = \dfrac{99{,}3 - 98{,}5}{740 - 720}(x - 720) \Leftrightarrow y = 0{,}04x - 28{,}8 + 98{,}5$

$y = 0{,}04x + 69{,}7$

$x = 735$ dus $y = 99{,}1\,°C$

b) $y - 700 = \dfrac{720 - 700}{98{,}5 - 97{,}7}(x - 97{,}7) \Leftrightarrow y = 25x - 2442{,}5 + 700$

$y = 25x - 1742{,}5$

$x = 98$ dus $y = 707{,}5$ mmHg

p 5.20 n°1

a) $$\left(\begin{array}{ccc|c} 1 & 1 & -2 & 10 \\ 4 & 2 & 2 & 1 \\ 6 & 2 & 3 & 4 \end{array}\right) \xrightarrow{R_3 \to R_3 - R_2} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 10 \\ 4 & 2 & 2 & 1 \\ 2 & 0 & 1 & 3 \end{array}\right) \xrightarrow{R_2 \to R_2 - 2R_3}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -2 & 10 \\ 0 & 2 & 0 & -5 \\ 2 & 0 & 1 & 3 \end{array}\right) \xrightarrow[R_2 \to \frac{1}{2} R_2]{R_3 \to R_3 - 2R_1} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 10 \\ 0 & 1 & 0 & -5/2 \\ 0 & -2 & 5 & -17 \end{array}\right) \xrightarrow[R_1 \to R_1 - R_2]{R_3 \to R_3 - 2R_2}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -2 & 25/2 \\ 0 & 1 & 0 & -5/2 \\ 0 & 0 & 5 & -22 \end{array}\right) \xrightarrow{R_3 \to \frac{1}{5} R_3} \left(\begin{array}{ccc|c} 1 & 0 & -2 & 25/2 \\ 0 & 1 & 0 & -5/2 \\ 0 & 0 & 1 & -22/5 \end{array}\right) \xrightarrow{R_1 \to R_1 + 2R_3}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 37/10 \\ 0 & 1 & 0 & -5/2 \\ 0 & 0 & 1 & -22/5 \end{array}\right) \quad \text{Opl: } (x,y,z) = \left(\frac{37}{10}, \frac{-5}{2}, \frac{-22}{5}\right)$$

b) $$\left(\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 3 & -1 & 2 & 7 \\ 5 & 3 & -4 & 2 \end{array}\right) \xrightarrow{R_3 \to R_3 - R_2 - 2R_1} \left(\begin{array}{ccc|c} 1 & 2 & -3 & 1 \\ 3 & -1 & 2 & 7 \\ 0 & 0 & 0 & 7 \end{array}\right) \quad \text{STRIJDIG}$$

c) $$\left(\begin{array}{cccc|c} 1 & 2 & -3 & 2 & 2 \\ 2 & 5 & -8 & 6 & 5 \\ 3 & 4 & -5 & 2 & 4 \end{array}\right) \xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to R_2 - 2R_1} \left(\begin{array}{cccc|c} 1 & 2 & -3 & 2 & 2 \\ 0 & 1 & -2 & 2 & 1 \\ 0 & -2 & 4 & -4 & -2 \end{array}\right) \xrightarrow{R_3 \to -\frac{1}{2} R_3}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -3 & 2 & 2 \\ 0 & 1 & -2 & 2 & 1 \\ 0 & 1 & -2 & 1 & 1 \end{array}\right) \xrightarrow{R_3 \to R_3 - R_2} \left(\begin{array}{cccc|c} 1 & 2 & -3 & 2 & 2 \\ 0 & 1 & -2 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{cases} x+2y-3z+2t=2 \\ y-2z+2t=1 \end{cases}$$ Stel $z=\lambda$ en $\mu=t$ met $\lambda,\mu \in \mathbb{R}$ willekeurig

$$\Leftrightarrow \begin{cases} x+2y-3\lambda+2\mu=2 \\ y-2\lambda+2\mu=1 \\ z=\lambda \\ t=\mu \end{cases} \Leftrightarrow \begin{cases} x+2(1+2\lambda-2\mu)-3\lambda+2\mu=2 \\ y=1+2\lambda-2\mu \\ z=\lambda \\ t=\mu \end{cases}$$

$$\Leftrightarrow \begin{cases} x+2+2\lambda-4\mu-3\lambda+2\mu=2 \\ y=1+2\lambda-2\mu \\ z=\lambda \\ \mu=t \end{cases}$$ Opl: $(-\lambda+2\mu,\ 1+2\lambda-2\mu,\ \lambda,\mu)$

d) $$\left(\begin{array}{ccc|c} 3 & 1 & 1 & 0 \\ 1 & 3 & 1 & 0 \\ 1 & 1 & 3 & 0 \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_3} \left(\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 1 & 3 & 1 & 0 \\ 3 & 1 & 1 & 0 \end{array}\right) \xrightarrow[R_2 \to R_2 - R_1]{R_3 \to R_3 - 3R_1} \left(\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & -2 & -8 & 0 \end{array}\right)$$

$$\xrightarrow[R_2 \to \frac{1}{2}R_2]{R_3 \to R_3 + R_2} \left(\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -10 & 0 \end{array}\right) \xrightarrow{R_3 \to \frac{1}{-10} R_3} \left(\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) R_2 \to R_2 + R_3$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) \xrightarrow{R_1 \to R_1 - R_2 - 3R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$$ opl: $(x,y,z)=(0,0,0)$

e) $\left(\begin{array}{ccc|c} 3 & -1 & 1 & 29 \\ 1 & 3 & 30 & 6 \\ 3 & -3 & 3 & 51 \end{array}\right) \xrightarrow{R_3 \to R_3 - R_1} \left(\begin{array}{ccc|c} 3 & -1 & 1 & 29 \\ 1 & 3 & 30 & 6 \\ 0 & -2 & 2 & 22 \end{array}\right) \xrightarrow{R_3 \to \frac{1}{2} R_3} \left(\begin{array}{ccc|c} 3 & -1 & 1 & 29 \\ 1 & 3 & 30 & 6 \\ 0 & -1 & 1 & 11 \end{array}\right)$

$\xrightarrow{R_1 \to R_1 - R_3} \left(\begin{array}{ccc|c} 3 & 0 & 0 & 18 \\ 1 & 3 & 30 & 6 \\ 0 & -1 & 1 & 11 \end{array}\right) \xrightarrow{R_1 \to \frac{1}{3} R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 1 & 3 & 30 & 6 \\ 0 & -1 & 1 & 11 \end{array}\right) \xrightarrow{R_2 \to R_2 - R_1}$

$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 3 & 30 & 0 \\ 0 & -1 & 1 & 11 \end{array}\right) \xrightarrow{R_2 \to \frac{1}{3} R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 10 & 0 \\ 0 & -1 & 1 & 11 \end{array}\right) \xrightarrow{R_3 \to R_3 + R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 10 & 0 \\ 0 & 0 & 11 & 11 \end{array}\right)$

$\xrightarrow{R_3 \to \frac{1}{11} R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 10 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right) \xrightarrow{R_2 \to R_2 - 10 R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -10 \\ 0 & 0 & 1 & 1 \end{array}\right)$

Opl : $(x, y, z) = (6, -10, 1)$

f) $\left(\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{array}\right)$ in R_3 : $x + u = 0$ dus $x = -u$

dit invullen in R_2 : $y + z - x = 0$

$x = y + z$

dit invullen in R_1 : $x + x = 1$

$x = \frac{1}{2}$ en $u = -\frac{1}{2}$

Stel $z = \lambda$, dan is $y = \frac{1}{2} - \lambda$

Opl : $(x, y, z, u) = (\frac{1}{2}, \frac{1}{2} - \lambda, \lambda, -\frac{1}{2})$ met $\lambda \in \mathbb{R}$

g) $\left(\begin{array}{ccc|c} 2 & 3 & 1 & 11 \\ 3 & 1 & 2 & 11 \\ 1 & 2 & 3 & 14 \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_3} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 14 \\ 3 & 1 & 2 & 11 \\ 2 & 3 & 1 & 11 \end{array}\right) \xrightarrow[R_3 \to R_3 - 2R_1]{R_2 \to R_2 - 3R_1} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 14 \\ 0 & -5 & -7 & -31 \\ 0 & -1 & -5 & -17 \end{array}\right)$

$\xrightarrow{R_2 \to R_2 - 5R_3} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 14 \\ 0 & 0 & 18 & 54 \\ 0 & -1 & -5 & -17 \end{array}\right) \xrightarrow{R_3 \leftrightarrow R_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 14 \\ 0 & -1 & -5 & -17 \\ 0 & 0 & 18 & 54 \end{array}\right) \xrightarrow{R_3 \to \frac{1}{18} R_3}$

$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 14 \\ 0 & -1 & -5 & -17 \\ 0 & 0 & 1 & 3 \end{array}\right) \xrightarrow{R_2 \to R_2 + 5R_3} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 14 \\ 0 & -1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right) \xrightarrow{R_2 \to -R_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 14 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right)$

$\xrightarrow{R_1 \to R_1 - 2R_1 - 3R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right)$ Opl: $(s, t, u) = (1, 2, 3)$

h) $\left(\begin{array}{ccc|c} 2 & 1 & -3 & 5 \\ 3 & -2 & 2 & 5 \\ 5 & -3 & -1 & 16 \end{array}\right) \xrightarrow[R_3 \to R_3 - 2R_1]{R_2 \to R_2 - R_1} \left(\begin{array}{ccc|c} 2 & 1 & -3 & 5 \\ 1 & -3 & 5 & 0 \\ 1 & -5 & 5 & 6 \end{array}\right) \xrightarrow{R_3 \to R_3 - R_2}$

$\left(\begin{array}{ccc|c} 2 & 1 & -3 & 5 \\ 1 & -3 & 5 & 0 \\ 0 & -2 & 0 & 6 \end{array}\right) \xrightarrow[R_1 \to R_2 - 2R_1]{R_3 \to -\frac{1}{2} R_3} \left(\begin{array}{ccc|c} 0 & 7 & -13 & 5 \\ 1 & -3 & 5 & 0 \\ 0 & 1 & 0 & 3 \end{array}\right) \xrightarrow{R_1 \to R_1 - 7R_3}$

$\left(\begin{array}{ccc|c} 0 & 0 & -13 & 26 \\ 1 & -3 & 5 & 0 \\ 0 & 1 & 0 & -3 \end{array}\right) \xrightarrow[R_2 \to R_2 + 3R_3 - 5R_1]{R_1 \to \frac{1}{-13} R_1} \left(\begin{array}{ccc|c} 0 & 0 & 1 & -2 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -3 \end{array}\right)$

Opl: $(x, y, z) = (1, -3, -2)$

j) $\left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 5 \\ 2 & -1 & 1 & 2 & 2 \\ 3 & 1 & 4 & 6 & 7 \end{array}\right) \xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to R_2 - 2R_1} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & -5 & -5 & -6 & -8 \\ 0 & -5 & -5 & -6 & -8 \end{pmatrix} \xrightarrow{R_3 \to R_2 + R_3}$

$\left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 5 \\ 0 & -5 & -5 & -6 & -8 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$ $\begin{cases} r + 2s + 3t + 4u = 5 \\ -5s - 5t - 6u = -8 \end{cases}$

Stel $t = \lambda$

$u = \mu$ met $\lambda, \mu \in \mathbb{R}$

Opl: $(r, s, t, u) = \left\{\left(-\lambda - \frac{8}{5}\mu + \frac{9}{5}, -\lambda - \frac{6}{5}\mu + \frac{8}{5}, \lambda, \mu\right) \mid \lambda, \mu \in \mathbb{R}\right\}$

k) $\left(\begin{array}{ccc|c} 1 & 2 & 2 & 2 \\ 3 & -2 & -1 & 5 \\ 2 & -5 & 3 & -4 \\ 1 & 4 & 6 & 0 \end{array}\right) \xrightarrow[R_4 \to R_4 - R_1]{\substack{R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 2R_1}} \left(\begin{array}{ccc|c} 1 & 2 & 2 & 2 \\ 0 & -8 & -7 & -1 \\ 0 & -9 & -1 & -8 \\ 0 & 2 & 4 & -2 \end{array}\right) \xrightarrow{R_2 \to R_2 - R_3}$

$\left(\begin{array}{ccc|c} 1 & 2 & 2 & 2 \\ 0 & 1 & -6 & 7 \\ 0 & -9 & -1 & -8 \\ 0 & 2 & 4 & -2 \end{array}\right) \xrightarrow{R_4 \to R_4 - 2R_2} \left(\begin{array}{ccc|c} 1 & 2 & 2 & 2 \\ 0 & 1 & -6 & 7 \\ 0 & -9 & -1 & -8 \\ 0 & 0 & 16 & -16 \end{array}\right) \xrightarrow{R_4 \to \frac{1}{16} R_4}$

$\left(\begin{array}{ccc|c} 1 & 2 & 2 & 2 \\ 0 & 1 & -6 & 7 \\ 0 & -9 & -1 & -8 \\ 0 & 0 & 1 & -1 \end{array}\right) \xrightarrow{R_3 \to R_3 + R_4} \left(\begin{array}{ccc|c} 1 & 2 & 2 & 2 \\ 0 & 1 & -6 & 7 \\ 0 & -9 & 0 & -9 \\ 0 & 0 & 1 & -1 \end{array}\right) \xrightarrow[R_1 - 2R_3 - 2R_4]{R_3 \to \frac{-1}{9} R_3}$

$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & -6 & 7 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & -1 \end{array}\right)$ Opl: $(x, y, z) = (2, 1, -1)$

l) $\left(\begin{array}{ccc|c} 5 & 2 & -3 & 1 \\ 2 & 1 & 4 & 5 \end{array}\right) \xrightarrow{R_1 \to R_1 - 2R_2} \left(\begin{array}{ccc|c} 1 & 0 & -5 & -9 \\ 2 & 1 & 4 & 5 \end{array}\right) \xrightarrow{R_2 \to R_2 - 2R_1}$

$\left(\begin{array}{ccc|c} 1 & 0 & -5 & -9 \\ 0 & 1 & 14 & 23 \end{array}\right)$ $\begin{cases} x - 5z = -9 \\ y + 14z = 23 \end{cases}$ Stel $z = \lambda \in \mathbb{R}$

opl $(x, y, z) = (11\lambda - 9, -26\lambda + 23, \lambda)$

m) $3x - 2y - 3z = 1$

Stel $z = \lambda$ en $y = \mu$

dan is $x = \dfrac{1 + 2\mu + 3\lambda}{3}$

p5.20 n°2

a) $3x + 2a = 1$

opl : $\left(\dfrac{1 - 2a}{3}\right)$

b) $ax + 3 = 6$

$x = \dfrac{3}{a}$

$\underline{a \neq 0}$: opl $= \dfrac{3}{a}$

$\underline{a = 0}$: $\emptyset$

c) $3(x - 1) + a(b + x) = x$

$\Leftrightarrow 3x - 3 + ab + ax = x$

$\Leftrightarrow 3x + ax - x = 3 - ab$

$2x + ax = 3 - ab$

$x = \dfrac{3 - ab}{2 + a}$

$\underline{a \neq -2, b \in \mathbb{R}}$: opl $= \dfrac{3 - ab}{2 + a}$

$\underline{a = -2, b \neq -3/2}$: opl $= \emptyset$

$\underline{a = -2, b = -3/2}$: opl $= \mathbb{R}$

d) $ax = bx$

$\underline{a \neq b}$: opl $= 0$

$\underline{a = b}$: opl $= \mathbb{R}$

p 5.21 n°6

a) $p_A = 20 + 0{,}3b + 0{,}1s$

b) $p_B = 25 + 0{,}2b + 0{,}05s$

c) $\begin{cases} 32 = 20 + 0{,}3b + 0{,}1s \\ 32 = 25 + 0{,}2b + 0{,}05s \end{cases}$ $\left(\begin{array}{cc|c} 30 & 10 & 1200 \\ 20 & 5 & 700 \end{array}\right) \xrightarrow{R_1 \to R_1 - 2R_2} \left(\begin{array}{cc|c} -10 & 0 & -200 \\ 20 & 5 & 700 \end{array}\right)$

$$\xrightarrow{R_1 \to \frac{-1}{10} R_1} \left(\begin{array}{cc|c} 1 & 0 & 10 \\ 20 & 5 & 700 \end{array}\right) \xrightarrow{R_2 \to R_2 - 20R_1} \left(\begin{array}{cc|c} 1 & 0 & 20 \\ 0 & 5 & 300 \end{array}\right) \xrightarrow{R_2 \to \frac{1}{5} R_2}$$

$$\left(\begin{array}{cc|c} 1 & 0 & 20 \\ 0 & 1 & 60 \end{array}\right)$$ dus opl $= (b, s) = \{(20, 60)\}$

p 5.21 n°7

$$\left(\begin{array}{cccc|c} 4 & 1 & 2 & 1 & 420 \\ 2 & 2 & 2 & 1 & 460 \\ 1 & 3 & 1 & 3 & 540 \\ 3 & 4 & 3 & 2 & 800 \end{array}\right) \xrightarrow{R_4 \to R_4 - R_2} \left(\begin{array}{cccc|c} 4 & 1 & 2 & 1 & 420 \\ 2 & 2 & 2 & 1 & 460 \\ 1 & 3 & 1 & 3 & 540 \\ 1 & 1 & 1 & 1 & 340 \end{array}\right) \xrightarrow{R_2 \to R_2 - R_4}$$

$$\left(\begin{array}{cccc|c} 4 & 1 & 2 & 1 & 420 \\ 1 & 0 & 1 & 0 & 120 \\ 1 & 3 & 1 & 3 & 540 \\ 1 & 2 & 1 & 1 & 340 \end{array}\right) \xrightarrow[R_4 \to R_4 - R_2]{R_3 \to R_3 - R_1} \left(\begin{array}{cccc|c} 4 & 1 & 2 & 1 & 420 \\ 1 & 0 & 1 & 0 & 120 \\ 0 & 3 & 0 & 3 & 420 \\ 0 & 2 & 0 & 1 & 220 \end{array}\right) \xrightarrow{R_1 \to \frac{1R_3}{3}}$$

$$\left(\begin{array}{cccc|c} 4 & 1 & 2 & 1 & 420 \\ 1 & 0 & 1 & 0 & 120 \\ 0 & 1 & 0 & 1 & 140 \\ 0 & 2 & 0 & 1 & 220 \end{array}\right) \xrightarrow[R_1 \to R_1 - R_3 - 2R_2]{R_4 \to R_4 - R_3} \left(\begin{array}{cccc|c} 2 & 0 & 0 & 0 & 40 \\ 1 & 0 & 1 & 0 & 120 \\ 0 & 1 & 0 & 1 & 140 \\ 0 & 1 & 0 & 0 & 80 \end{array}\right)$$

p 5.25 n°3

TB rang $(A) \leq$ rang $(A|B) \leq$ rang $(A) + 1$

Bewijs 1) als $AX = B$ juist 1 opl. heeft
2) als $AX = B$ strijdig is = geen opl
3) als $AX = B$ ∞ veel opl. heeft

$|AX| = B$
└ enkel oplossen als rang $(A|B)$ = rang A
$\iff$ rang $(A|B) <$ rang $(A) + 1$

p 5.28 n°1

a) Concurrerende goederen: de vraag naar goed 1 daalt als $p_1 \uparrow$
maar stijgt als $p_2 \uparrow$

b) $q_{v1} = q_{a1}$ $\qquad$ $q_{v2} = q_{a2}$

$10 - 2p_1 + p_2 = -3 + 2p_1$ $\qquad$ $5 + 2p_1 - 2p_2 = -2 + 3p_2$

$p_2 = 4p_1 - 13$ $\qquad$ $p_2 = \frac{2p_1 + 7}{5}$

$4p_1 - 13 = \frac{2p_1 + 7}{5}$

$p_1 = 4$

$p_2 = 4 \cdot 4 - 13 = 3$

$q_{a2} = -2 + 3 \cdot 3 = 7 = q_{v2}$

$q_{a1} = -3 + 2 \cdot 4 = 5 = q_{v1}$

p 5.29 n°2

a) $q_v = q_a$

$25 - p/2 = -50 + 2p$

$\begin{cases} p = 30 \\ q = 10 \end{cases}$

b) $p^c = p^p + 5$

$p = p^c \Rightarrow -50 + 2(p^c - 5) = 25 - p^c/2$

$-50 + 2p^c - 10 = 25 - p^c/2$

$2p^c + p^c/2 = 25 + 50 + 10$

$5/2\, p^c = 85$

$p^c = 34$

$q_v = 25 - \frac{34}{2} = 8 = q_v$

* marktprijs stijgt met 4 eenheden aangeboden
  & gevraagde hoeveelheid daalt met 2 eenheden
* 80% v/d belasting door consument $\quad$ ($34 - 30 = 4 \Rightarrow$ 4€ van 5€ = 80%)
  20% $\qquad$ producent

p6.23 n°1

$$\begin{cases} a-2b=2 \\ 3a-b=1 \end{cases} \qquad \begin{cases} a = 2+2b \\ a = \frac{1+b}{3} \end{cases} \qquad \begin{aligned} 2+2b &= \frac{1+b}{3} \\ 6+6b &= 1+b \\ 5b &= -5 \\ b &= -1 \quad \text{en } a=0 \end{aligned}$$

p6.23 n°2

$$2\begin{pmatrix} -2 & 1 \\ 1 & 4 \\ 3 & 6 \end{pmatrix} + 3\begin{pmatrix} -3 & -2 \\ 2 & -5 \\ 4 & 0 \end{pmatrix} = \begin{pmatrix} -4 & 2 \\ 2 & 8 \\ 6 & 12 \end{pmatrix} + \begin{pmatrix} -9 & -6 \\ 6 & -15 \\ 12 & 0 \end{pmatrix} = \begin{pmatrix} -13 & -4 \\ 8 & -7 \\ 18 & 12 \end{pmatrix}$$

p6.23 n°3

$$AB = \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 2 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 1 & 2 \\ 8 & 12 \end{pmatrix}$$

$$(AB)C = \begin{pmatrix} 3 & 4 \\ 1 & 2 \\ 8 & 12 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 3 \end{pmatrix} = \begin{pmatrix} 11 & 4 & 15 \\ 5 & 2 & 7 \\ 32 & 12 & 44 \end{pmatrix}$$

$$BC = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 3 \end{pmatrix} = \begin{pmatrix} 5 & 2 & 7 \\ 11 & 4 & 15 \end{pmatrix}$$

$$A(BC) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} 5 & 2 & 7 \\ 11 & 4 & 15 \end{pmatrix} = \begin{pmatrix} 11 & 4 & 15 \\ 5 & 2 & 7 \\ 32 & 12 & 44 \end{pmatrix}$$

$\Rightarrow (AB)C = A(BC)$

$\Rightarrow$ de matrixvermenigvuldiging is associatief

p6.24 n°4

$$AB = \begin{pmatrix} 2 & 3 \\ 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 3 & -2 \\ 0 & 1 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 9 & 8 \\ 1 & 3 & -2 \\ -1 & -2 & 6 \end{pmatrix}$$

$$BA = \begin{pmatrix} 1 & 3 & -2 \\ 0 & 1 & 4 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 1 & 0 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 1 \\ -3 & 4 \end{pmatrix}$$

$\Rightarrow AB \neq BA$

$\Rightarrow$ de matrixvermenigvuldiging is niet commutatief

p6.24 n°5

a)
$$\left(\begin{array}{ccc|ccc} 2 & 2 & 1 & 1 & 0 & 0 \\ 2 & 3 & 2 & 0 & 1 & 0 \\ 2 & 4 & 6 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_1 \to \frac{1}{2} R_1} \left(\begin{array}{ccc|ccc} 1 & 1 & 1/2 & 1/2 & 0 & 0 \\ 2 & 3 & 2 & 0 & 1 & 0 \\ 2 & 4 & 6 & 0 & 0 & 0 \end{array}\right) \xrightarrow{R_2 \to R_2 - 2R_1}$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 2 & 4 & 6 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_3 \to R_3 - 2R_1} \left(\begin{array}{ccc|ccc} 1 & 1 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 2 & 5 & -1 & 0 & 1 \end{array}\right) \xrightarrow{R_1 \to R_1 - R_2}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -1/2 & 3/2 & -1 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 2 & 5 & -1 & 0 & 1 \end{array}\right) \xrightarrow{R_3 \to R_3 - 2R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & -1/2 & 3/2 & -1 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 3 & 1 & -2 & 1 \end{array}\right) \xrightarrow{R_3 \to \frac{1}{3} R_3}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -1/2 & 3/2 & -1 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1/3 & -2/3 & 1/3 \end{array}\right) \xrightarrow{R_1 \to R_1 + \frac{1}{2} R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 5/3 & -4/3 & 1/6 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1/3 & -2/3 & 1/3 \end{array}\right)$$

p6.25 n°7

$$B = \begin{pmatrix} 8 & 38 & 14 \\ 6 & 32 & 4 \end{pmatrix} \qquad k_B = \begin{pmatrix} 0{,}10 \\ 0{,}05 \\ 0{,}15 \end{pmatrix} \qquad k_M = \begin{pmatrix} 1{,}25 \\ 0{,}08 \\ 0{,}40 \\ 0{,}12 \end{pmatrix}$$

$$M = \begin{pmatrix} 46 & 7 & 1 & 28 \\ 35 & 7 & 1 & 25 \end{pmatrix} \qquad C = \begin{pmatrix} 25 & 50 \end{pmatrix}$$

a) $B \cdot k_B = \begin{pmatrix} 4{,}8 \\ 2{,}8 \end{pmatrix}$

b) $M \cdot k_M + B \cdot k_B + \begin{pmatrix} 2{,}5 \\ 3 \end{pmatrix} = \begin{pmatrix} 61{,}82 \\ 47{,}71 \end{pmatrix} + \begin{pmatrix} 4{,}8 \\ 2{,}5 \end{pmatrix} + \begin{pmatrix} 2{,}5 \\ 3 \end{pmatrix} = \begin{pmatrix} 69{,}12 \\ 53{,}51 \end{pmatrix}$

c) $4{,}8 \cdot 25 + 2{,}8 \cdot 50 = 260$

d) $B \cdot C = T = (500, 2250, 550)$

e) $M \cdot C = H = (2900, 525, 75, 1950)$

f) 3931 $(M \cdot k_B) \cdot C$

p6.26 n°9

$$\begin{cases} x_n = \# \text{ consumenten A in jaar } n \\ y_n = \# \text{ consumenten B in jaar } n \\ z_n = \# \text{ consumenten C in jaar } n \end{cases}$$

$$\begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix} = \begin{pmatrix} 0{,}8 & 0{,}2 & 0{,}2 \\ 0{,}1 & 0{,}5 & 0{,}1 \\ 0{,}1 & 0{,}3 & 0{,}7 \end{pmatrix}^n \begin{pmatrix} 1000 \\ 1200 \\ 800 \end{pmatrix}$$

na 2j: A = 1320
B = 612
C = 1068

p6.27 n°11

a) $(A+B)(A-B) = A^2 - B^2$

$\rightarrow$ FOUT ; vb $A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}$

maar de stelling is wel juist onder de voorwaarde: $AB = BA$

b) $A^2 = A \Rightarrow A = O_m$ of $A = I_m$

$\rightarrow$ FOUT ; vb $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^2$

$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}^2$

c) $(I_n + A + A^2 + \ldots + A^k)(I_n - A) = I_n - A^{k+1}$ $\quad I \cdot (-A) = I$

$\underline{k=0}$ $\quad I_n(I_n - A) = I_n - A^{k+1=0}$ ok!

$\underline{k>0}$ $\quad I_n(I_n - A) + A(I_n - A) + A^2(I_n - A) + \ldots + A^k(I_n - A)$

$= I_n - \cancel{A} + \cancel{A} - \cancel{A^2} + \cancel{A^2} - \cancel{A^3} + \ldots + \cancel{A^k} - A^{k+1}$

ok!

d) Als A en B inverteerbaar zijn en $A \neq -B$, dan is $(A+B)$ inverteerbaar

$\rightarrow$ FOUT ; vb $A = \begin{pmatrix} 1 & 2 \\ -3 & -4 \end{pmatrix}$ $B = \begin{pmatrix} 5 & 8 \\ 3 & 4 \end{pmatrix}$

$A + B = \begin{pmatrix} 6 & 10 \\ 0 & 0 \end{pmatrix} \rightarrow$ NIET inverteerbaar

p 6.27 n°12

$$I_n - A^{k+1} = I_n - A \cdot \underbrace{A^k}_{=0} \text{ (zie gegeven)}$$

$$\underbrace{(I_n + A + A^2 + \ldots + A^k)}_{\text{inverse } (I_n - A)}(I_n - A) = I_n$$

p 6.27 n°13

a) $C = \begin{pmatrix} -5 & -4 & -3 \\ 0 & -3 & -6 \end{pmatrix}$

b) $AB = \begin{pmatrix} -5 & -4 & -3 \\ 0 & -3 & -6 \end{pmatrix}$ dus matrix v/d samenstelling $f \circ g$ = product f en g

p 6.26 n°8

a) $\begin{pmatrix} 15/16 & 1/8 \\ 1/16 & 7/8 \end{pmatrix} = M$

b) x_n = aantal werklozen in jaar n (met $n \in \mathbb{N}$)

y_n = aantal werkenden in jaar n

$$\begin{array}{cc} & \begin{array}{cc} W & NW \end{array} \\ \begin{array}{c} W \\ NW \end{array} \begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = & \begin{pmatrix} 15/16 & 1/8 \\ 1/16 & 7/8 \end{pmatrix} \begin{pmatrix} x_n \\ y_n \end{pmatrix} \end{array}$$

c) $\begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = \begin{pmatrix} 15/16 & 1/8 \\ 1/16 & 7/8 \end{pmatrix} \begin{pmatrix} 4 \\ 0{,}5 \end{pmatrix} = \begin{pmatrix} 31/32 \\ 113/32 \end{pmatrix}$

2j later: $M^2 \cdot \begin{pmatrix} 4 \\ 0{,}5 \end{pmatrix} = \begin{pmatrix} 1{,}3496 \\ 3{,}15 \end{pmatrix}$

p 7.13 n°1

a) $\det\begin{pmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 2 & 5 & 1 \end{pmatrix} = 2 + 12 + 60 + 12 - 15 + 8 = 79$

b) $1(-13) - 2(-10) + 3 \cdot 24 = 79$

c) $\det\begin{pmatrix} 2 & 5 & -1 \\ 8 & -4 & 6 \\ 1 & 2 & 3 \end{pmatrix} = -\det\begin{pmatrix} 1 & 2 & 3 \\ 8 & -4 & 6 \\ 2 & 5 & -1 \end{pmatrix} = -2\begin{pmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 2 & 5 & -1 \end{pmatrix}$

dus $-2 \cdot 79 = -158$

p 7.13 n°2

kolommen en rijen van een matrix geven een oppervlakte weer. Als 2 kolommen gelijk zijn, dan zijn de 2 vectoren gelijk. Een kolom is dus een vector die de plaats vh vlak aangeeft.

$\det\begin{pmatrix} a & a \\ b & b \end{pmatrix} = ab - ab = 0$

dus $|\text{opp in } \mathbb{R} (2\times 2)| = 0$ bij gelijke vectoren

p 7.13 n°3

singulier : $\det A = 0$

a) $A_1 = \begin{pmatrix} 1 & 2 & 4 \\ 3 & 1 & 6 \\ a & 3 & 2 \end{pmatrix} \rightarrow \det A_1 = 2 + 12a + 36 - 4a - 18 - 12 = 0$

$8a = -8$

$a = -1$

b)

$$\det A_2 = \det\begin{pmatrix} a+2 & 1 & 1 \\ 1 & a+2 & 1 \\ 1 & 1 & a+2 \end{pmatrix} \overset{R_3 \to R_3 - R_2}{=} \det\begin{pmatrix} a+2 & 1 & 1 \\ 1 & a+2 & 1 \\ 0 & -a-1 & a+1 \end{pmatrix}$$

$$\overset{K_3 \to K_3 + K_2}{=} \det\begin{pmatrix} a+2 & 1 & 2 \\ 1 & a+2 & a+3 \\ 0 & -a-1 & 0 \end{pmatrix}$$

$$\det A_2 = (-a-1)\big((a+2)(a+3)-2\big) = 0$$
$$= (-a-1)(a^2+5a+4) = 0$$
$$a = 1 \text{ of } a = -4$$

p 7.14 n°4

$$\det\begin{pmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{pmatrix} \underset{K_3 \to K_3 - K_1}{\overset{K_2 \to K_2 - K_1}{=}} \det\begin{pmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^2 & b^2-a^2 & c^2-a^2 \end{pmatrix}$$

$$\det A = (b-a)(c^2-a^2) - (b^2-a^2)(c-a)$$
$$= (b-a)(c-a)(c+a) - (b-a)(b+a)(c-a)$$
$$= (b-a)\big((c-a)(c+a) - (b-a)(c-a)\big)$$
$$= (b-a)(c-a)\big((c+a) - (b+a)\big)$$
$$= (b-a)(c-a)(c-b)$$

p 7.14 n°5

$$\det\begin{pmatrix} a+b+c+na & nb & nc \\ na & a+b+c+nb & nc \\ na & nb & a+b+c+nc \end{pmatrix} = (n+1)(a+b+c)^3$$

$$\det A \overset{\substack{R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1}}{=} \det \begin{pmatrix} a+b+c+na & nb & nc \\ -a-b-c & a+b+c & 0 \\ -a-b-c & 0 & a+b+c \end{pmatrix}$$

$$\overset{K_1 \to K_1 + K_2}{=} \det \begin{pmatrix} a+b+c+na+nb & nb & nc \\ 0 & a+b+c & 0 \\ -a-b-c & 0 & a+b+c \end{pmatrix}$$

$$\overset{K_1 \to K_1 + K_3}{=} \det \begin{pmatrix} a+b+c+na+nb+nc & nb & nc \\ 0 & a+b+c & 0 \\ 0 & 0 & a+b+c \end{pmatrix}$$

$$= \big(a(n+1)+b(n+1)+c(n+1)\big)(a+b+c)^2$$

$$= (n+1)(a+b+c)^3$$

p7.14 n°6

$$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \qquad A^t = \begin{pmatrix} a & d & g \\ b & e & f \\ c & f & i \end{pmatrix} \qquad -A = \begin{pmatrix} -a & -b & -c \\ -d & -e & -f \\ -g & -h & -i \end{pmatrix}$$

$A^t = -A$ dus

$$\begin{matrix} a = -a & d = -b & g = -c \\ b = -d & e = -e & h = -f \\ c = -g & -h = f & i = -i \end{matrix}$$

dan is $A = \begin{pmatrix} 0 & b & c \\ -b & 0 & f \\ -c & -f & 0 \end{pmatrix}$ en $\det A = -b(cf) + c(bf) = 0$

Geldt n̄t voor (2×2) matrix

p 7.17 n°7

$$\det A = 8 \xrightarrow{1} -8 \xrightarrow{2} -16 \xrightarrow{3} -16 \xrightarrow{4} 16$$

1 : det A wordt negatief

2 : det A verdubbelt

3 : det A verandert niet

4 : det A vermenigvuldigd met (-1)

niet diagonaliseerbaar als niet genoeg eigenwaarden

p8.13 n°1

a) $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$ λ is eigenwaarde $\Leftrightarrow \det(A - I_3\lambda) = 0$

$$\det\begin{pmatrix} -\lambda & 0 & 0 \\ 0 & 1-\lambda & 0 \\ 1 & 0 & 1-\lambda \end{pmatrix} = 0$$

$$\Leftrightarrow -\lambda\big((1-\lambda)(1-\lambda)\big) = -\lambda(1-\lambda)^2$$

* $\underline{\lambda = 0}$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{cases} y = 0 \\ x + z = 0 \end{cases} \quad \{(r, 0, -r) \mid r \in \mathbb{R}_0\}$$

* $\underline{\lambda = 1}$ → dubbele wortel

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{cases} -x = 0 \\ x = 0 \end{cases} \quad \{(0, r, s) \mid r, s \in \mathbb{R} \text{ en } (r, s) \neq (0,0)\}$$

→ diagonaliseerbaar

eigenvectoren als kolommen

eigenwaarden

QDQ^{-1} ontbinding: $Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$ $D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$ λ is eigenwaarde $\Leftrightarrow \det(A - \lambda I_3) = 0$

$$\det\begin{pmatrix} 1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 0 \\ 1 & 0 & 1-\lambda \end{pmatrix}$$

$$\Leftrightarrow (1-\lambda)^3 = 0$$

* $\underline{\lambda = 1}$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \{(0, r, s) \mid r \text{ en } s \in \mathbb{R} \text{ met } (r, s) \neq (0,0)\}$$

→ niet diagonaliseerbaar

c) $\begin{vmatrix} -3-\lambda & 1 & -1 \\ -7 & 5-\lambda & -1 \\ -6 & 6 & -2-\lambda \end{vmatrix} \underset{k_1+k_2}{\overset{k_1\to}{=}} \begin{vmatrix} -2-\lambda & 1 & -1 \\ -2-\lambda & 5-\lambda & -1 \\ 0 & 6 & -2-\lambda \end{vmatrix} = -1(\lambda+2) \begin{vmatrix} 1 & 1 & 1 \\ 1 & 5-x & 1 \\ 0 & 6 & x+2 \end{vmatrix}$

$\underset{k_3-k_1}{\overset{k_3\to}{=}} (\lambda+2) \begin{vmatrix} 1 & 1 & 0 \\ 1 & 5-\lambda & 0 \\ 0 & 6 & \lambda+2 \end{vmatrix} = (\lambda+2)^2(4-\lambda)$

* $\underline{\lambda=-2}$

$$\begin{pmatrix} -3+2 & 1 & -1 \\ -7 & 5+2 & -1 \\ -6 & 6 & -2+2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\begin{cases} -x+y-1 \\ -7x+7y-2=0 \\ -6x+6y=0 \end{cases} \Longleftrightarrow \begin{cases} -x+y-z=0 \\ -x+y-1/2\,z=0 \\ -x+y=0 \end{cases}$ $\{(r,r,0) \mid r \in \mathbb{R}_0\}$

* $\underline{\lambda=4}$

$$\begin{pmatrix} -3-4 & 1 & -1 \\ -7 & 5-4 & -1 \\ -6 & 6 & -2-4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\begin{cases} -7x+y-z=0 \\ -7x+y-z=0 \\ -6x+6y-6z=0 \end{cases}$ $\begin{cases} -x+y-z=0 \\ -x+y-z=0 \end{cases}$ $\{(0,r,-r) \mid r \in \mathbb{R}_0\}$

→ niet diagonaliseerbaar

p 8.13 n°2

de diagonaaltermen

p 9.13 n°1

$6x - 2y = 26 \rightarrow (3, -4)$ invullen

$6 \cdot 3 - 2 \cdot (-4) = 26$ ok!

p 9.13 n°2

* $a = (2,1)$ en $b = (3,-2)$

richtingsvector $= b - a = (1, -3)$

$$(x,y) = (2,1) + \lambda(1,-3)$$

$$\rightarrow \begin{cases} x = 2 + \lambda \\ y = 1 - 3\lambda \end{cases}$$

$$\rightarrow \begin{cases} \lambda = x - 2 \\ \lambda = \dfrac{y-1}{-3} \end{cases} \qquad x - 2 = \frac{y-1}{-3} \qquad 3x + y = 7$$

$$-3x + 6 = y - 1$$

$$y = -3x + 7$$

* $c = (7,0)$ en $d(1,4)$

richtingsvector $= d - c = (-6, 4)$

$$(x,y) = (7,0) + \lambda(-6,4)$$

$$\rightarrow \begin{cases} x = 7 - 6\lambda \\ y = 4\lambda \end{cases}$$

$$\rightarrow \begin{cases} \lambda = \dfrac{x-7}{-6} \\ \lambda = y/4 \end{cases} \qquad \frac{x-7}{-6} = y/4 \qquad 2x + 3y = 14$$

$$y = \left(\frac{-x}{6} + \frac{7}{6}\right)4 = -\frac{2}{3}x + \frac{14}{3}$$

* snijden ze?

$$\det\begin{pmatrix} 3 & 1 \\ 2 & 3 \end{pmatrix} = 7 \neq 0 \text{ dus ab en cd snijden elkaar}$$

$$-3x + 7 = -\frac{2}{3}x + \frac{14}{3}$$

$$\begin{cases} x = 1 \\ y = 4 \end{cases}$$

p9.13 n°3

Zelden zullen 3 vgl-en door 1 punt gaan :

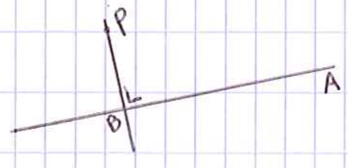
→ $\mathbb{R}^3$ is oneindig groot

→ twee rechten die niet evenwijdig zijn, snijden in een punt en dat die 3de rechte precies ook daardoor snijdt is zeer onwaarschijnlijk

p9.13 n°4

A : $3x + 2y = 8$

$p = (8,5)$



vgl van rechte Bp opstellen

$* \text{ rico } A \cdot \text{rico } Bp = -1$

$-3/2 \cdot \text{rico } Bp = -1$

$\text{rico } Bp = 2/3$

$3x + 2y = 8$

$2y = 8 - 3x$

$y = 4 - 3/2\, x$

$* \ y - y_1 = a(x - x_1)$

$y - 5 = 2/3(x - 8)$

$y - 5 = 2/3\, x - 16/3$

$Bp \Leftrightarrow y = 2/3\, x - 1/3$

snijpunt van A en Bp vinden

$2/3\, x - 1/3 = 4 - 3/2\, x$

$2/3\, x + 3/2\, x = 4 + 1/3$

$13/6\, x = 13/3$

$\begin{cases} x = 2 \\ y = 1 \end{cases}$

afstand tussen (8,5) en (2,1)

$= \sqrt{(8-2)^2 + (5-1)^2} = \sqrt{36 + 16} = \sqrt{52} = 2\sqrt{13}$

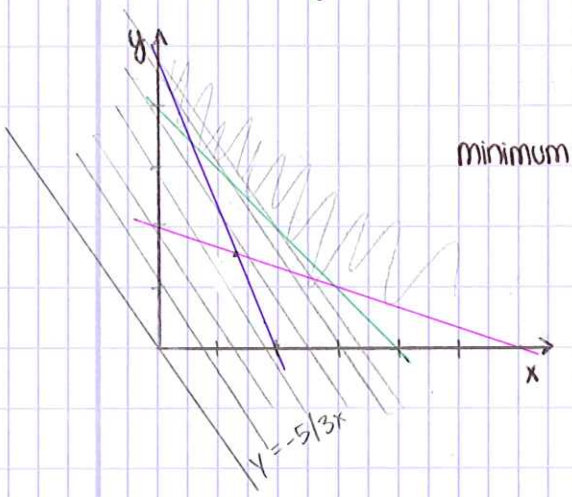
p9.13 n°5

$$\begin{cases} x = \text{hoeveelheid van } v_1 \\ y = \text{hoeveelheid van } v_2 \end{cases} \qquad \begin{cases} x \geq 0 \\ y \geq 0 \end{cases}$$

kost samenstelling : $k(x,y) = 15x + 9y \Rightarrow -\frac{5}{3}x = y$

Minimale behoeften :

$$\begin{cases} 2x + y \geq 400 & (\text{vitaminen}) \Rightarrow y \geq -2x + 400 \\ x + 2y \geq 500 & (\text{mineralen}) \Rightarrow y \geq -\frac{1}{2}x + 250 \\ 4x + 4y \geq 1400 & (\text{calorieën}) \Rightarrow y \geq -x + 250 \end{cases}$$



minimum w bereikt

$$\begin{cases} 2x + y \geq 400 \\ 4x + 4y \geq 1400 \end{cases}$$

$$\begin{pmatrix} 2 & 1 & 400 \\ 4 & 4 & 1400 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 350 \\ 2 & 1 & 400 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 350 \\ 0 & -1 & -300 \end{pmatrix}$$

$$\begin{cases} x + y = 350 \\ y = 300 \end{cases} \Rightarrow \begin{matrix} x = 50 \\ y = 300 \end{matrix}$$

$k(50, 300) = 15 \cdot 50 + 9 \cdot 300 = 3450 \Rightarrow 34,5$ euro

p9.13 n°6

HDT €400/rol
- 20 m^2 wol
- 40 m^2 nylon
- 40 man-uren

LDT €240/rol
- 40 m^2 nylon
- 20 man-uren

<u>beschikbare middelen</u> 1200 m^2 wol → 4€ / m^2

1000 m^2 nylon → €1,6 / m^2

€ 4,8 / uur → max 800-uren

$W(x,y)$ maximaliseren :

kost HDT = $20 \cdot 4 + 40 \cdot 1{,}6 + 40 \cdot 4{,}8 = 336$

kost LDT = $40 \cdot 1{,}6 + 20 \cdot 4{,}8 = 160$

$\Rightarrow W(x,y) = 400x - 336x + 240y - 160y$

$= 64x + 80y$

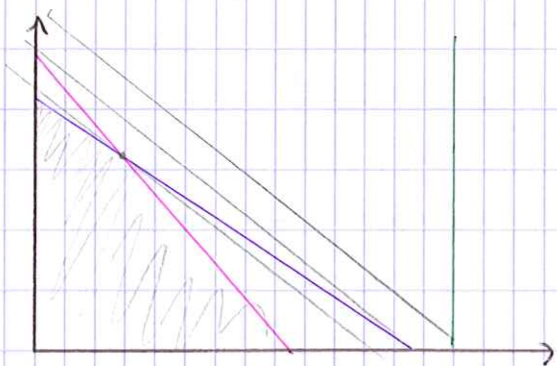
$y = -64/80\ x$

<u>ander voorwaarden</u> :

$$\begin{cases} 40x + 20y \leq 800 & \Rightarrow y = -2x + 40 \\ 20x \leq 1200 & \Rightarrow x = 60 \\ 40x + 40y \leq 1000 & \Rightarrow y = -x + 25 \end{cases}$$

en $x \geq 0$

$y \geq 0$



p 9.14 n°7

x = # witte sokken

y = # gekleurde sokken

$$\begin{cases} 4x + 2y = 15 \\ 8x + 2y = 20 \end{cases}$$

$W_{max} = 15x + 20y$

randvoorwaarden : $4x + 8y \leq 84$ met $x, y \geq 0$

$2x + 2y \leq 24$

$$\begin{pmatrix} 4 & 8 & 84 \\ 2 & 2 & 24 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 21 \\ 0 & -1 & -9 \end{pmatrix} \quad \begin{cases} x + 2y = 21 \\ y = 9 \end{cases} \Rightarrow \begin{matrix} x = 3 \\ y = 9 \end{matrix}$$

$\left.\begin{matrix} \text{3 keer € 15} \\ \text{9 keer € 20} \end{matrix}\right\}$ € 225

p 9.20 n°1

$$\begin{cases} x = 2+4t \\ y = 1-t \\ z = 2-2t \end{cases} \quad t \in \mathbb{R}$$

gelegen in vlak $x + 2y + z = 6$

$(2+4t) + 2(1-t) + (2-2t) = 6$

$2 + 4t + 2 - 2t + 2 - 2t = 6$

$6 = 6$ ok

p 9.20 n°2

Oplvz $\begin{cases} x + 2y - z = 3 \\ 2x - y + 3z = 1 \end{cases}$ = rechte door $(1,1,0)$ en rico $(-1,1,1)$

$$\begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & -1 & 3 & 1 \end{pmatrix} \xrightarrow{R_2 \to R_2 - 2R_1} \begin{pmatrix} 1 & 2 & -1 & 3 \\ 0 & -5 & 5 & -5 \end{pmatrix} \xrightarrow{R_2 \to 1/5 R_2} \begin{pmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & -1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 1 \end{pmatrix} \quad \begin{cases} x + z = 1 \\ y - z = 1 \end{cases}$$

Stel $z = \lambda$ met $\lambda \in \mathbb{R}$

$$\begin{cases} x = 1 - \lambda \\ y = 1 + \lambda \\ z = 0 + \lambda \end{cases}$$

$\downarrow$ punt $\quad \downarrow$ rico

p 9.20 n°3

a) $(1,2,-4)$ $(2,3,7)$ $(-4,-1,3)$

$a(x-1) + b(y-2) + c(z+4) = 0$

$a(2-1) + b(3-2) + c(7+4) = 0$

$$\begin{cases} a - b + 11c = 0 \\ 3a - 3b + 7c = 0 \end{cases}$$

kies $c = 3$

$a + b = -33$

$3a - 3b = -21 - 7$

$2a = -40 \Rightarrow a = -20$

$b = -33 - a = -13$

b) $(-7,1,0)\ (2,-1,3)\ (4,1,6)$

$$a(x+7)+b(y-1)+c(z-0)=0$$

$$\begin{cases} a(2+7)+b(-1-1)+c3=0 \\ a(4+7)+b(1-1)+c6=0 \end{cases} \Leftrightarrow \begin{cases} 9a-2b+3c=0 \\ 11a+6c=0 \end{cases}$$

$$c=-11 \quad \begin{cases} 9a-2b=33 \\ 11a=66 \end{cases} \quad a=6 \quad b=\frac{21}{2}$$

$$\Rightarrow 12x-21y+22z=63$$

p 9.20 n°4

a) $(2,3,-4)\ (2,0,-4)$

vectorvergelijking $(x,y,z)=(2,3,-4)+\lambda(0,1,0)$

parametervgl $\begin{cases} x=2 \\ y=3+\lambda \\ z=-4 \end{cases}$ met $\lambda \in \mathbb{R}$

cartesiaanse vgl $\begin{cases} x=2 \\ z=-4 \end{cases}$

b) $(2,1,3)\ (1,2,-1)$

vectorvgl $(x,y,z)=(2,1,3)+\lambda(1,-1,4)$

parametervgl $\begin{cases} x=2+\lambda \\ y=1-\lambda \\ z=3+4\lambda \end{cases}$ met $\lambda \in \mathbb{R}$

cartesiaanse vgl $\begin{cases} \lambda=x-2 \\ \lambda=-y+1 \\ \lambda=\frac{z-3}{4} \end{cases}$

$x-2=z-3$ | $x-2=-y+1$
$4x-8=z-3$ | $x+y=1+2$
$4x-z=5$ | $x+y=3$

$$\begin{cases} 4x-5=5 \\ x+y=3 \end{cases}$$

p9.20 n°5

$L \leftrightarrow \begin{cases} 2x - 3y - 1 = 0 \\ 2y + z + 4 = 0 \end{cases}$ $\quad P(3, 1, -2)$

$\begin{cases} 2x - 3y = a \\ 2y + z = b \end{cases}$ $\rightarrow$ punt P invullen geeft: $\begin{matrix} 2 \cdot 3 - 3 \cdot 1 = 3 \\ 2 \cdot 1 - 2 = 0 \end{matrix}$

dus $\begin{cases} 2x - 3y = 3 \\ 2y + z = 0 \end{cases}$

p9.20 n°6

a) $\begin{cases} x - 2y + 1 = 0 \\ 2y - z = 0 \end{cases}$ stel $y = \lambda$ dan $\begin{cases} x - 2\lambda + 1 = 0 \\ y = \lambda \\ 2\lambda - z = 0 \end{cases} \Leftrightarrow \begin{cases} x = 2\lambda - 1 \\ y = \lambda \\ z = 2\lambda \end{cases}$

b) $P(1,1,2)$, afstand $= 6$

$\sqrt{(x-1)^2 + (y-1)^2 + (z-2)^2} = 6$

$(2\lambda - 1 - 1)^2 + (\lambda - 1)^2 + (2\lambda - 2)^2 = 6^2$

$4\lambda^2 - 8\lambda + 4 + \lambda^2 - 2\lambda + 1 + 4\lambda^2 - 8\lambda + 4 = 36$

$6\lambda^2 - 18\lambda + 9 = 36$

$6\lambda^2 - 18\lambda - 27 = 0 \rightarrow D = (-18)^2 - 4 \cdot 9 (-27) = 1296$

$\lambda_{1,2} = \frac{18 \pm \sqrt{1296}}{2 \cdot 9} \begin{matrix} -1 \\ 3 \end{matrix}$

$\begin{cases} x = 2(-1) - 1 = -3 \\ y = -1 = -1 \\ z = 2(-1) = -2 \end{cases}$ $\quad \begin{cases} x = 2 \cdot 3 - 1 = 5 \\ y = 3 = 3 \\ z = 2 \cdot 3 = 6 \end{cases}$

$(-3, -1, -2)$ $(5, 3, 6)$

p9.21 n°7

afstand van P (5,6,-2) tot $V \leftrightarrow 2x + 3y - z = 6$

normaalvector van V = (2, 3, -1) → staat loodrecht op V

loodrechte door P (5,6,-2) : $\begin{cases} x = 5 + 2\lambda \\ y = 6 + 3\lambda \\ z = -2 - \lambda \end{cases}$ met $\lambda \in \mathbb{R}$

$2x + 3y - z = 6$

$2(5+2\lambda) + 3(6+3\lambda) - (-2-\lambda) = 6$

$10 + 4\lambda + 18 + 9\lambda + 2 + \lambda = 6$

$14\lambda = 6 - 30 = -24$

$\lambda = -\frac{12}{7}$

Snijpunt van V met loodlijn door P op V = $\begin{cases} x = 5 + 2(-12/7) = 11/7 \\ y = 6 + 3(-12/7) = 6/7 \\ z = -2 - (-12/7) = -2/7 \end{cases}$

afstand (5,6,-2) en (11/7, 6/7, -2/7)

$$\sqrt{\left(5-\frac{11}{7}\right)^2 + \left(6-\frac{6}{7}\right)^2 + \left(-2+\frac{2}{7}\right)^2} = \sqrt{\left(\frac{24}{7}\right)^2 + \left(\frac{36}{7}\right)^2 + \left(\frac{-12}{7}\right)^2}$$

$$= \sqrt{\frac{576}{49} + \frac{1296}{49} + \frac{144}{49}} = \sqrt{\frac{2016}{49}} = \frac{\sqrt{288}}{7} = \frac{12\sqrt{14}}{7}$$

p9.21 n°8

a) (3, 2, -3)

(1, 5, 0)

$\begin{cases} x = 1 + 2\lambda \\ y = 5 - 3\lambda \\ z = -3\lambda \end{cases}$ $\begin{cases} \lambda = \frac{x-1}{2} \\ \lambda = \frac{y-5}{-3} \\ \lambda = \frac{z}{-3} \end{cases}$

$$\frac{x-1}{2} = \frac{-z}{3}$$

$2z = -3(x-1)$

$2z = -3x+3$

$2z+3x = 3$

$$\frac{y-5}{-3} = \frac{z}{-3}$$

$-3(y-5) = -3z$

$-3y+15 = -3z$

$-3y+3z = -15$

$y-z = 15$

$$\begin{cases} 2z+3x = 3 \\ y-z = 15 \end{cases}$$

b) $(3,2,-3)$ normaalvector vh vlak (= richtingsvector)

$(1,5,0)$ $= (2,-3,-3)$

dus vlak $\leftrightarrow 2x-3y-3z = d$

$p \in$ vlak dus $2\cdot 8 - 3\cdot 0 - 3\cdot 4 = d$

$d = 4$

vgl vn vlak loodrecht op rechte door p : $2x-3y-3z = 4$

snijpunt vlak & rechte :

$$\begin{cases} x = 1+2\lambda \\ y = 5-3\lambda \\ z = -3\lambda \end{cases}$$

invullen in vgl vlak geeft : $2(1+2\lambda)-3(5-3\lambda)-3(-3\lambda) = 4$

$2+4\lambda-15+9\lambda+9\lambda-4 = 0$

$22\lambda = 17$

$\lambda = 17/22$

$$\begin{cases} x = 1+2(17/22) = 28/11 \\ y = 5-3(17/22) = 59/22 \\ z = -3(17/22) = -51/22 \end{cases}$$

afstand $\left(\frac{28}{11}, \frac{59}{22}, -\frac{51}{22}\right)$ tot $(8,0,4)$

$$\sqrt{\left(8-\frac{28}{11}\right)^2+\left(0-\frac{59}{22}\right)^2+\left(4+\frac{51}{22}\right)^2} = \sqrt{\left(\frac{60}{11}\right)^2+\left(-\frac{59}{11}\right)^2+\left(\frac{139}{22}\right)^2}$$

$$= \sqrt{\frac{3600}{121}+\frac{3481}{121}+\frac{19321}{484}} = \cancel{\sqrt{\frac{29693}{484}}} = \frac{\sqrt{37202}}{22}$$

p10.41 n°1

$\forall M \in \mathbb{R}, \exists n_0 \in \mathbb{N}, \forall n \in \mathbb{N} : n \geq n_0 \Rightarrow x_n < M$

p10.41 n°2

a) waar : begrensd als $m \leq x_n \leq M$ dus M = bovengrens
b) fout :
c) waar : definitie
d) fout : vb $\cos n$, $\sin n$, $(-1)^n$

p10.41 n°3

a) $x_n = 5 + \frac{1}{n}$

b) $x_n = \sin n$

c) bestaat niet

d) $x_n = (-1)^n \cdot n$
$x_n = 1 + n$

p10.41 n°4 (eigenschappen p10.22)

x_n convergeert naar 2 en $y_n = x_n + \frac{5}{n}$

$\lim\limits_{n \to \infty} \left(y_n = x_n + \frac{5}{n} \right) = 2 + 0 = 2$

p10.41 n°5 (eigenschappen p10.22)

x_n convergeert naar 2 en $\neq 0$, en $y_n = \frac{1}{2x_n}$

$\lim \left(y_n = \frac{1}{2x_n} \right) = \frac{\lim 1}{\lim 2x_n} = \frac{1}{2 \cdot 2} = \frac{1}{4}$

p10.42 n°6

Propositie : als $x_n \leq y_n$ dan $\lim_{n\to\infty} x_n \leq \lim_{n\to\infty} y_n$ (bij bestaande limieten)

$\rightarrow x_n < y_n, \overset{?}{\Rightarrow} \lim_{n\to\infty} x_n < \lim_{n\to\infty} y_n$? = FOUT !

tegenvoorbeeld : $x_n < y_n \Rightarrow \lim x_n = \lim y_n$

bij $\begin{cases} x_n = -\frac{1}{n} \\ y_n = \frac{1}{n} \end{cases}$

p10.42 n°7

! onbepaald : niet af te leiden over de limiet v/e quotiënt

p10.42 n°8

a) $\lim_{n\to\infty} (n^5 - n^4 + n^3) = \lim_{n\to\infty} n^5\left(1 - \frac{1}{n} + \frac{1}{n^2}\right) = \infty \cdot 1 = \infty$

b) $\lim_{n\to\infty} \frac{n}{3n^2-1} = \lim_{n\to\infty} \frac{1}{3n - 1/n} = \frac{1}{\infty - 0} = 0$

c) $\lim_{n\to\infty} \frac{2n+3}{n} = \lim_{n\to\infty} \frac{2 + 3/n}{1} = 2$

d) $\lim_{n\to\infty} \frac{2n}{n+3} = \lim_{n\to\infty} \frac{2}{1 + 3/n} = \frac{2}{1+0} = 2$

e) $\lim_{n\to\infty} \frac{4n^2+3}{2n^2+n+5} = \lim \frac{4 + 3/n^2}{2 + \frac{1}{n} + \frac{5}{n^2}} = 2 + 0 = 2$

f) $\lim_{n\to\infty} \frac{5 + \overbrace{\frac{1}{n^2}}^{=0}}{-\left(\frac{1}{5}\right)^n}$ = p10.31

$x_n = 5 + \frac{1}{n^2} \rightarrow \lim_{n\to\infty} = 5 + 0 = 5 \in \mathbb{R}_0^+ \cup \{+\infty\}$

$y_n = -(1/5)^n \rightarrow \lim_{n\to\infty} = 0 \rightarrow y_n < 0$

$\Big\}\ \lim_{n\to\infty} \frac{x_n}{y_n} = -\infty$

g) ! $\lim_{n\to\infty} \overset{\overset{?}{=0}}{2^{-n}} \sin n = 0$

↳ schommelt, ...

$-1 \leq \lim \sin n \leq 1$

$\lim_{n\to\infty} 2^{-n} = \lim_{n\to\infty} \frac{1}{2^n} = 0$

$\Big\}\ \lim 2^{-n} \sin n = 0$

h) $\lim\limits_{n\to\infty} \dfrac{(1/2)^n}{(1/3)^{n+1}} = \dfrac{\lim\limits_{n\to\infty} (1/2)^n}{\lim\limits_{n\to\infty} \left(\frac{1}{3}\right)^n \cdot \frac{1}{3}} = \lim\limits_{n\to\infty} \left(\frac{3}{2}\right)^n \cdot \lim\limits_{n\to\infty} 3 = \infty \cdot 3 = \infty$

! i) $\lim\limits_{n\to\infty} \dfrac{(-1)^n \cdot 2n}{n^2+5n+3} =$

$n^2+5n+3 \geqslant 0$
$-1 \leqslant (-1)^n \leqslant 1$
$2n : +\infty$

$= \lim\limits_{n\to\infty} \dfrac{\frac{(-1)^n \cdot 2n}{2n^2}}{1+\frac{5}{n}+\frac{3}{n^2}} = \dfrac{0}{1} = 0$

! j) $\lim\limits_{n\to\infty} \dfrac{(-1)^n}{2n-n^2} = \lim\limits_{n\to\infty} \dfrac{(-1)^n}{-1+2/n} = 0$

k) $\lim\limits_{n\to\infty} \dfrac{7}{\sqrt{n}+3} = \dfrac{7}{+\infty+3} = 0$

P10.42 n°9

a) FOUT want $\left.\begin{array}{l} \lim\limits_{n\to\infty} (-1)^n = -1 \text{ of } 1 \\ \lim\limits_{n\to\infty} n^2 = +\infty \end{array}\right\} \lim\limits_{n\to\infty} x_n = \infty$

$(n^2+(-1)^n)$

$x_n = n^2 + (-1)^n$

b) FOUT want $\left.\begin{array}{l} -1 \leqslant \lim\limits_{n\to\infty} \sin n \leqslant 1 \\ \lim\limits_{n\to\infty} \frac{1}{n} = 0 \end{array}\right\} \lim\limits_{n\to\infty} \frac{1}{n} \sin n = 0$

$\frac{1}{n} \sin n$

c) FOUT want $\lim\limits_{n\to\infty} \left(\dfrac{-7}{-8}\right)^n = \lim\limits_{n\to\infty} 0{,}875^n = 0$

$x_n = \dfrac{(-7)^n}{(-8)^n}$

INSLUITSTELLING

$-1 \leqslant \sin n \leqslant 1$

$-\dfrac{1}{n} \leqslant \dfrac{\sin n}{n} \leqslant \dfrac{1}{n}$ (alles vermenigvuldigen met $\frac{1}{n}$)

$\lim\limits_{n\to\infty} -\dfrac{1}{n} = \lim\limits_{n\to\infty} \dfrac{1}{n} = 0$ dus $\lim \dfrac{\sin n}{n} = 0$!

p11.13 n°1

a) $\log_a x = \frac{\log x}{\log a} \Rightarrow \log_3 20 = \frac{\log 20}{\log 3} = 2{,}72683$

b) $\log 20 = \log 10 + \log 2 \triangleq 1 + 0{,}30 = 1{,}30$

$\log 12 = \log 3 + \log 2 + \log 2 = 0{,}48 + 0{,}30 + 0{,}30 = 1{,}08$

$\log 0{,}16 = \log 2 - \log 12 = -0{,}77$

c) $a, b \in \mathbb{R}_0^+$ met $a \neq 1 \Rightarrow \log_{a^b} a^{(b^2)}$

$= b^2 \log_{a^b} a$

$= b \cdot \log_{a^b} a^b$

$= b$

d) i $a^{\log b} = b^{\log a}$ voor $a, b \in \mathbb{R}_0^+$

$\log a^{\log b} = \log b^{\log a}$

$\log b \cdot \log a \underset{\text{ok}}{=} \log a \cdot \log b$

ii $\log_{ab} c = \frac{\log_b c}{1 + \log_b a}$ als $a,b,c \in \mathbb{R}_0^+$ en $ab \neq 1 \neq b$

$\log_{ab} c = \frac{\log_b c}{\log_b ab} = \frac{\log_b c}{\log_b a + \log_b b} = \frac{\log_b c}{1 + \log_b a}$

iii $\log\left(\frac{a+b}{3}\right) = \frac{1}{2}(\log a + \log b)$ als $a,b \in \mathbb{R}_0^+$ en $a^2 + b^2 = 7ab$

$= \frac{1}{2}(\log a + \log b)$

$= \frac{1}{2} \log ab$

$2 \log\left(\frac{a+b}{3}\right) = \log ab$

$\log\left(\frac{a^2 + 2ab + b^2}{9}\right) = \log ab$

opgave: $a^2 + b^2 = 7ab$

$\log\left(\frac{9ab}{9}\right) = \log ab$

$\log ab = \log ab$

e) $\underline{\underline{i}}$ $e^{3\ln 2 - 2\ln 5} = e^{\ln 8 - \ln 25} = e^{\ln 8/25} = 8/25$

$\underline{\underline{ii}}$ $\ln \sqrt{e} = \ln e^{1/2} = 1/2$

$\underline{\underline{iii}}$ $\ln \dfrac{e^3\sqrt{e}}{e^{1/3}} = \ln \dfrac{e^3 e^{1/2}}{e^{1/3}} = \ln \dfrac{e^{7/2}}{e^{1/3}} = \ln e^{7/2 - 1/3} = 19/6$

$\underline{\underline{iv}}$ $\ln \dfrac{1}{\sqrt{ab^3}}$ als $\ln a = 2$ en $\ln b = 3$

$= \ln \dfrac{1}{a^{1/2} b^{3/2}} = -\ln a^{-1/2} b^{-3/2} = -\frac{1}{2}\ln a - \frac{3}{2}\ln b$

$= -\frac{1}{2} \cdot 2 - \frac{3}{2} \cdot 3 = -\frac{11}{2}$

f) $\underline{\underline{i}}$ $\ln x = \frac{1}{3}(\ln 16 + 2\ln 2)$
$= \frac{1}{3}(\ln 16 + \ln 4)$
$= \frac{1}{3}\ln 64$
$= \ln 64^{1/3}$
$x = \sqrt[3]{64}$
$= 4$

$\underline{\underline{ii}}$ $\ln x = 2(\ln 3 - \ln 5)$
$= 2(\ln 3/5)$
$= \ln(3/5)^2$
$x = 9/25$

$\underline{\underline{iii}}$ $3^x = e^2$
$\ln 3^x = \ln e^2 = 2$
$x = \dfrac{2}{\ln 3}$

$\underline{\underline{iv}}$ $x^{\ln x} = e$
$\ln x^{\ln x} = 1$
$\ln x \ln x = 1$ $= (\ln x)^2 = 1$
$\ln x = \dfrac{1}{\ln x}$ $\ln x = \pm 1$
$x = e$ of e^{-1} $\ln x = 1 \to x = e$
$\ln x = -1 \to x = 1/e$

p 11.13 n°2 + zie p 11.2

$f(0) = 1$ en $f(1) = a$

a) $\forall x_1, x_2 \in \mathbb{R} : f(x_1 + x_2) = f(x_1) f(x_2)$
relevante groei in $[0, x_1]$ en in $[x_2, x_2 + x_1]$ (zelfde lengte)
$f(x_1 + x_2)$ is dus $f(x_1)(x_2)$
want $\dfrac{f(x_0)f(x_1)}{f(x_0)} = \dfrac{f(x_0)f(x_1 + x_2)}{f(x_0)f(x_2)} \iff f(x_1)f(x_2) = f(x_1 + x_2)$

b) $\forall r \in \mathbb{Q} : f(r) = a^r$

$$a = f(1) = f\left(\frac{1}{n} + \frac{1}{n} + \frac{1}{n} + \ldots + \frac{1}{n}\right) \text{ termen}$$
$$= f\left(\frac{1}{n}\right)^n$$

Dus $f(1/n) = a^{1/n}$

$f(r) = a^n$

p 11.13 n° 3

$a \in \mathbb{R}_0^+ \setminus \{1\}$ dan $\forall x \in \mathbb{R}_0^+, \forall s \in \mathbb{R} : \log_a(x^s) = s \log_a x$

1) Stel $y = \log_a x$ dan $x = a^y$

$$x^s = a^{ys}$$
$$\log_a x^s = y \cdot s$$
$$= s \cdot \log_a x$$

2) $\log_a(x^s) = \log_a(x \cdot x \cdot \ldots \cdot x)$ s keer

$= \log_a x + \log_a x + \ldots$ s keer

$= s \log_a x$

p 11.13 n° 4

$\dfrac{\text{\# inwoners nu} \cdot \text{groeiwet}}{2} = 4 \text{ milj}$

$\dfrac{6 \text{milj} \cdot 1{,}016^n}{2} = 4 \text{ milj.}$

$1{,}016^n = 4/3$

$\log_{1{,}016} 4/3 = n$

$n = \dfrac{\log 4/3}{\log 1{,}016} = 18{,}123$

Dus in 18 j na 1999 => 2017

p11.14 n°5

$$1{,}5\,M = \log\left(\frac{E}{2{,}5\,10^4}\right)$$

* $M = 7{,}9 \Rightarrow 11{,}85 = \log \frac{E}{2{,}5 \cdot 10^4}$ dus $10^{11{,}85} = \frac{E}{2{,}5\,10^4}$

$\Rightarrow E = 10^{11} \cdot 10^{0{,}85} \cdot 2{,}5 \cdot 10^4$

$= 17{,}7 \cdot 10^{15}$

$= 1{,}77 \cdot 10^{16}$

* $M = 6{,}5 \Rightarrow E = 10^{9{,}75} \cdot 2{,}5 \cdot 10^4$

$= 1{,}41 \cdot 10^{14}$

* $M = 4{,}8 \Rightarrow E = 10^{7{,}2} \cdot 2{,}5 \cdot 10^4$

$= 3{,}96 \cdot 10^{11}$

* België tov Gujarat : $\frac{1{,}77 \cdot 10^{16}}{3{,}96 \cdot 10^{11}} = 0{,}45 \cdot 10^5$

* België tov Noord-Marokko : $\frac{1{,}41 \cdot 10^{14}}{3{,}96 \cdot 10^{11}} = 0{,}36 \cdot 10^3$

p11.14 n°6

$$h = 1{,}84 \cdot 10^4 \log \frac{p_0}{p}$$

a) $h = 1{,}84 \cdot 10^4 \log 2 = 5539$ meter

b) $\frac{8848}{1{,}84 \cdot 10^4} = \log \frac{p_0}{p}$

$\frac{p_0}{p} = 10^{\frac{8848}{1{,}84 \cdot 10^4}}$

$\frac{p_0}{p} = 3, \ldots$

$p = \frac{1}{3}\, p_0$

p 11.14 n°7   fout rekenfout

* Pakistan : 1980 → 85 299 000   wanneer dubbel van 1980 ?

1993 → 123 438 000   (= 170 598 000)

$W_0 = 85\ 299\ 000$

$W_t = 85\ 299\ 000 \cdot e^{bt} = 123\ 438\ 000$ e^{bt} met $t = 13 = 1993 - 1980$

$e^{bt} = 1{,}447121303$

$b = 0{,}\cancel{0264289442}$

90288944

wanneer $85299000 \cdot e^{bt} = 170\ 598\ 000$?

$e^{bt} = 2$

$bt = \ln 2$

$t = \dfrac{\ln 2}{0{,}0264289442} = 26{,}22$

=> Binnen 26,22 jaar

14,5 jaar.

p 11.14 n°8

$K(t) = k_0 \cdot e^{jt}$ $j_{eff} = e^j - 1$

$\frac{6500}{5000} = 1{,}3 = e^{jt}$ $= \left(1 + \frac{j}{n}\right)^n - 1 = 9\%$

$\frac{\ln 1{,}3}{3} = j = 0{,}0875$

p 11.14 n°9

$t = 0 \rightarrow$ bedrag k   continue

$t = 5 \rightarrow 1{,}5\,k$

$K(t) = k_0\, e^{jt}$

$1{,}5\,k = k \cdot e^{5j} \Rightarrow 1{,}5 = e^{5j}$

$\log_e 1{,}5 = 5j \Leftrightarrow \frac{\ln 1{,}5}{5} = j = 8{,}11\ \%$

p 11.15 n°10

$$\left(1+\frac{i}{n}\right)^n - 1 = \left(1+\frac{0{,}05}{4}\right)^4 - 1 = 0{,}050945 = i_{eff}$$

$$\frac{500}{1{,}050945} + \frac{500}{1{,}050945^2} + \frac{500}{1{,}050945^3} + \dots \quad \text{tem } 10$$

$$500 \cdot \frac{1-\left(1/1+i\right)^{t}}{i} = 500 \cdot \frac{1-\left(\frac{1}{1{,}0509453}\right)^{10}}{0{,}0509453}$$

$$= 500 \cdot 7{,}68641$$

$$= 3843{,}2$$

p11.19 n°2

$$\cos\left(\frac{\pi}{12}\right) = \frac{1+\sqrt{3}}{2\sqrt{2}}$$

$$\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \frac{1+\sqrt{3}}{2\sqrt{2}} \qquad \cos(x+y) = \cos x \cos y + \sin x \sin y$$

$$\cos\frac{\pi}{3}\cos\left(-\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{3}\right)\sin\left(-\frac{\pi}{4}\right) = \frac{1+\sqrt{3}}{2\sqrt{2}}$$

$$\frac{1}{2}\cdot\frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2}\cdot -\frac{1}{2} = \frac{1+\sqrt{3}}{2\sqrt{2}}$$

p11.19 n°1

$g(t) = a \sin bt$ verschuiven volgens vector $\vec{v}(c,d)$

amplitude $= a$

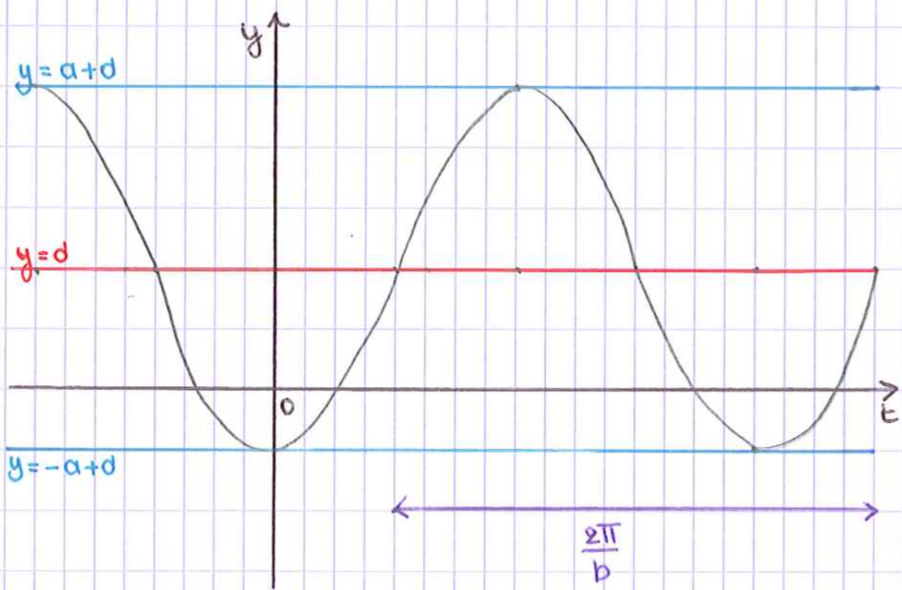
periode $= \frac{2\pi}{b}$

evenwichtslijn : $y = d$

min waarde $f(t) = d - a$

max waarde $f(t) = d + a$

startpunt periode $= P(c,d)$



p12.3 n°1

$f : \mathbb{R} \to \mathbb{R} : x \mapsto 2x^2 + 5$

Kies een $a \in \mathbb{R}$. Toon aan dat f continu is in a.

Kies willekeurige rij x_k die naar a convergeert.

Nu is $f(x_k) = 2x_k^2 + 5$.

($\to f(x_k)$ convergeert naar $f(a)$)

want $\lim_{k\to\infty} f(x_k) = \lim_{k\to\infty} (2x_k x_k + 5) = 5 + \lim_{x\to\infty} (2x_k x_k) = 5 + 2a^2 = f(a)$

Dus f is een continue fc want a is uit willekeurige definitiegebied gekozen en het was een willekeurige rij naar a

niet volledig

p12.4 n°2

$1 + 1/n$

Een fc $f : A \subseteq \mathbb{R} \to \mathbb{R}$ is niet continu in $a \in A$ indien er minstens 1 rij $(x_n)_{n\in\mathbb{N}}$ bestaat in A die naar a convergeert, maar zodat de rij $(f(x_n))_{n\in\mathbb{N}}$ niet naar $f(a)$ convergeert

1

$f(1)$

$f(1 + 1/n)$

↳ convergeert ≠ 1

niet volledig

p12.4 n°3

Neem een willekeurige $a \in A$. Merk op dat $a = (x, 0)$ voor een $x \in \mathbb{R}$ of dat $a = (0, y)$ voor een $y \in \mathbb{R}$.

We werken het argument hier uit voor $a = (x, 0)$ met $x \in \mathbb{R}$.

We zoeken dus een rij $(x_n, y_n)_{n\in\mathbb{N}} \in \mathbb{R}^2$ die convergeert naar $(x, 0)$, maar zodat de rij $(f(x_n, y_n))_{n\in\mathbb{N}}$ niet convergeert naar $f(x, 0) = 1$

Dan vinden we dat f overal continu is, behalve op $A = \{(x, y) \in \mathbb{R}^2 \mid xy = 0\}$

p 12. II n°1

$f : \mathbb{R} \to \mathbb{R} : x \mapsto c_n x^n + c_{n-1} x^{n-1} + \dots + c_1 x + c_0$ continu

f is opgebouwd uit 2 componentsfuncties, beide continu over $\mathbb{R}$

$$\begin{cases} f_1 : \mathbb{R} \to \mathbb{R} : x \mapsto c \\ f_2 : \mathbb{R} \to \mathbb{R} : x \mapsto x \end{cases}$$

Hiermee kunnen we een nieuwe fc definiëren:

$(f_1 \circ f_2)_n : \mathbb{R} \to \mathbb{R} : x \mapsto f_1(x)_n \circ f_2(x_n) = c_n x^n$

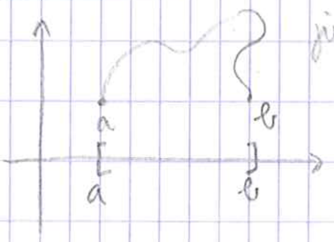
die dus eveneens continu is

Beschouw ook de functie $(f_1 f_2)_{n-1} : \mathbb{R} \to \mathbb{R} : x \mapsto f_1(x)_{n-1} \, f_2(x)_{n-1} = c_{n-1} x^{n-1}$
die continu is over $\mathbb{R}$

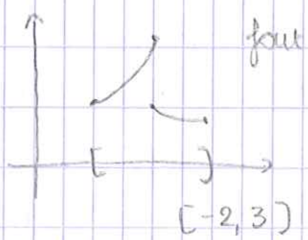
Dus definiëren we de fië $(f_1 f_2)_n + (f_1 f_2)_{n-1} : \mathbb{R} \to \mathbb{R} : x \mapsto c_n x^n + c_{n-1} x^{n-1}$ die ook continu is in $\mathbb{R}$

Dus onze veeltermfië, die bestaat uit bovenstaande componentsfiës, is ook continu.

p 12. II n°2



juist



fout $f(x) = 2x$ als $x \leq 1$
$= 2x + 1$ als $x > 1$

p 12. II n°3

$e^x = 3x$ heeft opl in $[0,1]$

Definieer $f : [0,1] \to \mathbb{R} : x \mapsto 3x - e^x$

Dan zal $\left.\begin{array}{l} f(0) = -1 < 0 \\ f(1) = 3 - e > 0 \end{array}\right\}$ teken verandert dus ergens zal nulpunt zitten!

andere methode: zoals Stephanie

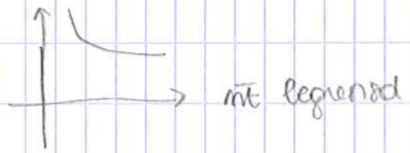
p12.12 n°5

niet begrensd : $f :]0,1[\to \mathbb{R} : x \to \ln x$ als $x > 0$

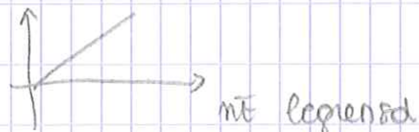
niet continu in $\emptyset$: $f : \mathbb{R} \to \mathbb{R} : x \mapsto \begin{cases} 1 & \text{als } x \geq 0 \\ 0 & \text{als } x < 0 \end{cases}$

OF

a) $]0,b] \Rightarrow f :]0,1] \to \mathbb{R} : x \to \frac{1}{x}$

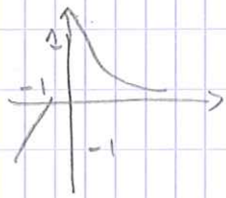


b) $]a, +\infty[\Rightarrow f :]0,\infty[\to \mathbb{R} : x \to x$



c) f is nt continu

$f\ [1,1] \to \mathbb{R} : x \mapsto \begin{cases} x & \text{als } x \leq 0 \\ \frac{1}{x} & \text{als } x > 0 \end{cases}$



p 12.23 n°1

a) $\lim_{x \to 1} \frac{x^3-1}{x-1} = \lim_{x \to 1} \frac{(\cancel{x-1})(x^2+x+1^2)}{(\cancel{x-1})} = 3$

b) $\lim_{x \to \infty} \frac{x^3-1}{x-1} = \lim_{x \to \infty} \frac{(x-1)}{(x-1)}(x^2+x+1) = \infty$

d) $\lim_{x \to \infty} \frac{x^6+5x^2+1}{x^4-x^6} = \lim_{x \to \infty} \left(\frac{x^6}{x^6}\left(\frac{1+5/x^4+1/x^6}{1/x^2-1}\right)\right) = \lim_{x \to \infty} \left(\frac{1+5/x^4+1/x^6}{1/x^2-1}\right)$

$= -\frac{1}{1} = -1$

f) $\lim_{x \to \infty} \sqrt{9x^2+1} - 3x = \lim_{x \to \infty} \frac{[(\sqrt{9x^2+1}-3x)(\sqrt{9x^2+1}+3x)]}{\sqrt{9x^2+1}+3x}$

$= \lim_{x \to \infty} \frac{(9x^2+1)-9x^2}{\sqrt{9x^2+1}+3x} = \lim_{x \to \infty} \frac{1}{\sqrt{9x^2+1}+3x} = 0$

p 12.23 n°2

$f = x \to 1/x$

$g = y \to e^y$

a) $\lim_{x \to \infty} e^{1/x} = e^{\lim_{x \to \infty}(1/x)} = e^0 = 1$

b) $\lim_{x \to -\infty} e^{(x^3)} = e^{\lim_{x \to -\infty} x^3} = e^{-\infty} = 0$

p 12.23 n°3

a) $\lim_{x \to \infty} e^{-x} \sin x$

$\left.\begin{array}{l} \lim_{x \to \infty} -e^{-x} = 0 \\ \lim_{x \to \infty} e^{-x} = 0 \end{array}\right\} \lim_{x \to \infty} \frac{\sin x}{e^x} = 0$

? b) $\lim_{x \to +\infty} x \sin \frac{1}{x}$ stel $y = \frac{1}{x}$ dan $\lim_{y \to \infty} \frac{\sin y}{y}$?

$\lim_{y \to \infty} \cos y \leq \lim_{y \to \infty} \frac{\sin y}{y} \leq \lim_{y \to \infty} 1$

?

$1 \leq \quad \ldots \quad \leq 1$

dus $\lim_{y \to \infty} \frac{\sin y}{y} = 1$

c) $\lim_{x \to 0} x^2 \sin \frac{1}{x} \Rightarrow \left.\begin{array}{l} \lim_{x \to 0} x^2(-1) = 0 \\ \\ \lim_{x \to 0} x^2(1) = 0 \end{array}\right\} \lim_{x \to 0} x^2 \sin \frac{1}{x} = 0$ (Sandwich)

d) $\lim_{x \to \infty} \frac{\cos x}{\sqrt{x}} \Rightarrow \left.\begin{array}{l} \lim_{x \to 0} \frac{-1}{\sqrt{x}} = 0 \\ \\ \lim_{x \to 0} \frac{1}{\sqrt{x}} = 0 \end{array}\right\} \lim_{x \to \infty} \frac{\cos x}{\sqrt{x}} = 0$

e) $\lim_{x \to \pm\infty} \frac{x + \sin x}{x} \Rightarrow \left.\begin{array}{l} \lim_{x \to \pm\infty} \frac{x-1}{x} = 1 \\ \\ \lim_{x \to \pm\infty} \frac{x+1}{x} = 1 \end{array}\right\} \lim_{x \to \pm\infty} \frac{x + \sin x}{x} = 1$ (*)

f) $\lim_{x \to \pm\infty} \frac{x + \sin x}{x + \cos x} \Rightarrow \left.\begin{array}{l} \lim_{x \to \pm\infty} \frac{x+1}{x-1} = 1 \\ \\ \lim_{x \to \pm\infty} \frac{x-1}{x+1} = 1 \end{array}\right\} \lim_{x \to \pm\infty} \frac{x + \sin x}{x + \cos x} = 1$

$$\frac{x-1}{x+1} < \frac{x + \sin x}{x + \cos x} \leq \frac{x+1}{x + \cos x} \leq \frac{x+1}{x-1}$$

(*) $\underset{\downarrow \atop 0}{-\frac{1}{x}} \leq \frac{\sin x}{x} \leq \underset{\downarrow \atop 0}{\frac{1}{x}}$

$= 1 + 0 = 1$

p 13. 18 n°1

a) $2x^5 - 4x^3 + 7 \to 10x^4 - 12x^2$

b) $a^x \to a^x \ln a$

c) $e^{-x^2} \to -2x e^{-x^2}$

d) $x^2 \sin x \to 2x \sin x + x^2 \cos x$

e) $\dfrac{1}{1+x^2} \to \dfrac{-2x}{(1+x^2)^2}$

f) $(1-2x)^7 \to 7(1-2x)^6 \cdot -2 = -14(1-2x)^6$

g) $\dfrac{1}{x^4+1} \to \dfrac{-4x^3}{(1+x^4)^2}$

h) $\dfrac{\sqrt{x}\ln x}{1+x^3} \to \dfrac{\left(\frac{1}{2}x^{-1/2}\ln x + \frac{\sqrt{x}}{x}\right)(1+x^3) - (\sqrt{x}\ln x)(3x^2)}{(1+x^3)^2}$

i) $(x+2)^3\sqrt{x^2+1} \to (3(x+2)^2)(\sqrt{x^2+1}) + ((x+2)^3 \frac{1}{2}(x^2+1)^{-1/2} \cdot 2x)$

$= (3x^2+12x+12)(\sqrt{x^2+1}) + \left(\dfrac{x(x+2)^3}{x^2+1}\right)^2$

j) $\dfrac{\sqrt{x}}{e^x+1} \to \dfrac{(1/2\, x^{-1/2})(e^x+1) - (\sqrt{x}\cdot e^x)}{(e^x+1)^2} = \dfrac{e^x(1-2x)+1}{2\sqrt{x}(e^x+1)^2}$

k) $|\sin x| \to -\cos x$ als $x \in \,](2k+1)\pi, (2k+2)\pi[$

$\cos x$ als $x \in \,]2k\pi, (2k+1)\pi[$

l) $-x\ln x \to -\ln x - \dfrac{x}{x} = -\ln x - 1$

m) $t^{4/3}(2t+1)^5 \to \frac{4}{3} t^{1/3}(2t+1)^5 + t^{4/3}\, 5(2t+1)^4 \cdot 2$

$= 4/3\, t^{1/3}(2t+1)^5 + 10\, t^{4/3}(2t+1)^4$

n) $\exp(\tan \theta/4) \to \exp(\tan \theta/4)\dfrac{1}{\cos^2(\theta/4)} \cdot \dfrac{1}{4}$ $\quad]0, 2\pi[$

o) $\ln(f(t)) \to \frac{f'(t)}{f(t)}$

p) $g(\pi r(x^2)h(x) \to g'(\pi r(x)^2)\, 2\pi r(x) \cdot r'(x)\, h(x) + g(\pi 2(x)^2)\, h'x$

p 13.19 n°2

$f: \mathbb{R} \to \mathbb{R} : x \mapsto \frac{x^2}{x^2+1}$ in $(1, f(1)) = (1, \frac{1}{2})$

$f'(x) = \frac{2x(x^2+1) - 2x^3}{(x^2+1)^2} = \frac{2x}{(x^2+1)^2}$

rico raaklijn $\Rightarrow f'(1) = \frac{1}{2}$

$g: \mathbb{R} \to \mathbb{R} : x \mapsto \underbrace{\frac{1}{2} + \frac{1}{2}(x-1)}_{f(a) + f'(a)(x-a)} = \frac{1}{2}x$

p 13.19 n°3 zie p13.12 voor meer uitleg (foutentheorie, 1e-orde benadering)

$f(x) \approx f(a) + f'(a)(x-a)$

a) $r_0 = 6$ fout $= 0{,}10$ $S = \pi r^2$

$\to f(r_0) \approx f(r) + f'(r)(r_0 - r)$

$f(r_0) - f(r) \approx f'(r)(r_0 - r)$

$\left.\begin{array}{l} f'(r) = 2\pi r \\ (r_0 - r) = 0{,}1 \end{array}\right\}$ $2\pi 6 \cdot 0{,}1 = 3{,}77$

$0{,}01 > \frac{\Delta S}{S} = \frac{r_0 S'(r_0)}{S(r_0)} \frac{\Delta r}{r_0} = 2\frac{\Delta r}{r_0}$

→ ~~[illegible]~~

99,5% nauwkeurig

b) kleinere fout dan 1% op opp ⇒ max fout van $\frac{1}{2}$% op straal ok

c) kost $= \frac{1}{100-p}$ Δk ? 75% → 77%

$\Delta k = k'(p)\,\Delta p = \frac{1}{(100-p)^2} \cdot 2 = \frac{1}{(100-75)^2} \cdot 2$

p 13.19 n°3

$$f(x) \approx f(a) + f'(a)(x-a)$$

a) $S = \pi r^2 \rightarrow S' = 2\pi r$
$r = 6$
fout $= r_0 - r = 0{,}10$

$$f(r_0) \approx f(r) + f'(r)(r_0 - r)$$
$$f(r_0) - f(r) \approx f'(r)(r_0 - r)$$
$$\approx 2\pi r \cdot 0{,}1$$
$$\approx 2\pi \cdot 6 \cdot 0{,}1 = 3{,}77$$

b) $$\frac{\Delta S}{S} = \frac{r_0 S'(r_0)}{S(r_0)} \frac{\Delta r}{r_0} = \frac{6(2\pi \cdot 6)}{\pi \cdot 6^2} \frac{\Delta r}{r_0} = 2 \frac{\Delta r}{r_0}$$

$$0{,}01 > \frac{\Delta S}{S} \rightarrow 0{,}01 > 2\frac{\Delta r}{r_0} \rightarrow 0{,}005 > \frac{\Delta r}{r_0}$$

→ mag maar 0,5% fout zijn
→ 99,5% nauwkeurig

c) $K = \frac{1}{100-p}$
$\Delta p = 2$

$$\Delta K = K'(p)\,\Delta p$$
$$= \frac{1}{(100-p)^2} \cdot 2 = \frac{1}{(100-75)^2} \cdot 2 = 0{,}0032 \text{ miljoen}$$
$$= 3200 \text{ euro}$$

procentueel: $K = \frac{1}{100-75} = 0{,}04 = 40.000$ euro

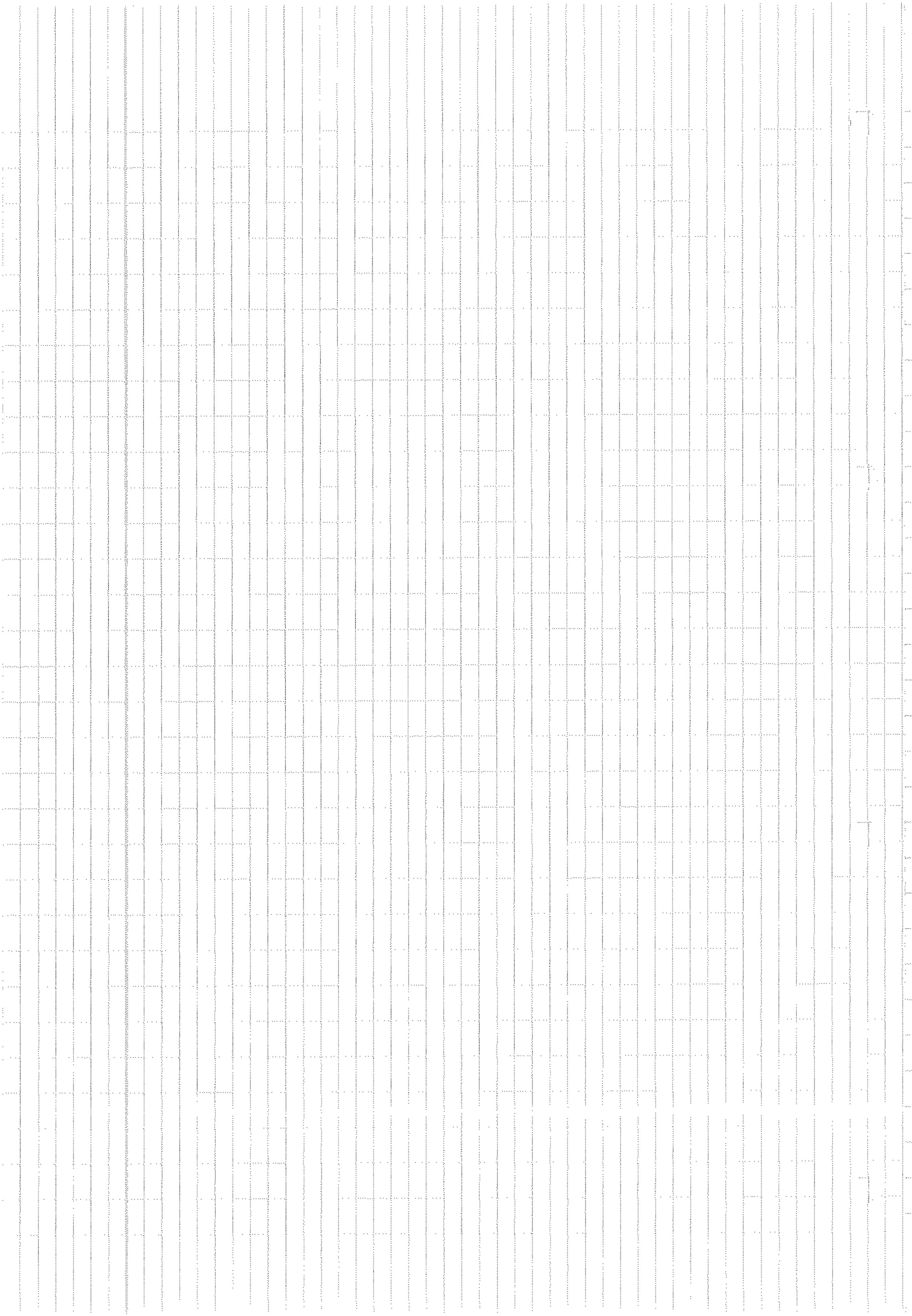
$\frac{0{,}0032}{0{,}04} = 0{,}08$ → met 8% toegenomen

p 13.20 n°4

$$V = \frac{4}{3}\pi r^3 \rightarrow V' = 4\pi r^2$$

a) $r = 5$
$r_0 - r = 0{,}2$

$$f(r_0) \approx f(r) + f'(r)(r_0 - r)$$
$$f(r_0) - f(r) \approx f'(r)(r_0 - r)$$
$$\approx 4\pi r^2 (r_0 - r)$$
$$\approx 4\pi \cdot 5^2 \cdot 0{,}2 = 62{,}831$$



p 13.26 n°1

Neen ! 1e : geen interval meer, maar heel $\mathbb{R}$

2e : orderelatie is versterkt naar $>$ ipv $\geq$

vb $f : \mathbb{R} \to \mathbb{R} : x \mapsto x^3$

→ monotoon stijgende functie op $\mathbb{R}$ maar $f'(x) > 0$ geldt niet voor alle $x \in \mathbb{R}$.

zie maar $f'(0) = 0$ voor $x = 0$

p 13.26 n°2

1) F = b F' = a F heeft n + 1 nulpunten
2) F = a F' = b F' heeft n nulpunten
3) F = a F' = b

p 13.27 n°3

a) $]3,4[: f' > 0$ dus FOUT
b) functie stijgt wel maar kan ook negatief zijn dus FOUT
c) FOUT
d) JUIST wel enkel relatief maximum
e) JUIST

p 13.27 n°4

TB voor alle $a, b \in \mathbb{R}$: $|\sin a - \sin b| < |a - b|$

Bewijs lagrange : $\sin'(c) = \dfrac{\sin(b) - \sin(a)}{b - a}$

$$\cos(c) = \frac{\sin(b) - \sin(a)}{b - a}$$

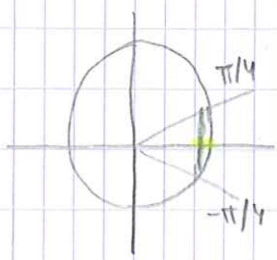
nu van beide abs. waarde want beide op reële as ($\mathbb{R}$) ↳ de grootte

a. $\begin{cases} \dfrac{|\sin b - \sin a|}{|b-a|} = |\cos c| \\ |\sin b - \sin a| < |a-b| \end{cases}$

p 13.27 n°5

TB voor alle $a, b \in [-\frac{\pi}{4}, \frac{\pi}{4}]$ geldt $|\tan b - \tan a| < 2|b-a|$

B Lagrange : $\tan' c = \dfrac{\tan b - \tan a}{b-a}$



$\frac{\sqrt{2}}{2} < \cos c < 1$

$\frac{1}{2} < \cos^2 c < 1$

$2 \geq \frac{1}{\cos^2 c} \geq 1$

$$\frac{1}{\cos^2 c} = \frac{\tan b - \tan a}{b-a}$$

$\frac{1}{\cos^2 c} \leq 2$ want in TB breng je $|b-a|$ naar linkerlid en $c \in -\frac{\pi}{4} ; \frac{\pi}{4}$

dus $\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

vul dit in in $\tan' c = \frac{1}{\cos^2 c} = \left(\frac{2}{\sqrt{2}}\right)^2 \leq 2$

↳ $1/0{,}5$ idd 2

wmbw !

p 13.27 n°6

Neem $x_a(t) : \mathbb{R}^+ \to \mathbb{R} : t \mapsto x_a(t)$ plaatsfunctie pers 1

$x_b(t) : \mathbb{R}^+ \to \mathbb{R} : t \mapsto x_b(t)$ plaatsfunctie pers 2

Neem willekeurig deelgebied van $\mathbb{R}^+$ vb $[0, T]$

Definieer nu verschilfunctie op het gesloten interval:

$$(x_a - x_b)(t) : [0,T] \to \mathbb{R} : t : x_a(t) - x_b(t)$$

Verschil in plaats van beide personen verloopt continu over definitiegebied $[0,T]$.
Deze verschilfunctie is afleidbaar & we verzwakken tot interval $(0,T)$

B uit ongerijmde : veronderstel dat ze meerdere keren kruisen op t_1 en t_2 waarbij we $t_1 < t_2$

dan: $x_a(t_1) - x_b(t_1) = 0$ en $x_a(t_2) = x_b(t_2)$

Er bestaat dan een $c \in (t_1, t_2)$: $x'_a(c) = x'_b(c)$

$\rightarrow$ minstens eenmaal = snelheid in (t_1, t_2)

maar we weten dat $x'_a(t) \neq x'_b(c)$ over alle $t \in \mathbb{R}^+$

wmbw!

p13.27 n°7

TB $a, b \in \mathbb{R}$ met $a > 0$

precies 1 $x \in \mathbb{R}$ waarvoor $x^3 + ax + b = 0$ $\qquad f' = 3x^2 + a$

B weten of functie x-as snijdt om opl. te krijgen

$\lim_{x \to \infty} f(x) = +\infty$ en $\lim_{x \to -\infty} f(x) = -\infty$

$\Rightarrow$ de x-as is minstens 1 keer gesneden

Stel: er is een element c van c_1 en c_2 waarvoor geldt: $f(c_1) = f(c_2) = 0$

Rolle: continue functie en begrensd gesloten interval

er is een element c van c_1 en c_2 waarvoor geldt dat $f'(c) = 0$

dit geeft: $f'(c) = 3c^2 + a = 0$ met $a > 0$

$\Rightarrow c^2 = -\frac{a}{3}$

Dit kan niet want LL positief en RL negatief

p 13.27 n°8

TW $\lim_{x \to \infty} e^x - x - 2 = \infty$ $\lim_{x \to -\infty} e^x - x - 2 = \infty$

moeten dus minstens 2 nulpunten zijn

$\lim_{x \to 0} 1 - 0 - 2 = -1 \to$ negatief

Rolle Stel er is een element c van c_1, c_2 en c_3 waarvoor geldt dat $f'(c) = 0$

$f'(c) = e^x - 1 = 0$

$e^x = 1$

$f : \mathbb{R} \to \mathbb{R} : x \mapsto e^x - x - 2$

Stel f heeft 3 nulpunten : x_1, x_2, x_3 met $x_1 < x_2 < x_3$

Dan $f(x_1) = f(x_2) = f(x_3) = 0$

Rolle : $\exists\, c_1 \in \,]x_1, x_2[\; : f'(c_1) = 0$

$\exists\, c_2 \in \,]x_2, x_3[\; : f'(c_2) = 0$

Maar $f'(x) = e^x - 1 = 0$

$e^x = 1$

$x = \ln 1 = 0$

f' heeft maar 1 nulpunt

STRIJDIG

p 13.42 n° 1

a) $f'(x) = 2(x^3+1)\cdot 3x^2 = 6x^5 + 6x^2$

$f''(x) = 30x^4 + 12x$

b) $f^{7\times 1469} = 0$

c) $f'(\theta) = -2\sin 2\theta \qquad f'''(\theta) = 8\sin 2\theta$

$f''(\theta) = -4\cos 2\theta \qquad f''''(\theta) = 16\cos 2\theta = 2^4 f$

$f^{(102)}(\theta) = -2^{102}\cos 2\theta \qquad \longrightarrow f^{(100)} = 2^{100}\cdot f$

$f^{(101)} = -2^{101}\sin 2\theta$

$\cancel{f^{(102)} = -2\sin^{102}\cos 2\theta} \qquad f^{(102)} = -2^{102}\cos 2\theta$

d) $y = (A+Bx)\cdot e^{2x}$

$y' = B\cdot e^{2x} + 2e^{2x}(A+Bx)$

$= e^{2x}(B + 2A + 2Bx)$

$y'' = 2e^{2x}(B + 2A + 2Bx) + 2Be^{2x}$

$= 2e^{2x}(B + 2A + 2Bx + B)$

$y'' - 4y' + 4y = 2e^{2x}(2B - 2A + 2Bx) - 4e^{2x}(B + 2A + 2Bx) + 4e^{2x}(A + Bx)$

$= 2e^{2x}(\cancel{2B} + \cancel{2A} + 2Bx - 2B - \cancel{4A} - 4Bx + \cancel{2A} + 2Bx)$

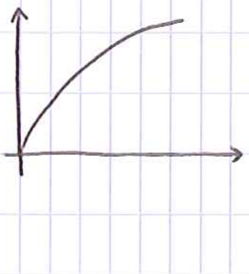
$= 2e^{2x}$

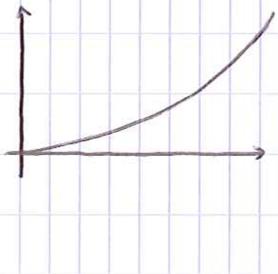
p 13.42 n° 2

$Q : \mathbb{R}^+ \to \mathbb{R} : q_A \mapsto Q(q_A)$

a) afnemende meeropbrengsten $Q'(q_A) \Rightarrow \oplus$ $Q''(q_A) \Rightarrow \ominus$ want je zegt

toenemende meeropbrengsten $Q'(q_A) \Rightarrow \oplus$ $Q''(q_A) \Rightarrow \oplus$





b) afnemende: $Q'(q_A) = \frac{2}{2q_A+1}$ $\quad Q''(q_A) = \frac{-4}{(2q_A+1)^2} < 0$

p 13.42 n°3

$f: \mathbb{R} \to \mathbb{R}: x \mapsto x^3 - 2x + 1$

$f(x) = (x-1)(x^2+x-1)$

	1	0	-2	1
1		1	1	-1
	1	1	-1	0

$D = 5$

nulpunten: $x_1 = 1$

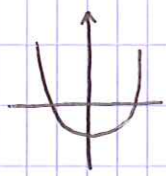
$x_2 = \frac{-1+\sqrt{5}}{2} = 0{,}618$

$x_3 = \frac{-1-\sqrt{5}}{2} = -1{,}618$

$f'(x) = 3x^2 - 2$

$3x^2 - 2 = 0$

$x = \pm\sqrt{\frac{2}{3}}$ < $0{,}816$ / $-0{,}816$



$f''(x) = 6x$

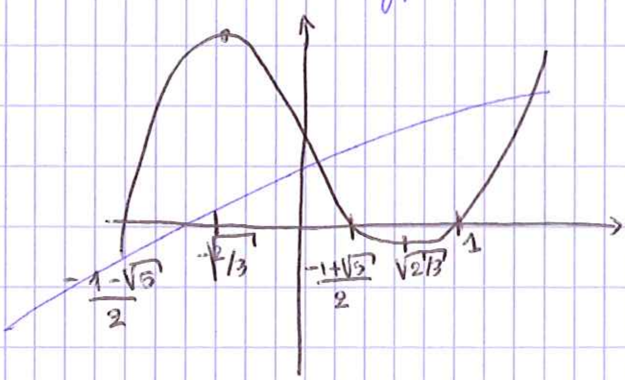
$x = 0$

x		$-\sqrt{2/3}$		0		$\sqrt{2/3}$	
$f'(x)$	+	0	−	−	−	0	+
$f''(x)$	−	−	−	0	+	+	+
$f(x)$	↗	MAX	↘	∄	↘	MIN	↗

buigpunt

$-0{,}544 + 1{,}63 + 1$

$0{,}544 - 1{,}63 + 1$



p 13.42 n°4

a) $f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

1 nulpunt dus 1 buigpunt namelijk $\left(\frac{-b}{3a}, f\left(\frac{-b}{3a}\right)\right)$

b) $\left[f(a-h) + f(a+h)\right]\frac{1}{2} \overset{(\text{uit } a)}{=} \left[f\left(\frac{-b}{3a} - h\right) + f\left(\frac{-b}{3a} + h\right)\right]\frac{1}{2}$

p 13.42 n°5

$f(x) = 5x^3 - 6x^2 + 2x + 1$ $\quad f(2) = 21$

$f'(x) = 15x^2 - 12x + 2$ $\quad f'(2) = 38$

$f''(x) = 30x - 12$ $\quad f''(2) = 48$

$f'''(x) = 30$ $\quad f'''(2) = 30$

$\rightarrow 5(x-2)^3 + 24(x-2)^2 + 38(x-2) + 21 \rightarrow$ 3e orde benadering :

$\frac{f'''(a)}{3!}(x-a)^3 + \frac{f''(a)}{2}(x-a)^2 + f'(a)(x-a) + f(a)$

p 13.42 n°6

Middelwaardestelling Taylor

$f : \mathbb{R} \rightarrow \mathbb{R} : x \mapsto \sin(x)$ benaderen door (rond $x = 0$)

$P_8 : \mathbb{R} \rightarrow \mathbb{R} : \sum_{n=0}^{8} \frac{f^n(0)}{n!}$

Gelijkheid tussen $f(x)$ en $P_8(x)$ bekom je door fouten "aan te nemen"

Deze zijn vd orde $R_8(c) = \frac{\sin^{10}(c)\, x^{10}}{10!}$

absolute fout is $x = \pi \Rightarrow R_8(\pi) = \frac{\pi^{10}}{10!}$

$\hookrightarrow$ groeit sneller

p 13.43 n°8

$f :]-\infty, 1[\to \mathbb{R} : x \mapsto \frac{1}{1-x}$ rond 0

$f(x) = \frac{1}{1-x}$ $\quad f(0) = 1$

$f'(x) = \frac{1}{(1-x)^2}$ $\quad f'(0) = 1$ $\qquad 0(1-x) - 1(-1) = 1$

$f''(x) = \frac{2(1-x)}{(1-x)^4} = \frac{2}{(1-x)^3}$ $\quad f''(0) = 2$ $\qquad 0(1-x)^2 + 1 \cdot 2(x-x)$

$f^{(3)}(x) = \frac{3(1-x)^2}{(1-x)^6} = \frac{3}{(1-x)^4}$ $\quad f^{(3)}(0) = 3$

$f^{(n)}(x) = \frac{n}{(1-x)^{n+1}}$ $\qquad f^{(n)}(0) = n$

p 13.43 n°9

$f :]-1, 1[\to \mathbb{R} : x \mapsto \ln(1+x)$

$f(x) = \ln(1+x)$ $\qquad f(0) = \ln 1 = 0$

$f'(x) = \frac{1}{1+x}$ $\qquad f'(0) = 1$

$f''(x) = -\frac{1}{(1+x)^2}$ $\qquad f''(0) = -1$

$f'''(x) = \frac{2(1+x)}{(1+x)^4} = \frac{2}{(1+x)^3}$ $\qquad f'''(0) = 2$

$f^{(4)}(x) = \frac{-6}{(1+x)^4}$ $\qquad f^{(4)}(0) = -6$

$f^{(n)}(x) = \frac{(x+1) \cdot (-1)^n}{(x+1)^n}$ $\qquad f^{(n)}(0) = -(-1)^n$

p 13.43 n°10

$f: \mathbb{R} \to \mathbb{R} : x \mapsto x \sin x$ rond 0

$f(x) = x \sin x$ $\quad f(0) = 0$

$f'(x) = \sin x + x\cos x$ $\quad f'(0) = 0$

$f''(x) = \cos x + \cos x - x \sin x$ $\quad f''(0) = 2$

$= 2\cos x - f(x)$

$f^{(3)}(x) = -2\sin x - \sin x - x\cos x$ $\quad f^{(3)}(0) = 0$

$= -3\sin x - x\cos x$

$f^{(4)}(x) = -3\cos x - \cos x + x \sin x$ $\quad f^{(4)}(0) = -4$

$= -4\cos x + f(x)$

p 13.45 n°1

a) $\lim\limits_{x \to 0} \dfrac{e^x - 1}{\ln(1+x)} \overset{0/0}{\underset{H}{=}} \lim\limits_{x \to 0} [e^x(1+x)] = 1$

b) $\lim\limits_{x \to 0} \dfrac{x - \sin x}{x^3} \overset{0/0}{\underset{H}{=}} \lim\limits_{x \to 0} \dfrac{1 - \cos x}{3x^2} \overset{0/0}{\underset{H}{=}} \lim\limits_{x \to 0} \dfrac{\sin x}{6x} \overset{0/0}{\underset{H}{=}} \lim\limits_{x \to 0} \dfrac{\cos x}{6} = \dfrac{1}{6}$

c) $\lim\limits_{x \to 1} \dfrac{\ln x}{x^2 - 1} \overset{0/0}{\underset{H}{=}} \lim\limits_{x \to 1} \dfrac{1}{2x^2} = \dfrac{1}{2}$

d) $\lim\limits_{x \to +\infty} \dfrac{e^x}{x^3 + 2x^2 + 1} \overset{\infty/\infty}{\underset{H}{=}} \lim\limits_{x \to +\infty} \dfrac{e^x}{3x^2 + 4x} \overset{+\infty/+\infty}{\underset{H}{=}} \lim\limits_{x \to +\infty} \dfrac{e^x}{6x + 4} \overset{+\infty/+\infty}{\underset{H}{=}} \lim\limits_{x \to +\infty} \dfrac{e^x}{6} = +\infty$

e) $\lim\limits_{x \to +\infty} e^{-x}\sqrt{x} = \lim\limits_{x \to +\infty} \dfrac{\sqrt{x}}{e^x} \overset{+\infty/+\infty}{\underset{H}{=}} \lim\limits_{x \to +\infty} -\dfrac{1}{2\sqrt{x}\, e^x} = -\infty$

f) $\lim\limits_{x \to \pi/2} \underbrace{(x - \pi/2) \cdot \tan x}_{0\,(+\infty) = \text{onbep. vorm}} = \lim\limits_{x \to \pi/2} \dfrac{(x - \pi/2)\sin x}{\cos x} \overset{0/0}{\underset{H}{=}} \lim\limits_{x \to \pi/2} -\dfrac{\sin x + (x - \pi/2)\cos x}{\sin x} = 1$

p 13.45 n°2

$$\lim_{x\to+\infty} e^{-x}\cdot x^n = 0 \quad \text{voor elke } n \in \mathbb{N}$$

* $$\lim_{x\to+\infty} \frac{x^n}{e^x} \overset{\frac{+\infty}{+\infty}}{\underset{H}{=}} \lim_{x\to+\infty} \frac{n\cdot x^{n-1}}{e^x} \underset{H}{=} \lim_{x\to+\infty} \frac{n\cdot(n-1)\cdot x^{n-2}}{e^x} = \lim_{x\to+\infty} \frac{n!}{e^x} = 0$$

* Stel $S = \left\{ n \in \mathbb{N} \mid \lim_{x\to+\infty} e^{-x} x^n = 0 \right\}$

TB : $S = \mathbb{N}$

B : $0 \in S$: $\lim_{x\to 0} \frac{x^n}{e^x} = \frac{0}{1} = 0$ Ok!

als $m \in S$, dan is $m+1 \in S$

$$\lim_{x\to+\infty} e^{-x} x^{m+1} = \lim_{x\to+\infty} \frac{x^m\cdot x}{e^x} \overset{\frac{+\infty}{+\infty}}{\underset{H}{=}} \lim_{x\to+\infty} \frac{(m+1)\,x^m}{e^x}$$

$$= (m+1)\cdot \lim_{x\to\infty} e^{-x} x^m = (m+1)\cdot 0 = 0$$ Ok!

p 13.45 n°3

?

$$\lim_{x\to+\infty} \frac{x+\sin x}{x} = \lim_{x\to+\infty} 1 + \lim_{x\to+\infty} \frac{\sin x}{x} \ldots$$

p 14.7 n°1

a) $f_1 : \mathbb{R} \times \mathbb{R}_0 \to \mathbb{R} : (x,y) \mapsto \frac{x^2}{y} + e^{xy}$

$D_1 f_1 : \mathbb{R} \times \mathbb{R}_0 \to \mathbb{R} : (x,y) \mapsto \frac{2x}{y} + ye^{xy}$

$D_2 f_1 : \mathbb{R} \times \mathbb{R}_0 \to \mathbb{R} : (x,y) \mapsto -\frac{x^2}{y^2} + xe^{xy}$

b) $f_2 : \mathbb{R}_0^+ \times \mathbb{R}_0^+ \to \mathbb{R} : (x,y) \mapsto \ln(x,y) \sin(1/x)$

$D_1 f_2 : \mathbb{R}_0^+ \times \mathbb{R}_0^+ \to \mathbb{R} : (x,y) \mapsto \frac{1}{x} \sin\left(\frac{1}{x}\right) - \frac{1}{x^2} \cos\left(\frac{1}{x}\right) \ln(xy)$

$D_2 f_2 : \mathbb{R}_0^+ \times \mathbb{R}_0^+ \to \mathbb{R} : (x,y) \mapsto \frac{1}{y} \sin\left(\frac{1}{x}\right)$

c) $f_3 : \mathbb{R}_0^2 \to \mathbb{R} : (r,\theta) \mapsto r^2 e^{r\cos\theta}$

$D_1 f_3 : \mathbb{R}_0^2 \to \mathbb{R} : (r,\theta) \mapsto 2r\, e^{r\cos\theta} + r^2 \cos\theta\, e^{r\cos\theta}$

$D_2 f_3 : \mathbb{R}_0^2 \to \mathbb{R} : (r,\theta) \mapsto -r^3 \sin\theta\, e^{r\cos\theta}$

d) $f_4 : \mathbb{R}_0^+ \times \mathbb{R}_0^+ \times \mathbb{R}_0^+ \to \mathbb{R} : (x_1,x_2,x_3) \mapsto x_1^2 \sin(x_2+x_3) + \ln\left(\frac{x_1}{x_2+x_3}\right)$

$D_1 f_4 : \mathbb{R}_0^+ \times \mathbb{R}_0^+ \times \mathbb{R}_0^+ \to \mathbb{R}\,(x_1,x_2,x_3) \mapsto 2x_1 \sin(x_2+x_3) + \frac{1}{x_1}$

$D_2 f_4 : \mapsto x_1^2 \cos(x_2+x_3) - \frac{1}{x_2+x_3}$

$D_3 f_4 : \mapsto x_1^2 \cos(x_2+x_3) - \frac{1}{x_2+x_3}$

e) $f_5 : \mathbb{R}_0 \times \mathbb{R}_0 \to \mathbb{R} : (x,y) \mapsto xy + 2x^3 + \frac{1}{x^2 y}$

$D_1 f_5 : \mathbb{R}_0 \times \mathbb{R}_0 \to \mathbb{R} : (x,y) \mapsto y + 6x^2 - \frac{2}{x^3 y}$

$D_2 f_5 : \mathbb{R}_0 \times \mathbb{R}_0 \to \mathbb{R} : (x,y) \mapsto x - \frac{1}{x^2 y^2}$

f) $f_6 : \mathbb{R} \times \mathbb{R}_0^+ \times \mathbb{R}_0^+ \to \mathbb{R} : (x,y,z) \mapsto z^2 \cos x \sin y + \ln(y/z) + e^{x^3 \sin(xz)}$

$D_1 f_6 : \quad \mapsto -z^2 \sin x \sin y + e^{x^3 \sin(xz)} (3x^2 \sin(xz) + x^3 \cos(xz))$ $\cdot z$

$D_2 f_6 : \quad \mapsto z^2 \cos x \cos y + 1/y$

$D_3 f_6 : \quad \mapsto 2z \cos x \sin y - \frac{1}{z} + e^{x^3 \sin(xz)} \cdot x^4 \cos(xz)$

p14.7 n°2

a) $f : \mathbb{R}^2 \to \mathbb{R} : (x,y) \mapsto \begin{cases} 1 & \text{als } xy = 0 \\ 0 & \text{als } xy \neq 0 \end{cases}$

$D_1 f(x,0) = 0$ voor alle $x \in \mathbb{R}$
$D_2 f(0,y) = 0$ voor alle $y \in \mathbb{R}$
$D_1 f(0,y)$ bestaat niet voor alle $y \in \mathbb{R}_0$
$D_2 f(x,0)$ bestaat niet voor alle $x \in \mathbb{R}_0$
$D_1 f(x,y) = D_2 f(x,y) = 0$ voor alle $(x,y) \in \mathbb{R}_0 \times \mathbb{R}_0$

b) $f : \mathbb{R}^2 \to \mathbb{R} : (x,y) \mapsto |x|$

$D_1 f(0,y)$ bestaat niet voor alle $y \in \mathbb{R}$
$D_1 f(x,y) = 1$ voor alle $x > 0$ en $y \in \mathbb{R}$
$D_1 f(x,y) = -1$ voor alle $x < 0$ en $y \in \mathbb{R}$
$D_2 f(x,y) = 0$ voor alle $(x,y) \in \mathbb{R}$

c) $f : \mathbb{R}^2 \to \mathbb{R} : (x,y) \mapsto |x-y|$

? $\longrightarrow$ $D_1 f(x,x)$ en $D_2 f(x,y)$ bestaan niet voor alle $x \in \mathbb{R}$
$D_1 f(x,y) = 1$ voor alle $x > y$
$D_1 f(x,y) = -1$ voor alle $x < y$
$D_2 f(x,y) = -1$ voor alle $x > y$
$D_2 f(x,y) = 1$ voor alle $x < y$

p 14.15 n°1

$$f : \mathbb{R}^3 \to \mathbb{R} : (x,y,z) \mapsto 2x - 3y + z + 5$$

a)

$$\lim_{(x,y,z)\to a} \frac{|f(x,y,z) - g(x,y,z)|}{\|(x,y,z) - a\|} = 0$$

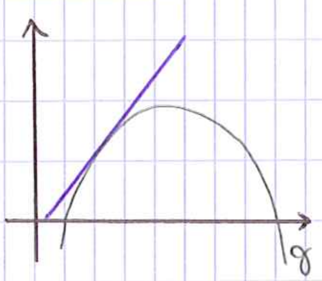
Als $f(x,y,z) \approx g(x,y,z)$, dan $f(x,y,z) - g(x,y,z) = 0$

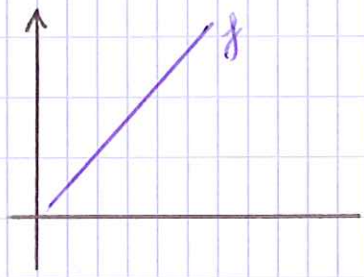
Dus $\lim_{(x,y,z)\to a} \frac{0}{\|(x,y,z)-a\|} = 0$

Dus $g(x,y,z) \to 2x - 3y + z + 5$

b) voorschrift g is onafhankelijk van a

p 14.15 n°2





1^e-orde ben. volgt fc

fc is lineair dus 1^e-orde ben. volgt fc

$\Rightarrow$ f heeft rond elk punt een 1^e-orde benadering en dat is f zelf.

p 14.15 n°3

$$f : \mathbb{R}^2 \to \mathbb{R} : (x,y) \mapsto 9 - x^2 - y^2 \qquad f(1,2) = 4$$

$$\lim_{(x,y)\to(1,2)} \frac{|f(x,y) - g(x,y)|}{\|(x,y) - (1,2)\|} = 0$$

$D_1 f(1,2)(x-1) = -2(x-1) = -2x+2$
$D_2 f(1,2)(y-2) = -4(y-2) = -4y+8$

$g: \mathbb{R}^2 \to \mathbb{R} : (x,y) \to f(a) + \sum_{i=1}^{n} D_i f(x,y)(x_i - a_i)$

$g: \mathbb{R}^2 \to \mathbb{R} : (x,y) \to 4 + (-2x+2) + (-4y+8)$
$\to 14 - 2x - 4y$

Als f een 1e-orde benadering heeft (namelijk g), dan moet

$$\lim_{(x,y)\to(1,2)} \frac{|f(x,y) - g(x,y)|}{\|(x,y) - (1,2)\|} = 0$$

$$\left.\begin{array}{l} f(1,2) = 4 \\ g(1,2) = 4 \end{array}\right\} f(x,y) - g(x,y) = 0 \qquad \text{Dus } \lim = 0$$

vgl raak~~lijn~~ VLAK: $z = -2x - 4y + 14$

p14.16 n°4

$f: \mathbb{R}^3 \to \mathbb{R} : (x,y,z) \to 4\sin(x(y+z)) - yz$
$f(0,1,2) = -2$

$D_1 f(0,1,2) = 4(y+z)\cos(x(y+z)) = 12$
$D_2 f(0,1,2) = 4x\cos(x(y+z)) - z = -2$
$D_3 f(0,1,2) = 4x\cos(x(y+z)) - y = -1$

$g: \mathbb{R}^3 \to \mathbb{R}\ (x,y,z) \mapsto f(0,1,2) + \sum_{i=1}^{n} D_i f(x,y,z)(x_i - a_i)$

$\mapsto -2 + 12(x-0) + (-2)(y-1) + (-1)(z-2)$
$\mapsto 12x - 2y - z + 2$

1e-orde vgl: $\lim_{(x,y,z)\to(0,1,2)} \dfrac{|f(x,y,z)-g(x,y,z)|}{\|(x,y,z)-(0,1,2)\|} = 0$

$$\left.\begin{array}{l} f(0,1,2) = -2 \\ g(0,1,2) = -2 \end{array}\right\} \begin{array}{l} f(0,1,2) - g(0,1,2) = 0 \\ \text{dus } \lim = 0 \end{array}$$

$\Rightarrow g = -12x - 2y - z + 2$

p 14.16 n°5

a) $f(x,y) = 4x^2 + y^2 - 3xy \qquad a = (1,1)$

$f(1,1) = 2$

$D_1 f(1,1) = 8x - 3y = 5$

$D_2 f(1,1) = 2y - 3x = -1$

$g(x) = 2 + 5(x-1) - 1(y-1) = 5x - y - 2$

b) $f(x,y) = \ln(1 + x^2 + y^2) \qquad a = (1,0)$

$f(1,0) = \ln 2$

$D_1 f(1,0) = \dfrac{1}{1 + x^2 + y^2} \cdot 2x = \dfrac{1}{2} \cdot 2x = x = 1$

$D_2 f(1,0) = \dfrac{1}{1 + x^2 + y^2} \cdot 2y = \dfrac{1}{2} \cdot 2y = y = 0$

$g(x) = \ln 2 + 1(x-1) + 0(y-0) = \ln 2 - 1 + x$

c) $f(x,y) = x^3 y + 4x^2 y^2 - xy^3 \qquad a = (-1,2)$

$f(-1,2) = -2 + 16 + 8 = 22$

$D_1 f(-1,2) = 3x^2 y + 8xy^2 - y^3 = -34$

$D_2 f(-1,2) = x^3 + 8y x^2 - 3y^2 x = 27$

$g(x) = 22 + (-34)(x+1) + 27(y-2)$

$\quad = -34x + 27y - 66$

d) $f(x,y) = e^{-(x^2+y^2)}$ $\qquad a = (0,0)$

$f(0,0) = 1$

$D_1 f(0,0) = -2x\left(e^{-(x^2+y^2)}\right) = 0$

$D_2 f(0,0) = -2y\left(e^{-(x^2+y^2)}\right) = 0$

$g(x) = 1$

p 14.16 n°6

$A : (L,B) \rightarrow A(L,B) = L \cdot B$

$\Delta A \approx \overset{D_2 A}{B} \cdot \Delta L + \overset{D_1 A}{L} \cdot \Delta B$

$\dfrac{\Delta A}{A} \approx \dfrac{B \cdot \Delta L + L \cdot \Delta B}{\underset{f(A)}{L \cdot B}} \approx \dfrac{\Delta L}{L} + \dfrac{\Delta B}{B}$

$\Rightarrow$ B = kortste zijde $\rightarrow$ fout groeit sneller want ΔB is absolute fout = absolute fout ΔL. Echter $B < L$ dus procentueel moet meetfout van breedte kleinste

p 14.16 n°7 zijn, immers $B < L \rightarrow$ fout groeit sneller want $\dfrac{\Delta B}{B}$

$Opp(a,b,\theta) = \frac{1}{2}\, ab \sin\theta$

$\Delta opp = D_1 Opp \cdot \Delta a + D_2 Opp\ \Delta b + D_3 Opp\ \Delta\theta$

$= 2{,}5 \cdot 0{,}1 + 5 \cdot 0{,}1 + 86{,}6 \cdot \dfrac{\pi}{180}$

$= 2\,26\ cm^2$

p 14.16 n°8

$R = \frac{1}{g} v^2 \sin 2\theta$

alleen v is variabele in vgl als steen kasteel raakt (gn heuvel)

$R = \frac{1}{g} v^2 \sin(2 \cdot 30°)$

$$g(v,\theta) = \frac{1}{g} v^2 \sin 2\theta$$

$$D_1 g = \frac{1}{g} 2v \sin 2\theta$$

$$D_2 g = 1/g \cdot v^2 \cdot 2\cos 2\theta \cdot \frac{\pi}{180}$$

$$0 = \frac{2v}{g} \sin \pi/_3 \, x + 2v^2/_g \cos \pi/3 \cdot \pi/_{180}$$

$\Delta v = x$ $\quad \frac{\sqrt{3}}{2} x = -v \frac{1}{2} \frac{\pi}{180} \iff x = -v \cdot \frac{1}{\sqrt{3}} \cdot \frac{\pi}{180}$

$$\iff x = -v \cdot 0{,}01$$

$\Rightarrow$ Katapultsnelheid met 1% verlagen

p 14.26 n°1

$f : \mathbb{R}^2 \to \mathbb{R} : (x,y) \mapsto x^3 + y^2$

$g : \mathbb{R} \to \mathbb{R}^2 : t \mapsto (\cos t, \sin t)$

$\Rightarrow (\cos t)^3 + (\sin t)^2$

$f : \mathbb{R} \to \mathbb{R} : t \mapsto (\cos t)^3 + (\sin t)^2$

$f' : \mathbb{R} \to \mathbb{R} : t \mapsto -3 \sin t (\cos t)^2 + 2 \cos t \sin t$

p 14.26 n°2

$f : \mathbb{R}^3 \to \mathbb{R} : (x,y,z) \mapsto xy + yz + xz$

$g : \mathbb{R}^3 \to \mathbb{R}^3 : (r, \theta, \varphi) \mapsto r\cos\theta \sin\varphi, r \sin\theta \sin\varphi, r\cos\varphi)$

$f \circ g : (r\cos\theta \sin\varphi \cdot r\sin\theta \sin\varphi) + (r\sin\theta\sin\varphi \cdot r\cos\varphi)$

$+ (r\cos\theta \sin\varphi \cdot r\cos\varphi)$

$= r^2 \cos\theta \sin\theta \sin^2\varphi + r^2 \sin\theta \sin\varphi \cos\varphi + r^2 \cos\theta \sin\varphi \cos\varphi$

$D_1(f\circ g) = 2r \cos\theta \sin\theta \sin^2\varphi + 2r \sin\theta \sin\varphi \cos\varphi + 2r\cos\varphi \sin\varphi \cdot \cos\varphi$

$D_2(f\circ g) = -r^2 \sin\theta\cos\theta \sin^2\varphi + r^2 \sin\theta \cos\varphi - r^2 \sin\theta \sin\varphi \cos\varphi$

p 14.26 n°3

$f : \mathbb{R}^3 \to \mathbb{R}^2 : (x_1, x_2, x_3) \mapsto (x_1 x_2 x_3, x_1^2 - x_2)$

$g : \mathbb{R}^2 \to \mathbb{R} : (y_1, y_2) \mapsto y_1^2 - y_2$

* $g \circ f : (x_1 x_2 x_3)^2 - (x_1^2 - x_2)$

$D_2(g\circ f) : 2x_1^2 \; x_2 x_3^2 + 1$

p 14.26 n°4

$g \circ f : f_1(u), f_2(u), f_3(u) - e^{f_2(u) f_3(u)}$

a) $(g\circ f)'(u) : f_1'(u) f_2(u) f_3(u)^2 + f_2(u)' (f_1(u) f_3(u)^2 +$
$f_3(u) e^{f_2(u) f_3(u)}) + f_3'(u) (2 f_1(u) f_2(u) + f_2(u) \cdot e^{f_2 f_3}$

b) voor $i \in \{1,2\}$ $D_i(g \circ f) : \mathbb{R}^2 \to \mathbb{R} : (u,u) \mapsto D_i(g\circ f)(u,v)$

$D_1(g\circ f)(u,v) = D_1 g(f(u,v))(2u+v) + D_3 g(f(u,v))\cos(u-v)$

$D_2(g\circ f)(u,v) = D_1 g(f(u,v)) \cdot u + D_2 g(f(u,v))e^{v} + D_3 g(f(u,v)) \cdot \cos(u-v)$

c) $(g\circ f)' : \mathbb{R} \to \mathbb{R} : t \mapsto (g\circ f)'(t)$

$(g\circ f)'(t) = 3 f_1(t)^2 \cdot f_2(t) \cdot t\cos t + D_1 h(f_1(t), t\cos t)e^{t\cos t} \cdot f_1'(t)$
$+ f_1(t)^3 \cdot f_2'(t) + \cos t + f_1(t)^3 \cdot f_2(t)$
$+ D_2 h(f_1(t), t\cos t)e^{t\cos t} + h(f_1(t), t\cos t)e^{t\cos t}$
$(\cos t \cdot \sin t)$

p 14.27 n°5

$g(t) = f(1+t^2, 2+t^3)$

$g'(t) = f(2t, 3t^2)$

$g'(0) = f(0)$

$g(h) - g(0) = f(1+h, 2+h^3) - f(1,2) \to$ continue afg dus 1e-orde ben.

$f(1+h^2, 2+h^3) - f(1,2) \approx D_1 f(\ldots)h^2 + D_2 f(\ldots)h^3$

$\approx$ betekent: $\lim_{h\to 0} \frac{[f(1+h^2, 2+h^3) - f(1,2) - D_1 \ldots h^2 - D_2 \ldots h^3]}{h} = 0$

$\lim_{h\to 0} \left(\frac{[f(1+h^2, 2+h^3) - f(1,2)]}{h} - D_1 f(.)h - D_2 f(\ldots)h^2 \right) = 0$

$\lim_{h\to 0} (\quad) = 0 = g'(0)$

p 14.27 n°6

voor $i \in \{1,2\}$ $D_i h : \mathbb{R}^2 \to \mathbb{R} : (x,y) \mapsto D_i h(x,y)$

$D_1 h(x,y) = 2 f(x+3y, 2x)(D_1 f(x+3y, 2x) + 2 D_2 f(x+3y, 2x))$

$D_2 h(x,y) = 6 f(x+3y, 2x)(D_1 f(x+3y, 2x))$

p 14.27 n°7

1) $f: x^2y^3 + (x+y)y^4$

$$(x\lambda)^2(y\lambda)^3 + (\lambda x + \lambda y)(y\lambda)^4 = \lambda^5 x^2 y^3 + \lambda^5 (x+y) y^4$$
$$= \lambda^5 (x^2 y^3 + (x+y) y^4)$$
$$= \lambda^5 f(x)$$

→ homogeen van graad 5

2) $f: \dfrac{y^2 z + xy^2}{x} + x^3$

$$\frac{(y\lambda)^2(\lambda z) + (\lambda z)(\lambda y)^2}{\lambda x} + (\lambda x)^3$$

$$= \frac{\lambda^3 (yz) + \lambda^3 (xy)}{\lambda x} + \lambda^3 x^3$$

$$= \lambda^2 \frac{(yz) + (xy)}{x} + \lambda^3 x^3$$ → niet homogeen

3) $f: x^2 y \ln\left(\frac{2x}{3y}\right)$

$$(x\lambda)^2 (y\lambda) \ln\left(\frac{2x\lambda}{3y\lambda}\right) = \lambda^3 xy \ln \frac{2x}{3y}$$ → homogeen v. graad 3

4) $f: r^2 \sin\left(\frac{r}{3\theta}\right) + \theta r$

$$(r\lambda)^2 \sin\left(\frac{r\lambda}{3\theta\lambda}\right) + \theta\lambda r\lambda = r^2\lambda^2 \sin\frac{r}{3\theta} + \lambda^2 \theta r = \lambda^2 (f(r,\theta))$$

→ homogeen van graad 2

p 14.28 n°8

$$q_1 = \frac{10y}{p_1 + 5p_2} \qquad \frac{10y\lambda}{p_1\lambda + 5p_2\lambda} = \frac{10y}{p_1 + 5p_2}\,\frac{\lambda}{\lambda}$$ → graad 0

p14. 28 n°9

a) $Q(k,L) = \frac{1}{2} K^{1/3} L^{2/3}$ $\frac{1}{2} K^{1/3} \lambda^{1/3} L^{2/3} \lambda^{2/3} = \lambda(Q(k,L)) \to$ grad 1

1.15.15

Oef^n M15

Oef 1

$f: [-2,3] \to \mathbb{R} : x \to x^4 - 2x^2 + 1$
$= (x^2-1)^2$

stel $t = x^2$

$f(t) = (t-1)^2$ nulpt^n: $t = 1$

$f'(t) = 2(t-1)$ $\Rightarrow x^2 = 1 \Rightarrow x = -1$ of $x = 1$

max: $f(-2) = 9$ $f(0) = 1$

$f(3) = 10$

$\Rightarrow$ globaal max in 3, lokaal max in -2 en 0, lokaal min in 1 en -1

Oef 2

$f_a : \mathbb{R} \to \mathbb{R} : x \to x^4 - 2ax^2 + a^2$
$= (x^2 - a)^2$

$f'(a)(x) = 2(x^2-a) \cdot 2x = 4x(x^2-a)$

lokale extrema bij $x = 0$, $x = \sqrt{a}$ en $x = -\sqrt{a}$

$\boxed{a > 0}$

	$-\sqrt{a}$		0		$\sqrt{a}$	
$f_a(x)$	$\searrow 0$	$\nearrow$	a^2	$\searrow$	0	$\nearrow$
$f'_a(x)$	$-$ 0	$+$	0	$-$	0	$+$

lok max in 0, glob min in $-\sqrt{a}$ en $\sqrt{a}$

$\boxed{a \leq 0}$

		0	
$f_a(x)$	$\searrow$	.	$\nearrow$
$f'_a(x)$	$-$	0	$+$

glob min in 0

Oef 3

$A(l,b) = l \cdot b = 2000 \Rightarrow l = \frac{2000}{b}$

$O(b) = 2l + b = \frac{4000}{b} + b$

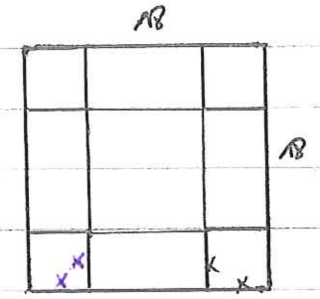
$O'(b) = 1 - \frac{4000}{b^2} = 0 \Rightarrow b^2 - 4000 = 0 \Rightarrow b = 20\sqrt{10}$

$l = \frac{2000}{b} = \frac{2000}{20\sqrt{10}} = \frac{100}{\sqrt{10}} = \frac{100\sqrt{10}}{10} = 10\sqrt{10}$

oef 5 p 15.15

$V(x) = l \cdot b \cdot h$

$= (18-2x)^2 \cdot x$



$V'(x) = -2(18-2x) \cdot 2 \cdot x + (18-2x)^2$

$= (18-2x)(-4x+18-2x)$

$= (18-2x)(18-6x)$

$V'(x) = 0 \iff 18-2x = 0$ of $18-6x = 0$

$x = 9$ $\qquad$ $x = 3$

Hoe??

oef 6 p 15.15

$V(x): [0,44] \subseteq \mathbb{R} \to \mathbb{R}: x \mapsto (44-2x)\left(\frac{80-3x}{2}\right) x$

$V'(x) = (44x-2x^2)\ \frac{1}{2}\ (80-3x)$

$V'(x) = (44-4x)\ \frac{1}{2}\ (80-3x) + (44x-2x^2)(3)$

$= (22-2x)(80-3x) + \ldots$

$V(x): [0,22] \subseteq \mathbb{R} \to \mathbb{R}: x \mapsto (80-3x)(22x-x^2)$

$V'(x) = -3(22x-x^2) + (22-2x)(80-3x)$

$= -66x + 3x^2 + 1760 - 66x - 160x + 6x^2$

$= 9x^2 - 292x + 1760$

$V'(x) = 0$

$\iff x_1 = \frac{292-148}{18} = 8$

$x_2 = 24{,}444$

$\Rightarrow V = 6272\ cm^3$

Oef 7 p 15.16

$V : \mathbb{R}^+ \to \mathbb{R} \mapsto V(p) = \dfrac{1000000}{(p+110)^2}$ kostprijs = €100

a) $\boxed{\cancel{V'(p) = \dfrac{-2(p+110).1000000}{(p+110)^4} = \dfrac{-2000000}{(p+110)^3} = 0 \Leftrightarrow}}$

$$f(p) = \frac{1000000p}{(p+110)^2} - \frac{100000000}{(p+110)^2} = p \cdot V(p) - 100 \cdot V(p)$$

$$f'(p) = \frac{1000000(p+110) - 2(1000000p - 100000000)}{(p+110)^3}$$

$f'(p) = 0 \Leftrightarrow p = €310$

max. winst 1 dag = €1190,48

stuks = 5.67

b) $f(p) = \dfrac{1000000p}{(p+110)^2} = p \cdot V(p)$

$$f'(p) = \frac{1000000p + 110000000 - 2000000p}{(p+110)^3}$$

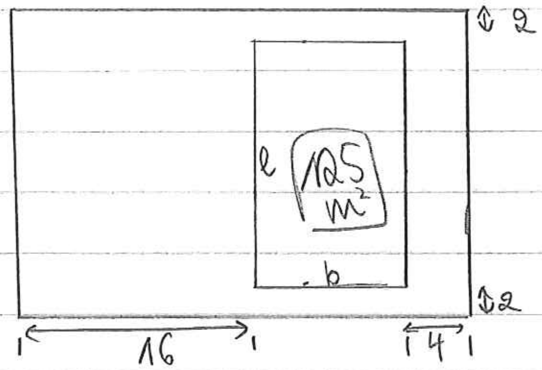
$f'(p) = 0 \Leftrightarrow p = €110$

max. daginkomsten = €2272,73

stuks = 20,67

winst = € 206,61

Oef 9 p 15.16



* $O(l,b) = (20+b)\cdot(4+l)$
* $l \cdot b = 125$

  $\Rightarrow l = 125/b$
* $O(b)$

  $= (20+b)\cdot(4+125/b) = 80 + \frac{2500}{b} + 4b + 125 = 205 + 4b + \frac{2500}{b}$

$O'(b) = 4 - \frac{2500}{b^2}$

$O'(b) = 0 \iff 4 - \frac{2500}{b^2} = 0 \iff 4b^2 - 2500 = 0$

$\iff b^2 = 625 \iff b = 25\text{ m}$ (g $-\frac{1}{2}$)

$\Downarrow$

$l = 5\text{ m}$

totale opp. terrein $= 45 \cdot 9 = 405\text{ m}^2$

Oef 8 p 15.16

$V(r,h) = \pi \cdot r^2 \cdot h$

$K(r,h) = 1{,}50 \cdot \pi \cdot r^2 + \pi \cdot r \cdot h$ $\qquad h = \frac{V}{\pi \cdot r^2}$

$= 1{,}50 \cdot \pi \cdot r^2 + \frac{V}{r}$

$K'(r) = 3\pi r - \frac{V}{r^2}$ $\qquad K'(r) = 0 \Leftrightarrow 3\pi r^3 = V$

$\Leftrightarrow r = \sqrt[3]{\frac{V}{3\pi}}$

$\Leftrightarrow h = \frac{V}{\pi \cdot \frac{V^{2/3}}{\pi^{2/3}}} = \sqrt[3]{\frac{V}{\pi}}$

$\frac{h}{r} = 3$

Oef 10 p 15.16

$K: \mathbb{R}^+ \to \mathbb{R}: q \mapsto K(q) = \frac{1}{3}q^3 - 100q + 10019$ (marktprijs = € 100)

$W: \mathbb{R}^+ \to \mathbb{R}: q \mapsto W(q) = -\frac{1}{3}q^3 + 200q - 10019$

$W'(q) = -q^2 + 200$ $\qquad W'(q) = 0 \Leftrightarrow q = \sqrt{200} = 10\sqrt{2}$

$W(10\sqrt{2}) = -8133{,}38$ €

Oef 11 p 15.16

$K: \mathbb{R}^+ \to \mathbb{R}: q \mapsto K(q) = \frac{1}{2}q^2 + 20q + 10$

$V(p) - 60 = -\frac{1}{2}p$

$\frac{q-60}{-\frac{1}{2}} = p = 120 - 2q$

$V: \mathbb{R}^+ \to \mathbb{R}: p \mapsto q = V(p) = 60 - \frac{1}{2}p$

$\forall p < 60$

TO - TK $\qquad$ accijns

$Wq = q(120-2q) - (\frac{1}{2}q^2 + 20q + 10 + \underbrace{tq}_{\text{accijns}})$

$W': \mathbb{R}^+ \to \mathbb{R}: q \to W'(q) = -5q + 100 - t = 0$

$\Rightarrow q = \frac{-t+100}{5}$

(invullen in $p = (120-2q) = \frac{2t}{5} + 80$)

inkomsten overheid $= tq$

ZOZ

Oef 1 p 15.28

* $f: \mathbb{R}^2 \to \mathbb{R} : (x,y) \mapsto x^2 + xy + y^2 + 3x - 3y + 4$

$D_1 f(x,y) = 2x + y + 3$ $\qquad$ $D_2 f(x,y) = x + 2y - 3$

$D^2_{11} f(x,y) = 2$ $\qquad$ $D^2_{22} f(x,y) = 2$

$D^2_{12} f(x,y) = 1$ $\qquad$ $D^2_{21} f(x,y) = 1$

kritieke punten: $\begin{cases} 2x + y + 3 = 0 \\ x + 2y - 3 = 0 \end{cases}$ $\begin{cases} 6 - 4y + y + 3 = 0 \\ x = 3 - 2y \end{cases}$ $\begin{cases} y = 3 \\ x = -3 \end{cases}$

$\Delta f(-3,3) = 2 \cdot 2 - 1 = 3$

$\Rightarrow$ minimum in $(-3,3)$

* $f: \mathbb{R}^2 \to \mathbb{R} : (x,y) \mapsto x^4 + y^4 + 4xy$

$D_1 f(x,y) = 4x^3 + 4y$ $\qquad$ $D_2 f(x,y) = 4y^3 + 4x$

$D^2_{11} f(x,y) = 12x^2$ $\qquad$ $D^2_{22} f(x,y) = 12y^2$

$D^2_{12} f(x,y) = 4$ $\qquad$ $D^2_{21} f(x,y) = 4$

kritieke punten: $\begin{cases} 4x^3 + 4y = 0 \\ 4y^3 + 4x = 0 \end{cases}$ $\begin{cases} y = -x^3 \\ -4x^9 + 4x = 0 \end{cases}$ $\begin{cases} y = -1 \\ x^9 = x = 1 \end{cases}$ $\begin{cases} y = 1 \\ x = 1 \end{cases}$

$(-1,1)$ $\quad$ $(1,-1)$ $\quad$ $(0,0)$

- $\Delta f(-1,1) = 12 \cdot 12 - 4 = 140$ en $D^2_{11} f(-1,1) > 0$ $\quad$ minimum
- $\Delta f(1,-1) = 140$ en $D^2_{11} f(1,-1) > 0$ $\quad$ minimum
- $\Delta f(0,0) = -4 < 0$ $\to$ zadelpunt

* $f: \mathbb{R}^2 \to \mathbb{R} : (x,y) \mapsto x^2 + 2xy$

$D_1 f(x,y) = 2x + 2y$ $\qquad$ $D_2 f(x,y) = 2x$

$D^2_{11} f(x,y) = 2$ $\qquad$ $D^2_{22} f(x,y) = 0$

$D^2_{12} f(x,y) = 2$ $\qquad$ $D^2_{21} f(x,y) = 2$

kritieke punten: $\begin{cases} 2x + 2y = 0 \\ 2x = 0 \end{cases}$ $\begin{cases} y = 0 \\ x = 0 \end{cases}$

$\Delta f(0,0) = 2 \cdot 0 - 4 = -2$ < 0 $\to$ zadelpunt

Oef 2 p 15.28

$f: \mathbb{R}^2 \to \mathbb{R} : (x,y) \mapsto 1 + x^2 - y^2$

$\Rightarrow$ afstand minimaliseren $d = \sqrt{x^2 + y^2 + f^2(x,y)}$ $(x-0)^2$ $(y-0)^2$

$\Rightarrow d : \mathbb{R}^2 \to \mathbb{R} : (x,y) \mapsto x^2 + y^2 + (1+x^2-y^2)^2$

$D_1 d(x,y) = 2x + 4x(1+x^2-y^2)$ $\quad D_2 d(x,y) = 2y - 4y(1+x^2+y^2)$

$D_{11}^2 d(x,y) = 2 + 4(1+x^2-y^2) + -2x \cdot 4x$ $\quad D_{22}^2 d(x,y) = 2 - 4(1+x^2+y^2) + 8y^2$

$= 2 + 4(1+x^2-y^2) - 8x^2$

$D_{12}^2 d(x,y) = -8xy$ $\quad D_{21}^2 d(x,y) = -8xy$

kritieke punten $\begin{cases} 2x + 4x(1+x^2-y^2) = 0 \\ 2y - 4y(1+x^2+y^2) = 0 \end{cases}$

- $(0,0)$: $\Delta f(0,0) = 6 \cdot (-2) + 0 = -12$ $\rightarrow$ zadelpunt
- $(0, \sqrt{2}/2)$: $\Delta f(0, \sqrt{2}/2) = 4 \cdot 0 + 0 = 0$ $\Big\}$ minimum
  $D_{11}^2 > 0$
- $(0, -\sqrt{2}/2)$: $\Delta f = 0$ & $D_{11}^2 > 0$ $\rightarrow$ minimum

$\Rightarrow \left(0, \frac{\sqrt{2}}{2}, \frac{1}{2}\right)$ en $\left(0, \frac{-\sqrt{2}}{2}, \frac{1}{2}\right)$

Oef 3 p 15.28

Doelf $K : \mathbb{R}^2 \to \mathbb{R} : (x_Q, x_R) \mapsto K(x_Q, x_R) = 15\,\|PQ\| + 10\,\|QR\| + 5(20 - x_R)$

Pyth $\|PQ\| = \sqrt{x_Q^2 + 2^2}$ $\quad \|QR\| = \sqrt{(x_R - x_Q)^2 + 1^2}$

$K^2 = 15(x_Q^2 + 4) + 10((x_R - x_Q)^2 + 1) + 5(20 - x_R)$

$= 15x_Q^2 + 60 + 10(x_R^2 - 2x_R x_Q + x_Q^2 + 1) + 100 - 5x_R$

$= 15x_Q^2 + 60 + 10x_R^2 - 20x_R x_Q + 10x_Q^2 + 10 + 100 - 5x_R$

$= 25x_Q^2 + 10x_R^2 - 20x_R x_Q - 5x_R + 170$

$D_1(K^2) = 50x_Q - 20x_R$ $\quad D_2(K^2) = 20x_R - 20x_Q - 5$

$D_{11} = 50$ $\quad D_{22} = 20$

$D_{12} = -20$

kritieke punten: $\begin{cases} 50x_Q - 20x_k = 0 \\ 20x_k - 20x_Q - 5 = 0 \end{cases} \Rightarrow \begin{matrix} x_Q = 1/6 \\ x_k = 5/12 \end{matrix}$

oef 4 p 15.28

$V = b \cdot l \cdot h = 64 \Leftrightarrow h = 64/bl$

opp. wanden:

$$A(b,l) = 2lb + 2bh + 2lh$$
$$= 2lb + \frac{128}{l} + \frac{128}{b}$$

$$\begin{matrix} D_1 A(b,l) = \\ D_2 A(b,l) = \end{matrix} \begin{cases} 2l - \frac{128}{b^2} = 0 \\ 2b - \frac{128}{l^2} = 0 \end{cases} \qquad \begin{cases} b = \frac{64}{l^2} \\ 2l - \frac{128 \cdot l^4}{64^2} = 0 \end{cases}$$

$$\begin{cases} b = 4 \\ l = 4 \end{cases}$$

Controle $\quad D^2_{11} = \frac{256}{b^3} \quad D^2_{22} = \frac{256}{l^3} \quad D^2_{12} = 2$

$$\left.\begin{matrix} \Delta f(4,4) = 4 \cdot 4 - 2 = 14 & > 0 \\ D^2_{12} = 2 & > 0 \end{matrix}\right\} \text{minimum}$$

oef 5 p 15.28

$$\frac{160R}{160 + R} + \frac{320P}{80 + P} \rightarrow \# \text{ stuks}$$

$$w = 4\left(\frac{160R}{160+R} + \frac{320P}{80+P}\right) - R - P$$

$$= \frac{640R}{160+R} + \frac{1280P}{80+P} - R - P$$

$$\begin{cases} D_1 w(R,P) = \dfrac{640(160+R) - 640R}{(160-R)^2} - 1 = 0 \\ D_2 w(R,P) = \dfrac{1280(80+P) - 1280P}{(80+P)^2} - 1 = 0 \end{cases} \qquad \begin{cases} 102400 = (160+R)^2 \\ 102400 = (80+P)^2 \end{cases}$$

$$\begin{cases} R = 160 \\ P = 240 \end{cases}$$

Opg 6 p 15.29

b) $K: \mathbb{R}^2 \to \mathbb{R}: (a,b) = \sum_{i=1}^{n} [y_i - (ax_i + b)]^2$

$$= \sum_i^n [y_i^2 + a^2 x_i^2 + b^2 - 2ax_iy_i - 2by_i + 2abx_i]$$

$$= \sum_i^n y_i^2 + a^2 \sum_i^n x_i^2 + b^2 n - 2a \sum_i^n x_iy_i - 2b \sum_i^n y_i + 2ab \sum_i^n x_i$$

kritieke punten $\begin{cases} 2a \sum_i^n x_i^2 + 2b \sum_i^n x_i - 2 \sum_i^n x_iy_i = 0 \\ 2a \sum_i^n x_i + 2bn - 2 \sum_i^n y_i = 0 \end{cases}$

$$a = \frac{n \sum x_iy_i - (\sum x_i)(\sum y_i)}{n \sum x_i^2 - (\sum x_i)^2} = \frac{5 \cdot 2339{,}2 - (330 \cdot 31{,}9)}{5 \cdot 238{,}25 - 1017{,}61}$$

$$= \frac{11696 - 10527}{1191{,}25 - 1017{,}61} = \frac{1169}{173{,}64} = 6{,}7323$$

$$b = \frac{1}{n}\left(\sum y_i - a \sum x_i\right) = \frac{1}{5}\left(330 - 31{,}9 \cdot a\right) = 23{,}0478$$

$$\Rightarrow y = 6{,}7323\,x + 23{,}0478$$

c) $y = 72{,}19$ zieken per 100.000 inwoners

Oef 7 p 15.29

* $f: \mathbb{R}^2 \to \mathbb{R},\ (x,y) \mapsto x^2 + y^4$

a) $\begin{cases} D_1 f(x,y) = 2x = 0 \\ D_2 f(x,y) = 4y^3 = 0 \end{cases}$ $\begin{cases} x = 0 \\ y = 0 \end{cases}$

b) $D^2_{11} f(x,y) = 2$ $D^2_{22} f(x,y) = 12y^2$

$D^2_{12} f(x,y) = 0$ $D^2_{21} f(x,y) = 0$

$\Delta f(0,0) = 0$

! c) $(0,0)$. elk andere koppel levert een hogere waarde op $\to$ minimum

* $f: \mathbb{R}^2 \to \mathbb{R}: (x,y) \mapsto x^2 - y^4$

a) $D_1 f(x,y) = 2x = 0$ $\begin{cases} x = 0 \\ y = 0 \end{cases}$

$D_2 f(x,y) = -4y^3 = 0$

b) idem

Oef 8 p 15.29

$$Q_1(p) = \begin{cases} 100-2p & \text{als } 0 \le p \le 50 \\ 0 & \text{als } p > 50 \end{cases}$$

$$Q_2(p) = \begin{cases} 90-3p & \text{als } 0 \le p \le 30 \\ 0 & \text{als } p > 30 \end{cases}$$

$K: \mathbb{R}^+ \to \mathbb{R}: q \mapsto K(q) = 20q$

b) $$Q(p): \mathbb{R}^+ \to \mathbb{R}: p \mapsto \begin{cases} (100-2p)+(90-3p) & \text{als } 0 \le p \le 30 \\ 100-2p & \text{als } 30 < p \le 50 \\ 0 & \text{als } p > 50 \end{cases}$$

$W(p) = p \cdot Q(p) - K(Q(p))$

$W'(p) = Q(p) + p \cdot Q'(p) - K'(Q(p)) \cdot Q'(p) = 0$

* $]0,30[$: $190 - 5p + p \cdot (-5) - 20 \cdot (-5) = 0$

  $290 - 10p = 0 \Leftrightarrow p = 29$

  $W = 405$

* $]30,50[$: $100 - 2p + p(-2) - 20(-2) = 0$

  $140 - 4p = 0 \Leftrightarrow \boxed{p = 35 \quad W = 450}$

$\longrightarrow p = 35$ en $(Q_1, Q_2) = (30, 0)$

a) ① $W'(p) = Q(p) + p \cdot Q'(p) - K'(Q(p)) \cdot Q'(p) = 0$

$= 100 - 2p + p(-2) - 20 \cdot (-2) = 0$

$140 - 4p = 0 \Leftrightarrow p = 35$

$W = 450$

$q = 30$

② $W'(p) = Q(p) + p \cdot Q'(p) - K'(Q(p)) \cdot Q'(p) = 0$

$= 90 - 3p + p(-3) - 20(-3) = 0$

$150 - 6p = 0 \Leftrightarrow p = 25$

$W = 75$

$q = 15$

waarom niet echt f?

$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$\left(\frac{12}{5}, \frac{16}{5}, -4\right)$ en $\left(\frac{18}{5}, \frac{24}{5}, 6\right)$

Oef 1 p 15.43

cirkel $(x-3)^2+(y-4)^2=1$

afstand tot $(0,0) = \sqrt{x^2+y^2}$

* MAX

maximaliseer $d: \mathbb{R}^2 \to \mathbb{R} : (x,y) \mapsto x^2+y^2$

onder $g(x) = (x-3)^2 + (y-4)^2 - 1 = 0$

$L(x,y,\lambda) = x^2+y^2 - \lambda(x-3)^2 + \lambda(y-4)^2 - \lambda$

$$\begin{cases} D_1(x,y,\lambda) = 2x - \lambda \cdot 2(x-3) = 0 \\ D_2(x,y,\lambda) = 2y + 2\lambda(y-4) = 0 \\ D_3(x,y,\lambda) = -(x-3)^2 + (y-4)^2 - 1 = 0 \end{cases} \qquad \begin{cases} x(2-2\lambda)+3\lambda = 0 \\ y(2+2\lambda) - 8\lambda = 0 \end{cases}$$

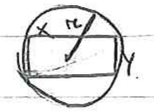
$$\begin{cases} x = \frac{-3\lambda}{2-2\lambda} \\ y = \frac{8\lambda}{2+2\lambda} \\ -\left(\frac{-3\lambda}{2-2\lambda}-3\right)^2 + \left(\frac{8\lambda}{2+2\lambda}-4\right)^2 - 1 = 0 \end{cases} \nearrow -\left(\frac{-3\lambda-3(2-2\lambda)}{2-2\lambda}\right)^2 + \left(\frac{8\lambda-4(2+2\lambda)}{2+2\lambda}\right)^2 = 1$$

$$\Leftrightarrow -\left(\frac{3\lambda-6}{2-2\lambda}\right)^2 + \left(\frac{-8}{2+2\lambda}\right)^2 = 1$$

$$\Leftrightarrow \frac{(-9\lambda^2+36\lambda-36)(2+2\lambda)^2 + 64(2-2\lambda)^2 - (2-2\lambda)^2(2+2\lambda)^2}{(2-2\lambda)^2(2+2\lambda)^2} = 0$$

$$\Leftrightarrow (2+2\lambda)^2(-9\lambda^2+36\lambda-36-4+8\lambda-4\lambda^2) + 64(2-2\lambda)^2 = 0$$

$$\Leftrightarrow (2+2\lambda)^2(-13\lambda^2+44\lambda-40) + (2-2\lambda)^2 64 = 0$$

Ćw 2 p 15.43 

max. pp

na okręgu: $x^2+y^2-4r^2=0$

$f(x,y)=xy$

$L(x,y,\lambda)=xy-\lambda x^2-\lambda y^2+4\lambda r^2$

$$\begin{cases} D_1L(x,y,\lambda)=y-2\lambda x=0 \\ D_2L(x,y,\lambda)=x-2\lambda y=0 \\ D_3L(x,y,\lambda)=-x^2-y^2+4r^2=0 \end{cases} \quad \begin{cases} y=2\lambda x \\ x-4\lambda^2x=0 \\ -x^2-y^2+4r^2=0 \end{cases} \quad \begin{cases} x(1-4\lambda^2)=0 \end{cases}$$

$$* \cancel{\begin{cases} x=0 \\ y=0 \\ \lambda= \end{cases}} \quad \begin{cases} 1-4\lambda^2=0 \end{cases} \Leftrightarrow \lambda=1/2 \text{ g } -1/2$$

a) $\lambda=1/2$ $\begin{cases} y=x \\ -2x^2+4r^2=0 \end{cases}$ $\begin{cases} x=y \\ \quad=\sqrt{2}\,r \end{cases}$

b) $\lambda=-1/2$ $y=-x$ $\left\{\begin{matrix} x=\sqrt{2}\,r \\ y=-\sqrt{2}\,r \end{matrix}\right\}$ kan Nt!

$\rightarrow (\sqrt{2}\,r, \sqrt{2}\,r, 1/2)$

$r^2=x^2+y^2$

$\nabla g(x^*,y^*)\neq(0,0)$?

$\nabla g(x,y)=(2x,2y)=$

$\Rightarrow pp=2r^2$

oef 3 p 15.44

$R = 100\,000$

$q_1 = 50 + \sqrt{9R_1}$ en $q_2 = 70 + \sqrt{4R_2}$

$p_1 = 500$ $\quad p_2 = 300$ $\quad$ } per stuk

$v_1 = 300$ $\quad v_2 = 100$

$F_1 = 50$ $\quad F_2 = 50$ $\quad$ } voor de hele productie

$$\begin{aligned}
\text{max. winst} = f(R_1, R_2) &= q_1 p_1 + q_2 p_2 - q_1 v_1 - q_2 v_2 - F_1 - F_2 \\
&= q_1 (p_1 - v_1) + q_2 (p_2 - v_2) - 100 \\
&= 200 (50 + 3\sqrt{R_1}) + 200 (70 + 2\sqrt{R_2}) - 100 \\
&= 200 (120 + 3\sqrt{R_1} + 2\sqrt{R_2}) - 100
\end{aligned}$$

maximaliseren onder $g(R_1, R_2) = R_1 + R_2 - 100.000 = 0$

$$L(R_1, R_2, \lambda) = 200 (120 + 3\sqrt{R_1} + 2\sqrt{R_2}) - 100 - \lambda R_1 - \lambda R_2 + \lambda 100\,000$$

$$\begin{cases} D_1 L(R_1, R_2, \lambda) = 300 / \sqrt{R_1} - \lambda = 0 \\ D_2 L(R_1, R_2, \lambda) = 200 / \sqrt{R_2} - \lambda = 0 \\ D_3 L(R_1, R_2, \lambda) = -R_1 - R_2 + 100\,000 = 0 \end{cases} \qquad \begin{cases} R_1 = 90\,000 / \lambda^2 \\ R_2 = 40\,000 / \lambda^2 \\ -90\,000/\lambda^2 - 40\,000/\lambda^2 + 100\,000 = 0 \end{cases}$$

$$\begin{cases} R_1 = 90\,000 / \lambda^2 \\ R_2 = 40\,000 / \lambda^2 \\ -130\,000 + 100\,000 \lambda^2 = 0 \end{cases} \qquad \begin{cases} R_1 = 69\,230{,}77 \\ R_2 = 30\,769{,}23 \\ \lambda^2 = 1{,}3 \Leftrightarrow \lambda = 1{,}14 \end{cases}$$

$\Rightarrow$ winst $= 251\,935{,}085$

controle: $D^2_{11} L(R_1, R_2, \lambda) = -150 \cdot R_1^{-3/2} = -150 / \sqrt{R_1^3}$

$D^2_{22} L(R_1, R_2, \lambda) = -100 / \sqrt{R_2^3}$

$D^2_{12} L(R_1, R_2, \lambda) = 0$

$\Delta L(R_1, R_2, \lambda) = \dfrac{15000}{\sqrt{R_1^3} \cdot \sqrt{R_2^3}} > 0$ en $D^2_{11} < 0$ $\Rightarrow$ maximum

Oef 4 p 15.44

product 1 : $4\left(\frac{160R_1}{160+R_1} + \frac{320P_1}{80+P_1}\right)$

product 2 : $9\left(\frac{40R_2}{40+R_2} + \frac{120P_2}{30+P_2}\right)$

a) $\rightarrow W_1(R_1,P_1) = \frac{640R_1}{160+R_1} + \frac{1280P_1}{80+P_1} - R_1 - P_1$

$$\begin{cases} D_1^W(R_1,P_1) = \frac{640(160+R_1)-640R_1}{(160+R_1)^2} - 1 = 0 \\ D_2^W(R_1,P_1) = \frac{1280(80+P_1)-1280P_1}{(80+P_1)^2} - 1 = 0 \end{cases}$$

$\begin{cases} R_1 = 160 \\ P_1 = 240 \end{cases}$

$\Downarrow$

$W = 880$

$\rightarrow W_2(R_2,P_2) = \frac{360R_2}{40+R_2} + \frac{1080P_2}{30+P_2} - R_2 - P_2$

$$\begin{cases} D_1W(R_2,P_2) = \frac{360(40+R_2)-360R_2}{(40+R_2)^2} - 1 = 0 \\ D_2W_2(R_2,P_2) = \frac{1080(30+P_2)-1080P_2}{(30+P_2)^2} - 1 = 0 \end{cases}$$

$\begin{cases} R_2 = 80 \\ P_2 = 150 \end{cases}$

$\Downarrow$

$W = 910$

$\Rightarrow W = 1790$

b) $R+P = 48000 \quad \Rightarrow R_2 = 48000 - R_1 - P$

Q15 p. 16.45

$h : I \subseteq \mathbb{R} \to \mathbb{R}$ en $\boxed{h(x) + x \cdot e^{h(x)} = 3}$ * $\forall x \in I$

$f : \mathbb{R}^2 \to \mathbb{R}$ en $F : I \to \mathbb{R} : x \mapsto F(x) = f(x, h(x))$

a) $F'(x^*) = 0 \Rightarrow (x^* + e^{-h(x^*)}) \cdot \partial f / \partial x \; (x^*, h(x^*)) = \partial f / \partial y \; (x^*, h(x^*))$

TB : $D_1 f(x^*, h(x^*))(e^{-h(x^*)} + x) = D_2 f(x^*, h(x^*))$

Opl : Omdat $F'(x^*) = 0$, moet $\dfrac{\partial f}{\partial x}(x^*, h(x^*)) = 0$

Of : $D_1 f(x^*, h(x^*)) + h'(x^*) \cdot D_2 f(x^*, h(x^*)) = 0$ **

* afleiden naar x.

$\Rightarrow h'(x) + e^{h(x)} + x \cdot h'(x) \cdot e^{h(x)} = 0$

$\Leftrightarrow h'(x) = \dfrac{-e^{h(x)}}{1 + e^{h(x)}}$

$\Rightarrow$ $h'(x)$ invullen in **

$\rightarrow D_1 f(x^*, h(x^*)) - \dfrac{e^{h(x)}}{1 + e^{h(x)}} \cdot D_2 f(x^*, h(x^*)) = 0$

= het TB, enkel nog herschrijven