Wiskunde voor economen – Proefexamen (november 2013)

Vraag 1.1 (aantonen hint)

$$\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1} \qquad n \in \mathbb{N}_0$$

$$RL = \frac{1}{n} - \frac{1}{n+1} \quad \text{(zet op dezelfde noemer)}$$

$$= \frac{1.(n+1)}{n\cdot(n+1)} - \frac{1.n}{(n+1)\cdot n} \quad \text{(uitwerken)}$$

$$= \frac{(n+1)-n}{n(n+1)}$$

$$= \frac{\cancel{n}+1-\cancel{n}}{n(n+1)}$$

$$= \frac{1}{n(n+1)} = LL$$

Vraag 1.2 (uitwerken sommatie)

Bereken de uitdrukking $\sum_{n=1}^{2013} \frac{1}{n(n+1)}$

We kunnen zeggen dat:

$$\sum_{n=1}^{2013} \frac{1}{n(n+1)} = \sum_{n=1}^{2013} \frac{1}{n} - \sum_{n=1}^{2013} \frac{1}{n+1}$$

$$= \left(\frac{1}{1} + \cancel{\frac{1}{2}} + \cancel{\frac{1}{3}} + \dots + \cancel{\frac{1}{2013}}\right)$$

$$- \left(\cancel{\frac{1}{2}} + \cancel{\frac{1}{3}} + \dots + \cancel{\frac{1}{2013}} + \frac{1}{2014}\right)$$

$$= \frac{1}{1} - \frac{1}{2014} = \frac{2014-1}{2014} = \frac{2013}{2014}$$

Vraag 2

$f: \mathbb{R}^2 \to \mathbb{R} : (x,y) \mapsto |x| - y^2$

Niveaulijn $N_c = \{ y = \pm\sqrt{|x|-c} \mid (x,y) \in \mathbb{R}^2 \}$

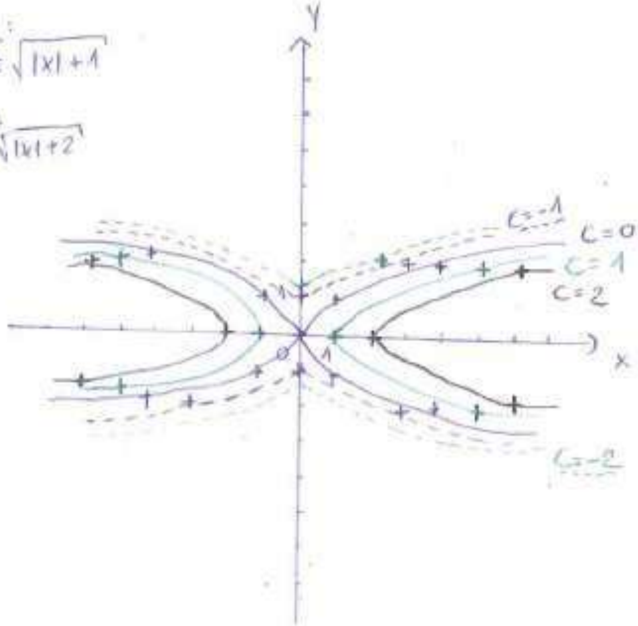
Kies $\underline{c=0}$:
$y = \pm\sqrt{|x|}$

$\underline{c=1}$:
$y = \pm\sqrt{|x|-1}$

$\underline{c=2}$:
$y = \pm\sqrt{|x|-2}$

$\underline{c=-1}$:
$y = \pm\sqrt{|x|+1}$

$\underline{c=-2}$:
$y = \pm\sqrt{|x|+2}$



Vraag 3

$$\leadsto \left(\begin{array}{ccc|c} 1 & 1 & -a & 0 \\ 1 & 4 & 2a & 3 \\ a & (a+1) & (a-1) & a \end{array}\right)$$

$$\xrightarrow[R_3 \to R_3 - aR_1]{R_2 \to R_2 - R_1} \left(\begin{array}{ccc|c} 1 & 1 & -a & 0 \\ 0 & 3 & 3a & 3 \\ 0 & 1 & a-1+a^2 & a \end{array}\right)$$

$$\xrightarrow[R_2 \to R_2/3]{} \left(\begin{array}{ccc|c} 1 & 1 & -a & 0 \\ 0 & 1 & a & 1 \\ 0 & 1 & a^2+a-1 & a \end{array}\right)$$

$$\xrightarrow[R_3 \to R_3 - R_2]{} \left(\begin{array}{ccc|c} 1 & 1 & -a & 0 \\ 0 & 1 & a & 1 \\ 0 & 0 & a^2-1 & a-1 \end{array}\right)$$

Geval 1.1: $a^2 - 1 = 0 \Leftrightarrow a = 1$

$$\leadsto \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$z = \lambda$

$y + z = 1 \Leftrightarrow y = 1 - \lambda$

$x + y - z = 0 \Leftrightarrow x = \lambda - 1 + \lambda = 2\lambda - 1$

OPLOSSING

$V = \{(2\lambda - 1, 1 - \lambda, \lambda) \mid \forall \lambda \in \mathbb{R}\}$

Geval 1.2 : $a^2 - 1 = 0 \iff a = -1$

$$\rightsquigarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & -2 \end{array}\right)$$

Dit stelsel is vals, want $0x + 0y + 0z \neq -2$

oplossing

$V = \emptyset$

Geval 2 : $a^2 - 1 \neq 0 \Rightarrow a \neq \{1, -1\}$

$a \in \mathbb{R} \setminus \{1, -1\}$

$$\xrightarrow{R_3 \leftarrow R_3 / a^2 - 1} \left(\begin{array}{ccc|c} 1 & 1 & -a & 0 \\ 0 & 1 & a & 1 \\ 0 & 0 & 1 & \frac{(a-1)}{a^2-1} \end{array}\right)$$

$$z = \frac{a-1}{a^2-1} = \frac{\cancel{a-1}}{(\cancel{a-1})(a+1)} = \frac{1}{a+1}$$

$$y + az = 1 \iff y = 1 - a\left(\frac{1}{a+1}\right) = 1 - \frac{a}{a+1} = \frac{\cancel{a}+1-\cancel{a}}{a+1}$$

$$\iff y = \frac{1}{a+1}$$

$$x + y - az = 0 \iff x = az - y \iff x = a\left(\frac{1}{a+1}\right) - \left(\frac{1}{a+1}\right)$$

$$\iff x = \frac{a-1}{a+1}$$

oplossing

$$V = \left\{ \left(\frac{a-1}{a+1}, \frac{1}{a+1}, \frac{1}{a+1} \right) \middle| \ \forall a \in \mathbb{R} \setminus \{1, -1\} \right\}$$

Meerkeuzevragen

1C – 2C – 3C