Opdrachten 4.2.4. p.445

1 a) $U_1 = \{(x,y,z) \in \mathbb{R}^3 \mid z = 0\}$

ja: w° $v, u \in U_1$, $\lambda, \mu \in \mathbb{R}$ ✓

$\lambda u + \mu v = \lambda(x_1, y_1, 0) + \mu(x_2, y_2, 0) = (\lambda x_1 + \mu x_2, \lambda y_1 + \mu y_2, 0) \in U_1$

b) $U_2 = \{(x,y,z) \in \mathbb{R}^3 \mid x, y, z \in \mathbb{Q}\}$

nee: stel $u = (1, 2, 3)$ $\lambda = \sqrt{2}$ ✓

dan is $\lambda u = (\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}) \notin U_2$.

d) $U_4 = \{(x,y,z) \in \mathbb{R}^3 \mid x + y = 1\}$ $x = 1 - y$

~~ja~~ ja? $\lambda u + \mu v = \lambda\{1 - y_1, y_1, z_1\} + \mu\{1 - y_2, y_2, z_2\}$

$\Rightarrow$ nee: $u = (2, 1, 3)$ $\lambda = 0$ $\Rightarrow \lambda u = (0,0,0) \notin U_4$ ✓

f) $U_6 = \{(x,y,z) \in \mathbb{R}^3 \mid -x + 2y + 3z = 0\}$

$\Leftrightarrow x = 2y + 3z$

ja? $\lambda(2y_1 + 3z_1, y_1, z_1) + \mu(2y_2 + 3z_2, y_2, z_2)$

$\Rightarrow$ nee? $u = (5, 1, 1)$ $v = (7, 2, 1)$ $\lambda = 2$

$u + v = (12, 3, 2)$, ~~2~~ λu ~~nnn~~ $= (10, 2, 2)$

ja? $(2\lambda y_1 + 3\lambda z_1 + 2\mu y_2 + 3\mu z_2, \lambda y_1 + \mu y_2, \lambda z_1 + \mu z_2)$

$= (2(\lambda y_1 + \mu y_2) + 3(\lambda z_1 + \mu z_2), \ldots, \ldots)$

$\Rightarrow$ JA ✓

g) $U_7 = \{(x,y,z) \in \mathbb{R}^3 \mid x + y + z = 0\}$

$\Rightarrow$ JA ✓

h) $U_8 = \{(x,y,z) \in \mathbb{R}^3 \mid x + y + 2z \leq 10\}$

Nee: $u = \{1, 2, 3\}$ $\longrightarrow$ $5u = \{5, 10, 15\} \notin U_8$. ✓

2 a) $U_1 = \{f : \mathbb{R} \to \mathbb{R} \mid f(5) = 0\}$

Kies $f, g \in U_1$ $\lambda, \mu \in \mathbb{R}$

~~...~~ als $x = 5$ is $\lambda f(5) + \mu g(5) = 0$ $\longrightarrow h(5) \in U_1$ ✓

b) $U_2 = \{f : \mathbb{R} \to \mathbb{R} \mid f \text{ is begrensd}\}$

$h : \mathbb{R} \to \mathbb{R}$ $x \longmapsto \lambda f(x) + \mu g(x)$.

$\lim_{x \to \infty} h(x) = \lim_{x \to \infty} (\lambda f(x) + \mu g(x)) = \lambda \lim_{x \to \infty} f(x) + \mu \lim_{x \to \infty} g(x)$

omdat f en g begrensd zijn zal h ook begrensd zijn ✓

c) $U_3 = \{ f : \mathbb{R} \to \mathbb{R} \mid f(0) = f(1)$

$h : \mathbb{R} \to \mathbb{R} : x \longmapsto \lambda f(x) + \mu g(x)$.

$h(0) = \lambda f(0) + \mu g(0) = \lambda f(1) + \mu g(1) = h(1) \longrightarrow h \in U_3$ V

d) $U_4 = \{ f : \mathbb{R} \to \mathbb{R} \mid \forall x \in \mathbb{R} : f(x) \leq 5 \}$

stel $f : \mathbb{R} \to \mathbb{R} : x \longmapsto 5$

$g : \mathbb{R} \to \mathbb{R} : x \longmapsto 4$.

dan is $h : \mathbb{R} \to \mathbb{R} : x \longmapsto f(x) + g(x) = 9 \longrightarrow h \notin U_4$ V

e) ...

f) $U_6 = \{ f : \mathbb{R} \to \mathbb{R} \mid f$ heeft cont. 2^{de} afg $\}$

$h''(x) = \lambda f''(x) + \mu g''(x) \Rightarrow$ optelling v. cont functies blijft cont

dus $h \in U_6$. V

3 a) alle rijen met lim $+\infty$

Nee: stel $\lim_{n\to\infty} x_n = +\infty$ en $\lambda = -1$

dan is $\lim_{n\to\infty} -1 \cdot x_n = -1 \cdot \lim_{n\to\infty} x_n = -\infty$

b) alle rijen met lim $+\infty$ of $-\infty$

nee stel $\lim_{n\to\infty} x_n = +\infty$ en $\lim_{n\to\infty} y_n = -\infty$ V

dan is $\lim_{n\to\infty} (x_n + y_n) =$ onbepaald

c) de rijen $(x_n)_{n\in\mathbb{N}} : \exists n_0 \in \mathbb{N} \; \forall n \in \mathbb{N} : n \geq n_0 \Rightarrow x_n = 0$

ja: kies w° x_n en y_n die hieraan voldoen, $\lambda, \mu \in \mathbb{R}$ V

~~dan zal $\lim_{n\to\infty} (\lambda x_n + \mu y_n) = \lambda \lim_{n\to\infty} x_n + \mu \lim_{n\to\infty} y_n$~~

kies n_1 zodat $x_{n_1} = 0$, kies n_2 zodat $y_{n_2} = 0$.

stel $m = \max\{n_1, n_2\}$

dan zal $\lambda x_m + \mu y_m = 0 \rightarrow \in$

4 $X + Y = \{ x + y \mid x \in X, y \in Y \}$

kies w° $u, v \in X + Y$ $\lambda, \mu \in \mathbb{R}$ met $u = x_1 + y_1$ en $v = x_2 + y_2$

dan is $\lambda u + \mu v = \lambda x_1 + \lambda y_1 + \mu x_2 + \mu y_2$

$= \underbrace{\lambda x_1 + \mu x_2}_{\in X} + \underbrace{\lambda y_1 + \mu y_2}_{\in Y} \rightarrow$ deelruimte

$X \cap Y = \{ x \in V \mid x \in X$ en $x \in Y \}$

$\lambda u + \mu v = \lambda x_1 + \mu x_2 \Rightarrow \in X$ want x_1 en $x_2 \in X$

$\in Y$ want x_1 en $x_2 \in Y \Rightarrow \in X \cap Y$

Werkcollege 1. : * vectorruimten
* deelruimte v/e vectorruimte.

voorbereiding

1c : $U = \{(x,y,z) \in \mathbb{R}^3 \mid x \geq 0\}$

deelruimte als $U \neq \emptyset$ en als $\forall u,v \in U . \forall \lambda, \mu \in \mathbb{R} : \lambda u + \mu v \in U$

nee : kies $u = (1,1,1)$ $\lambda = -1$
$v = (0,0,0)$ $\mu = 0$.

dan is $\lambda u + \mu v = (-1,-1,-1)$ ~~wat~~ $\notin U$.

1e : $U = \{(x,y,z) \in \mathbb{R}^3 \mid x = 2y\}$

~~nee : kies $u = (2,1,3)$ $\lambda = 2$~~
~~$v = (4,4,2,3)$ $\mu = 3$~~
~~$\lambda u + \mu v = (4,2,6$~~

ja : kies w° $u,v \in U$ $\lambda, \mu \in \mathbb{R}$:

$\lambda u + \mu v = \lambda(2y_1, y_1, z_1) + \mu(2y_2, y_2, z_2)$

$= (2(\lambda y_1 + \mu y_2), \lambda y_1 + \mu y_2, \lambda z_1 + \mu z_2) \in U$

2e : $U = \{f : \mathbb{R} \to \mathbb{R} \mid \forall x \in \mathbb{R} : f(x) = f(-x)\}$.

ja : kies w° $f,g \in U$, $\lambda, \mu \in \mathbb{R}$.

definieer $h : \mathbb{R} \to \mathbb{R} : x \longmapsto \lambda f(x) + \mu g(x)$

dan is $h(-x) = \lambda f(-x) + \mu g(-x) = \lambda f(x) + \mu g(x) = h(x)$

dus $h \in U$.

zelfstandige sessie 1.

p 437 oef 6.

$\left(\mathbb{R}, \mathbb{R}[X]^n, +\right)$

optelling : $+ : \mathbb{R}[X]^n \longrightarrow \mathbb{R}[X]^n : (u,v) \longmapsto u+v$

sc. verm. $\mathbb{R} \times \mathbb{R}[X]^n \longrightarrow \mathbb{R}[X]^n : (\lambda, v) \longmapsto \lambda v.$

(1) (4) (5) (6) (7) (8)

A. (2) $m = 0 \cdot X^n + 0 \cdot X^{n-1} + \ldots + 0 \cdot X + 0$ = nulvector.

(3) kies w: $X \in \mathbb{R}[X]^n$ ~~dan~~ $X = a_1 \cdot X^n + a_2 \cdot X^{n-1} + \ldots + a_{n-1} \cdot X^1 + a_n$

neem $Y \in \mathbb{R}[X]^n$ $Y = -a_1 \cdot X^n + (-a_2) \cdot X^{n-1} + \ldots$ ~~m~~ $+ (-a_{n-1}) \cdot X + (-a_n)$.

$\mathbb{R}(X)^n$ is ook gesloten voor het nemen van lin. comb.

kies w: $X, Y \in \mathbb{R}[X]^n$ $\lambda, \mu \in \mathbb{R}$ $(X = a_1 \cdot X^n + a_2 \cdot X^{n-1} + \ldots + a_n)$
$(Y = b_1 \cdot X^n + \ldots + b_n)$.

$\lambda X + \mu Y \in \mathbb{R}[X]^n$

want $= (\lambda a_1 + \mu b_1) \cdot X^n + \ldots + (\lambda a_n + \mu b_n)$.

B: er zou geen nulvector zijn omdat deze van graad nul is.

oef 9 p. 447.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & & \ddots & \\ a_{m1} & & & a_{mn} \end{bmatrix} \qquad B \in \text{vct}\{A_1, \ldots, A_n\}$$

$A_1 \quad A_2 \quad \cdots \quad A_n$

$AX = B$ is oplosbaar als er een $X^* \in \mathbb{R}^n$ is zodat

$A \cdot X^* = B$

WAT BETEKENT HET DAT $B \in \text{vct}\{A_1, \ldots, A_n\}$

= dat B geschreven kan w als een lin comb. van $\{A_1, \ldots, A_n\}$ = dus dat

WAT BETEKENT $AX = B$ is OPLOSBAAR?

= dat er een $X = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n$ is waarvoor

$A \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = B$ dus dat ~~$a_{11} \ldots + a_{1n} \cdot x_n + \ldots + a_{m1} \cdot x$~~

$b_j = a_{j1} x_1 + a_{j2} x_2 + \ldots + a_{jn} \cdot x_n$

~~m~~ met a_{ji} het j^{de} element van A_i

oef 10 p. 447.

$$\left[\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 \end{array}\right]$$

dus $x_4 = -2x_5$ neem ~~x_4~~ $x_5 = \lambda$ dan is $x_4 = -2\lambda$

dus $x_2 - x_3 + x_4 - x_5 = x_2 - x_3 - 2\lambda - \lambda = 0$

$x_2 = x_3 + 3\lambda$ neem $x_3 = \mu$

$x_2 = \mu + 3\lambda$.

dus $x_1 + \mu + 3\lambda + \mu - 2\lambda + \lambda = 0$

$x_1 = -2\mu - 2\lambda$

dus Opl $= \{(-2\mu - 2\lambda, \mu + 3\lambda, \mu, -2\lambda, \lambda) \in \mathbb{R}^5 \mid \mu, \lambda \in \mathbb{R}\}$

$= \text{vct}(\{(-2, 1, 1, 0, 0), (-2, 3, 0, -2, 1)\})$

oef 8 p 447

Kies wo $n \in \mathbb{N}$:

dan $e_1 = (1, 0, \ldots)$

$e_2 = (0, 1, 0, \ldots)$

$\vdots$

$e_n = (0, \ldots, 0, 1, 0, \ldots)$.

+

a $\text{vct}(\{e^n \in \mathbb{R} \mid n \in \mathbb{N}\}) = \{(x_n)_{n \in \mathbb{N}} \in \mathbb{R} \mid (x_n)_{n \in \mathbb{N}} = (\lambda_1, \ldots, \lambda_n, 0, \ldots)$ met $\lambda_1, \ldots, \lambda_n \in \mathbb{R}\}$

$\boxed{13}$

⑤ $D = \{(1,-1,2,3),(2,1,-1,4),(0,-3,5,2)\}$

$\text{ver}(D) = a(1,-1,2,3) + b(2,1,-1,4) + c(0,-3,5,2)$ ver $a,b,c \in \mathbb{R}$.

$\Rightarrow$ ¿$(3\lambda, 3, \lambda-5, 7\lambda-2)$?

$$\begin{cases} a + 2b = 3\lambda \\ -a + b - 3c = 3 \\ 2a - b + 5c = \lambda - 5 \\ 3a + 4b + 2c = 7\lambda - 2 \end{cases}$$

$$\begin{array}{c} \left[\begin{array}{cccc|c} 1 & 2 & 0 & -3 & 0 \\ -1 & 1 & -3 & 0 & 3 \\ 2 & -1 & 5 & -1 & -5 \\ 3 & 4 & 2 & -7 & -2 \end{array}\right] \end{array} \begin{array}{c} \cancel{6}\ 0 \\ 0 \\ 0 \\ 0 \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & 0 & -3 & 0 \\ 0 & 3 & -3 & -3 & 3 \\ 0 & -5 & 5 & +5 & -5 \\ 0 & -2 & 2 & \cancel{11}\ 2 & -2 \end{array}\right] \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \end{array} \sim \left[\begin{array}{cccc|c} 1 & 2 & 0 & -3 & 0 \\ 0 & 1 & -1 & -1 & 1 \\ 0 & -1 & 1 & 1 & -1 \\ 0 & -1 & 1 & 1 & -1 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & 0 & -3 & 0 \\ 0 & 1 & -1 & -1 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 2 & -1 & -2 \\ 0 & 1 & -1 & -1 & 1 \end{array}\right]$$

$$\begin{cases} \dfrac{a + 2b}{3} = \lambda \\ b - c - 1 = \lambda \end{cases} \qquad \begin{cases} b = \lambda + c + 1 \\ 3\lambda = a + 2\lambda + 2c + 2 \end{cases}$$

dus $\lambda = a + 2c + 2$.

¿$(5,1,0,11)$?

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & \cancel{3} & 5 \\ -1 & 1 & -3 & \cancel{0} & 1 \\ 2 & -1 & 5 & \cancel{-1} & 0 \\ 3 & 4 & 2 & \cancel{-7} & 11 \end{array}\right] \begin{array}{c} 8 \\ -2 \\ 6 \\ 20 \end{array} \sim \left[\begin{array}{ccc|c} 1 & 2 & 0 & 5 \\ 0 & 3 & -3 & 6 \\ 0 & -5 & 5 & 10 \\ 0 & -2 & 2 & -4 \end{array}\right] \begin{array}{c} 8 \\ 6 \\ -10 \\ -4 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 0 & 5 \\ 0 & \cancel{3}\ 1 & -1 & 2 \end{array}\right] 2 \qquad \Rightarrow \underline{SA}$$

⑥ $\text{ver}(\{(1,-3,2),(2,-1,1)\})$

¿$(1,k,5)$?

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ -3 & -1 & \cancel{k}\ 1 & 0 \\ 2 & 1 & 0 & 5 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 2 & 1 & 0 & 5 \\ 3 & 1 & 1 & 0 \end{array}\right] \begin{array}{c} 4 \\ 8 \\ 5 \end{array} \sim \left[\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 0 & \cancel{-7}\ 1 & 0 & \cancel{3}\ 1 \\ 0 & -5 & 1 & -3 \end{array}\right] \begin{array}{c} 4 \\ 0 \\ -7 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} -1 & 0 & 0 & -3 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 8 \end{array}\right] \begin{array}{c} -4 \\ 0 \\ 7 \end{array} \qquad k = -8$$

7 $\text{vct}(\{1,2,3\},\{4,5,6\}) \; ? \; \text{vct}(\{(2,1,0),(1,1,1)\})$

$\Rightarrow$ w° elementen moeten in beide voorkomen: kies $a, b \in \mathbb{R}$

$a(1,2,3) + b(4,5,6)$ is ~~...~~ $\in \text{vct}_1$

er is een $c, d \in \mathbb{R}$ zodat:

$a(1,2,3) + b(4,5,6) = c(2,1,0) + d(1,1,1)$

dus $\begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} a+4b \\ 2a+5b \\ 3a+6b \end{bmatrix}$ $\qquad \left[\begin{array}{cc|c} 2 & 1 & a+4b \\ 1 & 1 & 2a+5b \\ 0 & 1 & 3a+6b \end{array}\right]$

$\sim \left[\begin{array}{cc|c} 2 & 1 & a+4b \\ 0 & 1 & 4a+10b-a-4b = 3a+6b \\ 0 & 1 & 3a+6b \end{array}\right]$

$\sim \left[\begin{array}{cc|c} 2 & 0 & a+4b-3a-6b = -2a-2b \\ 0 & 1 & 3a+6b \end{array}\right]$

dus $c = -a-b$

en $d = 3a+6b$

8

9

opdrachten 4.3.1.13 p458

1 (a) $\{(1,2,0),(2,-1,1),(1,7,-1)\}$

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 7 \\ 0 & 1 & -1 \end{bmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\det \begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 7 \\ 0 & 1 & -1 \end{pmatrix} \begin{matrix} 1 & 2 \\ 2 & = \\ 0 & 1 \end{matrix} \qquad -1+2-7+4 = 0 \qquad \Rightarrow \text{lin. afh.}$$

$$(1,7,-1) = 3(1,2,0) - 1(2,-1,1).$$

(b) $\{(1,1,-1,2), \dots\}$ $\Rightarrow$ lin. afh.

(c) $\{1+X, 1+X^2, X+X^2\}$ — $\Rightarrow$ ~~lin~~

?

$a(1+x)+b(1+x^2)+c(x+x^2)=0.$

$$\begin{cases} a+b=0 \\ a+c=0 \\ b+c=0 \end{cases}$$ dus ~~a=b~~ $a=b=c=0.$

$\Downarrow$

VRIJ

(d) $\left\{\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 3 & 1 \end{pmatrix}, \begin{pmatrix} 5 & 2 \\ 8 & 0 \end{pmatrix}\right\}$

$$a\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + b\begin{pmatrix} -1 & 0 \\ 3 & 1 \end{pmatrix} + c\begin{pmatrix} 5 & 2 \\ 8 & 0 \end{pmatrix} = 0$$

$$\begin{pmatrix} a-b+5c & 2a+2c \\ 3a+3b+8c & 4a+b \end{pmatrix} = 0$$

dus
$$\begin{cases} a-b+5c=0 \\ 2a+2c=0 \\ 3a+3b+8c=0 \\ 4a+b=0 \end{cases}$$

~~det~~

$$\det \begin{pmatrix} 1 & -1 & 5 \\ 2 & 0 & 2 \\ 3 & 3 & 8 \end{pmatrix} \begin{matrix} 1 & -1 \\ 2 & \\ 3 & 3 \end{matrix}$$
$= -6+30-6+16 \neq 0.$

dat als de eerste 3 vgl^n als samen vrij zijn zal de laatste vgl. geen verschil maken(?)

$\Rightarrow$ lin. onafh.

② (a) $\{x \longmapsto \sin x \;,\; x \longmapsto \sin 2x,\; x \longmapsto \sin 3x\}$

zie les

(b) $\{x \longmapsto e^{2x}\cos x,\; x \longmapsto e^{2x}\sin x\}$

$\lambda e^{2x}\cos x + \mu e^{2x}\sin x = 0$

$\lambda 2 \cdot e^{2x} \cdot (-\sin x) + 2\mu e^{2x}\cos x = 0.$

neem $x = 0$

dan $\left\{\begin{array}{l} \lambda \cdot 1 \cdot 1 = 0 \\ 2\mu = 0 \end{array}\right\}$ λ en μ moeten 0 zijn $\Rightarrow$ vrij

(c) $\{x \longmapsto \cos 2x,\; x \longmapsto \cos^2 x,\; x \longmapsto \sin^2 x\}$

$$\begin{cases} \lambda_1 \cos 2x + \lambda_2 \cos^2 x + \lambda_3 \sin^2 x = 0. \\ -2\lambda_1 \sin 2x + 2\lambda_2 \cos x \cdot (-\sin x) + 2\lambda_3 \cos x \sin x = 0 \\ -4\lambda_1 \cos 2x \cancel{+} -2\lambda_2 (-\sin x \cdot \sin x + \cos x \cdot \cos x) + 2\lambda_3 (-\sin^2 x + \cos^2 x) = 0. \\ \quad + 2\lambda_2 (\cancel{\cos^2 x}\ \sin^2 x - \cos^2 x) \end{cases}$$

$x = 0$

$$\begin{cases} \lambda_1 + \lambda_2 = 0 \\ -4\lambda_1 - 2\lambda_2 + 2\lambda_3 = 0. \end{cases}$$

$\Rightarrow (8\lambda_1 \sin 2x)' + (2\lambda_2 \sin^2 x)'' \cancel{+} - (2\lambda_2 \cos^2 x)'' + (-2\lambda_3 \sin^2 x)'' + (2\lambda_3 \cos^2(x))'' = 0.$

$\cancel{x=0}$ $\cancel{+16\lambda_1 \cos 2x} + 4\lambda_2 (\cos^2 x - \sin^2 x) + 4\lambda_2 (\cos^2 x \;4 - \sin^2 x)$

$-32\lambda_1 \sin 2x \qquad + 8\lambda_3 (\sin^2 x \cdot \cos^2 x) = 0.$

$$\begin{cases} \lambda_1 + \lambda_2 = 0 \\ \cancel{4\lambda_1}\ 2\lambda_1 + \lambda_2 + \lambda_3 = 0 \\ 8\lambda_2 - 8\lambda_3 = 0 \qquad = \lambda_2 - \lambda_3. \end{cases}$$

$$\det \begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & -1 \end{pmatrix} \begin{matrix} 1 & 1 \\ 2 & 1 \\ 0 & 1 \end{matrix} = -1 - 1 + 2 = 0 \Rightarrow \cancel{\text{vrij}}\ \text{vrij}$$

$\overset{\shortparallel}{\wedge}$

$$\begin{cases} \lambda_1 + \lambda_2 = 0 \\ x = \pi/2 \quad \cancel{\cos 2x} - \lambda_1 + \lambda_3 = 0 \\ x = 2\pi \quad \cancel{\lambda_1 + \lambda_2} \\ x = 3\pi/2 \quad -4\lambda_1 - 2\lambda_2 + 2\lambda_3 = 0 \end{cases}$$

$$\det \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ -4 & -2 & 2 \end{bmatrix} = 0$$

$\cos 2x = \cancel{\sin^2 x}\ \cos^2 x - \sin^2 x.$

zelfstandige sessie 2.

SB

5

extra oef. 1.

$$U = \text{vct}\left\{ M_1 = \begin{pmatrix} 0 & 0 \\ 0 & 6 \end{pmatrix}, M_2 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, M_3 = \begin{pmatrix} 0 & 0 \\ 0 & 3 \end{pmatrix}, M_4 = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix} \right\}$$

1. $\{M_1, M_2\}$ $\{M_1, M_2, M_3\}$, $\{M_1, M_2, M_3, M_4\}$ $\longrightarrow$ voort
2. $\{\{M_1\}$, $\{M_1, M_2\}$, $\{\}$ $\longrightarrow$ vrij
   $\{\}$ $\longrightarrow$ lege verzameling
3. $\{M_1, M_2\}$, $\{M_2 M_3\}$, $\{M_3, M_4\}$ $\longrightarrow$ basis
4. $\dim U = 2.$

extra oef 2.

$f : \mathbb{R} \to \mathbb{R} : x \longmapsto 2x$

$g : \mathbb{R} \to \mathbb{R} : x \longmapsto 3 - x^2$

$h : \mathbb{R} \to \mathbb{R} : x \longmapsto 6 - 8x - 2x^2.$

$h = -4f + 2g$

hieruit volgt dat h te schrijven als een lin. comb. v. f. en g

dus dat $\{f, g, h\}$ niet vrij is

extra oef 3

$f : \mathbb{R} \to \mathbb{R} : x \longmapsto e^{2x}$

$g : \mathbb{R} \to \mathbb{R} : x \longmapsto xe^{2x}$

$h : \mathbb{R} \to \mathbb{R} : x \longmapsto 4e^{4x}$

$$\begin{cases} a \cdot e^{2x} + b \cdot xe^{2x} + ce^{4x} = 0 \\ a \cdot 2e^{2x} + b \cdot 2xe^{2x} + 4ce^{4x} = 0 \\ \end{cases}$$

$$\begin{cases} a + \cancel{b}c = 0 \\ 2a + 4c = 0 \\ e^2 \cdot a + e^2 b + e^4 c = 0 \end{cases} \quad \det\begin{bmatrix} 1 & 0 & 1 \\ 2 & 0 & 4 \\ e^2 & e^2 & e^4 \end{bmatrix} \neq 0$$

dus is vrij

opg 4

S = deelverzameling v. $\mathbb{R}^{2\times 2}$ van alle symmetrische matrices $\begin{bmatrix} a_1 & b_1 \\ b_1 & c_1 \end{bmatrix}$

① $\begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix}$

② kies w° $v_1, v_2 \in S$ $\lambda, \mu \in \mathbb{R}$:
dan geldt:

$$\lambda v_1 + \mu v_2 = \begin{bmatrix} \lambda a_1 + \mu a_2 & \lambda b_1 + \mu b_2 \\ \lambda b_1 + \mu b_2 & \lambda c_1 + \mu c_2 \end{bmatrix} \in S$$

③ $B = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$

④ $U = \text{Vct} \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}$

③ (a) $\left\{ \left(1/n+1\right)_{n\in\mathbb{N}}, \left(\frac{1}{n+2}\right)_{n\in\mathbb{N}}, \left(\frac{1}{n+3}\right)_{n\in\mathbb{N}} \right\}$

$$\begin{array}{l} n=0 \\ n=1 \\ n=2 \end{array} \quad \begin{cases} a\cdot\frac{1}{1} + b\cdot\frac{1}{2} + c\,\frac{1}{3} = 0 \\ a\,\frac{1}{2} + b\,\frac{1}{3} + c\,\frac{1}{4} = 0 \\ a\,\frac{1}{3} + b\,\frac{1}{4} + c\,\frac{1}{5} = 0. \end{cases}$$

$$\det\begin{pmatrix} 1 & 1/2 & 1/3 \\ 1/2 & 1/3 & 1/4 \\ 1/3 & 1/4 & 1/5 \end{pmatrix} = 1/15 + 1/24 + 1/24 - 1/27 - 1/20 - 1/16 \neq 0$$

$\Rightarrow$ vrij

(b) $\left\{ \left(\frac{1}{n+1}\right)_{n\in\mathbb{N}}, \left(\frac{1}{n+2}\right)_{n\in\mathbb{N}}, \left(\frac{1}{(n+1)(n+2)}\right)_{n\in\mathbb{N}} \right\}$ $\longrightarrow = \frac{1}{n+1} - \frac{1}{n+2}.$

$\downarrow$

lin afh.

$$\begin{array}{l} n=0 \\ n=1 \\ n=2 \end{array} \quad \det\begin{pmatrix} 1 & 1/2 & 1/2 \\ 1/2 & 1/3 & 1/6 \\ 1/3 & 1/4 & 1/12 \end{pmatrix} \begin{matrix} 1 & 1/2 \\ 1/2 & 1/3 \\ 1/3 & 1/4 \end{matrix} = \frac{1}{36} + \frac{1}{36} + \frac{1}{18} - \frac{1}{18} - \frac{1}{24} - \frac{1}{48}\ \cancel{\neq 0}\ = 0.$$

$\cancel{-\frac{1}{144}}$

$A(n+2) + B(n+1) = \cancel{0}\ 1.$

$$\begin{cases} A + B = 0 \\ \cancel{A+B}\ 2A + B = 1. \end{cases} \qquad A = -B.$$

$A = 1$ en $B = -1$

(c) $\left\{ \left(\frac{1}{n+1}\right)_{n\in\mathbb{N}}, \left(\frac{1}{(n+1)^2}\right)_{n\in\mathbb{N}}, \left(\frac{1}{(n+1)^3}\right)_{n\in\mathbb{N}} \right\}$ $\cancel{\Rightarrow \text{vrij}}$

$$\det\begin{pmatrix} 1 & 1 & 1 \\ 2 & 4 & 8 \\ 3 & 9 & 27 \end{pmatrix} \begin{matrix} 1 & 1 \\ 2 & 4 \\ 3 & 9 \end{matrix} = = 12 \neq 0$$

$\Rightarrow$ vrij

④ (a) $(\mathbb{R}, \mathbb{R}^{m\times n}, +)$

basis $= \left\{ \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \ddots & & \\ 0 & & & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & & \ddots & \\ 0 & & & 0 \end{bmatrix}, \ldots, \begin{bmatrix} 0 & \cdots & 0 & 1 \\ \vdots & & & 0 \\ 0 & & & 0 \end{bmatrix}, \begin{bmatrix} 0 & \cdots & 0 \\ 1 & & \vdots \\ 0 & & \\ 0 & & 0 \end{bmatrix} \ldots \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 1 \end{bmatrix} \right.$

$\Rightarrow$ dimensie $= m\cdot n$

(b) $(\mathbb{R}, C(\mathbb{R}), +) \longrightarrow$ alle functies ~~uit~~

dimensie = oneindig, er kunnen steeds functies aan de basis toegevoegd w.

stel $n\in\mathbb{N}$ met $n = \dim$ ~~...~~ $C(\mathbb{R})$. Kies nu een vrij deel F in $C(\mathbb{R})$: $\{ \cancel{\mu}\, x \longmapsto 1,\ x \longmapsto x,\ x \longmapsto x^2, \ldots, x \longmapsto x^{n+1} \}$

dan is $\# F > n$, waardoor n niet de dimensie van $C(\mathbb{R})$ kan zijn

(5) $\{(0,2,4,1), (1,-1,3,1), (1,5,5,1), (0,8,-4,1)\} = D$

is vrij: $\det\begin{pmatrix} 0 & 1 & 1 & 0 \\ 2 & -1 & 5 & 8 \\ 4 & 3 & 5 & -4 \\ 1 & 1 & 1 & 1 \end{pmatrix} \neq 0$ (?)

is voortbrengend deel v. $\mathbb{R}^4$

$vct(D) = \{a(0,2,4,1) + b(1,-1,3,1) + c(1,5,5,1) + d(0,8,-4,1) \mid a,b,c,d \in \mathbb{R}\}$

elke element v. $\mathbb{R}^4$ is te schrijven als een lineaire comb. van D

Kies w: $(x,y,z,w) \in \mathbb{R}^4$.

dan $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 2 & -1 & 5 & 8 \\ 4 & 3 & 5 & -4 \\ 1 & 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$.

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & w \\ 0 & 1 & 1 & 0 & x \\ 2 & -1 & 5 & 8 & y \\ 4 & 3 & 5 & -4 & z \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & w \\ 0 & 1 & 1 & 0 & x \\ 0 & -3 & 3 & 6 & y-2w \\ 0 & -1 & 1 & -8 & z-4w \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & w-x \\ 0 & 1 & 1 & 0 & x \\ 0 & 0 & 6 & 6 & y-2w+3x \\ 0 & 0 & 2 & -8 & z-4w+y-2w = z+y-6w \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 1w - 1x \\ 0 & 6 & 0 & -6 & 6x - y + 2w - 3x = 3x-y+2w \\ 0 & 0 & 6 & 6 & y-2w+3x \\ 0 & 0 & 0 & -60 & 6z+6y-36w-2y+4w-6x \end{array}\right]$$

$= 6z + 4y - 32w - 6x$

$$\sim \left[\begin{array}{cccc|c} -60 & 0 & 0 & 0 & -60w + 60x - 6z - 4y + 32w + 6x \\ 0 & -360 & 0 & 0 & -60(3x - y + 2w) + 6(6z + 4y - 32w - 6x) \\ 0 & 0 & -360 & 0 & -60(y - 2w + 3x) - 6(6z + 4y - 32w - 6x) \\ 0 & 0 & 0 & -60 & 6z + 4y - 32w - 6x \end{array}\right]$$

$\Rightarrow (1,0,0,0) = \frac{7}{5}v_4 + \frac{13}{15}v_1 - \frac{13}{15}v_2 + \frac{8}{15}v_3$

(5) (8B15)

<u>VRij</u>: $\det\begin{pmatrix} 0 & 1 & 1 & 0 \\ 2 & -1 & 5 & 8 \\ 4 & 3 & 5 & -4 \\ 1 & 1 & 1 & 1 \end{pmatrix} = 36 \neq 0.$

<u>voortbrengend</u>

kies w° element in $\mathbb{R}^4$: ~~(a,b,c,d)~~ (x_1, x_2, x_3, x_4).

dan zou $\lambda_1(0,2,4,1) + \lambda_2(1,-1,3,1)$
$+ \lambda_3(1,5,5,1) + \lambda_4(0,8,-4,1) = (x_1, x_2, x_3, x_4)$

met $\lambda_1, \lambda_2, \lambda_3, \lambda_4 \in \mathbb{R}$.

$$\left[\begin{array}{cccc|c} 0 & 1 & 1 & 0 & x_1 \\ 2 & -1 & 5 & 8 & x_2 \\ 4 & 3 & 5 & -4 & x_3 \\ 1 & 1 & 1 & 1 & x_4 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & x_4 \\ 0 & 1 & 1 & 0 & x_1 \\ 2 & -1 & 5 & 8 & x_2 \\ 4 & 3 & 5 & -4 & x_3 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & x_4 \\ 0 & 1 & 1 & 0 & x_1 \\ 0 & -3 & 3 & 6 & x_2 - 2x_4 \\ 0 & -1 & 1 & -8 & x_3 - 4x_4 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & x_4 - x_1 \\ 0 & 1 & 1 & 0 & x_1 \\ 0 & 0 & 6 & 6 & x_2 - 2x_4 + 3x_1 \\ 0 & 0 & 2 & -8 & x_3 - 4x_4 + x_1 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 6 & 0 & 0 & 6 & 6x_4 - 6x_1 \\ 0 & 6 & 0 & -6 & 6x_1 \\ 0 & 0 & 6 & 6 & x_2 - 2x_4 + 3x_1 \\ 0 & 0 & 0 & -60 & 6x_3 - 24x_4 + \cancel{6x_1} - 2x_2 + 4x_4 - \cancel{6x_1} \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & x_4 - x_1 \\ 0 & 1 & 0 & -1 & x_1 \\ 0 & 0 & 6 & 6 & x_2 - 2x_4 + 3x_1 \\ 0 & 0 & 0 & -60 & 6x_3 - 20x_4 - 2x_2 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} -60 & 0 & 0 & 0 & 60x_1 - 60x_4 - 6x_3 + 20x_4 + 2x_2 \\ 0 & -60 & 0 & 0 & -60x_1 + 6x_3 - 20x_4 - 2x_2 \\ 0 & 0 & -360 & 0 & -60x_2 + 120x_4 - 180x_1 - 36x_3 + 120x_4 + 12x_2 \\ 0 & 0 & 0 & -60 & 6x_3 - 20x_4 - 2x_2 \end{array}\right]$$

dus ~~als~~ $\lambda_1 = \dfrac{40x_4 + 6x_3 - 2x_2 - 60x_1}{60}$ $\quad \lambda_3 = \dfrac{-180x_1 - 48x_2 - 36x_3 + 240x_4}{-360}$

$\lambda_2 = \dfrac{-60x_1 - 2x_2 + 6x_3 - 20x_4}{-60}$ $\quad \lambda_4 = \dfrac{6x_3 - 20x_4 - 2x_2}{-60}$

dus voor $x_? = (1,0,0,0)$ $\lambda = \left(-1, \frac{1}{2}, \frac{1}{2}, 0\right)$

9

6) $U = vct\{(4,4,-4), (1,-1,2), (3,1,0)\}$

$-2(1,-1,2) = (-2,2,-4) + 2(3,1,0)$

$+ (6,2,0) = (4,4,-4)$

dimensie = 2 met basis $B = \{(1,-1,2), (3,1,0)\}$

B is vrij: $\lambda(1,-1,2) + \mu(3,1,0) = 0$

dus $\begin{cases} \lambda + 3\mu = 0 \\ -\lambda + \mu = 0 \\ 2\lambda = 0 \end{cases}$ dus $\lambda = 0 = \mu$.

B is voortbrengend: $(4,4,-4) = -2(1,-1,2) + 2(3,1,0)$

* $(1,0,0)$ ~~no~~ behoort niet tot U want

~~Als~~ $\begin{cases} 2\lambda = 0 \\ \mu - \lambda = 0 \end{cases}$ ~~maakt...~~ heeft geen opl.

* $(-4,-10,13)$: $\begin{cases} \lambda + 3\mu = -4 \\ \mu - \lambda = -10 \\ 2\lambda = 13 \end{cases}$ dus $\lambda = \frac{13}{2}$, $\mu = -3{,}5$

7) $\{(1,3,-1), (3,-1,1), (3,4,-1)\}$ is ~~basis~~

1) ~~vrij~~ nietvrij det $\begin{bmatrix} 1 & 3 & 3 \\ 3 & -1 & 4 \\ -1 & 1 & -1 \end{bmatrix} \cancel{\neq 0} = 0$

$\Rightarrow \lambda_1(1,3,-1) + \lambda_2(3,-1,1) = (3,4,-1)$

$\begin{cases} \lambda_1 + 3\lambda_2 = 3 \\ 3\lambda_1 + \lambda_2 = 4 \\ -\lambda_1 + \lambda_2 = -1 \end{cases}$ → klopt.

$3\lambda_1 + 1 - \lambda_1 = 4$, $\lambda_1 = 3/2$

$\lambda_2 = -1 + \lambda_1$

$\lambda_2 = 1/2$.

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~~2) voortbreng. kies $w \in U$ dan is $v = a(1,3,-1) + b(3,-1,1) + c(3,4,-1)$ met $a,b,c \in \mathbb{R}$~~

~~dus ook voortbr.~~

~~vrij~~ ~~...~~

8) ~~a)~~ $\begin{cases} z = x + 2y \\ z = -y - 2x \end{cases} \sim \begin{cases} 3x = -3y \\ z = y \end{cases}$ $x = -y$

alle elementen v. V zijn van de vorm $(-y, y, y)$ $y \in \mathbb{R}$

kies 2 elementen $u, v \in V$ en $\lambda, \mu \in \mathbb{R}$ dan is

$\lambda u + \mu v = (-(\lambda y_1 + \mu y_2), \lambda y_1 + \mu y_2, \lambda y_1 + \mu y_2) \in V$

basis $B = \{(-1,1,1)\}$ ⇒ dimensie = 1.

* vrij: ~~...~~ triviaal
* voortbr. kies w $v \in V$: $v = (-y, y, y)$ $y \in \mathbb{R}$

dan is $v = yB$

(9) dus $\lambda_1(x+y)+\lambda_2(y+z)+\lambda_3(x+z)=0$ $\quad \lambda_1,\lambda_2,\lambda_3 \in \mathbb{R}$ (10)

$\lambda_1 \neq \lambda_2 \neq \lambda_3 \neq 0$

ges: $\mu_1 x + \mu_2 y + \mu_3 z = 0$ dan is $\mu_1 = \mu_2 = \mu_3 = 0$. met $\mu_1, \mu_2, \mu_3 \in \mathbb{R}$.

$\cancel{\lambda_1 x + \lambda_2 y + \lambda_3 z +}$

$x(\lambda_1+\lambda_3) + y(\lambda_1+\lambda_2) + z(\lambda_2+\lambda_3) = 0$

dus $\lambda_1+\lambda_3 = \lambda_1+\lambda_2 = \lambda_2+\lambda_3 = 0$.

$\begin{cases}\lambda_1 = -\lambda_3\\ \lambda_1 = -\lambda_2 \\ \lambda_3 = -\lambda_2\end{cases}$ dus $\begin{cases}\lambda_1 = -\lambda_3 \\ \lambda_1 = \lambda_3\end{cases}$ dus $\lambda_1 = \lambda_2 = \lambda_3 = 0$.

(10) a) $B = \{(1,0,0),(0,1,0)\}$

b) $B = /$

c) $B = /$

d) $B = /$

e) $B = \{(2,1,0),(0,0,1)\}$

f) $B = \{(2,1,0),(3,0,2)\}$

g) $B = \{(-1,1,0),(-1,0,1)\}$

h) $B = /$

(11) stel ~~V~~ V is ~~voortbrengend door eindig een~~. een eindige basis v $(\mathbb{R},\mathbb{R},+)$ door $B = \{e_n \in N \mid n \in N$

stel $n = \#V$ $\quad$ e_n is rij met nullen behalve 1 op n^{de} plaats.

dan zal de rij e_{m+1} niet tot de basis behoren.

wat tegenstrijdig is met het feit dat V een basis is.

$\Rightarrow \dim \mathbb{R} = \infty$

(10 g) $\det\begin{pmatrix}-1 & 1\\ -1 & 0\end{pmatrix} = 1 \neq 0 \Rightarrow$ vrij

* voortbr.: $\forall$ $(x,y,z) \in U_7$:

$(-y-z, y, z) = y v_1 + z v_2$.

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7 $\text{basis}_1 = \{(1,3,-1),(3,-1,1)\}$

vrij: als $\lambda_1(1,3,-1) + \lambda_2(3,-1,1) = 0$

$$\begin{cases} \lambda_1 + 3\lambda_2 = 0 & \text{dus } \lambda_1 = -3\lambda_2 \\ 3\lambda_1 - \lambda_2 = 0 & \text{dus } -\cancel{9}\,10\lambda_2 = 0. \\ -\lambda_1 + \lambda_2 = 0 & \text{dus } \lambda_1 = \lambda_2. \end{cases}$$

dus $\lambda_1 = \lambda_2 = 0$.

voortbrengend.

kies w° $x = (x_1, x_2, x_3) \in U$

dan is $x = \lambda_1(1,3,-1) + \lambda_2(3,-1,1) + \lambda_3(3,4,-1)$.

$= \lambda_1(1,3,-1) + \lambda_2(3,-1,1) + \lambda_3 \cdot \left(\frac{3}{2}(1,3,-1) + \frac{1}{2}(3,-1,1)\right)$

dus x valt te schrijven als lin comb van basis_1.

$\text{basis}_2 = \{(2,6,-2),(6,-2,2)\}$ ... → DUH.

opdrachten 4.4.9 p470

K11

(1) $f_1: \mathbb{R} \to \mathbb{R}: x \longmapsto 3x$

f is lin als $L(\lambda_1 v_1 + \lambda_2 v_2) = \lambda_1 L(v_1) + \lambda_2 L(v_2)$

Kies w° $u, v \in \mathbb{R}$ en $\lambda, \mu \in \mathbb{R}$:

$f_1(\lambda u + \mu v) = 3(\lambda u + \mu \cdot v)$

$= \lambda 3u + \mu \cdot 3v = \lambda f(u) + \mu \cdot f_1(v) \Rightarrow$ LIN.

$f_2: \mathbb{R} \to \mathbb{R}: x \longmapsto ~~2x~~ \; 2x + 1$

$f_2(3 \cdot 2) = 2 \cdot 6 + 1 \qquad 3 f_2(2) = 3(4+1)$

$= 13 \qquad = 15$

$\Rightarrow$ niet lin.

$f_3: \mathbb{R}^2 \to \mathbb{R}: (x, y) \longmapsto x + y$

Kies w° $u, v \in \mathbb{R}^2$ $\lambda, \mu \in \mathbb{R}$ met $u = (x_1, y_1)$ $v = (x_2, y_2)$

$f_3(\lambda u + \mu v) = f_3((\lambda x_1 + \mu x_2, \lambda y_1 + \mu y_2))$

$= \lambda(x_1 + y_1) + \mu(x_2 + y_2) = \lambda f_3(u) + \mu f_3(v)$

$\Rightarrow$ LIN: $A = (1 \;\; 1)$

$f_4: \mathbb{R}^3 \to \mathbb{R}: (x, y, z) \longmapsto 5x - 2y + 3z.$

$v_1, v_2, ~~v_3~~ \in \mathbb{R}^3 \qquad v_i = (x_i, y_i, z_i)$

$f_4(\lambda_1 v_1 + \lambda_2 v_2) = \ldots = \ldots \Rightarrow$ LIN: $A = (5 \;\; -2 \;\; 3)$

$f_5: \mathbb{R}^2 \to \mathbb{R}: (x, y) \longmapsto |x - y|$

$f_5(\lambda_1 v_1 + \lambda_2 v_2) = |\lambda_1 x_1 + \lambda_2 x_2 - \lambda_1 y_1 - \lambda_2 y_2|$

$v_1 = (-1, -2) \qquad f_5(v_1 + v_2) = |2 - (-6)| = 8$

$v_2 = (3, 4) \qquad f_5(v_1) + f_5(v_2) = |1| + |-1| = 2.$

$\Rightarrow$ NIET LIN

$f_6: \mathbb{R}^2 \to \mathbb{R}^2: (x, y) \longmapsto (x - y, x + y)$

$f_6(\ldots) = (\lambda_1 x_1 + \lambda_2 x_2 - (\lambda_1 y_1 + \lambda_2 y_2), \lambda_1 x_1 + \lambda_2 x_2 + \lambda_1 y_1 + \lambda_2 y_2)$

$v = (\lambda_1 x_1 + \lambda_2 x_2, \lambda_1 y_1 + \lambda_2 y_2)$

$\Rightarrow$ LIN: $A = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

$f_7: \mathbb{R}^2 \to \mathbb{R}^2: (x, y) \longmapsto (x + 2y, 1)$

$f_7(2(1,1)) = f_7(6, 1)$

$2 f_7(1,1) = (6, 2)$

$\Rightarrow$ NIET LIN.

f_8: ...

triviaal $\Rightarrow$ LIN. $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \\ 1 & -1 & -1 \end{pmatrix}$

~~f8~~ f_9 : ...

$\Rightarrow$ NIET LIN

f_{10}

$\Rightarrow$ LIN $\quad A = \begin{pmatrix} 0 & 3 & 1 \\ 1 & -1 & -1 \end{pmatrix}$

$f_{11} : \mathbb{R}^2 \to \mathbb{R}^3 : (x,y) \longmapsto (\sin x, 7y, xy)$

$f\left((\pi,0) + (2\pi,0)\right) = (\sin 3\pi, 0, 0)$

$= (-1, 0, 0)$

$f(\pi,0) + f(2\pi,0) = (1,0,0) \quad \Rightarrow$ NIET LIN

$f_{12} : \mathbb{R}[X]^2 \to \mathbb{R}[X]^3 : a + bX + cX^2 \longmapsto c + 2a\,X + (a+b+c)X^3$

$f(v) = (\lambda_1 c_1 + \lambda_2 c_2) + 2(\lambda_1 a_1 + \lambda_2 a_2)X + \lambda_1(a_1+b_1+c_1)X^3 + \lambda_2(a_2+b_2+c_2)X^3$

$v = \lambda_1 a_1 + \lambda_2 a_2 + (\lambda_1 b_1 + \lambda_2 b_2)X + (\lambda_1 c_1 + \lambda_2 c_2)X^2$

$\Rightarrow$ LIN. $A =$

(basis $= \{1, X, X^2\}$ in $\mathbb{R}[X]^2$ en $\{1, X, X^2, X^3\}$ in $\mathbb{R}[X]^3$)

$\Rightarrow A = \begin{bmatrix} 0 & 0 & 1 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$\Rightarrow A \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} c \\ 2a \\ 0 \\ a+b+c \end{pmatrix}$

$f_{13} : \mathbb{R}^{2\times 2} \to \mathbb{R}^2 : \begin{pmatrix} a & b \\ c & d \end{pmatrix} \longmapsto (a+c, b-d)$

$v = \lambda_1 \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \lambda_2 \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$

$f(v) = \left(\lambda_1 a_1 + \lambda_2 a_2 + \lambda_1 c_1 + \lambda_2 c_2, \lambda_1(b_1 - d_1) + \lambda_2(b_2 - d_2)\right)$

$= \left(\lambda_1(a_1+c_1) + \lambda_2(a_2+c_2), \ldots\right)$

$= \lambda_1 f(v_1) + \lambda_2 f(v_2) \Rightarrow$ lin

STANDAARDBASIS v. $\mathbb{R}^2$ $= \{(1,0), (0,1)\}$

?? $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \Rightarrow * \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$ ~~$? \times 2 \times 2 = 2 \times 1$~~ ??

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2) $f(1,0,0) = (-3,-13,-17)$

$f(0,1,0) = (-2,-5,-8)$

$f(0,0,1) = (2,7,10)$

A t.o.v. standaardbasis is

$$= \begin{bmatrix} -3 & -2 & 2 \\ -13 & -5 & 7 \\ -17 & -8 & 10 \end{bmatrix}.$$

$f(1,-1,1) = (1,-1,1) \longrightarrow$ t.o.v. nieuwe basis $= 1\,(1,-1,1)$

$f(1,2,3) = (-1,-2,-3) \longrightarrow$ t.o.v. nieuwe basis $= -1\,(1,2,3)$

$f(0,1,1) = (0,2,2) \longrightarrow$ t.o.v. nieuwe basis $= 2\,(0,1,1)$

dus A t.o.v nieuwe basis

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

② $f: \mathbb{R}^3 \to \mathbb{R}^3 : (x,y,z) \longmapsto (-3x - 2y + 2z, -13x - 5y + 7z, -17x - 8y + 10z)$.

$$A = \begin{bmatrix} -3 & -2 & +2 \\ -13 & -5 & +7 \\ -17 & -8 & +10 \end{bmatrix}$$

als basis $\{(1,-1,1), (1,2,3), (0,1,1)\} = B$

dus $a_n(1,-1,1) + b(1,2,3) + c(0,1,1) = v$

stel $g: \mathbb{R}^3 \to \mathbb{R}^3 : (a,b,c) \longmapsto (a+b, -a+2b+c, a+3b+c)$.

$$B = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 2 & 1 \\ 1 & 3 & 1 \end{bmatrix}$$

$$A \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = f(x,y,z) = B \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

dus
$$\begin{aligned} -3x - 2y + 2z &= a + b \\ -13x - 5y + 7z &= -a + 2b + c \\ -17x - 8y + 10z &= a + 3b + c. \end{aligned}$$

$\Rightarrow$ stelsel:

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & -3x - 2y + 2z \\ -1 & 2 & 1 & -13x - 5y + 7z \\ 1 & 3 & 1 & -17x - 8y + 10z \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & -3x - 2y + 2z \\ 0 & 3 & 1 & -16x - \cancel{3}7y + \cancel{5}9z \\ 0 & \cancel{3}2 & 1 & -4x - 3y + 3z \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 3 & 0 & -1 & 7x + y - 3z \\ 0 & 3 & 1 & -16x - 7y + 9z \\ 0 & 0 & 1 & 20x + 5y - 9z \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 3 & 0 & 0 & 27x + 6y - 12z \\ 0 & 3 & 0 & +36x + 12y - 18z \\ 0 & 0 & 1 & 20x + 5y - 9z \end{array}\right]$$

$$\begin{aligned} a &= 9x + 3y - 4z \\ b &= 12x + 4y - 6z \\ c &= 20x + 5y - 9z \end{aligned}$$

??

THEORIE:

$$\begin{bmatrix} \cdots \end{bmatrix}$$

kolommen $= L(e_j)$

dus
$$A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & -2 & 2 \\ 1 & -3 & 2 \end{bmatrix}$$

dan is
$$A \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} a - b \\ -a + 2b + 2c \\ a - 3b + 2c \end{pmatrix}$$

stel bv. dat $a = b = c = 1$.
(is punt $(2,2,5)$)
$f(p) = (0, \cancel{12}, 0)$ volgens st. B.
-1

$= \cancel{(12, 8, 0)}$ ~~volgens B~~

2,2,5 → (1,1,1) ⟶ (0,3,0) ↑
b=3

pideevel: f volgens basis B: $f(1,-1,1) = (1,-1,1)$ $f(0,1,1) = (0,2,2)$
$f(1,2,3) = (-1,-2,-3) = (0,-1,0)$ volgens B.

$\Rightarrow$ volgens B: $A: \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ ⟶ volgens b $= (0,-1,0)$

③ $f: \mathbb{R}^2 \to \mathbb{R}^2$:

$f(1,2) = (0,-1)$

$f(-1,1) = (2,1)$

$$\begin{cases} a \cdot 1 + b \cdot 2 = 0 \\ c + 2d = -1 \\ -a + b = 2 \\ -c + d = 1 \end{cases}$$

$$\left[\begin{array}{cc|cc} 1 & 2 & 0 & -1 \\ -1 & 1 & 2 & 1 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & 2 & 0 & -1 \\ 0 & 3 & 2 & 0 \end{array}\right] \sim \left[\begin{array}{cc|cc} 3 & 0 & -4 & -3 \\ 0 & 3 & 2 & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 0 & -4/3 & -1 \\ 0 & 1 & 2/3 & 0 \end{array}\right]$$

$a = -4/3$ $\quad c = -1$

$b = 2/3$ $\quad d = 0$

$(x,y) \longmapsto (-4/3\, x + 2/3\, y \;, -x)$

④ $f: \mathbb{R}[x]^2 \to \mathbb{R}[x]^3 : a + bX + cX^2 \longmapsto c + (a+b)X + (a+c)X^2 + (b+c)X^3$

a) $v = (\lambda_1 a_1 + \lambda_2 a_2) + (\lambda_1 b_1 + \lambda_2 b_2)X + (\lambda_1 c_1 + \lambda_2 c_2)X^2$

$f(v) = \lambda_1 c_1 + (\lambda_1 a_1 + \lambda_1 b_1)X + (\lambda_1 a_1 + \lambda_1 c_1)X^2 + (\lambda_1 b_1 + \lambda_1 c_1)X^3$

$\quad + \lambda_2 c_2 + (\lambda_2 a_2 + \lambda_2 b_2)X + (\lambda_2 a_2 + \lambda_2 c_2)X^2 + (\lambda_2 b_2 + \lambda_2 c_2)X^3$

... TRIV.

b) matrix t.o.v. standaard b. $= \{1, x, x^2\}$

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} c \\ a+b \\ a+c \\ b+c \end{pmatrix}$$

matrix t.o.v. basissen $B = \{1+x, 1+x^2, x+x^2\}$ in $\mathbb{R}[x]^2$

$B' = \{1, 1+x, 1+x^2, 1+x^3\}$ in $\mathbb{R}[x]^3$.

$$\begin{array}{ccc} \mathbb{R}[X]^2 & \xrightarrow{f} & \mathbb{R}[X]^3 \\ \downarrow co_B & & \uparrow co_{B'} \\ \mathbb{R}^3 & \xrightarrow{\tilde{f}} & \mathbb{R}^4 \end{array}$$

① we vullen de basis van B in, in f:

$f(1+x) = 2x + x^2 + x^3$ $\sim f(1,0,0) = (0,2,1,1)$ volgens stB

$f(1+x^2) = 1 + \cancel{x} + 2x^2 + x^3$ $\sim \cancel{f(0,1,0)} = (-4,2,1,1)$ volgens $\mathbb{R}[x]$ B'

$f(0,1,0) = (-3,1,2,1)$

$f(x+x^2) = 1 + x + x^2 + 2x^3$

$f(0,0,1) = (-3,1,1,2)$

de matrix is dus

$$A = \begin{bmatrix} -4 & -3 & -3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

?

5 a) B is vrij:

als $\lambda_1 v_1 + \lambda_2 v_2 + \lambda_3 v_3 + \lambda_4 \cdot v_4 = 0$

is $\begin{cases} \lambda_1 \cdot 0 + \lambda_2 \cdot 0 + \lambda_3 \cdot 0 + \lambda_4 = 0 \\ \lambda_2 + 2\lambda_3 + \lambda_4 = 0 \\ \lambda_1 + 2\lambda_2 + 3\lambda_3 - \lambda_4 = 0 \\ \lambda_3 + \lambda_4 = 0 \end{cases}$ dus is $\lambda_4 = 0$, $\lambda_3 = 0$, $\lambda_2 = 0$, $\lambda_1 = 0$.

B is voortbrengend.

Kies $w^\circ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathbb{R}^{2\times 2}$ dan is

$$\begin{cases} \lambda_4 = a \\ \lambda_3 + \lambda_4 = d \\ \lambda_2 + 2\lambda_3 + \lambda_4 = b \\ \lambda_1 + 2\lambda_2 + 3\lambda_3 - \lambda_4 = c \end{cases}$$

b) $\lambda_1 v_1 + \lambda_2 v_2 + \lambda_3 v_3 + \lambda_4 v_4 = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

$\lambda_4 = 1 \quad \lambda_2 = 1$

$\lambda_3 = -1 \quad \lambda_1 = 2$.

dus $f\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 0 & 1 & 1 \\ 0 & 3 & 1 & 0 \\ 2 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} 2 \\ 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \\ 2 \\ 5 \end{pmatrix}$ t.o.v. B.

~~tegens de standaard~~

DUS $f\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = 5v_1 + 4v_2 + 2v_3 + 5v_4$

$= \begin{pmatrix} 5 & 13 \\ 14 & 7 \end{pmatrix}$

6 a) geg: L is vrij en lin. $L(\lambda_1 v_1 + \lambda_2 v_2) = \lambda_1 L(v_1) + \lambda_2 L(v_2)$

dus $\forall v_1, v_2 \in V : L(v_1) = L(v_2) \Rightarrow v_1 = v_2$.

$\lambda_1 L(e_1) + \dots + \lambda_n L(e_n) = 0$

$= L(\lambda_1 e_1 + \dots + \lambda_n e_n)$

nu is $L(0) = 0$ omdat L lineair is dus is

$L(0) = L(\lambda_1 e_1 + \dots + \lambda_n e_n)$

dus $0 = \sum_{i=1}^{n} \lambda_i e_i$ omdat $\{e_1, \dots, e_n\}$ is vrij is, is

$\lambda_1 = \lambda_2 = \dots = \lambda_n = 0$.

dus is $\{L(e_1), L(e_2), \dots, L(e_n)\}$ vrij

b) stel dat L niet injectief is.

$L: \mathbb{R}^2 \to \mathbb{R}: (x,y) \longmapsto x+y$ (L is lin want $L(\lambda_1 v_1 + \lambda_2 v_2)$ $= \lambda_1(x_1+y_1) + \lambda_2(x_2+y_2)$)

nu is $L(2,1) = 3$ maar $(2,1) \neq (1,2)$

en is $L(1,2) = 3$ dus L is niet injectief

gelijkaardig is ook

$M: \mathbb{R}^4 \to \mathbb{R}: (x,y,z,\omega) \longmapsto x+y+z+\omega$ niet injectief en wel lin.

neem nu de standaard basis van $\mathbb{R}^2$: dan is

$L(e_1) = 1$, maar $L(e_2) = 1$

dus is $\{L(e_1), L(e_2)\} = \{1, 1\}$ wat duidelijk geen vrij deel van $\mathbb{R}$ is

7 alle functie $F: \mathbb{R} \to \mathbb{R}$ waarvoor $DF = f$ met D-afleidsoperator

F_p = particuliere oplossing

TOON AAN alle oplossing zijn van de vorm $F = F_p + C$

met C de cte functie

$\operatorname{Ker} D$ = alle cte functies.

~~Stelling~~ ① Bewijs $F_p + C$ een opl is.

$D(F_p + C) = DF_p + 0$ ~~en~~ dus

omdat F_p een opl. is, is ~~D(~~$F_p + C$ dus ook een opl.

② Bewijs dat alle opl. $F = F_p + C$.

door ~~$F-F_p$~~ te tonen $F - F_p$ - cte functie

$D(F - F_p) = D(F) - D(F_p)$

$= f - f = 0$ → afgeleide is 0 dus cte functie

zelfstandige serie 3.

extra oef 1.

$f: \mathbb{R}^{2\times 2} \longrightarrow \mathbb{R}[X]^2$

$B_1 = \left\{ M_1 = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}\ M_2 = \begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}\ M_3 = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}\ M_4 = \begin{pmatrix} 0 & 1 \\ 0 & -1 \end{pmatrix} \right\}$

$B_2 = \{1 + X, 1 - X, X^2\}$

$A = \begin{pmatrix} 1 & 0 & 3 & 1 \\ 0 & 2 & 0 & 1 \\ -2 & 3 & -6 & 1 \end{pmatrix}$

① $f\begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix} = A \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} = 2(1-X) + 3 \cdot X^2$

$= 2 - 2x + 3x^2$

② $f\begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix} = f(M_1 + M_4) = A \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = 2(1+x) + 1 - x - x^2$

$= 3 + x - x^2.$

③ ① f is injectief $\Leftrightarrow$ Ker $f = \{0\}$

dus vindt een $B \in \mathbb{R}^{2\times 2}$ waarvoor $f(B) = 0$ met $B \neq 0$

$B = aM_1 + bM_2 + cM_3 + dM_4.$

$A \cdot \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = 0 = \begin{pmatrix} a + 3c + d \\ 2b + d \\ -2a + 3b - 6c + d \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$= (a + 3c + d)(1 + x) + (a\ 2b + d)(1 - x) + (-2a + 3b - 6c + d)x^2$

$= a + 2b + 3c + 2d + X(a - 2b + 3c) + X^2(-2a + 3b - 6c + d)$

$\begin{cases} a + 2b + 3c + 2d = 0 \\ a - 2b + 3c = 0 \\ -2a + 3b - 6c + d = 0 \end{cases} \quad \left[\begin{array}{cccc|c} 1 & 2 & 3 & 2 & 0 \\ 1 & -2 & 3 & 0 & 0 \\ -2 & 3 & -6 & 1 & 0 \end{array}\right]$

$\sim \left[\begin{array}{cccc|c} 1 & 2 & 3 & 2 & 0 \\ 0 & \cancel{-4}\,2 & 0 & \cancel{-2}\,1 & 0 \\ 0 & 7 & 0 & 5 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 2 & 0 & 6 & 2 & 0 \\ 0 & 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & 0 \end{array}\right] \Rightarrow$ niet enkel nuloplossing.

$\left.\begin{array}{l} a = -3c \\ b = 0 \\ d = 0 \end{array}\right\}$ $f(-3M_1 + M_3) = 0$

$-3M_1 + M_3 = \begin{pmatrix} -6 & 1 \\ 0 & -2 \end{pmatrix}$

② f is inj $\Leftrightarrow \forall v_1, v_2 \in \mathbb{R}^{2\times 2} : f(v_1) = f(v_2) \Rightarrow v_1 = v_2.$

$f\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = 0$

$f\begin{pmatrix} -6 & 1 \\ 0 & -2 \end{pmatrix} = 0$

extra oef 2.

$$T: \mathbb{R}^{2\times 3} \longrightarrow \mathbb{R}: \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} \longmapsto \big((a^3+d)n^3 + (b+e)n^2 + (c-f)n + 2a\big)_{n\in\mathbb{N}}$$

① $T\begin{pmatrix} 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}_n = -2n^2 - n \quad \forall n \in \mathbb{N}$

$\lim_{n\to\infty} (-2n^2 - n) = -\infty$

② Ker T

$$\begin{cases} a^3 + d = 0 \\ b + e = 0 \\ c - f = 0 \\ 2a = 0 \end{cases}$$

dus $\text{Ker}\, T = \left\{ \begin{pmatrix} 0 & \lambda & \mu \\ 0 & -\lambda & \mu \end{pmatrix} \in \mathbb{R}^{2\times 3} \;\middle|\; \lambda, \mu \in \mathbb{R} \right\}$

basis $= \left\{ \begin{pmatrix} 0 & 1 & 0 \\ 0 & -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right\}$

③ Im T = verzameling van alle el$^{\bar{n}}$ van $\mathbb{R}$ waarvan er een $v \in \mathbb{R}^{2\times 3}$: $T(v) = w$.

basis $= \left\{ (1)_{n\in\mathbb{N}}, (n)_{n\in\mathbb{N}}, (n^2)_{n\in\mathbb{N}}, (n^3)_{n\in\mathbb{N}} \right\}$

want elk element van Im T is te schrijven als $(v)_n = w(1)_{n\in\mathbb{N}} + x(n)_{n\in\mathbb{N}} + y(n^2)_{n\in\mathbb{N}} + z(n^3)_{n\in\mathbb{N}}$

met $w, x, y, z \in \mathbb{R}$

namelijk als $a = w/2$, $b = y$, $c = x$, $d = z - \left(\frac{w}{2}\right)^3$

en $e = 0 = f$

opdrachten 4.5.5 p494

1 (a) $y_{n+1} - y_n = e^n$

(1) karakteristieke vgl:

$\lambda - 1 = 0$
$\lambda = 1$ } algemene opl. van de homogene vgl: $y_n = C(1)^n = C$ met $C \in \mathbb{R}$.

(2) veronderstel $y_n = \alpha e^n$

$\alpha e^{n+1} - \alpha e^n = e^n$

$$\alpha = \frac{e^n}{e^{n+1} - e^n} = \frac{1}{e-1}$$

dus de algemene oplossing is $y_n = \dfrac{e^n}{e-1} + C$

(b) $y_{n+1} + 3y_n = 4$

(1) $\lambda + 3 = 0$
$\lambda = -3$ } $y_n = (-3)^n \cdot C$ met $C \in \mathbb{R}$

(2) stel $y_n = \alpha$

dan is $\alpha + 3\alpha = 4$.
$\alpha = 1$.

dus is de algemene opl. $y_n = 1 + (-3)^n \cdot C$ met $C \in \mathbb{R}$.

(c) $2y_{n+1} - y_n = 6$

(1) $2\lambda - 1 = 0$
$\lambda = 1/2$ } $y_n = \left(\frac{1}{2}\right)^n \cdot C$ met $C \in \mathbb{R}$.

(2) $y_n = \alpha$
$2\alpha - \alpha = 6$
$\alpha = 6$ } $y_n = 6 + \left(\frac{1}{2}\right)^n \cdot C$ met $C \in \mathbb{R}$.

(d) $y_{n+1} = 0{,}2\, y_n + 4$ $\lambda - 0{,}2 = 0$ dus $\lambda = 0{,}2$.

(1) ~~$\lambda = 0{,}2 + 4 = 4{,}2 \rightarrow y_n = (4{,}2)^n \cdot C$~~ met $C \in \mathbb{R}$.
$y_n = \left(\frac{1}{5}\right)^n \cdot C$

(2) $y_n = \alpha$
$\alpha = 0{,}2\alpha + 4$.
$0{,}8\alpha = 4$
$\alpha = 5$ } $y_n = 5 + \cancel{(4{,}2)^n}\ \frac{1}{5^n} \cdot C$ met $C \in \mathbb{R}$.

4
1
9
2
5
3

e) $y_{n+1} - 2y_n = n$

(1) $\lambda - 2 = 0$, $\lambda = 2$ $\Big\}$ $y_n^{(0)} = 2^n \cdot C$ met $C \in \mathbb{R}$

(2) stel $y_n = \alpha n + \beta$

$\alpha(n+1) + \beta - 2(\alpha(n) + \beta) = n$

$\alpha n + \alpha + \beta - 2\alpha n - 2\beta = n$

$-\alpha n + \alpha - \beta = n$

dus $\begin{cases} -\alpha = 1 \\ \alpha - \beta = 0 \end{cases}$ dus $\alpha = \beta = -1$.

dus algemeen: $y_n = -n - 1 + 2^n \cdot C$ met $C \in \mathbb{R}$.

f) $y_{n+1} - (n+1)y_n = 1$

(1) $y_{n+1} - (n+1)y_n = 0$

$y_n = (n+1-1)y_{n-1} = n \cdot (n-1)y_{n-2} = n(n-1) \dots 2 \cdot 1 \cdot y_0$

$= y_0 \cdot n!$

dus $y_n = C \cdot n!$ met $C \in \mathbb{R}$

(2) $y_{n+1} = 1 + (n+1)y_n$

$y_n = 1 + ny_{n-1} = 1 + n + n(n-1)y_{n-2} = \dots$

$\sum_{i=0}^{n} \frac{n!}{(n-i)!} + n!\,y_0$ dus $y_n = n! \sum_{i=0}^{n} \frac{1}{i!}$

g) $y_{n+1} - e^{2n} y_n = 0$

$y_n = e^{2(n-1)} y_{n-1} = \frac{e^{2n}}{e} \cdot e^{2n-2} \cdot y_{n-2} = e^{4n-3} e^{2n-3} y_{n-3}$

$= \frac{e^{2n}}{e} \cdot y_{n-1} = e^{\cancel{2n-1+2n-2}\ 4n-3} y_{n-2}$

$= e^{8n-6-4} y_{n-4}$

$= e^{10n-10-5} y_{n-5}$

$= e^{10n-15} y_{n-5}$

$= e^{2n^2 - \sum_{i=1}^{n} i} y_0$

dus algemeen $y_n = e^{2n^2 - \sum_{i=1}^{n} i} \cdot C$ met $C \in \mathbb{R}$.

h) $(n+2)y_{n+1} - (n+1)y_n = n+1$

$n - (n-1)$

(1) $y_n = \frac{n \cdot y_{n-1}}{n+1} = \frac{n}{n+1} \cdot \frac{n-1}{n} y_{n-2} = \frac{n-1}{n+1} \cdot \frac{n-2}{n-1} \cdot y_{n-3}$

$= \frac{1}{n+1} y_0$

$\Rightarrow y_n = \frac{1}{n+1} \cdot C$ met $C \in \mathbb{R}$

(2) y_n ? ? ? y stel $y_n = \alpha n + \beta$ dan is $\alpha = \frac{1}{2}$, $\beta = 0$.

dus $y_n^p = \frac{n}{2}$.

(2) (a) $y_{n+1} - (1-2p)y_n = p$ met $y_0 = 1-p$ met $p \in \mathbb{R}$

(1) $\lambda - (1-2p) = 0$
$\lambda = 1-2p$ $\Big\}$ $y_n^{(0)} = (1-2p)^n \cdot C$ met $C \in \mathbb{R}$

(2) stel $y_n = \alpha$ dan:
$\alpha - (1-2p)\alpha = p$
$\alpha(2p) = p$
$\alpha = 1/2$

$y_n = 1/2 + (1-2p)^n \cdot C$ met $C \in \mathbb{R}$.

(3) $y_0 = \frac{1}{2} + C = 1-p$
$C = \frac{1}{2} - p$

dus is $y_n = \frac{1}{2} + (1-2p)^n \cdot (\frac{1}{2} - p)$

(b) $y_{n+1} - (n+1)y_n = (n+1)!$ met $y_0 = 1$

(1) $y_n = n\, y_{n-1} = n(n-1)y_{n-2}$
$= n!\, y_0 = n!\, C$ met $C \in \mathbb{R}$.

(2) $y_1 = (0+1)! + (0+1) \cdot y_0 = 2$ $\cdot 3$
$y_2 = (1+1)! + (1+1)\, 2 = 6$ $\cdot 4$
$y_3 = (2+1)! + (2+1)\, 6 = 3! + 18 = 24$ $\cdot 5$
$y_4 = 4! + 4 \cdot 24 = 120$ $\cdot 6$
$y_5 = 5! + 5 \cdot 120 = 720$

$y_n = n! + n \cdot \alpha_n$

$y_n = y_{n-1} \cdot (n+1)$

$y_n = (n+1)!$ ~~dus vergeten~~

(3) 75 000 €, jaarlijkse interest v. 8% die maandelijks is samengesteld.
$\Rightarrow$ effectieve maandelijkse van $\frac{2}{3}$%

a) $75000 = a_{\overline{n}|} \cdot \frac{v^n - 1}{(v'-1)v^n}$

$y_n = y_{n-1}(1 + \frac{2}{300}) - A$

$75000 = y_0$
$y_1 = y_0 - (A - y_0 \cdot \frac{2}{3}\%)$

b) $A = (1 + \frac{2}{300})\, y_{n-1} - y_n \quad \forall n \in \mathbb{N}$

c) (1) $0 = -\lambda + 1 + \frac{2}{300}$
$\lambda = \frac{302}{300}$ $\Big\}$ $y_n^{(0)} = \left(\frac{302}{300}\right)^n \cdot C$ met $C \in \mathbb{R}$

(2) stel $y_n = \alpha$
$A = \frac{302}{300}\alpha - \frac{300}{300}\alpha$
$\alpha = 150A$

$y_n = 150A - \left(\frac{302}{300}\right)^n \cdot C$ met $C \in \mathbb{R}$
$y_0 = 150A - C = 75000$
$C = 150A - 75000$

DUS $y_n = 150A\left(1 - \left(\frac{302}{300}\right)^n\right) + \left(\frac{302}{300}\right)^n \cdot 75000$

(3) $y_{240} = 0 = A\,150\left(1 - \left(\frac{302}{300}\right)^{240}\right)$

$A = 627{,}33$

4 Kapitaal K , jaarlijkse intrest 6% $\Rightarrow$ eff. maandelijkse intrest $= \frac{1}{2}$%
niet belangrijk ...

$an = 20\,000$

a) $K_n = K_{n-1} \cdot 1{,}06 - 20\,000$

b)(1) $\lambda - 1{,}06 = 0$
$\lambda = 1{,}06$ $\Big\}$ $K_n = (1{,}06)^n \cdot C$ met $C \in \mathbb{R}$.

(2) stel $K_n = \alpha$ $\forall n \in \mathbb{N}$

$\alpha - 1{,}06\alpha = -20\,000$
$\alpha = \frac{1\,000\,000}{3}$ $\Big\}$ $K_n = \frac{1\,000\,000}{3} + (1{,}06)^n \cdot C$ met $C \in \mathbb{R}$

c) wat is K_0 zodat $\forall n \in \mathbb{N}$ $K_n > 0$

$K_0 = \frac{1000\,000}{3} + C$

$\frac{10^6}{3} + C \cdot 1{,}06^n > 0$ $\forall n \in \mathbb{N}$

$C > \frac{-10^6}{3} \cdot \frac{1}{1{,}06^n}$ $\Leftrightarrow$ als $\lim\limits_{n\to\infty} K_n > 0$
met $C \geq 0$

$\lim\left(\frac{10^6}{3} + C \cdot 1{,}06^n\right) \geq 0$

$C \cdot \lim(1{,}06^n) > \frac{-10^6}{3}$

$C > \frac{-10^6}{3 \cdot \lim(1{,}06^n)}$

dus moet $C = K_0 - \frac{10^6}{3} > 0$
$K_0 > \frac{10^6}{3}$.

d) $K_n = K_{n-1} \cdot 1{,}06 - 20\,000 \cdot 1{,}02^n$

stel $K_n = \alpha \cdot 1{,}02^n$ $\forall n \in \mathbb{N}$

dan is $-\alpha \cdot 1{,}02^n + \alpha \cdot 1{,}02^{n-1} \cdot 1{,}06 = 20\,000 \cdot 1{,}02^n$

$\alpha\left(\frac{1{,}06}{1{,}02} - 1\right) = 20\,000$

$\alpha = 510\,000$

dus $K_n = 510\,000 + (1{,}06)^n \cdot C$ met $C \in \mathbb{R}$

$K_n > 0$ $\forall n \in \mathbb{N}$ als $K_0 > 510\,000$

(20

a) $K_n = K_{n-1} \cdot 1{,}06 - 20\,000$ oef 4

b) $y_n - y_{n-1} \cdot 1{,}06 = -20\,000$. met $y_0 = K$ (beginkap).

y^h: $\lambda - 1{,}06 = 0$

$\lambda = 1{,}06$

$$\left.\begin{matrix} \lambda - 1{,}06 = 0 \\ \lambda = 1{,}06 \end{matrix}\right\} y_n^h = (1{,}06)^n \cdot C$$

y^p: $y_n = \alpha$

$\alpha - \alpha \cdot 1{,}06 = -20\,000$

$\alpha(0{,}06) = 20\,000$

$\alpha = \dfrac{20\,000}{0{,}06}$

$y = \dfrac{20\,000}{0{,}06} + (1{,}06)^n \cdot C$.

$y_0 = \dfrac{20\,000}{0{,}06} + C = K$.

dus $C = K - \dfrac{20\,000}{0{,}06}$.

dus algemeen: $K_n = \dfrac{20\,000}{0{,}06} + \left(K - \dfrac{20\,000}{0{,}06}\right) \cdot 1{,}06^n$.

c) $\lim\limits_{n \to \infty} K_n > 0$ moet $K - \dfrac{20\,000}{0{,}06} \geq 0$

dus $K \geq \dfrac{20\,000}{0{,}06}$.

d) * $K_n = K_{n-1} \cdot 1{,}06 - 20\,000 \cdot 1{,}02^{n-1}$ $K_0 = K$.

* y^h: $\lambda - 1{,}06 = 0$. $\Rightarrow y_n^h = 1{,}06^n \cdot C$.

y^p: $= \alpha \cdot 1{,}02^{n-1} = \alpha \cdot \dfrac{1{,}02^n}{1{,}02}$.

$\dfrac{\alpha \cdot \cancel{1{,}02^{n-1}}}{1{,}02} \cdot 1{,}06 - \alpha \cdot \cancel{1{,}02^{n-1}} = 20\,000 \cdot \cancel{1{,}02^{n-1}}$

$\alpha\left(\dfrac{1{,}06}{1{,}02} - 1\right) = 20\,000$.

$\alpha = 510\,000$.

dus $y = 510\,000 \cdot 1{,}02^{n-1} + 1{,}06^n \cdot C$

$y_0 = 500\,000 + C = K$ dus $C = K - 500\,000$.

$y = 1{,}06^n (K - 500\,000) + 500\,000 \cdot 1{,}02^n$.

Zelfstandige 4.

extra oef 1

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 6 & 6 & 6 \\ 4 & 6 & 8 \end{pmatrix}$$

① basis voor Rij(A)

is er een $\lambda, \mu \in \mathbb{R}$ $(6,6,6) = v_1 \cdot \lambda + v_2 \cdot \mu$

$$\begin{cases} \lambda + 4\mu = 6 \\ 2\lambda + 5\mu = 6 \\ 3\lambda + 6\mu = 6 \end{cases} \quad \ldots$$

A omvormen : $\begin{pmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -6 & -12 \\ 0 & -2 & -4 \end{pmatrix}$ dus basis $= \{(1,2,3),(0,1,2)\}$

② kolomrang(A) = rijrang(A) = 2

③ oplosbaar als rang(A) = rang(A|B)

$$\text{rang}(A|B) = \text{rang}\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 8 \\ 6 & 6 & 6 & 12 \\ 4 & 6 & 8 & 12 \end{pmatrix} = \text{rang}\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -6 & -8 \\ 0 & -6 & -12 & -12 \\ 0 & -2 & -4 & -4 \end{pmatrix}$$

$$= \text{rang}\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 3 & 6 & 8 \\ 0 & 1 & 2 & 2 \end{pmatrix} = \text{rang}\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 3 & 6 & 8 \\ 0 & 0 & 0 & -2 \end{pmatrix} = 3 \Rightarrow \text{niet oplosbaar.}$$

④ $L : \mathbb{R}^3 \to \mathbb{R}^4 : X \longmapsto AX$

$\text{Im}(L) = \text{Kol}(A)$ dus $\dim(\text{Im}(L)) = \dim(\text{Kol}(A)) = \text{kolomrang}(A) = 2.$

extra oef 2.

$A \in \mathbb{R}^{7\times 7}$ en A is inverteerbaar

rijrang(A) = # kolommen A omdat A inverteerbaar is
$= 7$

extra oef 3.

$U = \text{vct}\{(1,6,0,2,0,3), (0,1,2,0,3,0), (1,8,4,2,6,3), (-2,5,1,-1,3,0)$
$(1,1,1,1,1,1), (1,2,3,1,4,1)\}$

$$A = \begin{bmatrix} 1 & 6 & 0 & 2 & 0 & 3 \\ 0 & 1 & 2 & 0 & 3 & 0 \\ 1 & 8 & 4 & 2 & 6 & 3 \\ -2 & 5 & 1 & -1 & 3 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 1 & 4 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 6 & 0 & 2 & 0 & 3 \\ 0 & 1 & 2 & 0 & 3 & 0 \\ \cancel{0} & \cancel{2} & \cancel{4} & \cancel{0} & \cancel{6} & \cancel{0} \\ 0 & 17 & 1 & 3 & 3 & +6 \\ 0 & -5 & 1 & -1 & 1 & -2 \\ 0 & -4 & 3 & -1 & 4 & -2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 6 & 0 & 2 & 0 & 3 \\ 0 & 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & -33 & 3 & -48 & 6 \\ 0 & 0 & \cancel{11} & \cancel{-1} & \cancel{16} & \cancel{-2} \\ 0 & 0 & \cancel{11} & \cancel{-1} & \cancel{16} & \cancel{-2} \end{bmatrix}$$

rang(A) = dim(U)
$= 3.$

(5) $a_1 = 1$

$a_2 = 3 = a_1 + a_1 + a_1 = 2a_1 + a_1$

$a_3 = 7 = a_2 + a_1 + a_2 = 2a_2 + a_1 = 2(2a_1 + a_1) + a_1$

$a_4 = 15 = a_3 + a_1 + a_3$

dus $a_n = 2a_{n-1} + 1$

(1) $\lambda - 2 = 0 \Rightarrow a_n^{(0)} = 2^n \cdot C$ met $C \in \mathbb{R}$

(2) stel $a_n = \alpha \ \forall n \in \mathbb{N}$

$$\left.\begin{array}{l}\alpha - 2\alpha = 1 \\ \alpha = -1\end{array}\right\} a_n = -1 + 2^n \cdot C \text{ met } C \in \mathbb{R}$$

(3) nu is $a_1 = 1 = -1 + 2 \cdot C$

dus is $C = 1$

$\Rightarrow a_n = 2^n - 1 \quad \forall n \in \mathbb{N}$

(6) (a) $y_{n+2} - 2\cos\varphi \, y_{n+1} + y_n = 0$?

dus $\lambda^2 - 2\cos\varphi \, \lambda + 1 = 0$

$D = 4\cos^2\varphi - 4$ ~~$= -4\sin^2\varphi$~~

* stel $\varphi = k\pi$ met $k \in \mathbb{Z}$

dan is $\lambda_1 = \lambda_2 = \cos\varphi$ met $A, B \in \mathbb{R}$.

en is $y_n = (A + Bn)\cos^n\varphi$

* stel $\varphi \neq k\pi$ met $k \in \mathbb{Z}$

dan is $\lambda_{1,2} = \cos\varphi \pm i\sqrt{1 - \cos^2\varphi}$

$r = \sqrt{a^2 + b^2}$ $\quad \theta = \operatorname{Bgcos}\left(\frac{a}{\sqrt{a^2+b^2}}\right)$

$\sqrt{\cos^2\varphi + 1 - \cos^2\varphi} = 1 \quad = \operatorname{Bgcos}\left(\frac{\cos\varphi}{\sqrt{\cos^2\varphi + 1 - \cos^2\varphi}}\right) = \varphi$

en is $y_n = \underbrace{r^n}_{=1}(A\cos n\underbrace{\theta}_{=\varphi} + B\sin n\underbrace{\theta}_{=\varphi})$ met $A, B \in \mathbb{R}$.

(b) $y_{n+2} - 2y_{n+1} + 3y_n = 4$

(1) $\lambda^2 - 2\lambda + 3 = 0$

$D = 4 - 12 = -8$

$\lambda = \frac{2 \pm 2i\sqrt{2}}{2} = 1 \pm i\sqrt{2}$

$r = \sqrt{1 + 2} = \sqrt{3}$

$\theta = \cos\theta = \frac{1}{\sqrt{3}}$

$\theta = \operatorname{Bgcos}\left(\frac{\sqrt{3}}{3}\right)$

dan is $y_n^{(0)} = \left(\frac{\sqrt{3}}{3}\right)^n \cdot \left(A\sin n \cdot \operatorname{Bgcos}\left(\frac{\sqrt{3}}{3}\right) + B\cos n \cdot \operatorname{Bgcos}\left(\frac{\sqrt{3}}{3}\right)\right)$ $A, B \in \mathbb{R}$

(2) stel $y_n = \alpha$

$\alpha - 2\alpha + 3\alpha = 4$

$\alpha = 2$

dus is $y_n = 2 + y_n^{(0)}$

(c) $y_{n+2} - 2y_{n+1} + 4y_n = 4$

(1) $\lambda^2 - 2\lambda + 4 = 0$

$D = 4 - 16 = -12$.

$\lambda = 1 \pm i\sqrt{3}$

$r = 2$ $\cos\theta = \frac{1}{2}$ dus $\theta = \pi/3$

$y_n^{(0)} = 2^n\left(A\cdot\cos\frac{n\pi}{3} + B\sin\frac{n\cdot\pi}{3}\right)$ met $A, B \in \mathbb{R}$

(2) stel $y_n = \alpha$ $\forall n \in \mathbb{N}$

dan $\alpha - 2\alpha + 4\alpha = 4$

$3\alpha = 4$

$\alpha = 4/3$

waaruit volgt dat $y_n = 4/3 + 2^n\left(A\cdot\cos\frac{n\cdot\pi}{3} + B\sin\frac{n\cdot\pi}{3}\right)$ met $A, B \in \mathbb{R}$

d) $y_{n+2} - 4y_{n+1} + 4y_n = 7$

(1) $\lambda^2 - 4\lambda + 4 = 0$

$S = 4$ $P = 4$ $\Rightarrow \lambda_1 = \lambda_2 = 2$

(1) $y_n^{(0)} = (A + Bn)2^n$ met $A, B \in \mathbb{R}$.

(2) stel $y_n = \alpha$

$\alpha - 4\alpha + 4\alpha = 7$ dus $\alpha = 7$ $\Big\}$ $y_n = 7 + y_n^{(0)}$ $\forall n \in \mathbb{N}$

e) $y_{n+2} - 4y_{n+1} + 4y_n = 2^n$

(1) $y_n^{(0)} = (A + Bn)2^n$ met $A, B \in \mathbb{R}$

(2) stel $y_n = \alpha 2^n$

$\alpha 2^{n+2} - 4\alpha 2^{n+1} + 4\alpha 2^n = 2^n$

$4\alpha - 8\alpha + 4\alpha = 0$ $\longrightarrow$ werkt niet

stel $y_n = \alpha n 2^n$

dan is $\alpha(n+2)\cdot 4\cancel{2^n} - 4\cdot 2\alpha(n+1)\cdot\cancel{2^n} + 4\alpha n\cancel{2^n} = \cancel{2^n}\,1$

$4\alpha n + 8\alpha - 8\alpha n - 8\alpha + 4\alpha n = 0$ $\longrightarrow$ werkt niet

dan stel $y_n = \alpha n^2 2^n$

dan is $\alpha(n+2)^2\cdot 4\cdot\cancel{2^n} - 8\alpha(n+1)^2\cancel{2^n} + 4\alpha n^2\cancel{2^n} = \cancel{2^n}\,1$

$4\alpha\cdot(n^2+4n+4) - 8\alpha(n^2+2n+1) + 4\alpha n^2 = 1$

$16\alpha n + 16\alpha - 16\alpha n - 8\alpha = 1$

$\alpha = 1/8$

Dus is $y_n = \frac{1}{8}n^2 2^n + (A + Bn)\,2^n$ met $A, B \in \mathbb{R}$

f) $y_{n+2} - 5y_{n+1} + 6y_n = 2 + 4n$

(1) $\lambda^2 - 5\lambda + 6 = 0$

$D = 25 - 24 = 1$

$\lambda_1 = \frac{5+1}{2} = 3 \quad \lambda_2 = \frac{5-1}{2} = 2.$

dan is $y_n^{(0)} = A \cdot 3^n + B \cdot 2^n$ met $A, B \in \mathbb{R}$.

(2) stel $y_n = \alpha n + \beta \quad \forall n \in \mathbb{N}$

$\alpha(n+2) + \beta - 5(\alpha(n+1) + \beta) + 6(\alpha n + \beta) = 2 + 4n$

$\alpha n + 2\alpha + \beta - 5\alpha n - 5\alpha - 5\beta + 6\alpha n + 6\beta = 2 + 4n$

$2\alpha n - 3\alpha + 2\beta = 4n + 2.$

dus: $\begin{cases} 2\alpha = 4 \\ -3\alpha + 2\beta = 2 \end{cases}$ $\alpha = 2$ $\quad 2\beta = 8 \quad \beta = 3$

waaruit volgt dat $y_n = 2n + 4 + 3^n \cdot A + 2^n B$ met $A, B \in \mathbb{R}$.

g) $y_{n+2} + 2y_{n+1} - 3y_n = n^3 + 1$

(1) $\lambda^2 + 2\lambda - 3 = 0$

$D = 4 + 12 = 16$

$\lambda = \frac{-2 \pm 4}{2} \quad \lambda_1 = 1, \quad \lambda_2 = -3$

$y_n^{(0)} = (-3)^n \cdot A + B$ met $A, B \in \mathbb{R}$.

(2) stel $y_n = \alpha n^3 + \beta n^2 + \gamma n + \delta \quad \forall n \in \mathbb{N}$, met $\alpha, \beta, \gamma, \delta \in \mathbb{R}$ nader te bepalen.

dan is

$\alpha(n+2)^3 + \beta(n+2)^2 + \gamma(n+2) + \delta + 2\alpha(n+1)^3 + 2\beta(n+1)^2 + 2\gamma(n+1) + 2\delta$

$- 3\alpha n^3 - 3\beta n^2 - 3\gamma n - 3\delta = n^3 + 1.$

$(n+2)^3 =$

$= \cancel{\alpha n^3} + 6\alpha n^2 + 12\alpha n + 8\alpha + \cancel{\beta n^2} + 4\beta n + 4\beta + \cancel{\gamma n} + 2\gamma + \cancel{\delta}$

$(+\cancel{\alpha n^3} + 3\alpha n^2 + 3\alpha n + \alpha + \cancel{\beta n^2} + 2\beta n + \beta + \cancel{\gamma n} + \gamma + \cancel{\delta}) \cdot 2.$

$\cancel{-3\alpha n^3} - \cancel{3\beta n^2} - \cancel{3\gamma n} - \cancel{3\delta}$

$= 12\alpha n^2 + 18\alpha n + 8\beta n + 10\alpha + 6\beta + 4\gamma$

dus moet $\begin{cases} 12\alpha = 1 & \Rightarrow \alpha = 1/12 \\ 18\alpha + 8\beta = 0 & \beta = -3/16 \\ 10\alpha + 6\beta = \ldots \end{cases}$

$\Rightarrow$ WERKT NIET

1 is een oplossing van de homog. vergel. $\Rightarrow 1 \cdot n^3 \Rightarrow$ zal niet werken.

stel $y_n = \alpha n^4 + \beta n^3 + \gamma n^2 + \delta n + \varepsilon \quad \forall n \in \mathbb{N}$

dan is $n^3 + 1$

$= \cancel{\alpha n^4} + 8\alpha n^3 + 24\alpha n^2 + 32\alpha n + 16\alpha \cancel{+\beta n^3} + 6\beta n^2 + 12\beta n + 8\beta$
$\cancel{+\gamma n^2} + 4\gamma n + 4\gamma \cancel{+\delta n} + 2\delta \cancel{+\varepsilon}$
$\cancel{+2\alpha n^4} + 8\alpha n^3 + 12\alpha n^2 + 8\alpha n + 2\alpha \cancel{+2\beta n^3} + 3\beta n^2 + 6\beta n + 2\beta$
$+ \cancel{2\gamma n^2} + 4\gamma n + 2\gamma + \cancel{2\delta n} + 2\delta + \cancel{2\varepsilon}$
$\cancel{-3\alpha n^4} \cancel{-3\beta n^3} \cancel{-3\gamma n^2} \cancel{-3\delta n} \cancel{-3\varepsilon}$

dus moet
$$\begin{cases} 16\alpha = 1 \Rightarrow \alpha = 1/16 \\ 36\alpha + 12\beta = 0 \Rightarrow \beta = -3/16 \\ 40\alpha + 18\beta + 8\gamma = 0 \Rightarrow \gamma = 7/64 \\ 18\alpha + 10\beta + 6\gamma + 4\delta = 1 \Rightarrow \delta = \frac{35}{128} \end{cases}$$

DUS $y_n = \frac{n^4}{16} - \frac{3n^3}{16} + \frac{7n^2}{64} + \frac{35n}{128} + (-3)^n A + B$ met $A, B \in \mathbb{R}$

h) $y_{n+2} - 3y_{n+1} + 2y_n = 4^n + 3n^2$

(1) $\lambda^2 - 3\lambda + 2 = 0$
$S = 3 \quad P = 2 \Rightarrow \lambda_1 = 1, \; \lambda_2 = 2$ } $y_n^{(0)} = A + 2^n B$ met $A, B \in \mathbb{R}$

(2) stel $y_n = (\alpha n^2 + \beta n + \gamma) + \delta 4^n$ (?) $+\varepsilon n^3$

dan is $4^n + 3n^2$

$= \cancel{\alpha n^2} + 4\alpha n + 4\alpha + \cancel{\beta n} + 2\beta + \cancel{\gamma} + 16\delta 4^n \cancel{+\varepsilon n^3} + 6\varepsilon n^2 + 12\varepsilon n + 8\varepsilon$
$\cancel{-3\alpha n^2} - 6\alpha n - 3\alpha \cancel{-3\beta n} - 3\beta \cancel{-3\gamma} - 12\delta 4^n \cancel{-3\varepsilon n^3} - 9\varepsilon n^2 - 9\varepsilon n - 3\varepsilon$
$\cancel{+2\alpha n^2} + \cancel{2\beta n} + \cancel{2\gamma} + 2\delta 4^n + \cancel{2\varepsilon n^3}$

dus moet
$$\begin{cases} 6\delta = 1 \Rightarrow \delta = 1/6 \\ -3\varepsilon = 3 \Rightarrow \varepsilon = -1 \\ 3\varepsilon - 2\alpha = 0 \Rightarrow \alpha = -3/2 \\ \alpha - \beta + 5\varepsilon = 0 \Rightarrow \beta = -6,5 \end{cases}$$

DUS $y_n = -n^3 - \frac{3}{2}n^2 - \frac{13}{2}n + \frac{4^n}{6} + A + 2^n B$ met $A, B \in \mathbb{R}$

(i) $y_{n+2} - 6y_{n+1} + 8y_n = 2 + 3n^2 - 5\cdot 3^n$

(1) $\lambda^2 - 6\lambda + 8 = 0$

$s = 6 \quad p = 8 \quad \Rightarrow \lambda_1 = 4, \ \lambda_2 = 2.$ $\Big\}$ $y_n^{(0)} = 2^n \cdot A + 4^n \cdot B$ met $A, B \in \mathbb{R}$

(2) stel $y_n = \alpha n^2 + \beta n + \gamma + \delta 3^n$

dan is $\alpha n^2 + 4\alpha n + 4\alpha + \beta n + 2\beta + \gamma + 9\delta \cdot 3^n$

$\cancel{+\alpha n^2 + 2\alpha}$

$-6\alpha n^2 - 12\alpha n - 6\alpha - 6\beta n - 6\beta - 6\gamma - 18\delta 3^n$

$+8\alpha n^2 \qquad + 8\beta n \qquad + 8\gamma + 8\delta 3^n \quad = 2 + 3n^2 - 5 3^n$

dus moet: $\begin{cases} -\delta = -5 \Rightarrow \delta = 5 \\ 3\alpha = 3 \Rightarrow \alpha = 1 \\ -8\alpha + 3\beta = 0 \Rightarrow \beta = 8/3 \\ -2\alpha - 4\beta + 3\gamma = 2 \Rightarrow \gamma = \frac{44}{9} \end{cases}$

Dus $y_n = n^2 + 8/3 \cdot n + 44/9 + 5 \cdot 3^n + 2^n \cdot A + 4^n \cdot B$ met $A, B \in \mathbb{R}$.

7 $y_{n+2} + (n+2)y_{n+1} - 6(n+2)(n+1)y_n = 0$

a) · homogene vergl, en lineair!

· van de 2de orde

b) deze zal 2 vrijheidsgraden hebben.

c) stel $y_n = n! \cdot \lambda^n$ dan is

$\cancel{(n+2)!}\lambda^{n+2} + \cancel{(n+2)(n+1)!}\lambda^{n+1} - 6\cancel{(n+2)(n+1) \cdot n!}\lambda^n = 0$

$\lambda^2 + \lambda - 6 = 0$

$s = -1 \quad p = -6 \quad \Rightarrow \lambda_1 = -3, \ \lambda_2 = 2$

dus $y_n^{(1)} = n!(-3)^n \cdot$ ~~A met A∈R~~ $\Big\}$ y^1 en y^2 vormen de basis van de oplossing verzameling.

$y_n^{(2)} = n!\, 2^n$

hieruit volgt dus dat $y_n = A \cdot n!\, 2^n + B \cdot n!(-3)^n$ met $A, B \in \mathbb{R}$

8 a) $y_n = 2^n \cdot A + (-3)^n \cdot B + 6 \cdot 4^n$

(1) $\lambda^2 + a \cdot \lambda + b = 0$

$\lambda = \frac{-a \pm \sqrt{a^2 - 4b}}{2} \longrightarrow \lambda_1 = -3 = \frac{+a + \sqrt{a^2-4b}}{2}$

$\lambda_2 = 2 = \frac{-a + \sqrt{a^2-4b}}{2}$

dus $\begin{cases} (6 - a)^2 = a^2 - 4b \\ (4 + a)^2 = a^2 - 4b \end{cases}$ $\begin{cases} 6 - a = 4 + a \\ -4b = 24 \end{cases}$ $\begin{matrix} a = 1 \\ b = -6 \end{matrix}$ $\Big\}$ $\begin{matrix} s = -1 \\ p = -6. \end{matrix}$

(2) stel $y_n = 6 \cdot 4^n$

dan is $96 \cdot 4^n + 24 \cdot 4^n - 36 \cdot 4^n = \alpha \cdot 4^n.$

$\alpha = 84 \quad \Rightarrow y_{n+2} + y_{n+1} - 6y_n = 84 \cdot 4^n$

b) $\lim_{n\to\infty} 2^n y_n = 5$. $y_{n+2} + a y_{n+1} + b y_n = ?$

~~uit de pls $y_n = \frac{5}{2^n} = 5\left(\frac{1}{2}\right)^n$ → valt wel uit te werken maar komt naar 1 opl.~~

~~→ geen lek met hogh. graden~~

zie 26 BIS

9

$c_n = \alpha + \beta(c_{n-1} - c_{n-2})$ met $\alpha, \beta \in \mathbb{R}^+_0$

(1) $c_n - \beta c_{n-1} + \beta c_{n-2} = \alpha$

(1) $\lambda^2 - \beta\lambda + \beta = 0$

$D = \beta^2 - 4\beta = \beta(\beta - 4)$

$\underline{\beta > 4}$

$\lambda_{1,2} = \frac{\beta \pm \sqrt{\beta^2 - 4\beta}}{2}$ dus

$y_n^{(0)} = A\cdot\lambda_1^n + B\lambda_2^n$ met $A, B \in \mathbb{R}$

↳ afrond (?)

$\underline{\beta = 4}$

$\lambda_1 = \beta/2 = 2$.

$y_n^{(0)} = (A + nB)2^n$ met $A, B \in \mathbb{R}$

$\underline{\beta < 4}$

$\lambda = \frac{\beta \pm i\sqrt{4\beta - \beta^2}}{2}$ $R = \sqrt{\frac{\beta^2}{4} + \frac{4\beta - \beta^2}{4}} = \frac{2\sqrt{\beta}}{2}$

$\theta = \text{Bgcos}\left(\frac{\sqrt{\beta}}{2}\right)$

$y_n^{(0)} = (2\sqrt{\beta})^n (A\cos n\theta + B\sin n\theta)$ met $A, B \in \mathbb{R}$

(2) stel $y_n = a$ $\forall n \in \mathbb{N}$

$a - \beta\cdot a + \beta\cdot a = \alpha$

dan is $a = \alpha$

DUS algemeen is

$c_n = \alpha + y_n^{(0)}$.

(2) $\underline{\beta > 4}$???

$\lim_{n\to\infty} c_n = \alpha + A\lambda_1^n + B\lambda_2^n$

$= +\infty$ ($-\infty$ hier realistisch)

$\underline{\beta = 4}$.

$\lim_{n\to\infty} c_n = +\infty$

(0<) $\underline{\beta < 4}$:

$\beta > 1 \to \lim_{n\to\infty} c_n = +\infty$

$\beta = 1$: oscillatie (rond α)

$\beta < 1 \to \lim_{n\to\infty} c_n = \alpha$

P. 26 BIS

$\lim 2^n \cdot y_n = 5$.

* de $\lim 2^n \cdot y_{h,n} = 0$

dus bv. $y_{h,n} = \frac{1}{4^n} \cdot A + \frac{1}{8^n} \cdot B$

* de $\lim 2^n \cdot y_{p,n} = 5$.

dus $y_{p,n} = 5 \cdot \frac{1}{2^n}$.

$\Rightarrow \frac{1}{4}$ en $\frac{1}{8}$ zijn opl. v/d kwadr. vgl.

$$\left(\lambda - \frac{1}{4}\right)\left(\lambda - \frac{1}{8}\right) = 0.$$

$$\lambda^2 - \frac{3}{8}\lambda + \frac{1}{32} = 0$$

$$y_{n+2} - \frac{3}{8} y_{n+1} + \frac{1}{32} y_n = \alpha \cdot \frac{1}{2^n}.$$

$$\frac{5}{4\cancel{2^n}} - \frac{15}{8 \cdot 2 \cdot \cancel{2^n}} + \frac{5}{32 \cdot \cancel{2^n}} = \alpha \cdot \frac{1}{\cancel{2^n}}.$$

$$\alpha = \frac{15}{32}.$$

① db: $a = (2,1)$ $b = (3,-2)$

param.: $\begin{cases} x = (1-\lambda)x_1 + \lambda x_2 \\ y = (1-\lambda)y_1 + \lambda y_2 \end{cases}$

parameter vgl: $\begin{cases} x = (1-\lambda)2 + 3\lambda \\ \quad = \lambda + 2 \\ y = (1-\lambda) - 2\lambda \\ \quad = -3\lambda + 1 \end{cases}$ met $\lambda \in \mathbb{R}$

$x - 2 = \frac{y+1}{3}$

$3x + y = 7$

carth. vgl: $(x-2)(-2-1) = (y-1)(3-2)$

$-3(x-2) = y - 1$ dus $-3x - y = -7$

$3x + y = 7$

$y = -3x + 7$ ✓

cd: $c = (7,0)$ $d = (1,4)$

param.: $\begin{cases} x = (1-\lambda)\cdot 7 + \lambda \\ \quad = -6\lambda + 7 \\ y = 4\lambda \end{cases}$ met $\lambda \in \mathbb{R}$

carth. vgl: $\frac{7-x}{6} = \frac{y}{4}$ $2x + 3y = 14$.

$28 - 4x = 6y$

$y = \frac{-2x + 14}{3}$ ✓

$\Rightarrow$ snijdend in (x,y) $\Leftrightarrow$ $\begin{cases} 3x + y = 7 \\ 2x + 3y = 14 \end{cases}$

$$\left[\begin{array}{cc|c} 3 & 1 & 7 \\ 2 & 3 & 14 \end{array}\right] \sim \left[\begin{array}{cc|c} 3 & 1 & 7 \\ 0 & 7 & 28 \end{array}\right] \sim \left[\begin{array}{cc|c} 3 & 1 & 7 \\ 0 & 1 & 4 \end{array}\right] \sim \left[\begin{array}{cc|c} 3 & 0 & 3 \\ 0 & 1 & 4 \end{array}\right]$$

$42 - 14 = 28$

$(x,y) = (1,4)$

② A: $3x + 2y = 8$ $\Rightarrow$ wat is afstand tussen A en p

$p = (8,5)$ = afstand tussen p en snijpunt v. A en rechte loodrecht op A door p. $\rightarrow$ schrijf B.

richtingsvector v. A $= (b,-a) = (2,-3)$

richtingsvector v. B $= (a,b) = (3,2)$. $(\rightarrow a' = -2,\ b' = 3)$

dus $B \leftrightarrow -2x + 3y = c$

als $p \in B$ dan $-2\cdot 8 + 3\cdot 5 = -1 = c$

dus $B \Leftrightarrow -2x + 3y = -1$.

snijpunt v. A en B: $(x,y) \Leftrightarrow \begin{cases} 3x + 2y = 8 \\ -2x + 3y = -1 \end{cases}$

$$\left[\begin{array}{cc|c} 3 & 2 & 8 \\ -2 & 3 & -1 \end{array}\right] \sim \left[\begin{array}{cc|c} 3 & 2 & 8 \\ 0 & 13 & 13 \end{array}\right] \sim \left[\begin{array}{cc|c} 3 & 0 & 6 \\ 0 & 1 & 1 \end{array}\right]$$

$\Rightarrow$ snijdt in $(2,1)$

afstand $\|\ \|$ tussen p en $(2,1)$ $= \sqrt{(8-2)^2 + (5-1)^2} = \sqrt{36 + 16}$

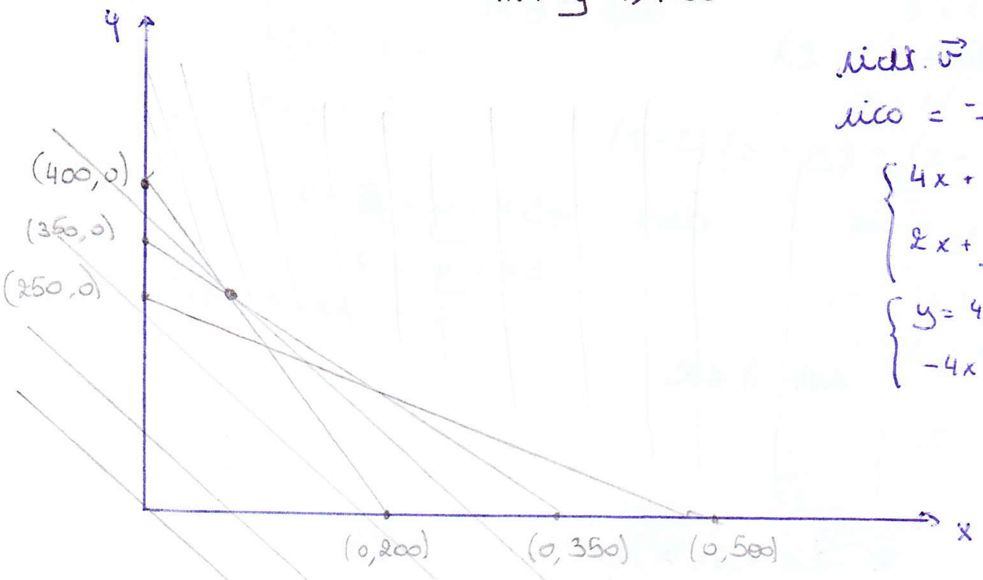
$= \sqrt{52}$

③ ~~w(x,y)~~ = | $K(x,y) = 15x + 9y$ minimaliseren

randvoorw. $2x + y \geq 400$ pos. voorw. $x \geq 0$

$x + 2y \geq 500$ $y \geq 0$

$4x + 4y \geq 1400$



richt. $\vec{v}$ $K(x,y) = (9,-15)$

rico $= -\frac{a}{b} = -\frac{15}{9} = -\frac{5}{3}$

$$\begin{cases} 4x + 4y = 1400 \\ 2x + y = 400 \end{cases}$$

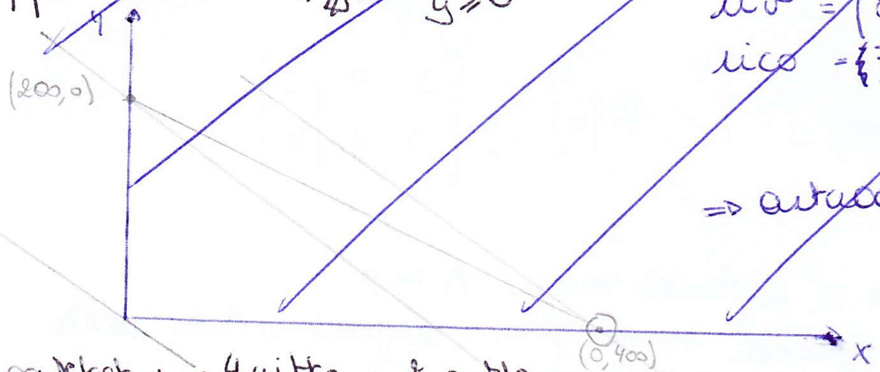
$$\begin{cases} y = 400 - 2x \\ -4x = -200 \end{cases} \quad \boxed{\begin{matrix} y = 300 \\ x = 50 \end{matrix}}$$

$K(50,300) = 34,5$ €

④ | $w(x,y) = 400x + 240y - 192x - 96y - 144x - 64y = 64x + 80y$

randvoorw: $40x + 20y \leq 800$

pos. voorw: $x \geq 0$ $y \geq 0$



ri $\vec{v} = (80,-64)$

rico $= -\frac{4}{5}$

Zie p. 28 BIS

⇒ antwoord. 0 HDT

400 LDT.

⑤ pakket 1 = 4 witte + 2 gekleurde = 15 €

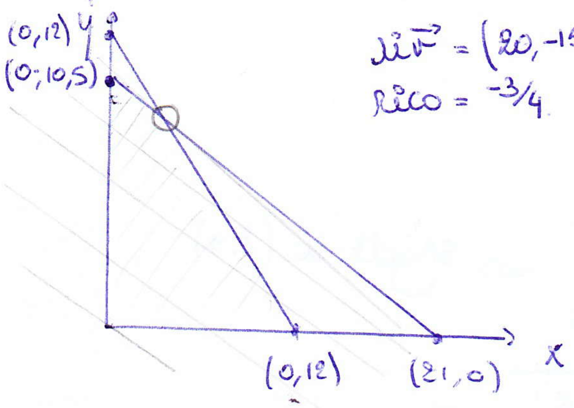
pakket 2 = 8 witte + 2 gekleurde = 20 €

voorraad: 24 ~~witte~~ gekleurde + 84 witte

| $w(x,y) = 15x + 20y$

randvoorw: $4x + 8y \leq 84$ $2x + 2y \leq 24$

pos. voorw: $x \geq 0$ $y \geq 0$.



ri $\vec{v} = (20,-15)$

Rico $= -3/4$

$$\begin{cases} 2x + 2y = 24 \\ x + 2y = 21 \end{cases} \quad x + y = 12 \quad -x = -3 \quad x = 3, \; y = 9$$

$x = 12 - y$

$y = 12 - x$

3 pakketten 1

9 pakketten 2.

4

p28 B15

1200 wol

1000 nylon

$p_{HDT} = 400$

$p_{LDT} = 240$

1 HDT = 20 wol + 40 nylon + 40 uur

1 LDT = 40 nylon + 20 uur.

~~totaal~~ 800 uur

$p_{1\,uur} = 4{,}8$

1 wol = 4   1 nylon = 1,6.

max winst = 64 HDT + 80 LDT

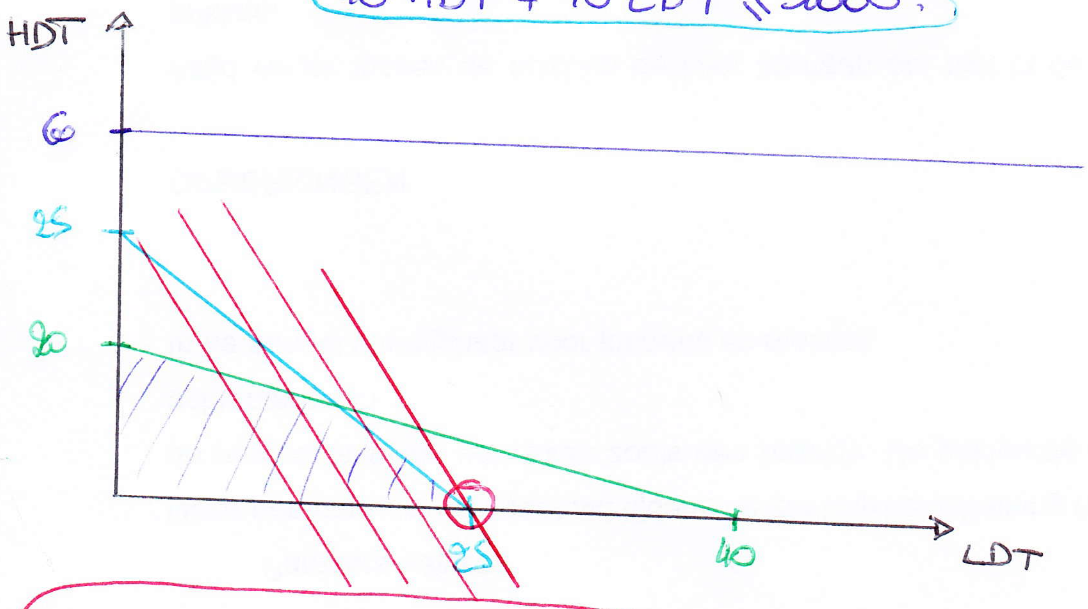
onder vw: 40 HDT + 20 LDT ≤ 800

20 HDT ≤ 1200

40 HDT + 40 LDT ≤ 1000.

LDT ≥ 0

HDT ≥ 0



$$HDT = -\frac{80}{64} LDT + winst$$

⇒ 25 LDT

0 HDT

opdrachten 5.2.3. p. 517

1. $\begin{cases} x = p_1 + a_1\lambda + b_1\mu \\ y = p_2 + a_2\lambda + b_2\mu \\ z = p_3 + a_3\lambda + b_3\mu \end{cases}$ $\qquad \det\begin{pmatrix} a_1 & b_1 & x-p_1 \\ a_2 & b_2 & y-p_2 \\ a_3 & b_3 & z-p_3 \end{pmatrix} = 0$

dus $a_1b_2z - a_1b_2p_3 + a_3b_1y - a_3b_1p_2 + a_2b_3x - a_2b_3p_1$

$- a_3b_2x + a_3b_2p_1 - a_1b_3y + a_1b_3p_2 - a_2b_1z + a_2b_1p_3 = 0$

dus $x\overbrace{(a_2b_3 - a_3b_2)}^{a} + y\overbrace{(a_3b_1 - a_1b_3)}^{b} + z\overbrace{(a_1b_2 - a_2b_1)}^{c}$

$\underbrace{= p_3(a_1b_2 - a_2b_1) + p_2(a_3b_1 - a_1b_3) + p_1(a_2b_3 - a_3b_2)}_{d}$

dus $\det\begin{pmatrix} x & y & z \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{pmatrix} = \det\begin{pmatrix} p_1 & p_2 & p_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{pmatrix}$

2. a) $(1,2,-4)$, $(2,3,7)$, $(4,-1,3)$

punt $(1,2,-4)$
en $\vec{v}_1$ $(1,1,11)$
en $\vec{v}_2$ $(3,-3,7)$

$\det\begin{pmatrix} x & y & z \\ 1 & 1 & 11 \\ 3 & -3 & 7 \end{pmatrix} = \det\begin{pmatrix} 1 & 2 & -4 \\ 1 & 1 & 11 \\ 3 & -3 & 7 \end{pmatrix}$

$40x + 26y - 6z = 116$

b) $(-7,1,0)$, $(2,-1,3)$, $(4,1,6)$

punt $(-7,1,0)$
$\vec{v}_1$ $(+9,-2,3)$
$\vec{v}_2$ $(+11,0,6)$

$\det\begin{pmatrix} x & y & z \\ 9 & -2 & 3 \\ 11 & 0 & 6 \end{pmatrix} = \det\begin{pmatrix} -7 & 1 & 0 \\ 9 & -2 & 3 \\ 11 & 0 & 6 \end{pmatrix}$

$\cancel{-12x - 21y - 22z = 63}$

$-12x - 21y + 22z = 63$

3. a) $(2,3,-4)$, $(2,0,-4)$

vect. vgl: $v = (2,3,-4) + \lambda((2,0,-4) - (2,3,-4))$

$= (2,3,-4) + \lambda(0,-3,0)$ met $\lambda \in \mathbb{R}$.

param. vgl: $\begin{cases} x = 2 \\ y = \lambda \\ z = -4 \end{cases}$ met $\lambda \in \mathbb{R}$.

cart. vgl. $\begin{cases} x = 2 \\ z = -4 \end{cases}$

b) $(2,1,3)$, $(1,2,-1)$

vect. vgl: $v = (2,1,3) + \lambda(-1,1,-4)$ met $\lambda \in \mathbb{R}$

param. vgl: $\begin{cases} x = 2 - \lambda \\ y = 1 + \lambda \\ z = 3 - 4\lambda \end{cases}$ met $\lambda \in \mathbb{R}$.

$\lambda = 2 - x = \frac{x-2}{-1}$
$\lambda = y - 1 = \frac{y-1}{1}$
$\lambda = \frac{3-z}{4} = \frac{z-3}{-4}$

cart vgl: $\begin{cases} x + y = 3 \\ 4y + z = 4 \end{cases}$

$y = 3 - x$

$\begin{cases} x + y = 3 \\ z - 4x = -8 \end{cases}$

④ $\frac{x+1}{3} = \frac{y+3}{2} = \frac{z-2}{-4}$ en punt $p = (3,1,-2) \Rightarrow$ rechte B door p, $\parallel L$. (30

$L_0 \leftrightarrow \begin{cases} x = 3\lambda \\ y = 2\lambda \\ z = -4\lambda \end{cases}$ met $\lambda \in \mathbb{R}$. ~~$B \leftrightarrow \begin{cases} 3 = 3\lambda + a \\ 1 = 2\lambda + b \\ -2 = -4\lambda + c \end{cases}$~~

dus ri $\vec{v} = (3,2,-4)$

punt $= (3,1,-2)$

dus ~~vr~~ $B \leftrightarrow \begin{cases} x = 3 + 3\lambda \\ y = 1 + 2\lambda \\ z = -2 - 4\lambda \end{cases}$ $\frac{x-3}{3} = \frac{y-1}{2} = \frac{z+2}{-4}$.

$\begin{cases} 2x - 3y = 3 \\ -4y - 2z = 0 \end{cases}$

⑤ $\begin{cases} x - 2y + 1 = 0 \\ 2y - z = 0 \end{cases}$

a) param. vgl. : $\begin{cases} y = \frac{x+1}{2} \\ y = \frac{z}{2} \end{cases}$ dus $\begin{cases} x = -1 + 2\lambda \\ y = \lambda \\ z = 2\lambda \end{cases}$ met $\lambda \in \mathbb{R}$

b) afstand tot $p = (1,1,2)$ is 6 ;

$\sqrt{(1-(-1+2\lambda))^2 + (1-\lambda)^2 + (2-2\lambda)^2} = 6$

$= \cancel{\sqrt{\ldots}} = \sqrt{2 \cdot 4(1-\lambda)^2 + (1-\lambda)^2} = \pm 3(1-\lambda) = \pm(3-3\lambda)$

dus $\lambda_1 = \frac{3-6}{3} = -1$, $\lambda_2 = 3$ } de punten $(-3,-1,-2)$ en $(5,3,6)$

⑥ $p = (5,6,-2)$

$V \leftrightarrow 2x + 3y - z = 6$. $\Rightarrow$ doorsnede van normaalvector $v = (2,3,-1)$ door p en V.

rechte $a \leftrightarrow \begin{cases} x = 5 + 2\lambda \\ y = 6 + 3\lambda \\ z = -2 - \lambda \end{cases}$ met $\lambda \in \mathbb{R}$.

$a \cap V \leftrightarrow \begin{cases} x = \ldots \quad z \ldots \\ y = \ldots \\ 2(5+2\lambda) + 3(6+3\lambda) - 1(-2-\lambda) = 6. \end{cases}$

$\lambda = -24/14 = -\frac{12}{7}$

doorsnede $= \left(\frac{11}{7}, \frac{6}{7}, \frac{-2}{7}\right)$

afstand $= \frac{1}{7}\sqrt{(35-11)^2 + (42-6)^2 + (-14+2)^2}$

$= \frac{12\sqrt{14}}{7}$

~~$\Rightarrow$ afstand $\approx 6{,}59$~~ ✗

⑦ $v_1 = (3,2,-3)$ $p = (8,0,4)$

$v_2 = (1,5,0)$

$a \leftrightarrow \begin{cases} x = 1 + 2\lambda \\ y = 5 - 3\lambda \\ z = -3\lambda \end{cases}$ carth. vgl: $\frac{x-1}{2} = \frac{y-5}{-3} = \frac{z}{-3}$ $\begin{cases} -3x + 3 = 2y - 10 \\ \Leftrightarrow y - z = 5 \end{cases}$ $3x + 2y = 13$ ✓

ri $\vec{v} = (2,-3,-3) \Rightarrow$ normaalvector : ~~...~~ (x,y,z) met $2x - 3y - 3z = \delta$

$\delta = 16 - 12 = 4 \to P$ ingevuld. $\Rightarrow$ een VLAK.

$a \cap$ normaalvlak $\leftrightarrow \begin{cases} 2 + 4\lambda - 15 + 9\lambda + 9\lambda = 4 \quad \lambda = \frac{17}{22} \\ x = \frac{28}{11} \quad z = \frac{-51}{22}. \\ y = 59/22 \end{cases}$

afstand $= 8{,}77 = \sqrt{(8-x)^2 + (-y)^2 + (4-z)^2} = \frac{\sqrt{37202}}{22}$

opdracht 5

extra oefening 1

$$\det\begin{pmatrix} 1/3-\lambda & 1/6 \\ 2/3 & 5/6-\lambda \end{pmatrix} = 0$$

$$(1/3-\lambda)(5/6-\lambda) - \frac{2}{18} = 0$$

$$\frac{5}{18} - \frac{\lambda}{3} - \frac{5\lambda}{6} + \lambda^2 - \frac{2}{18} = 0$$

$$6\lambda^2 - \frac{7\lambda}{\cancel{6}} + \frac{1}{\cancel{6}} = 0 \qquad S = \frac{7}{6} \quad P = \frac{1}{6}$$

$D = 49 - 24 = 25$

$\lambda_1 = \frac{7+5}{12} = 1$

$\lambda_2 = \frac{7-5}{12} = 1/6 \longrightarrow$ nutteloos...

$$\begin{pmatrix} -2/3 & 1/6 \\ 2/3 & -1/6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$\frac{2}{3}x_1 = \frac{1}{6}x_2$ dus $x_2 = 4x_1$

dus oplossing $= \{(4a, a) \mid a \in \mathbb{R}_0\}$

vector die voldoet aan $\sum_{i=1}^{m} x_i = 1$

dus $4a + a = 1$ dus $a = 1/5$

dus $x^* = \left\{\frac{4}{5}, \frac{1}{5}\right\}$

(2,3) omvormen naar veelvoud van $\sum_{i=1}^{2} y_i = 1$

$\alpha y_1 = 2$ dus $y_1 = \frac{2}{\alpha}$ en $y_2 = \frac{3}{\alpha}$

$\alpha y_2 = 3$

dus $\frac{2}{\alpha} + \frac{3}{\alpha} = 1$

$2\alpha + 3\alpha = 1$ dus $\alpha = 1/5$

waaruit volgt dat $(2,3) = \frac{1}{5}\left(\frac{2}{5}, \frac{3}{5}\right)$

dus $\lim_{n\to\infty} P^n \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \frac{1}{5}\lim_{n\to\infty} P^n \begin{pmatrix} 2/5 \\ 3/5 \end{pmatrix}$

$$= \frac{1}{5}\begin{pmatrix} 4/5 \\ 1/5 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

extra oef 2.

150 landen:

2 categorieën:

1) laagconj → 70% kans volgend jaar nog steeds l.

2) hoog conj → 80%

dus

A) $P = \begin{bmatrix} 0,7 & 0,2 \\ 0,3 & 0,8 \end{bmatrix} \rightarrow 0,3x_1 = 0,2x_2$

dus eigenvectr bij eigenw 1 = $\{(\frac{2}{3}a, a) \mid a \in \mathbb{R}_0\}$

$\Rightarrow x^* = \{\frac{2}{5}, \frac{3}{5}\}$

$\lim_{n\to\infty} P^n \begin{pmatrix} 50 \\ 100 \end{pmatrix} = 150 \cdot \lim_{n\to\infty} P^n \begin{pmatrix} 1/3 \\ 2/3 \end{pmatrix}$

$= \begin{pmatrix} 60 \\ 90 \end{pmatrix} \Rightarrow$ 60 in laagconj, 90 in hoogconj

B) $\boxed{P^{10} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = ?}$

$\det \begin{pmatrix} 0,7-\lambda & 0,2 \\ 0,3 & 0,8-\lambda \end{pmatrix} = 0 \Leftrightarrow \frac{56}{100} - \frac{7}{10}\lambda - \frac{8}{10}\lambda + \lambda^2 - \frac{6}{100} = 0$

$\lambda^2 - \frac{3}{2}\lambda + \frac{1}{2} = 0$

$S = \frac{3}{2} \quad P = \frac{1}{2}.$

$\lambda_1 = 1$

$\lambda_2 = \frac{1}{2}$

det $\begin{pmatrix} 0,2 & 0,2 \\ 0,3 & 0,3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\quad x_1 = -x_2$

dus $x = \{(-a, a) \mid a \in \mathbb{R}_0\}$

$P^{10} = \begin{pmatrix} -1 & 3 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 1/2 & 0 \\ 0 & 1 \end{pmatrix}^{10} \begin{pmatrix} -0,4 & 0,6 \\ 0,2 & 0,2 \end{pmatrix}$

$= \begin{pmatrix} -\frac{1}{2^{10}} & 3 \\ \frac{1}{2^{10}} & 2 \end{pmatrix} \begin{pmatrix} -0,4 & 0,6 \\ 0,2 & 0,2. \end{pmatrix}$

$= \begin{pmatrix} 0,60039\ (y) & 0,599414\ (z) \\ 0,3996\ (t) & 0,4005859\ (a) \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

dus $P^{10} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0,5994 \\ 0,4006 \end{pmatrix} \Rightarrow$ 59,94% laagconj. 40,06% hoogconj.

opdrachten 6.3 p 546

1

1) $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$ ~~det~~ $\begin{vmatrix} \cancel{0 \;\; 0 \;\; 0} \\ \cancel{0 \;\; 1 \;\; 0} \\ \cancel{1 \;\; 0 \;\; 1} \end{vmatrix} =$

$\det \begin{pmatrix} -\lambda & 0 & 0 \\ 0 & 1-\lambda & 0 \\ 1 & 0 & 1-\lambda \end{pmatrix} = -\lambda \cdot (1-\lambda)^2 = 0 \rightarrow$ karakteristieke vgl.

$\lambda = 1$ of $\lambda = 0$

voor eigenw. $\lambda = 1$:

$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Leftrightarrow \begin{cases} -x_1 = 0 \\ x_1 = 0 \end{cases}$

~~dus $x = (0\ a\ b)$~~
~~KAN NIET~~

$\Rightarrow x = (0, a, b)$ met $a, b \in \mathbb{R}$.
en a of b verschillend v. 0

voor eigenw $\lambda = 0$

$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Leftrightarrow \begin{cases} x_1 + x_3 = 0 \\ x_2 = 0 \end{cases}$ dus $x = (a, 0, -a)$ met $a \in \mathbb{R}_0$

diagonaliseerbaar? : ja $Q = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix}$ en $D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\Rightarrow \{(0,0,1), (0,1,0), (1,0,-1)\}$ vormt een basis van $\mathbb{R}^m$

2) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

karakteristieke vgl:

$\det \begin{pmatrix} 1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 0 \\ 1 & 0 & 1-\lambda \end{pmatrix} = 0 = (1-\lambda)^3$

dus $\lambda = 1$.

$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$ dus $x = (0, a, b)$ met $a, b \in \mathbb{R}$ en a of b verschillend van 0

diagonaliseerbaar?
nee: er valt geen basis v. $\mathbb{R}^3$ te construeren: ~~stel~~ stel 2 eigenv. ~~0~~: $(0,1,0)$ en $(0,0,1)$, dan kunnen alle overige vectoren geschreven w als een lin. comb. van deze 2.

3) $\begin{pmatrix} -3 & 1 & -1 \\ -7 & 5 & -1 \\ -6 & 6 & -2 \end{pmatrix}$ kar. $\det \begin{vmatrix} -3-\lambda & 1 & -1 \\ -7 & 5-\lambda & -1 \\ -6 & 6 & -2-\lambda \end{vmatrix} = 0$

$-\lambda^3 + 12\lambda + 16 = 0$ dus $\lambda_1 = \lambda_2 = -2$
$\lambda_3 = 4$.

eigenw. -2: $\begin{cases} -x_1 + x_2 - x_3 = 0 \\ x_3 = 0 \end{cases}$ dus $x = (a, a, 0)$ met $a \in \mathbb{R}_0$

eigenw: 4: $\begin{cases} 7x_1 - x_2 + x_3 = 0 \\ -x_2 + x_3 = 0 \end{cases}$ dus $x = (0, b, b)$ met $b \in \mathbb{R}_0$

$\Rightarrow$ niet diagonaliseerbaar

② $A \in \mathbb{R}^{m\times m}$ en is diagonaliseerbaar

en $\forall$ eigenwaardes $\lambda : |\lambda| < 1$

TB: $\lim_{n\to\infty} A^n = 0$

omdat A diagonaliseerbaar is, is er een basis v. $\mathbb{C}^m$ in de vorm van $B = \{e^{(1)}, \dots, e^{(m)}\}$ met $e^{(j)}$ een eigenvector v. A, bijhorende bij eigenwaarde λ_j met $j = 1, \dots, m$

① Kies nu ω° $x = (x_1, \dots, x_m) \in \mathbb{C}^m$ met $x_i \geq 0 \ \forall i$ (en $\sum_{i=1}^{m} x_i = 1$)

omdat B een basis vormt v. $\mathbb{C}^m$ valt x te schrijven als:

$$x = \sum_{j=1}^{m} \alpha_j e^{(j)} \quad \text{met } \alpha_1, \dots, \alpha_m \in \mathbb{C}$$

door de componenten van x op te tellen bekomen we:

$$\sum_{i=1}^{m} x_i = \sum_{i=1}^{m} \left(\sum_{j=1}^{m} \alpha_j e_i^{(j)} \right)$$

$$= \sum_{j=1}^{m} \alpha_j \overbrace{\left(\sum_{i=1}^{m} e_i^{(j)} \right)}^{=0} = 0$$

② poging 2: $A = Q^{-1} \cdot D \cdot Q$ met Q de inverteerbare $m \times m$ matrix: B vormt kolom

D een $m \times m$ diagonaalmatrix: eigenwaardes $\lambda_1, \dots, \lambda_m$ op de diagonaal.

dus is $\lim_{n\to\infty} A^n = \lim Q^{-1} \cdot D^n \cdot Q$

nu is $D^n = \begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_m \end{pmatrix}^n = \begin{pmatrix} \lambda_1^n & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & \lambda_m^n \end{pmatrix}$ dus is de $\lim_{n\to\infty} D^n = 0$

waaruit volgt dat $\lim_{n\to\infty} Q^{-1} \cdot D^n \cdot Q = Q^{-1} \cdot \lim_{n\to\infty} D^n \cdot Q$

③ $A = \begin{pmatrix} 0,25 & 0 & 0 \\ 0,50 & 0,18 & 0 \\ 0 & 0,75 & 0,02 \end{pmatrix}$ X_0 $B = \begin{pmatrix} 200 \\ 0 \\ 0 \end{pmatrix}$ $= 0$ $X_1 = \begin{pmatrix} 240 \\ 0 \\ 0 \end{pmatrix}$

eigenwaarden: $\det \begin{pmatrix} 0,25-\lambda & 0 & 0 \\ 0,50 & 0,18-\lambda & 0 \\ 0 & 0,75 & 0,02-\lambda \end{pmatrix} = 0$

$\Rightarrow$ karakt. vgl: $(0,25-\lambda)(0,18-\lambda)(0,02-\lambda) = 0$

* $\lambda = 0,25$: $\begin{cases} 0,5 \cdot x_1 = 0,07 x_2 \\ 0,23 x_3 = 0,75 x_2 \end{cases}$ dus $x = (0,14\, a;\ a;\ \underbrace{3,26}_{=75/23} a)$ met $a \in \mathbb{R}_0$
* $\lambda = 0,18$: $\begin{cases} x_1 = 0 \\ x_3 = 4,6875 x_2 \end{cases}$ dus $x = (0;\ b;\ \underbrace{4,6875}_{=75/16} b)$ met $b \in \mathbb{R}_0$
* $\lambda = 0,02$: $\begin{cases} x_1 = x_2 = 0 \end{cases}$ dus $x = (0, 0, c)$ met $c \in \mathbb{R}_0$

$\lim_{n\to\infty} x_n = (X_0 - (\mathbb{1}_3 - A)^{-1} \cdot B) \cdot \underbrace{\lim_{n\to\infty} A^n}_{=0} + (\mathbb{1}_3 - A)^{-1} \cdot B$ (zie p. 403)

$(\mathbb{1}_3 - A)^{-1} \cdot \begin{pmatrix} 200 \\ 0 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 266,67 \\ 162,6 \\ 124,9 \end{pmatrix}$

④ in het begin: $\left(A\cdot\begin{pmatrix}x_1\\ \vdots\\ x_m\end{pmatrix}\right)^t = \left(\lambda\begin{pmatrix}x_1\\ \vdots\\ x_m\end{pmatrix}\right)^t$

$= (x_1 \cdots x_m)\cdot A = \lambda\cdot(x_1 \cdots x_m)$

⑤ $A, B \in \mathbb{R}^{m\times m}$: $B = C\cdot A\cdot C^{-1}$ met $C \in \mathbb{R}^{m\times m}$ en C inverteerbaar

λ is een eigenwaarde van B als

① $\det(B-\lambda \mathbb{1}_m) = 0$ ② $(B-\lambda\mathbb{1}_m)\begin{pmatrix}x_1\\ \vdots\\ x_m\end{pmatrix} = 0$

$= \det(C\cdot A\cdot C^{-1} - \lambda\mathbb{1}_m)$

TB: $\det(A-\lambda\mathbb{1}_m) = 0$

$= \det(C^{-1}B\cdot C - \lambda\mathbb{1}_m) = 0$

?

?

$B\cdot C = C\cdot A$

$A\cdot v = \lambda\cdot v$

$C\cdot A\cdot v = C\cdot\lambda\cdot v$

$\Longrightarrow$ $B\cdot C\cdot v = \lambda C\cdot v$

dus λ EW met $C\cdot v$ eigenvector.

⑥ kies w' bovendriehoeksmatrix $A \in \mathbb{R}^{m\times m}$:

$A = \begin{pmatrix}\lambda_1 & 0 & 0 & \cdots & 0\\ 0 & \lambda_2 & 0 & \cdots & 0\\ \vdots & \vdots & & & \vdots\\ 0 & 0 & \cdots & & \lambda_m\end{pmatrix} \longrightarrow \det(A-\lambda\mathbb{1}_m) = 0$

dan is $\lambda = \lambda_1, \lambda_2, \dots \lambda_m$

⑦ $\{v_1, v_2, \dots, v_n\}$ is basis v. V

$e_1 = \frac{v_1}{\|v_1\|}$ $\tilde{v}_2 = v_2 - \langle e_1, v_2\rangle e_1$

a) TB: $\langle e_1, \tilde{v}_2\rangle = 0$ en $\tilde{v}_2 \neq 0$

①

$= \left\langle \frac{v_1}{\|v_1\|}, v_2 - \left\langle\frac{v_1}{\|v_1\|}, v_2\right\rangle\frac{v_1}{\|v_1\|}\right\rangle$

$= \frac{v_{1,1}\cdot\left(v_{2,1} - \frac{\langle v_1, v_2\rangle v_{1,1}}{\|v_1\|^2}\right)}{\|v_1\|} + \dots + \frac{v_{1,m}\cdot\left(v_{2,m} - \frac{\langle v_1,v_2\rangle v_{1,m}}{\|v_1\|^2}\right)}{\|v_1\|}$

$= \frac{\|v_1\|^2\cdot(v_{1,1}v_{2,1} + \dots + v_{1,m}v_{2,m}) - (v_{1,1}v_{2,1} + \dots + v_{1,m}v_{2,m})(v_{1,1}^2 + \dots + v_{1,m}^2)}{\|v_1\|^3}$

$= \frac{\langle v_1, v_2\rangle\left(\|v_1\|^2 - \|v_1\|^2\right)}{\|v_1\|^3} = 0$

② $\tilde{v}_2 = v_2 - \frac{\langle v_1, v_2\rangle\cdot v_1}{\|v_1\|^2} = \|v_1\|^2 v_2 - v_1(v_{1,1}\cdot v_{2,1} + \dots + v_{1,m}\cdot v_{2,m})$

als $\tilde{v}_2 = 0$ zou $v_1 = \frac{\|v_1\|^2 v_2}{\langle v_1, v_2\rangle}$

wat niet mogelijk is aangezien $\{v_1, v_2, \dots, v_n\}$ een vrij deel is

$e_2 = \dfrac{\tilde{v}_2}{\|\tilde{v}_2\|} \qquad \tilde{v}_3 = v_3 - \langle e_1, v_3\rangle e_1 - \langle e_2, v_3\rangle e_2$

b) TB: $\langle e_1, \tilde{v}_3\rangle = \langle e_2, \tilde{v}_3\rangle = 0$ en $\tilde{v}_3 \neq 0$

(1) $\left\langle \dfrac{v_1}{\|v_1\|}, v_3 - \dfrac{\langle v_1, v_3\rangle \cdot v_1}{\|v_1\|^2} - \dfrac{\langle \tilde{v}_2, v_3\rangle \cdot \tilde{v}_2}{\|\tilde{v}_2\|} \right\rangle$ $\|\tilde{v}_2\|^2$

$= \dfrac{\langle v_1, \|v_1\|^2 \|\tilde{v}_2\|^2 v_3 - v_1 \langle v_1, v_3\rangle - \tilde{v}_2 \langle \tilde{v}_2, v_3\rangle \cdot \|v_1\|^2\rangle}{\|v_1\|^3 \|\tilde{v}_2\|^2}$

$= \dfrac{\langle v_1, v_3\rangle}{\|v_1\|} - \dfrac{\cancel{\|v_1\|}\langle v_1, v_3\rangle}{\|v_1\|^{\cancel{3}\,2}} - \dfrac{\langle v_1, \tilde{v}_2\rangle \langle \tilde{v}_2, v_3\rangle}{\|v_1\| \; \|\tilde{v}_2\|^2}$

$= \dfrac{\|v_1\|}{\|v_1\|} \qquad = \dfrac{\langle v_1, v_3\rangle}{\|v_1\|}$

$= \dfrac{\|v_1\| \langle v_1, v_3\rangle \cdot \|\tilde{v}_2\|^2 - \|\tilde{v}_2\|^2 \langle v_1, v_3\rangle - \|v_1\| \langle v_1, \tilde{v}_2\rangle \langle \tilde{v}_2, v_3\rangle}{\|v_1\|^2 \|\tilde{v}_2\|^2}$

$\|\tilde{v}_2\|^2 =$

$= \cancel{\|\tilde{v}_2\|^2 \langle v_1, v_3\rangle (\|v_1\| - 1 - \|v_1\|)}$

$= \dfrac{\cancel{\|\tilde{v}_2\|^2} \langle v_1, v_3\rangle \left(\|v_1\| \|\tilde{v}_2\|^2 - \|\tilde{v}_2\|^2 - 1\right)}{\|v_1\|^2 \|\tilde{v}_2\|^2} \ldots$

poging 2:

$\langle e_1, \tilde{v}_3\rangle$

$= \left\langle \dfrac{v_1}{\|v_1\|}, v_3 - \dfrac{\langle v_1, v_3\rangle v_1}{\|v_1\|^2} - \dfrac{\langle \tilde{v}_2, v_3\rangle \tilde{v}_2}{\|\tilde{v}_2\|^2} \right\rangle$

$= \cancel{\dfrac{\langle v_1, v_3\rangle}{\|v_1\|}} - \cancel{\dfrac{\langle v_1, v_3\rangle 1}{\|v_1\|}} - \dfrac{\langle v_1, \tilde{v}_2\rangle \langle \tilde{v}_2, v_3\rangle}{\|v_1\|^2 \; \|\tilde{v}_2\|^2}$

$= -\dfrac{\langle \tilde{v}_2, v_3\rangle}{\|v_1\| \; \|\tilde{v}_2\|^2} \left(\left\langle v_1, v_2 - \dfrac{\langle v_1, v_2\rangle v_1}{\|v_1\|^2} \right\rangle \right) = 0$ $\quad \| \quad 0$

$\langle e_2, \tilde{v}_3\rangle$

$= \cancel{\dfrac{\langle \tilde{v}_2, v_3\rangle}{\|\tilde{v}_2\|}} - \dfrac{\langle v_1, v_3\rangle \langle v_1, \tilde{v}_2\rangle}{\|v_1\|^2 \cdot \|\tilde{v}_2\|} - \cancel{\dfrac{\|\tilde{v}_2\|^2 \langle \tilde{v}_2, v_3\rangle}{\|\tilde{v}_2\|^3}}$ $\quad \langle v_1, \tilde{v}_2\rangle = 0$

$= 0$

stel dat $\tilde{v}_3 = 0$

dan zou $v_3 = \dfrac{\langle e_1, v_3\rangle}{\|v_1\|} \cdot v_1 + \dfrac{\langle e_2, v_3\rangle}{\|\tilde{v}_2\|} \cdot \left(v_2 - \dfrac{\langle e_1, v_2\rangle \cdot v_1}{\|v_1\|}\right)$

waaruit zou volgen dat v_3 te schrijven valt als een lin. comb. van v_1 en v_2, wat strijdig is met het feit dat ~~dit~~ een basis vrij is

$e_3 = \frac{\tilde{v}_3}{\|\tilde{v}_3\|}$ $\tilde{v}_j = v_j - \sum_{i=1}^{j-1} \langle e_i, v_j \rangle e_i$ ⚠ eerst w° $j \in \{2, \dots, k\}$

c) TB: $\langle e_i, \tilde{v}_j \rangle = 0 \quad \forall i = 1, \dots, j-1$ en dat $\tilde{v}_j \neq 0$

$\Rightarrow$ bewijzen dat geldt $\forall j$

① $\boxed{j = 2}$ ② $j = p$ ~~+1~~ $\Rightarrow \boxed{j = p+1}$

de stelling geldt voor $j = 2$ want $\langle e_1, \tilde{v}_2 \rangle = 0$ (zie (a))

② stel dat $\langle e_i, \tilde{v}_p \rangle = 0 \quad \forall i = 1, \dots, p-1$ en $\tilde{v}_p \neq 0$

dan is $\langle e_i, \tilde{v}_{p+1} \rangle$

... en dan ?

IDEE:

* a): relatie v. v_3 t.o.v. v_1 en e_1

a en

...

d) $\{e_1, e_2, \dots, e_k\}$ is een orthonormale basis v. V.

① voortbrengend: volgt uit lin. onafh.

② vrij: omdat elke e_i opgebouwd is ~~als~~ als lin comb van vorige e_j's ~~en~~ gesommeerd met v_i, kan geen enkele e_i met $i \in \{1, \dots, k\}$ geschreven w als lin. comb. van andere basisvectoren.

③ orthonormaal

TB: $\langle e_i, e_j \rangle = 0 \quad \forall i, j \in \{1, \dots, k\}$ met $i \neq j$

stel $i > j$ dan

$\langle e_i, e_j \rangle = \langle e_i, \frac{\tilde{v}_j}{\|\tilde{v}_j\|} \rangle$ en dit is 0 aangezien $\langle e_i, \tilde{v}_j \rangle = 0 \quad \forall i \in \{1, \dots, j-1\}$

(8) 6.2.2.4. als $p_{ij} > 0 \ \forall \, i,j = 1 \ldots m$

(1) slechts 1 eigenvector die bij eigenwaarde 1 hoort

(2) voor alle eigenw. v. λ v. P geldt: $\forall |\lambda| < 1$

~~TERRAIN~~ & ERROR?

$$P = \begin{pmatrix} 1 & 0 & & \\ 0 & 1 & & \\ & & 0 & 1 \\ & & 1 & 0 \end{pmatrix}$$

(9) $P = \begin{pmatrix} -1/2 & 1/4 & 1/2 \\ 1/4 & 1/4 & -1/4 \\ 1/4 & 1/2 & 1/4 \end{pmatrix}$ a) eigenwaarden:

$$\det \begin{pmatrix} -1/2-\lambda & -1/4 & -1/2 \\ -1/4 & -1/4-\lambda & -1/4 \\ -1/4 & -1/2 & -1/4-\lambda \end{pmatrix} = 0$$

$$(-1/2-\lambda)(1/4-\lambda)^2 + \frac{1}{64} + \frac{1}{16} - \frac{1}{8}(1/4-\lambda) - \frac{1}{8}(1/2-\lambda) - \frac{1}{16}(1/4-\lambda) = 0$$

$= -\lambda^3 + \lambda^2 = 0$

$= \lambda^2(\lambda - 1)$ dus $\lambda_1 = \lambda_2 = 0 \longrightarrow$ eigenvectoren: $(a, 0, -a)$ met $a \in \mathbb{R}_0$

$\lambda_3 = 1 \longrightarrow$ eigenvectoren: $(\frac{7}{5}a, \frac{4}{5}a, a)$ met $a \in \mathbb{R}_0$

b) deze eigenvect. vormen geen basis v. $\mathbb{R}^3$

$$P^2 = \begin{pmatrix} 0,4375 & 0,4375 & 0,4375 \\ 0,25 & 0,25 & 0,25 \\ 0,3125 & 0,3125 & 0,3125 \end{pmatrix} \Rightarrow \text{voor } a = 0,3125$$

(10) $\begin{pmatrix} x_{n+1} \\ y_{n+1} \\ z_{n+1} \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 3 & -1 & 1 \\ -1 & 3 & -5 \\ 0 & 0 & -2 \end{pmatrix} \begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix}$ met $x_0 = 2$ & $y_0 = 1$ $z_0 = 1$

$z_{n+1} = \frac{-1}{2} z_n$

$\begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix} = \frac{1}{4^n} \underbrace{\begin{pmatrix} 3 & -1 & 1 \\ -1 & 3 & -5 \\ 0 & 0 & -2 \end{pmatrix}^n}_{=A} \cdot \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$ $\quad z_n = \left(\frac{-1}{2}\right)^n \cdot z_0 = \left(\frac{-1}{2}\right)^n$

$A = Q \cdot D \cdot Q^{-1}$ als A diag.

* eigenw: $\lambda_1 = -2$ $\lambda_3 = 4$ (karakt. vgl: $-\lambda^3 + 4\lambda^2 + 4\lambda - 16$)

$\lambda_2 = 2$

* eigenvect: $\lambda_1 \longrightarrow \begin{pmatrix} 5 & -1 & 1 \\ -1 & 5 & -5 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \rightarrow (0, a, a)$ met $a \in \mathbb{R}_0$ vb $(0,1,1)$

$\lambda_2 \longrightarrow \begin{pmatrix} 1 & -1 & 1 \\ -1 & 1 & -5 \\ 0 & 0 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \longrightarrow (b, b, 0)$ met $b \in \mathbb{R}_0$ $(1,1,0)$

$\lambda_3 \longrightarrow \begin{pmatrix} -1 & -1 & 1 \\ -1 & -1 & -5 \\ 0 & 0 & -6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \rightarrow (c, -c, 0)$ met $c \in \mathbb{R}_0$

$$A^n = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} -2^n & 0 & 0 \\ 0 & 2^n & 0 \\ 0 & 0 & 4^n \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & 0 & 0 \end{pmatrix}^{-1} \quad = \begin{pmatrix} 0 & 0 & 1 \\ 1/2 & 1/2 & -1/2 \\ 1/2 & -1/2 & 1/2 \end{pmatrix}$$

$$\lim_{n\to\infty} \begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix} = Q \begin{pmatrix} \frac{-1}{2^n} & 0 & 0 \\ 0 & \frac{1}{2^n} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot Q^{-1} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

$$\lim_{n\to\infty}\begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 & 1 \\ 1/2 & 1/2 & -1/2 \\ 1/2 & -1/2 & 1/2 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1/2 & -1/2 & 1/2 \\ -1/2 & 1/2 & -1/2 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 - 1/2 + \frac{1}{2} \\ -1 + 1/2 - \frac{1}{2} \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}.$$

(11) muskusratten:

0j	→ 0,6 jongen	⇒ overleving:	1/2
1j	→ 3,5 "		75%
2j	→ 2,4 "		75%
3j	→ 1,2 "		0%

$$A = \begin{bmatrix} 0{,}6 & 3{,}5 & 2{,}4 & 1{,}2 \\ 0{,}5 & 0 & 0 & 0 \\ 0 & 0{,}75 & 0 & 0 \\ 0 & 0 & 0{,}75 & 0 \end{bmatrix}$$

(1) eigenw.

$$\det\begin{pmatrix} 0{,}6-\lambda & 3{,}5 & 2{,}4 & 1{,}2 \\ 0{,}5 & -\lambda & 0 & 0 \\ 0 & 0{,}75 & -\lambda & 0 \\ 0 & 0 & 0{,}75 & -\lambda \end{pmatrix} = 0$$

$$= -\frac{3}{4}\det\begin{pmatrix} 0{,}6-\lambda & 2{,}4 & 1{,}2 \\ 0{,}5 & 0 & 0 \\ 0 & 0{,}75 & -\lambda \end{pmatrix} + \lambda \det\begin{pmatrix} 0{,}6-\lambda & 3{,}5 & 1{,}2 \\ 0{,}5 & -\lambda & 0 \\ 0 & 0 & -\lambda \end{pmatrix}$$

...

~~dus $\lambda_1 = 0$~~

$\lambda_2 = 1{,}8725$ .. $(\approx 1{,}86)$

$\lambda_3 = -0{,}82$

$\lambda_4 = -0{,}22 + 0{,}42\,i$

$\lambda_1 = -0{,}22 - 0{,}42\,i$

b) ja

c) $\lambda = 1,86$.

$$\begin{pmatrix} -1,26 & 3,5 & 2,4 & 1,2 \\ 0,5 & -1,86 & 0 & 0 \\ 0 & 0,75 & -1,86 & 0 \\ 0 & 0 & 0,75 & -1,86 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$A \sim \begin{pmatrix} -1,26 & 3,5 & 2,4 & 1,2 \\ 0 & 0,5936 & 0 & 0 \\ 0 & 0 & -1,104096 & 0 \\ 0 & 0 & 0 & 2,0536 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

12 $Ax = x$ met A een stochastische matrix

dan zal 1 een eigenwaarde zijn van A met bijhorende eigenvector x

uit prop 6.2.2.3. zal volgen dat er dus een ~~pa~~ $x \in \mathbb{R}^m$ is (met m afm. v. A) met enkel pos. comp.

(anders kies ω^0 x^* eigenv. bij 1 dan zal $x = (|x_1^*|, \dots, |x_m^*|)$ ook een eigenv. zijn bij 1)

(11) $A = \begin{bmatrix} 0{,}6 & 3{,}5 & 2{,}4 & 1{,}2 \\ 0{,}5 & 0 & 0 & 0 \\ 0 & 0{,}75 & 0 & 0 \\ 0 & 0 & 0{,}75 & 0 \end{bmatrix}$

$$\det \begin{pmatrix} 0{,}6-\lambda & 3{,}5 & 2{,}4 & 1{,}2 \\ 0{,}5 & 0-\lambda & 0 & 0 \\ 0 & 0{,}75 & -\lambda & 0 \\ 0 & 0 & 0{,}75 & -\lambda \end{pmatrix} = \frac{-3}{4} \det \begin{pmatrix} 0{,}6-\lambda & 2{,}4 & 1{,}2 \\ 0{,}5 & 0 & 0 \\ 0 & 0{,}75 & -\lambda \end{pmatrix}$$

$$-\lambda \det \begin{pmatrix} 0{,}6-\lambda & 3{,}5 & 1{,}2 \\ 0{,}5 & -\lambda & 0 \\ 0 & 0 & -\lambda \end{pmatrix}$$

$$= \frac{-3}{4} \cdot \frac{-1}{2} \cdot \det \begin{pmatrix} 2{,}4 & 1{,}2 \\ 0{,}75 & -\lambda \end{pmatrix}$$

$$-\lambda \cdot -\lambda \cdot \det \begin{pmatrix} 0{,}6-\lambda & 3{,}5 \\ 0{,}5 & -\lambda \end{pmatrix} = \frac{3}{8}\left(-2{,}4\lambda - \overbrace{1{,}2 \cdot 0{,}75}^{=0{,}9}\right).$$

$$+ \lambda^2\left(-\lambda(0{,}6-\lambda) - \frac{3{,}5}{2}\right)$$

$$= -\frac{7{,}2}{8}\lambda - \frac{2{,}7}{8} - 0{,}6\lambda^3 + \lambda^4 - \frac{7}{4}\lambda^2 = 0.$$

c) $\begin{pmatrix} -1{,}26 & 3{,}5 & 2{,}4 & 1{,}2 \\ 0{,}5 & -1{,}86 & 0 & 0 \\ 0 & 0{,}75 & -1{,}86 & 0 \\ 0 & 0 & 0{,}75 & -1{,}86 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$$\sim \left[\begin{array}{cccc|c} -1{,}26 & 3{,}5 & 2{,}4 & 1{,}2 & 0 \\ 0 & 0{,}5936 & -1{,}2 & -0{,}6 & 0 \\ 0 & \cancel{0{,}045}\ 0{,}75 & \cancel{2{,}3436}\ -1{,}86 & 0 & 0 \\ 0 & 0 & 0{,}75 & -1{,}86 & 0 \end{array}\right] \sim \begin{bmatrix} -0{,}747936 & 0 & & \\ 0 & 0{,}5936 & \dots & \text{BLABLA} \\ 0 & 0 & & \\ 0 & 0 & & \end{bmatrix}$$

check: $\mathbb{R}(22{,}88 ; 6{,}15 ; 2{,}48 ; 1)$

$\Rightarrow$ ok!

d) $x_0 = a_1 \cdot e_1 + a_2 \cdot e_2 + a_3 \cdot e_3 + a_4 \cdot e_4$ met e_i de eigenvector bij EW λ_i

$\lim_{n\to\infty} x^n = \lim_{n\to\infty} Q \cdot D^n \cdot Q^{-1} \cdot x_0$ met Q = de matrix met de EV^n opn in de kolommen.

$$= Q \lim_{n\to\infty} \begin{pmatrix} 1{,}86^n & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cdot Q^{-1} \cdot x_0$$

=

$$X^n = A \cdot X^{n-1}.$$

$$X^n = A^n \cdot X_0 = Q \cdot D^n \cdot Q^{-1} \cdot X_0.$$

$$= \begin{pmatrix} e_1 & e_2 & e_3 & e_4 \end{pmatrix} \cdot D^n \cdot \begin{pmatrix} e_1 \cdot e_2 \cdot e_3 \cdot e_4 \end{pmatrix}^{-1} \cdot X_0.$$

= als n groot is $\bar{\omega}$ $D^n = \begin{pmatrix} 1{,}86^n & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

schrijf e_{ij} het j-de element van eigenvector i.

$$X^n = \begin{pmatrix} 1{,}86^n \cdot e_{1,1} & 0 & 0 & 0 \\ 1{,}86^n e_{1,2} & 0 & 0 & 0 \\ 1{,}86^n e_{1,3} & 0 & 0 & 0 \\ 1{,}86^n e_{1,4} & 0 & 0 & 0 \end{pmatrix} \cdot Q^{-1} \cdot X_0.$$

$$= 1{,}86^n \cdot \left[\begin{pmatrix} e_1 & 0 & 0 & 0 \end{pmatrix} \cdot Q^{-1} \cdot \left(a_1 e_1 + a_2 e_2 + a_3 e_3 + a_4 e_4 \right) \right]$$

. . .

opdrachten 1.1.3. impliciete functies

PS73.

oef. 1.

$$x \cdot f^{3}(x) - 2x f^2(x) + 3x^2 f(x) + 6x^3 = 6.$$

Opmerking Hendrik

$$* \quad f^3(x) + 3 f^2(x) \cdot f'(x) - 2 f^2(x) - 4x f(x) f'(x)$$

$$+ 6x f(x) + 3x^2 f'(x) + 18x^2 = 0.$$

$\boxed{x=1}$

$$\cancel{f^3(1)} + \cancel{3 f^2(1) \cdot f'(1)} - \cancel{2 f^2(1)} - 0 + 0 + 3 \cdot f'(1) + 18 = 0$$

dus $f'(1) = -6.$

$$* \quad 3 f^2(x) \cdot f'(x) + 3 f^2(x) f''(x) + 6 f(x) (f'(x))^2$$

$$- 4 f(x) f'(x) - 4 f(x) f'(x) - 4x \underbrace{(f(x) \cdot f'(x))'}_{((f'(x))^2 + f(x) \cdot f''(x))} + 6 f(x) + 6x f'(x)$$

$$+ 6x f'(x) + 3x^2 f''(x) + 36x = 0$$

$\boxed{x=1}$

$$-4 (f'(1))^2 + \overset{12}{\cancel{6}} f'(1) + 3 f''(1) + 36 = 0.$$

$$f''(1) = 60$$

oef 2.

$$3 \cancel{f(1,1)} - 1 + 2 f(1,1)^3 - \cancel{3 f(1,1)} = 1$$

$$\boxed{f(1,1) = 1}$$

naar x

$$3x^2 \cdot D_1 f(x,y) + 6x f(x,y) - 2xy^2 + 6 D_1 f(x,y) \cdot f(x,y)^2$$

$$- 3y D_1 f(x,y) = 0$$

$$\cancel{3 D_1 f(1,1)} + 6 f(1,1) - 2 + 6 f(1,1)^2 D_1 f(1,1) - \cancel{3 D_1 f(1,1)} = 0$$

$$4 + 6 D_1 f(1,1) = 0$$

$$D_1 f(1,1) = -2/3$$

1ste orde naar y

$$\cancel{3x^2 D^2_{12} f(x,y) + 6x D_2 f(x,y)}$$

$$D_2 f(x,y) = 5/6.$$

2de orde naar y

$$3x^2 D^2_{12} f(x,y) + 6x D_2 f(x,y) - 4xy + 6 D^2_{12} f(x,y) \cdot f(x,y)^2$$

$$+ 6 D_1 f(x,y) \cdot 2 f(x,y) D_2 f(x,y)$$

$$- 3 D_1 f(x,y) - 3y D^2_{12} f(x,y) = 0$$

$$3 D^2_{12} f(1,1) + 6 D_2 f(1,1) - 4 + 6 D^2_{12} f(1,1) \cdot f(1,1)^2 + 6 D_1 f(1,1) \cdot 2 f(1,1) \cdot D_2 f(1,1)$$ (4)

$$-3 D_1 f(1,1) - 3 D^2_{12} f(1,1) = 0$$

$$D^2_{12} f(1,1) = -\frac{23}{18}$$

oef 3.

$$Q(K,L) = \left(\frac{1}{2} K^{1/2} + \frac{1}{2} L^{1/2}\right)^2.$$

stel $Q(K,L) = c$ met $c \in \mathbb{R}$.

dus $\left(\frac{1}{2} K^{1/2} + \frac{1}{2} L^{1/2}\right)^2 = c.$

$$\frac{1}{2} K^{1/2} + \frac{1}{2} L^{1/2} = \sqrt{c}$$

naar K afleiden:

$$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{K^{1/2}} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{L^{1/2}} \cdot \frac{\delta L}{\delta K} = 0$$

dus $\frac{\delta L}{\delta K} = -\frac{L^{1/2}}{K^{1/2}}$

$$MTS = -\frac{2 \cdot \left(\frac{1}{2} K^{1/2} + \frac{1}{2} L^{1/2}\right) \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{K^{1/2}}}{2 \cdot \quad '' \quad \cdot \frac{1}{4} \cdot \frac{1}{L^{1/2}}}$$

$$= -\frac{L^{1/2}}{K^{1/2}}.$$

oef 4.

$n \in \mathbb{N}_0$

$F : \mathbb{R}^{n+1} \to \mathbb{R} : (x,y) \longmapsto F(x,y)$ met $x \in \mathbb{R}^n = (x_1, \dots, x_n)$

$f : A \subseteq \mathbb{R}^n \to \mathbb{R} : x \longmapsto f(x) = y$

$h : A \subseteq \mathbb{R}^n \to \mathbb{R}^{n+1} : x \longmapsto (x, f(x))$

$\tilde{f} : A \subseteq \mathbb{R}^n \to \mathbb{R} : x \longmapsto F(x, f(x))$ dus $\tilde{f} = F \circ h$

$D_i \tilde{f}(x) = 0 \quad \forall x \in A$ met $i \in \{1, \dots, n\}$

$$D_i \tilde{f}(x) = D_1 F(x; f(x)) \cdot D_i h_1(x)^{=0} + \dots + D_i F(x; f(x)) \cdot D_i h_i(x)^{=1}$$
$$+ \dots + D_n F(x; f(x)) \text{ in } D_i h_n(x) + \frac{\delta F}{\delta y}(x; f(x)) \cdot D_i h_{n+1}(x)$$

$$\text{dus } 0 = \frac{\delta F(x; f(x))}{\delta x_i} + \frac{\delta F}{\delta y}(x; f(x)) \cdot \frac{\delta f(x)}{\delta x_i}$$

oef 5.

~~neem kans~~ ~~nee~~ $n = 1$.

... DVH.

oef 7

afleiden naar M:

$$\begin{cases} \frac{\delta C}{\delta M}^{a} + \frac{\delta I}{\delta M}^{b} + \frac{\delta G}{\delta M}^{=0} - \frac{\delta Y}{\delta M}^{c} = 0 \\ C'(Y-T) \cdot \frac{\delta Y}{\delta M}^{c} - \frac{\delta C}{\delta M}^{a} = 0 \\ i'(r) \cdot \frac{\delta r}{\delta M}^{d} - \frac{\delta I}{\delta M}^{b} = 0 \\ \frac{\delta m}{\delta r} \cdot \frac{\delta r}{\delta M}^{d} + \frac{\delta m}{\delta Y} \frac{\delta Y}{\delta M}^{c} - 1 = 0. \end{cases}$$

$$\begin{cases} a + b - c = 0 \\ c'(Y-T) \cdot c - a = 0 \\ i'(r)\, d - b = 0 \\ \frac{\delta M}{\delta r} \cdot d + \frac{\delta m}{\delta r} \cdot c = 1. \end{cases}$$

$$\begin{cases} a = c'(Y-T) \cdot c \\ b = i'(r)\, d \\ c\,(c'(Y-T) - 1) + i'(r)\, d = 0 \\ \frac{\delta M}{\delta r}\left(-c \cdot \frac{(c'(Y-T)-1)}{i'(r)} + c\right) = 1. \end{cases}$$

$$\frac{\delta m}{\delta r}\left(c + c\, i'(r) - c \cdot c'(Y-T)\right) = 1\, i'(r).$$

$$i'(r) \quad \frac{\delta m}{\delta r} \cdot c \cdot \left(1 + i'(r) - c'(Y-T)\right) = i'(r)$$

$$c = -\frac{i'(r)}{D} > 0$$

lemniscaat v. Bernoulli

$F: \mathbb{R}^2 \to \mathbb{R} : (x,y) \mapsto (x^2+y^2)^2 - 2(x^2-y^2)$

① snijpunten v. N_0 met horizontale as.

$y=0$: $(x^2)^2 - 2x^2 = 0$

dus $x = 0 \vee (x^2-2) = 0$
$x^2 = 2$
$x = \pm\sqrt{2}.$

$p_1 = (-\sqrt{2}, 0)$
$p_2 = (0,0)$
$p_3 = (\sqrt{2}, 0)$

② $p_4 = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\left(\frac{3}{4}+\frac{1}{4}\right)^2 - 2\cdot\left(\frac{3}{4}-\frac{1}{4}\right) = 1 - 2\cdot\frac{1}{2} = 0 \Rightarrow$ behoort tot N_0

③ $\nabla F\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right) = \left(\frac{\delta F}{\delta x}\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), \frac{\delta F}{\delta y}\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)\right)$

$= \begin{pmatrix} 2(x^2+y^2)\cdot 2x - 2\cdot 2x \\ 2(x^2+y^2)\cdot 2y + 4y \end{pmatrix} = (0, 4)$

$\Rightarrow \langle \nabla F\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), \underset{v}{\mu}(0) \rangle = 0$

$0 \cdot v_1 + 4 \cdot v_2 = 0.$

$\vec{v} = (a, 0)$ met $a \in \mathbb{R}_0$

④ in p_1 niet: $\frac{\delta F}{\delta y}(x,y) = 0.$

in p_2 niet: $\frac{\delta F}{\delta y}(0,0) = 0$

in p_4 wel: ① $\frac{\delta F}{\delta y}\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right) = 4$

② $\exists\, c \in \mathbb{R}^2$ namelijk $(0,0)$

③ F heeft c⁰ part. afgel. (zie ③)

⑤ p_4: blablabla : $\frac{\delta f}{\delta x}(x) = \frac{-0}{4} = 0$ ⇓

opdrachten p. 585: pos/neg definiete matrices.

oef 1

$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{pmatrix}$ ① $\det\begin{pmatrix} 1-\lambda & 1 & 1 \\ 1 & 2-\lambda & 3 \\ 1 & 3 & 6-\lambda \end{pmatrix} = 0$

$= 6 - \cancel{\lambda} 19\lambda + 9\lambda^2 - \lambda^3 + 3 + 3 = -\lambda^3 + 9\lambda^2 - \cancel{\ldots}\ 9\lambda + 1$

$- 2 + \lambda - 9 + 9\lambda - 6 + \lambda$

	-1	9	-9	1
1		-1	8	-1
	-1	8	-1	0

$= (x^2 - 8x + 1)(x - 1) \Rightarrow \lambda_2$ en λ_3: $4 \pm \sqrt{15} > 0$

$\Rightarrow$ alle eigenwaarden zijn strikt positief

② $D_1 = \det(1) = 1 > 0$

$D_2 = \det\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} = 1 > 0$

$D_3 = \det\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{pmatrix} = 1 > 0$

oef 2.

$\begin{pmatrix} 1 & 0 & 3 \\ 9 & 2 & 5 \\ 1 & 1 & 1 \end{pmatrix}$ $\left.\begin{array}{l} D_1 = 1 > 0 \\ D_2 = 2 > 0 \\ D_3 = 18 > 0. \end{array}\right\}$ positief definiet

oef 3

$q(x,y,z) = x^2 + (a^2+b)y^2 + (1+9b+c)z^2 - 2axy$
$- (2a + 6b)yz + 2xz.$

$= \underset{1\times 3}{(x,y,z)} \cdot \underset{3\times 3}{A} \cdot \underset{3\times 1}{\begin{pmatrix} x \\ y \\ z \end{pmatrix}} = (x,y,z) \cdot \begin{pmatrix} a_{11}x + a_{12}y + a_{13}z \\ a_{12}x \quad \ldots \quad " \quad \cdot \\ a_{13}x \quad " \quad \ldots \quad + a_{33}z \end{pmatrix}$

$= a_{11}x^2 + 2(a_{12}xy) + 2(a_{13}xz) + 2a_{23}yz$
$+ a_{22}y^2 + a_{33}z^2.$

Dus:

$a_{11} = 1 \qquad a_{12} = -a$

$a_{22} = a^2 + b \qquad a_{13} = 1$

$a_{33} = 1 + 9b + c \qquad a_{23} = -a + 3b.$

dus $A = \begin{pmatrix} 1 & -a & 1 \\ -a & a^2+b & -a+3b \\ 1 & -a+3b & 1+9b+c \end{pmatrix}$

waarvit volgt:

$D_1 = 1 \qquad D_2 = a^2+b-a^2 = b$

$D_3 = b \cdot c$

$\Rightarrow$ voor b en $c \in \mathbb{R}_0^+$ zal q positief definiet zijn

alle rest: noch pos, noch neg.

oef 8:

a) fout. vb $\begin{pmatrix} 1 & -2 \\ 4 & 5 \end{pmatrix} \longrightarrow \left.\begin{matrix} D_2 = 5+8 = 13. \\ D_1 = 1 \end{matrix}\right\}$ pos. definiet

b) fout vb. $\begin{pmatrix} 1 & 2 \\ 4 & 5 \end{pmatrix} \longrightarrow \left.\begin{matrix} D_1 = 1 \\ D_2 = -3 \end{matrix}\right\}$ niet pos, niet neg. definiet opdrachten: vrije extrema : 2.3.5 p. 594.

oef 1

a) $f : \mathbb{R}^3 \to \mathbb{R} : (x,y,z) \longmapsto \frac{3}{2}(x^2+y^2+z^2) - xy + xz + yz$.

$D_1 f(x,y,z) = 3x - y + z$.

$D_2 f(x,y,z) = 3y - x + z$

$D_3 f(x,y,z) = 3z + x + y$

} kritiek punt in $(0,0,0)$.

$H_f(x,y,z) = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 3 & 1 \\ 1 & 1 & 3 \end{pmatrix}$ $D_1 > 0$, $D_2 = 10 > 0$, $D_3 = 16 > 0$ } $H_f(x,y,z)$ is positief definiet $\forall (x,y,z) \in \mathbb{R}^3 \Rightarrow$ convex.

$\Rightarrow$ globaal min

b) $f : \mathbb{R}^3 \to \mathbb{R} : (x,y,z) \longmapsto x^2 + y^2 + z^2 - 2xyz$.

$D_1 f(x,y,z) = 2x - 2yz$

$D_2 f(x,y,z) = 2y - 2xz$ $\Rightarrow$ $\begin{cases} x = yz \\ y = xz \\ z = xy \end{cases}$ $x = y = z = 1$, $= 0$.

$D_3 f(x,y,z) = 2z - 2xy$

$\begin{cases} x = y \cdot x \cdot y \\ \dots \end{cases}$

$H_f(x,y,z) = \begin{pmatrix} 2 & -2z & -2y \\ -2z & 2 & -2x \\ -2y & -2x & 2 \end{pmatrix}$

$H_f(0,0,0) = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \longrightarrow$ positief definiet $\Rightarrow$ minimum

$H_f(1,1,1) = \begin{pmatrix} 2 & -2 & -2 \\ -2 & 2 & -2 \\ -2 & -2 & 2 \end{pmatrix}$ $D_1 > 0$, $D_2 = 0$, $D_3 = \dots$ } $\Rightarrow$ EW: $\lambda_1 = -2$ $\lambda_2 = \lambda_3 = 4$

$\Rightarrow$ nog pos. nog neg definiet

c) $f : \mathbb{R}^3 \to \mathbb{R} : (x,y,z) \longmapsto (x^2+y^2+z^2) \cdot e^{-xy}$

$D_1 f(x,y,z) = (x^2+y^2+z^2) \cdot e^{-xy} \cdot -y + 2x \cdot e^{-xy}$

$D_2 f(x,y,z) = (x^2+y^2+z^2) \cdot e^{-xy} \cdot -x + 2y \cdot e^{-xy}$

$D_3 f(x,y,z) = 2z \cdot e^{-xy}$

$\begin{cases} 2z \cdot e^{-xy} = 0 \Rightarrow z = 0 \\ 2x \cdot \cancel{e^{-xy}} = y(x^2+y^2 \cancel{+z^2}) \cdot \cancel{e^{-xy}} \\ x = y \end{cases}$ dus $\cancel{2}x \cdot \cancel{e^{-2x}} = \cancel{2}x^3 \cdot \cancel{e^{-2x}}$

$x = 0 \lor x = 1$

$\Rightarrow$ 2 extrema: $(1,1,0)$ en $(0,0,0)$.

$H_f(x,y,z) = \Big(-y \cdot D_1 f(x,y,z) + 2 \cdot e^{-xy} + 2x \cdot e^{-xy} \cdot -y \quad \dots$

PFF)

oef 2 $\quad f'(0) < 0$

$g: \mathbb{R}^3 \to \mathbb{R}: (x,y,z) \longmapsto f(\underbrace{x^2 + 2y^2 + 3z^2}_{\alpha})$

$D_1 g(x,y,z) = f'(x^2 + 2y^2 + 3z^2) \cdot 2x$

$D_2 g(x,y,z) = \quad '' \quad \cdot 4y$

$D_3 g(x,y,z) = \quad '' \quad \cdot 6z$

$$H_f g(x,y,z) = \begin{pmatrix} 2x \cdot f''(\alpha) \cdot 2x + 2f'(\alpha) & 2x \cdot f''(\alpha) \cdot 4y & 2x\, f''(\alpha) \cdot 6z \\ 4y \cdot f''(\alpha) \cdot 2x & 4y \cdot f''(\alpha) \cdot 4y + 4f'(\alpha) & \text{BLA} \\ \text{BLA} & \text{BLA} & \text{BLA} + 6 \cdot f'(\alpha) \end{pmatrix}$$

$$H_f g(0,0,0) = \begin{pmatrix} 2 \cdot f'(0) & 0 & 0 \\ 0 & 4f'(0) & 0 \\ 0 & 0 & 6f'(0) \end{pmatrix} \longrightarrow \left.\begin{matrix} D_1 < 0 \\ D_2 > 0 \\ D_3 < 0 \end{matrix}\right\} \text{negatief definiet}$$

$\Rightarrow$ lokaal max in $(0,0,0)$

oef 3

$f: \mathbb{R}^3 \to \mathbb{R}: (a,b,c) \longmapsto \sum_{i=1}^{n} (ax_i + by_i + c - z_i)^2$

$D_1 f(a,b,c) = \cancel{\sum_{i=1}^{n} 2 \cdot x_i} \quad 2 \sum_{i=1}^{n} (ax_i + by_i + c - z_i) \cdot \cancel{\sum_{i=1}^{n}} x_i$

$D_2 f(a,b,c) = \cancel{\sum_{i=1}^{n} 2y_i} \quad 2 \sum_{i=1}^{n} (ax_i + by_i + c - z_i) \cdot \cancel{\sum_{i=1}^{n}} y_i$

$D_3 f(a,b,c) = 2 \sum_{i=1}^{n} (ax_i + by_i + c - z_i)$

~~$H_f(a,b,c) = 2\sum_{i=1}^{n}(ax_i + by_i + c - z_i) + 2\sum_{i=1}^{n} a \cdot x_i \; 2\sum_{i=1}^{n} b \cdot x_i$~~ $\quad 2$

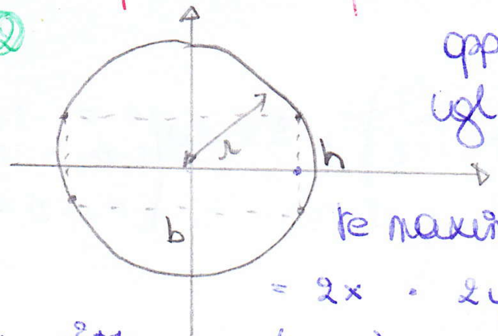
kritieke punten

$$\begin{cases} \sum (ax + by + c - z)\, x = 0 \\ \sum (ax + by + c - z)\, y = 0 \\ \sum (ax + by + c - z) = 0 \end{cases} \qquad \begin{cases} a \cdot \sum_{i=1}^{n} x_i^2 + b \cdot \sum_{i=1}^{n} y_i x_i + c \cdot \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i z_i \\ a \cdot \sum_{i=1}^{n} x_i y_i + b \cdot \sum_{i=1}^{n} y_i^2 + c \cdot \sum_{i=1}^{n} y_i = \sum_{i=1}^{n} y_i z_i \\ a \sum_{i=1}^{n} x_i + b \sum_{i=1}^{n} y_i + c \cdot n = \sum_{i=1}^{n} z_i \end{cases}$$

$$H_f(a,b,c) = \begin{pmatrix} 2\sum_{i=1}^{n} x_i^2 & 2\sum_{i=1}^{n} x_i \cdot y_i & 2\sum_{i=1}^{n} x_i \\ 2\sum_{i=1}^{n} x_i y_i & 2\sum_{i=1}^{n} y_i^2 & 2\sum_{i=1}^{n} y_i \\ 2\sum_{i=1}^{n} x_i & 2\sum_{i=1}^{n} y_i & 2n \end{pmatrix} \quad \begin{matrix} D_1 > 0 \\ D_2 > 0 \; (?) \\ D_3 > 0 \; (???) \end{matrix}$$

opdrachten p. 624: gebonden extrema

2



opp = b · h.

vgl v/d cirkel : $x^2+y^2=r^2$.

te maximaliseren functie

$= 2x \cdot 2y$

$L : \mathbb{R}^{2+1} \to \mathbb{R} : (x,y,\lambda) \longmapsto 2x2y - \lambda(x^2+y^2-r^2)$

$$\begin{cases} \frac{dL}{dx}(x,y,\lambda) = 4y - 2\lambda x = 0 & y = \frac{\lambda x}{2} & \text{dus } y = x. \\ \frac{dL}{dy}(x,y,\lambda) = 4x - 2\lambda y = 0 & 4x = \lambda^2 x. & \to \text{als } x \neq 0 \text{ dan } \lambda = \pm 2. \\ \frac{dL}{d\lambda}(x,y,\lambda) = r^2 - x^2 - y^2 = 0 & r^2 = 2x^2 & \cancel{\lambda} \ \frac{r}{\sqrt{2}} \end{cases}$$

$x = \pm\cancel{\sqrt{2r}} = y$.

$\Rightarrow$ dus extremum ~~mogelijk~~ als $x = y = \cancel{\sqrt{2r}}$. $\frac{r}{\sqrt{2}}$

dan is de opp $= 4 \cdot \cancel{r} \cdot \frac{r^2}{2} = \cancel{8r}.\ 2r^2$.

$\Delta = \det\begin{pmatrix} D_{11}L(x,y,\lambda) & D_{12}L & D_{13}L \\ D_{21}L(x,y,\lambda) & D_{22}L & D_{23}L \\ D_{31}L(x,y,\lambda) & D_{32}L & D_{33}L \end{pmatrix}$

$= \det\begin{pmatrix} -4 & 4 & -\frac{2r}{\sqrt{2}} \\ 4 & -4 & -\frac{2r}{\sqrt{2}} \\ -\frac{2r}{\sqrt{2}} & -\frac{2r}{\sqrt{2}} & 0 \end{pmatrix}$

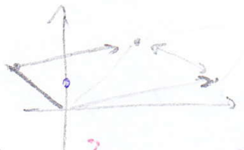
$= \left(\frac{2r}{\sqrt{2}}\left(\frac{8r}{\sqrt{2}}\right) + \frac{2r}{\sqrt{2}}\left(\frac{8r}{\sqrt{2}}\right)\right)$

$\Delta > 0$

$\Downarrow$ ~~min~~ ~~max~~ max

3 $x^2+y^2+z^2=1$

$(1,2,3)$



~~max~~ max. ~~λ~~ van ~~$x^2+y^2+z^2$~~ $(x-1)^2+(y-2)^2+(z-3)^2$

als $x^2+y^2+z^2=1$.

$L : \mathbb{R}^4 \to \mathbb{R} : (x,y,z,\lambda) \longmapsto \cancel{x^2+y^2+z^2}\ (x-1)^2+(y-2)^2+(z-3)^2 - \lambda(x^2+y^2+z^2-1)$

$$\begin{cases} \frac{dL}{dx} = \cancel{2x - \lambda 2x}\ 2(x-1) - 2x\cdot\lambda = 0 & 2x\lambda = 2x & \text{als } x \neq 0 \\ & \lambda = 1 & \\ \frac{dL}{dy} = \cancel{2y - \lambda 2y}\ 2y - 4 - 2y\lambda = 0 & & \\ \frac{dL}{dz} = \cancel{2z - \lambda 2z}\ 2z - 6 - 2z\lambda = 0 & & \\ \frac{dL}{d\lambda} = 1 - x^2 - y^2 - z^2 = 0 & & \end{cases}$$

$$\begin{cases} x(1-\lambda) = 1 & x - 1 = x\lambda \quad 1 - \frac{2}{y} & \lambda = \frac{x-1}{x} = 1 - \frac{1}{x} \text{ als } x \neq 0. & \\ y(1-\lambda) = 2 & \lambda = \frac{y-2}{y} \text{ als } y \neq 0 & \text{dus } \frac{1}{x} = \frac{2}{y} & y = 2x. \\ z(1-\lambda) = 3 & \lambda = \frac{z-3}{z} \text{ als } z \neq 0. & \text{dus } \frac{1}{x} = \frac{3}{z} & z = 3x \\ x^2+y^2+z^2 = 1. & & & \end{cases}$$

dus $x^2 + 4x^2 + 9x^2 = 1$.

$14x^2 = 1$

$x = \pm\sqrt{\frac{1}{14}}$ $\qquad y = \pm 2\sqrt{\frac{1}{14}}$ $\qquad z = \pm 3\sqrt{\frac{1}{14}}$

$\lambda = \pm(1 \mp \sqrt{14})$

$$\Delta\left(\sqrt{\tfrac{1}{14}},\ 2\sqrt{\tfrac{1}{14}},\ 3\sqrt{\tfrac{1}{14}},\ \lambda\right) =$$

$$\det\begin{pmatrix} 2-2\lambda & 0 & 0 & -2x \\ 0 & 2-2\lambda & 0 & -2y \\ 0 & 0 & 2-2\lambda & -2z \\ -2x & -2y & -2z & 0 \end{pmatrix} = (2-2\lambda)\det\begin{pmatrix} 2-2\lambda & 0 & -2y \\ 0 & 2-2\lambda & -2z \\ -2y & -2z & 0 \end{pmatrix} + 2x\det\begin{pmatrix} 0 & 0 & -2x \\ 2-2\lambda & 0 & -2y \\ 0 & 2-2\lambda & -2z \end{pmatrix}$$

$$= (2-2\lambda)^2\left(-4y^2-4z^2\right) + 2x(-2x)(2-2\lambda)^2$$

$$= (2-2\lambda)^2 \cdot -4\left(x^2+y^2+z^2\right)$$

$$\Delta\left(\sqrt{\tfrac{1}{14}},\ 2\sqrt{\tfrac{1}{14}},\ 3\sqrt{\tfrac{1}{14}},\ 1-\sqrt{14}\right) = -224 < 0 \Rightarrow \text{minimum}$$

$$\Delta\left(-\sqrt{\tfrac{1}{14}}, \dots\right) = -224 \rightarrow \text{ook minimum?}$$

④ | ~~max~~ min. $x^2+y^2+z^2$
onder voorw. $\{ \dots$

$$L: \mathbb{R}^5 \to \mathbb{R} : (x,y,z,\lambda,\mu) \longmapsto x^2+y^2+z^2 - \lambda\left(x^2-xy+y^2-z^2-1\right) - \mu\left(x^2+y^2-1\right)$$

$$\begin{cases} \frac{dL}{dx} = 2x - 2\lambda x + \lambda y - 2\mu x = 0 \\ \frac{dL}{dy} = 2y - 2\lambda y + \lambda x - 2\mu y = 0 \\ \frac{dL}{dz} = 2z + 2z\lambda = 0 \end{cases}$$

als $z \neq 0$ $\lambda = -1$

$$y = \frac{2\lambda x + 2\mu x - 2x}{\lambda}$$

$$= 2x\left(\frac{\lambda + \mu - 1}{\lambda}\right)$$

$$\frac{dL}{d\lambda} = 1 - x^2 + xy - y^2 + z^2 = 0$$

$$\frac{dL}{d\mu} = 1 - x^2 - y^2 = 0$$

$$y = \pm\sqrt{1-x^2}$$

$$z = \pm\sqrt{x^2+y^2-xy-1}$$

als $z = 0$

$$\begin{cases} \lambda = 0 \\ 2x = 2\mu x \\ 2y = 2\mu y \\ y = \pm\sqrt{1-x^2} \\ x^2+xy-y^2 = 1 \end{cases}$$

$x \neq 0$ want $x = 0$ ⇒ $y = 1$ tegenstrijdig ~~en visa versa~~

$$\begin{cases} z = 0 \\ \lambda = 0 \\ \mu = 1 \\ y = \pm\sqrt{1-x^2} \\ x^2 + x\sqrt{1-x^2} - (1-x^2) = 1 \\ 2x^2 + x\sqrt{1-x^2} = 2 \end{cases}$$

③ max en min $\sqrt{(x-1)^2+(y-2)^2+(z-3)^2} = (x-1)^2+(y-2)^2+(z-3)^2$.

onder VW $x^2+y^2+z^2=1$.

① nodige VW

$L : \mathbb{R}^4 \to \mathbb{R} : (x,y,z,\lambda) \longmapsto (x-1)^2+(y-2)^2+(z-3)^2-\lambda(x^2+y^2+z^2-1)$.

$$\begin{cases} \frac{dL}{dx} = 2(x-1) - 2\lambda x \\ \frac{dL}{dy} = 2(y-2) - 2\lambda y \\ \frac{dL}{dz} = 2(z-3) - 2\lambda z \end{cases} \qquad \frac{dL}{d\lambda} = 1 - x^2 - y^2 - z^2.$$

$\lambda = \frac{x-1}{x} = \frac{y-2}{y} = \frac{z-3}{z}$. dus $1-\frac{1}{x} = 1-\frac{2}{y} = 1-\frac{3}{z}$.

dus $2x = y$ en $\cancel{3y = 2z}$ en $z = 3x$.

dus $1 - x^2 - 4x^2 - 9x^2 = 0$

dus $x = \pm\sqrt{\frac{1}{14}}$, $y = \pm 2\sqrt{\frac{1}{14}}$, $z = \pm 3\sqrt{\frac{1}{14}}$ $\Big\}\ \lambda = 1 \mp \sqrt{14}$.

② voldoende VW.

$$\Delta(x,y,z,\lambda) = \det\begin{pmatrix} 2-2\lambda & 0 & 0 & -2x \\ 0 & 2-2\lambda & 0 & -2y \\ 0 & 0 & 2-2\lambda & -2z \\ -2x & -2y & -2z & 0 \end{pmatrix}$$

$$= (2-2\lambda)\det\begin{pmatrix} 2-2\lambda & 0 & -2y \\ 0 & 2-2\lambda & -2z \\ -2y & -2z & 0 \end{pmatrix}$$

$$+ 2x \det\begin{pmatrix} 0 & 2-2\lambda & 0 \\ 0 & 0 & 2-2\lambda \\ -2x & -2y & -2z \end{pmatrix}$$

$$= (2-2\lambda)^2 \det\begin{pmatrix} 2-2\lambda & -2z \\ -2z & 0 \end{pmatrix} - 2y(2-2\lambda)\det\begin{pmatrix} 0 & 2-2\lambda \\ -2y & -2z \end{pmatrix}$$

$$+ 2x\,(2-2\lambda)\det\begin{pmatrix} 0 & 2-2\lambda \\ -2x & -2z \end{pmatrix} = -(2-2\lambda)^2 \cdot 4z^2 - 4y^2(2-2\lambda)^2 - 4x^2(2-2\lambda)^2.$$

$\Delta\left(\sqrt{\frac{1}{14}}, 2\sqrt{\frac{1}{14}}, 3\sqrt{\frac{1}{14}}, 1-\sqrt{14}\right) = -224 \longrightarrow$ minimum.

$\Delta\left(-\sqrt{\frac{1}{14}}, -2\sqrt{\frac{1}{14}}, -3\sqrt{\frac{1}{14}}, 1+\sqrt{14}\right) = -224 \longrightarrow$ ook minimum

(4) min. $x^2+y^2+z^2$

onder VW: $g_1(x,y,z) = x^2 - xy + y^2 - z^2 = 1.$

$g_2(x,y,z) = x^2+y^2 = 1.$

$L: \mathbb{R}^{3+2} \to \mathbb{R}: (x,y,z,\lambda,\mu) = x^2+y^2+z^2 - \lambda(g_1(x,y,z)-1)$
$- \mu(g_2(x,y,z)-1).$

$$\begin{cases} \frac{dL}{dx} = 2x - 2\lambda x + \lambda y - 2\mu x. \\ \quad = 2x + \lambda(y-2x) - 2\mu x! \\ \frac{dL}{dy} = 2y - 2\lambda y + \lambda x - 2\mu y \\ \frac{dL}{dz} = 2z + 2\lambda z. \end{cases}$$

$\frac{dL}{d\lambda} = 1 - g_1(x,y,z)$

$\frac{dL}{d\mu} = 1 - g_2(x,y,z)$

$$\begin{cases} z = -\lambda z. \\ x^2 - xy + y^2 = 1. \\ x^2+y^2 = 1. \end{cases}$$ stel $z = 0$

$1 = -xy + 1.$

$-xy = 0.$

dus $x=0 \vee y=0.$

$\Rightarrow$ stel $x = 0.$ $\to$ $y^2 = 1$ dus $y = \pm 1.$

stel $y = 0$ $\to$ $x^2 = 1$ dus $x = \pm 1.$

stel $z \neq 0$

$$\begin{cases} \lambda = -1 \\ -xy + z^2 = 0 \\ 4x - y - 2\mu x = 0 \\ 4y - x - 2\mu y = 0 \\ x^2+y^2 = 1 \end{cases}$$

dus $z^2 = xy$ dus $z = \pm\sqrt{xy}$

$2\mu x = 4x - y$

$2\mu y = 4y - x.$

$y = 4x - 2\mu x. = \sqrt{1-x^2}$

$2\mu \cdot 4x \mp 2\mu \cdot 2\mu x = 16x - 8\mu x - x.$

stel $x \neq 0$

$$\begin{cases} \mu = \frac{4x-y}{2x} \\ \frac{(4x-y)y}{x} = 4y - x \end{cases}$$

$\Rightarrow$ tegenstrijdig!

⑥ $\left| \begin{array}{l} \min \quad y = 4x_1 + 5x_2 + 7x_3 \\ \text{onder NV: } U(x) = 20 \end{array} \right.$

$$\begin{cases} \lambda = \dfrac{4}{0{,}3} \dfrac{x_1^{0,7}}{x_2^{0,5} \cdot x_3^{0,2}} & \lambda = \dfrac{5}{0{,}5} \dfrac{x_2^{0,5}}{x_1^{0,3} x_3^{0,2}} \\ \lambda = \dfrac{7}{0{,}2} \dfrac{x_3^{0,8}}{x_1^{0,3} x_2^{0,5}} & \end{cases}$$

dus $4x_1 = 10{,}5\, x_3$ en $3{,}5\, x_3 = 10 x_2$

dus $20 = \underbrace{\left(\frac{10{,}5}{4}\right)^{0,3}}_{x_1} \cdot x_3^{0,3} \cdot (3{,}5)^{0,5} \cdot x_3^{0,5} \cdot x_3^{0,2}$

.. $x_3 = 8$

$x_1 = \frac{3}{4} x_2$ dus $x_3 = \frac{3}{10{,}5} x_2$

$$20 = \left(\frac{3}{4}\right)^{0,3} x_2^{0,3}\, x_2^{0,5} \left(\frac{3}{10{,}5}\right)^{0,2} x_2^{0,2}$$

$x_2 = 28$ $\qquad x_1 = 21.$

$$\Delta(x;\lambda) = \begin{pmatrix} 0 & D_1 f & D_2 f & D_3 f \\ \dfrac{0{,}3 \cdot x_2^{0,5}\, x_3^{0,2}}{x_1^{0,7}}\ a & 0{,}21 \cdot x_1^{-1,7} & 0 & 0 \\ \dfrac{0{,}5\, x_1^{0,3} x_3^{0,2}}{x_2^{0,5}}\ b & 0 & 0{,}25\, x_2^{-1,5} & 0 \\ \dfrac{0{,}2\, x_1^{0,3}\, x_2^{0,5}}{x_3^{0,8}}\ c & 0 & 0 & 0{,}16\, x_3^{-1,8} \end{pmatrix}$$

$$= b \det \begin{pmatrix} a & b & c \\ 0{,}21\, x_1^{-1,7} & 0 & 0 \\ 0 & 0 & 0{,}16\, x_3^{-1,8} \end{pmatrix} + \begin{vmatrix} 0 & a & c \\ a & 0{,}21\, x_1^{-1,7} & 0 \\ c & 0 & 0{,}16\, x_3^{-1,8} \end{vmatrix} \cdot 0{,}25\, x_2^{-1,5}$$

$$= -b^2\, 0{,}21\, x_1^{-1,7}\, 0{,}16\, x_3^{-1,8} + 0{,}25\, x_2^{-1,5} \left(-c^2\, 0{,}21\, x_1^{-1,7} - a^2\, 0{,}16\, x_3^{-1,8}\right)$$

$< 0.$

$\Rightarrow$ min.

7 b) $R_1 + R_2 + P_1 + P_2 = 48\cancel{000}$; $g(x) = \ldots - 48\cancel{000}$

$L: \mathbb{R}^5 \longrightarrow \mathbb{R} : (R, P, \lambda) \longmapsto W(R;P) - \lambda \cdot g(R;P).$

totale winst

$$= 400\left(\frac{160R_1}{160+R_1} + \frac{320P_1}{80+P_1}\right) + 900\left(\frac{40R_2}{40+R_2} + \frac{120P_2}{30+P_2}\right) - 100\left(R_2 + R_1 + P_1 + P_2\right)$$

max $W(x)$

onder vw $g(x) = 0$.

$$\begin{cases} \dfrac{dL}{dR_1} = \dfrac{400 \cdot 160 \cdot 160}{(160+R_1)^2} - 100 - \lambda & \dfrac{dL}{dR_2} = \dfrac{900 \cdot 40^2}{(40+R_2)^2} - 100 - \lambda \\ \dfrac{dL}{dP_1} = \dfrac{400 \cdot 320 \cdot 80}{(80+P_1)^2} - 100 - \lambda & \dfrac{dL}{dP_2} = \dfrac{900 \cdot 120 \cdot 30}{(30+P_2)^2} - 100 - \lambda \\ \dfrac{dL}{d\lambda} = 48\cancel{000} - R_1 - R_2 - P_1 - P_2. \end{cases}$$

$$\begin{cases} \cancel{3200}(160+R_1) = \cancel{3200}(80+P_1). & P_1 = 80 + R_1. \\ 1200(160+R_1) = 3200(40+R_2). & \cancel{60}^{20} + \frac{3}{8}R_1 - \cancel{40} = R_2. \\ 1800(160+R_1) = 3200(30+P_2) & \cancel{90}^{60} + \frac{9}{16}R_1 - \cancel{30} = P_2. \\ 480\cancel{00} = R_1 + 80 + R_1 + 20 + \frac{3}{8}R_1 + 60 + \frac{9}{16}R_1. \end{cases}$$

$320 = \frac{47}{16}R_1 \qquad R_1 = \frac{5120}{47} \quad \ldots$

...

8 inhoud = b · h · d

= 2x · 2y · 2z

max inhoud

onder VW: $g(x,y,z) = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 = 0.$

$\frac{dL}{dx} = 8yz - \frac{2x\lambda}{a^2}$ ① $\frac{dL}{dz} = 8xy - \frac{2\lambda z}{c^2}$ ③

$\frac{dL}{dy} = 8xz - \frac{2\lambda y}{b^2}$ ② $\frac{dL}{d\lambda} = 1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2}$

$\lambda = \frac{4a^2yz}{x}$ ① $= \frac{4b^2xz}{y}$ ② $= \frac{4xyc^2}{z}$ ③

$x^2 = y^2 \cdot \frac{a^2}{b^2}$ $z^2 = y^2 \cdot \frac{c^2}{b^2}$

$x = y \cdot \frac{a}{b}$ $z = y \cdot \frac{c}{b}$

dus $1 = y^2 \cdot \left(\frac{\cancel{a^2}\,1}{b^2} + \frac{1}{b^2} + \frac{\cancel{c^2}\,1}{b^2} \right)$ $y = \frac{b}{\sqrt{\cancel{a^2}_1 + \cancel{c^2}_1 + 1}}$ $z = \frac{c}{\sqrt{\cancel{a^2}_1 + \cancel{b^2}_1 + 1}}$

$\frac{8abc}{\cancel{a^2+c^2+1}\ 3\sqrt{3}}$ $x = \frac{a}{\sqrt{\cancel{a^2}_1 + \cancel{c^2}_1 + 1}}$

(9) $h: I \subseteq \mathbb{R} \longrightarrow \mathbb{R}$ $\quad 0 \in I$

$h(x) + x e^{h(x)} = 3 \quad \forall x \in I$

$F: I \to \mathbb{R} : x \longmapsto f(x, h(x))$

a) TB: als F een LE in $x^* \in I$

dan $\frac{\delta f}{\delta x}(x^*, h(x^*)) \cdot (x^* + e^{-h(x^*)}) = \frac{\delta f}{\delta y}(x^*, h(x^*))$

$F'(x^*) = D_1 f(x^*, h(x^*)) + D_2 f(x^*, h(x^*)) \cdot h'(x^*) = 0.$

$h'(x) + e^{h(x)} + x e^{h(x)} \cdot h'(x) = 0.$

$h'(x) = \frac{-\cancel{x} e^{h(x)}}{1 + x e^{h(x)}}$

dus $D_2 f(x^*, h(x^*)) = D_1 f(x^*, h(x^*)) \cdot \frac{1 + x^* e^{h(x^*)}}{e^{h(x^*)}}$

b) $y + x e^y = 3.$

$L: \mathbb{R}^3 \longrightarrow \mathbb{R} : (x, y, \lambda) \longmapsto f(x,y) - \lambda(y + x e^y - 3)$

$\frac{dL}{dx} = D_1 f(x,y) - \lambda e^y$ $\qquad \frac{dL}{d\lambda} = 3 - y - x e^y.$

$\frac{dL}{dy} = D_2 f(x,y) - \lambda - \lambda x e^y$

y is door de randvoorwaarde bepaald door x.

oefeningen p. 643 : inleiding integralen → eig^n.

① a) waar : $\int f'(x)dx = f(x)$.

b) waar :

c) niet waar

d) waar.

③ $g(x) = \int_1^{x^2} f(t)dt = F(x^2) - F(1)$ met F als $F' = f$.

$g'(x) = 2x f(x^2) - f(1)\cdot 0 = 2x f(x^2)$

$h(x) = \int_x^{x+1} f(t)dt$

$h'(x) = f(x+1) - f(x)$

* x is bovengrens van A als $\forall a \in A : a \leq x$
* als er een kleinste bovengrens is = supremum van A.

⑥ $W'(t) = D(t) - A(t)$

$W(t) = \int_0^t (D(x) - A(x))dx + W_0$.

oefeningen p. 636 : def v. integralen.

① bewijs dat $f : [0,1] \to \mathbb{R} : x \mapsto 2$ Riemann integreerbaar is.

neem een verdeling $P = \{0; 0{,}5; 1\}$

dan is $\underline{S}(f,P) = 0{,}5 \cdot 2 + 0{,}5 \cdot 2 = 2$.

dan is $\overline{S}(f,P) = 0{,}5 \cdot 2 + 0{,}5 \cdot 2 = 2$.

schrijf $P_n = \{x_0 = 0, x_1, \dots, x_{n-1}, x_n = 1\}$ met $n \in \mathbb{N}_0$

zodat $x_j - x_{j-1} = \frac{1}{n}$. $\forall j \in \{1, \dots, n\}$

dus $\underline{S}(f,P_n) = \sum_{i=1}^{n} \frac{1}{n} \cdot 2 = 2 = \underline{S}(f)$

$= \overline{S}(f,P_n) = \overline{S}(f)$

② ...

③ ik zou denken dat je niet meteen zulke functie kunt vinden.

⑧ ⟹ als f R.I is : $\underline{S}(f)$ = kleinste bovengrens v. $\{\underline{S}(f,P) \mid P \text{ verd. v. } [a,b]\}$

kies $\varepsilon > 0$ dan is $\varepsilon > 0 = \underline{S}(f) - \overline{S}(f)$

⟸

BLH BCA.

⑥ als $f_{\geq 0\ \forall x \in [a,b]}$ R.I zullen ook alle benedensommen ≥ 0 dus ...

oef p. 654: 3.4.4.

oef 1:

(3) $\int \frac{dx}{\sqrt{4x-x^2-3}} = \int \frac{dx}{\sqrt{1-(x-2)^2}}$ subst: $y = x-2$, $dy = dx$.

$= \int \frac{dy}{\sqrt{1-y^2}} = \text{Bgsin}(x-2) + C$ met $C \in \mathbb{R}$.

(4) $\int \frac{x\,dx}{x^4+1}$ subst: $y = x^2$, $\frac{1}{2}dy = x\,dx$

$= \frac{1}{2}\int \frac{dy}{y^2+1} = \frac{\text{Bgtan}\, x^2}{2} + C$ met $C \in \mathbb{R}$.

oef 2.

(3) $\int \ln x\, dx =$ part. int: $u(x) = \ln x \Rightarrow u'(x) = \frac{1}{x}$, $v'(x) = 1 \Rightarrow v(x) = x$

$= x\ln x - \int dx = x\ln x - x + C$ met $C \in \mathbb{R}$.

(4) $\int \sqrt{1-x^2}\,dx$

$u(x) = \sqrt{1-x^2}$ $u'(x) = \frac{x}{\sqrt{1-x^2}}$

$v'(x) = 1$ $v(x) = x$

$= x\sqrt{1-x^2} - \int \frac{+x^2+1-1}{\sqrt{1-x^2}}dx$.

$u(x) = x$ $u'(x) = 1$

$v'(x) = \frac{x}{\sqrt{1-x^2}}$ $v(x) = -\sqrt{1-x^2}$

$\int \frac{x}{\sqrt{1-x^2}}dx$. $-x^2 = y$, $dy = -2x\,dx$

$= -\frac{1}{2}\int \frac{1}{\sqrt{1+y}}dy = -\sqrt{1+y} + C$ met $C \in \mathbb{R}$

$= x\sqrt{1-x^2} + x\sqrt{1-x^2} - \int \sqrt{1-x^2}\,dx + C$

dus $2\int \sqrt{1-x^2}\,dx = 2x\sqrt{1-x^2} + C$

(5) $\int e^x \sin x \, dx$

$u(x) = \sin x \qquad u'(x) = \cos x$

$v'(x) = e^x \qquad v(x) = e^x.$

$= e^x \sin x - \int e^x \cos x \, dx.$

$u(x) = \cos x \qquad u'(x) = -\sin x$

$v'(x) = e^x \qquad v(x) = e^x.$

$= e^x \sin x - e^x \cos x + \int e^x . (-\sin x) \, dx. + C$ met $C \in \mathbb{R}$.

$2 \int e^x \sin x = e^x \sin x - e^x \cos x + C.$

(6) $\int e^x \cos x \, dx. \ \ldots$

oef 3

(3) $\int \frac{dx}{4x^2 - 4x + 7} = \int \frac{dx}{(x-2)^2 + 3} = \frac{1}{3} \int \frac{dx}{\left(\frac{x-2}{\sqrt{3}}\right)^2 + 1}$

$= \frac{1}{3} \text{Bgtan}\left(\frac{x-2}{\sqrt{3}}\right) + C$

(4) $\int \frac{x^2 + x + 1}{x^2 + 1} dx.$

$$\begin{array}{l|l} x^2 + x + 1 & x^2 + 1 \\ \hline -\ x^2 + 1 & 1 \\ \hline x - 1 + 1 & \end{array}$$

$\Rightarrow = \int \frac{(x^2+1)1 + x}{x^2 + 1} dx = x + \int \frac{x}{x^2+1} dx.$

$x^2 = y$

$\frac{dy}{2} = x \, dx.$

$= x + \frac{1}{2} \int \frac{1}{y+1} dy$

$= x + \frac{\ln(x^2+1)}{2} + C$

(5) $\int \frac{3x + 2 + 1 - 1}{x^2 + 2x + 1} dx.$

$= 3 \int \frac{x + 1 - 1}{(x+1)^2} dx + 2 \int \frac{1}{(x+1)^2} dx.$

$= 3 \int \frac{1}{x+1} dx - \int \frac{1}{(x+1)^2} dx$

$= 3 \ln(x+1) + \frac{1}{(x+1)} + C.$

(6) $\displaystyle\int \frac{2x^2+3}{2x^2+1}\,dx = x + 2\int \frac{1}{(\sqrt{2}x)^2+1}\,dx$ $\quad y=\sqrt{2}x$, $dy=\sqrt{2}\,dx$

$$\begin{array}{r|l} 2x^2+3 & 2x^2+1 \\ \hline 2x^2+1 & 1 \\ \hline 2 & \end{array}$$

$= x + \sqrt{2}\displaystyle\int \frac{1}{y^2+1}\,dy$

$= x + \sqrt{2}\,\mathrm{Bgtan}(\sqrt{2}\cdot x) + C.$

(7) $\displaystyle\int \frac{\ln x}{x\ln x - x}\,dx.$ $\quad y=\ln x$, $dy=\frac{1}{x}\,dx.$

$= \displaystyle\int \frac{\ln x}{x(\ln x - 1)}\,dx.$

$= \displaystyle\int \frac{y+1-1}{y-1}\,dy.$

$= \ln x + \ln(\ln x - 1) + C.$

(8) $\displaystyle\int \frac{e^x}{1+e^{2x}}\,dx$

$y=e^x$
$dy=e^x\,dx$

$I = \displaystyle\int \frac{1}{1+y^2}\,dy$

$= \mathrm{Bgtan}(e^x) + C.$

(9) $\displaystyle\int x\,\mathrm{Bgtan}\,x\,dx$

$u(x)=\mathrm{Bgtan}\,x \Rightarrow u'(x)=\frac{1}{1+x^2}$ $\quad$ dus $\quad I = \dfrac{x^2\,\mathrm{Bgtan}\,x}{2} - \dfrac{1}{2}\displaystyle\int \frac{x^2+1-1}{x^2+1}\,dx.$

$v'(x)=x \Rightarrow v(x)=\frac{x^2}{2}$

$I = \dfrac{x^2\,\mathrm{Bgtan}\,x}{2} - \dfrac{1}{2}x + \dfrac{1}{2}\mathrm{Bgtan}\,x + C$

(10) $\displaystyle\int x^3 e^{x^2}\,dx$ $\quad x^2=y$, $x\,dx=\frac{dy}{2}$

$I = \dfrac{\int y e^y\,dy}{2}$ $\quad u(y)=y \Rightarrow u'(y)=1$, $v'(y)=e^y \Rightarrow v(y)=e^y$

$= \dfrac{y\cdot e^y - \int e^y\,dy}{2} = \dfrac{(y-1)e^y}{2} + C$

$= \dfrac{(x^2-1)e^{x^2}}{2}.$

(11) $\displaystyle\int e^{-x}\cos 3x\,dx.$

$u(x)=\cos 3x \Rightarrow u'(x)=-3\sin 3x$

$v'(x)=e^{-x} \Rightarrow v(x)=-e^{-x}.$

$I = -\cos 3x\cdot e^{-x} - 3\displaystyle\int e^{-x}\sin 3x\,dx$

$u(x)=\sin 3x \Rightarrow 3\cos 3x$

$v'(x)=e^{-x} \Rightarrow v(x)=-e^{-x}.$

$I = -\cos 3x\cdot e^{-x} + 3e^{-x}\sin 3x + C$

$-9\displaystyle\int \cos 3x\cdot e^{-x}\,dx.$

$10\,I = \dfrac{3e^{-x}\sin 3x - \cos 3x\cdot e^{-x}}{10}.$

(12) $\int (\ln x)^3 \, dx.$

$u(x) = (\ln x)^3 \Rightarrow u'(x) = 3(\ln x)^2 \cdot \frac{1}{x}$

$v'(x) = 1 \Rightarrow v(x) = x$

$I = x(\ln x)^3 - 3\int (\ln x)^2 dx.$

$u(x) = (\ln x)^2 \Rightarrow u'(x) = 2\ln x \cdot \frac{1}{x}$

$v'(x) = 1 \Rightarrow v(x) = x$

$I = x(\ln x)^3 - 3(\ln x)^2 \cdot x + 6\int \ln x \, dx.$

$u(x) = \ln x \Rightarrow u'(x) = \frac{1}{x}.$

$v'(x) = 1 \rightarrow v(x) = x$

$I = x(\ln x)^3 - 3(\ln x)^2 + 6\ln x \cdot x - 6x.$
$+ C$

(13) $\int \sin(\ln x) \, dx$

$u(x) = \sin(\ln x) \Rightarrow u'(x) = \cos(\ln x) \cdot \frac{1}{x}$

$v'(x) = 1 \Rightarrow v(x) = x$

$I = x \sin(\ln x) - \int \cos(\ln x) \, dx$

$u(x) = \cos(\ln x) \rightarrow u'(x) = -\sin(\ln x) \cdot \frac{1}{x}.$

$v'(x) = 1 \rightarrow v(x) = x$

$I = x\sin(\ln x) - x\cos(\ln x) - \int \sin(\ln x) \, dx.$

$I = \frac{x(\sin(\ln x) - \cos(\ln x))}{2} + C.$

(14) $\int \frac{1}{e^x + e^{-x}} dx. = \int \frac{e^x}{e^{2x}+1} dx$ $\quad y = e^x$ $\quad dy = e^x dx.$

$= \int \frac{1}{y^2+1} dy = \arctan e^x + C.$

(15) $\int \frac{\sin x}{1+\cos x} dx.$ $\quad y = \cos x$ $\quad dy = -\sin dx.$

$I = -\int \frac{1}{1+y} dy = -\ln(\cos x + 1) + C.$

(16) $\int x(1+x)^{3/2} dx.$

$u(x) = x \Rightarrow u'(x) = 1$

$v'(x) = (1+x)^{3/2} \Rightarrow v(x) = \frac{2}{5}(1+x)^{5/2}$

$= \frac{2x}{5}(1+x)^{5/2} - \frac{2}{5}\int (1+x)^{5/2} \cdot dx.$

$= \frac{2x(1+x)^{5/2}}{5} - \frac{4}{35} \cdot (1+x)^{7/2} + C.$

(17) $\int \frac{e^x}{5(e^x-1)} dx. = \frac{1}{5}\int \frac{1}{y-1} dy$

$y = e^x$
$dy = e^x dx.$

$= \frac{1}{5}\ln(e^x - 1) + C.$

(18) $\int \frac{3x}{(x^2-1)^{3/2}} dx$ $\quad y = x^2$ $\quad \frac{dy}{2} = x\,dx.$

$= \frac{3}{2}\int \frac{1}{(y-1)^{3/2}} dx.$

$= \frac{-3}{\sqrt{x^2-1}} dx + C.$

(4)

(1) $\int_1^e \ln(x)\,dx$

$u(x) = \ln(x) \Rightarrow u'(x) = \frac{1}{x}$

$v'(x) = 1 \Rightarrow v(x) = x.$

$I = x\ln(x) - x \Big|_{x=1}^{e} = e \cdot 1 - e - 0 + 1$

(2) $\int_0^4 \frac{dx}{1+\sqrt{x}}$

$\sqrt{x} = y$

$dy = \frac{1}{2\sqrt{x}}dx$

$dx = 2y\,dy$

$I = 2\int \frac{2y+1-1}{1+y}dy = 2y - 2\ln(1+y) + C.$

dus $2\sqrt{x} - 2\ln(1+\sqrt{x})\Big|_{x=0}^{4}$

$= 4 - 2\ln 3$

(3) $\int_0^{\pi/2} \sin^2(x)\cos x\,dx.$

$y = \sin x$ $\quad x=\pi/2 \to y=1$, $\quad x=0 \to y=0$

$dy = \cos x\,dx$

$I = \int_0^1 y^2\,dy = \frac{y^3}{3}\Big|_{y=0}^{1} = \frac{1}{3}$

(4) $\int_0^1 x^3 \cdot e^{(-x^2)}dx. = \frac{1}{2}\int y \cdot e^{-y}dy$

$y = x^2$

$dy = 2x\,dx.$

$u(y) = y \Rightarrow u'(y) = 1$

$v'(y) = e^{-y} \Rightarrow v(y) = -e^{-y}$

$= \frac{1}{2}\left(-ye^{-y}\,1 - e^{-y}\right)\Big|_{y=0}^{1}$

$= -e + \frac{1}{2}$

(5) $\int_0^1 \frac{x}{(4+x^2)^2}dx.$

$y = x^2 \quad dy = 2x\,dx$

$I = \frac{1}{2}\int_0^1 \frac{1}{(4+y)^2}dy$ $= -\frac{1}{2}\cdot\frac{1}{4+x^2}\Big|_{x=0}^{1}$

$= \frac{1}{2}\left(\frac{-1}{4+x^2}\Big|_{x=0}^{1}\right) = \left(\frac{1}{4} - \frac{1}{5}\right)\cdot\frac{1}{2}.$

$= 1/40$

(6) $\int_0^2 \frac{2x+1+1-1}{\sqrt{x+1}}dx.$

$= 2\int_0^2 \sqrt{x+1}\,dx - 1\int_0^2 \frac{1}{\sqrt{x+1}}dx.$

$= \left(2\cdot\frac{2}{3}\cdot(x+1)^{3/2} - 2\cdot\sqrt{x+1}\right)\Big|_{x=0}^{2}$

$= \frac{4}{3}\;3\cdot\sqrt{3} - 2\sqrt{3} - \frac{4}{3} + 2 = 2\sqrt{3} + \frac{2}{3}$

$\int_0^2 (|x| + |1-x|)\,dx$

$= \frac{x^2}{2}\Big|_{x=0}^{2} + \int_0^1 (1-x)dx - \int_1^2 (1-x)dx.$

$= 2 \; \frac{(1-x)^2}{2}\Big|_{x=0}^{1} + \frac{(1-x)^2}{2}\Big|_{x=1}^{2}$

$= 2 + \frac{1}{2} + \frac{1}{2} = 3.$

(7) $\int_0^3 |x^2 - 3x + 2|\,dx$

$S = 3 \quad P = 2.$ $\quad \frac{x^3}{3} - \frac{3x^2}{2} + 2x\Big|_{x=0}^{1}$

$= \int_0^1 (x^2-3x+2)dx - \int_1^2 (x^2-3x+2)dx + \int_2^3 (x^2-3x+2)dx.$

$= 5/6 \quad -1/6 \quad +5/6 = 11/6.$

6) $N'(t) = 6000\,t^2 - 75t^4$

$N(0) = 1000.$

$$N(t) = N(0) + \int_0^t N'(x)\,dx = 1000 + 2000\,t^3 - 15t^5.$$

7) $$\int_0^1 \frac{f(x)}{f(x)+f(1-x)}\,dx = \int_0^1 \frac{f(x)+f(1-x)-f(1-x)}{f(x)+f(1-x)}\,dx$$

$$= \cancel{\int_0^1 \frac{f(1-y)}{f(1-y)+f(y)}\,dy} = \int_0^1 dx - \int_0^1 \frac{f(1-x)}{f(x)+f(1-x)}\,dx.$$

$$2\int_0^1 \frac{f(x)}{f(x)+f(1-x)}\,dx = 1$$

dus $$\int_0^1 \frac{f(x)}{f(x)+f(1-x)} = \frac{1}{2}.$$

8) ~~zeg nu F(x) de functie F die voldoet aan F' = f, dus F'(a) = f(a).~~

$$\cancel{\int_{-a}^{a} f(x) = F(a) - F(-a)}$$

$$\cancel{F(x) = \int f(x)\,dx}$$

$$\cancel{F(-x) = \int f(-x)\,dx} \qquad -x = y, \quad dy = -dx.$$

$$\cancel{= \int -f(y)\,dy}$$

$$\cancel{= -F(-x) + C.}$$

$$\int_{-a}^{a} f = -\int_{-a}^{a} f(x)\,dx \qquad -x = y \quad dx = -dy$$

$$= -\int_{+a}^{-a} f(-y)\,dy$$

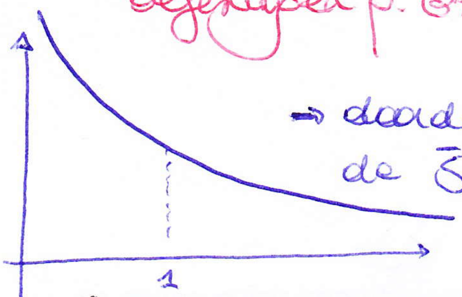
$$= \int_{-a}^{a} f(-y)\,dy$$

$$= -\int_{-a}^{a} f(y)\,dy = -\int_{-a}^{a} f$$

dus $\int_{-a}^{a} f$ moet $= 0.$

oefeningen p. 673. oneigenlijke integralen.

1)



→ doordat we in een eindig # delen verdelen zal de $\bar{S}(f,P)$ altijd $+\infty$ zijn terwijl de $\underline{S}(f,P)$ wél eindig kan zijn.

2) c) $\displaystyle\int_1^{+\infty} \frac{dx}{x^{2/3}} = 3\sqrt[3]{x}\Big|_{x=1}^{+\infty} = \lim_{b\to+\infty} 3\sqrt[3]{b} - 3$

$= +\infty.$

d) $\displaystyle\int_0^1 \frac{dx}{x^{2/3}} = \lim_{\substack{c\to 0\\ >}} \left(3\sqrt[3]{x}\Big|_0^1\right)$ $= 3$

e) $\displaystyle\int_0^5 \frac{dx}{5-x} = \lim_{\substack{c\to 5\\ <}} \left(\ln(5-x)\Big|_{x=0}^{c}\right) = \lim_{\substack{c\to 0\\ >}} \ln(c) - \ln(5)$ $= -\infty.$

f) $\displaystyle\int_{-\infty}^{+\infty} \frac{dx}{1+4x^2} = \lim_{b\to+\infty}\left(\text{Bgtan}\,2x\Big|_{x=0}^{b}\right) + \lim_{c\to-\infty}\left(\text{Bgtan}\,2x\Big|_{c}^{0}\right).$

$= \frac{\pi}{2} - \left(-\frac{\pi}{2}\right)$!

g) $\displaystyle\int_0^{+\infty} xe^{-x}\,dx.$

$= \left(-xe^{-x} + \int e^{-x}dx\right)\Big|_{x=0}^{+\infty} \overset{\lim_{b\to+\infty}}{=} -xe^{-x} - e^{-x}\Big|_{x=0}^{b}$

$= 1$

h) $\displaystyle\int_1^{+\infty} \frac{1}{e^x-1}\,dx$ $\displaystyle\int \frac{1+e^x-e^x}{e^x-1}dx = -\int\frac{e^x-1}{e^x-1}dx + \int\frac{e^x}{e^x-1}dx$ $y = e^x$, $dy = e^x dx$

$= -x + \ln(e^x-1) + C.$

$\displaystyle= \lim_{b\to+\infty}\left(-x + \ln(e^x-1)\Big|_{x=1}^{b}\right) = \lim_{b\to\infty}\left(-b + \ln(e^b-1)\right) + 1 - \ln(e-1).$

$= \lim$ ~~$-b$~~ $\ln(e^b)$

$= 1 - \ln(e-1).$!

(i) $\int_0^1 \frac{1}{e^x - 1}\,dx = 1 - \ln(e-1) - \lim_{b \overset{>}{\to} 0}\left(b - \ln(e^0 - 1)\right)$.
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad + \quad -\infty$

(j) $\int_{-\infty}^{+\infty} x e^{-x^2}\,dx$. $= -\infty$ $\qquad \int x e^{-x^2} dx = \frac{1}{2}\int e^{-y} dy = \frac{-e^{-x^2}}{2} + C$.

$x^2 = y$
$dy = 2x\,dx$

$= \lim_{b \to +\infty}\left(\frac{-1}{2e^{-x^2}}\right)\Big|_{x=0}^{b} + \lim_{c \to -\infty}\left(\frac{-1}{2e^{-x^2}}\Big|_{x=c}^{0}\right)$.

$= 0 + 0 = 0$

(k) $\int_0^1 \ln x\,dx$. $\qquad u(x) = \ln(x) \Rightarrow u'(x) = \frac{1}{x}$
$\qquad v'(x) = 1 \Rightarrow v(x) = x$

dus $\int \ln x\,dx = x\ln x - x$.

$I = \lim_{b \overset{>}{\to} 0}\left(x\ln x - x \Big|_{x=b}^{1}\right) = -1 - \lim_{b \to 0} b\ln b = -1$.

(l) $\int_0^{\pi/2} \frac{\cos x}{\sqrt{1-\sin x}}\,dx$ $\qquad \int \frac{\cos x}{\sqrt{1-\sin x}}\,dx$ $\qquad -\sin x = y$
$\qquad dy = -\cos x\,dx$.

$= \int \frac{-dy}{\sqrt{1+y}} = 2\sqrt{1-\sin x} + C$.

$= \lim_{b \to \pi/2}\left(2\sqrt{1-\sin x}\Big|_{x=0}^{b}\right)$

$= 2$.

(3) $\ln(1-x) \leqslant \ln(1-x^2) \leqslant -\ln(1-x)$.

$1-x \overset{①}{\leqslant} 1-x^2 \overset{②}{\leqslant} (1-x)^{-1}$

① $1 \leqslant \frac{(1-x)(1+x)}{(1-x)} \longrightarrow$ klopt

$\int_0^1 -\ln(1-x)\,dx$.
$\downarrow$
convert.

dus ... ook convert.

② $(1-x)^2(1+x) \leqslant 1$.

$(1 - 2x + x^2)(1+x) - 1 \leqslant 0$

$1 - 2x + x^2 + x - 2x^2 + x^3 - 1 \leqslant 0$

$x^3 - x^2 - x \leqslant 0$.

$x(x^2 - x - 1)$. $D = 1 + 4$ $\qquad x = \frac{1 \pm \sqrt{5}}{2} \to$ $\quad \frac{1-\sqrt{5}}{2} \quad 0 \quad 1 \quad \frac{1+\sqrt{5}}{2}$ $\quad + \quad - \quad +$ $\quad \nearrow$ klopt.

④ a) $\int_1^{+\infty} \frac{2+\sin x}{x^2}\,dx$ $\leq \int_1^{+\infty} \frac{3}{x^2}\,dx$

$+ \cancel{\int_1^{+\infty}\frac{2}{x^2}\,dx + \int_1^{+\infty}\frac{\sin x}{x^2}\,dx} \Rightarrow$ $= 3 \lim_{b\to+\infty} (-1)\cdot\frac{1}{x}\Big|_{x=1}^{b}$

$\geq \int_1^{+\infty}\frac{1}{x^2}\,dx$ $= 3$.

$= \lim_{b} 1 \Rightarrow$ convergent

b)

c) $\int_0^1 \frac{\sqrt{1+x^2}}{\sqrt{x}}\,dx \leq \int_0^1 \frac{\sqrt{2}}{\sqrt{x}}\,dx = \lim_{b\to 0}\left(\sqrt{2}\cdot 2\cdot\sqrt{x}\Big|_{x=0}^{1}\right)$

$\geq \int_0^1 \frac{1}{\sqrt{x}}\,dx = 2.$ $= 2\sqrt{2}.$

d) $\int_{-\infty}^0 \frac{x}{1+e^{-x}}\,dx = \int_{-\infty}^0 \frac{e^x\cdot x}{e^x+1}\,dx \leq \int_{-\infty}^0 \frac{e^x}{e^x+1}\,dx$

e) $\int_0^{\pi/4} \frac{\cos x}{x}\,dx \leq \int_0^{\pi/4}\frac{1}{x}\,dx = \lim_{b\to 0} \ln(x)\Big|_{x=b}^{\pi/4}$

$= \ln\left(\frac{\pi}{4}\right) - (-\infty)$

$\geq \int_0^{\pi/4} \frac{\frac{\sqrt{2}}{2}}{x}\,dx = \frac{\sqrt{2}}{2}\left(\ln\frac{\pi}{4} + \infty\right) = +\infty$

f) $\int_0^{+\infty} \frac{\sin x}{e^x-1}\,dx \leq \int_0^{+\infty}\frac{1}{e^x-1}\,dx$ *

⑤ $\int_0^{+\infty} \frac{\sin^2 x}{x^2} = \int_0^{+\infty}\frac{\sin x}{x}.$

oefeningen p640: toepassingen integralen.

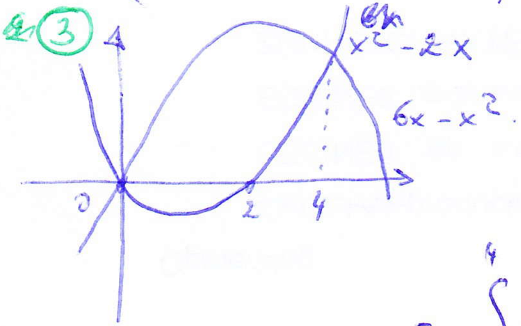
(1) $y = 2x^2$, $y = 2x+4$

$$\begin{cases} x^2 - x - 2 = 0 & S = 1 \quad P = -2 \quad x_1 = 2 \text{ en } x_2 = -1 \\ y_1 = 8 \quad y_2 = 2 \end{cases}$$

$$\text{opp} = \int_{-1}^{2}(2x+4)\,dx - \int_{-1}^{2} 2x^2\,dx$$

$$= \left(-\frac{2}{3}x^3 + x^2 + 4x\right)\Big|_{x=-1}^{2} = \frac{20}{3} + \frac{7}{3} = \frac{27}{3} = 9.$$

(3)



$$\text{opp} = -\int_0^2 (x^2-2x)\,dx + \int_0^4 (6x-x^2)\,dx - \int_2^4 x^2-2x\,dx.$$

$$= \int_0^4 6x - x^2 - x^2 + 2x \; dx = \int_0^4 (-2x^2+8x)\,dx.$$

$$= -\frac{2x^3}{3} + 4x^2 \Big|_{x=0}^{4} = 64/3$$

(4) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$

$$y = \pm\sqrt{1-\frac{x^2}{a^2}}\cdot b = 0$$

$$1 = \frac{x^2}{a^2} \qquad x = -a \text{ en } x = +a.$$

$$\text{dus } 2\int_{-a}^{a}\sqrt{1-\frac{x^2}{a^2}}\; b\,dx = \frac{2b}{a}\int_{-a}^{a}\sqrt{a^2-x^2}\,dx.$$

$$= \frac{2b}{a}\left(\int_{-a}^{a}\frac{a^2}{\sqrt{a^2-x^2}}\,dx - \int_{-a}^{a}\frac{x^2}{\sqrt{a^2-x^2}}\,dx\right) \rightarrow$$

$\sqrt{a^2-x^2} = y \rightarrow x = \sqrt{a^2-y^2}$

$dy = \frac{-x}{2\sqrt{a^2-x^2}}\,dx.$

= ...

(5) $A_m = \int_1^m x^{-4/3}\,dx = \frac{-3}{\sqrt[3]{m}} + 3$

dus $\lim_{m\to\infty} A_m = 3.$

⑥ a) $y=\sqrt{x}$.

volume rond x $= \pi \cdot \int_0^4 \sqrt{x}^2 \, dx = \frac{\pi x^2}{2}\Big|_{x=0}^{4} = 8\pi$

volume rond y $= \pi\int_0^2 (y^2)^2 \, dy = \frac{\pi y^5}{5}\Big|_{y=0}^{2} = \frac{32\pi}{5}$

⑧ $y^2 = 4x/3$ $\quad -2 \le y \le 2$ rond $x_0 = 3$

$x = \frac{3y^2}{4}$ $\quad$ straal $= \left|\frac{3y^2}{4} - 3\right|$

$y^2 = 2$ dus $y = 2$ of $y = -2$.

dus volume $= \pi \int_{-2}^{2} \left(3 - \frac{3}{4}y^2\right)^2 dy$

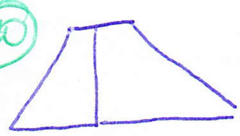
$= \pi \int_{-2}^{2} 9 - \frac{9}{2}y^2 + \frac{9}{16}y^4 \, dy$

$= \pi \left(9y - \frac{3}{2}y^3 + \frac{9}{80}y^5 \Big|_{y=-2}^{2}\right)$

$= \frac{96}{5}\pi$

⑨ volume = opp cirkel · omtrek van band.

$= r^2 \cdot \pi \cdot 2R\pi = 2r^2\pi^2 R$.

⑩  = omwenteling van 

$r_B = \sqrt{B/\pi}$ en $r_G = \sqrt{G/\pi}$

dan is ~~volume~~ ~~opp knop~~ · ~~omtrek~~ hoogte op punt x met $x \in [0, h]$

$= \cancel{r_B + \frac{(r_G - r_B)}{h} x}$

~~dus is het volume~~ $= \cancel{\pi \int_0^1 \left(r_B + (r_G - r_B)x\right)^2 dx}$.

$= \cancel{\pi \int_0^1 r_B^2 + 2r_B(r_G - r_B)x + (r_G - r_B)^2}$

$= r_B + \frac{r_G - r_B}{h} \cdot x$

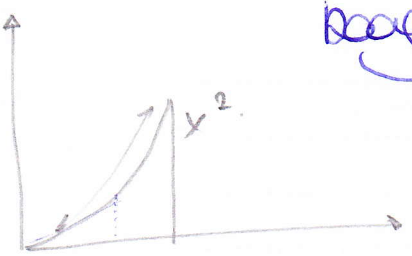
~~dus volum~~ m $\quad R^2 = r_B^2 + \frac{2r_B(r_G - r_B)x}{h} + \frac{x^2}{h^2}\left(r_G^2 - r_B \cdot r_G \cdot 2 + r_B^2\right)$.

$$V = \pi \int_0^h r_B^2 + \frac{2x}{h} \cdot \frac{r_B(r_G - r_B)}{h} + x^2\, \frac{r_G^2 - 2r_B r_G + r_B^2}{h^2}\, dx.$$

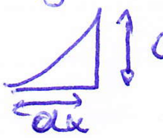
$$= \pi r_B^2 \cdot h + \pi h^{\cancel{2}} \cdot \frac{r_B(r_G - r_B)}{\cancel{h}} + \frac{h^3}{3}\pi\, \frac{r_G^2 - 2r_B \cdot r_G + r_B^2}{h^2}$$

$$= \frac{h}{3}\left(\cancel{3B} + 3\cdot\left(\sqrt{B\cdot G} - \cancel{B}\right) + G - \cancel{2\sqrt{BG}} + B\right).$$

14



booglengte van $y = x^2$ voor $0 \leq x \leq 2$.



$$ds = \sqrt{dx^2 + dy^2}$$

$$= \sqrt{1 + \left(\frac{dy}{dx}(t)\right)^2}\, dt$$

dus is booglengte $= \int_0^2 \sqrt{1 + (2t)^2}\, dt$

$u(x) = \sqrt{1+(2t)^2}$

$u'(x) = \frac{\cancel{8t}^{\,4}}{\cancel{2}\sqrt{1+(2t)^2}}$

$\cancel{= \frac{1}{2} \cdot \frac{2}{3} \cdot (1 + 2t)^{3/2} \Big|_{t=0}^{2}}$

$\cancel{= \frac{2}{3}\sqrt{5}^3 - \frac{1}{3}}$

$v'(x) = 1$

$v(x) = t$

$I = t\sqrt{1+(2t)^2} - 4\int \frac{(2t)^2 + 1 - 1}{\sqrt{1+(2t)^2}}\, dt.$

$\cancel{t =}$

$\cancel{1 + (2t)^2 = u}$

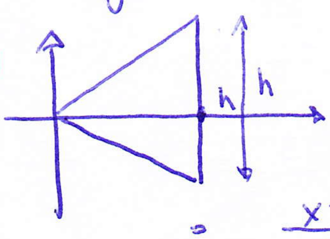
$\cancel{du = 8t\, dt.}$

$= t\sqrt{1+(2t)^2} - I + \int \frac{1}{\sqrt{1+(2t)^2}} = \frac{\ln\left|2t + \sqrt{1+(2t)^2}\right|}{2}$

$I = \frac{t\sqrt{1+(2t)^2}}{2} + \frac{\ln\left|2t + \sqrt{1+4t^2}\right|}{4}$

dus lengte $= \frac{2\sqrt{1+16}}{2} + \frac{\ln\left|4 + \sqrt{17}\right|}{4}$

15



volume $= \int_0^h \frac{x^2\sqrt{3}}{4}\, dx$

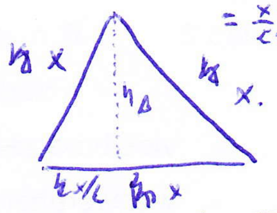
$= \frac{x^3}{12}\sqrt{3}\Big|_{x=0}^{h}$

$= \frac{h^3\sqrt{3}}{12}.$

$h_\Delta^2 + (x/2)^2 = x^2$

dus $h_\Delta = \sqrt{x^2 - \frac{x^2}{4}}$

$= \frac{x}{2}\sqrt{3}.$



dus opp $= \frac{x \cdot \frac{x}{2}\sqrt{3}}{2}$

prisma volume $= \frac{h^2 \cdot \sqrt{3}}{4} \cdot h = \frac{h^3\sqrt{3}}{4}$

(17) $D(q) = 18 - 3q \quad S(q) = 3q + 6.$

$q = 2 \quad p = 12$

$$CS = \int_0^2 D(q)\,dq - p \cdot q = 18q - \frac{3q^2}{2}\Big|_{q=0}^{2} - 24 = 6$$

$$PS = \cancel{\int} pq - \int_0^2 \cancel{?}S(q)\,dq = 24 - \left(\frac{3q^2}{2} + 6q\Big|_{q=0}^{2}\right) = 6.$$

(19) $n = 8 \quad i = 7\%$

$\cancel{K}$ $ar = 1000000$

$$AW = \int_0^8 \cancel{?} R(t)\, e^{-jt}\, dt.$$

$$= 1000000 \cancel{\int_0^8 \frac{1}{e^{0.08t}}dt} \left(\frac{-e^{-\cancel{8}7\% \cdot t}}{7\%}\Big|_{t=0}^{8}\right)$$

$$= 1000000 \cdot \left(\cancel{4r} - \frac{1 - e^{-\cancel{64}56\%}}{7\%}\right)$$

$= \cancel{4727047{,}58}$ €

$= 6125585$ €

(20) $w(t) = 100 + t^2$

$w_2(t) = 220 + 2t.$

$\cancel{w}$ $100 + t^2 - 2t - 220 = \ldots t^2 - 2t - 120 = 0$ $\cancel{D=}$

$\cancel{D = 4 + 880}$ $\quad s = 2 \quad p = -120$

$t_1 = -10 \quad t_2 = 12.$

a) 12 jaar

b) $\int_0^{12} w_1(t)\,dt = 1776 \qquad \int_0^{12} w_2(t)\,dt = 2784 \Rightarrow 1008000$ € meer geld.

c) $\cancel{w}\ W_1(t) = 100t + \frac{t^3}{3} \qquad W_2(t) = 220t + t^2.$

$\cancel{t\left(\frac{t^2}{3} - 3t - 360\right) = 0 \quad s = 3 \quad p = -360}$ $\cancel{?}$

$t\left(t^2 - 3t - 360\right) = 0$

$t = \frac{3 \pm 3\sqrt{161}}{2} \Rightarrow t_1 = 20{,}53$

(21) $\cancel{\int_0^1 R(t)\, e^{-jt}\, dt = 25000.} \quad \cancel{= K\int_T^1 e^{0.036}dt = K\left(\frac{-1}{0.03} e^{0.03t}\Big|_{t=0}^{1}\right)}$ $= 1{,}015.$

K = BW van kapitaal, IS NIET verder gestort - BW van 25000 € die continu is weggehaald.

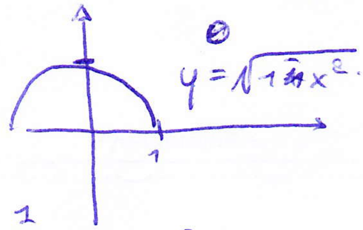
$$K = K \cdot e^{jT} - \int_0^1 25000\, e^{j(T-t)}\,dt$$

$\cancel{K = \frac{25000}{e^{0.03} - 1}}$ $\quad K = \frac{1 \cdot 25000 \cdot e^{j}}{e^{j} - 1}\left(\frac{-1}{je^{jt}}\Big|_{t=0}^{1}\right) = \dfrac{25000 \cdot e^{j}\left(\frac{1}{j} - \frac{1}{e^{j} \cdot j}\right)}{e^{j} - 1}$

$= \frac{25000}{j}$

opdrachten p 704 : meervoudige integratie.

① $\iint_D xy\,dx\,dy$



$$= \int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} y\,dy\,dx = \int_{-1}^{1} \frac{1-x^2}{2}\,\cancel{dx}\;dx$$

$$= \frac{x}{2} - \frac{x^3}{6}\Big|_{x=-1}^{1}$$

$$= \frac{1}{2} - \frac{1}{6} + \frac{1}{2} - \frac{1}{6} = 1 - \frac{1}{3} = 2/3$$

② a. $\iint_D (x+y)\,dx\,dy$

$$(x-1)^2+(y-1)^2=1 \qquad y = \pm\sqrt{1-x^2+2x-1}+1$$

$$x^2-2x+1+y^2-2y+1=1 \qquad = \pm\sqrt{2x-x^2}+1$$

$$x(x-2)+y(y-2)=-1$$

$$\int_0^2\int_{1-\sqrt{2x-x^2}}^{1+\sqrt{2x-x^2}}(x+y)\,dy\,dx = \int_0^2 \left(xy+\frac{y^2}{2}\Big|_{-\sqrt{2x-x^2}+1}^{\sqrt{2x-x^2}+1}\right)dx$$

$$= \int_0^2 x\left(\sqrt{2x-x^2}+1\right)+\frac{\left(\sqrt{2x-x^2}+1\right)^2}{2} - \left(x\left(-\sqrt{2x-x^2}+1\right)+\frac{\left(-\sqrt{2x-x^2}+1\right)^2}{2}\right)dx$$

$$= \int_0^2 2x\sqrt{2x-x^2} + \frac{\cancel{2x-x^2}+2\sqrt{2x-x^2}\cancel{+1}}{\cancel{2}} - \frac{\cancel{2x-x^2}-2\sqrt{2x-x^2}\cancel{+1}}{\cancel{2}}\,dx$$

$$= \int_0^2 2\sqrt{2x-x^2}\,(x+1)\,dx$$

enz.

$$\int\sqrt{2x-x^2+1-1}\,dx$$

$$\int\sqrt{1-(x-1)^2}\,dx$$

③ $\int_{-1}^{1}\int_{-1}^{1}(2-x^2-y^2)\,dx\,dy = \ldots$

opdrachten p. 713 ⊘: inl. dif vgl$^{\bar{n}}$.

klanten v. conc. komen erbij bed.

① $$\begin{cases} m(0) = m_0 \\ m'(t) = \dfrac{(1-m(t))\cdot p - m(t)\,q}{100} \end{cases}$$ eigen kl. lopen weg

$$= \frac{p - m(t)\cdot p - m(t)\,q}{100}$$

$$m'(t) = 0 = p - m(t)\,p - m(t)\,q = 0$$

$$p = m(t)(p+q)$$

$$m(t) = \frac{p}{p+q}.$$

③ DUH.

④ DUH.

② $$\begin{cases} V(0) = 0 \\ V'(t) = \lambda(340 - 4V(t)) \end{cases}$$

1ste graad dif vgl: oef p. 738.

1 b) $(x-2)y' = y + 2(x-2)^3$

$y' - \frac{y}{x-2} = 2(x-2)^2$

$\mu(x) = e^{-\ln(x-2)} + C = \frac{1}{x-2}$

$\frac{y'}{x-2} - \frac{y}{(x-2)^2} = 2(x-2)$

$\left(y \cdot \frac{1}{x-2}\right)' = 2x-4$

$y = (x-2)\cdot\left((x^2-4x) + C\right)$

c) $y' + 2xy = -xy^4$

$y' = -x(2y + y^4)$

$\int \frac{dy}{2y+y^4} = \int -x\,dx$

$z = y^{-3} \quad z' = \frac{-3}{y^4}\cdot y'$

$\frac{-3}{y^4}y' + \frac{-6x}{y^3} = 3x$

$z' - 6x\cdot z = 3x$

$z' = 3x(1+2z)$

$\int \frac{1}{1+2z}dz = \int 3x\,dx$

$\frac{\ln(1+2z)}{2} = \frac{3x^2}{2} + C$

$1+2z = e^{3x^2}\cdot e^c$

$z = \frac{e^{3x^2}\cdot e^c - 1}{2}$

$y = \sqrt[3]{\frac{2}{D\cdot e^{3x^2}-1}}$ ✓

d) $x\ln x - xy' + 3y = 0$

$y' - \frac{3y}{x} = \ln x$

$\mu(x) = e^{-3\ln x} = x^{-3}$

dus vgl: $y'/x^3 - 3y/x^4 = \frac{\ln x}{x^3}$

$\left(\frac{y}{x^3}\right)' = \frac{\ln x}{x^3}$

dus $\frac{y}{x^3} = \int \frac{\ln x}{x^3}dx$

$u(x) = \ln x \Rightarrow u'(x) = 1/x$

$v'(x) = \frac{1}{x^3} \Rightarrow v(x) = \frac{-1}{2x^2}$

$= \frac{-\ln x}{2x^2} + \frac{1}{2}\int\frac{1}{x^3}dx = \frac{-\ln x}{2x^2} - \frac{1}{4x^2} + C$

dus $y = x^3\left(C - \frac{\ln x}{2x^2} - \frac{1}{4x^2}\right)$

e) $y' = \frac{1}{x} \cdot \frac{4y}{y-3}$

$\frac{1}{4}\int \frac{y-3}{y}\,dy = \int \frac{1}{x}\,dx$

$\frac{1}{4}y - \cancel{\frac{1}{4}}\frac{3}{4}\ln y = \ln x + C$

f) $(x+1)y' - (x+1)^4 - 2y = 0$

$\mu(x) = \exp\int \frac{-2}{x+1}\,dx$

$= e^{-2\ln(x+1)}$

$= \frac{1}{(x+1)^2}$

vgl: $\frac{y'}{(x+1)^2} - \frac{2y}{(x+1)^3} = x+1$

$\frac{y}{(x+1)^2} = \int (x+1)\,dx$

$y = (x+1)^2 \cdot \left(\frac{x^2}{2} + x + C\right)$

g) $y' = \frac{x}{y} + xy$

$= x\left(\frac{1}{y} + y\right)$

$y'\left(\frac{y}{1+y^2}\right) = x$

$\int \frac{y}{1+y^2}\,dy = \int x\,dx$

$u = y^2$

$du = \cancel{2y}\ 2y\,dy$

$\frac{1}{2}\ln(1+y^2) = \frac{x^2}{2} + C$

$y^2 = e^{x^2+C}$ dus $y = \sqrt{e^{x^2} + C}$

h) $y' + y = (\cos x - \sin x)y^2$

$z = \frac{1}{y}$ $z' = \frac{-1}{y^2}\cdot y'$

$\frac{-y'}{y^2} - \frac{1}{y} = \sin x - \cos x$

$z' - z = \sin x - \cos x$

$\mu(x) = \exp\int -\,dx$

$= e^{-x}$

$z \cdot e^{-x} = \int \sin x - \cos x\,dx$

$= \cancel{\cos x} \quad \frac{-\sin x - \cos x + C}{e^{-x}}$

$y = \frac{e^{-x}}{C - \sin x - \cos x}$

i) $x^2(y+1) + y^2(x-1)y' = 0$

$y' = \frac{x^2(y+1)}{(1-x)y^2}$ dus $\int \frac{y^2}{y+1}\,dy = \int \frac{x^2}{1-x}\,dx$

dus $\int (y-1)\,dy + \int \frac{1}{y+1}\,dy = -\int (x+1)\,dx - \int \frac{1}{x-1}\,dx$

$\frac{y^2}{2} - y + \ln(y+1) = \frac{-x^2}{2} - x - \ln|x-1| + C$

j) $-3(x^2+1)y' + 2xy = (x^2+1)e^x y^4$. $z = y^{-3}$

$y' - \frac{2x}{3(x^2+1)} y = \frac{-e^x}{+3} y^4$ $z' = \frac{-3}{y^4}\cdot y'$

$\frac{-3y'}{y^4} + \frac{2x}{x^2+1}\cdot\frac{1}{y^3} = e^x$. $z' + z\cdot\frac{2x}{x^2+1} = e^x$.

$\mu(x) = \exp\int\frac{2x}{x^2+1}dx$. $x^2 = u$, $du = 2x\,dx$.

dus $z'(x^2+1) + z\cdot 2x(x^2+1) = e^x\cdot(x^2+1)$ $= x^2+1$.

$\left(z\cdot(x^2+1)\right)' = e^x(x^2+1)$.

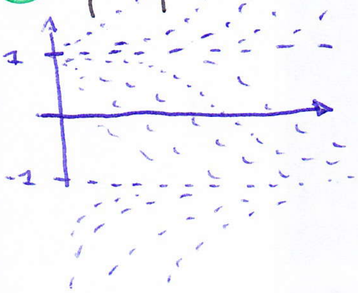
$z = \frac{\int e^x(x^2+1)\,dx}{x^2+1}$. $u(x) = x^2+1 \Rightarrow u'(x) = 2x$; $v'(x) = e^x \Rightarrow v(x) = e^x$.

$= e^x - \frac{2\int e^x\cdot x\,dx}{x^2+1}$ $u(x) = x \Rightarrow u'(x) = 1$; $v'(x) = e^x \Rightarrow v(x) = e^x$.

$= e^x - \frac{2xe^x}{x^2+1} + \frac{2e^x}{x^2+1} + c$.

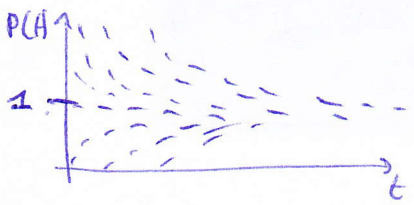
$y = \left(e^x + \frac{-2xe^x + 2e^x}{x^2+1} + c\right)^{-1/3}$

(2) $y' = y^2 - 1$



$\Rightarrow +\infty$ als $y_0 > 1$, 1 als $y_0 = 1$, -1 als $y_0 < 1$.

(8) a) $\begin{cases} P(0) = P_0 \\ P'(t) = \lambda\left(\frac{3}{1+2P(t)} + 1 - 2P(t)\right) \end{cases}$ met $\lambda \in \mathbb{R}^+$

b) 

$\Rightarrow$ hangt niet af v. P_0.

$\cancel{6}$ 6 $V(1) = v_1$

a) $\eta = \cancel{\ldots} = \frac{p}{V} \cdot V'(p) = -\lambda \cdot p$

dus $\begin{cases} V(1) = v_1 \\ V'(p) = -\lambda \cdot V(p) \end{cases}$ $\qquad \mu = \exp \int \lambda \, dp = e^{\lambda p}$

$y' \cdot e^{\lambda p} + \lambda e^{\lambda p} \cdot y = 0$

$\left(e^{\lambda p} \cdot V(p)\right)' = 0$

$V(p) = \frac{C}{e^{\lambda p}}$ $\qquad V(1) = \frac{C}{e^{\lambda}} = v_1$ $\qquad$ dus $V(p) = v_1 e^{\lambda(1-p)}$.

b) $\eta = \frac{p}{V} \cdot V'(p) = k$ $\qquad$ dus $C = v_1 \cdot e^{\lambda}$ met $k \in \mathbb{R}^-$ $\qquad \Rightarrow$ als $k \in \mathbb{R}^-$ dan is $V(p) = v_1 e^{k(p-1)}$.

dus $\begin{cases} V'(p) = \frac{k \cdot V(p)}{p} \\ V(1) = v_1 \end{cases}$ $\qquad \mu(p) = \exp \int \frac{-k}{p} \, dp$

$= \cancel{\exp} e^{-k \ln p}$

$= p^{-k}$

dus $\left(V(p) \cdot p^{-k}\right)' = 0$

$V(p) = \cancel{\frac{k}{p}} \, C \cdot p^{k}$. $\qquad$ nu is $V(1) = v_1 = C$

dus $V(p) = v_1 \cdot p^{k}$.

7 a) $\begin{cases} n(0) = n_0 \\ n'(t) = \lambda \cdot n(t)\,(1 - n(t)) = \lambda n(t) - \lambda n^2(t) \end{cases}$

$\Rightarrow n(t) = \frac{e^{\lambda t}}{e^{\lambda t} + C}$

b) $n_0 = 0{,}05$ $\qquad n(10) = 0{,}10$

dus $n(0) = \frac{1}{1 + C} = \frac{1}{20}$ $\qquad$ dus $20 - 1 = C$

$C = 19$.

$\cancel{0{,}10}\ \frac{1}{10} = \frac{e^{\lambda 10}}{e^{10\lambda} + 19}$ $\qquad 10 e^{10\lambda} = e^{10\lambda} + 19$.

$e^{10\lambda} = \frac{19}{9}$

$\lambda = \frac{\ln(19/9)}{10}$ $\qquad$ } JUIST !

c) $n(t^*) = \frac{1}{2} = \frac{e^{\lambda t}}{e^{t\lambda} + 19}$.

$e^{t\lambda} = 19$ $\qquad$ ~~ln~~ $t = \frac{\ln(19)}{\lambda} = 39{,}4$.

(8) $\begin{cases} m(0) = m_0 \\ m'(t) = \dfrac{p - m(t)(p+q)}{100} \end{cases}$

$$m'(t) + m(t)\frac{(p+q)}{100} = \frac{p}{100}$$

$$\mu(t) = \exp\int \frac{p+q}{100}\,dt = e^{\frac{p+q}{100}t}$$

dus $\left(m(t)\cdot e^{\frac{p+q}{100}t}\right)' = \int \dfrac{p \cdot e^{\frac{p+q}{100}t}}{100}$

$$m(t) = \frac{\int \frac{p \cdot e^{\frac{p+q}{100}t}}{100}\,dt}{e^{\frac{p+q}{100}t}} = \frac{p\,100}{100(p+q)} + \frac{C}{e^{\frac{p+q}{100}t}}$$

$m(0) = m_0 = \dfrac{p}{\cancel{100}\,p+q} + C$ dus $C = m_0 - \dfrac{p}{p+q}$

(9) $\begin{cases} v(0) = 0 \\ v'(t) = \lambda(340 - v(t)) \end{cases}$

$y' + \lambda y = 340\lambda$ $\quad \mu(t) = e^{\lambda t}$

$(y \cdot e^{\lambda t})' = 340\lambda \cdot e^{\lambda t}$

$y = 340 + \dfrac{C}{e^{\lambda t}}$ $\quad y(0) = 0 = 340 + \dfrac{C}{\cancel{1}}$

dus $C = -340$.

dus $v(t) = 340 - \dfrac{340}{e^{\lambda t}}$

$v(3{,}8) = 100 = 340 - \dfrac{340}{e^{\lambda \cdot 3{,}8}}$ $\quad 340 = 240\, e^{\lambda \cdot 3{,}8} = 17/12$.

$\lambda = \dfrac{\ln(340/240)}{3{,}8} = 0{,}0917$.

$v(t^*_{200}) = 200 =$

10 $\begin{cases} o(0) = 0 \\ o'(t) = \lambda \cdot (750 - o(t)) \end{cases}$

$$o(t) = 750 - \frac{750}{e^{\lambda t}}$$

$o(5) = 150$ dan is $\lambda = \dfrac{\ln\left(\frac{750}{750-150}\right)}{5} = 0{,}0446$

$o(15) = \cancel{365}\ 366$ g.

11 $\begin{cases} n'(t) = k n(t) (N - n(t)) - a \\ n(0) = n_0 \end{cases}$ $N = 100$, $k = 1/1000$, $\alpha = 1{,}6$.

$$y' = kyN - ky^2 - a.$$

$$\int \frac{1}{kyN - ky^2 - a/k} dy = \int k dx. = x \cdot 1000$$

$$I = \cancel{\int \frac{1}{y^2 - yN + \frac{a}{k}} dy} = -\int \frac{1}{y^2 - 100y + 1600}$$

nu is $\dfrac{1}{(y-80)(y-20)} = \dfrac{A}{(y-80)} + \dfrac{B}{(y-20)} = \dfrac{1}{60(y-80)} - \dfrac{1}{60(y-20)}.$

$$\begin{cases} A + B = 0 & \text{dus } A = -B \\ -20A - 80B = 1 & -20A + 80A = 1 \\ & A = 1/60 \end{cases}$$

$$I = \frac{1}{60}\left(\ln(y-20) - \ln(y-80)\right) = \frac{1}{60} \ln\left(\frac{y-20}{y-80}\right)$$

dus $\dfrac{y-20}{y-80} = e^{60\,000x}$

$$y - 20 = (y-80) e^{60\,000x + C}.$$

$$y(1 - e^{60000x} e^{C}) = 20 - 80 e^{60000x} D.$$

$$y = \frac{20 - 80 e^{60\,000x} D}{1 - e^{60\,000x} D}.$$

...

12

$$\begin{cases} m(0) = 100 \\ m'(t) = 3 - \dfrac{m(t)}{V(t)} \cdot 2 \end{cases}$$

$$y' + \frac{2}{200+t}\, y = 3 \qquad \mu(t) = e^{\ln(200+t)^2} = (200+t)^2$$

vgl: $(200+t)^2 y' + 2(200+t)y = 3(200+t)^2$

$$\left((200+t)^2 \cdot y\right)' = 3(200+t)^2$$

$$y = \frac{\frac{3}{3}(200+t)^3 + C}{(200+t)^2} = 200 + t + \frac{C}{(200+t)^2}$$

$$m(0) = 100 = 200 + \frac{C}{200^2}$$

$$-100 \cdot 200^2 = C$$

dus $m(t) = 200 + t - \dfrac{100 \cdot 200^2}{(200+t)^2}$

$$m(300) = 484$$

13

$$\begin{cases} m(0) = 6300 \\ m'(t) = 45 - \dfrac{m(t)}{V(t)} \cdot 1 \end{cases} \qquad V(t) = 900 + 2t = \cancel{6300}\ 3600$$

$$t = \cancel{2700}\ 1350$$

$$y' + \frac{1}{900+2t} \cdot y = 45 \qquad \mu(t) = \cancel{e^{900+}}\ e^{\frac{1}{2}\ln(900+2t)} = (900+2t)^{1/2}$$

$$(900+2t)^{1/2} \cdot y' + \frac{y}{(900+2t)^{1/2}} = 45 \cdot (900+2t)^{1/2}$$

$$\left((900+2t)^{1/2} \cdot y\right)' = \int 45 \cdot (900+2t)^{1/2}\, dt$$

$$y = \frac{\frac{45}{2} \cdot \frac{2}{3}(900+2t)^{3/2} + C}{(900+2t)^{1/2}} = 15(900+2t) + \frac{C}{(900+2t)^{1/2}}$$

$$= 13500 + 30t + \frac{C}{(900+2t)^{1/2}}$$

$$y(0) = 6300 = 13500 + \frac{C}{30}$$

$$C = \cancel{240}\ 216000$$

$$y(\cancel{2700}\ 1350) = 13500 + 30 \cdot \cancel{2700}\ 1350 - \frac{\cancel{240}\ 216000}{(900 + 2 \cdot \cancel{2700}\ 1350)^{1/2}} = 50400$$

$$\Rightarrow 14\ g/l$$

(15) $\begin{cases} \ell(0) = \ell_0 \\ \ell(t) = \end{cases}$ $\qquad V'(t) = \lambda \cdot Opp = \lambda \cdot 6 \cdot z^2.$

$V(t) = z^3 \quad V'(t) = 3z^2(t) \cdot z'(t).$

dus $\quad z'(t) \cdot 3z^2(t) = 6\lambda \cdot z^2(t).$

$$\boxed{z'(t) = 2\lambda}$$

$$\longrightarrow z(t) = 2\lambda t + C$$

b) $z(0) = 0{,}5 = C$

$z(4) = 1 = 8\lambda + 0{,}5 \quad \Leftrightarrow \lambda = \frac{1}{16}$

$z(t^*) = 2 = \frac{2t^*}{16} + 0{,}5 \quad \Leftrightarrow t^* = 8 \cdot 1{,}5 = 12$

$\Rightarrow 8$ min

(16) $C_0 = 20 \cdot C_{max}$ $\quad$ a) $\begin{cases} c(0) = C_0 = \frac{m_0}{V} \\ c'(t) = \frac{C_{in} \cdot d_0}{V} - \frac{c(t) d_0}{V} \end{cases}$

$C_{in} = \frac{1}{20} C_{max}.$

b) $y' + \frac{d_0}{V} y = \frac{C_{in} d_0}{V}$

$\mu(t) = e^{\frac{d_0}{V} t}$

$\left(y \cdot e^{d_0/V \cdot t}\right)' = \frac{C_{in} \cdot d_0}{V} \cdot e^{d_0/V \cdot t}$

$$y = \frac{C_{in} \cdot e^{\frac{d_0}{V} \cdot t} + C}{e^{d_0/V \cdot t}} = C_{in} + \frac{C}{e^{d_0/V \cdot t}}$$

$c(0) = C_0 = C_{in} + C. \quad$ dus $\quad C = C_0 - C_{in}$

c) $c(t^*) = C_{max} = C_{in} + \frac{C_0 - C_{in}}{e^{d_0/V \cdot t}} \qquad = \frac{399}{20} C_{max}.$

$$e^{d_0/V \cdot t} = \frac{C_0 - C_{in}}{C_{max} - C_{in}}$$

$$t = \frac{V}{d_0} \cdot \ln\left|\frac{C_0 - C_{in}}{C_{max} - C_{in}}\right| = \frac{V}{d_0} \ln(21)$$

$C_{max} - C_{in} = \frac{19}{20} C_{max}$

vervolg 16

d) $t^* = \frac{V}{d_0+x}\cdot\gamma$ e) $K(x) = K_0\cdot x + K_1\cdot x + \frac{V\gamma}{d_0+x} + \frac{S\cdot V}{d_0+x}\cdot\gamma$.

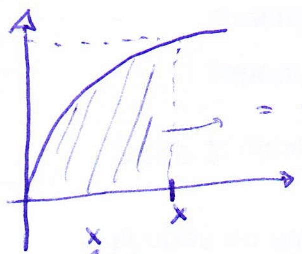
f) $K(x) - K(0) =$

$$\frac{d_0(d_0+x)K_0 x + K_1\cdot x\cdot d_0\cdot V\gamma + S\cdot V\cdot\gamma\cdot d_0 - S\cdot V\cdot\gamma(d_0+x)}{d_0(d_0+x)}$$

$$= \frac{\cancel{x}\left(x(d_0K_0) + d_0^2K_0 + K_1 d_0\cdot V\cdot\gamma - S\cdot V\cdot\gamma\right)}{\cancel{d_0(d_0+x)}} < 0$$

$$\frac{xK_0}{d_0} + K_0 + \frac{K_1\cdot V\cdot\gamma}{d_0} < \frac{V\cdot\gamma}{d_0^2}\cdot S$$

17



$= \frac{2}{3}$ van rechthoek.

dus $\int_0^x f(x)\,dx = \frac{2}{3}\cdot f(x)\cdot x$. schrijf $F' = f$

$F(x) - F(0) = \frac{2}{3} f(x)\cdot x$.

$f(x) - f(0) = \frac{2}{3}\left(f(x) + x\cdot f'(x)\right)$

$\frac{1}{3} f(x) = \frac{2}{3} x f'(x) + f(0)$.

$f(x) = 2\cdot x f'(x) + 3f(0)$.

$y' - \frac{y}{2x} = \frac{-3}{2x} f(0)$ $\mu(x) = e^{-\frac{1}{2}\ln 2x} = \frac{1}{\sqrt{2x}}$.

$\frac{y'}{\sqrt{2x}} - \frac{y}{(2x)^{3/2}} = \frac{-3}{(2x)^{3/2}} f(0)$.

$\int\left(\frac{y}{\sqrt{2x}}\right)' = -3f(0)\int\frac{1}{\sqrt{2x}^3}\,dx$.

$y = -3f(0)\cdot\cancel{\sqrt{2x}}\cdot\frac{-2}{1}\cdot\frac{1}{2}\cdot\frac{1}{\cancel{\sqrt{2x}}} + \sqrt{2x}\cdot C$.

$= f(0)\cdot 3 + \sqrt{2x}\cdot C$.

opdrachten 4.1.4: oefeningen

oefening 1.

$$\begin{cases} m(0) = m_0 \\ m'(t) = \frac{p}{100}\left(1 - m(t)\right) - \frac{q}{100} m(t) \end{cases}$$

$$= \frac{p}{100} - m(t)\left(\frac{p+q}{100}\right)$$

$$\int \frac{dy}{y} = \int \frac{p-q}{100} dt$$

$$\ln|y| = \frac{p-q}{100} t$$

$$y = e^{\frac{p-q}{100}t + C} = m(t)$$

$$m(0) = e^{c} = m_0$$

$$c = \ln(m_0)$$

oefening 2.

$$\begin{cases} v(0) = 0 \\ v'(t) = \alpha \cdot (340 - v(t)) \quad \text{met } \alpha \in \mathbb{R}^+_0 \end{cases}$$

oefening 3.

a) $y' + 2xy^2 = 0$ en $y = \frac{1}{1+x^2}$ dan is $y' = \frac{-1}{(1+x^2)^2} \cdot 2x$.

$$\frac{-2x}{(1+x^2)^2} + 2xy^2 = 0 \rightarrow \text{ok.}$$

b) $y'' + 2y' + y = 1$ en $y = 1 + 2xe^{-x}$. dan is $y' = 2e^{-x} - 2xe^{-x}$

$$y'' = -2e^{-x} - 2e^{-x} + 2xe^{-x} = -4e^{-x} + 2xe^{-x}$$

$$-4e^{-x} + 2xe^{-x} + 4e^{-x} - 4xe^{-x} + 1 + 2xe^{-x} = 1 \longrightarrow \text{ok.}$$

oef 4

$\lambda = -1$ en $\lambda = 2$.

opdrachten 4.2.7: oefeningen p.738.

oef 1

a) $y' + 2xy = 4x$. $\quad y' + p(x) \cdot y = q(x)$

$$\mu(x) = \exp \int 2x \, dx$$

$$= e^{x^2} \quad \text{dus } e^{x^2} y' + e^{x^2} \cdot 2xy = e^{x^2} \cdot 4x.$$

$$\underbrace{e^{x^2} \cdot y}_{=(e^{x^2} \cdot y)'} = \int e^{x^2} \cdot 4x \, dx. \longrightarrow \begin{array}{l} t = x^2 \\ dt = 2x \, dx. \end{array}$$

$$y = \frac{2e^{x^2} + C}{e^{x^2}} = 2 + \frac{C}{e^{x^2}}$$

b) $(x-2)\,y' = y + 2(x-2)^3$

$\forall x \neq 2$:

$$y' - \frac{y}{x-2} = 2(x-2)^2$$

$$\mu(x) = \exp \int \frac{-1}{x-2}\,dx = e^{\ln(x-2)} = -(x-2)$$

$\Rightarrow$ niet nodig, wel nodig ...

$$y + (-x+2)\,y' = -2(x-2)^3$$

$$\left(y \cdot (2-x)\right)' = -2(x-2)^3$$

$$y = \frac{\frac{-1}{2}(x-2)^4}{2-x} = \frac{(x-2)^4}{2x-4} = \frac{(x-2)^3}{2}$$

TEST: $y' = \cancel{\frac{2(x-2)}{(x-2)}} = \frac{3}{2}(x-2)^2$

dan is $\frac{(x-2)^3}{2} \cdot \frac{3}{2} =$ fout

* $(x-2)\,y' - y = 2(x-2)^3$

$$\mu(x) = \exp \int \frac{-1}{x-2}\,dx = e^{-\ln(x-2)} = \frac{1}{x-2}$$

$$\frac{y'}{x-2} - \frac{y}{(x-2)^2} = 2(x-2)$$

$$\left(y \cdot \frac{1}{x-2}\right)' = 2(x-2)$$

$$\frac{y}{x-2} = \frac{2x^2}{2} - 4x + c$$

$$y = (x-2)(x^2 - 4x + c) \quad \text{met } c \in \mathbb{R}$$

opdrachten p 764: dif. vgl v/d 2de orde

1 a) $y'' + y = e^{3x}$

y_h: $\lambda^2 + 1 = 0$, $\lambda = \pm i$ $\Big\}$ $y_h = \cancel{e^{0x}}\left(\cos(1x)\cdot A + \sin(1x)\cdot B\right)$

$= A\cos x + B\sin x$.

y_p: $\lambda e^{3x} = y_p$.

$9\lambda e^{3x} + \lambda e^{3x} = e^{3x}$

$\lambda = \frac{1}{10}$ $\Big\}$ $y = A\cos x + B\sin x + \frac{e^{3x}}{10}$

b) $y'' - 5y' + 6y = 6x - 1$

y_h: $\lambda^2 - 5\lambda + 6 = 0$, $s = 5$, $p = 6$ $\Big\}$ $y_h = e^{2x}\cdot A + Be^{3x}$

y_p: $\alpha x + \beta = y_p$

$0 - 5\alpha + 6\alpha x + 6\beta = 6x - 1$

dus $\alpha = 1$

$-5 + 6\beta = -1$ $\beta = \frac{2}{3}$ $\Big\}$ $y = x + \frac{2}{3} + y_h$.

c) $y'' + 9y = 5\cos x$

$\lambda^2 + 9 = 0$ dus $\lambda = \pm 3i$ $\Big\}$ $y_h = A\cos 3x + B\sin 3x$

y_p: $\alpha\cos x + \beta\sin x = y_p$.

$\alpha(-\cos x) - \beta\sin x + 9\alpha\cos x + 9\beta\sin x = 5\cos x$.

$\beta = 0$ $\alpha = \frac{5}{8}$ $\Rightarrow y_p = \frac{5}{8}\cos x$.

d) ...

e) $y'' - 6y' + 9y = e^{3x}$

$\lambda^2 - 6\lambda + 9 = 0$, $s = 6$, $p = 9$, $\lambda_1 = \lambda_2 = 3$ $\Big\}$ $y = e^{3x}(A + Bx)$

y_p: $\alpha e^{3x} = y_p$

$9\alpha e^{3x} - 18\alpha e^{3x} + 9\alpha e^{3x} = e^{3x}$ $\longrightarrow$ FOUT

$\cdot x$: $\alpha x e^{3x} = y_p$

$\cancel{9\alpha e^{3x}}$ ...

$\left(\alpha e^{3x} + \alpha x\cdot 3e^{3x}\right)' - 6\left(\alpha e^{3x} + \alpha x\cdot 3e^{3x}\right) + 9\alpha x e^{3x} = e^{3x}$.

$3\alpha e^{3x} + 3\alpha e^{3x} + \cancel{9\alpha\cdot x\cdot e^{3x}}$
$- 6\alpha e^{3x} - \cancel{6\alpha\cdot x\, 3e^{3x}}$
$+ \cancel{9\alpha x e^{3x}} = e^{3x}$

$\Rightarrow$ nog fout

want multiplicteit 2

e) $y'' - 6y' + 9y = e^{3x}$

$y_p = \alpha x^2 e^{3x}$

$\alpha\left(2xe^{3x} + x^2 \cdot 3e^{3x}\right)' - 6\alpha\left(2xe^{3x} + x^2 \cdot 3e^{3x}\right) + 9\alpha \cdot x^2 \cdot e^{3x} = e^{3x}.$

$\alpha\left(2e^{3x} + \cancel{6xe^{3x}} + \cancel{2x \cdot 3e^{3x}} + \cancel{x^2 \cdot 9e^{3x}}\right) - \cancel{12\alpha \cdot xe^{3x}} - \cancel{18x^2 \cdot \alpha \cdot e^{3x}}$

$+ \cancel{9\alpha \cdot x^2 \cdot e^{3x}} = e^{3x}$

$e^{3x} \cdot 2\alpha = e^{3x}$ dus $\alpha = \frac{1}{2}$ dus $y_p = \frac{1}{2} x^2 e^{3x}.$

f) $y'' + y = \cos x + x^2.$

$\lambda^2 + 1 = 0$ dus $\lambda = \pm i \Rightarrow y_h = A\cos x + B\sin x$

$\to a + bi = i \to$ komt voor in homogene.

$y_p = (\alpha \cos x + \beta \sin x) \cdot x + \gamma x^2 + \delta x + \eta$

$\left((\alpha \cos x + \beta \sin x) + x(-\alpha \sin x + \beta \cos x) + 2\gamma x + \delta\right)'$

$+ x \cdot \alpha \cos x + x \cdot \beta \sin x + \gamma x^2 + \delta x + \eta = \cos x + x^2.$

$-\alpha \sin x + \beta \cos x + (-\alpha \sin x + \beta \cos x) + \cancel{x(-\alpha \cos x - \beta \sin x)} + 2\gamma$

$\cancel{+ x \cdot \alpha \cos x + x \beta \sin x} + \gamma x^2 + \delta x + \eta = \cos x + x^2.$

$\alpha = 0 \quad \beta = \frac{1}{2} \quad \gamma = 1 \quad 2\gamma + \eta = 0$

dus $y_p = x \frac{1}{2} \sin x + x^2 - 2$ $\quad \eta = -2.$

g) $y'' + 2y' - 2y = 2x + 2e^x \cos x.$

y_h: $\lambda^2 + 2\lambda - 2 = 0$ $\quad D = 4 + 2 \cdot 4 = \cancel{20}\ 12$

$S = -2 \quad P = -2.$ $\quad \lambda_1 = \frac{-4 + \sqrt{12}}{2}$ en $\lambda_2 = \frac{-4 - \sqrt{12}}{2}.$

dus $y_h = \cancel{A e^{\frac{-4+\sqrt{10}}{2}x} + B e^{\frac{-4-\sqrt{10}}{2}x}}$

$= Ae^{-1+\sqrt{3}} + Be^{-1-\sqrt{3}}$

$y_p = \alpha x + \beta + e^x \cdot \gamma \cos x + e^x \delta \sin x.$

$+\delta(e^x \sin x + e^x \cos x)'$

$\gamma(e^x \cos x + -\sin x \cdot e^x)' + 2\gamma(e^x \cos x + -e^x \sin x) + 2\alpha$
$+ 2\delta(e^x \sin x + e^x \cos x).$

$- 2\alpha x - 2\beta - 2e^x \gamma \cos x = 2x + 2e^x \cos x.$
$- 2e^x \delta \sin x.$

$+\delta(\cancel{e^x \sin x} + e^x \cos x + e^x \cos x - \cancel{e^x \sin x})$

$\gamma(\cancel{e^x \cos x} + -\sin x e^x - \sin x \cdot e^x - \cancel{\cos x \cdot e^x}) + \cancel{2\gamma e^x \cos x}$
$\cancel{+ 2\delta e^x \sin x}$

$- 2\gamma e^x \sin x + 2\alpha - 2\alpha x - 2\beta - \cancel{2e^x \gamma \cos x} = 2x + 2e^x \cos x.$
$+ 2\delta e^x \cos x. \qquad - \cancel{2e^x \delta \sin x}$

$- 4\gamma e^x \sin + 2\alpha - 2\alpha x - 2\beta = 2x + 2e^x \cos x$
$+ 4\delta e^x \cos x$

dus $\gamma = 0 \quad \delta = \frac{1}{2} \quad \alpha = -1$ dus $-2 - 2\beta = 0$
$\beta = -1.$

dus $y_p = -x - 1 + \frac{e^x}{2} \sin x.$

h) $y'' + 4y = e^{-x} \cos 2x.$

y_h $\lambda^2 + 4 = 0$ dus $\lambda_1 = \pm 2i$

dus $y_h = A \cos 2x + B \sin 2x.$

$y_p = \alpha e^{-x} \cos 2x + \beta e^{-x} \sin 2x.$

$\alpha(-e^{-x} \cos 2x + e^{-x} \cdot (-\sin 2x) \cdot 2)' + \beta(-e^{-x} \sin 2x + 2e^{-x} \cos 2x)'$
$+ 4y = q(x).$

$\alpha(e^{-x} \cos 2x + e^{-x} \sin 2x \cdot 2 + e^{-x} \sin 2x \cdot 2 - \cancel{e^{-x} \cos 2x \cdot 4})$
$+ \beta(e^{-x} \sin 2x + - e^{-x} \cos 2x \cdot 2 - 2e^{-x} \cos 2x - \cancel{4e^{-x} \sin 2x}).$

$+ \cancel{4\alpha e^{-x} \cos 2x} + \cancel{4\beta e^{-x} \sin 2x} = e^{-x} \cos 2x.$

schrijf $p = e^{-x} \cos 2x \quad q = e^{-x} \sin 2x.$

$\alpha p + 4\alpha q + \beta q - 4\beta p = p.$

dus $\alpha - 4\beta = 1 \qquad \alpha = 1 + 4\beta \qquad \alpha = \frac{1}{17}.$
$4\alpha + \beta = 0 \qquad 4 + 16\beta + \beta = 0 \qquad \beta = \frac{-4}{17}$

$y_p = \frac{1}{17} e^{-x} \cos 2x$
$\frac{-4}{17} e^{-x} \sin 2x$

i) $y'' - 2y' + y = xe^{x}$

y_h: $\lambda^2 - 2\lambda + 1 = 0$. $\lambda_1 = \lambda_2 = 1$

$S = 2 \quad P = 1$ donc $y_h = e^{x}(A + xB)$

$y_p = x^2e^x(\alpha x + \beta) = x^3\alpha e^x + \beta x^2 e^x$.

$= \alpha(3x^2e^x + x^3e^x)' + \beta(2xe^x + x^2e^x)' - 2\alpha(3x^2e^x + x^3e^x) +$

$- 2\beta(2xe^x + x^2e^x) + \alpha x^3 e^x + \beta x^2 e^x = xe^x.$

$= \alpha(6xe^x + \cancel{3x^2e^x} + \cancel{3x^2e^x} \cancel{+x^3e^x}) - 2\alpha(\cancel{6x^2e^x} + \cancel{2x^3e^x})$

$+ \beta(2e^x + \cancel{2xe^x} + \cancel{2xe^x} \cancel{+x^2e^x}) - 2\beta(\cancel{4xe^x} \cancel{+2x^2e^x})$

$+ \cancel{\alpha x^3 e^x} + \cancel{\beta x^2 e^x} = xe^x$

$6\alpha \cdot xe^x + 2\beta \cdot e^x = xe^x.$

$\alpha = \frac{1}{6}$ donc $y_p = \frac{1}{6}x^3e^x.$

j) $y'' + 2y' + 2y = e^{-x}\sin x.$

$\lambda^2 + 2\lambda + 2 = 0$ donc $\lambda_1 = \frac{-2 + 2i}{2}$ $\lambda_2 = \frac{-2 - 2i}{2}$

$D = 4 - 8 = -4$ $= i - 1$ $= -1 - i$

donc $y_h = e^{-x}(A\cos x + B\sin x).$

$y_p = xe^{-x}\sin x \cdot \alpha + \beta xe^{-x}\cos x.$

$\alpha(e^{-x}\sin x + x(e^{-x}\sin x)')' + \beta(e^{-x}\cos x + x(e^{-x}\cos x)')'$

$+ 2\alpha(e^{-x}\sin x - xe^{-x}\sin x + xe^{-x}\cos x)$

$+ 2\beta(e^{-x}\cos x - xe^{-x}\cos x - xe^{-x}\sin x) + 2y = g(x)$

$\alpha(-e^{-x}\sin x + e^{-x}\cos x - e^{-x}\sin x \cancel{+ xe^{-x}\sin x} - xe^{-x}\cos x$

$+ e^{-x}\cos x - xe^{-x}\cos x \cancel{- xe^{-x}\sin x})$

$+\beta(-e^{-x}\cos x - e^{-x}\sin x - e^{-x}\cos x \cancel{+ xe^{-x}\cos x} \cancel{+ xe^{-x}\sin x}$

$- e^{-x}\sin x \cancel{+ xe^{-x}\sin x} - xe^{-x}\cos x)$

$+ 2\alpha(e^{-x}\sin x \cancel{- xe^{-x}\sin x} + xe^{-x}\cos x) + 2\beta(e^{-x}\cos x - xe^{-x}\cos x \cancel{- xe^{-x}\sin x})$

$+ \cancel{2xe^{-x}\sin x \cdot \alpha} + \cancel{2\beta xe^{-x}\cos x} = g(x)$

j) $2\alpha e^{-x}\cos x - 2\beta e^{-x}\sin x = e^{-x}\sin x.$

leuk

$\alpha = 0 \qquad \beta = \frac{-1}{2}.$

$\Rightarrow y_p = \frac{-1}{2} x e^{-x}\cos x.$

k). $y'' - y' + 6y = 6\cos 3x - 5e^{2x}.$

y_h $\lambda^2 - \lambda + 6 = 0 \qquad D = 1 - 24 = \sqrt{-23}.$

$S = 1 \quad p = 6 \qquad \lambda_1 = \frac{1 + i\sqrt{23}}{2} \quad \lambda_2 = \frac{1 - i\sqrt{23}}{2}.$

dus $y_h = e^{\frac{1}{2}x}\left(A\cos\frac{\sqrt{23}}{2}x + B\sin\frac{\sqrt{23}}{2}x\right)$

y_p $y = \alpha\cos 3x + \beta\sin 3x + \gamma e^{2x}.$

° $\cancel{\alpha\ 1\sin}\ (-3\alpha\sin 3x + 3\beta\cos 3x + 2\gamma e^{2x})'$

$-y' + 6y = q(x).$

° $-9\alpha \cancel{\ldots}\cos 3x - 9\beta\sin 3x + 4\gamma e^{2x}$

$+ 3\alpha\sin 3x - 3\beta\cos 3x - 2\gamma e^{2x}$

$+ 6\alpha\cos 3x + 6\beta\sin 3x + 6\gamma e^{2x} = 6\cos 3x - 5e^{2x}.$

schrijf $p = \cos 3x \quad q = \sin 3x \quad t = e^{2x}.$

$-3\alpha p - 3\beta q + 8\gamma t + 3\cancel{\alpha q} - 3\beta p = 6p - 5t.$

dus $\cancel{t}\ \gamma = \frac{-5}{8}$

$$\begin{cases} -3\alpha - 3\beta = 6 \\ -3\beta + 3\alpha = 0 \end{cases}$$ dus $\alpha = \beta$ dus $\alpha = -1 = \beta.$

$y_p = -\cos 3x - \sin 3x - \frac{5}{8}e^{2x}.$

e) $y'' + 2y' + y = e^{-x}$ $\quad y(0) = 1 \quad y'(0) = 0$

y_h $\lambda^2 + 2\lambda + 1 = 0$

$S = -2 \quad P = 1$ dcs $\lambda_1 = \lambda_2 = -1$.

$y_h = e^{-x}(A + Bx)$

$y_p = \alpha e^{-x} \cdot x^2$.

$\alpha(-e^{-x} \cdot x^2 + e^{-x} \cdot 2x)' + 2y' + y = q(x)$.

$\alpha(\cancel{e^{-x} \cdot x^2} - \cancel{2xe^{-x}} - e^{-x} \cdot 2x + e^{-x} \cdot 2) + 2\alpha(\cancel{-e^{-x} \cdot x^2} + \cancel{e^{-x} \cdot 2x})$

$+ \cancel{\alpha e^{-x} \cdot x^2} = q(x)$

$2\alpha e^{-x} = e^{-x}$ dcs $\alpha = \frac{1}{2}$.

dcs $y = e^{-x}(A + Bx) + \frac{e^{-x} \cdot x^2}{2}$.

$\begin{cases} y(0) = 1 \cdot A = 1. \text{ dcs } A = 0. \\ y'(x) = e^{-x} \cdot B - e^{-x} \cdot Bx + \frac{1}{2}(-e^{-x} x^2 + e^{-x} \cdot 2x). \end{cases}$

$y'(0) = B - \cancel{e^{-x}} 1 = 0$

dcs $B = e^{-1 \times 0} = 1$

dcs $y = e^{-x}(1 + x) + \frac{e^{-x} \cdot x^2}{2}$.

2 a) $y''' - 5y'' + 8y' - 4y = 5e^{2x}$

$\lambda^3 - 5\lambda^2 + 8\lambda - 4 = 0. \qquad = (\lambda - 1)(\lambda^2 - 4\lambda + 4)$

$$\begin{array}{c|cccc} & 1 & -5 & 8 & -4 \\ 1 & & 1 & -4 & 4 \\ \hline & 1 & -4 & 4 & \lfloor 0 \end{array}$$

$S = 4 \quad P = 4.$

dus $\lambda_2 = \lambda_3 = 2.$

dus $y_h = e^{2x}(A + Bx) + e^{x} \cdot C$

$y_p = \alpha x^2 e^{2x} \Rightarrow \cancel{\alpha(2xe^{2x} + x^2 2e^{2x})''' - 5\alpha(}$

$\cancel{8\alpha e^{2x} - 8 \ldots xe^{2x} + 16xe^{2x}}$

$q(x) = \cancel{-4q} + 8\alpha(2xe^{2x} + 2x^2e^{2x}) - 5\alpha(2e^{2x} + \cancel{4}xe^{2x} + 4xe^{2x} + 4x^2e^{2x})$
$-4\alpha x^2 e^{2x}$

$+\alpha(4e^{2x} + \cancel{4}e^{2x} + 8xe^{2x} + 4e^{2x} + 8xe^{2x} + 8xe^{2x} + 8x^2e^{2x}).$

schrijf $p = x^2e^{2x} \quad q = xe^{2x} \quad t = e^{2x}.$

$5t = \alpha(\cancel{-4p} + \cancel{16q} + \cancel{16p} - 10t - \cancel{40q} - \cancel{20p}$

$+ 12t + \cancel{16q} + \cancel{8q} + \cancel{8p}$

$\frac{5t}{2t} = \alpha$ dus $\alpha = \frac{5}{2}$

dus $y_p = \frac{5}{2}x^2e^{2x}.$

b) $y^{(4)} - 2y'' + y = 4(\sin x + \cos x) - 8e^{3x}.$

$\lambda^4 - 2\lambda^2 + 1 = 0$

$t^2 - 2t + 1 = 0$ dan is $r = 1$ dus $\lambda_1 = \lambda_2 = 1$

$\lambda_3 = \lambda_4 = -1$

$y_h = e^{x}(A + Bx) + e^{-x}(C + Dx).$

$y_p = \alpha \sin x + \beta \cos x + \gamma e^{3x}.$

9 b (vervolg)

$q(x) = \alpha \sin x + \beta \cos x + \gamma e^{3x} - 2 \cdot (\alpha \cos x - \beta \sin x + 3\gamma e^{3x})'$

$+ (\alpha - \alpha \sin x - \beta \cos x + 9\gamma e^{3x})''$

$q(x) = y - 2y'' + (-\alpha \cos x + \beta \sin x + 27\gamma e^{3x})'$

schrijf $d = \cos x \quad e = \sin x \quad f = e^{3x}$

dan is

$4(d + e) - 8f = \alpha e + \beta d + \gamma f$ ~~...~~

$+ 2\alpha e + 2\beta d - 18\gamma f$

$+ \alpha e + \beta d + 81\gamma f$

$-8f = 1\gamma f - 18\gamma f + 81\gamma f \qquad -8 = 64\gamma$

$\begin{cases} 4 = 4\beta & \beta = 1 \\ 4 = 4\alpha & \alpha = 1 \end{cases} \qquad \gamma = \frac{-1}{8}$

dus $y_p = \sin x + \cos x + \frac{-1}{8} e^{3x}$

3 $y'' + \frac{2}{x} y' = 6.$

schrijf $z = y'$ dan is $z' + \frac{2}{x} z = 6.$

$x^2 z' + 2xz = 6x^2.$

$(x^2 z)' = 6x^2.$

$z = \frac{2x^3 + C}{x^2} = x + \frac{C}{x^2}$

$y = \int (x + \frac{C}{x^2}) dx$

$= x^2 - \frac{C}{x} + D$

④ $y'' + \frac{2x}{x^2+1} y' = x \qquad y(0) = 5$

$z' + \frac{2x}{x^2+1} z = x$

$(x^2+1) \cdot z' + 2x\, z = x \cdot (x^2+1)$

$((x^2+1) \cdot z)' = x^3 + x.$

$z = \cancel{\frac{x^2}{x^2+1}} = \dfrac{\frac{x^4}{4} + \frac{2x^2}{4} + C}{x^2+1}.$

$y = \frac{1}{4} \int \frac{x^4 + x^2}{x^2+1} dx + \frac{1}{4} \int \frac{x^2+1-1}{x^2+1} dx + \int \frac{C}{x^2+1} dx.$

$= \frac{1}{4} \left(\int x^2 dx + \int dx - \int \frac{1}{x^2+1} \right) + C \operatorname{Bgtan} x + D$

$= \frac{x^3}{12} + \frac{x}{4} - \frac{\operatorname{Bgtan} x}{4} + C \operatorname{Bgtan} x + D$! fout.

$y(0) = D = 5$

$\Rightarrow$ nee: $E = C - \frac{1}{4}$

$\Rightarrow y = \frac{x^3}{12} + \frac{x}{4} + E \operatorname{Bgtan} x + 5.$!!

⑤ Zoek $a, b, q(x)$ zodat $y'' + a y' + b y = q(x)$.

zodat $\lim_{x\to\infty} y = +\infty = \lim_{x\to\infty} y_h + y_p = \lim_{x\to\infty} y_h + \lim_{x\to\infty} y_p$

y_h: stel $y_h = e^{-x} \cdot A + e^{-2x} \cdot B$ dan zijn -1 en -2 opl. v/d.

karakteristieke vgl.: $(\lambda - 1)(\lambda - 2) = 0$

$\lambda^2 - 3\lambda + 2 = 0.$

dus $a = -3$ en $b = 2$.

y_p: stel $y_p = 7$ dan is $2 \cdot 7 = q(x)$

$q(x) = 14.$

dus $y'' - 3y' + 2y = 14.$

6) $y = ~~A~~ \; e^{3x}(A - x + x^2) + Be^{-2x}$.

$= e^{3x} \, x(x-1) + Ae^{3x} + Be^{-2x}$.

dus 3 en −2 zijn oplossingen v/d k. vgl.

$= (\lambda - 3)(\lambda + 2) = \lambda^2 - \lambda - 6 = 0$

dus homogene vgl: $y'' - y' - 6y = 0$.

$y_p = (\alpha x + \beta) x e^{3x}$ met $\alpha = 1$ en $\beta = -1$

$q(x) = (\gamma x + \delta) \cdot e^{3x}$.

$\cancel{y''} \cancel{-6x^2e^{3x}} + 6xe^{3x} - 1(2xe^{3x} \cancel{+3x^2e^{3x}} + e^{3x} + 3xe^{3x}) + 2e^{3x} + 6xe^{3x}$

$+ 6xe^{3x} \cancel{+9x^2e^{3x}} + 3e^{3x} + 3e^{3x} + 9xe^{3x} = \gamma xe^{3x} + \delta e^{3x}$

schrijf $xe^{3x} = p$ en $e^{3x} = q$ ~~en en~~ $\cancel{x^2e^{3x}}$

$6p - 2p - q - 3p + 2q + 6p$

$+ 6p + 3q + 3q + 9p = \gamma p + \delta q$.

$22 = \gamma$

$7 = \delta$ dus vgl: $y'' - y' - 6y = 22xe^{3x} + 7e^{3x}$.

7) omdat we een basis nodig hebben van de oplossings verzameling ⇒ lin. onafh.

b): te dom. : de functie y_2 kan niet geschreven w̄ als λy_1 met $\lambda \in \mathbb{R}$.

8) $y'' - \frac{2}{x}y' + \frac{2}{x^2}y = 0$

a) homogeen

b) 2 vrijheidsgraden.

d. $\lambda \cdot (\lambda - 1) \cdot x^{\lambda - 2} - \frac{2}{x} \cdot \lambda \cdot x^{\lambda - 1} + \frac{2}{x^2} x^\lambda = 0$

$\cancel{\lambda^2 x^{\lambda} \ldots} \; x^{\lambda-2} \quad \lambda^2 - \lambda - 2\lambda + 2 = 0$.

$\lambda^2 - 3\lambda + 2 = 0$

$S = 3 \quad P = 2 \qquad \lambda_1 = 2$

$\lambda_2 = 1$.

dus $y = A \cdot x + B \cdot x^2$.

(9) $y'' - 2y' + \left(1 - \frac{2}{x^2}\right)y = 0$ met $x > 0$

2de orde homogeen → 2 vrijheidsgraden.

$$\left(1-\frac{2}{x^2}\right)\cdot\left(e^x\cdot x^\lambda\right) + -2\left(\cancel{e^x x^\lambda} + \cancel{\lambda x^{\lambda-1}}\cdot e^x\right) + \cancel{e^x x^\lambda} + \cancel{\lambda x^{\lambda-1}} e^x + \cancel{\lambda e^x x^{\lambda-1}} + \lambda(\lambda-1)x^{\lambda-2}e^x = 0.$$

$$-2e^x\cdot x^{\lambda-2} + \lambda^2 e^x x^{\lambda-2} - \lambda x^{\lambda-2}e^x = 0.$$

dus $\lambda^2 - \lambda - 2 = 0$

$S = 1$ $\quad P = -2.$ $\quad \lambda_1 = -1$, $\lambda_2 = 2.$

dus $y = A\cdot e^x\cdot x^{-1} + Be^x\cdot x^2.$

(10) $(x^2+1)y'' - 2xy' + 2y = 0.$

$y_1(x) = x \quad y''(x) = 0 \quad y'(x) = 1.$ dus $-2x + 2x = 0.$

$y_2(x) = \mu(x)\cdot x. \quad y_2'(x) = \mu(x) + x\mu'(x) \quad y_2''(x) = \mu'(x) + \mu'(x) + x\mu''(x)$

$= 2\mu'(x) + x\mu''(x)$

$$(x^2+1)\left(2\mu'(x) + x\mu''(x)\right) - 2x\cdot\left(\cancel{\mu(x)} + x\mu'(x)\right) + \cancel{2\mu(x)\cdot x} = 0$$

$$x^3\cdot\mu''(x) + \cancel{2\mu'(x)\cdot x^2} + x\mu''(x) + 2\mu'(x) - \cancel{2x^2\mu'(x)} = 0.$$

$$\mu''(x)\left(x^3 + x\right) + 2\mu'(x) = 0.$$

schrijf [illegible]: $z = \mu'$

dan is [illegible] $z'(x^3+x) + 2z = 0.$

$z' = -2z\cdot\frac{1}{x^3+x} \qquad z' + \frac{2}{x^3+x}$

$$\int\frac{-1}{2z}\,dz = \int\frac{1}{x^3+x}\,dx = \int\frac{1}{x}\,dx - \int\frac{x}{x^2+1}\,dx$$

$$= \int\frac{1}{x(x^2+1)} = \cancel{\int\frac{-1}{x}\,dx + \int\frac{1}{2}\frac{1}{x+1}\,dx + \int\frac{1}{2}\frac{1}{x-1}\,dx.}$$

$$\cancel{\frac{-1}{2}\ln z = -1\ln x + \frac{1}{2}\left(\ln(x+1) + \ln(x-1)\right)}$$

$$\cancel{= \ln\frac{1}{x} + \ln\left(\sqrt{x^2+1}\right)}$$

$$\ln z^{-1/2} = \ln x - \ln(x^2+1)^{1/2}$$

$$z^{-1/2} = \frac{x}{\sqrt{x^2+1}} \qquad z = \frac{x^2+1}{x^2} \qquad \int z\,dx = x - \frac{1}{x} = \mu(x)$$

dus $y = Ax + B(x^2-1)$

(11) $y'' + \left(\frac{1}{x} - 2\right)y' + \left(1 - \frac{1}{x}\right)y = 0$

$y_2(x) = \mu(x)e^x$ $\quad y_2'(x) = \mu'(x)e^x + \mu(x)e^x$ $\quad y_2''(x) = \mu''(x)e^x + 2\mu'(x)\cdot e^x + \mu(x)e^x$

$\mu''(x)e^x + \cancel{2\mu'(x)e^x} + \cancel{\mu(x)e^x}$

$+ \frac{\mu'(x)e^x}{x} + \cancel{\frac{\mu(x)e^x}{x}} - \cancel{2\mu'(x)e^x} - \cancel{2\mu(x)e^x} + \cancel{\mu(x)e^x} - \cancel{\frac{\mu(x)e^x}{x}} = 0$

$z = \mu'$

$z' + \frac{z}{x} = 0.$ $\quad$ schrijf $p(x) = \exp\int \frac{1}{x}dx = x.$

$z' = \frac{-z}{x}$

$\int \frac{-1}{z}dz = \int \frac{1}{x}dx$

$\ln z^{-1} = \ln x.$

$\frac{1}{z} = x.$ $\quad$ dus $z = \frac{1}{x} \to \mu = \int z = \ln x.$

dus $y = e^x A + Be^x \ln(x)$

(12) $\begin{cases} P(0) = 40 \\ P'(0) = 0 \\ P'(t) = \frac{1}{10}\left(60 - 10P(t) + \frac{2P'(t)}{10} - \frac{P''(t)}{10} - 10P(t) + 20\right). \end{cases}$

dus $y' = 6 - y + \frac{1}{5}y' - \frac{y''}{10} - y$

$y'' + 8y' + 20y = 60.$

(y_h) $\lambda^2 + 8\lambda + 20 = 0.$

$D = -16.$ $\quad \lambda_{1,2} = \frac{-8 \pm 4i}{2} = -4 \pm 2i$ $\Rightarrow y_h = e^{-4x}(A\cos 2x + B\sin 2x)$

(y_p) $y_p = \beta$

$20\beta = 60$ $\quad \beta = 3.$ $\quad$ dus $y = 3 + e^{-4x}\cdot(A\cos 2x + B\sin 2x)$

$y(0) = 40 = 3 + A$ $\quad$ dus $A = 37$

$y'(0) = 0 = -4e^{-4x}\cdot(A\cos 2x + B\sin 2x)$

$+ e^{-4x}\cdot 2A\sin 2x + e^{-4x}\, 2B\cos 2x$

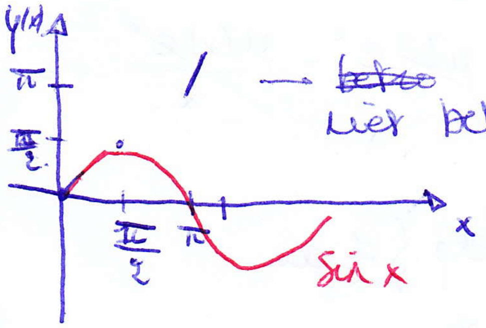
$y'(0) = -4A + 2B = 0$ $\quad$ dus $P(t) = 3 + e^{-4x}(37\cos 2x + 74\sin 2x)$

$B = 2\cdot 37 = 74$

oef p 778 : numerieke ber. diff. vgl^n.

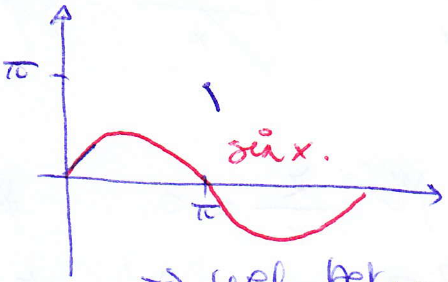
(2) $\begin{cases} y' - 10y = \cos x - 10\sin x \\ y(0) = 0 \end{cases}$ $\begin{cases} y' + 10y = \cos x + 10\sin x \\ y(0) = 0 \end{cases}$

b) $y' = \cos x - 10\sin x + 10y$



/ → ~~betto~~ niet betrouwbaar

$y' = \cos x + 10\sin x - 10y$



\ sin x.

⇒ wel betrouwbaar

A) $\lambda - 10 = 0$
$\lambda = 10$. $\Big\}$ $y_h = A \cdot e^{10x}$.

(y_p) : = ~~cos x − 2 sin x~~
sin x.

$\Rightarrow y = \sin x + A \cdot e^{10x}$

vanaf $A \neq 0$ zullen we exponentieel verschillen v. $x \mapsto \sin(x)$

B) $y_h = \dfrac{A}{\cancel{100}\, e^{10x}}$.

dus $y = \sin x + \dfrac{A}{e^{10x}}$.

bij welke A we ook welen we zullen steeds dichter bij sin x komen.