## CHAPTER 1: Introduction, Measurement, Estimating

## Responses to Questions

(a) A particular person's foot. Merits: reproducible. Drawbacks: not accessible to the general public; not invariable (could change size with age, time of day, etc.); not indestructible.
(b) Any person's foot. Merits: accessible. Drawbacks: not reproducible (different people have different size feet); not invariable (could change size with age, time of day, etc.); not indestructible.
Neither of these options would make a good standard.
2. The number of digits you present in your answer should represent the precision with which you know a measurement; it says very little about the accuracy of the measurement. For example, if you measure the length of a table to great precision, but with a measuring instrument that is not calibrated correctly, you will not measure accurately.
3. The writers of the sign converted 3000 ft to meters without taking significant figures into account. To be consistent, the elevation should be reported as 900 m .
4. The distance in miles is given to one significant figure and the distance in kilometers is given to five significant figures! The figure in kilometers indicates more precision than really exists or than is meaningful. The last digit represents a distance on the same order of magnitude as the car's length!
5. If you are asked to measure a flower bed, and you report that it is "four," you haven't given enough information for your answer to be useful. There is a large difference between a flower bed that is 4 m long and one that is 4 ft long. Units are necessary to give meaning to the numerical answer.
6. Imagine the jar cut into slices each about the thickness of a marble. By looking through the bottom of the jar, you can roughly count how many marbles are in one slice. Then estimate the height of the jar in slices, or in marbles. By symmetry, we assume that all marbles are the same size and shape. Therefore the total number of marbles in the jar will be the product of the number of marbles per slice and the number of slices.
7. You should report a result of 8.32 cm . Your measurement had three significant figures. When you multiply by 2 , you are really multiplying by the integer 2 , which is exact. The number of significant figures is determined by your measurement.
8. The correct number of significant figures is three: $\sin 30.0^{\circ}=0.500$.
9. You only need to measure the other ingredients to within $10 \%$ as well.
10. Useful assumptions include the population of the city, the fraction of people who own cars, the average number of visits to a mechanic that each car makes in a year, the average number of weeks a mechanic works in a year, and the average number of cars each mechanic can see in a week.
(a) There are about 800,000 people in San Francisco. Assume that half of them have cars. If each of these 400,000 cars needs servicing twice a year, then there are 800,000 visits to mechanics in a year. If mechanics typically work 50 weeks a year, then about 16,000 cars would need to be seen each week. Assume that on average, a mechanic can work on 4 cars per day, or 20 cars a week. The final estimate, then, is 800 car mechanics in San Francisco.
(b) Answers will vary.
11. One common way is to observe Venus at a time when a line drawn from Earth to Venus is perpendicular to a line connecting Venus to the Sun. Then Earth, Venus, and the Sun are at the vertices of a right triangle, with Venus at the $90^{\circ}$ angle. (This configuration will result in the greatest angular distance between Venus and the Sun, as seen from Earth.) One can then measure the distance to
 Venus, using radar, and measure the angular distance between Venus and the Sun. From this information you can use trigonometry to calculate the length of the leg of the triangle that is the distance from Earth to the Sun.
12. No. Length must be included as a base quantity.

## Solutions to Problems

1. (a) 14 billion years $=1.4 \times 10^{10}$ years
(b) $\left(1.4 \times 10^{10} \mathrm{y}\right)\left(3.156 \times 10^{7} \mathrm{~s} / 1 \mathrm{y}\right)=4.4 \times 10^{17} \mathrm{~s}$
2. (a) 214

3 significant figures
(b) 81.60

4 significant figures
(c) 7.03

3 significant figures
(d) 0.03

1 significant figure
(e) 0.0086

2 significant figures
(f) 3236
(g) 8700

4 significant figures
2 significant figures
3. (a) $1.156=1.156 \times 10^{\circ}$
(b) $21.8=2.18 \times 10^{1}$
(c) $0.0068=6.8 \times 10^{-3}$
(d) $328.65=3.2865 \times 10^{2}$
(e) $0.219=2.19 \times 10^{-1}$
(f) $444=4.44 \times 10^{2}$
4. (a) $8.69 \times 10^{4}=86,900$
(b) $9.1 \times 10^{3}=9,100$
(c) $8.8 \times 10^{-1}=0.88$
(d) $4.76 \times 10^{2}=476$
(e) $3.62 \times 10^{-5}=0.0000362$
5. $\%$ uncertainty $=\frac{0.25 \mathrm{~m}}{5.48 \mathrm{~m}} \times 100 \%=4.6 \%$
6. (a) $\%$ uncertainty $=\frac{0.2 \mathrm{~s}}{5 \mathrm{~s}} \times 100 \%=4 \%$
(b) $\%$ uncertainty $=\frac{0.2 \mathrm{~s}}{50 \mathrm{~s}} \times 100 \%=0.4 \%$
(c) $\%$ uncertainty $=\frac{0.2 \mathrm{~s}}{300 \mathrm{~s}} \times 100 \%=0.07 \%$
7. To add values with significant figures, adjust all values to be added so that their exponents are all the same.

$$
\begin{aligned}
& \left(9.2 \times 10^{3} \mathrm{~s}\right)+\left(8.3 \times 10^{4} \mathrm{~s}\right)+\left(0.008 \times 10^{6} \mathrm{~s}\right)=\left(9.2 \times 10^{3} \mathrm{~s}\right)+\left(83 \times 10^{3} \mathrm{~s}\right)+\left(8 \times 10^{3} \mathrm{~s}\right) \\
& \quad=(9.2+83+8) \times 10^{3} \mathrm{~s}=100.2 \times 10^{3} \mathrm{~s}=1.00 \times 10^{5} \mathrm{~s}
\end{aligned}
$$

When adding, keep the least accurate value, and so keep to the "ones" place in the last set of parentheses.
8. $\left(2.079 \times 10^{2} \mathrm{~m}\right)\left(0.082 \times 10^{-1}\right)=1.7 \mathrm{~m}$. When multiplying, the result should have as many digits as the number with the least number of significant digits used in the calculation.
9.

| $\theta$ (radians) | $\sin (\theta)$ | $\tan (\theta)$ |
| :---: | :--- | :--- |
| 0 | 0.00 | 0.00 |
| 0.10 | 0.10 | 0.10 |
| 0.12 | 0.12 | 0.12 |
| 0.20 | 0.20 | 0.20 |
| 0.24 | 0.24 | 0.24 |
| 0.25 | 0.25 | 0.26 |

Keeping 2 significant figures in the angle, and expressing the angle in radians, the largest angle that has the same sine and tangent is 0.24 radians. In degrees, the largest angle (keeping 2 significant figure) is $12^{\circ}$. The spreadsheet used for this problem can be found on the Media Manager, with filename
"PSE4_ISM_CH01.XLS," on tab "Problem 1.9."
10. To find the approximate uncertainty in the volume, calculate the volume for the minimum radius and the volume for the maximum radius. Subtract the extreme volumes. The uncertainty in the volume is then half this variation in volume.

$$
\begin{aligned}
& V_{\text {specified }}=\frac{4}{3} \pi r_{\text {specified }}^{3}=\frac{4}{3} \pi(0.84 \mathrm{~m})^{3}=2.483 \mathrm{~m}^{3} \\
& V_{\min }=\frac{4}{3} \pi r_{\min }^{3}=\frac{4}{3} \pi(0.80 \mathrm{~m})^{3}=2.145 \mathrm{~m}^{3} \\
& V_{\max }=\frac{4}{3} \pi r_{\max }^{3}=\frac{4}{3} \pi(0.88 \mathrm{~m})^{3}=2.855 \mathrm{~m}^{3} \\
& \Delta V=\frac{1}{2}\left(V_{\max }-V_{\min }\right)=\frac{1}{2}\left(2.855 \mathrm{~m}^{3}-2.145 \mathrm{~m}^{3}\right)=0.355 \mathrm{~m}^{3}
\end{aligned}
$$

The percent uncertainty is $\frac{\Delta V}{V_{\text {specified }}}=\frac{0.355 \mathrm{~m}^{3}}{2.483 \mathrm{~m}^{3}} \times 100=14.3 \approx 14 \%$.
11. (a) 286.6 mm
(b) $85 \mu \mathrm{~V}$
(c) 760 mg
(d) 60.0 ps
(e) 22.5 fm
(f) 2.50 gigavolts
$286.6 \times 10^{-3} \mathrm{~m}$
$85 \times 10^{-6} \mathrm{~V}$
$760 \times 10^{-6} \mathrm{~kg}$
$60.0 \times 10^{-12} \mathrm{~s}$
$22.5 \times 10^{-15} \mathrm{~m}$
$2.5 \times 10^{9}$ volts

12. (a) $1 \times 10^{6}$ volts

$$
1 \text { megavolt }=1 \mathrm{Mvolt}
$$

2 micrometers $=2 \mu \mathrm{~m}$
6 kilodays $=6$ kdays
18 hectobucks $=18$ hbucks or 1.8 kilobucks
80 nanoseconds $=80 \mathrm{~ns}$
13. Assuming a height of 5 feet 10 inches, then $5^{\prime} 10^{\prime \prime}=(70 \mathrm{in})(1 \mathrm{~m} / 39.37 \mathrm{in})=1.8 \mathrm{~m}$. Assuming a weight of 165 lbs , then $(165 \mathrm{lbs})(0.456 \mathrm{~kg} / 1 \mathrm{lb})=75.2 \mathrm{~kg}$. Technically, pounds and mass measure two separate properties. To make this conversion, we have to assume that we are at a location where the acceleration due to gravity is $9.80 \mathrm{~m} / \mathrm{s}^{2}$.
14. (a) 93 million miles $=\left(93 \times 10^{6}\right.$ miles $)(1610 \mathrm{~m} / 1$ mile $)=1.5 \times 10^{11} \mathrm{~m}$
(b) $1.5 \times 10^{11} \mathrm{~m}=150 \times 10^{9} \mathrm{~m}=150$ gigameters or $1.5 \times 10^{11} \mathrm{~m}=0.15 \times 10^{12} \mathrm{~m}=0.15$ terameters
15. (a) $1 \mathrm{ft}^{2}=\left(1 \mathrm{ft}^{2}\right)(1 \mathrm{yd} / 3 \mathrm{ft})^{2}=0.111 \mathrm{yd}^{2}$, and so the conversion factor is $\frac{0.111 \mathrm{yd}^{2}}{1 \mathrm{ft}^{2}}$.
(b) $1 \mathrm{~m}^{2}=\left(1 \mathrm{~m}^{2}\right)(3.28 \mathrm{ft} / 1 \mathrm{~m})^{2}=10.8 \mathrm{ft}^{2}$, and so the conversion factor is $\frac{10.8 \mathrm{ft}^{2}}{1 \mathrm{~m}^{2}}$.
16. Use the speed of the airplane to convert the travel distance into a time. $d=v t$, so $t=d / v$.

$$
t=d / v=1.00 \mathrm{~km}\left(\frac{1 \mathrm{~h}}{950 \mathrm{~km}}\right)\left(\frac{3600 \mathrm{~s}}{1 \mathrm{~h}}\right)=3.8 \mathrm{~s}
$$

17. (a) $1.0 \times 10^{-10} \mathrm{~m}=\left(1.0 \times 10^{-10} \mathrm{~m}\right)(39.37 \mathrm{in} / 1 \mathrm{~m})=3.9 \times 10^{-9} \mathrm{in}$
(b) $\quad(1.0 \mathrm{~cm})\left(\frac{1 \mathrm{~m}}{100 \mathrm{~cm}}\right)\left(\frac{1 \text { atom }}{1.0 \times 10^{-10} \mathrm{~m}}\right)=1.0 \times 10^{8}$ atoms
18. To add values with significant figures, adjust all values to be added so that their units are all the same.

$$
1.80 \mathrm{~m}+142.5 \mathrm{~cm}+5.34 \times 10^{5} \mu \mathrm{~m}=1.80 \mathrm{~m}+1.425 \mathrm{~m}+0.534 \mathrm{~m}=3.759 \mathrm{~m}=3.76 \mathrm{~m}
$$

When adding, the final result is to be no more accurate than the least accurate number used. In this case, that is the first measurement, which is accurate to the hundredths place when expressed in meters.
19. (a) $(1 \mathrm{~km} / \mathrm{h})\left(\frac{0.621 \mathrm{mi}}{1 \mathrm{~km}}\right)=0.621 \mathrm{mi} / \mathrm{h}$, and so the conversion factor is $\frac{0.621 \mathrm{mi} / \mathrm{h}}{1 \mathrm{~km} / \mathrm{h}}$.
(b) $(1 \mathrm{~m} / \mathrm{s})\left(\frac{3.28 \mathrm{ft}}{1 \mathrm{~m}}\right)=3.28 \mathrm{ft} / \mathrm{s}$, and so the conversion factor is $\frac{3.28 \mathrm{ft} / \mathrm{s}}{1 \mathrm{~m} / \mathrm{s}}$.
(c) $(1 \mathrm{~km} / \mathrm{h})\left(\frac{1000 \mathrm{~m}}{1 \mathrm{~km}}\right)\left(\frac{1 \mathrm{~h}}{3600 \mathrm{~s}}\right)=0.278 \mathrm{~m} / \mathrm{s}$, and so the conversion factor is $\frac{0.278 \mathrm{~m} / \mathrm{s}}{1 \mathrm{~km} / \mathrm{h}}$.
20. One mile is $1.61 \times 10^{3} \mathrm{~m}$. It is 110 m longer than a $1500-\mathrm{m}$ race. The percentage difference is calculated here.

$$
\frac{110 \mathrm{~m}}{1500 \mathrm{~m}} \times 100 \%=7.3 \%
$$

21. (a) Find the distance by multiplying the speed times the time.

$$
1.00 \mathrm{ly}=\left(2.998 \times 10^{8} \mathrm{~m} / \mathrm{s}\right)\left(3.156 \times 10^{7} \mathrm{~s}\right)=9.462 \times 10^{15} \mathrm{~m} \approx 9.46 \times 10^{15} \mathrm{~m}
$$

(b) Do a unit conversion from ly to AU.

$$
(1.00 \mathrm{ly})\left(\frac{9.462 \times 10^{15} \mathrm{~m}}{1.00 \mathrm{ly}}\right)\left(\frac{1 \mathrm{AU}}{1.50 \times 10^{11} \mathrm{~m}}\right)=6.31 \times 10^{4} \mathrm{AU}
$$

(c) $\left(2.998 \times 10^{8} \mathrm{~m} / \mathrm{s}\right)\left(\frac{1 \mathrm{AU}}{1.50 \times 10^{11} \mathrm{~m}}\right)\left(\frac{3600 \mathrm{~s}}{1 \mathrm{hr}}\right)=7.20 \mathrm{AU} / \mathrm{h}$
22. $\left(82 \times 10^{9}\right.$ bytes $) \times \frac{1 \text { char }}{1 \text { byte }} \times \frac{1 \text { min }}{180 \text { char }} \times \frac{1 \text { hour }}{60 \text { min }} \times \frac{1 \text { day }}{8 \text { hour }} \times \frac{1 \text { year }}{365.25 \text { days }}=2598$ years $\approx 2600$ years
23. The surface area of a sphere is found by $A=4 \pi r^{2}=4 \pi(d / 2)^{2}=\pi d^{2}$.
(a) $A_{\text {Moon }}=\pi D_{\text {Moon }}^{2}=\pi\left(3.48 \times 10^{6} \mathrm{~m}\right)^{2}=3.80 \times 10^{13} \mathrm{~m}^{2}$
(b) $\frac{A_{\text {Earth }}}{A_{\text {Moon }}}=\frac{\pi D_{\text {Earth }}^{2}}{\pi D_{\text {Moon }}^{2}}=\left(\frac{D_{\text {Earth }}}{D_{\text {Moon }}}\right)^{2}=\left(\frac{R_{\text {Earth }}}{R_{\text {Moon }}}\right)^{2}=\left(\frac{6.38 \times 10^{6} \mathrm{~m}}{1.74 \times 10^{6} \mathrm{~m}}\right)^{2}=13.4$
24. (a) $2800=2.8 \times 10^{3} \approx 1 \times 10^{3}=10^{3}$
(b) $86.30 \times 10^{2}=8.630 \times 10^{3} \approx 10 \times 10^{3}=10^{4}$
(c) $0.0076=7.6 \times 10^{-3} \approx 10 \times 10^{-3}=10^{-2}$
(d) $15.0 \times 10^{8}=1.5 \times 10^{9} \approx 1 \times 10^{9}=10^{9}$
25. The textbook is approximately 25 cm deep and 5 cm wide. With books on both sides of a shelf, the shelf would need to be about 50 cm deep. If the aisle is 1.5 meter wide, then about $1 / 4$ of the floor space is covered by shelving. The number of books on a single shelf level is then $\frac{1}{4}\left(3500 \mathrm{~m}^{2}\right)\left(\frac{1 \text { book }}{(0.25 \mathrm{~m})(0.05 \mathrm{~m})}\right)=7.0 \times 10^{4}$ books. With 8 shelves of books, the total number of books stored is as follows.

$$
\left(7.0 \times 10^{4} \frac{\text { books }}{\text { shelf level }}\right)(8 \text { shelves }) \approx 6 \times 10^{5} \text { books }
$$

26. The distance across the United States is about 3000 miles.

$$
(3000 \mathrm{mi})(1 \mathrm{~km} / 0.621 \mathrm{mi})(1 \mathrm{hr} / 10 \mathrm{~km}) \approx 500 \mathrm{hr}
$$

Of course, it would take more time on the clock for the runner to run across the U.S. The runner could obviously not run for 500 hours non-stop. If they could run for 5 hours a day, then it would take about 100 days for them to cross the country.
27. A commonly accepted measure is that a person should drink eight 8 -oz. glasses of water each day. That is about 2 quarts, or 2 liters of water per day. Approximate the lifetime as 70 years.

$$
(70 \mathrm{y})(365 \mathrm{~d} / 1 \mathrm{y})(2 \mathrm{~L} / 1 \mathrm{~d}) \approx 5 \times 10^{4} \mathrm{~L}
$$

28. An NCAA-regulation football field is 360 feet long (including the end zones) and 160 feet wide, which is about 110 meters by 50 meters, or $5500 \mathrm{~m}^{2}$. The mower has a cutting width of 0.5 meters. Thus the distance to be walked is as follows.

$$
d=\frac{\text { area }}{\text { width }}=\frac{5500 \mathrm{~m}^{2}}{0.5 \mathrm{~m}}=11000 \mathrm{~m}=11 \mathrm{~km}
$$

At a speed of $1 \mathrm{~km} / \mathrm{hr}$, then it will take about 11 h to mow the field.
29. In estimating the number of dentists, the assumptions and estimates needed are:
the population of the city
the number of patients that a dentist sees in a day
the number of days that a dentist works in a year
the number of times that each person visits the dentist each year
We estimate that a dentist can see 10 patients a day, that a dentist works 225 days a year, and that each person visits the dentist twice per year.
(a) For San Francisco, the population as of 2001 was about 1.7 million, so we estimate the population at two million people. The number of dentists is found by the following calculation.

$$
\left(2 \times 10^{6} \text { people }\right)\left(\frac{2 \frac{\text { visits }}{\text { year }}}{1 \text { person }}\right)\left(\frac{1 \text { yr }}{225 \text { workdays }}\right)\left(\frac{1 \text { dentist }}{10 \frac{\text { visits }}{\text { workday }}}\right) \approx 1800 \text { dentists }
$$

(b) For Marion, Indiana, the population is about 50,000. The number of dentists is found by a similar calculation to that in part (a), and would be 45 dentists. There are about 50 dentists listed in the 2005 yellow pages.
30. Assume that the tires last for 5 years, and so there is a tread wearing of $0.2 \mathrm{~cm} / \mathrm{year}$. Assume the average tire has a radius of 40 cm , and a width of 10 cm . Thus the volume of rubber that is becoming pollution each year from one tire is the surface area of the tire, times the thickness per year
that is wearing. Also assume that there are $1.5 \times 10^{8}$ automobiles in the country - approximately one automobile for every two people. And there are 4 tires per automobile. The mass wear per year is given by the following calculation.

$$
\begin{aligned}
\left(\frac{\text { mass }}{\text { year }}\right) & =\left(\frac{\text { surface area }}{\text { tire }}\right)\left(\frac{\text { thickness wear }}{\text { year }}\right)(\text { density of rubber })(\# \text { of tires }) \\
& =\left[\frac{2 \pi(0.4 \mathrm{~m})(0.1 \mathrm{~m})}{1 \text { tire }}\right](0.002 \mathrm{~m} / \mathrm{y})\left(1200 \mathrm{~kg} / \mathrm{m}^{3}\right)\left(6.0 \times 10^{8} \text { tires }\right)=4 \times 10^{8} \mathrm{~kg} / \mathrm{y}
\end{aligned}
$$

31. Consider the diagram shown (not to scale). The balloon is a distance $h$ above the surface of the Earth, and the tangent line from the balloon height to the surface of the earth indicates the location of the horizon, a distance $d$ away from the balloon. Use the Pythagorean theorem.

$$
\begin{aligned}
& (r+h)^{2}=r^{2}+d^{2} \rightarrow r^{2}+2 r h+h^{2}=r^{2}+d^{2} \\
& 2 r h+h^{2}=d^{2} \rightarrow d=\sqrt{2 r h+h^{2}} \\
& d=\sqrt{2\left(6.4 \times 10^{6} \mathrm{~m}\right)(200 \mathrm{~m})+(200 \mathrm{~m})^{2}}=5.1 \times 10^{4} \mathrm{~m} \approx 5 \times 10^{4} \mathrm{~m}(\approx 80 \mathrm{mi})
\end{aligned}
$$


32. At $\$ 1,000$ per day, you would earn $\$ 30,000$ in the 30 days. With the other pay method, you would get $\$ 0.01\left(2^{t-1}\right)$ on the $t^{\text {th }}$ day. On the first day, you get $\$ 0.01\left(2^{1-1}\right)=\$ 0.01$. On the second day, you get $\$ 0.01\left(2^{2-1}\right)=\$ 0.02$. On the third day, you get $\$ 0.01\left(2^{3-1}\right)=\$ 0.04$. On the $30^{\text {th }}$ day, you get $\$ 0.01\left(2^{30-1}\right)=\$ 5.4 \times 10^{6}$, which is over 5 million dollars. Get paid by the second method.
33. In the figure in the textbook, the distance $d$ is perpendicular to the vertical radius. Thus there is a right triangle, with legs of $d$ and $R$, and a hypotenuse of $R+h$. Since $h \ll R, h^{2} \ll 2 R h$.

$$
\begin{aligned}
& d^{2}+R^{2}=(R+h)^{2}=R^{2}+2 R h+h^{2} \rightarrow d^{2}=2 R h+h^{2} \rightarrow d^{2} \approx 2 R h \rightarrow \\
& R=\frac{d^{2}}{2 h}=\frac{(4400 \mathrm{~m})^{2}}{2(1.5 \mathrm{~m})}=6.5 \times 10^{6} \mathrm{~m}
\end{aligned}
$$

A better measurement gives $R=6.38 \times 10^{6} \mathrm{~m}$.
34. To see the Sun "disappear," your line of sight to the top of the Sun is tangent to the Earth's surface. Initially, you are lying down at point A , and you see the first sunset. Then you stand up, elevating your eyes by the height $h$. While standing, your line of sight is tangent to the Earth's surface at point B, and so that is the direction to the second sunset. The angle $\theta$ is the angle through which the Sun appears to move relative to the Earth during the time to be measured. The distance $d$ is the distance from your eyes when standing to point B .
Use the Pythagorean theorem for the following
 relationship.

$$
d^{2}+R^{2}=(R+h)^{2}=R^{2}+2 R h+h^{2} \rightarrow d^{2}=2 R h+h^{2}
$$

The distance $h$ is much smaller than the distance $R$, and so $h^{2} \ll 2 R h$ which leads to $d^{2} \approx 2 R h$. We also have from the same triangle that $d / R=\tan \theta$, and so $d=R \tan \theta$. Combining these two relationships gives $d^{2} \approx 2 R h=R^{2} \tan ^{2} \theta$, and so $R=\frac{2 h}{\tan ^{2} \theta}$.
The angle $\theta$ can be found from the height change and the radius of the Earth. The elapsed time between the two sightings can then be found from the angle, knowing that a full revolution takes 24 hours.

$$
\begin{aligned}
& R=\frac{2 h}{\tan ^{2} \theta} \rightarrow \theta=\tan ^{-1} \sqrt{\frac{2 h}{R}}=\tan ^{-1} \sqrt{\frac{2(1.3 \mathrm{~m})}{6.38 \times 10^{6} \mathrm{~m}}}=\left(3.66 \times 10^{-2}\right)^{\circ} \\
& \frac{\theta}{360^{\circ}}=\frac{t \mathrm{sec}}{24 \mathrm{~h} \times \frac{3600 \mathrm{~s}}{1 \mathrm{~h}}} \rightarrow \\
& t=\left(\frac{\theta}{360^{\circ}}\right)\left(24 \mathrm{~h} \times \frac{3600 \mathrm{~s}}{1 \mathrm{~h}}\right)=\left(\frac{\left(3.66 \times 10^{-2}\right)^{\circ}}{360^{\circ}}\right)\left(24 \mathrm{~h} \times \frac{3600 \mathrm{~s}}{1 \mathrm{~h}}\right)=8.8 \mathrm{~s}
\end{aligned}
$$

35. Density units $=\frac{\text { mass units }}{\text { volume units }}=\left[\frac{M}{L^{3}}\right]$
36. (a) For the equation $v=A t^{3}-B t$, the units of $A t^{3}$ must be the same as the units of $v$. So the units of $A$ must be the same as the units of $v / t^{3}$, which would be $L / T^{4}$. Also, the units of $B t$ must be the same as the units of $v$. So the units of $B$ must be the same as the units of $v / t$, which would be $L / T^{2}$.
(b) For $A$, the SI units would be $\mathrm{m} / \mathrm{s}^{4}$, and for B, the SI units would be $\mathrm{m} / \mathrm{s}^{2}$.
37. (a) The quantity $v t^{2}$ has units of $(\mathrm{m} / \mathrm{s})\left(\mathrm{s}^{2}\right)=\mathrm{m} \cdot \mathrm{s}$, which do not match with the units of meters for $x$. The quantity 2 at has units $\left(\mathrm{m} / \mathrm{s}^{2}\right)(\mathrm{s})=\mathrm{m} / \mathrm{s}$, which also do not match with the units of meters for $x$. Thus this equation cannot be correct.
(b) The quantity $v_{0} t$ has units of $(\mathrm{m} / \mathrm{s})(\mathrm{s})=\mathrm{m}$, and $\frac{1}{2} a t^{2}$ has units of $\left(\mathrm{m} / \mathrm{s}^{2}\right)\left(\mathrm{s}^{2}\right)=\mathrm{m}$. Thus, since each term has units of meters, this equation can be correct.
(c) The quantity $v_{0} t$ has units of $(\mathrm{m} / \mathrm{s})(\mathrm{s})=\mathrm{m}$, and $2 a t^{2}$ has units of $\left(\mathrm{m} / \mathrm{s}^{2}\right)\left(\mathrm{s}^{2}\right)=\mathrm{m}$. Thus, since each term has units of meters, this equation can be correct.
38. $t_{p}=\sqrt{\frac{G h}{c^{5}}} \rightarrow \sqrt{\frac{\left[\frac{L^{3}}{M T^{2}}\right]\left[\frac{M L^{2}}{T}\right]}{\left[\frac{L}{T}\right]^{5}}}=\sqrt{\left[\frac{L^{3} L^{2} T^{5} M}{M T^{3} L^{5}}\right]}=\sqrt{\left[\frac{T^{5}}{T^{3}}\right]}=\sqrt{\left[T^{2}\right]}=[T]$
39. The percentage accuracy is $\frac{2 \mathrm{~m}}{2 \times 10^{7} \mathrm{~m}} \times 100 \%=1 \times 10^{-5} \%$. The distance of $20,000,000 \mathrm{~m}$ needs to be distinguishable from $20,000,002 \mathrm{~m}$, which means that 8 significant figures are needed in the distance measurements.
40. Multiply the number of chips per wafer times the number of wafers that can be made from a cylinder.

$$
\left(100 \frac{\text { chips }}{\text { wafer }}\right)\left(\frac{1 \text { wafer }}{0.300 \mathrm{~mm}}\right)\left(\frac{250 \mathrm{~mm}}{1 \text { cylinder }}\right)=83,000 \frac{\text { chips }}{\text { cylinder }}
$$

41. (a) \# of seconds in 1.00 y :

$$
\begin{aligned}
& 1.00 \mathrm{y}=(1.00 \mathrm{y})\left(\frac{3.156 \times 10^{7} \mathrm{~s}}{1 \mathrm{y}}\right)=3.16 \times 10^{7} \mathrm{~s} \\
& 1.00 \mathrm{y}=(1.00 \mathrm{y})\left(\frac{3.156 \times 10^{7} \mathrm{~s}}{1 \mathrm{y}}\right)\left(\frac{1 \times 10^{9} \mathrm{~ns}}{1 \mathrm{~s}}\right)= \\
& 1.00 \mathrm{~s}=(1.00 \mathrm{~s})\left(\frac{1 \mathrm{y}}{3.156 \times 10^{7} \mathrm{~s}}\right)=3.17 \times 10^{-8} \mathrm{y}
\end{aligned}
$$

$$
\text { (b) \# of nanoseconds in } 1.00 \mathrm{y}: \quad 1.00 \mathrm{y}=(1.00 \mathrm{y})\left(\frac{3.156 \times 10^{7} \mathrm{~s}}{1 \mathrm{y}}\right)\left(\frac{1 \times 10^{9} \mathrm{~ns}}{1 \mathrm{~s}}\right)=3.16 \times 10^{16} \mathrm{~ns}
$$

(c) \# of years in 1.00 s :
42. Since the meter is longer than the yard, the soccer field is longer than the football field.

$$
\begin{aligned}
& L_{\text {soccer }}-L_{\text {football }}=100 \mathrm{~m} \times \frac{1.09 \mathrm{yd}}{1 \mathrm{~m}}-100 \mathrm{yd}=9 \mathrm{yd} \\
& L_{\text {socecer }}-L_{\text {football }}=100 \mathrm{~m}-100 \mathrm{yd} \times \frac{1 \mathrm{~m}}{1.09 \mathrm{yd}}=8 \mathrm{~m}
\end{aligned}
$$

Since the soccer field is 109 yd compare to the 100 -yd football field, the soccer field is $9 \%$ longer than the football field.
43. Assume that the alveoli are spherical, and that the volume of a typical human lung is about 2 liters, which is $.002 \mathrm{~m}^{3}$. The diameter can be found from the volume of a sphere, $\frac{4}{3} \pi r^{3}$.

$$
\begin{aligned}
& \frac{4}{3} \pi r^{3}=\frac{4}{3} \pi(d / 2)^{3}=\frac{\pi d^{3}}{6} \\
& \left(3 \times 10^{8}\right) \pi \frac{d^{3}}{6}=2 \times 10^{-3} \mathrm{~m}^{3} \rightarrow d=\left[\frac{6\left(2 \times 10^{-3}\right)}{3 \times 10^{8} \pi} \mathrm{~m}^{3}\right]^{1 / 3}=2 \times 10^{-4} \mathrm{~m}
\end{aligned}
$$

44. 1 hectare $=(1$ hectare $)\left(\frac{1.000 \times 10^{4} \mathrm{~m}^{2}}{1 \text { hectare }}\right)\left(\frac{3.281 \mathrm{ft}}{1 \mathrm{~m}}\right)^{2}\left(\frac{1 \text { acre }}{4.356 \times 10^{4} \mathrm{ft}^{2}}\right)=2.471 \mathrm{acres}$
45. There are about $3 \times 10^{8}$ people in the United States. Assume that half of them have cars, that they each drive 12,000 miles per year, and their cars get 20 miles per gallon of gasoline.

$$
\left(3 \times 10^{8} \text { people }\right)\left(\frac{1 \text { automobile }}{2 \text { people }}\right)\left(\frac{12,000 \mathrm{mi} / \text { auto }}{1 \mathrm{y}}\right)\left(\frac{1 \text { gallon }}{20 \mathrm{mi}}\right) \approx 1 \times 10^{11} \mathrm{gal} / \mathrm{y}
$$

46. (a) $\left(\frac{10^{-15} \mathrm{~kg}}{1 \text { bacterium }}\right)\left(\frac{1 \text { proton or neutron }}{10^{-27} \mathrm{~kg}}\right)=10^{12}$ protons or neutrons
(b) $\left(\frac{10^{-17} \mathrm{~kg}}{1 \text { DNA molecule }}\right)\left(\frac{1 \text { proton or neutron }}{10^{-27} \mathrm{~kg}}\right)=10^{10}$ protons or neutrons
(c) $\left(\frac{10^{2} \mathrm{~kg}}{1 \text { human }}\right)\left(\frac{1 \text { proton or neutron }}{10^{-27} \mathrm{~kg}}\right)=10^{29}$ protons or neutrons
(d) $\left(\frac{10^{41} \mathrm{~kg}}{1 \text { galaxy }}\right)\left(\frac{1 \text { proton or neutron }}{10^{-27} \mathrm{~kg}}\right)=10^{68}$ protons or neutrons
47. The volume of water used by the people can be calculated as follows:

$$
\left(4 \times 10^{4} \text { people }\right)\left(\frac{1200 \mathrm{~L} / \text { day }}{4 \text { people }}\right)\left(\frac{365 \text { day }}{1 \mathrm{y}}\right)\left(\frac{1000 \mathrm{~cm}^{3}}{1 \mathrm{~L}}\right)\left(\frac{1 \mathrm{~km}}{10^{5} \mathrm{~cm}}\right)^{3}=4.38 \times 10^{-3} \mathrm{~km}^{3} / \mathrm{y}
$$

The depth of water is found by dividing the volume by the area.

$$
d=\frac{V}{A}=\frac{4.38 \times 10^{-3} \mathrm{~km}^{3} / \mathrm{y}}{50 \mathrm{~km}^{2}}=\left(8.76 \times 10^{-5} \frac{\mathrm{~km}}{\mathrm{y}}\right)\left(\frac{10^{5} \mathrm{~cm}}{1 \mathrm{~km}}\right)=8.76 \mathrm{~cm} / \mathrm{y} \approx 9 \mathrm{~cm} / \mathrm{y}
$$

48. Approximate the gumball machine as a rectangular box with a square cross-sectional area. In counting gumballs across the bottom, there are about 10 in a row. Thus we estimate that one layer contains about 100 gumballs. In counting vertically, we see that there are about 15 rows. Thus we estimate that there are 1500 gumballs in the machine.
49. Make the estimate that each person has 1.5 loads of laundry per week, and that there are 300 million people in the United States.

$$
\left(300 \times 10^{6} \text { people }\right) \times \frac{1.5 \text { loads } / \text { week }}{1 \text { person }} \times \frac{52 \text { weeks }}{1 \mathrm{y}} \times \frac{0.1 \mathrm{~kg}}{1 \text { load }}=2.34 \times 10^{9} \frac{\mathrm{~kg}}{\mathrm{y}} \approx 2 \times 10^{9} \frac{\mathrm{~kg}}{\mathrm{y}}
$$

50. The volume of a sphere is given by $V=\frac{4}{3} \pi r^{3}$, and so the radius is $r=\left(\frac{3 V}{4 \pi}\right)^{1 / 3}$. For a 1-ton rock, the volume is calculated from the density, and then the diameter from the volume.

$$
\begin{aligned}
& V=(1 \mathrm{~T})\left(\frac{2000 \mathrm{lb}}{1 \mathrm{~T}}\right)\left(\frac{1 \mathrm{ft}^{3}}{186 \mathrm{lb}}\right)=10.8 \mathrm{ft}^{3} \\
& d=2 r=2\left(\frac{3 V}{4 \pi}\right)^{1 / 3}=2\left[\frac{3\left(10.8 \mathrm{ft}^{3}\right)}{4 \pi}\right]^{1 / 3}=2.74 \mathrm{ft} \approx 3 \mathrm{ft}
\end{aligned}
$$

51. $\left(783.216 \times 10^{6}\right.$ bytes $) \times \frac{8 \text { bits }}{1 \text { byte }} \times \frac{1 \mathrm{sec}}{1.4 \times 10^{6} \mathrm{bits}} \times \frac{1 \mathrm{~min}}{60 \mathrm{sec}}=74.592 \mathrm{~min} \approx 75 \mathrm{~min}$
52. A pencil has a diameter of about 0.7 cm . If held about 0.75 m from the eye, it can just block out the Moon. The ratio of pencil diameter to arm length is the same as the ratio of Moon diameter to Moon distance. From the diagram, we have the following ratios.


$$
\frac{\text { Pencil diameter }}{\text { Pencil distance }}=\frac{\text { Moon diameter }}{\text { Moon distance }} \rightarrow
$$

$$
\text { Moon diameter }=\frac{\text { Pencil diameter }}{\text { Pencil distance }}(\text { Moon distance })=\frac{7 \times 10^{-3} \mathrm{~m}}{0.75 \mathrm{~m}}\left(3.8 \times 10^{5} \mathrm{~km}\right) \approx 3500 \mathrm{~km}
$$

The actual value is 3480 km .
53. To calculate the mass of water, we need to find the volume of water, and then convert the volume to mass. The volume of water is the area of the city $\left(40 \mathrm{~km}^{2}\right)$ times the depth of the water $(1.0 \mathrm{~cm})$.

$$
\left[\left(4 \times 10^{1} \mathrm{~km}^{2}\right)\left(\frac{10^{5} \mathrm{~cm}}{1 \mathrm{~km}}\right)^{2}\right](1.0 \mathrm{~cm})\left(\frac{10^{-3} \mathrm{~kg}}{1 \mathrm{~cm}^{3}}\right)\left(\frac{1 \text { metric ton }}{10^{3} \mathrm{~kg}}\right)=4 \times 10^{5} \text { metric tons }
$$

To find the number of gallons, convert the volume to gallons.

$$
\left[\left(4 \times 10^{1} \mathrm{~km}^{2}\right)\left(\frac{10^{5} \mathrm{~cm}}{1 \mathrm{~km}}\right)^{2}\right](1.0 \mathrm{~cm})\left(\frac{1 \mathrm{~L}}{1 \times 10^{3} \mathrm{~cm}^{3}}\right)\left(\frac{1 \mathrm{gal}}{3.78 \mathrm{~L}}\right)=1.06 \times 10^{8} \mathrm{gal} \approx 1 \times 10^{8} \mathrm{gal}
$$

54. A cubit is about a half of a meter, by measuring several people's forearms. Thus the dimensions of Noah's ark would be 150 m long, 25 m wide, 15 m high. The volume of the ark is found by multiplying the three dimensions.

$$
V=(150 \mathrm{~m})(25 \mathrm{~m})(15 \mathrm{~m})=5.625 \times 10^{4} \mathrm{~m}^{3} \approx 6 \times 10^{4} \mathrm{~m}^{3}
$$

55. The person walks $4 \mathrm{~km} / \mathrm{h}, 10$ hours each day. The radius of the Earth is about 6380 km , and the distance around the Earth at the equator is the circumference, $2 \pi R_{\text {Earth }}$. We assume that the person can "walk on water," and so ignore the existence of the oceans.

$$
2 \pi(6380 \mathrm{~km})\left(\frac{1 \mathrm{~h}}{4 \mathrm{~km}}\right)\left(\frac{1 \mathrm{~d}}{10 \mathrm{~h}}\right)=1 \times 10^{3} \mathrm{~d}
$$

56. The volume of the oil will be the area times the thickness. The area is $\pi r^{2}=\pi(d / 2)^{2}$, and so

$$
V=\pi(d / 2)^{2} t \rightarrow d=2 \sqrt{\frac{V}{\pi t}}=2 \sqrt{\frac{1000 \mathrm{~cm}^{3}\left(\frac{1 \mathrm{~m}}{100 \mathrm{~cm}}\right)^{3}}{\pi\left(2 \times 10^{-10} \mathrm{~m}\right)}}=3 \times 10^{3} \mathrm{~m} .
$$

57. Consider the diagram shown. Let $\ell$ represent is the distance she walks upstream, which is about 120 yards. Find the distance across the river from the diagram.

$$
\begin{aligned}
& \tan 60^{\circ}=\frac{d}{\ell} \rightarrow d=\ell \tan 60^{\circ}=(120 \mathrm{yd}) \tan 60^{\circ}=210 \mathrm{yd} \\
& (210 \mathrm{yd})\left(\frac{3 \mathrm{ft}}{1 \mathrm{yd}}\right)\left(\frac{0.305 \mathrm{~m}}{1 \mathrm{ft}}\right)=190 \mathrm{~m}
\end{aligned}
$$


58. $\left(\frac{8 \mathrm{~s}}{1 \mathrm{y}}\right)\left(\frac{1 \mathrm{y}}{3.156 \times 10^{7} \mathrm{~s}}\right) \times 100 \%=3 \times 10^{-5} \%$
59. (a) $1.0 \AA=\left(1.0 \AA \begin{array}{c}\circ \\ \hline\end{array}\left(\frac{10^{-10} \mathrm{~m}}{1 \AA}\right)\left(\frac{1 \mathrm{~nm}}{10^{-9} \mathrm{~m}}\right)=0.10 \mathrm{~nm}\right.$
(b) $1.0 \AA=(1.0 \AA)\left(\frac{10^{-10} \mathrm{~m}}{1 \AA}\right)\left(\frac{1 \mathrm{fm}}{10^{-15} \mathrm{~m}}\right)=1.0 \times 10^{5} \mathrm{fm}$
(c) $1.0 \mathrm{~m}=(1.0 \mathrm{~m})\left(\frac{1 \AA^{\circ}}{10^{-10} \mathrm{~m}}\right)=1.0 \times 10^{10} \mathrm{~A}$
(d) $1.0 \mathrm{ly}=(1.0 \mathrm{ly})\left(\frac{9.46 \times 10^{15} \mathrm{~m}}{1 \mathrm{ly}}\right)\left(\frac{1 \AA^{\circ}}{10^{-10} \mathrm{~m}}\right)=9.5 \times 10^{25} \mathrm{~A}$
60. The volume of a sphere is found by $V=\frac{4}{3} \pi r^{3}$.

$$
\begin{aligned}
& V_{\text {Moon }}=\frac{4}{3} \pi R_{\text {Moon }}^{3}=\frac{4}{3} \pi\left(1.74 \times 10^{6} \mathrm{~m}\right)^{3}=2.21 \times 10^{19} \mathrm{~m}^{3} \\
& \frac{V_{\text {Earth }}}{V_{\text {Moon }}}=\frac{\frac{4}{3} \pi R_{\text {Earth }}^{3}}{\frac{4}{3} \pi R_{\text {Moon }}^{3}}=\left(\frac{R_{\text {Earth }}}{R_{\text {Moon }}}\right)^{3}=\left(\frac{6.38 \times 10^{6} \mathrm{~m}}{1.74 \times 10^{6} \mathrm{~m}}\right)^{3}=49.3
\end{aligned}
$$

Thus it would take about 49.3 Moons to create a volume equal to that of the Earth.
61. (a) Note that $\sin 15.0^{\circ}=0.259$ and $\sin 15.5^{\circ}=0.267$, and so $\Delta \sin \theta=0.267-0.259=0.008$.

$$
\left(\frac{\Delta \theta}{\theta}\right) 100=\left(\frac{0.5^{\circ}}{15.0^{\circ}}\right) 100=3 \% \quad\left(\frac{\Delta \sin \theta}{\sin \theta}\right) 100=\left(\frac{8 \times 10^{-3}}{0.259}\right) 100=3 \%
$$

(b) Note that $\sin 75.0^{\circ}=0.966$ and $\sin 75.5^{\circ}=0.968$, and so $\Delta \sin \theta=0.968-0.966=0.002$.

$$
\left(\frac{\Delta \theta}{\theta}\right) 100=\left(\frac{0.5^{\circ}}{75.0^{\circ}}\right) 100=0.7 \% \quad\left(\frac{\Delta \sin \theta}{\sin \theta}\right) 100=\left(\frac{2 \times 10^{-3}}{0.966}\right) 100=0.2 \%
$$

A consequence of this result is that when using a protractor, and you have a fixed uncertainty in the angle ( $\pm 0.5^{\circ}$ in this case), you should measure the angles from a reference line that gives a large angle measurement rather than a small one. Note above that the angles around $75^{\circ}$ had only a $0.2 \%$ error in $\sin \theta$, while the angles around $15^{\circ}$ had a $3 \%$ error in $\sin \theta$.
62. Utilize the fact that walking totally around the Earth along the meridian would trace out a circle whose full $360^{\circ}$ would equal the circumference of the Earth.

$$
(1 \text { minute })\left(\frac{1^{\circ}}{60 \text { minute }}\right)\left(\frac{2 \pi\left(6.38 \times 10^{3} \mathrm{~km}\right)}{360^{\circ}}\right)\left(\frac{0.621 \mathrm{mi}}{1 \mathrm{~km}}\right)=1.15 \mathrm{mi}
$$

63. Consider the body to be a cylinder, about 170 cm tall $\left(\approx 5^{\prime} 7^{\prime \prime}\right)$, and about 12 cm in cross-sectional radius (which corresponds to a 30 -inch waist). The volume of a cylinder is given by the area of the cross section times the height.

$$
V=\pi r^{2} h=\pi(0.12 \mathrm{~m})^{2}(1.7 \mathrm{~m})=7.69 \times 10^{-2} \mathrm{~m}^{3} \approx 8 \times 10^{-2} \mathrm{~m}^{3}
$$

64. The maximum number of buses would be needed during rush hour. We assume that a bus can hold 50 passengers.
(a) The current population of Washington, D.C. is about half a million people. We estimate that $10 \%$ of them ride the bus during rush hour.

$$
50,000 \text { passengers } \times \frac{1 \text { bus }}{50 \text { passengers }} \times \frac{1 \text { driver }}{1 \text { bus }} \approx 1000 \text { drivers }
$$

(b) For Marion, Indiana, the population is about 50,000 . Because the town is so much smaller geographically, we estimate that only $5 \%$ of the current population rides the bus during rush hour.

$$
2500 \text { passengers } \times \frac{1 \text { bus }}{50 \text { passengers }} \times \frac{1 \text { driver }}{1 \text { bus }} \approx 50 \text { drivers }
$$

65. The units for each term must be in liters, since the volume is in liters.

$$
\begin{aligned}
& {[\text { units of } 4.1][\mathrm{m}]=[\mathrm{L}] \rightarrow[\text { units of } 4.1]=\frac{\mathrm{L}}{\mathrm{~m}}} \\
& {[\text { units of } 0.018][\mathrm{y}]=[\mathrm{L}] \rightarrow[\text { units of } 0.018]=\frac{\mathrm{L}}{\mathrm{y}}} \\
& {[\text { units of } 2.69]=\mathrm{L}}
\end{aligned}
$$

66. density $=\frac{\text { mass }}{\text { volume }}=\frac{8 \mathrm{~g}}{2.8325 \mathrm{~cm}^{3}}=2.82 \mathrm{~g} / \mathrm{cm}^{3} \approx 3 \mathrm{~g} / \mathrm{cm}^{3}$
67. (a) $\frac{\mathrm{SA}_{\text {Earth }}}{\mathrm{SA}_{\text {Moon }}}=\frac{4 \pi R_{\text {Earth }}^{2}}{4 \pi R_{\text {Moon }}^{2}}=\frac{R_{\text {Earth }}^{2}}{R_{\text {Moon }}^{2}}=\frac{\left(6.38 \times 10^{3} \mathrm{~km}\right)^{2}}{\left(1.74 \times 10^{3} \mathrm{~km}\right)^{2}}=13.4$
(b) $\frac{\mathrm{V}_{\text {Earth }}}{\mathrm{V}_{\text {Moon }}}=\frac{\frac{4}{3} \pi R_{\text {Earth }}^{3}}{\frac{4}{3} \pi R_{\text {Moon }}^{3}}=\frac{R_{\text {Earth }}^{3}}{R_{\text {Moon }}^{3}}=\frac{\left(6.38 \times 10^{3} \mathrm{~km}\right)^{3}}{\left(1.74 \times 10^{3} \mathrm{~km}\right)^{3}}=49.3$
68. $\frac{\# \text { atoms }}{\mathrm{m}^{2}}=\frac{6.02 \times 10^{23} \text { atoms }}{4 \pi R_{\text {Earth }}^{2}}=\frac{6.02 \times 10^{23} \text { atoms }}{4 \pi\left(6.38 \times 10^{6} \mathrm{~m}\right)^{2}}=1.18 \times 10^{9} \frac{\text { atoms }}{\mathrm{m}^{2}}$
69. Multiply the volume of a spherical universe times the density of matter, adjusted to ordinary matter. The volume of a sphere is $\frac{4}{3} \pi r^{3}$.

$$
\begin{aligned}
m & =\rho V=\left(1 \times 10^{-26} \mathrm{~kg} / \mathrm{m}^{3}\right) \frac{4}{3} \pi\left(\left(13.7 \times 10^{9} \mathrm{ly}\right) \times \frac{9.46 \times 10^{15} \mathrm{~m}}{11 \mathrm{y}}\right)^{3}(0.04) \\
& =3.65 \times 10^{51} \mathrm{~kg} \approx 4 \times 10^{51} \mathrm{~kg}
\end{aligned}
$$

## CHAPTER 2: Describing Motion: Kinematics in One Dimension

## Responses to Questions

1. A car speedometer measures only speed, since it gives no indication of the direction in which the car is traveling.
2. If the velocity of an object is constant, the speed must also be constant. (A constant velocity means that the speed and direction are both constant.) If the speed of an object is constant, the velocity CAN vary. For example, a car traveling around a curve at constant speed has a varying velocity, since the direction of the velocity vector is changing.
3. When an object moves with constant velocity, the average velocity and the instantaneous velocity are the same at all times.
4. No, if one object has a greater speed than a second object, it does not necessarily have a greater acceleration. For example, consider a speeding car, traveling at constant velocity, which passes a stopped police car. The police car will accelerate from rest to try to catch the speeder. The speeding car has a greater speed than the police car (at least initially!), but has zero acceleration. The police car will have an initial speed of zero, but a large acceleration.
5. The accelerations of the motorcycle and the bicycle are the same, assuming that both objects travel in a straight line. Acceleration is the change in velocity divided by the change in time. The magnitude of the change in velocity in each case is the same, $10 \mathrm{~km} / \mathrm{h}$, so over the same time interval the accelerations will be equal.
6. Yes, for example, a car that is traveling northward and slowing down has a northward velocity and a southward acceleration.
7. Yes. If the velocity and the acceleration have different signs (opposite directions), then the object is slowing down. For example, a ball thrown upward has a positive velocity and a negative acceleration while it is going up. A car traveling in the negative $x$-direction and braking has a negative velocity and a positive acceleration.
8. Both velocity and acceleration are negative in the case of a car traveling in the negative $x$-direction and speeding up. If the upward direction is chosen as $+y$, a falling object has negative velocity and negative acceleration.
9. Car A is going faster at this instant and is covering more distance per unit time, so car A is passing car B. (Car B is accelerating faster and will eventually overtake car A.)
10. Yes. Remember that acceleration is a change in velocity per unit time, or a rate of change in velocity. So, velocity can be increasing while the rate of increase goes down. For example, suppose a car is traveling at $40 \mathrm{~km} / \mathrm{h}$ and a second later is going $50 \mathrm{~km} / \mathrm{h}$. One second after that, the car's speed is $55 \mathrm{~km} / \mathrm{h}$. The car's speed was increasing the entire time, but its acceleration in the second time interval was lower than in the first time interval.
11. If there were no air resistance, the ball's only acceleration during flight would be the acceleration due to gravity, so the ball would land in the catcher's mitt with the same speed it had when it left the bat, $120 \mathrm{~km} / \mathrm{h}$. The path of the ball as it rises and then falls would be symmetric.
12. (a) If air resistance is negligible, the acceleration of a freely falling object stays the same as the object falls toward the ground. (Note that the object's speed increases, but since it increases at a constant rate, the acceleration is constant.)
(b) In the presence of air resistance, the acceleration decreases. (Air resistance increases as speed increases. If the object falls far enough, the acceleration will go to zero and the velocity will become constant. See Section 5-6.)
13. Average speed is the displacement divided by the time. If the distances from $A$ to $B$ and from $B$ to $C$ are equal, then you spend more time traveling at $70 \mathrm{~km} / \mathrm{h}$ than at $90 \mathrm{~km} / \mathrm{h}$, so your average speed should be less than $80 \mathrm{~km} / \mathrm{h}$. If the distance from A to B (or B to C) is $x$, then the total distance traveled is $2 x$. The total time required to travel this distance is $x / 70$ plus $x / 90$. Then
$\bar{v}=\frac{d}{t}=\frac{2 x}{x / 70+x / 90}=\frac{2(90)(70)}{90+70}=79 \mathrm{~km} / \mathrm{h}$.
14. Yes. For example, a rock thrown straight up in the air has a constant, nonzero acceleration due to gravity for its entire flight. However, at the highest point it momentarily has a zero velocity. A car, at the moment it starts moving from rest, has zero velocity and nonzero acceleration.
15. Yes. Anytime the velocity is constant, the acceleration is zero. For example, a car traveling at a constant $90 \mathrm{~km} / \mathrm{h}$ in a straight line has nonzero velocity and zero acceleration.
16. A rock falling from a cliff has a constant acceleration IF we neglect air resistance. An elevator moving from the second floor to the fifth floor making stops along the way does NOT have a constant acceleration. Its acceleration will change in magnitude and direction as the elevator starts and stops. The dish resting on a table has a constant acceleration (zero).
17. The time between clinks gets smaller and smaller. The bolts all start from rest and all have the same acceleration, so at any moment in time, they will all have the same speed. However, they have different distances to travel in reaching the floor and therefore will be falling for different lengths of time. The later a bolt hits, the longer it has been accelerating and therefore the faster it is moving. The time intervals between impacts decrease since the higher a bolt is on the string, the faster it is moving as it reaches the floor. In order for the clinks to occur at equal time intervals, the higher the bolt, the further it must be tied from its neighbor. Can you guess the ratio of lengths?
18. The slope of the position versus time curve is the velocity. The object starts at the origin with a constant velocity (and therefore zero acceleration), which it maintains for about 20 s. For the next 10 s , the positive curvature of the graph indicates the object has a positive acceleration; its speed is increasing. From 30 s to 45 s , the graph has a negative curvature; the object uniformly slows to a stop, changes direction, and then moves backwards with increasing speed. During this time interval its acceleration is negative, since the object is slowing down while traveling in the positive direction and then speeding up while traveling in the negative direction. For the final 5 s shown, the object continues moving in the negative direction but slows down, which gives it a positive acceleration. During the 50 s shown, the object travels from the origin to a point 20 m away, and then back 10 m to end up 10 m from the starting position.
19. The object begins with a speed of $14 \mathrm{~m} / \mathrm{s}$ and increases in speed with constant positive acceleration from $t=0$ until $t=45 \mathrm{~s}$. The acceleration then begins to decrease, goes to zero at $t=50 \mathrm{~s}$, and then goes negative. The object slows down from $t=50 \mathrm{~s}$ to $t=90 \mathrm{~s}$, and is at rest from $t=90 \mathrm{~s}$ to $t=108$ s . At that point the acceleration becomes positive again and the velocity increases from $t=108 \mathrm{~s}$ to $t=130 \mathrm{~s}$.

## Solutions to Problems

1. The distance of travel (displacement) can be found by rearranging Eq. 2-2 for the average velocity. Also note that the units of the velocity and the time are not the same, so the speed units will be converted.

$$
\bar{v}=\frac{\Delta x}{\Delta t} \rightarrow \Delta x=\bar{v} \Delta t=(110 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~h}}{3600 \mathrm{~s}}\right)(2.0 \mathrm{~s})=0.061 \mathrm{~km}=61 \mathrm{~m}
$$

2. The average speed is given by Eq. 2-2.

$$
\bar{v}=\Delta x / \Delta t=235 \mathrm{~km} / 3.25 \mathrm{~h}=72.3 \mathrm{~km} / \mathrm{h}
$$

3. The average velocity is given by Eq. 2.2.

$$
\bar{v}=\frac{\Delta x}{\Delta t}=\frac{8.5 \mathrm{~cm}-4.3 \mathrm{~cm}}{4.5 \mathrm{~s}-(-2.0 \mathrm{~s})}=\frac{4.2 \mathrm{~cm}}{6.5 \mathrm{~s}}=0.65 \mathrm{~cm} / \mathrm{s}
$$

The average speed cannot be calculated. To calculate the average speed, we would need to know the actual distance traveled, and it is not given. We only have the displacement.
4. The average velocity is given by Eq. 2-2.

$$
\bar{v}=\frac{\Delta x}{\Delta t}=\frac{-4.2 \mathrm{~cm}-3.4 \mathrm{~cm}}{5.1 \mathrm{~s}-3.0 \mathrm{~s}}=\frac{-7.6 \mathrm{~cm}}{2.1 \mathrm{~s}}=-3.6 \mathrm{~cm} / \mathrm{s}
$$

The negative sign indicates the direction.
5. The speed of sound is intimated in the problem as 1 mile per 5 seconds. The speed is calculated as follows.

$$
\text { speed }=\frac{\text { distance }}{\text { time }}=\left(\frac{1 \mathrm{mi}}{5 \mathrm{~s}}\right)\left(\frac{1610 \mathrm{~m}}{1 \mathrm{mi}}\right)=300 \mathrm{~m} / \mathrm{s}
$$

The speed of $300 \mathrm{~m} / \mathrm{s}$ would imply the sound traveling a distance of 900 meters (which is approximately 1 km ) in 3 seconds. So the rule could be approximated as 1 km every 3 seconds.
6. The time for the first part of the trip is calculated from the initial speed and the first distance.

$$
\bar{v}_{1}=\frac{\Delta x_{1}}{\Delta t_{1}} \rightarrow \Delta t_{1}=\frac{\Delta x_{1}}{\bar{v}_{1}}=\frac{130 \mathrm{~km}}{95 \mathrm{~km} / \mathrm{h}}=1.37 \mathrm{~h}=82 \mathrm{~min}
$$

The time for the second part of the trip is now calculated.

$$
\Delta t_{2}=\Delta t_{\text {total }}-\Delta t_{1}=3.33 \mathrm{~h}-1.37 \mathrm{~h}=1.96 \mathrm{~h}=118 \mathrm{~min}
$$

The distance for the second part of the trip is calculated from the average speed for that part of the trip and the time for that part of the trip.

$$
\bar{v}_{2}=\frac{\Delta x_{2}}{\Delta t_{2}} \rightarrow \Delta x_{2}=\bar{v}_{2} \Delta t_{2}=(65 \mathrm{~km} / \mathrm{h})(1.96 \mathrm{~h})=127.5 \mathrm{~km}=1.3 \times 10^{2} \mathrm{~km}
$$

(a) The total distance is then $\Delta x_{\text {total }}=\Delta x_{1}+\Delta x_{2}=130 \mathrm{~km}+127.5 \mathrm{~km}=257.5 \mathrm{~km} \approx 2.6 \times 10^{2} \mathrm{~km}$.
(b) The average speed is NOT the average of the two speeds. Use the definition of average speed, Eq. 2-2.

$$
\bar{v}=\frac{\Delta x_{\text {total }}}{\Delta t_{\text {total }}}=\frac{257.5 \mathrm{~km}}{3.33 \mathrm{~h}}=77 \mathrm{~km} / \mathrm{h}
$$

7. The distance traveled is $116 \mathrm{~km}+\frac{1}{2}(116 \mathrm{~km})=174 \mathrm{~km}$, and the displacement is $116 \mathrm{~km}-\frac{1}{2}(116 \mathrm{~km})=58 \mathrm{~km}$. The total time is $14.0 \mathrm{~s}+4.8 \mathrm{~s}=18.8 \mathrm{~s}$.
(a) Average speed $=\frac{\text { distance }}{\text { time elapsed }}=\frac{174 \mathrm{~m}}{18.8 \mathrm{~s}}=9.26 \mathrm{~m} / \mathrm{s}$
(b) Average velocity $=v_{\text {avg }}=\frac{\text { displacement }}{\text { time elapsed }}=\frac{58 \mathrm{~m}}{18.8 \mathrm{~s}}=3.1 \mathrm{~m} / \mathrm{s}$
8. (a)


The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH02.XLS", on tab "Problem 2.8a".
(b) The average velocity is the displacement divided by the elapsed time.

$$
\bar{v}=\frac{x(3.0)-x(0.0)}{3.0 \mathrm{~s}-0.0 \mathrm{~s}}=\frac{\left[34+10(3.0)-2(3.0)^{3}\right] \mathrm{m}-(34 \mathrm{~m})}{3.0 \mathrm{~s}}=-8.0 \mathrm{~m} / \mathrm{s}
$$

(c) The instantaneous velocity is given by the derivative of the position function.

$$
v=\frac{d x}{d t}=\left(10-6 t^{2}\right) \mathrm{m} / \mathrm{s} \quad 10-6 t^{2}=0 \rightarrow t=\sqrt{\frac{5}{3}} \mathrm{~s}=1.3 \mathrm{~s}
$$

This can be seen from the graph as the "highest" point on the graph.
9. Slightly different answers may be obtained since the data comes from reading the graph.
(a) The instantaneous velocity is given by the slope of the tangent line to the curve. At $t=10.0 \mathrm{~s}$, the slope is approximately $v(10) \approx \frac{3 \mathrm{~m}-0}{10.0 \mathrm{~s}-0}=0.3 \mathrm{~m} / \mathrm{s}$.
(b) At $t=30.0 \mathrm{~s}$, the slope of the tangent line to the curve, and thus the instantaneous velocity, is approximately $v(30) \approx \frac{22 \mathrm{~m}-10 \mathrm{~m}}{35 \mathrm{~s}-25 \mathrm{~s}}=1.2 \mathrm{~m} / \mathrm{s}$.
(c) The average velocity is given by $\bar{v}=\frac{x(5)-x(0)}{5.0 \mathrm{~s}-0 \mathrm{~s}}=\frac{1.5 \mathrm{~m}-0}{5.0 \mathrm{~s}}=0.30 \mathrm{~m} / \mathrm{s}$.
(d) The average velocity is given by $\bar{v}=\frac{x(30)-x(25)}{30.0 \mathrm{~s}-25.0 \mathrm{~s}}=\frac{16 \mathrm{~m}-9 \mathrm{~m}}{5.0 \mathrm{~s}}=1.4 \mathrm{~m} / \mathrm{s}$.
(e) The average velocity is given by $\bar{v}=\frac{x(50)-x(40)}{50.0 \mathrm{~s}-40.0 \mathrm{~s}}=\frac{10 \mathrm{~m}-19.5 \mathrm{~m}}{10.0 \mathrm{~s}}=-0.95 \mathrm{~m} / \mathrm{s}$.
10. (a) Multiply the reading rate times the bit density to find the bit reading rate.

$$
N=\frac{1.2 \mathrm{~m}}{1 \mathrm{~s}} \times \frac{1 \mathrm{bit}}{0.28 \times 10^{-6} \mathrm{~m}}=4.3 \times 10^{6} \mathrm{bits} / \mathrm{s}
$$

(b) The number of excess bits is $N-N_{0}$.

$$
\begin{aligned}
& N-N_{0}=4.3 \times 10^{6} \mathrm{bits} / \mathrm{s}-1.4 \times 10^{6} \mathrm{bits} / \mathrm{s}=2.9 \times 10^{6} \mathrm{bits} / \mathrm{s} \\
& \frac{N-N_{0}}{N}=\frac{2.9 \times 10^{6} \mathrm{bits} / \mathrm{s}}{4.3 \times 10^{6} \mathrm{bits} / \mathrm{s}}=0.67=67 \%
\end{aligned}
$$

11. Both objects will have the same time of travel. If the truck travels a distance $\Delta x_{\text {truck }}$, then the distance the car travels will be $\Delta x_{\text {car }}=\Delta x_{\text {truck }}+110 \mathrm{~m}$. Use Eq. 2-2 for average speed, $\bar{v}=\Delta x / \Delta t$, solve for time, and equate the two times.

$$
\Delta t=\frac{\Delta x_{\text {tuck }}}{\bar{v}_{\text {truck }}}=\frac{\Delta x_{\text {car }}}{\bar{v}_{\text {car }}} \quad \frac{\Delta x_{\text {tuck }}}{75 \mathrm{~km} / \mathrm{h}}=\frac{\Delta x_{\text {tuck }}+110 \mathrm{~m}}{95 \mathrm{~km} / \mathrm{h}}
$$

Solving for $\Delta x_{\text {truck }}$ gives $\Delta x_{\text {truck }}=(110 \mathrm{~m}) \frac{(75 \mathrm{~km} / \mathrm{h})}{(95 \mathrm{~km} / \mathrm{h}-75 \mathrm{~km} / \mathrm{h})}=412.5 \mathrm{~m}$.
The time of travel is $\Delta t=\frac{\Delta x_{\text {truck }}}{\bar{v}_{\text {truck }}}=\left(\frac{412.5 \mathrm{~m}}{75000 \mathrm{~m} / \mathrm{h}}\right)\left(\frac{60 \mathrm{~min}}{1 \mathrm{~h}}\right)=0.33 \mathrm{~min}=19.8 \mathrm{~s}=2.0 \times 10^{1} \mathrm{~s}$.
Also note that $\Delta t=\frac{\Delta x_{\text {car }}}{\bar{v}_{\text {car }}}=\left(\frac{412.5 \mathrm{~m}+110 \mathrm{~m}}{95000 \mathrm{~m} / \mathrm{h}}\right)\left(\frac{60 \mathrm{~min}}{1 \mathrm{~h}}\right)=0.33 \mathrm{~min}=20 \mathrm{~s}$.

## ALTERNATE SOLUTION:

The speed of the car relative to the truck is $95 \mathrm{~km} / \mathrm{h}-75 \mathrm{~km} / \mathrm{h}=20 \mathrm{~km} / \mathrm{h}$. In the reference frame of the truck, the car must travel 110 m to catch it.

$$
\Delta t=\frac{0.11 \mathrm{~km}}{20 \mathrm{~km} / \mathrm{h}}\left(\frac{3600 \mathrm{~s}}{1 \mathrm{~h}}\right)=19.8 \mathrm{~s}
$$

12. Since the locomotives have the same speed, they each travel half the distance, 4.25 km . Find the time of travel from the average speed.

$$
\bar{v}=\frac{\Delta x}{\Delta t} \rightarrow \Delta t=\frac{\Delta x}{\bar{v}}=\frac{4.25 \mathrm{~km}}{95 \mathrm{~km} / \mathrm{h}}=0.0447 \mathrm{~h}\left(\frac{60 \mathrm{~min}}{1 \mathrm{~h}}\right)=2.68 \mathrm{~min} \approx 2.7 \mathrm{~min}
$$

13. (a) The area between the concentric circles is equal to the length times the width of the spiral path.

$$
\begin{aligned}
& \pi R_{2}^{2}-\pi R_{1}^{2}=w \ell \rightarrow \\
& \ell=\frac{\pi\left(R_{2}^{2}-R_{1}^{2}\right)}{w}=\frac{\pi\left[(0.058 \mathrm{~m})^{2}-(0.025 \mathrm{~m})^{2}\right]}{1.6 \times 10^{-6} \mathrm{~m}}=5.378 \times 10^{3} \mathrm{~m} \approx 5400 \mathrm{~m}
\end{aligned}
$$

(b) $5.378 \times 10^{3} \mathrm{~m}\left(\frac{1 \mathrm{~s}}{1.25 \mathrm{~m}}\right)\left(\frac{1 \mathrm{~min}}{60 \mathrm{~s}}\right)=72 \mathrm{~min}$
14. The average speed for each segment of the trip is given by $\bar{v}=\frac{\Delta x}{\Delta t}$, so $\Delta t=\frac{\Delta x}{\bar{v}}$ for each segment. For the first segment, $\Delta t_{1}=\frac{\Delta x_{1}}{\bar{v}_{1}}=\frac{3100 \mathrm{~km}}{720 \mathrm{~km} / \mathrm{h}}=4.306 \mathrm{~h}$. For the second segment, $\Delta t_{2}=\frac{\Delta x_{2}}{\bar{v}_{2}}=\frac{2800 \mathrm{~km}}{990 \mathrm{~km} / \mathrm{h}}=2.828 \mathrm{~h} .$.
Thus the total time is $\Delta t_{\text {tot }}=\Delta t_{1}+\Delta t_{2}=4.306 \mathrm{~h}+2.828 \mathrm{~h}=7.134 \mathrm{~h} \approx 7.1 \mathrm{~h}$.
The average speed of the plane for the entire trip is $\bar{v}=\frac{\Delta x_{\text {tot }}}{\Delta t_{\text {tot }}}=\frac{3100 \mathrm{~km}+2800 \mathrm{~km}}{7.134 \mathrm{~h}}=827 \mathrm{~km} / \mathrm{h}$ $\approx 830 \mathrm{~km} / \mathrm{h}$.
15. The distance traveled is 500 km ( 250 km outgoing, 250 km return, keep 2 significant figures). The displacement $(\Delta x)$ is 0 because the ending point is the same as the starting point.
(a) To find the average speed, we need the distance traveled ( 500 km ) and the total time elapsed.

During the outgoing portion, $\bar{v}_{1}=\frac{\Delta x_{1}}{\Delta t_{1}}$ and so $\Delta t_{1}=\frac{\Delta x_{1}}{\bar{v}_{1}}=\frac{250 \mathrm{~km}}{95 \mathrm{~km} / \mathrm{h}}=2.632 \mathrm{~h}$. During the
return portion, $\bar{v}_{2}=\frac{\Delta x_{2}}{\Delta t_{2}}$, and so $\Delta t_{2}=\frac{\Delta x_{2}}{\bar{v}_{2}}=\frac{250 \mathrm{~km}}{55 \mathrm{~km} / \mathrm{h}}=4.545 \mathrm{~h}$. Thus the total time,
including lunch, is $\Delta t_{\text {total }}=\Delta t_{1}+\Delta t_{\text {lunch }}+\Delta t_{2}=8.177 \mathrm{~h}$.

$$
\bar{v}=\frac{\Delta x_{\text {total }}}{\Delta t_{\text {total }}}=\frac{500 \mathrm{~km}}{8.177 \mathrm{~h}}=61 \mathrm{~km} / \mathrm{h}
$$

(b) Average velocity $=\bar{v}=\Delta x / \Delta t=0$
16. We are given that $x(t)=2.0 \mathrm{~m}-(3.6 \mathrm{~m} / \mathrm{s}) t+\left(1.1 \mathrm{~m} / \mathrm{s}^{2}\right) t^{2}$.
(a) $x(1.0 \mathrm{~s})=2.0 \mathrm{~m}-(3.6 \mathrm{~m} / \mathrm{s})(1.0 \mathrm{~s})+\left(1.1 \mathrm{~m} / \mathrm{s}^{2}\right)(1.0 \mathrm{~s})^{2}=-0.5 \mathrm{~m}$ $x(2.0 \mathrm{~s})=2.0 \mathrm{~m}-(3.6 \mathrm{~m} / \mathrm{s})(2.0 \mathrm{~s})+\left(1.1 \mathrm{~m} / \mathrm{s}^{2}\right)(2.0 \mathrm{~s})^{2}=-0.8 \mathrm{~m}$ $x(3.0 \mathrm{~s})=2.0 \mathrm{~m}-(3.6 \mathrm{~m} / \mathrm{s})(3.0 \mathrm{~s})+\left(1.1 \mathrm{~m} / \mathrm{s}^{2}\right)(3.0 \mathrm{~s})^{2}=1.1 \mathrm{~m}$
(b) $\bar{v}=\frac{\Delta x}{\Delta t}=\frac{1.1 \mathrm{~m}-(-0.5 \mathrm{~m})}{2.0 \mathrm{~s}}=0.80 \mathrm{~m} / \mathrm{s}$
(c) The instantaneous velocity is given by $v(t)=\frac{d x(t)}{d t}=-3.6 \mathrm{~m} / \mathrm{s}+\left(2.2 \mathrm{~m} / \mathrm{s}^{2}\right) t$.

$$
\begin{aligned}
& v(2.0 \mathrm{~s})=-3.6 \mathrm{~m} / \mathrm{s}+\left(2.2 \mathrm{~m} / \mathrm{s}^{2}\right)(2.0 \mathrm{~s})=0.8 \mathrm{~m} / \mathrm{s} \\
& v(3.0 \mathrm{~s})=-3.6 \mathrm{~m} / \mathrm{s}+\left(2.2 \mathrm{~m} / \mathrm{s}^{2}\right)(3.0 \mathrm{~s})=3.0 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

17. The distance traveled is $120 \mathrm{~m}+\frac{1}{2}(120 \mathrm{~m})=180 \mathrm{~m}$, and the displacement is
$120 \mathrm{~m}-\frac{1}{2}(120 \mathrm{~m})=60 \mathrm{~m}$. The total time is $8.4 \mathrm{~s}+\frac{1}{3}(8.4 \mathrm{~s})=11.2 \mathrm{~s}$.
(a) Average speed $=\frac{\text { distance }}{\text { time elapsed }}=\frac{180 \mathrm{~m}}{11.2 \mathrm{~s}}=16 \mathrm{~m} / \mathrm{s}$
(b) Average velocity $=v_{\text {avg }}=\frac{\text { displacement }}{\text { time elapsed }}=\frac{60 \mathrm{~m}}{11.2 \mathrm{~s}}=+5 \mathrm{~m} / \mathrm{s}$ (in original direction)(1 sig fig)
18. For the car to pass the train, the car must travel the length of the train AND the distance the train travels. The distance the car travels can thus be written as either $d_{\text {car }}=v_{\text {cat }} t=(95 \mathrm{~km} / \mathrm{h}) t$ or $d_{\text {car }}=\ell_{\text {train }}+v_{\text {train }} t=1.10 \mathrm{~km}+(75 \mathrm{~km} / \mathrm{h}) t$. To solve for the time, equate these two expressions for the distance the car travels.

$$
(95 \mathrm{~km} / \mathrm{h}) t=1.10 \mathrm{~km}+(75 \mathrm{~km} / \mathrm{h}) t \rightarrow t=\frac{1.10 \mathrm{~km}}{20 \mathrm{~km} / \mathrm{h}}=0.055 \mathrm{~h}=3.3 \mathrm{~min}
$$

The distance the car travels during this time is $d=(95 \mathrm{~km} / \mathrm{h})(0.055 \mathrm{~h})=5.225 \mathrm{~km} \approx 5.2 \mathrm{~km}$.
If the train is traveling the opposite direction from the car, then the car must travel the length of the train MINUS the distance the train travels. Thus the distance the car travels can be written as either $d_{\text {car }}=(95 \mathrm{~km} / \mathrm{h}) t$ or $d_{\text {car }}=1.10 \mathrm{~km}-(75 \mathrm{~km} / \mathrm{h}) t$. To solve for the time, equate these two expressions for the distance the car travels.

$$
(95 \mathrm{~km} / \mathrm{h}) t=1.10 \mathrm{~km}-(75 \mathrm{~km} / \mathrm{h}) t \rightarrow t=\frac{1.10 \mathrm{~km}}{170 \mathrm{~km} / \mathrm{h}}=6.47 \times 10^{-3} \mathrm{~h}=23.3 \mathrm{~s}
$$

The distance the car travels during this time is $d=(95 \mathrm{~km} / \mathrm{h})\left(6.47 \times 10^{-3} \mathrm{~h}\right)=0.61 \mathrm{~km}$.
19. The average speed of sound is given by $v_{\text {sound }}=\Delta x / \Delta t$, and so the time for the sound to travel from the end of the lane back to the bowler is $\Delta t_{\text {sound }}=\frac{\Delta x}{v_{\text {sound }}}=\frac{16.5 \mathrm{~m}}{340 \mathrm{~m} / \mathrm{s}}=4.85 \times 10^{-2} \mathrm{~s}$. Thus the time for the ball to travel from the bowler to the end of the lane is given by $\Delta t_{\text {ball }}=\Delta t_{\text {total }}-\Delta t_{\text {sound }}=$ $2.50 \mathrm{~s}-4.85 \times 10^{-2} \mathrm{~s}=2.4515 \mathrm{~s}$. And so the speed of the ball is as follows.

$$
v_{\text {ball }}=\frac{\Delta x}{\Delta t_{\text {ball }}}=\frac{16.5 \mathrm{~m}}{2.4515 \mathrm{~s}}=6.73 \mathrm{~m} / \mathrm{s} .
$$

20. The average acceleration is found from Eq. 2-5.

$$
\bar{a}=\frac{\Delta v}{\Delta t}=\frac{95 \mathrm{~km} / \mathrm{h}-0 \mathrm{~km} / \mathrm{h}}{4.5 \mathrm{~s}}=\frac{(95 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)}{4.5 \mathrm{~s}}=5.9 \mathrm{~m} / \mathrm{s}^{2}
$$

21. The time can be found from the average acceleration, $\bar{a}=\Delta v / \Delta t$.

$$
\Delta t=\frac{\Delta v}{\bar{a}}=\frac{110 \mathrm{~km} / \mathrm{h}-80 \mathrm{~km} / \mathrm{h}}{1.8 \mathrm{~m} / \mathrm{s}^{2}}=\frac{(30 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)}{1.8 \mathrm{~m} / \mathrm{s}^{2}}=4.630 \mathrm{~s} \approx 5 \mathrm{~s}
$$

22. (a) The average acceleration of the sprinter is $\bar{a}=\frac{\Delta v}{\Delta t}=\frac{9.00 \mathrm{~m} / \mathrm{s}-0.00 \mathrm{~m} / \mathrm{s}}{1.28 \mathrm{~s}}=7.03 \mathrm{~m} / \mathrm{s}^{2}$.
(b) $\bar{a}=\left(7.03 \mathrm{~m} / \mathrm{s}^{2}\right)\left(\frac{1 \mathrm{~km}}{1000 \mathrm{~m}}\right)\left(\frac{3600 \mathrm{~s}}{1 \mathrm{~h}}\right)^{2}=9.11 \times 10^{4} \mathrm{~km} / \mathrm{h}^{2}$
23. Slightly different answers may be obtained since the data comes from reading the graph.
(a) The greatest velocity is found at the highest point on the graph, which is at $t \approx 48 \mathrm{~s}$.
(b) The indication of a constant velocity on a velocity-time graph is a slope of 0 , which occurs from $t=90 \mathrm{~s}$ to $t \approx 108 \mathrm{~s}$.
(c) The indication of a constant acceleration on a velocity-time graph is a constant slope, which occurs from $t=0 \mathrm{~s}$ to $t \approx 42 \mathrm{~s}$, again from $t \approx 65 \mathrm{~s}$ to $t \approx 83 \mathrm{~s}$, and again from $t=90 \mathrm{~s}$ to $t \approx 108 \mathrm{~s}$.
(d) The magnitude of the acceleration is greatest when the magnitude of the slope is greatest, which occurs from $t \approx 65 \mathrm{~s}$ to $t \approx 83 \mathrm{~s}$.
24. The initial velocity of the car is the average speed of the car before it accelerates.

$$
\bar{v}=\frac{\Delta x}{\Delta t}=\frac{110 \mathrm{~m}}{5.0 \mathrm{~s}}=22 \mathrm{~m} / \mathrm{s}=v_{0}
$$

The final speed is $v=0$, and the time to stop is 4.0 s . Use Eq. $2-12 \mathrm{a}$ to find the acceleration.

$$
v=v_{0}+a t \rightarrow a=\frac{v-v_{0}}{t}=\frac{0-22 \mathrm{~m} / \mathrm{s}}{4.0 \mathrm{~s}}=-5.5 \mathrm{~m} / \mathrm{s}^{2}
$$

Thus the magnitude of the acceleration is $5.5 \mathrm{~m} / \mathrm{s}^{2}$, or $\left(5.5 \mathrm{~m} / \mathrm{s}^{2}\right)\left(\frac{1 g}{9.80 \mathrm{~m} / \mathrm{s}^{2}}\right)=0.56 g^{\prime} \mathrm{s}$.
25. (a) $\bar{v}=\frac{\Delta x}{\Delta t}=\frac{385 \mathrm{~m}-25 \mathrm{~m}}{20.0 \mathrm{~s}-3.0 \mathrm{~s}}=21.2 \mathrm{~m} / \mathrm{s}$
(b) $\bar{a}=\frac{\Delta v}{\Delta t}=\frac{45.0 \mathrm{~m} / \mathrm{s}-11.0 \mathrm{~m} / \mathrm{s}}{20.0 \mathrm{~s}-3.0 \mathrm{~s}}=2.00 \mathrm{~m} / \mathrm{s}^{2}$
26. Slightly different answers may be obtained since the data comes from reading the graph. We assume that the short, nearly horizontal portions of the graph are the times that shifting is occurring, and those times are not counted as being "in" a certain gear.
(a) The average acceleration in $2^{\text {nd }}$ gear is given by $\bar{a}_{2}=\frac{\Delta v_{2}}{\Delta t_{2}}=\frac{24 \mathrm{~m} / \mathrm{s}-14 \mathrm{~m} / \mathrm{s}}{8 \mathrm{~s}-4 \mathrm{~s}}=2.5 \mathrm{~m} / \mathrm{s}^{2}$.
(b) The average acceleration in $4^{\text {th }}$ gear is given by $\bar{a}_{4}=\frac{\Delta v_{4}}{\Delta t_{4}}=\frac{44 \mathrm{~m} / \mathrm{s}-37 \mathrm{~m} / \mathrm{s}}{27 \mathrm{~s}-16 \mathrm{~s}}=0.6 \mathrm{~m} / \mathrm{s}^{2}$.
(c) The average acceleration through the first four gears is given by $\bar{a}=\frac{\Delta v}{\Delta t}=$

$$
\bar{a}=\frac{\Delta v}{\Delta t}=\frac{44 \mathrm{~m} / \mathrm{s}-0 \mathrm{~m} / \mathrm{s}}{27 \mathrm{~s}-0 \mathrm{~s}}=1.6 \mathrm{~m} / \mathrm{s}^{2} .
$$

27. The acceleration is the second derivative of the position function.

$$
x=6.8 t+8.5 t^{2} \rightarrow v=\frac{d x}{d t}=6.8+17.0 t \rightarrow a=\frac{d^{2} x}{d t^{2}}=\frac{d v}{d t}=17.0 \mathrm{~m} / \mathrm{s}^{2}
$$

28. To estimate the velocity, find the average velocity over each time interval, and assume that the car had that velocity at the midpoint of the time interval. To estimate the acceleration, find the average acceleration over each time interval, and assume that the car had that acceleration at the midpoint of the time interval. A sample of each calculation is shown.

From 2.00 s to 2.50 s , for average velocity:

$$
\begin{aligned}
& t_{\mathrm{mid}}=\frac{2.50 \mathrm{~s}+2.00 \mathrm{~s}}{2}=2.25 \mathrm{~s} \\
& v_{\mathrm{avg}}=\frac{\Delta x}{\Delta t}=\frac{13.79 \mathrm{~m}-8.55 \mathrm{~m}}{2.50 \mathrm{~s}-2.00 \mathrm{~s}}=\frac{5.24 \mathrm{~m}}{0.50 \mathrm{~s}}=10.48 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

From 2.25 s to 2.75 s , for average acceleration:

$$
\begin{aligned}
t_{\mathrm{mid}} & =\frac{2.25 \mathrm{~s}+2.75 \mathrm{~s}}{2}=2.50 \mathrm{~s} \\
a_{\mathrm{avg}} & =\frac{\Delta v}{\Delta t}=\frac{13.14 \mathrm{~m} / \mathrm{s}-10.48 \mathrm{~m} / \mathrm{s}}{2.75 \mathrm{~s}-2.25 \mathrm{~s}}=\frac{2.66 \mathrm{~m} / \mathrm{s}}{0.50 \mathrm{~s}} \\
& =5.32 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

Table of Calculations

| $t$ (s) | $x(\mathrm{~m})$ | $t(\mathrm{~s})$ | $v(\mathrm{~m} / \mathrm{s})$ | $t(\mathrm{~s})$ | $a\left(\mathrm{~m} / \mathrm{s}^{2}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.063 | 3.52 |
|  |  | 0.125 | 0.44 |  |  |
| 0.25 | 0.11 |  |  | 0.25 | 3.84 |
|  |  | 0.375 | 1.40 |  |  |
| 0.50 | 0.46 |  |  | 0.50 | 4.00 |
|  |  | 0.625 | 2.40 |  |  |
| 0.75 | 1.06 |  |  | 0.75 | 4.48 |
|  |  | 0.875 | 3.52 |  |  |
| 1.00 | 1.94 |  |  | 1.06 | 4.91 |
|  |  | 1.25 | 5.36 |  |  |
| 1.50 | 4.62 |  |  | 1.50 | 5.00 |
|  |  | 1.75 | 7.86 |  |  |
| 2.00 | 8.55 |  |  | 2.00 | 5.24 |
|  |  | 2.25 | 10.48 |  |  |
| 2.50 | 13.79 |  |  | 2.50 | 5.32 |
|  |  | 2.75 | 13.14 |  |  |
| 3.00 | 20.36 |  |  | 3.00 | 5.52 |
|  |  | 3.25 | 15.90 |  |  |
| 3.50 | 28.31 |  |  | 3.50 | 5.56 |
|  |  | 3.75 | 18.68 |  |  |
| 4.00 | 37.65 |  |  | 4.00 | 5.52 |
|  |  | 4.25 | 21.44 |  |  |
| 4.50 | 48.37 |  |  | 4.50 | 4.84 |
|  |  | 4.75 | 23.86 |  |  |
| 5.00 | 60.30 |  |  | 5.00 | 4.12 |
|  |  | 5.25 | 25.92 |  |  |
| 5.50 | 73.26 |  |  | 5.50 | 3.76 |
|  |  | 5.75 | 27.80 |  |  |
| 6.00 | 87.16 |  |  |  |  |



The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH02.XLS," on tab "Problem 2.28."
29. (a) Since the units of $A$ times the units of $t$ must equal meters, the units of $A$ must be $\mathrm{m} / \mathrm{s}$.

Since the units of $B$ times the units of $t^{2}$ must equal meters, the units of $B$ must be

$$
\mathrm{m} / \mathrm{s}^{2} \text {. }
$$

(b) The acceleration is the second derivative of the position function.

$$
x=A t+B t^{2} \rightarrow v=\frac{d x}{d t}=A+2 B t \rightarrow a=\frac{d^{2} x}{d t^{2}}=\frac{d v}{d t}=2 B \mathrm{~m} / \mathrm{s}^{2}
$$

(c) $v=A+2 B t \rightarrow v(5)=(A+10 B) \mathrm{m} / \mathrm{s} \quad a=2 B \mathrm{~m} / \mathrm{s}^{2}$
(d) The velocity is the derivative of the position function.

$$
x=A t+B t^{-3} \rightarrow v=\frac{d x}{d t}=A-3 B t^{-4}
$$

30. The acceleration can be found from Eq. 2-12c.

$$
v^{2}=v_{0}^{2}+2 a\left(x-x_{0}\right) \rightarrow a=\frac{v^{2}-v_{0}^{2}}{2\left(x-x_{0}\right)}=\frac{0-(25 \mathrm{~m} / \mathrm{s})^{2}}{2(85 \mathrm{~m})}=-3.7 \mathrm{~m} / \mathrm{s}^{2}
$$

31. By definition, the acceleration is $a=\frac{v-v_{0}}{t}=\frac{21 \mathrm{~m} / \mathrm{s}-12 \mathrm{~m} / \mathrm{s}}{6.0 \mathrm{~s}}=1.5 \mathrm{~m} / \mathrm{s}^{2}$.

The distance of travel can be found from Eq. 2-12b.

$$
x-x_{0}=v_{0} t+\frac{1}{2} a t^{2}=(12 \mathrm{~m} / \mathrm{s})(6.0 \mathrm{~s})+\frac{1}{2}\left(1.5 \mathrm{~m} / \mathrm{s}^{2}\right)(6.0 \mathrm{~s})^{2}=99 \mathrm{~m}
$$

32. Assume that the plane starts from rest. The runway distance is found by solving Eq. 2-12c for $x-x_{0}$.

$$
v^{2}=v_{0}^{2}+2 a\left(x-x_{0}\right) \rightarrow x-x_{0}=\frac{v^{2}-v_{0}^{2}}{2 a}=\frac{(32 \mathrm{~m} / \mathrm{s})^{2}-0}{2\left(3.0 \mathrm{~m} / \mathrm{s}^{2}\right)}=1.7 \times 10^{2} \mathrm{~m}
$$

33. For the baseball, $v_{0}=0, x-x_{0}=3.5 \mathrm{~m}$, and the final speed of the baseball (during the throwing motion) is $v=41 \mathrm{~m} / \mathrm{s}$. The acceleration is found from Eq. 2-12c.

$$
v^{2}=v_{0}^{2}+2 a\left(x-x_{0}\right) \rightarrow a=\frac{v^{2}-v_{0}^{2}}{2\left(x-x_{0}\right)}=\frac{(41 \mathrm{~m} / \mathrm{s})^{2}-0}{2(3.5 \mathrm{~m})}=240 \mathrm{~m} / \mathrm{s}^{2}
$$

34. The average velocity is defined by Eq. 2-2, $\bar{v}=\frac{\Delta x}{\Delta t}=\frac{x-x_{0}}{t}$. Compare this expression to Eq. 2$12 \mathrm{~d}, \bar{v}=\frac{1}{2}\left(v+v_{0}\right)$. A relation for the velocity is found by integrating the expression for the acceleration, since the acceleration is the derivative of the velocity. Assume the velocity is $v_{0}$ at time $t=0$.

$$
a=A+B t=\frac{d v}{d t} \rightarrow d v=(A+B t) d t \rightarrow \int_{v_{0}}^{v} d v=\int_{0}^{t}(A+B t) d t \rightarrow v=v_{0}+A t+\frac{1}{2} B t^{2}
$$

Find an expression for the position by integrating the velocity, assuming that $x=x_{0}$ at time $t=0$.

$$
\begin{aligned}
& v=v_{0}+A t+\frac{1}{2} B t^{2}=\frac{d x}{d t} \rightarrow d x=\left(v_{0}+A t+\frac{1}{2} B t^{2}\right) d t \rightarrow \\
& \int_{x_{0}}^{x} d x=\int_{0}^{t}\left(v_{0}+A t+\frac{1}{2} B t^{2}\right) d t \rightarrow x-x_{0}=v_{0} t+\frac{1}{2} A t^{2}+\frac{1}{6} B t^{3}
\end{aligned}
$$

Compare $\frac{x-x_{0}}{t}$ to $\frac{1}{2}\left(v+v_{0}\right)$.

$$
\begin{aligned}
& \bar{v}=\frac{x-x_{0}}{t}=\frac{v_{0} t+\frac{1}{2} A t^{2}+\frac{1}{6} B t^{3}}{t}=v_{0}+\frac{1}{2} A t+\frac{1}{6} B t^{2} \\
& \frac{1}{2}\left(v+v_{0}\right)=\frac{v_{0}+v_{0}+A t+\frac{1}{2} B t^{2}}{2}=v_{0}+\frac{1}{2} A t+\frac{1}{4} B t^{2}
\end{aligned}
$$

They are different, so $\bar{v} \neq \frac{1}{2}\left(v+v_{0}\right)$.
35. The sprinter starts from rest. The average acceleration is found from Eq. 2-12c.

$$
v^{2}=v_{0}^{2}+2 a\left(x-x_{0}\right) \rightarrow a=\frac{v^{2}-v_{0}^{2}}{2\left(x-x_{0}\right)}=\frac{(11.5 \mathrm{~m} / \mathrm{s})^{2}-0}{2(15.0 \mathrm{~m})}=4.408 \mathrm{~m} / \mathrm{s}^{2} \approx 4.41 \mathrm{~m} / \mathrm{s}^{2}
$$

Her elapsed time is found by solving Eq. 2-12a for time.

$$
v=v_{0}+a t \rightarrow t=\frac{v-v_{0}}{a}=\frac{11.5 \mathrm{~m} / \mathrm{s}-0}{4.408 \mathrm{~m} / \mathrm{s}^{2}}=2.61 \mathrm{~s}
$$

36. Calculate the distance that the car travels during the reaction time and the deceleration.

$$
\begin{aligned}
& \Delta x_{1}=v_{0} \Delta t=(18.0 \mathrm{~m} / \mathrm{s})(0.200 \mathrm{~s})=3.6 \mathrm{~m} \\
& v^{2}=v_{0}^{2}+2 a \Delta x_{2} \rightarrow \Delta x_{2}=\frac{v^{2}-v_{0}^{2}}{2 a}=\frac{0-(18.0 \mathrm{~m} / \mathrm{s})^{2}}{2\left(-3.65 \mathrm{~m} / \mathrm{s}^{2}\right)}=44.4 \mathrm{~m} \\
& \Delta x=3.6 \mathrm{~m}+44.4 \mathrm{~m}=48.0 \mathrm{~m}
\end{aligned}
$$

He will NOT be able to stop in time.
37. The words "slows down uniformly" implies that the car has a constant acceleration. The distance of travel is found from combining Eqs. 2-2 and 2-9.

$$
x-x_{0}=\frac{v_{0}+v}{2} t=\left(\frac{18.0 \mathrm{~m} / \mathrm{s}+0 \mathrm{~m} / \mathrm{s}}{2}\right)(5.00 \mathrm{sec})=45.0 \mathrm{~m}
$$

38. The final velocity of the car is zero. The initial velocity is found from Eq. 2-12c with $v=0$ and solving for $v_{0}$. Note that the acceleration is negative.

$$
v^{2}=v_{0}^{2}+2 a\left(x-x_{0}\right) \rightarrow v_{0}=\sqrt{v^{2}-2 a\left(x-x_{0}\right)}=\sqrt{0-2\left(-4.00 \mathrm{~m} / \mathrm{s}^{2}\right)(85 \mathrm{~m})}=26 \mathrm{~m} / \mathrm{s}
$$

39. (a) The final velocity of the car is 0 . The distance is found from Eq. 2-12c with an acceleration of $a=-0.50 \mathrm{~m} / \mathrm{s}^{2}$ and an initial velocity of $85 \mathrm{~km} / \mathrm{h}$.

$$
x-x_{0}=\frac{v^{2}-v_{0}^{2}}{2 a}=\frac{0-\left[(85 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)\right]^{2}}{2\left(-0.50 \mathrm{~m} / \mathrm{s}^{2}\right)}=557 \mathrm{~m} \approx 560 \mathrm{~m}
$$

(b) The time to stop is found from Eq. 2-12a.

$$
t=\frac{v-v_{0}}{a}=\frac{0-\left[(85 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)\right]}{\left(-0.50 \mathrm{~m} / \mathrm{s}^{2}\right)}=47.22 \mathrm{~s} \approx 47 \mathrm{~s}
$$

(c) Take $x_{0}=x(t=0)=0 \mathrm{~m}$. Use Eq. 2-12b, with $a=-0.50 \mathrm{~m} / \mathrm{s}^{2}$ and an initial velocity of $85 \mathrm{~km} / \mathrm{h}$. The first second is from $t=0 \mathrm{~s}$ to $t=1 \mathrm{~s}$, and the fifth second is from $t=4 \mathrm{~s}$ to $t=5 \mathrm{~s}$.

$$
\begin{aligned}
& x(0)=0 ; x(1)=0+(85 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)(1 \mathrm{~s})+\frac{1}{2}\left(-0.50 \mathrm{~m} / \mathrm{s}^{2}\right)(1 \mathrm{~s})^{2}=23.36 \mathrm{~m} \rightarrow \\
& x(1)-x(0)=23 \mathrm{~m} \\
& x(4)=0+(85 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)(4 \mathrm{~s})+\frac{1}{2}\left(-0.50 \mathrm{~m} / \mathrm{s}^{2}\right)(4 \mathrm{~s})^{2}=90.44 \mathrm{~m} \\
& x(5)=0+(85 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)(5 \mathrm{~s})+\frac{1}{2}\left(-0.50 \mathrm{~m} / \mathrm{s}^{2}\right)(5 \mathrm{~s})^{2}=111.81 \mathrm{~m} \\
& x(5)-x(4)=111.81 \mathrm{~m}-90.44 \mathrm{~m}=21.37 \mathrm{~m} \approx 21 \mathrm{~m}
\end{aligned}
$$

40. The final velocity of the driver is zero. The acceleration is found from Eq. 2-12c with $v=0$ and solving for $a$.

$$
a=\frac{v^{2}-v_{0}^{2}}{2\left(x-x_{0}\right)}=\frac{0-\left[(105 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)\right]^{2}}{2(0.80 \mathrm{~m})}=-531.7 \mathrm{~m} / \mathrm{s}^{2} \approx-5.3 \times 10^{2} \mathrm{~m} / \mathrm{s}^{2}
$$

Converting to " $g$ ' s ": $a=\frac{-531.7 \mathrm{~m} / \mathrm{s}^{2}}{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) / g}=-54 g^{\prime} \mathrm{s}$
41. The origin is the location of the car at the beginning of the reaction time. The initial speed of the car is $(95 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)=26.39 \mathrm{~m} / \mathrm{s}$. The location where the brakes are applied is found from the equation for motion at constant velocity: $x_{0}=v_{0} t_{R}=(26.39 \mathrm{~m} / \mathrm{s})(1.0 \mathrm{~s})=26.39 \mathrm{~m}$. This is now the starting location for the application of the brakes. In each case, the final speed is 0 .
(a) Solve Eq. 2-12c for the final location.

$$
v^{2}=v_{0}^{2}+2 a\left(x-x_{0}\right) \rightarrow x=x_{0}+\frac{v^{2}-v_{0}^{2}}{2 a}=26.39 \mathrm{~m}+\frac{0-(26.39 \mathrm{~m} / \mathrm{s})^{2}}{2\left(-5.0 \mathrm{~m} / \mathrm{s}^{2}\right)}=96 \mathrm{~m}
$$

(b) Solve Eq. 2-12c for the final location with the second acceleration.

$$
x=x_{0}+\frac{v^{2}-v_{0}^{2}}{2 a}=26.39 \mathrm{~m}+\frac{0-(26.39 \mathrm{~m} / \mathrm{s})^{2}}{2\left(-7.0 \mathrm{~m} / \mathrm{s}^{2}\right)}=76 \mathrm{~m}
$$

42. Calculate the acceleration from the velocity-time data using Eq. 2-12a, and then use Eq. 2-12b to calculate the displacement at $t=2.0 \mathrm{~s}$ and $t=6.0 \mathrm{~s}$. The initial velocity is $v_{0}=65 \mathrm{~m} / \mathrm{s}$.

$$
\begin{aligned}
& a=\frac{v-v_{0}}{t}=\frac{162 \mathrm{~m} / \mathrm{s}-65 \mathrm{~m} / \mathrm{s}}{10.0 \mathrm{~s}}=9.7 \mathrm{~m} / \mathrm{s}^{2} \quad x=x_{0}+v_{0} t+\frac{1}{2} a t^{2} \rightarrow \\
& x(6.0 \mathrm{~s})-x(2.0 \mathrm{~s})=\left[\left(x_{0}+v_{0}(6.0 \mathrm{~s})+\frac{1}{2} a(6.0 \mathrm{~s})^{2}\right)-\left(x_{0}+v_{0}(2.0 \mathrm{~s})+\frac{1}{2} a(2.0 \mathrm{~s})^{2}\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
& =v_{0}(6.0 \mathrm{~s}-2.0 \mathrm{~s})+\frac{1}{2} a\left[(6.0 \mathrm{~s})^{2}-(2.0 \mathrm{~s})^{2}\right]=(65 \mathrm{~m} / \mathrm{s})(4.0 \mathrm{~s})+\frac{1}{2}\left(9.7 \mathrm{~m} / \mathrm{s}^{2}\right)\left(32 \mathrm{~s}^{2}\right) \\
& =415 \mathrm{~m} \approx 4.2 \times 10^{2} \mathrm{~m}
\end{aligned}
$$

43. Use the information for the first 180 m to find the acceleration, and the information for the full motion to find the final velocity. For the first segment, the train has $v_{0}=0 \mathrm{~m} / \mathrm{s}, v_{1}=23 \mathrm{~m} / \mathrm{s}$, and a displacement of $x_{1}-x_{0}=180 \mathrm{~m}$. Find the acceleration from Eq. 2-12c.

$$
v_{1}^{2}=v_{0}^{2}+2 a\left(x_{1}-x_{0}\right) \rightarrow a=\frac{v_{1}^{2}-v_{0}^{2}}{2\left(x_{1}-x_{0}\right)}=\frac{(23 \mathrm{~m} / \mathrm{s})^{2}-0}{2(180 \mathrm{~m})}=1.469 \mathrm{~m} / \mathrm{s}^{2}
$$

Find the speed of the train after it has traveled the total distance (total displacement of $x_{2}-x_{0}=255 \mathrm{~m}$ ) using Eq. 2-12c.

$$
v_{2}^{2}=v_{0}^{2}+2 a\left(x_{2}-x_{0}\right) \rightarrow v_{2}=\sqrt{v_{0}^{2}+2 a\left(x_{2}-x_{0}\right)}=\sqrt{2\left(1.469 \mathrm{~m} / \mathrm{s}^{2}\right)(255 \mathrm{~m})}=27 \mathrm{~m} / \mathrm{s}
$$

44. Define the origin to be the location where the speeder passes the police car. Start a timer at the instant that the speeder passes the police car, and find another time that both cars have the same displacement from the origin.

For the speeder, traveling with a constant speed, the displacement is given by the following.

$$
\Delta x_{s}=v_{s} t=(135 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)(t)=(37.5 t) \mathrm{m}
$$

For the police car, the displacement is given by two components. The first part is the distance traveled at the initially constant speed during the 1 second of reaction time.

$$
\Delta x_{p 1}=v_{p 1}(1.00 \mathrm{~s})=(95 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)(1.00 \mathrm{~s})=26.39 \mathrm{~m}
$$

The second part of the police car displacement is that during the accelerated motion, which lasts for $(t-1.00) \mathrm{s}$. So this second part of the police car displacement, using Eq. 2-12b, is given as follows.

$$
\Delta x_{p 2}=v_{p 1}(t-1.00)+\frac{1}{2} a_{p}(t-1.00)^{2}=\left[(26.39 \mathrm{~m} / \mathrm{s})(t-1.00)+\frac{1}{2}\left(2.00 \mathrm{~m} / \mathrm{s}^{2}\right)(t-1.00)^{2}\right] \mathrm{m}
$$

So the total police car displacement is $\Delta x_{p}=\Delta x_{p 1}+\Delta x_{p 2}=\left(26.39+26.39(t-1.00)+(t-1.00)^{2}\right) \mathrm{m}$. Now set the two displacements equal, and solve for the time.

$$
\begin{aligned}
& 26.39+26.39(t-1.00)+(t-1.00)^{2}=37.5 t \quad \rightarrow \quad t^{2}-13.11 t+1.00=0 \\
& t=\frac{13.11 \pm \sqrt{(13.11)^{2}-4.00}}{2}=7.67 \times 10^{-2} \mathrm{~s}, 13.0 \mathrm{~s}
\end{aligned}
$$

The answer that is approximately 0 s corresponds to the fact that both vehicles had the same displacement of zero when the time was 0 . The reason it is not exactly zero is rounding of previous values. The answer of 13.0 s is the time for the police car to overtake the speeder.

As a check on the answer, the speeder travels $\Delta x_{s}=(37.5 \mathrm{~m} / \mathrm{s})(13.0 \mathrm{~s})=488 \mathrm{~m}$, and the police car travels $\Delta x_{p}=\left[26.39+26.39(12.0)+(12.0)^{2}\right] \mathrm{m}=487 \mathrm{~m}$. The difference is due to rounding.
45. Define the origin to be the location where the speeder passes the police car. Start a timer at the instant that the speeder passes the police car. Both cars have the same displacement 8.00 s after the initial passing by the speeder.

For the speeder, traveling with a constant speed, the displacement is given by $\Delta x_{s}=v_{s} t=\left(8.00 v_{s}\right) \mathrm{m}$.
For the police car, the displacement is given by two components. The first part is the distance traveled at the initially constant speed during the 1.00 s of reaction time.

$$
\Delta x_{p 1}=v_{p 1}(1.00 \mathrm{~s})=(95 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)(1.00 \mathrm{~s})=26.39 \mathrm{~m}
$$

The second part of the police car displacement is that during the accelerated motion, which lasts for 7.00 s . So this second part of the police car displacement, using Eq. 2-12b, is given by the following.

$$
\Delta x_{p 2}=v_{p 1}(7.00 \mathrm{~s})+\frac{1}{2} a_{p}(7.00 \mathrm{~s})^{2}=(26.39 \mathrm{~m} / \mathrm{s})(7.00 \mathrm{~s})+\frac{1}{2}\left(2.00 \mathrm{~m} / \mathrm{s}^{2}\right)(7.00 \mathrm{~s})^{2}=233.73 \mathrm{~m}
$$

Thus the total police car displacement is $\Delta x_{p}=\Delta x_{p 1}+\Delta x_{p 2}=(26.39+233.73) \mathrm{m}=260.12 \mathrm{~m}$.
Now set the two displacements equal, and solve for the speeder's velocity.

$$
\left(8.00 v_{s}\right) \mathrm{m}=260.12 \mathrm{~m} \rightarrow v_{s}=(32.5 \mathrm{~m} / \mathrm{s})\left(\frac{3.6 \mathrm{~km} / \mathrm{h}}{1 \mathrm{~m} / \mathrm{s}}\right)=117 \mathrm{~km} / \mathrm{h}
$$

46. During the final part of the race, the runner must have a displacement of 1100 m in a time of 180 s $(3.0 \mathrm{~min})$. Assume that the starting speed for the final part is the same as the average speed thus far.

$$
\bar{v}=\frac{\Delta x}{\Delta t}=\frac{8900 \mathrm{~m}}{(27 \times 60) \mathrm{s}}=5.494 \mathrm{~m} / \mathrm{s}=v_{0}
$$

The runner will accomplish this by accelerating from speed $v_{0}$ to speed $v$ for $t$ seconds, covering a distance $d_{1}$, and then running at a constant speed of $v$ for $(180-t)$ seconds, covering a distance $d_{2}$. We have these relationships from Eq. 2-12a and Eq. 2-12b.

$$
\begin{aligned}
& v=v_{o}+a t \quad d_{1}=v_{o} t+\frac{1}{2} a t^{2} \quad d_{2}=v(180-t)=\left(v_{0}+a t\right)(180-t) \\
& 1100 \mathrm{~m}=d_{1}+d_{2}=v_{o} t+\frac{1}{2} a t^{2}+\left(v_{0}+a t\right)(180-t) \rightarrow 1100 \mathrm{~m}=180 v_{0}+180 a t-\frac{1}{2} a t^{2} \\
& 1100 \mathrm{~m}=(180 \mathrm{~s})(5.494 \mathrm{~m} / \mathrm{s})+(180 \mathrm{~s})\left(0.2 \mathrm{~m} / \mathrm{s}^{2}\right) t-\frac{1}{2}\left(0.2 \mathrm{~m} / \mathrm{s}^{2}\right) t^{2} \\
& 0.1 t^{2}-36 t+111=0 \quad t=357 \mathrm{~s}, 3.11 \mathrm{~s}
\end{aligned}
$$

Since we must have $t<180 \mathrm{~s}$, the solution is $t=3.1 \mathrm{~s}$.
47. For the runners to cross the finish line side-by-side means they must both reach the finish line in the same amount of time from their current positions. Take Mary's current location as the origin. Use Eq. 2-12b.

For Sally: $\quad 22=5+5 t+\frac{1}{2}(-.5) t^{2} \rightarrow t^{2}-20 t+68=0 \rightarrow$

$$
t=\frac{20 \pm \sqrt{20^{2}-4(68)}}{2}=4.343 \mathrm{~s}, 15.66 \mathrm{~s}
$$

The first time is the time she first crosses the finish line, and so is the time to be used for the problem. Now find Mary's acceleration so that she crosses the finish line in that same amount of time.
For Mary: $\quad 22=0+4 t+\frac{1}{2} a t^{2} \quad \rightarrow \quad a=\frac{22-4 t}{\frac{1}{2} t^{2}}=\frac{22-4(4.343)}{\frac{1}{2}(4.343)^{2}}=0.49 \mathrm{~m} / \mathrm{s}^{2}$
48. Choose downward to be the positive direction, and take $y_{0}=0$ at the top of the cliff. The initial velocity is $v_{0}=0$, and the acceleration is $a=9.80 \mathrm{~m} / \mathrm{s}^{2}$. The displacement is found from Eq. 212b, with $x$ replaced by $y$.

$$
y=y_{0}+v_{0} t+\frac{1}{2} a t^{2} \rightarrow y-0=0+\frac{1}{2}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(3.75 \mathrm{~s})^{2} \rightarrow y=68.9 \mathrm{~m}
$$

49. Choose downward to be the positive direction. The initial velocity is $v_{0}=0$, the final velocity is $v=(55 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)=15.28 \mathrm{~m} / \mathrm{s}$, and the acceleration is $a=9.80 \mathrm{~m} / \mathrm{s}^{2}$. The time can be found by solving Eq. 2-12a for the time.

$$
v=v_{0}+a t \rightarrow t=\frac{v-v_{0}}{a}=\frac{15.28 \mathrm{~m} / \mathrm{s}-0}{9.80 \mathrm{~m} / \mathrm{s}^{2}}=1.6 \mathrm{~s}
$$

50. Choose downward to be the positive direction, and take $y_{0}=0$ to be at the top of the Empire State Building. The initial velocity is $v_{0}=0$, and the acceleration is $a=9.80 \mathrm{~m} / \mathrm{s}^{2}$.
(a) The elapsed time can be found from Eq. 2-12b, with $x$ replaced by $y$.

$$
y-y_{0}=v_{0} t+\frac{1}{2} a t^{2} \quad \rightarrow \quad t=\sqrt{\frac{2 y}{a}}=\sqrt{\frac{2(380 \mathrm{~m})}{9.80 \mathrm{~m} / \mathrm{s}^{2}}}=8.806 \mathrm{~s} \approx 8.8 \mathrm{~s} .
$$

(b) The final velocity can be found from Eq. 2-12a.

$$
v=v_{0}+a t=0+\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(8.806 \mathrm{~s})=86 \mathrm{~m} / \mathrm{s}
$$

51. Choose upward to be the positive direction, and take $y_{0}=0$ to be at the height where the ball was hit. For the upward path, $v_{0}=20 \mathrm{~m} / \mathrm{s}, v=0$ at the top of the path, and $a=-9.80 \mathrm{~m} / \mathrm{s}^{2}$.
(a) The displacement can be found from Eq. 2-12c, with $x$ replaced by $y$.

$$
v^{2}=v_{0}^{2}+2 a\left(y-y_{0}\right) \rightarrow y=y_{0}+\frac{v^{2}-v_{0}^{2}}{2 a}=0+\frac{0-(20 \mathrm{~m} / \mathrm{s})^{2}}{2\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=20 \mathrm{~m}
$$

(b) The time of flight can be found from Eq. 2-12b, with $x$ replaced by $y$, using a displacement of 0 for the displacement of the ball returning to the height from which it was hit.

$$
y=y_{0}+v_{0} t+\frac{1}{2} a t^{2}=0 \quad \rightarrow \quad t\left(v_{0}+\frac{1}{2} a t\right)=0 \quad \rightarrow \quad t=0, t=\frac{2 v_{0}}{-a}=\frac{2(20 \mathrm{~m} / \mathrm{s})}{9.80 \mathrm{~m} / \mathrm{s}^{2}}=4 \mathrm{~s}
$$

The result of $t=0 \mathrm{~s}$ is the time for the original displacement of zero (when the ball was hit), and the result of $t=4 \mathrm{~s}$ is the time to return to the original displacement. Thus the answer is $t=4 \mathrm{~s}$.
52. Choose upward to be the positive direction, and take $y_{0}=0$ to be the height from which the ball was thrown. The acceleration is $a=-9.80 \mathrm{~m} / \mathrm{s}^{2}$. The displacement upon catching the ball is 0 , assuming it was caught at the same height from which it was thrown. The starting speed can be found from Eq. 2-12b, with $x$ replaced by $y$.

$$
\begin{aligned}
& y=y_{0}+v_{0} t+\frac{1}{2} a t^{2}=0 \rightarrow \\
& v_{0}=\frac{y-y_{0}-\frac{1}{2} a t^{2}}{t}=-\frac{1}{2} a t=-\frac{1}{2}\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(3.2 \mathrm{~s})=15.68 \mathrm{~m} / \mathrm{s} \approx 16 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

The height can be calculated from Eq. 2-12c, with a final velocity of $v=0$ at the top of the path.

$$
v^{2}=v_{0}^{2}+2 a\left(y-y_{0}\right) \rightarrow y=y_{0}+\frac{v^{2}-v_{0}^{2}}{2 a}=0+\frac{0-(15.68 \mathrm{~m} / \mathrm{s})^{2}}{2\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=12.54 \mathrm{~m} \approx 13 \mathrm{~m}
$$

53. Choose downward to be the positive direction, and take $y_{0}=0$ to be at the maximum height of the kangaroo. Consider just the downward motion of the kangaroo. Then the displacement is $y=1.65 \mathrm{~m}$, the acceleration is $a=9.80 \mathrm{~m} / \mathrm{s}^{2}$, and the initial velocity is $v_{0}=0 \mathrm{~m} / \mathrm{s}$. Use Eq. 212 b to calculate the time for the kangaroo to fall back to the ground. The total time is then twice the falling time.

$$
\begin{aligned}
& y=y_{0}+v_{0} t+\frac{1}{2} a t^{2}=0 \rightarrow y=\frac{1}{2} a t^{2} \rightarrow t_{\text {fall }}=\sqrt{\frac{2 y}{a}} \rightarrow \\
& t_{\text {total }}=2 \sqrt{\frac{2 y}{a}}=2 \sqrt{\frac{2(1.65 \mathrm{~m})}{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}}=1.16 \mathrm{~s}
\end{aligned}
$$

54. Choose upward to be the positive direction, and take $y_{0}=0$ to be at the floor level, where the jump starts. For the upward path, $y=1.2 \mathrm{~m}, v=0$ at the top of the path, and $a=-9.80 \mathrm{~m} / \mathrm{s}^{2}$.
(a) The initial speed can be found from Eq. 2-12c, with $x$ replaced by $y$.

$$
\begin{aligned}
& v^{2}=v_{0}^{2}+2 a\left(y-y_{0}\right) \rightarrow \\
& v_{0}=\sqrt{v^{2}-2 a\left(y-y_{0}\right)}=\sqrt{-2 a y}=\sqrt{-2\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(1.2 \mathrm{~m})}=4.8497 \mathrm{~m} / \mathrm{s} \approx 4.8 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(b) The time of flight can be found from Eq. 2-12b, with $x$ replaced by $y$, using a displacement of 0 for the displacement of the jumper returning to the original height.

$$
\begin{aligned}
& y=y_{0}+v_{0} t+\frac{1}{2} a t^{2}=0 \rightarrow t\left(v_{0}+\frac{1}{2} a t\right)=0 \rightarrow \\
& t=0, t=\frac{2 v_{0}}{-a}=\frac{2(4.897 \mathrm{~m} / \mathrm{s})}{9.80 \mathrm{~m} / \mathrm{s}^{2}}=0.99 \mathrm{~s}
\end{aligned}
$$

The result of $t=0 \mathrm{~s}$ is the time for the original displacement of zero (when the jumper started to jump), and the result of $t=0.99 \mathrm{~s}$ is the time to return to the original displacement. Thus the answer is $t=0.99$ seconds.
55. Choose downward to be the positive direction, and take $y_{0}=0$ to be the height where the object was released. The initial velocity is $v_{0}=-5.10 \mathrm{~m} / \mathrm{s}$, the acceleration is $a=9.80 \mathrm{~m} / \mathrm{s}^{2}$, and the displacement of the package will be $y=105 \mathrm{~m}$. The time to reach the ground can be found from Eq. 2-12b, with $x$ replaced by $y$.

$$
\begin{aligned}
& y=y_{0}+v_{0} t+\frac{1}{2} a t^{2} \rightarrow t^{2}+\frac{2 v_{0}}{a} t-\frac{2 y}{a}=0 \rightarrow t^{2}+\frac{2(-5.10 \mathrm{~m} / \mathrm{s})}{9.80 \mathrm{~m} / \mathrm{s}^{2}} t-\frac{2(105 \mathrm{~m})}{9.80 \mathrm{~m} / \mathrm{s}^{2}}=0 \rightarrow \\
& t=5.18 \mathrm{~s},-4.14 \mathrm{~s}
\end{aligned}
$$

The correct time is the positive answer, $t=5.18 \mathrm{~s}$.
56. Choose downward to be the positive direction, and take $y_{0}=0$ to be the height from which the object is released. The initial velocity is $v_{0}=0$, and the acceleration is $a=g$. Then we can calculate the position as a function of time from Eq. 2-12b, with $x$ replaced by $y$, as $y(t)=\frac{1}{2} g t^{2}$. At the end of each second, the position would be as follows.

$$
y(0)=0 ; \quad y(1)=\frac{1}{2} g ; \quad y(2)=\frac{1}{2} g(2)^{2}=4 y(1) ; \quad y(3)=\frac{1}{2} g(3)^{2}=9 y(1)
$$

The distance traveled during each second can be found by subtracting two adjacent position values from the above list.

$$
d(1)=y(1)-y(0)=y(1) ; \quad d(2)=y(2)-y(1)=3 y(1) ; \quad d(3)=y(3)-y(2)=5 y(1)
$$

We could do this in general.

$$
\begin{aligned}
& y(n)=\frac{1}{2} g n^{2} \quad y(n+1)=\frac{1}{2} g(n+1)^{2} \\
& d(n+1)=y(n+1)-y(n)=\frac{1}{2} g(n+1)^{2}-\frac{1}{2} g n^{2}=\frac{1}{2} g\left((n+1)^{2}-n^{2}\right) \\
& \quad=\frac{1}{2} g\left(n^{2}+2 n+1-n^{2}\right)=\frac{1}{2} g(2 n+1)
\end{aligned}
$$

The value of $(2 n+1)$ is always odd, in the sequence $1,3,5,7, \ldots$.
57. Choose upward to be the positive direction, and $y_{0}=0$ to be the level from which the ball was thrown. The initial velocity is $v_{0}$, the instantaneous velocity is $v=14 \mathrm{~m} / \mathrm{s}$, the acceleration is $a=-9.80 \mathrm{~m} / \mathrm{s}^{2}$, and the location of the window is $y=23 \mathrm{~m}$.
(a) Using Eq. 2-12c and substituting $y$ for $x$, we have

$$
\begin{aligned}
& v^{2}=v_{0}^{2}+2 a\left(y-y_{0}\right) \rightarrow \\
& v_{0}= \pm \sqrt{v^{2}-2 a\left(y-y_{0}\right)}= \pm \sqrt{(14 \mathrm{~m} / \mathrm{s})^{2}-2\left(-9.8 \mathrm{~m} / \mathrm{s}^{2}\right)(23 \mathrm{~m})}=25.43 \mathrm{~m} / \mathrm{s} \approx 25 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Choose the positive value because the initial direction is upward.
(b) At the top of its path, the velocity will be 0 , and so we can use the initial velocity as found above, along with Eq. 2-12c.

$$
v^{2}=v_{0}^{2}+2 a\left(y-y_{0}\right) \rightarrow y=y_{0}+\frac{v^{2}-v_{0}^{2}}{2 a}=0+\frac{0-(25.43 \mathrm{~m} / \mathrm{s})^{2}}{2\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=33 \mathrm{~m}
$$

(c) We want the time elapsed from throwing (speed $v_{0}=25.43 \mathrm{~m} / \mathrm{s}$ ) to reaching the window (speed $v=14 \mathrm{~m} / \mathrm{s}$ ). Using Eq. 2-12a, we have the following.

$$
v=v_{0}+a t \rightarrow t=\frac{v-v_{0}}{a}=\frac{14 \mathrm{~m} / \mathrm{s}-25.43 \mathrm{~m} / \mathrm{s}}{-9.80 \mathrm{~m} / \mathrm{s}^{2}}=1.166 \mathrm{~s} \approx 1.2 \mathrm{~s}
$$

(d) We want the time elapsed from the window (speed $v_{0}=14 \mathrm{~m} / \mathrm{s}$ ) to reaching the street (speed $v=-25.43 \mathrm{~m} / \mathrm{s}$ ). Using Eq. 2-12a, we have the following.

$$
v=v_{0}+a t \rightarrow t=\frac{v-v_{0}}{a}=\frac{-25.43 \mathrm{~m} / \mathrm{s}-14 \mathrm{~m} / \mathrm{s}}{-9.80 \mathrm{~m} / \mathrm{s}^{2}}=4.0 \mathrm{~s}
$$

This is the elapsed time after passing the window. The total time of flight of the baseball from passing the window to reaching the street is $4.0 \mathrm{~s}+1.2 \mathrm{~s}=5.2 \mathrm{~s}$.
58. (a) Choose upward to be the positive direction, and $y_{0}=0$ at the ground. The rocket has $v_{0}=0$, $a=3.2 \mathrm{~m} / \mathrm{s}^{2}$, and $y=950 \mathrm{~m}$ when it runs out of fuel. Find the velocity of the rocket when it runs out of fuel from Eq 2-12c, with $x$ replaced by $y$.

$$
\begin{aligned}
& v_{950 \mathrm{~m}}^{2}=v_{0}^{2}+2 a\left(y-y_{0}\right) \rightarrow \\
& v_{950 \mathrm{~m}}= \pm \sqrt{v_{0}^{2}+2 a\left(y-y_{0}\right)}= \pm \sqrt{0+2\left(3.2 \mathrm{~m} / \mathrm{s}^{2}\right)(950 \mathrm{~m})}=77.97 \mathrm{~m} / \mathrm{s} \approx 78 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

The positive root is chosen since the rocket is moving upwards when it runs out of fuel.
(b) The time to reach the 950 m location can be found from Eq. 2-12a.

$$
v_{950 \mathrm{~m}}=v_{0}+a t_{950 \mathrm{~m}} \rightarrow t_{950 \mathrm{~m}}=\frac{v_{950 \mathrm{~m}}-v_{0}}{a}=\frac{77.97 \mathrm{~m} / \mathrm{s}-0}{3.2 \mathrm{~m} / \mathrm{s}^{2}}=24.37 \mathrm{~s} \approx 24 \mathrm{~s}
$$

(c) For this part of the problem, the rocket will have an initial velocity $v_{0}=77.97 \mathrm{~m} / \mathrm{s}$, an acceleration of $a=-9.80 \mathrm{~m} / \mathrm{s}^{2}$, and a final velocity of $v=0$ at its maximum altitude. The altitude reached from the out-of-fuel point can be found from Eq. 2-12c.

$$
\begin{aligned}
& v^{2}=v_{950 \mathrm{~m}}^{2}+2 a(y-950 \mathrm{~m}) \rightarrow \\
& y_{\text {max }}=950 \mathrm{~m}+\frac{0-v_{950 \mathrm{~m}}^{2}}{2 a}=950 \mathrm{~m}+\frac{-(77.97 \mathrm{~m} / \mathrm{s})^{2}}{2\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=950 \mathrm{~m}+310 \mathrm{~m}=1260 \mathrm{~m}
\end{aligned}
$$

(d) The time for the "coasting" portion of the flight can be found from Eq. 2-12a.

$$
v=v_{950 \mathrm{~m}}+a t_{\text {coast }} \rightarrow t_{\text {coast }}=\frac{v-v_{0}}{a}=\frac{0-77.97 \mathrm{~m} / \mathrm{s}}{-9.80 \mathrm{~m} / \mathrm{s}^{2}}=7.96 \mathrm{~s}
$$

Thus the total time to reach the maximum altitude is $t=24.37 \mathrm{~s}+7.96 \mathrm{~s}=32.33 \mathrm{~s} \approx 32 \mathrm{~s}$.
(e) For the falling motion of the rocket, $v_{0}=0 \mathrm{~m} / \mathrm{s}, a=-9.80 \mathrm{~m} / \mathrm{s}^{2}$, and the displacement is -1260 m (it falls from a height of 1260 m to the ground). Find the velocity upon reaching the Earth from Eq. 2-12c.

$$
\begin{aligned}
& v^{2}=v_{0}^{2}+2 a\left(y-y_{0}\right) \rightarrow \\
& v= \pm \sqrt{v_{0}^{2}+2 a\left(y-y_{0}\right)}= \pm \sqrt{0+2\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(-1260 \mathrm{~m})}=-157 \mathrm{~m} / \mathrm{s} \approx-160 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

The negative root was chosen because the rocket is moving downward, which is the negative direction.
(f) The time for the rocket to fall back to the Earth is found from Eq. 2-12a.

$$
v=v_{0}+a t \rightarrow t_{\text {fall }}=\frac{v-v_{0}}{a}=\frac{-157 \mathrm{~m} / \mathrm{s}-0}{-9.80 \mathrm{~m} / \mathrm{s}^{2}}=16.0 \mathrm{~s}
$$

Thus the total time for the entire flight is $t=32.33 \mathrm{~s}+16.0 \mathrm{~s}=48.33 \mathrm{~s} \approx 48 \mathrm{~s}$. .
59. (a) Choose $y=0$ to be the ground level, and positive to be upward. Then $y=0 \mathrm{~m}$, $y_{0}=15 \mathrm{~m}, a=-g$, and $t=0.83 \mathrm{~s}$ describe the motion of the balloon. Use Eq. 2-12b.

$$
\begin{aligned}
& y=y_{0}+v_{0} t+\frac{1}{2} a t^{2} \rightarrow \\
& v_{0}=\frac{y-y_{0}-\frac{1}{2} a t^{2}}{t}=\frac{0-15 \mathrm{~m}-\frac{1}{2}\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.83 \mathrm{~s})^{2}}{(0.83 \mathrm{~s})}=-14 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

So the speed is $14 \mathrm{~m} / \mathrm{s}$.
(b) Consider the change in velocity from being released to being at Roger's room, using Eq. 2-12c.

$$
v^{2}=v_{0}^{2}+2 a \Delta y \rightarrow \Delta y=\frac{v^{2}-v_{0}^{2}}{2 a}=\frac{-(-14 \mathrm{~m} / \mathrm{s})^{2}}{2\left(-9.8 \mathrm{~m} / \mathrm{s}^{2}\right)}=10 \mathrm{~m}
$$

Thus the balloons are coming from 2 floors above Roger, and so the fifth floor.
60. Choose upward to be the positive direction, and $y_{0}=0$ to be the height from which the stone is thrown. We have $v_{0}=24.0 \mathrm{~m} / \mathrm{s}, a=-9.80 \mathrm{~m} / \mathrm{s}^{2}$, and $y-y_{0}=13.0 \mathrm{~m}$.
(a) The velocity can be found from Eq, 2-12c, with $x$ replaced by $y$.

$$
\begin{aligned}
& v^{2}=v_{0}^{2}+2 a\left(y-y_{0}\right)=0 \rightarrow \\
& v= \pm \sqrt{v_{0}^{2}+2 a y}= \pm \sqrt{(24.0 \mathrm{~m} / \mathrm{s})^{2}+2\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(13.0 \mathrm{~m})}= \pm 17.9 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Thus the speed is $|v|=17.9 \mathrm{~m} / \mathrm{s}$.
(b) The time to reach that height can be found from Eq. 2-12b.

$$
\begin{aligned}
& y=y_{0}+v_{0} t+\frac{1}{2} a t^{2} \rightarrow t^{2}+\frac{2(24.0 \mathrm{~m} / \mathrm{s})}{-9.80 \mathrm{~m} / \mathrm{s}^{2}} t+\frac{2(-13.0 \mathrm{~m})}{-9.80 \mathrm{~m} / \mathrm{s}^{2}}=0 \rightarrow \\
& t^{2}-4.898 t+2.653=0 \rightarrow t=4.28 \mathrm{~s}, 0.620 \mathrm{~s}
\end{aligned}
$$

(c) There are two times at which the object reaches that height - once on the way up $(t=0.620 \mathrm{~s})$, and once on the way down $(t=4.28 \mathrm{~s})$.
61. Choose downward to be the positive direction, and $y_{0}=0$ to be the height from which the stone is dropped. Call the location of the top of the window $y_{w}$, and the time for the stone to fall from release to the top of the window is $t_{w}$. Since the stone is dropped from rest, using Eq. 2-12b with $y$ substituting for $x$, we have $y_{w}=y_{0}+v_{0} t+\frac{1}{2} a t^{2}=0+0+\frac{1}{2} g t_{w}^{2}$. The location of the bottom of the window is $y_{w}+2.2 \mathrm{~m}$, and the time for the stone to fall from release to the bottom of the window is $t_{w}+0.33 \mathrm{~s}$. Since the stone is dropped from rest, using Eq. 2-12b, we have the following:
$y_{w}+2.2 \mathrm{~m}=y_{0}+v_{0}+\frac{1}{2} a t^{2}=0+0+\frac{1}{2} g\left(t_{w}+0.33 \mathrm{~s}\right)^{2}$. Substitute the first expression for $y_{w}$ into the second expression.

$$
\frac{1}{2} g t_{w}^{2}+2.2 \mathrm{~m}=\frac{1}{2} g\left(t_{w}+0.33 \mathrm{~s}\right)^{2} \rightarrow t_{w}=0.515 \mathrm{~s}
$$

Use this time in the first equation to get the height above the top of the window from which the stone fell.

$$
y_{w}=\frac{1}{2} g t_{w}^{2}=\frac{1}{2}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.515 \mathrm{~s})^{2}=1.3 \mathrm{~m}
$$

62. Choose upward to be the positive direction, and $y_{0}=0$ to be the location of the nozzle. The initial velocity is $v_{0}$, the acceleration is $a=-9.80 \mathrm{~m} / \mathrm{s}^{2}$, the final location is $y=-1.5 \mathrm{~m}$, and the time of flight is $t=2.0 \mathrm{~s}$. Using Eq. $2-12 \mathrm{~b}$ and substituting $y$ for $x$ gives the following.

$$
y=y_{0}+v_{0} t+\frac{1}{2} a t^{2} \rightarrow v_{0}=\frac{y-\frac{1}{2} a t^{2}}{t}=\frac{-1.5 \mathrm{~m}-\frac{1}{2}\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(2.0 \mathrm{~s})^{2}}{2.0 \mathrm{~s}}=9.1 \mathrm{~m} / \mathrm{s}
$$

63. Choose up to be the positive direction, so $a=-g$. Let the ground be the $y=0$ location. As an intermediate result, the velocity at the bottom of the window can be found from the data given. Assume the rocket is at the bottom of the window at $t=0$, and use Eq. 2-12b.

$$
\begin{aligned}
& y_{\substack{\text { top of } \\
\text { window }}}=y_{\text {bottom of }}^{\text {window }}+\cdots v_{\substack{\text { bottom of } \\
\text { window } \\
t_{\text {pass }} \\
\text { window }}}+\frac{1}{2} a t_{\substack{\text { pass } \\
\text { window }}}^{2} \rightarrow \\
& 10.0 \mathrm{~m}=8.0 \mathrm{~m}+v_{\substack{\text { botomo of } \\
\text { window }}}(0.15 \mathrm{~s})+\frac{1}{2}\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.15 \mathrm{~s})^{2} \rightarrow \underset{\substack{\text { botom of } \\
\text { window }}}{ }=14.07 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Now use the velocity at the bottom of the window with Eq. 2-12c to find the launch velocity, assuming the launch velocity was achieved at the ground level.

$$
\begin{aligned}
& v_{\text {botom of }}^{2}=v_{\text {launch }}^{2}+2 a\left(y-y_{0}\right) \rightarrow \\
& v_{\text {lindownch }}^{2}=\sqrt{v_{\text {bototom of }}^{2}-2 a\left(y-y_{0}\right)}=\sqrt{(14.07 \mathrm{~m} / \mathrm{s})^{2}+2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(8.0 \mathrm{~m})}=18.84 \mathrm{~m} / \mathrm{s} \\
& \quad \approx 18.8 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

The maximum height can also be found from Eq. 2-12c, using the launch velocity and a velocity of 0 at the maximum height.

$$
\begin{aligned}
& v_{\substack{\text { maximum } \\
\text { height }}}^{2}=v_{\text {launch }}^{2}+2 a\left(y_{\text {max }}-y_{0}\right) \rightarrow \\
& y_{\max }=y_{0}+\frac{v_{\text {maximum }}^{2}-v_{\text {launch }}^{2}}{2 a}=\frac{-(18.84 \mathrm{~m} / \mathrm{s})^{2}}{2\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=18.1 \mathrm{~m}
\end{aligned}
$$

64. Choose up to be the positive direction. Let the bottom of the cliff be the $y=0$ location. The equation of motion for the dropped ball is $y_{\text {ball }}=y_{0}+v_{0} t+\frac{1}{2} a t^{2}=50.0 \mathrm{~m}+\frac{1}{2}\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right) t^{2}$. The equation of motion for the thrown stone is $y_{\text {stone }}=y_{0}+v_{0} t+\frac{1}{2} a t^{2}=(24.0 \mathrm{~m} / \mathrm{s}) t+\frac{1}{2}\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right) t^{2}$. Set the two equations equal and solve for the time of the collision. Then use that time to find the location of either object.

$$
\begin{aligned}
& y_{\text {ball }}=y_{\text {stone }} \rightarrow 50.0 \mathrm{~m}+\frac{1}{2}\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right) t^{2}=(24.0 \mathrm{~m} / \mathrm{s}) t+\frac{1}{2}\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right) t^{2} \rightarrow \\
& 50.0 \mathrm{~m}=(24.0 \mathrm{~m} / \mathrm{s}) t \rightarrow t=\frac{50.0 \mathrm{~m}}{24.0 \mathrm{~m} / \mathrm{s}}=2.083 \mathrm{~s} \\
& y_{\text {ball }}=y_{0}+v_{0} t+\frac{1}{2} a t^{2}=50.0 \mathrm{~m}+\frac{1}{2}\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(2.083 \mathrm{~s})^{2}=28.7 \mathrm{~m}
\end{aligned}
$$

65. For the falling rock, choose downward to be the positive direction, and $y_{0}=0$ to be the height from which the stone is dropped. The initial velocity is $v_{0}=0 \mathrm{~m} / \mathrm{s}$, the acceleration is $a=g$, the displacement is $y=H$, and the time of fall is $t_{1}$. Using Eq. 2-12b with $y$ substituting for $x$, we have $H=y_{0}+v_{0} t+\frac{1}{2} t^{2}=0+0+\frac{1}{2} g t_{1}^{2}$. For the sound wave, use the constant speed equation that $v_{s}=\frac{\Delta x}{\Delta t}=\frac{H}{T-t_{1}}$, which can be rearranged to give $t_{1}=T-\frac{H}{v_{s}}$, where $T=3.4 \mathrm{~s}$ is the total time elapsed from dropping the rock to hearing the sound. Insert this expression for $t_{1}$ into the equation for $H$ from the stone, and solve for $H$.

$$
\begin{aligned}
& H=\frac{1}{2} g\left(T-\frac{H}{v_{s}}\right)^{2} \rightarrow \frac{g}{2 v_{s}^{2}} H^{2}-\left(\frac{g T}{v_{s}}+1\right) H+\frac{1}{2} g T^{2}=0 \rightarrow \\
& 4.239 \times 10^{-5} H^{2}-1.098 H+56.64=0 \rightarrow H=51.7 \mathrm{~m}, 2.59 \times 10^{4} \mathrm{~m}
\end{aligned}
$$

If the larger answer is used in $t_{1}=T-\frac{H}{v_{s}}$, a negative time of fall results, and so the physically correct answer is $H=52 \mathrm{~m}$.
66. (a) Choose up to be the positive direction. Let the throwing height of both objects be the $y=0$ location, and so $y_{0}=0$ for both objects. The acceleration of both objects is $a=-g$. The equation of motion for the rock, using Eq. 2-12b, is $y_{\text {rock }}=y_{0}+v_{0 \text { rock }} t+\frac{1}{2} a t^{2}=v_{0 \text { rock }} t-\frac{1}{2} g t^{2}$, where $t$ is the time elapsed from the throwing of the rock. The equation of motion for the ball, being thrown 1.00 s later, is $y_{\text {ball }}=y_{0}+v_{0 \text { ball }}(t-1.00 \mathrm{~s})+\frac{1}{2} a(t-1.00 \mathrm{~s})^{2}=$ $v_{0 \text { ball }}(t-1.00 \mathrm{~s})-\frac{1}{2} g(t-1.00 \mathrm{~s})^{2}$. Set the two equations equal (meaning the two objects are at the same place) and solve for the time of the collision.

$$
\begin{aligned}
& y_{\text {rock }}=y_{\text {ball }} \rightarrow v_{0 \text { rock }} t-\frac{1}{2} g t^{2}=v_{0 \text { ball }}(t-1.00 \mathrm{~s})-\frac{1}{2} g(t-1.00 \mathrm{~s})^{2} \rightarrow \\
& (12.0 \mathrm{~m} / \mathrm{s}) t-\frac{1}{2}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) t^{2}=(18.0 \mathrm{~m} / \mathrm{s})(t-1.00 \mathrm{~s})-\frac{1}{2}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(t-1.00 \mathrm{~s})^{2} \rightarrow \\
& (15.8 \mathrm{~m} / \mathrm{s}) t=(22.9 \mathrm{~m}) \rightarrow t=1.45 \mathrm{~s}
\end{aligned}
$$

(b) Use the time for the collision to find the position of either object.

$$
y_{\text {rock }}=v_{0 \text { rock }} t-\frac{1}{2} g t^{2}=(12.0 \mathrm{~m} / \mathrm{s})(1.45 \mathrm{~s})-\frac{1}{2}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(1.45 \mathrm{~s})^{2}=7.10 \mathrm{~m}
$$

(c) Now the ball is thrown first, and so $y_{\text {ball }}=v_{0 \text { ball }} t-\frac{1}{2} g t^{2}$ and $y_{\text {rock }}=v_{0 \text { rock }}(t-1.00 \mathrm{~s})-\frac{1}{2} g(t-1.00 \mathrm{~s})^{2}$. Again set the two equations equal to find the time of collision.

$$
\begin{aligned}
& y_{\text {ball }}=y_{\text {rock }} \rightarrow v_{0 \text { ball }} t-\frac{1}{2} g t^{2}=v_{0 \text { rock }}(t-1.00 \mathrm{~s})-\frac{1}{2} g(t-1.00 \mathrm{~s})^{2} \rightarrow \\
& (18.0 \mathrm{~m} / \mathrm{s}) t-\frac{1}{2}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) t^{2}=(12.0 \mathrm{~m} / \mathrm{s})(t-1.00 \mathrm{~s})-\frac{1}{2}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(t-1.00 \mathrm{~s})^{2} \rightarrow \\
& (3.80 \mathrm{~m} / \mathrm{s}) t=16.9 \mathrm{~m} \rightarrow t=4.45 \mathrm{~s}
\end{aligned}
$$

But this answer can be deceptive. Where do the objects collide?

$$
y_{\text {ball }}=v_{0 \text { ball }} t-\frac{1}{2} g t^{2}=(18.0 \mathrm{~m} / \mathrm{s})(4.45 \mathrm{~s})-\frac{1}{2}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(4.45 \mathrm{~s})^{2}=-16.9 \mathrm{~m}
$$

Thus, assuming they were thrown from ground level, they collide below ground level, which cannot happen. Thus they never collide.
67. The displacement is found from the integral of the velocity, over the given time interval.

$$
\begin{aligned}
\Delta x & =\int_{t_{1}}^{t_{2}} v d t=\int_{t=1.5 \mathrm{~s}}^{t=3.1 \mathrm{~s}}(25+18 t) d t=\left.\left(25 t+9 t^{2}\right)\right|_{t=1.5 \mathrm{~s}} ^{t=3.1 \mathrm{~s}}=\left[25(3.1)+9(3.1)^{2}\right]-\left[25(1.5)+9(1.5)^{2}\right] \\
& =106 \mathrm{~m}
\end{aligned}
$$

68. (a) The speed is the integral of the acceleration.

$$
\begin{aligned}
& a=\frac{d v}{d t} \rightarrow d v=a d t \rightarrow d v=A \sqrt{t} d t \rightarrow \int_{v_{0}}^{v} d v=A \int_{0}^{t} \sqrt{t} d t \rightarrow \\
& v-v_{0}=\frac{2}{3} A t^{3 / 2} \rightarrow v=v_{0}+\frac{2}{3} A t^{3 / 2} \rightarrow v=7.5 \mathrm{~m} / \mathrm{s}+\frac{2}{3}\left(2.0 \mathrm{~m} / \mathrm{s}^{5 / 2}\right) t^{3 / 2}
\end{aligned}
$$

(b) The displacement is the integral of the velocity.

$$
\begin{aligned}
& v=\frac{d x}{d t} \rightarrow d x=v d t \rightarrow d x=\left(v_{0}+\frac{2}{3} A t^{3 / 2}\right) d t \rightarrow \\
& \int_{0 \mathrm{~m}}^{x} d x=\int_{0}^{t}\left(v_{0}+\frac{2}{3} A t^{3 / 2}\right) d t \rightarrow x=v_{0} t+\frac{2}{3} \frac{2}{5} A t^{5 / 2}=(7.5 \mathrm{~m} / \mathrm{s}) t+\frac{4}{15}\left(2.0 \mathrm{~m} / \mathrm{s}^{5 / 2}\right) t^{5 / 2}
\end{aligned}
$$

(c) $a(t=5.0 \mathrm{~s})=\left(2.0 \mathrm{~m} / \mathrm{s}^{5 / 2}\right) \sqrt{5.0 \mathrm{~s}}=4.5 \mathrm{~m} / \mathrm{s}^{2}$
$v(t=5.0 \mathrm{~s})=7.5 \mathrm{~m} / \mathrm{s}+\frac{2}{3}\left(2.0 \mathrm{~m} / \mathrm{s}^{5 / 2}\right)(5.0 \mathrm{~s})^{3 / 2}=22.41 \mathrm{~m} / \mathrm{s} \approx 22 \mathrm{~m} / \mathrm{s}$
$x(t=5.0 \mathrm{~s})=(7.5 \mathrm{~m} / \mathrm{s})(5.0 \mathrm{~s})+\frac{4}{15}\left(2.0 \mathrm{~m} / \mathrm{s}^{5 / 2}\right)(5.0 \mathrm{~s})^{5 / 2}=67.31 \mathrm{~m} \approx 67 \mathrm{~m}$
69. (a) The velocity is found by integrating the acceleration with respect to time. Note that with the substitution given in the hint, the initial value of $u$ is $u_{0}=g-k v_{0}=g$.

$$
a=\frac{d v}{d t} \rightarrow d v=a d t \rightarrow d v=(g-k v) d t \rightarrow \frac{d v}{g-k v}=d t
$$

Now make the substitution that $u \equiv g-k v$.

$$
\begin{aligned}
& u \equiv g-k v \rightarrow d v=-\frac{d u}{k} \quad \frac{d v}{g-k v}=d t \rightarrow-\frac{d u}{k} \frac{1}{u}=d t \rightarrow \frac{d u}{u}=-k d t \\
& \int_{g}^{u} \frac{d u}{u}=-\left.k \int_{0}^{t} d t \rightarrow \ln u\right|_{g} ^{u}=-k t \rightarrow \ln \frac{u}{g}=-k t \rightarrow u=g e^{-k t}=g-k v \rightarrow \\
& v=\frac{g}{k}\left(1-e^{-k t}\right)
\end{aligned}
$$

(b) As $t$ goes to infinity, the value of the velocity is $v_{\text {term }}=\lim _{t \rightarrow \infty} \frac{g}{k}\left(1-e^{-k t}\right)=\frac{g}{k}$. We also note that if the acceleration is zero (which happens at terminal velocity), then $a=g-k v=0 \rightarrow$ $v_{\text {term }}=\frac{g}{k}$.
70. (a) The train's constant speed is $v_{\text {train }}=5.0 \mathrm{~m} / \mathrm{s}$, and the location of the empty box car as a function of time is given by $x_{\text {train }}=v_{\text {train }} t=(5.0 \mathrm{~m} / \mathrm{s}) t$. The fugitive has $v_{0}=0 \mathrm{~m} / \mathrm{s}$ and $a=1.2 \mathrm{~m} / \mathrm{s}^{2}$ until his final speed is $6.0 \mathrm{~m} / \mathrm{s}$. The elapsed time during the acceleration is $t_{\text {acc }}=\frac{v-v_{0}}{a}=\frac{6.0 \mathrm{~m} / \mathrm{s}}{1.2 \mathrm{~m} / \mathrm{s}^{2}}=5.0 \mathrm{~s}$. Let the origin be the location of the fugitive when he starts to run. The first possibility to consider is, "Can the fugitive catch the empty box car before he reaches his maximum speed?" During the fugitive's acceleration, his location as a function of time is given by Eq. 2-12b, $x_{\text {fugitive }}=x_{0}+v_{0} t+\frac{1}{2} a t^{2}=0+0+\frac{1}{2}\left(1.2 \mathrm{~m} / \mathrm{s}^{2}\right) t^{2}$. For him to catch
the train, we must have $x_{\text {train }}=x_{\text {fugitive }} \rightarrow(5.0 \mathrm{~m} / \mathrm{s}) t=\frac{1}{2}\left(1.2 \mathrm{~m} / \mathrm{s}^{2}\right) t^{2}$. The solutions of this are $t=0 \mathrm{~s}, 8.3 \mathrm{~s}$. Thus the fugitive cannot catch the car during his 5.0 s of acceleration.

Now the equation of motion of the fugitive changes. After the 5.0 s of acceleration, he runs with a constant speed of $6.0 \mathrm{~m} / \mathrm{s}$. Thus his location is now given (for times $t>5 \mathrm{~s}$ ) by the following.

$$
x_{\text {fuytive }}=\frac{1}{2}\left(1.2 \mathrm{~m} / \mathrm{s}^{2}\right)(5.0 \mathrm{~s})^{2}+(6.0 \mathrm{~m} / \mathrm{s})(t-5.0 \mathrm{~s})=(6.0 \mathrm{~m} / \mathrm{s}) t-15.0 \mathrm{~m}
$$

So now, for the fugitive to catch the train, we again set the locations equal.

$$
x_{\text {train }}=x_{\text {fugitive }} \rightarrow(5.0 \mathrm{~m} / \mathrm{s}) t=(6.0 \mathrm{~m} / \mathrm{s}) t-15.0 \mathrm{~m} \rightarrow t=15.0 \mathrm{~s}
$$

(b) The distance traveled to reach the box car is given by the following.

$$
x_{\text {fugtive }}(t=15.0 \mathrm{~s})=(6.0 \mathrm{~m} / \mathrm{s})(15.0 \mathrm{~s})-15.0 \mathrm{~m}=75 \mathrm{~m}
$$

71. Choose the upward direction to be positive, and $y_{0}=0$ to be the level from which the object was thrown. The initial velocity is $v_{0}$ and the velocity at the top of the path is $v=0 \mathrm{~m} / \mathrm{s}$. The height at the top of the path can be found from Eq. 2-12c with $x$ replaced by $y$.

$$
v^{2}=v_{0}^{2}+2 a\left(y-y_{0}\right) \rightarrow y-y_{0}=\frac{-v_{0}^{2}}{2 a}
$$

From this we see that the displacement is inversely proportional to the acceleration, and so if the acceleration is reduced by a factor of 6 by going to the Moon, and the initial velocity is unchanged, the displacement increases by a factor of 6 .
72. (a) For the free-falling part of the motion, choose downward to be the positive direction, and $y_{0}=0$ to be the height from which the person jumped. The initial velocity is $v_{0}=0$, acceleration is $a=9.80 \mathrm{~m} / \mathrm{s}^{2}$, and the location of the net is $y=15.0 \mathrm{~m}$. Find the speed upon reaching the net from Eq. 2-12c with $x$ replaced by $y$.

$$
v^{2}=v_{0}^{2}+2 a\left(y-y_{()}\right) \rightarrow \quad v= \pm \sqrt{0+2 a(y-0)}= \pm \sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(15.0 \mathrm{~m})}=17.1 \mathrm{~m} / \mathrm{s}
$$

The positive root is selected since the person is moving downward.
For the net-stretching part of the motion, choose downward to be the positive direction, and $y_{0}=15.0 \mathrm{~m}$ to be the height at which the person first contacts the net. The initial velocity is $v_{0}=17.1 \mathrm{~m} / \mathrm{s}$, the final velocity is $v=0$, and the location at the stretched position is $y=16.0 \mathrm{~m}$. Find the acceleration from Eq. 2-12c with $x$ replaced by $y$.

$$
v^{2}=v_{0}^{2}+2 a\left(y-y_{0}\right) \rightarrow a=\frac{v^{2}-v_{0}^{2}}{2\left(y-y_{0}\right)}=\frac{0^{2}-(17.1 \mathrm{~m} / \mathrm{s})^{2}}{2(1.0 \mathrm{~m})}=-150 \mathrm{~m} / \mathrm{s}^{2}
$$

(b) For the acceleration to be smaller, in the above equation we see that the displacement should be larger. This means that the net should be "loosened".
73. The initial velocity of the car is $v_{0}=(100 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)=27.8 \mathrm{~m} / \mathrm{s}$. Choose $x_{0}=0$ to be the location at which the deceleration begins. We have $v=0 \mathrm{~m} / \mathrm{s}$ and $a=-30 g=-294 \mathrm{~m} / \mathrm{s}^{2}$. Find the displacement from Eq. 2-12c.

$$
v^{2}=v_{0}^{2}+2 a\left(x-x_{0}\right) \rightarrow x=x_{0}+\frac{v^{2}-v_{0}^{2}}{2 a}=0+\frac{0-(27.8 \mathrm{~m} / \mathrm{s})^{2}}{2\left(-2.94 \times 10^{2} \mathrm{~m} / \mathrm{s}^{2}\right)}=1.31 \mathrm{~m} \approx 1.3 \mathrm{~m}
$$

74. Choose downward to be the positive direction, and $y_{0}=0$ to be at the start of the pelican's dive. The pelican has an initial velocity is $v_{0}=0$, an acceleration of $a=g$, and a final location of $y=16.0 \mathrm{~m}$. Find the total time of the pelican's dive from Eq. $2-12 \mathrm{~b}$, with $x$ replaced by $y$.

$$
y=y_{0}+v_{0} t+\frac{1}{2} a t^{2} \rightarrow y=0+0+\frac{1}{2} a t^{2} \rightarrow t_{\text {dive }}=\sqrt{\frac{2 y}{a}}=\sqrt{\frac{2(16.0 \mathrm{~m})}{9.80 \mathrm{~m} / \mathrm{s}^{2}}}=1.81 \mathrm{~s}
$$

The fish can take evasive action if he sees the pelican at a time of $1.81 \mathrm{~s}-0.20 \mathrm{~s}=1.61 \mathrm{~s}$ into the dive. Find the location of the pelican at that time from Eq. 2-12b.

$$
y=y_{0}+v_{0} t+\frac{1}{2} a t=0+0+\frac{1}{2}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(1.61 \mathrm{~s})^{2}=12.7 \mathrm{~m}
$$

Thus the fish must spot the pelican at a minimum height from the surface of the water of $16.0 \mathrm{~m}-12.7 \mathrm{~m}=3.3 \mathrm{~m}$.
75. (a) Choose downward to be the positive direction, and $y_{0}=0$ to be the level from which the car was dropped. The initial velocity is $v_{0}=0$, the final location is $y=H$, and the acceleration is $a=g$. Find the final velocity from Eq. 2-12c, replacing $x$ with $y$.

$$
v^{2}=v_{0}^{2}+2 a\left(y-y_{0}\right) \rightarrow v= \pm \sqrt{v_{0}^{2}+2 a\left(y-y_{0}\right)}= \pm \sqrt{2 g H}
$$

The speed is the magnitude of the velocity, $v=\sqrt{2 g H}$.
(b) Solving the above equation for the height, we have that $H=\frac{v^{2}}{2 g}$. Thus for a collision of

$$
\begin{gathered}
v=(50 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)=13.89 \mathrm{~m} / \mathrm{s}, \text { the corresponding height is as follows. } \\
H=\frac{v^{2}}{2 g}=\frac{(13.89 \mathrm{~m} / \mathrm{s})^{2}}{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=9.84 \mathrm{~m} \approx 10 \mathrm{~m}
\end{gathered}
$$

(c) For a collision of $v=(100 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)=27.78 \mathrm{~m} / \mathrm{s}$, the corresponding height is as follow.

$$
H=\frac{v^{2}}{2 g}=\frac{(27.78 \mathrm{~m} / \mathrm{s})^{2}}{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=39.37 \mathrm{~m} \approx 40 \mathrm{~m}
$$

76. Choose downward to be the positive direction, and $y_{0}=0$ to be at the roof from which the stones are dropped. The first stone has an initial velocity of $v_{0}=0$ and an acceleration of $a=g$. Eqs. 212a and $2-12 \mathrm{~b}$ (with $x$ replaced by $y$ ) give the velocity and location, respectively, of the first stone as a function of time.

$$
v=v_{0}+a t \rightarrow v_{1}=g t_{1} \quad y=y_{0}+v_{0} t+\frac{1}{2} a t^{2} \quad \rightarrow y_{1}=\frac{1}{2} g t_{1}^{2}
$$

The second stone has the same initial conditions, but its elapsed time $t-1.50 \mathrm{~s}$, and so has velocity and location equations as follows.

$$
v_{2}=g\left(t_{1}-1.50 \mathrm{~s}\right) \quad y_{2}=\frac{1}{2} g\left(t_{1}-1.50 \mathrm{~s}\right)^{2}
$$

The second stone reaches a speed of $v_{2}=12.0 \mathrm{~m} / \mathrm{s}$ at a time given by the following.

$$
t_{1}=1.50 \mathrm{~s}+\frac{v_{2}}{g}=1.50 \mathrm{~s}+\frac{12.0 \mathrm{~m} / \mathrm{s}}{9.80 \mathrm{~m} / \mathrm{s}^{2}}=2.72 \mathrm{~s}
$$

The location of the first stone at that time is $y_{1}=\frac{1}{2} g t_{1}^{2}=\frac{1}{2}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(2.72 \mathrm{~s})^{2}=36.4 \mathrm{~m}$. .
The location of the second stone at that time is $y_{2}=\frac{1}{2} g\left(t_{1}-1.50 \mathrm{~s}\right)^{2}=$ $\frac{1}{2}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(2.72-1.50 \mathrm{~s})^{2}=7.35 \mathrm{~m}$. Thus the distance between the two stones is $y_{1}-y_{2}=36.4 \mathrm{~m}-7.35 \mathrm{~m}=29.0 \mathrm{~m}$.
77. The initial velocity is $v_{0}=(15 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)=4.17 \mathrm{~m} / \mathrm{s}$. The final velocity is $v_{0}=(75 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)=20.83 \mathrm{~m} / \mathrm{s}$. The displacement is $x-x_{0}=4.0 \mathrm{~km}=4000 \mathrm{~m}$. Find the average acceleration from Eq. 2-12c.

$$
v^{2}=v_{0}^{2}+2 a\left(x-x_{0}\right) \rightarrow a=\frac{v^{2}-v_{0}^{2}}{2\left(x-x_{0}\right)}=\frac{(20.83 \mathrm{~m} / \mathrm{s})^{2}-(4.17 \mathrm{~m} / \mathrm{s})^{2}}{2(4000 \mathrm{~m})}=5.2 \times 10^{-2} \mathrm{~m} / \mathrm{s}^{2}
$$

78. The speed limit is $50 \mathrm{~km} / \mathrm{h}\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)=13.89 \mathrm{~m} / \mathrm{s}$.
(a) For your motion, you would need to travel $(10+15+50+15+70+15) \mathrm{m}=175 \mathrm{~m}$ to get the front of the car all the way through the third intersection. The time to travel the 175 m is found using the distance and the constant speed.

$$
\Delta x=\bar{v} \Delta t \rightarrow \Delta t=\frac{\Delta x}{\bar{v}}=\frac{175 \mathrm{~m}}{13.89 \mathrm{~m} / \mathrm{s}}=12.60 \mathrm{~s}
$$

Yes, you can make it through all three lights without stopping.
(b) The second car needs to travel 165 m before the third light turns red. This car accelerates from $v_{0}=0 \mathrm{~m} / \mathrm{s}$ to a maximum of $v=13.89 \mathrm{~m} / \mathrm{s}$ with $a=2.0 \mathrm{~m} / \mathrm{s}^{2}$. Use Eq. 2-12a to determine the duration of that acceleration.

$$
v=v_{0}+a t \rightarrow t_{\mathrm{acc}}=\frac{v-v_{0}}{a}=\frac{13.89 \mathrm{~m} / \mathrm{s}-0 \mathrm{~m} / \mathrm{s}}{2.0 \mathrm{~m} / \mathrm{s}^{2}}=6.94 \mathrm{~s}
$$

The distance traveled during that time is found from Eq. 2-12b.

$$
\left(x-x_{0}\right)_{\mathrm{acc}}=v_{0} t_{\mathrm{acc}}+\frac{1}{2} a t_{\mathrm{acc}}^{2}=0+\frac{1}{2}\left(2.0 \mathrm{~m} / \mathrm{s}^{2}\right)(6.94 \mathrm{~s})^{2}=48.2 \mathrm{~m}
$$

Since 6.94 s have elapsed, there are $13-6.94=6.06 \mathrm{~s}$ remaining to clear the intersection. The car travels another 6.06 s at a speed of $13.89 \mathrm{~m} / \mathrm{s}$, covering a distance of $\Delta x_{\substack{\text { constant } \\ \text { speed }}}=v_{\text {avg }} t=$
$(13.89 \mathrm{~m} / \mathrm{s})(6.06 \mathrm{~s})=84.2 \mathrm{~m}$. Thus the total distance is $48.2 \mathrm{~m}+84.2 \mathrm{~m}=132.4 \mathrm{~m}$. No, the car cannot make it through all three lights without stopping.

The car has to travel another 32.6 m to clear the third intersection, and is traveling at a speed of $13.89 \mathrm{~m} / \mathrm{s}$. Thus the care would enter the intersection a time $t=\frac{\Delta x}{v}=\frac{32.6 \mathrm{~m}}{13.89 \mathrm{~m} / \mathrm{s}}=2.3 \mathrm{~s}$ after the light turns red.
79. First consider the "uphill lie," in which the ball is being putted down the hill. Choose $x_{0}=0$ to be the ball's original location, and the direction of the ball's travel as the positive direction. The final velocity of the ball is $v=0 \mathrm{~m} / \mathrm{s}$, the acceleration of the ball is $a=-1.8 \mathrm{~m} / \mathrm{s}^{2}$, and the displacement of the ball will be $x-x_{0}=6.0 \mathrm{~m}$ for the first case and $x-x_{0}=8.0 \mathrm{~m}$ for the second case. Find the initial velocity of the ball from Eq. 2-12c.

$$
v^{2}=v_{0}^{2}+2 a\left(x-x_{0}\right) \rightarrow v_{0}=\sqrt{v^{2}-2 a\left(x-x_{0}\right)}=\left\{\begin{array}{l}
\sqrt{0-2\left(-1.8 \mathrm{~m} / \mathrm{s}^{2}\right)(6.0 \mathrm{~m})}=4.6 \mathrm{~m} / \mathrm{s} \\
\sqrt{0-2\left(-1.8 \mathrm{~m} / \mathrm{s}^{2}\right)(8.0 \mathrm{~m})}=5.4 \mathrm{~m} / \mathrm{s}
\end{array}\right.
$$

The range of acceptable velocities for the uphill lie is $4.6 \mathrm{~m} / \mathrm{s}$ to $5.4 \mathrm{~m} / \mathrm{s}$, a spread of $0.8 \mathrm{~m} / \mathrm{s}$.
Now consider the "downhill lie," in which the ball is being putted up the hill. Use a very similar setup for the problem, with the basic difference being that the acceleration of the ball is now $a=-2.8 \mathrm{~m} / \mathrm{s}^{2}$. Find the initial velocity of the ball from Eq. 2-12c.

$$
v^{2}=v_{0}^{2}+2 a\left(x-x_{0}\right) \rightarrow v_{0}=\sqrt{v^{2}-2 a\left(x-x_{0}\right)}=\left\{\begin{array}{l}
\sqrt{0-2\left(-2.8 \mathrm{~m} / \mathrm{s}^{2}\right)(6.0 \mathrm{~m})}=5.8 \mathrm{~m} / \mathrm{s} \\
\sqrt{0-2\left(-2.8 \mathrm{~m} / \mathrm{s}^{2}\right)(8.0 \mathrm{~m})}=6.7 \mathrm{~m} / \mathrm{s}
\end{array}\right.
$$

The range of acceptable velocities for the downhill lie is $5.8 \mathrm{~m} / \mathrm{s}$ to $6.7 \mathrm{~m} / \mathrm{s}$, a spread of $0.9 \mathrm{~m} / \mathrm{s}$.
Because the range of acceptable velocities is smaller for putting down the hill, more control in putting is necessary, and so putting the ball downhill (the "uphill lie") is more difficult.
80. To find the distance, we divide the motion of the robot into three segments. First, the initial acceleration from rest; second, motion at constant speed; and third, deceleration back to rest.

$$
\begin{aligned}
& d_{1}=v_{0} t+\frac{1}{2} a_{1} t_{1}^{2}=0+\frac{1}{2}\left(0.20 \mathrm{~m} / \mathrm{s}^{2}\right)(5.0 \mathrm{~s})^{2}=2.5 \mathrm{~m} \quad v_{1}=a_{1} t_{1}=\left(0.20 \mathrm{~m} / \mathrm{s}^{2}\right)(5.0 \mathrm{~s})=1.0 \mathrm{~m} / \mathrm{s} \\
& d_{2}=v_{1} t_{2}=(1.0 \mathrm{~m} / \mathrm{s})(68 \mathrm{~s})=68 \mathrm{~m} \quad v_{2}=v_{1}=1.0 \mathrm{~m} / \mathrm{s} \\
& d_{3}=v_{2} t_{3}+\frac{1}{2} a_{1} t_{1}^{2}=(1.0 \mathrm{~m} / \mathrm{s})(2.5 \mathrm{~s})+\frac{1}{2}\left(-0.40 \mathrm{~m} / \mathrm{s}^{2}\right)(2.5 \mathrm{~s})^{2}=1.25 \mathrm{~m} \\
& d=d_{1}+d_{2}+d_{3}=2.5 \mathrm{~m}+68 \mathrm{~m}+1.25 \mathrm{~m}=71.75 \mathrm{~m} \approx 72 \mathrm{~m}
\end{aligned}
$$

81. Choose downward to be the positive direction, and $y_{0}=0$ to be at the top of the cliff. The initial velocity is $v_{0}=-12.5 \mathrm{~m} / \mathrm{s}$, the acceleration is $a=9.80 \mathrm{~m} / \mathrm{s}^{2}$, and the final location is $y=75.0 \mathrm{~m}$.
(a) Using Eq. 2-12b and substituting $y$ for $x$, we have the following.

$$
y=y_{0}+v_{0} t+\frac{1}{2} a t^{2} \rightarrow\left(4.9 \mathrm{~m} / \mathrm{s}^{2}\right) t^{2}-(12.5 \mathrm{~m} / \mathrm{s}) t-75.0 \mathrm{~m}=0 \rightarrow t=-2.839 \mathrm{~s}, 5.390 \mathrm{~s}
$$

The positive answer is the physical answer: $t=5.39 \mathrm{~s}$.
(b) Using Eq. 2-12a, we have $v=v_{0}+a t=-12.5 \mathrm{~m} / \mathrm{s}+\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(5.390 \mathrm{~s})=40.3 \mathrm{~m} / \mathrm{s}$.
(c) The total distance traveled will be the distance up plus the distance down. The distance down will be 75.0 m more than the distance up. To find the distance up, use the fact that the speed at the top of the path will be 0 . Using Eq. $2-12 \mathrm{c}$ we have the following.

$$
v^{2}=v_{0}^{2}+2 a\left(y-y_{0}\right) \rightarrow y=y_{0}+\frac{v^{2}-v_{0}^{2}}{2 a}=0+\frac{0-(-12.5 \mathrm{~m} / \mathrm{s})^{2}}{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=-7.97 \mathrm{~m}
$$

Thus the distance up is 7.97 m , the distance down is 82.97 m , and the total distance traveled is 90.9 m .
82. (a) In the interval from A to B , it is moving in the negative direction, because its displacement is negative.
(b) In the interval from A to B , it is speeding up, because the magnitude of its slope is increasing (changing from less steep to more steep).
(c) In the interval from A to B , the acceleration is negative, because the graph is concave down, indicating that the slope is getting more negative, and thus the acceleration is negative.
(d) In the interval from D to E , it is moving in the positive direction, because the displacement is positive.
(e) In the interval from D to E, it is speeding up, because the magnitude of its slope is increasing (changing from less steep to more steep).
(f) In the interval from D to E , the acceleration is positive, because the graph is concave upward, indicating the slope is getting more positive, and thus the acceleration is positive.
(g) In the interval from C to D , the object is not moving in either direction.

The velocity and acceleration are both 0 .
83. This problem can be analyzed as a series of three one-dimensional motions: the acceleration phase, the constant speed phase, and the deceleration phase. The maximum speed of the train is as follows.

$$
(95 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)=26.39 \mathrm{~m} / \mathrm{s}
$$

In the acceleration phase, the initial velocity is $v_{0}=0 \mathrm{~m} / \mathrm{s}$, the acceleration is $a=1.1 \mathrm{~m} / \mathrm{s}^{2}$, and the final velocity is $v=26.39 \mathrm{~m} / \mathrm{s}$. Find the elapsed time for the acceleration phase from Eq. 2-12a.

$$
v=v_{0}+a t \rightarrow t_{\mathrm{acc}}=\frac{v-v_{0}}{a}=\frac{26.39 \mathrm{~m} / \mathrm{s}-0}{1.1 \mathrm{~m} / \mathrm{s}^{2}}=23.99 \mathrm{~s}
$$

Find the displacement during the acceleration phase from Eq. 2-12b.

$$
\left(x-x_{0}\right)_{\mathrm{acc}}=v_{0} t+\frac{1}{2} a t^{2}=0+\frac{1}{2}\left(1.1 \mathrm{~m} / \mathrm{s}^{2}\right)(23.99 \mathrm{~s})^{2}=316.5 \mathrm{~m}
$$

In the deceleration phase, the initial velocity is $v_{0}=26.39 \mathrm{~m} / \mathrm{s}$, the acceleration is $a=-2.0 \mathrm{~m} / \mathrm{s}^{2}$, and the final velocity is $v=0 \mathrm{~m} / \mathrm{s}$. Find the elapsed time for the deceleration phase from Eq. 2-12a.

$$
v=v_{0}+a t \rightarrow t_{\mathrm{dec}}=\frac{v-v_{0}}{a}=\frac{0-26.39 \mathrm{~m} / \mathrm{s}}{-2.0 \mathrm{~m} / \mathrm{s}^{2}}=13.20 \mathrm{~s}
$$

Find the distance traveled during the deceleration phase from Eq. 2-12b.

$$
\left(x-x_{0}\right)_{\mathrm{dec}}=v_{0} t+\frac{1}{2} a t^{2}=(26.39 \mathrm{~m} / \mathrm{s})(13.20 \mathrm{~s})+\frac{1}{2}\left(-2.0 \mathrm{~m} / \mathrm{s}^{2}\right)(13.20 \mathrm{~s})^{2}=174.1 \mathrm{~m}
$$

The total elapsed time and distance traveled for the acceleration / deceleration phases are:
$t_{\mathrm{acc}}+t_{\mathrm{dec}}=23.99 \mathrm{~s}+13.20 \mathrm{~s}=37.19 \mathrm{~s}$
$\left(x-x_{0}\right)_{\mathrm{acc}}+\left(x-x_{0}\right)_{\mathrm{dec}}=316.5 \mathrm{~m}+174.1 \mathrm{~m}=491 \mathrm{~m}$
(a) If the stations are spaced $1.80 \mathrm{~km}=1800 \mathrm{~m}$ apart, then there is a total of $\frac{9000 \mathrm{~m}}{1800 \mathrm{~m}}=5$ interstation segments. A train making the entire trip would thus have a total of 5 inter-station segments and 4 stops of 22 s each at the intermediate stations. Since 491 m is traveled during acceleration and deceleration, $1800 \mathrm{~m}-491 \mathrm{~m}=1309 \mathrm{~m}$ of each segment is traveled at an average speed of $\bar{v}=26.39 \mathrm{~m} / \mathrm{s}$. The time for that 1309 m is given by $\Delta x=\bar{v} \Delta t \rightarrow$ $\Delta t_{\substack{\text { constant } \\ \text { speed }}}=\frac{\Delta x}{\bar{v}}=\frac{1309 \mathrm{~m}}{26.39 \mathrm{~m} / \mathrm{s}}=49.60 \mathrm{~s}$. Thus a total inter-station segment will take $37.19 \mathrm{~s}+$ $49.60 \mathrm{~s}=86.79 \mathrm{~s}$. With 5 inter-station segments of 86.79 s each, and 4 stops of 22 s each, the total time is given by $t_{0.8 \mathrm{~km}}=5(86.79 \mathrm{~s})+4(22 \mathrm{~s})=522 \mathrm{~s}=8.7 \mathrm{~min}$.
(b) If the stations are spaced $3.0 \mathrm{~km}=3000 \mathrm{~m}$ apart, then there is a total of $\frac{9000 \mathrm{~m}}{3000 \mathrm{~m}}=3$ interstation segments. A train making the entire trip would thus have a total of 3 inter-station segments and 2 stops of 22 s each at the intermediate stations. Since 491 m is traveled during acceleration and deceleration, $3000 \mathrm{~m}-491 \mathrm{~m}=2509 \mathrm{~m}$ of each segment is traveled at an average speed of $\bar{v}=26.39 \mathrm{~m} / \mathrm{s}$. The time for that 2509 m is given by $d=\bar{v} t \rightarrow$ $t=\frac{d}{\bar{v}}=\frac{2509 \mathrm{~m}}{26.39 \mathrm{~m} / \mathrm{s}}=95.07 \mathrm{~s}$. Thus a total inter-station segment will take $37.19 \mathrm{~s}+95.07 \mathrm{~s}=$ 132.3 s . With 3 inter-station segments of 132.3 s each, and 2 stops of 22 s each, the total time is $t_{3.0 \mathrm{~km}}=3(132.3 \mathrm{~s})+2(22 \mathrm{~s})=441 \mathrm{~s}=7.3 \mathrm{~min}$.
84. For the motion in the air, choose downward to be the positive direction, and $y_{0}=0$ to be at the height of the diving board. The diver has $v_{0}=0$ (assuming the diver does not jump upward or downward), $a=g=9.80 \mathrm{~m} / \mathrm{s}^{2}$, and $y=4.0 \mathrm{~m}$ when reaching the surface of the water. Find the diver's speed at the water's surface from Eq. $2-12 \mathrm{c}$, with $x$ replaced by $y$.

$$
\begin{aligned}
& v^{2}=v_{0}^{2}+2 a\left(y-y_{0}\right) x \rightarrow \\
& v= \pm \sqrt{v_{0}^{2}+2 a\left(y-y_{0}\right)}=\sqrt{0+2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(4.0 \mathrm{~m})}=8.85 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

For the motion in the water, again choose down to be positive, but redefine $y_{0}=0$ to be at the surface of the water. For this motion, $v_{0}=8.85 \mathrm{~m} / \mathrm{s}, v=0$, and $y-y_{0}=2.0 \mathrm{~m}$. Find the acceleration from Eq. 2-12c, with $x$ replaced by $y$.

$$
v^{2}=v_{0}^{2}+2 a\left(y-y_{0}\right) \rightarrow a=\frac{v^{2}-v_{0}^{2}}{2\left(y-y_{0}\right) x}=\frac{0-(8.85 \mathrm{~m} / \mathrm{s})^{2}}{2(2.0 \mathrm{~m})}=-19.6 \mathrm{~m} / \mathrm{s}^{2} \approx-20 \mathrm{~m} / \mathrm{s}^{2}
$$

The negative sign indicates that the acceleration is directed upwards.
85. Choose upward to be the positive direction, and the origin to be at the level where the ball was thrown. The velocity at the top of the ball's path will be $v=0$, and the ball will have an acceleration of $a=-g$. If the maximum height that the ball reaches is $y=H$, then the relationship
between the initial velocity and the maximum height can be found from Eq. $2-12 \mathrm{c}$, with $x$ replaced by $y$.

$$
v^{2}=v_{0}^{2}+2 a\left(y-y_{0}\right) \rightarrow 0=v_{0}^{2}+2(-g) H \rightarrow H=v_{0}^{2} / 2 g
$$

It is given that $v_{0 \text { Bill }}=1.5 v_{0 \text { Joe }}$, so $\frac{H_{\text {Bill }}}{H_{\text {Joe }}}=\frac{\left(v_{0 \text { Bill }}\right)^{2} / 2 g}{\left(v_{0 \text { Joe }}\right)^{2} / 2 g}=\frac{\left(v_{0 \text { Bill }}\right)^{2}}{\left(v_{0 \text { Joe }}\right)^{2}}=1.5^{2}=2.25 \approx 2.3$.
86. The $v$ vs. $t$ graph is found by taking the slope of the $x$ vs. $t$ graph. Both graphs are shown here.


87. The car's initial speed is $v_{o}=(45 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)=12.5 \mathrm{~m} / \mathrm{s}$.

Case I: trying to stop. The constraint is, with the braking deceleration of the car $\left(a=-5.8 \mathrm{~m} / \mathrm{s}^{2}\right)$, can the car stop in a 28 m displacement? The 2.0 seconds has no relation to this part of the problem. Using Eq. 2-12c, the distance traveled during braking is as follows.

$$
\left(x-x_{0}\right)=\frac{v^{2}-v_{0}^{2}}{2 a}=\frac{0-(12.5 \mathrm{~m} / \mathrm{s})^{2}}{2\left(-5.8 \mathrm{~m} / \mathrm{s}^{2}\right)}=13.5 \mathrm{~m} \rightarrow \text { She can stop the car in time. }
$$

Case II: crossing the intersection. The constraint is, with the given acceleration of the car $\left[a=\left(\frac{65 \mathrm{~km} / \mathrm{h}-45 \mathrm{~km} / \mathrm{h}}{6.0 \mathrm{~s}}\right)\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)=0.9259 \mathrm{~m} / \mathrm{s}^{2}\right]$, can she get through the intersection
(travel 43 meters) in the 2.0 seconds before the light turns red? Using Eq. 2-12b, the distance traveled during the 2.0 sec is as follows.

$$
\left(x-x_{0}\right)=v_{0} t+\frac{1}{2} a t^{2}=(12.5 \mathrm{~m} / \mathrm{s})(2.0 \mathrm{~s})+\frac{1}{2}\left(0.927 \mathrm{~m} / \mathrm{s}^{2}\right)(2.0 \mathrm{~s})^{2}=26.9 \mathrm{~m}
$$

## She should stop.

88. The critical condition is that the total distance covered by the passing car and the approaching car must be less than 400 m so that they do not collide. The passing car has a total displacement composed of several individual parts. These are: i) the 10 m of clear room at the rear of the truck, ii) the 20 m length of the truck, iii) the 10 m of clear room at the front of the truck, and iv) the distance the truck travels. Since the truck travels at a speed of $\bar{v}=25 \mathrm{~m} / \mathrm{s}$, the truck will have a displacement of $\Delta x_{\text {truck }}=(25 \mathrm{~m} / \mathrm{s}) t$. Thus the total displacement of the car during passing is $\Delta x_{\substack{\text { passing } \\ \text { car }}}=40 \mathrm{~m}+(25 \mathrm{~m} / \mathrm{s}) t$.

To express the motion of the car, we choose the origin to be at the location of the passing car when the decision to pass is made. For the passing car, we have an initial velocity of $v_{0}=25 \mathrm{~m} / \mathrm{s}$ and an acceleration of $a=1.0 \mathrm{~m} / \mathrm{s}^{2}$. Find $\Delta x_{\substack{\text { passing } \\ \text { car }}}$ from Eq. 2-12b.

$$
\Delta x_{\substack{\text { passing } \\ \text { car }}}=x_{c}-x_{0}=v_{0} t+\frac{1}{2} a t=(25 \mathrm{~m} / \mathrm{s}) t+\frac{1}{2}\left(1.0 \mathrm{~m} / \mathrm{s}^{2}\right) t^{2}
$$

Set the two expressions for $\Delta x_{\substack{\text { passing } \\ \text { car }}}$ equal to each other in order to find the time required to pass.

$$
\begin{aligned}
& 40 \mathrm{~m}+(25 \mathrm{~m} / \mathrm{s}) t_{\text {pass }}=(25 \mathrm{~m} / \mathrm{s}) t_{\mathrm{pass}}+\frac{1}{2}\left(1.0 \mathrm{~m} / \mathrm{s}^{2}\right) t_{\mathrm{pass}}^{2} \rightarrow 40 \mathrm{~m}=\frac{1}{2}\left(1.0 \mathrm{~m} / \mathrm{s}^{2}\right) t_{\mathrm{pass}}^{2} \rightarrow \\
& t_{\text {pass }}=\sqrt{80 \mathrm{~s}^{2}}=8.94 \mathrm{~s}
\end{aligned}
$$

Calculate the displacements of the two cars during this time.

$$
\begin{aligned}
& \Delta x_{\substack{\text { passing } \\
\text { car }}}=40 \mathrm{~m}+(25 \mathrm{~m} / \mathrm{s})(8.94 \mathrm{~s})=264 \mathrm{~m} \\
& \Delta x_{\substack{\text { approaching } \\
\text { car }}}=v_{\substack{\text { approaching } \\
\text { car }}} t=(25 \mathrm{~m} / \mathrm{s})(8.94 \mathrm{~s})=224 \mathrm{~m}
\end{aligned}
$$

Thus the two cars together have covered a total distance of 488 m , which is more than allowed.
The car should not pass.
89. Choose downward to be the positive direction, and $y_{0}=0$ to be at the height of the bridge. Agent Bond has an initial velocity of $v_{0}=0$, an acceleration of $a=g$, and will have a displacement of $y=13 \mathrm{~m}-1.5 \mathrm{~m}=11.5 \mathrm{~m}$. Find the time of fall from Eq. 2-12b with $x$ replaced by $y$.

$$
y=y_{0}+v_{0} t+\frac{1}{2} a t^{2} \rightarrow t=\sqrt{\frac{2 y}{a}}=\sqrt{\frac{2(11.5 \mathrm{~m})}{9.80 \mathrm{~m} / \mathrm{s}^{2}}}=1.532 \mathrm{~s}
$$

If the truck is approaching with $v=25 \mathrm{~m} / \mathrm{s}$, then he needs to jump when the truck is a distance away given by $d=v t=(25 \mathrm{~m} / \mathrm{s})(1.532 \mathrm{~s})=38.3 \mathrm{~m}$. Convert this distance into "poles."

$$
d=(38.3 \mathrm{~m})(1 \text { pole } / 25 \mathrm{~m})=1.53 \text { poles }
$$

So he should jump when the truck is about 1.5 poles away from the bridge.
90. Take the origin to be the location where the speeder passes the police car. The speeder's constant speed is $v_{\text {speeder }}=(130 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)=36.1 \mathrm{~m} / \mathrm{s}$, and the location of the speeder as a function of time is given by $x_{\text {speeder }}=v_{\text {speeder }} t_{\text {speeder }}=(36.1 \mathrm{~m} / \mathrm{s}) t_{\text {speeder }}$. The police car has an initial velocity of $v_{0}=0 \mathrm{~m} / \mathrm{s}$ and a constant acceleration of $a_{\text {police }}$. The location of the police car as a function of time is given by Eq. 2-12b: $x_{\text {police }}=v_{0} t+\frac{1}{2} a t^{2}=\frac{1}{2} a_{\text {police }} t_{\text {police }}^{2}$.
(a) The position vs. time graphs would qualitatively look like the graph shown here.
(b) The time to overtake the speeder occurs when the speeder has gone a distance of 750 m . The time is found using the speeder's equation from above.


$$
750 \mathrm{~m}=(36.1 \mathrm{~m} / \mathrm{s}) t_{\text {speceder }} \rightarrow t_{\text {speceder }}=\frac{750 \mathrm{~m}}{36.1 \mathrm{~m} / \mathrm{s}}=20.8 \mathrm{~s} \approx 21 \mathrm{~s}
$$

(c) The police car's acceleration can be calculated knowing that the police car also had gone a distance of 750 m in a time of 22.5 s .

$$
750 \mathrm{~m}=\frac{1}{2} a_{p}(20.8 \mathrm{~s})^{2} \rightarrow a_{p}=\frac{2(750 \mathrm{~m})}{(20.8 \mathrm{~s})^{2}}=3.47 \mathrm{~m} / \mathrm{s}^{2} \approx 3.5 \mathrm{~m} / \mathrm{s}^{2}
$$

(d) The speed of the police car at the overtaking point can be found from Eq. 2-12a.

$$
v=v_{0}+a t=0+\left(3.47 \mathrm{~m} / \mathrm{s}^{2}\right)(20.8 \mathrm{~s})=72.2 \mathrm{~m} / \mathrm{s} \approx 72 \mathrm{~m} / \mathrm{s}
$$

Note that this is exactly twice the speed of the speeder.
91. The speed of the conveyor belt is given by $d=\bar{v} \Delta t \rightarrow \bar{v}=\frac{d}{\Delta t}=\frac{1.1 \mathrm{~m}}{2.5 \mathrm{~min}}=0.44 \mathrm{~m} / \mathrm{min}$. The rate of burger production, assuming the spacing given is center to center, can be found as follows.

$$
\left(\frac{1 \text { burger }}{0.15 \mathrm{~m}}\right)\left(\frac{0.44 \mathrm{~m}}{1 \mathrm{~min}}\right)=2.9 \frac{\text { burgers }}{\min }
$$

92. Choose downward to be the positive direction, and the origin to be at the top of the building. The barometer has $y_{0}=0, v_{0}=0$, and $a=g=9.8 \mathrm{~m} / \mathrm{s}^{2}$. Use Eq. $2-12 \mathrm{~b}$ to find the height of the building, with $x$ replaced by $y$.

$$
\begin{aligned}
& y=y_{0}+v_{0} t+\frac{1}{2} a t^{2}=0+0+\frac{1}{2}\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right) t^{2} \\
& y_{t=2.0}=\frac{1}{2}\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)(2.0 \mathrm{~s})^{2}=20 \mathrm{~m} \quad y_{t=2.3}=\frac{1}{2}\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)(2.3 \mathrm{~s})^{2}=26 \mathrm{~m}
\end{aligned}
$$

The difference in the estimates is 6 m . If we assume the height of the building is the average of the two measurements, then the \% difference in the two values is $\frac{6 \mathrm{~m}}{23 \mathrm{~m}} \times 100=26 \%$.
93. (a) The two bicycles will have the same velocity at any time when the instantaneous slopes of their $x$ vs. $t$ graphs are the same. That occurs near the time $t_{1}$ as marked on the graph.
(b) Bicycle A has the larger acceleration, because its graph is concave upward, indicating a positive acceleration. Bicycle B has no acceleration because its graph has a constant slope.

(c) The bicycles are passing each other at the times when the two graphs cross, because they both have the same position at that time. The graph with the steepest slope is the faster bicycle, and so is the one that is passing at that instant. So at the first crossing, bicycle B is passing bicycle A. At the second crossing, bicycle A is passing bicycle $B$.
(d) Bicycle B has the highest instantaneous velocity at all times until the time $t_{1}$, where both graphs have the same slope. For all times after $t_{1}$, bicycle A has the highest instantaneous velocity. The largest instantaneous velocity is for bicycle A at the latest time shown on the graph.
(e) The bicycles appear to have the same average velocity. If the starting point of the graph for a particular bicycle is connected to the ending point with a straight line, the slope of that line is the average velocity. Both appear to have the same slope for that "average" line.
94. In this problem, note that $a<0$ and $x>0$. Take your starting position as 0 . Then your position is given by Eq. $2-12 \mathrm{~b}, x_{1}=v_{M} t+\frac{1}{2} a t^{2}$, and the other car's position is given by $x_{2}=x+v_{A} t$. Set the two positions equal to each other and solve for the time of collision. If this time is negative or imaginary, then there will be no collision.

$$
\begin{aligned}
& x_{1}=x_{2} \rightarrow v_{M} t+\frac{1}{2} a t^{2}=x+v_{A} t \rightarrow \frac{1}{2} a t^{2}+\left(v_{M}-v_{A}\right) t-x=0 \\
& t=\frac{\left(v_{A}-v_{M}\right) \pm \sqrt{\left(v_{M}-v_{A}\right)^{2}-4 \frac{1}{2} a(-x)}}{2 \frac{1}{2} a}
\end{aligned}
$$

No collision: $\left(v_{M}-v_{A}\right)^{2}-4 \frac{1}{2} a(-x)<0 \rightarrow x>\frac{\left(v_{M}-v_{A}\right)^{2}}{-2 a}$
95. The velocities were changed from $\mathrm{km} / \mathrm{h}$ to $\mathrm{m} / \mathrm{s}$ by multiplying the conversion factor that $1 \mathrm{~km} / \mathrm{hr}=$ $1 / 3.6 \mathrm{~m} / \mathrm{s}$.
(a) The average acceleration for each interval is calculated by $a=\Delta v / \Delta t$, and taken to be the acceleration at the midpoint of the time interval. In the spreadsheet, $a_{n+\frac{1}{2}}=\frac{v_{n+1}-v_{n}}{t_{n+1}-t_{n}}$. The accelerations are shown in the table below.
(b) The position at the end of each interval is calculated by $x_{n+1}=x_{n}+\frac{1}{2}\left(v_{n}+v_{n+1}\right)\left(t_{n+1}-t_{n}\right)$.

This can also be represented as $x=x_{0}+\bar{v} \Delta t$. These are shown in the table below.

| $t$ (s) | $v(\mathrm{~km} / \mathrm{h})$ | $v(\mathrm{~m} / \mathrm{s})$ | $t(\mathrm{~s})$ | $a\left(\mathrm{~m} / \mathrm{s}^{2}\right)$ | $t$ (s) | $x(\mathrm{~m})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | 0.0 | 0.0 |  |  | 0.0 | 0.00 |
| 0.5 | 6.0 | 1.7 | 0.25 | 3.33 | 0.5 | 0.42 |
| 1.0 | 13.2 | 3.7 | 0.75 | 4.00 | 1.0 | 1.75 |
| 1.5 | 22.3 | 6.2 | 1.25 | 5.06 | 1.5 | 4.22 |
| 2.0 | 32.2 | 8.9 | 1.75 | 5.50 | 2.0 | 8.00 |
| 2.5 | 43.0 | 11.9 | 2.25 | 6.00 | 2.5 | 13.22 |
| 3.0 | 53.5 | 14.9 | 2.75 | 5.83 | 3.0 | 19.92 |
| 3.5 | 62.6 | 17.4 | 3.25 | 5.06 | 3.5 | 27.99 |
| 4.0 | 70.6 | 19.6 | 3.75 | 4.44 | 4.0 | 37.24 |
| 4.5 | 78.4 | 21.8 | 4.25 | 4.33 | 4.5 | 47.58 |
| 5.0 | 85.1 | 23.6 | 4.75 | 3.72 | 5.0 | 58.94 |

(c) The graphs are shown below. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH02.XLS," on tab "Problem 2.95c."


96. For this problem, a spreadsheet was designed. The columns of the spreadsheet are time, acceleration, velocity, and displacement. The time starts at 0 and with each interval is incremented by 1.00 s . The acceleration at each time is from the data given in the problem. The velocity at each time is found by multiplying the average of the accelerations at the current time and the previous time, by the time interval, and then adding that to the previous velocity. Thus $v_{n+1}=v_{n}+\frac{1}{2}\left(a_{n}+a_{n+1}\right)\left(t_{n+1}-t_{n}\right)$. The displacement from the starting position at each time interval is calculated by a constant acceleration model, where the acceleration is as given above. Thus the positions is calculated as follows.

$$
x_{n+1}=x_{n}+v_{n}\left(t_{n+1}-t_{n}\right)+\frac{1}{2}\left[\frac{1}{2}\left(a_{n}+a_{n+1}\right)\right]\left(t_{n+1}-t_{n}\right)^{2}
$$

The table of values is reproduced here.
(a) $v(17.00)=30.3 \mathrm{~m} / \mathrm{s}$
(b) $x(17.00)=305 \mathrm{~m}$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH02.XLS," on tab "Problem 2.96."

| $t(\mathrm{~s})$ | $a\left(\mathrm{~m} / \mathrm{s}^{2}\right)$ | $v(\mathrm{~m} / \mathrm{s})$ | $x(\mathrm{~m})$ |
| ---: | ---: | ---: | ---: |
| 0.0 | 1.25 | 0.0 | 0 |
| 1.0 | 1.58 | 1.4 | 1 |
| 2.0 | 1.96 | 3.2 | 3 |
| 3.0 | 2.40 | 5.4 | 7 |
| 4.0 | 2.66 | 7.9 | 14 |
| 5.0 | 2.70 | 10.6 | 23 |
| 6.0 | 2.74 | 13.3 | 35 |
| 7.0 | 2.72 | 16.0 | 50 |
| 8.0 | 2.60 | 18.7 | 67 |
| 9.0 | 2.30 | 21.1 | 87 |
| 10.0 | 2.04 | 23.3 | 109 |
| 11.0 | 1.76 | 25.2 | 133 |
| 12.0 | 1.41 | 26.8 | 159 |
| 13.0 | 1.09 | 28.0 | 187 |
| 14.0 | 0.86 | 29.0 | 215 |
| 15.0 | 0.51 | 29.7 | 245 |
| 16.0 | 0.28 | 30.1 | 275 |
| 17.0 | 0.10 | 30.3 | 305 |

97. (a) For each segment of the path, the time is given by the distance divided by the speed.

$$
\begin{aligned}
t & =t_{\text {land }}+t_{\mathrm{pool}}=\frac{d_{\text {land }}}{v_{\text {land }}}+\frac{d_{\mathrm{pool}}}{v_{\mathrm{pool}}} \\
& =\frac{x}{v_{R}}+\frac{\sqrt{D^{2}+(d-x)^{2}}}{v_{S}}
\end{aligned}
$$

(b) The graph is shown here. The minimum time occurs at a distance along the pool of about $x=6.8 \mathrm{~m}$.


An analytic differentiation to solve for the minimum point gives $x=6.76 \mathrm{~m}$.
The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH02.XLS," on tab "Problem 2.97b."

## CHAPTER 3: Kinematics in Two or Three Dimensions; Vectors

## Responses to Questions

1. No. Velocity is a vector quantity, with a magnitude and direction. If two vectors have different directions, they cannot be equal.
2. No. The car may be traveling at a constant speed of $60 \mathrm{~km} / \mathrm{h}$ and going around a curve, in which case it would be accelerating.
3. Automobile races that begin and end at the same place; a round-trip by car from New York to San Francisco and back; a balloon flight around the world.
4. The length of the displacement vector is the straight-line distance between the beginning point and the ending point of the trip and therefore the shortest distance between the two points. If the path is a straight line, then the length of the displacement vector is the same as the length of the path. If the path is curved or consists of different straight line segments, then the distance from beginning to end will be less than the path length. Therefore, the displacement vector can never be longer than the length of the path traveled, but it can be shorter.
5. The player and the ball have the same displacement.
6. $\quad V$ is the magnitude of the vector $\overrightarrow{\mathbf{V}}$; it is not necessarily larger than the magnitudes $V_{1}$ and $V_{2}$. For instance, if $\overrightarrow{\mathbf{V}}_{1}$ and $\overrightarrow{\mathbf{V}}_{2}$ have the same magnitude as each other and are in opposite directions, then $V$ is zero.
7. The maximum magnitude of the sum is 7.5 km , in the case where the vectors are parallel. The minimum magnitude of the sum is 0.5 km , in the case where the vectors are antiparallel.
8. No. The only way that two vectors can add up to give the zero vector is if they have the same magnitude and point in exactly opposite directions. However, three vectors of unequal magnitudes can add up to the zero vector. As a one-dimensional example, a vector 10 units long in the positive $x$ direction added to two vectors of 4 and 6 units each in the negative $x$ direction will result in the zero vector. In two dimensions, consider any three vectors that when added form a triangle.
9. (a) Yes. In three dimensions, the magnitude of a vector is the square root of the sum of the squares of the components. If two of the components are zero, the magnitude of the vector is equal to the magnitude of the remaining component.
(b) No.
10. Yes. A particle traveling around a curve while maintaining a constant speed is accelerating because its direction is changing. A particle with a constant velocity cannot be accelerating, since the velocity is not changing in magnitude or direction.
11. The odometer and the speedometer of the car both measure scalar quantities (distance and speed, respectively).
12. Launch the rock with a horizontal velocity from a known height over level ground. Use the equations for projectile motion in the $y$-direction to find the time the rock is in the air. (Note that the initial velocity has a zero $y$-component.) Use this time and the horizontal distance the rock travels in the
equation for $x$-direction projectile motion to find the speed in the $x$-direction, which is the speed the slingshot imparts. The meter stick is used to measure the initial height and the horizontal distance the rock travels.
13. No. The arrow will fall toward the ground as it travels toward the target, so it should be aimed above the target. Generally, the farther you are from the target, the higher above the target the arrow should be aimed, up to a maximum launch angle of $45^{\circ}$. (The maximum range of a projectile that starts and stops at the same height occurs when the launch angle is $45^{\circ}$.)
14. As long as air resistance is negligible, the horizontal component of the projectile's velocity remains constant until it hits the ground. It is in the air longer than 2.0 s , so the value of the horizontal component of its velocity at 1.0 s and 2.0 s is the same.
15. A projectile has the least speed at the top of its path. At that point the vertical speed is zero. The horizontal speed remains constant throughout the flight, if we neglect the effects of air resistance.
16. If the bullet was fired from the ground, then the $y$-component of its velocity slowed considerably by the time it reached an altitude of 2.0 km , because of both acceleration due to gravity (downward) and air resistance. The $x$-component of its velocity would have slowed due to air resistance as well. Therefore, the bullet could have been traveling slowly enough to be caught!
17. (a) Cannonball A, because it has a larger initial vertical velocity component.
(b) Cannonball A, same reason.
(c) It depends. If $\theta_{\mathrm{A}}<45^{\circ}$, cannonball A will travel farther. If $\theta_{\mathrm{B}}>45^{\circ}$, cannonball B will travel farther. If $\theta_{\mathrm{A}}>45^{\circ}$ and $\theta_{\mathrm{B}}<45^{\circ}$, the cannonball whose angle is closest to $45^{\circ}$ will travel farther.
18. (a) The ball lands back in her hand.
(b) The ball lands behind her hand.
(c) The ball lands in front of her hand.
(d) The ball lands beside her hand, to the outside of the curve.
(e) The ball lands behind her hand, if air resistance is not negligible.
19. This is a question of relative velocity. From the point of view of an observer on the ground, both trains are moving in the same direction (forward), but at different speeds. From your point of view on the faster train, the slower train (and the ground) will appear to be moving backward. (The ground will be moving backward faster than the slower train!)
20. The time it takes to cross the river depends on the component of velocity in the direction straight across the river. Imagine a river running to the east and rowers beginning on the south bank. Let the still water speed of both rowers be $v$. Then the rower who heads due north (straight across the river) has a northward velocity component $v$. The rower who heads upstream, though, has a northward velocity component of less than $v$. Therefore, the rower heading straight across reaches the opposite shore first. (However, she won't end up straight across from where she started!)
21. As you run forward, the umbrella also moves forward and stops raindrops that are at its height above the ground. Raindrops that have already passed the height of the umbrella continue to move toward the ground unimpeded. As you run, you move into the space where the raindrops are continuing to fall (below the umbrella). Some of them will hit your legs and you will get wet.

## Solutions to Problems

1. The resultant vector displacement of the car is given by $\overrightarrow{\mathbf{D}}_{\mathrm{R}}=\overrightarrow{\mathbf{D}}_{\text {west }}+\overrightarrow{\mathbf{D}}_{\text {soulur- }}^{\text {west }}$. The westward displacement is

$225+78 \cos 45^{\circ}=280.2 \mathrm{~km}$ and the south displacement is
$78 \sin 45^{\circ}=55.2 \mathrm{~km}$. The resultant displacement has a magnitude of $\sqrt{280.2^{2}+55.2^{2}}=286 \mathrm{~km}$.
The direction is $\theta=\tan ^{-1} 55.2 / 280.2=11^{\circ}$ south of west.
2. The truck has a displacement of $28+(-26)=2$ blocks north and 16 blocks east. The resultant has a magnitude of $\sqrt{2^{2}+16^{2}}=16.1$ blocks $\approx 16$ blocks and a direction of $\tan ^{-1} 2 / 16=7^{\circ}$ north of east.

3. Given that $V_{x}=7.80$ units and $V_{y}=-6.40$ units, the magnitude of $\overrightarrow{\mathbf{V}}$ is given by $V=\sqrt{V_{x}^{2}+V_{y}^{2}}=\sqrt{7.80^{2}+(-6.40)^{2}}=10.1$ units. The direction is given by $\theta=\tan ^{-1} \frac{-6.40}{7.80}=-39.4^{\circ}, 39.4^{\circ}$ below the positive $x$-axis.

4. The vectors for the problem are drawn approximately to scale. The resultant has a length of 17.5 m and a direction $19^{\circ}$ north of east. If calculations are done, the actual resultant should be 17 m at $23^{\circ}$ north of east.

5. (a) See the accompanying diagram
(b) $V_{x}=-24.8 \cos 23.4^{\circ}=-22.8$ units $V_{y}=24.8 \sin 23.4^{\circ}=9.85$ units
(c) $V=\sqrt{V_{x}^{2}+V_{y}^{2}}=\sqrt{(-22.8)^{2}+(9.85)^{2}}=24.8$ units

$$
\theta=\tan ^{-1} \frac{9.85}{22.8}=23.4^{\circ} \text { above the }-x \text { axis }
$$

6. We see from the diagram that $\overrightarrow{\mathbf{A}}=6.8 \hat{\mathbf{i}}$ and $\overrightarrow{\mathbf{B}}=-5.5 \hat{\mathbf{i}}$.
(a) $\overrightarrow{\mathbf{C}}=\overrightarrow{\mathbf{A}}+\overrightarrow{\mathbf{B}}=6.8 \hat{\mathbf{i}}+(-5.5) \hat{\mathbf{i}}=1.3 \hat{\mathbf{i}}$. The magnitude is 1.3 units, and the direction is $+x$.
(b) $\overrightarrow{\mathbf{C}}=\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{B}}=6.8 \hat{\mathbf{i}}-(-5.5) \hat{\mathbf{i}}=12.3 \hat{\mathbf{i}}$. The magnitude is 12.3 units, and the direction is $+x$.
(c) $\overrightarrow{\mathbf{C}}=\overrightarrow{\mathbf{B}}-\overrightarrow{\mathbf{A}}=(-5.5) \hat{\mathbf{i}}-6.8 \hat{\mathbf{i}}=-12.3 \hat{\mathbf{i}}$. The magnitude is 12.3 units, and the direction is $-x$.
7. (a) $v_{\text {north }}=(835 \mathrm{~km} / \mathrm{h})\left(\cos 41.5^{\circ}\right)=625 \mathrm{~km} / \mathrm{h} \quad v_{\text {west }}=(835 \mathrm{~km} / \mathrm{h})\left(\sin 41.5^{\circ}\right)=553 \mathrm{~km} / \mathrm{h}$
(b) $\Delta d_{\text {north }}=v_{\text {north }} t=(625 \mathrm{~km} / \mathrm{h})(2.50 \mathrm{~h})=1560 \mathrm{~km}$ $\Delta d_{\text {west }}=v_{\text {west }} t=(553 \mathrm{~km} / \mathrm{h})(2.50 \mathrm{~h})=1380 \mathrm{~km}$
8. (a) $\quad \overrightarrow{\mathbf{V}}_{1}=-6.0 \hat{\mathbf{i}}+8.0 \hat{\mathbf{j}} \quad V_{1}=\sqrt{6.0^{2}+8.0^{2}}=10.0 \quad \theta=\tan ^{-1} \frac{8.0}{-6.0}=127^{\circ}$
(b) $\quad \overrightarrow{\mathbf{V}}_{2}=4.5 \hat{\mathbf{i}}-5.0 \hat{\mathbf{j}} \quad V_{2}=\sqrt{4.5^{2}+5.0^{2}}=6.7 \quad \theta=\tan ^{-1} \frac{-5.0}{4.5}=312^{\circ}$
(c) $\overrightarrow{\mathbf{V}}_{1}+\overrightarrow{\mathbf{V}}_{2}=(-6.0 \hat{\mathbf{i}}+8.0 \hat{\mathbf{j}})+(4.5 \hat{\mathbf{i}}-5.0 \hat{\mathbf{j}})=-1.5 \hat{\mathbf{i}}+3.0 \hat{\mathbf{j}}$

$$
\left|\overrightarrow{\mathbf{V}}_{1}+\overrightarrow{\mathbf{V}}_{2}\right|=\sqrt{1.5^{2}+3.0^{2}}=3.4 \quad \theta=\tan ^{-1} \frac{3.0}{-1.5}=117^{\circ}
$$

(d) $\overrightarrow{\mathbf{V}}_{2}-\overrightarrow{\mathbf{V}}_{1}=(4.5 \hat{\mathbf{i}}-5.0 \hat{\mathbf{j}})-(-6.0 \hat{\mathbf{i}}+8.0 \hat{\mathbf{j}})=10.5 \hat{\mathbf{i}}-13.0 \hat{\mathbf{j}}$
$\left|\overrightarrow{\mathbf{V}}_{2}-\overrightarrow{\mathbf{V}}_{1}\right|=\sqrt{10.5^{2}+13.0^{2}}=16.7 \quad \theta=\tan ^{-1} \frac{-13.0}{10.5}=309^{\circ}$
9. (a) $\overrightarrow{\mathbf{V}}_{1}+\overrightarrow{\mathbf{V}}_{2}+\overrightarrow{\mathbf{V}}_{3}=(4.0 \hat{\mathbf{i}}-8.0 \hat{\mathbf{j}})+(1.0 \hat{\mathbf{i}}+1.0 \hat{\mathbf{j}})+(-2.0 \hat{\mathbf{i}}+4.0 \hat{\mathbf{j}})=3.0 \hat{\mathbf{i}}-3.0 \hat{\mathbf{j}}$ $\left|\overrightarrow{\mathbf{V}}_{1}+\overrightarrow{\mathbf{V}}_{2}+\overrightarrow{\mathbf{V}}_{3}\right|=\sqrt{3.0^{2}+3.0^{2}}=4.2 \quad \theta=\tan ^{-1} \frac{-3.0}{3.0}=315^{\circ}$
(b) $\overrightarrow{\mathbf{V}}_{1}-\overrightarrow{\mathbf{V}}_{2}+\overrightarrow{\mathbf{V}}_{3}=(4.0 \hat{\mathbf{i}}-8.0 \hat{\mathbf{j}})-(1.0 \hat{\mathbf{i}}+1.0 \hat{\mathbf{j}})+(-2.0 \hat{\mathbf{i}}+4.0 \hat{\mathbf{j}})=1.0 \hat{\mathbf{i}}-5.0 \hat{\mathbf{j}}$ $\left|\overrightarrow{\mathbf{V}}_{1}-\overrightarrow{\mathbf{V}}_{2}+\overrightarrow{\mathbf{V}}_{3}\right|=\sqrt{1.0^{2}+5.0^{2}}=5.1 \quad \theta=\tan ^{-1} \frac{-5.0}{1.0}=280^{\circ}$
10. $A_{x}=44.0 \cos 28.0^{\circ}=38.85 \quad A_{y}=44.0 \sin 28.0^{\circ}=20.66$
$B_{x}=-26.5 \cos 56.0^{\circ}=-14.82 \quad B_{y}=26.5 \sin 56.0^{\circ}=21.97$
$C_{x}=31.0 \cos 270^{\circ}=0.0$
$C_{y}=31.0 \sin 270^{\circ}=-31.0$
(a) $(\overrightarrow{\mathbf{A}}+\overrightarrow{\mathbf{B}}+\overrightarrow{\mathbf{C}})_{x}=38.85+(-14.82)+0.0=24.03=24.0$

$$
(\overrightarrow{\mathbf{A}}+\overrightarrow{\mathbf{B}}+\overrightarrow{\mathbf{C}})_{y}=20.66+21.97+(-31.0)=11.63=11.6
$$

(b) $|\overrightarrow{\mathbf{A}}+\overrightarrow{\mathbf{B}}+\overrightarrow{\mathbf{C}}|=\sqrt{(24.03)^{2}+(11.63)^{2}}=26.7 \quad \theta=\tan ^{-1} \frac{11.63}{24.03}=25.8^{\circ}$
11. $A_{x}=44.0 \cos 28.0^{\circ}=38.85 \quad A_{y}=44.0 \sin 28.0^{\circ}=20.66$
$B_{x}=-26.5 \cos 56.0^{\circ}=-14.82 \quad B_{y}=26.5 \sin 56.0^{\circ}=21.97$
(a) $(\overrightarrow{\mathbf{B}}-\overrightarrow{\mathbf{A}})_{x}=(-14.82)-38.85=-53.67 \quad(\overrightarrow{\mathbf{B}}-\overrightarrow{\mathbf{A}})_{y}=21.97-20.66=1.31$

Note that since the $x$ component is negative and the $y$ component is positive, the vector is in the $2^{\text {nd }}$ quadrant.

$$
\overrightarrow{\mathbf{B}}-\overrightarrow{\mathbf{A}}=-53.7 \hat{\mathbf{i}}+1.31 \hat{\mathbf{j}}
$$

$$
|\overrightarrow{\mathbf{B}}-\overrightarrow{\mathbf{A}}|=\sqrt{(-53.67)^{2}+(1.31)^{2}}=53.7 \quad \theta_{B-A}=\tan ^{-1} \frac{1.31}{-53.67}=1.4^{\circ} \text { above }-x \text { axis }
$$

(b) $(\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{B}})_{x}=38.85-(-14.82)=53.67 \quad(\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{B}})_{y}=20.66-21.97=-1.31$

Note that since the $x$ component is positive and the $y$ component is negative, the vector is in the $4^{\text {th }}$ quadrant.

$$
\begin{aligned}
& \overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{B}}=53.7 \hat{\mathbf{i}}-1.31 \hat{\mathbf{j}} \\
& |\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{B}}|=\sqrt{(53.67)^{2}+(-1.31)^{2}}=53.7 \quad \theta=\tan ^{-1} \frac{-1.31}{53.7}=1.4^{\circ} \text { below }+x \text { axis }
\end{aligned}
$$

Comparing the results shows that $\overrightarrow{\mathbf{B}}-\overrightarrow{\mathbf{A}}=-(\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{B}})$.
12. $A_{x}=44.0 \cos 28.0^{\circ}=38.85$

$$
A_{y}=44.0 \sin 28.0^{\circ}=20.66
$$

$C_{x}=31.0 \cos 270^{\circ}=0.0$
$C_{y}=31.0 \sin 270^{\circ}=-31.0$
$(\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{C}})_{x}=38.85-0.0=38.85 \quad(\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{C}})_{y}=20.66-(-31.0)=51.66$
$\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{C}}=38.8 \hat{\mathbf{i}}+51.7 \hat{\mathbf{j}}$
$|\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{C}}|=\sqrt{(38.85)^{2}+(51.66)^{2}}=64.6 \quad \theta=\tan ^{-1} \frac{51.66}{38.85}=53.1^{\circ}$
13. $A_{x}=44.0 \cos 28.0^{\circ}=38.85$
$A_{y}=44.0 \sin 28.0^{\circ}=20.66$
$B_{x}=-26.5 \cos 56.0^{\circ}=-14.82$
$B_{y}=26.5 \sin 56.0^{\circ}=21.97$
$C_{x}=31.0 \cos 270^{\circ}=0.0$
$C_{y}=31.0 \sin 270^{\circ}=-31.0$
(a) $(\overrightarrow{\mathbf{B}}-2 \overrightarrow{\mathbf{A}})_{x}=-14.82-2(38.85)=-92.52 \quad(\overrightarrow{\mathbf{B}}-2 \overrightarrow{\mathbf{A}})_{y}=21.97-2(20.66)=-19.35$

Note that since both components are negative, the vector is in the $3^{\text {rd }}$ quadrant.

$$
\overrightarrow{\mathbf{B}}-2 \overrightarrow{\mathbf{A}}=-92.5 \hat{\mathbf{i}}-19.4 \hat{\mathbf{j}}
$$

$$
|\overrightarrow{\mathbf{B}}-2 \overrightarrow{\mathbf{A}}|=\sqrt{(-92.52)^{2}+(-19.35)^{2}}=94.5 \quad \theta=\tan ^{-1} \frac{-19.35}{-92.52}=11.8^{\circ} \text { below }-x \text { axis }
$$

(b) $(2 \overrightarrow{\mathbf{A}}-3 \overrightarrow{\mathbf{B}}+2 \overrightarrow{\mathbf{C}})_{x}=2(38.85)-3(-14.82)+2(0.0)=122.16$
$(2 \overrightarrow{\mathbf{A}}-3 \overrightarrow{\mathbf{B}}+2 \overrightarrow{\mathbf{C}})_{y}=2(20.66)-3(21.97)+2(-31.0)=-86.59$
Note that since the $x$ component is positive and the $y$ component is negative, the vector is in the $4^{\text {th }}$ quadrant.

$$
\begin{aligned}
& 2 \overrightarrow{\mathbf{A}}-3 \overrightarrow{\mathbf{B}}+2 \overrightarrow{\mathbf{C}}=122 \hat{\mathbf{i}}-86.6 \hat{\mathbf{j}} \\
& |2 \overrightarrow{\mathbf{A}}-3 \overrightarrow{\mathbf{B}}+2 \overrightarrow{\mathbf{C}}|=\sqrt{(122.16)^{2}+(-86.59)^{2}}=150 \quad \theta=\tan ^{-1} \frac{-86.59}{122.16}=35.3^{\circ} \text { below }+x \text { axis }
\end{aligned}
$$

14. $A_{x}=44.0 \cos 28.0^{\circ}=38.85$
$B_{x}=-26.5 \cos 56.0^{\circ}=-14.82$
$C_{x}=31.0 \cos 270^{\circ}=0.0$
$A_{y}=44.0 \sin 28.0^{\circ}=20.66$
$B_{y}=26.5 \sin 56.0^{\circ}=21.97$
$C_{y}=31.0 \sin 270^{\circ}=-31.0$
(a) $(\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{B}}+\overrightarrow{\mathbf{C}})_{x}=38.85-(-14.82)+0.0=53.67$

$$
(\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{B}}+\overrightarrow{\mathbf{C}})_{y}=20.66-21.97+(-31.0)=-32.31
$$

Note that since the $x$ component is positive and the $y$ component is negative, the vector is in the $4^{\text {th }}$ quadrant.

$$
\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{B}}+\overrightarrow{\mathbf{C}}=53.7 \hat{\mathbf{i}}-32.3 \hat{\mathbf{j}}
$$

$$
|\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{B}}+\overrightarrow{\mathbf{C}}|=\sqrt{(53.67)^{2}+(-32.31)^{2}}=62.6 \quad \theta=\tan ^{-1} \frac{-32.31}{53.67}=31.0^{\circ} \text { below }+x \text { axis }
$$

(b) $(\overrightarrow{\mathbf{A}}+\overrightarrow{\mathbf{B}}-\overrightarrow{\mathbf{C}})_{x}=38.85+(-14.82)-0.0=24.03$
$(\overrightarrow{\mathbf{A}}+\overrightarrow{\mathbf{B}}-\overrightarrow{\mathbf{C}})_{y}=20.66+21.97-(-31.0)=73.63$
$\overrightarrow{\mathbf{A}}+\overrightarrow{\mathbf{B}}-\overrightarrow{\mathbf{C}}=24.0 \hat{\mathbf{i}}+73.6 \hat{\mathbf{j}}$
$|\overrightarrow{\mathbf{A}}+\overrightarrow{\mathbf{B}}-\overrightarrow{\mathbf{C}}|=\sqrt{(24.03)^{2}+(73.63)^{2}}=77.5 \quad \theta=\tan ^{-1} \frac{73.63}{24.03}=71.9^{\circ}$
(c) $(\overrightarrow{\mathbf{C}}-\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{B}})_{x}=0.0-38.85-(-14.82)=-24.03$
$(\overrightarrow{\mathbf{C}}-\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{B}})_{y}=-31.0-20.66-21.97=-73.63$
Note that since both components are negative, the vector is in the $3^{\text {rd }}$ quadrant.

$$
\begin{aligned}
& \overrightarrow{\mathbf{C}}-\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{B}}=-24.0 \hat{\mathbf{i}}-73.6 \hat{\mathbf{j}} \\
& |\overrightarrow{\mathbf{C}}-\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{B}}|=\sqrt{(-24.03)^{2}+(-73.63)^{2}}=77.5 \quad \theta=\tan ^{-1} \frac{-73.63}{-24.03}=71.9^{\circ} \text { below }-x \mathrm{axis}
\end{aligned}
$$

Note that the answer to $(c)$ is the exact opposite of the answer to $(b)$.
15. The $x$ component is negative and the $y$ component is positive, since the summit is to the west of north. The angle measured counterclockwise from the positive $x$ axis would be $122.4^{\circ}$. Thus the components are found to be as follows.

$$
x=4580 \cos 122.4^{\circ}=-2454 \mathrm{~m} \quad y=4580 \sin 122.4^{\circ}=3867 \mathrm{~m} \quad z=2450 \mathrm{~m}
$$

$$
\overrightarrow{\mathbf{r}}=-2450 \mathrm{~m} \hat{\mathbf{i}}+3870 \mathrm{~m} \hat{\mathbf{j}}+2450 \mathrm{~m} \hat{\mathbf{k}} \quad|\overrightarrow{\mathbf{r}}|=\sqrt{(-2454)^{2}+(4580)^{2}+(2450)^{2}}=5190 \mathrm{~m}
$$

16. (a) Use the Pythagorean theorem to find the possible $x$ components.

$$
90.0^{2}=x^{2}+(-55.0)^{2} \rightarrow x^{2}=5075 \rightarrow x= \pm 71.2 \text { units }
$$

(b) Express each vector in component form, with $\overrightarrow{\mathbf{V}}$ the vector to be determined.

$$
\begin{aligned}
& (71.2 \hat{\mathbf{i}}-55.0 \hat{\mathbf{j}})+\left(V_{x} \hat{\mathbf{i}}+V_{y} \hat{\mathbf{j}}\right)=-80.0 \hat{\mathbf{i}}+0.0 \hat{\mathbf{j}} \rightarrow \\
& V_{x}=(-80.0-71.2)=-151.2 \quad V_{y}=55.0 \\
& \overrightarrow{\mathbf{V}}=-151.2 \hat{\mathbf{i}}+55.0 \hat{\mathbf{j}}
\end{aligned}
$$

17. Differentiate the position vector in order to determine the velocity, and differentiate the velocity in order to determine the acceleration.

$$
\begin{aligned}
& \overrightarrow{\mathbf{r}}=\left(9.60 t \hat{\mathbf{i}}+8.85 \hat{\mathbf{j}}-1.00 t^{2} \hat{\mathbf{k}}\right) \mathrm{m} \rightarrow \overrightarrow{\mathbf{v}}=\frac{d \overrightarrow{\mathbf{r}}}{d t}=(9.60 \hat{\mathbf{i}}-2.00 t \hat{\mathbf{k}}) \mathrm{m} / \mathrm{s} \rightarrow \\
& \overrightarrow{\mathbf{a}}=\frac{d \overrightarrow{\mathbf{v}}}{d t}=-2.00 \hat{\mathbf{k}} \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

18. The average velocity is found from the displacement at the two times.

$$
\begin{aligned}
\overrightarrow{\mathbf{v}}_{\text {avg }} & =\frac{\overrightarrow{\mathbf{r}}\left(t_{2}\right)-\overrightarrow{\mathbf{r}}\left(t_{1}\right)}{t_{2}-t_{1}} \\
& =\frac{\left[\left(9.60(3.00) \hat{\mathbf{i}}+8.85 \hat{\mathbf{j}}-(3.00)^{2} \hat{\mathbf{k}}\right) \mathrm{m}\right]-\left[\left(9.60(1.00) \hat{\mathbf{i}}+8.85 \hat{\mathbf{j}}-(1.00)^{2} \hat{\mathbf{k}}\right) \mathrm{m}\right]}{2.00 \mathrm{~s}} \\
& =(9.60 \hat{\mathbf{i}}-4.00 \hat{\mathbf{k}}) \mathrm{m} / \mathrm{s}
\end{aligned}
$$

The magnitude of the instantaneous velocity is found from the velocity vector.

$$
\begin{aligned}
& \overrightarrow{\mathbf{v}}=\frac{d \overrightarrow{\mathbf{r}}}{d t}=(9.60 \hat{\mathbf{i}}-2.00 t \hat{\mathbf{k}}) \mathrm{m} / \mathrm{s} \\
& \overrightarrow{\mathbf{v}}(2.00)=(9.60 \hat{\mathbf{i}}-(2.00)(2.00) \hat{\mathbf{k}}) \mathrm{m} / \mathrm{s}=(9.60 \hat{\mathbf{i}}-4.00 \hat{\mathbf{k}}) \mathrm{m} / \mathrm{s} \rightarrow \\
& v=\sqrt{(9.60)^{2}+(4.00)^{2}} \mathrm{~m} / \mathrm{s}=10.4 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Note that, since the acceleration of this object is constant, the average velocity over the time interval is equal to the instantaneous velocity at the midpoint of the time interval.
19. From the original position vector, we have $x=9.60 t, y=8.85, z=-1.00 t^{2}$. Thus $z=-\left(\frac{x}{9.60}\right)^{2}=-a x^{2}, y=8.85$. This is the equation for a parabola in the $x-z$ plane that has its vertex at coordinate $(0,8.85,0)$ and opens downward.
20. (a) Average velocity is displacement divided by elapsed time. Since the displacement is not known, the average velocity cannot be determined. A special case exists in the case of constant acceleration, where the average velocity is the numeric average of the initial and final velocities. But this is not specified as motion with constant acceleration, and so that special case cannot be assumed.
(b) Define east as the positive $x$-direction, and north as the positive $y$-direction. The average acceleration is the change in velocity divided by the elapsed time.

$$
\begin{aligned}
& \overrightarrow{\mathbf{a}}_{\text {avg }}=\frac{\Delta \overrightarrow{\mathbf{v}}}{\Delta t}=\frac{27.5 \hat{\mathbf{i}} \mathrm{~m} / \mathrm{s}-(-18.0 \hat{\mathbf{j} ~ \mathrm{~m}} / \mathrm{s})}{8.00 \mathrm{~s}}=3.44 \hat{\mathbf{i}} \mathrm{~m} / \mathrm{s}^{2}+2.25 \hat{\mathbf{j} ~ \mathrm{~m}} / \mathrm{s}^{2} \\
& \left|\overrightarrow{\mathbf{a}}_{\text {avg }}\right|=\sqrt{\left(3.44 \mathrm{~m} / \mathrm{s}^{2}\right)^{2}+\left(2.25 \mathrm{~m} / \mathrm{s}^{2}\right)^{2}}=4.11 \mathrm{~m} / \mathrm{s}^{2} \quad \theta=\tan ^{-1} \frac{2.25}{3.44}=33.2^{\circ}
\end{aligned}
$$

(c) Average speed is distance traveled divided by elapsed time. Since the distance traveled is not known, the average speed cannot be determined.
21. Note that the acceleration vector is constant, and so Eqs. 3-13a and 3-13b are applicable. Also $\overrightarrow{\mathbf{v}}_{0}=0$ and $\overrightarrow{\mathbf{r}}_{0}=0$.
(a) $\overrightarrow{\mathbf{v}}=\overrightarrow{\mathbf{v}}_{0}+\overrightarrow{\mathbf{a}} t=(4.0 t \hat{\mathbf{i}}+3.0 t \hat{\mathbf{j}}) \mathrm{m} / \mathrm{s} \rightarrow v_{x}=4.0 t \mathrm{~m} / \mathrm{s}, v_{y}=3.0 t \mathrm{~m} / \mathrm{s}$
(b) $v=\sqrt{v_{x}^{2}+v_{y}^{2}}=\sqrt{(4.0 t \mathrm{~m} / \mathrm{s})^{2}+(3.0 t \mathrm{~m} / \mathrm{s})^{2}}=5.0 t \mathrm{~m} / \mathrm{s}$
(c) $\overrightarrow{\mathbf{r}}=\overrightarrow{\mathbf{r}}_{0}+\overrightarrow{\mathbf{v}}_{0} t+\frac{1}{2} \overrightarrow{\mathbf{a}} t^{2}=\left(2.0 t^{2} \hat{\mathbf{i}}+1.5 t^{2} \hat{\mathbf{j}}\right) \mathrm{m}$
(d)

$$
v_{x}(2.0)=8.0 \mathrm{~m} / \mathrm{s}, v_{y}(2.0)=6.0 \mathrm{~m} / \mathrm{s}, v(2.0)=10.0 \mathrm{~m} / \mathrm{s}, \overrightarrow{\mathbf{r}}(2.0)=(8.0 \hat{\mathbf{i}}+6.0 \hat{\mathbf{j}}) \mathrm{m}
$$

22. Choose downward to be the positive $y$ direction for this problem. Her acceleration is directed along the slope.
(a) The vertical component of her acceleration is directed downward, and its magnitude will be given by $a_{y}=a \sin \theta=\left(1.80 \mathrm{~m} / \mathrm{s}^{2}\right) \sin 30.0^{\circ}=0.900 \mathrm{~m} / \mathrm{s}^{2}$.
(b) The time to reach the bottom of the hill is calculated from Eq. 2-12b, with a $y$ displacement of $325 \mathrm{~m}, v_{y 0}=0$, and $a_{y}=0.900 \mathrm{~m} / \mathrm{s}^{2}$.

$$
\begin{aligned}
& y=y_{0}+v_{y 0} t+\frac{1}{2} a_{y} t^{2} \rightarrow 325 \mathrm{~m}=0+0+\frac{1}{2}\left(0.900 \mathrm{~m} / \mathrm{s}^{2}\right)(t)^{2} \rightarrow \\
& t=\sqrt{\frac{2(325 \mathrm{~m})}{\left(0.900 \mathrm{~m} / \mathrm{s}^{2}\right)}}=26.9 \mathrm{~s}
\end{aligned}
$$

23. The three displacements for the ant are shown in the diagram, along with the net displacement. In $x$ and $y$ components, they are $+10.0 \mathrm{~cm} \hat{\mathbf{i}}, \quad\left(10.0 \cos 30.0^{\circ} \hat{\mathbf{i}}+10.0 \sin 30.0^{\circ} \hat{\mathbf{j}}\right) \mathrm{cm}$, and $\left(10.0 \cos 100^{\circ} \hat{\mathbf{i}}+10.0 \sin 100^{\circ} \hat{\mathbf{j}}\right) \mathrm{cm}$. To find the average velocity, divide the net displacement by the elapsed time.
(a) $\Delta \overrightarrow{\mathbf{r}}=+10.0 \mathrm{~cm} \hat{\mathbf{i}}+\left(10.0 \cos 30.0^{\circ} \hat{\mathbf{i}}+10.0 \sin 30.0^{\circ} \hat{\mathbf{j}}\right) \mathrm{cm}$


$$
+\left(10.0 \cos 100^{\circ} \hat{\mathbf{i}}+10.0 \sin 100^{\circ} \hat{\mathbf{j}}\right) \mathrm{cm}=(16.92 \hat{\mathbf{i}}+14.85 \hat{\mathbf{j}}) \mathrm{cm}
$$

$$
\overrightarrow{\mathbf{v}}_{\text {avg }}=\frac{\Delta \overrightarrow{\mathbf{r}}}{\Delta t}=\frac{(16.92 \hat{\mathbf{i}}+14.85 \hat{\mathbf{j}}) \mathrm{cm}}{2.00 \mathrm{~s}+1.80 \mathrm{~s}+1.55 \mathrm{~s}}=(3.16 \hat{\mathbf{i}}+2.78 \hat{\mathbf{j}}) \mathrm{cm} / \mathrm{s}
$$

(b) $\left|\overrightarrow{\mathbf{v}}_{\text {avg }}\right|=\sqrt{(3.16 \mathrm{~cm} / \mathrm{s})^{2}+(2.78 \mathrm{~cm} / \mathrm{s})^{2}}=4.21 \mathrm{~cm} / \mathrm{s}$

$$
\theta=\tan ^{-1} \frac{v_{y}}{v_{x}}=\tan ^{-1} \frac{2.78}{3.16}=41.3^{\circ}
$$

24. Since the acceleration vector is constant, Eqs. 3-13a and 3-13b are applicable. The particle reaches its maximum $x$ coordinate when the $x$ velocity is 0 . Note that $\overrightarrow{\mathbf{v}}_{0}=5.0 \mathrm{~m} / \mathrm{s} \hat{\mathbf{i}}$ and $\overrightarrow{\mathbf{r}}_{0}=0$.

$$
\begin{aligned}
& \overrightarrow{\mathbf{v}}=\overrightarrow{\mathbf{v}}_{0}+\overrightarrow{\mathbf{a}} t=5.0 \hat{\mathbf{i}} \mathrm{~m} / \mathrm{s}+(-3.0 t \hat{\mathbf{i}}+4.5 t \hat{\mathbf{j}}) \mathrm{m} / \mathrm{s} \\
& v_{x}=(5.0-3.0 t) \mathrm{m} / \mathrm{s} \rightarrow v_{x}=0=\left(5.0-3.0 t_{x-\max }\right) \mathrm{m} / \mathrm{s} \rightarrow t_{x-\max }=\frac{5.0 \mathrm{~m} / \mathrm{s}}{3.0 \mathrm{~m} / \mathrm{s}^{2}}=1.67 \mathrm{~s} \\
& \overrightarrow{\mathbf{v}}\left(t_{x-\max }\right)=5.0 \hat{\mathbf{i}} \mathrm{~m} / \mathrm{s}+[-3.0(1.67) \hat{\mathbf{i}}+4.5(1.67) t \hat{\mathbf{j}}] \mathrm{m} / \mathrm{s}=7.5 \mathrm{~m} / \mathrm{s} \hat{\mathbf{j}} \\
& \overrightarrow{\mathbf{r}}=\overrightarrow{\mathbf{r}}_{0}+\overrightarrow{\mathbf{v}}_{0} t+\frac{1}{2} \overrightarrow{\mathbf{a}} t^{2}=5.0 t \hat{\mathbf{i}} \mathrm{~m}+\frac{1}{2}\left(-3.0 t^{2} \hat{\mathbf{i}}+4.5 t^{2} \hat{\mathbf{j}}\right) \mathrm{m} \\
& \overrightarrow{\mathbf{r}}\left(t_{x-\max }\right)=5.0(1.67) \hat{\mathbf{i}} \mathrm{m} / \mathrm{s}+\frac{1}{2}\left[-3.0(1.67)^{2} \hat{\mathbf{i}}+4.5(1.67)^{2} \hat{\mathbf{j}}\right] \mathrm{m}=4.2 \hat{\mathbf{i}} \mathrm{~m}+6.3 \hat{\mathbf{j} ~ \mathrm{~m}}
\end{aligned}
$$

25. (a) Differentiate the position vector, $\overrightarrow{\mathbf{r}}=\left(3.0 t^{2} \hat{\mathbf{i}}-6.0 t^{3} \hat{\mathbf{j}}\right) \mathrm{m}$, with respect to time in order to find the velocity and the acceleration.

$$
\overrightarrow{\mathbf{v}}=\frac{d \overrightarrow{\mathbf{r}}}{d t}=\left(6.0 t \hat{\mathbf{i}}-18.0 t^{2} \hat{\mathbf{j}}\right) \mathrm{m} / \mathrm{s} \quad \overrightarrow{\mathbf{a}}=\frac{d \overrightarrow{\mathbf{v}}}{d t}=(6.0 \hat{\mathbf{i}}-36.0 t \hat{\mathbf{j}}) \mathrm{m} / \mathrm{s}^{2}
$$

(b) $\quad \overrightarrow{\mathbf{r}}(2.5 \mathrm{~s})=\left[3.0(2.5)^{2} \hat{\mathbf{i}}-6.0(2.5)^{3} \hat{\mathbf{j}}\right] \mathrm{m}=(19 \hat{\mathbf{i}}-94 \hat{\mathbf{j}}) \mathrm{m}$
$\overrightarrow{\mathbf{v}}(2.5 \mathrm{~s})=\left[6.0(2.5) \hat{\mathbf{i}}-18.0(2.5)^{2} \hat{\mathbf{j}}\right] \mathrm{m} / \mathrm{s}=(15 \hat{\mathbf{i}}-110 \hat{\mathbf{j}}) \mathrm{m} / \mathrm{s}$
26. The position vector can be found from Eq. 3-13b, since the acceleration vector is constant. The time at which the object comes to rest is found by setting the velocity vector equal to 0 . Both components of the velocity must be 0 at the same time for the object to be at rest.

$$
\begin{aligned}
& \overrightarrow{\mathbf{v}}=\overrightarrow{\mathbf{v}}_{0}+\overrightarrow{\mathbf{a}} t=(-14 \hat{\mathbf{i}}-7.0 \hat{\mathbf{j}}) \mathrm{m} / \mathrm{s}+(6.0 t \hat{\mathbf{i}}+3.0 t \hat{\mathbf{j}}) \mathrm{m} / \mathrm{s}=[(-14+6.0 t) \hat{\mathbf{i}}+(-7.0+3.0 t) \hat{\mathbf{j}}] \mathrm{m} / \mathrm{s} \\
& \overrightarrow{\mathbf{v}}_{\text {rest }}=(0.0 \hat{\mathbf{i}}+0.0 \hat{\mathbf{j}}) \mathrm{m} / \mathrm{s}=[(-14+6.0 t) \hat{\mathbf{i}}+(-7.0+3.0 t) \hat{\mathbf{j}}] \mathrm{m} / \mathrm{s} \rightarrow \\
& \left(v_{x}\right)_{\text {rest }}=0.0=-14+6.0 t \rightarrow t=\frac{14}{6.0} \mathrm{~s}=\frac{7}{3} \mathrm{~s} \\
& \left(v_{y}\right)_{\text {rest }}=0.0=-7.0+3.0 t \rightarrow t=\frac{7.0}{3.0} \mathrm{~s}=\frac{7}{3} \mathrm{~s}
\end{aligned}
$$

Since both components of velocity are 0 at $t=\frac{7}{3} \mathrm{~s}$, the object is at rest at that time.

$$
\begin{aligned}
\overrightarrow{\mathbf{r}} & =\overrightarrow{\mathbf{r}}_{0}+\overrightarrow{\mathbf{v}}_{0} t+\frac{1}{2} \overrightarrow{\mathbf{a}} t^{2}=(0.0 \hat{\mathbf{i}}+0.0 \hat{\mathbf{j}}) \mathrm{m}+(-14 t \hat{\mathbf{i}}-7.0 t \hat{\mathbf{j}}) \mathrm{m}+\frac{1}{2}\left(6.0 t^{2} \hat{\mathbf{i}}+3.0 t^{2} \hat{\mathbf{j}}\right) \mathrm{m} \\
& =\left(-14\left(\frac{7}{3}\right) \hat{\mathbf{i}}-7.0\left(\frac{7}{3}\right) \hat{\mathbf{j}}\right) \mathrm{m}+\frac{1}{2}\left(6.0\left(\frac{7}{3}\right)^{2} \hat{\mathbf{i}}+3.0\left(\frac{7}{3}\right)^{2} \hat{\mathbf{j}}\right) \mathrm{m} \\
& =\left(-14\left(\frac{7}{3}\right)+\frac{1}{2} 6.0\left(\frac{7}{3}\right)^{2}\right) \hat{\mathbf{i}} \mathrm{m}+\left(-7.0\left(\frac{7}{3}\right)+\frac{1}{2} 3.0\left(\frac{7}{3}\right)^{2}\right) \hat{\mathbf{j}} \mathrm{m} \\
& =(-16.3 \hat{\mathbf{i}}-8.16 \hat{\mathbf{j}}) \mathrm{m} \approx(-16.3 \hat{\mathbf{i}}-8.2 \hat{\mathbf{j}}) \mathrm{m}
\end{aligned}
$$

27. Find the position at $t=5.0 \mathrm{~s}$, and then subtract the initial point from that new location.

$$
\begin{aligned}
& \overrightarrow{\mathbf{r}}(5.0)=\left[5.0(5.0)+6.0(5.0)^{2}\right] \mathrm{m} \hat{\mathbf{i}}+\left[7.0-3.0(5.0)^{3}\right] \mathrm{m} \hat{\mathbf{j}}=175 \mathrm{~m} \hat{\mathbf{i}}-368 \mathrm{~m} \hat{\mathbf{j}} \\
& \Delta \overrightarrow{\mathbf{r}}=(175.0 \mathrm{~m} \hat{\mathbf{i}}-368.0 \mathrm{~m} \hat{\mathbf{j}})-(0.0 \mathrm{~m} \hat{\mathbf{i}}+7.0 \mathrm{~m} \hat{\mathbf{j}})=175 \mathrm{~m} \hat{\mathbf{i}}-375 \mathrm{~m} \hat{\mathbf{j}} \\
& |\Delta \overrightarrow{\mathbf{r}}|=\sqrt{(175 \mathrm{~m})^{2}+(-375 \mathrm{~m})^{2}}=414 \mathrm{~m} \quad \theta=\tan ^{-1} \frac{-375}{175}=-65.0^{\circ}
\end{aligned}
$$

28. Choose downward to be the positive $y$ direction. The origin will be at the point where the tiger leaps from the rock. In the horizontal direction, $v_{x 0}=3.2 \mathrm{~m} / \mathrm{s}$ and $a_{x}=0$. In the vertical direction, $v_{y 0}=0, a_{y}=9.80 \mathrm{~m} / \mathrm{s}^{2}, y_{0}=0$, and the final location $y=7.5 \mathrm{~m}$. The time for the tiger to reach the ground is found from applying Eq. $2-12 b$ to the vertical motion.

$$
y=y_{0}+v_{y 0} t+\frac{1}{2} a_{y} t^{2} \rightarrow 7.5 \mathrm{~m}=0+0+\frac{1}{2}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) t^{2} \rightarrow t=\sqrt{\frac{2(7.5 \mathrm{~m})}{9.80 \mathrm{~m} / \mathrm{s}^{2}}}=1.24 \mathrm{sec}
$$

The horizontal displacement is calculated from the constant horizontal velocity.

$$
\Delta x=v_{x} t=(3.2 \mathrm{~m} / \mathrm{s})(1.24 \mathrm{sec})=4.0 \mathrm{~m}
$$

29. Choose downward to be the positive $y$ direction. The origin will be at the point where the diver dives from the cliff. In the horizontal direction, $v_{x 0}=2.3 \mathrm{~m} / \mathrm{s}$ and $a_{x}=0$. In the vertical direction, $v_{y 0}=0, a_{y}=9.80 \mathrm{~m} / \mathrm{s}^{2}, y_{0}=0$, and the time of flight is $t=3.0 \mathrm{~s}$. The height of the cliff is found from applying Eq. 2-12b to the vertical motion.

$$
y=y_{0}+v_{y 0} t+\frac{1}{2} a_{y} t^{2} \rightarrow y=0+0+\frac{1}{2}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(3.0 \mathrm{~s})^{2}=44 \mathrm{~m}
$$

The distance from the base of the cliff to where the diver hits the water is found from the horizontal motion at constant velocity:

$$
\Delta x=v_{x} t=(2.3 \mathrm{~m} / \mathrm{s})(3.0 \mathrm{~s})=6.9 \mathrm{~m}
$$

30. Apply the range formula from Example 3-10: $R=\frac{v_{0}^{2} \sin 2 \theta_{0}}{g}$. If the launching speed and angle are held constant, the range is inversely proportional to the value of $g$. The acceleration due to gravity on the Moon is $1 / 6^{\text {th }}$ that on Earth.

$$
\begin{aligned}
& R_{\text {Earth }}=\frac{v_{0}^{2} \sin 2 \theta_{0}}{g_{\text {Earth }}} \quad R_{\text {Moon }}=\frac{v_{0}^{2} \sin 2 \theta_{0}}{g_{\text {Moon }}} \rightarrow R_{\text {Earth }} g_{\text {Earth }}=R_{\text {Moon }} g_{\text {Moon }} \\
& R_{\text {Moon }}=R_{\text {Earth }} \frac{g_{\text {Earth }}}{g_{\text {Moon }}}=6 R_{\text {Earth }}
\end{aligned}
$$

Thus on the Moon, the person can jump 6 times farther.
31. Apply the range formula from Example 3-10.

$$
\begin{aligned}
& R=\frac{v_{0}^{2} \sin 2 \theta_{0}}{g} \rightarrow \\
& \sin 2 \theta_{0}=\frac{R g}{v_{0}^{2}}=\frac{(2.5 \mathrm{~m})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{(6.5 \mathrm{~m} / \mathrm{s})^{2}}=0.5799 \\
& 2 \theta_{0}=\sin ^{-1} 0.5799 \rightarrow \theta_{0}=18^{\circ}, 72^{\circ}
\end{aligned}
$$

There are two angles because each angle gives the
 same range. If one angle is $\theta=45^{\circ}+\delta$, then $\theta=45^{\circ}-\delta$ is also a solution. The two paths are shown in the graph.
32. Choose downward to be the positive $y$ direction. The origin will be at the point where the ball is thrown from the roof of the building. In the vertical direction, $v_{y 0}=0, a_{y}=9.80 \mathrm{~m} / \mathrm{s}^{2}, y_{0}=0$, and the displacement is 9.0 m . The time of flight is found from applying Eq. 2-12b to the vertical motion.

$$
y=y_{0}+v_{y 0} t+\frac{1}{2} a_{y} t^{2} \quad \rightarrow \quad 9.0 \mathrm{~m}=\frac{1}{2}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) t^{2} \quad \rightarrow \quad t=\sqrt{\frac{2(9.0 \mathrm{~m})}{9.80 \mathrm{~m} / \mathrm{s}^{2}}}=1.355 \mathrm{sec}
$$

The horizontal speed (which is the initial speed) is found from the horizontal motion at constant velocity.

$$
\Delta x=v_{x} t \rightarrow v_{x}=\Delta x / t=9.5 \mathrm{~m} / 1.355 \mathrm{~s}=7.0 \mathrm{~m} / \mathrm{s}
$$

33. Choose the point at which the football is kicked the origin, and choose upward to be the positive $y$ direction. When the football reaches the ground again, the $y$ displacement is 0 . For the football, $v_{y 0}=\left(18.0 \sin 38.0^{\circ}\right) \mathrm{m} / \mathrm{s}, a_{y}=-9.80 \mathrm{~m} / \mathrm{s}^{2}$, and the final $y$ velocity will be the opposite of the starting $y$ velocity. Use Eq. 2-12a to find the time of flight.

$$
v_{y}=v_{y 0}+a t \rightarrow t=\frac{v_{y}-v_{y 0}}{a}=\frac{\left(-18.0 \sin 38.0^{\circ}\right) \mathrm{m} / \mathrm{s}-\left(18.0 \sin 38.0^{\circ}\right) \mathrm{m} / \mathrm{s}}{-9.80 \mathrm{~m} / \mathrm{s}^{2}}=2.26 \mathrm{~s}
$$

34. Choose downward to be the positive $y$ direction. The origin is the point where the ball is thrown from the roof of the building. In the vertical direction $v_{y 0}=0, y_{0}=0$, and $a_{y}=9.80 \mathrm{~m} / \mathrm{s}^{2}$. The initial horizontal velocity is $23.7 \mathrm{~m} / \mathrm{s}$ and the horizontal range is 31.0 m . The time of flight is found from the horizontal motion at constant velocity.

$$
\Delta x=v_{x} t \quad \rightarrow \quad t=\Delta x / v_{x}=31.0 \mathrm{~m} / 23.7 \mathrm{~m} / \mathrm{s}=1.308 \mathrm{~s}
$$

The vertical displacement, which is the height of the building, is found by applying Eq. 2-12b to the vertical motion.

$$
y=y_{0}+v_{y 0} t+\frac{1}{2} a_{y} t^{2} \rightarrow y=0+0+\frac{1}{2}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(1.308 \mathrm{~s})^{2}=8.38 \mathrm{~m}
$$

35. Choose the origin to be the point of release of the shot put. Choose upward to be the positive $y$ direction. Then $y_{0}=0, v_{y 0}=\left(14.4 \sin 34.0^{\circ}\right) \mathrm{m} / \mathrm{s}=8.05 \mathrm{~m} / \mathrm{s}, a_{y}=-9.80 \mathrm{~m} / \mathrm{s}^{2}$, and $y=-2.10 \mathrm{~m}$ at the end of the motion. Use Eq. $2-12 b$ to find the time of flight.

$$
\begin{aligned}
& y=y_{0}+v_{y 0} t+\frac{1}{2} a_{y} t^{2} \rightarrow \frac{1}{2} a_{y} t^{2}+v_{y 0} t-y=0 \rightarrow \\
& t=\frac{-v_{y 0} \pm \sqrt{v_{y 0}^{2}-4\left(\frac{1}{2} a_{y}\right)(-y)}}{2 \frac{1}{2} a_{y}}=\frac{-8.05 \pm \sqrt{(8.05)^{2}-2(-9.80)(2.10)}}{-9.80}=1.872 \mathrm{~s},-0.2290 \mathrm{~s}
\end{aligned}
$$

Choose the positive result since the time must be greater than 0 . Now calculate the horizontal distance traveled using the horizontal motion at constant velocity.

$$
\Delta x=v_{x} t=\left[\left(14.4 \cos 34.0^{\circ}\right) \mathrm{m} / \mathrm{s}\right](1.872 \mathrm{~s})=22.3 \mathrm{~m}
$$

36. Choose the origin to be the point of launch, and upwards to be the positive $y$ direction. The initial velocity of the projectile is $v_{0}$, the launching angle is $\theta_{0}, a_{y}=-g, y_{0}=0$, and $v_{y 0}=v_{0} \sin \theta_{0}$. Eq. $2-12 \mathrm{a}$ is used to find the time required to reach the highest point, at which $v_{y}=0$.

$$
v_{y}=v_{y 0}+a t_{\text {up }} \quad \rightarrow \quad t_{\text {up }}=\frac{v_{y}-v_{y 0}}{a}=\frac{0-v_{0} \sin \theta_{0}}{-g}=\frac{v_{0} \sin \theta_{0}}{g}
$$

Eq. 2-12c is used to find the height at this highest point.

$$
v_{y}^{2}=v_{y 0}^{2}+2 a_{y}\left(y_{\max }-y_{0}\right) \quad \rightarrow \quad y_{\max }=y_{0}+\frac{v_{y}^{2}-v_{y 0}^{2}}{2 a_{y}}=0+\frac{-v_{0}^{2} \sin ^{2} \theta_{0}}{-2 g}=\frac{v_{0}^{2} \sin ^{2} \theta_{0}}{2 g}
$$

Eq. $2-12 \mathrm{~b}$ is used to find the time for the object to fall the other part of the path, with a starting $y$ velocity of 0 and a starting height of $y_{0}=\frac{v_{0}^{2} \sin ^{2} \theta_{0}}{2 g}$.

$$
y=y_{o}+v_{y 0} t+\frac{1}{2} a_{y} t^{2} \quad \rightarrow \quad 0=\frac{v_{0}^{2} \sin ^{2} \theta_{0}}{2 g}+0 t_{\mathrm{down}}-\frac{1}{2} g t_{\mathrm{down}}^{2} \quad \rightarrow \quad t_{\mathrm{down}}=\frac{v_{0} \sin \theta_{0}}{g}
$$

A comparison shows that $t_{\text {up }}=t_{\text {down }}$.
37. When shooting the gun vertically, half the time of flight is spent moving upwards. Thus the upwards flight takes 2.0 s . Choose upward as the positive $y$ direction. Since at the top of the flight, the vertical velocity is zero, find the launching velocity from Eq. 2-12a.

$$
v_{y}=v_{y 0}+a t \rightarrow v_{y 0}=v_{y}-a t=0-\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(2.0 \mathrm{~s})=19.6 \mathrm{~m} / \mathrm{s}
$$

Using this initial velocity and an angle of $45^{\circ}$ in the range formula (from Example 3-10) will give the maximum range for the gun.

$$
R=\frac{v_{0}^{2} \sin 2 \theta_{0}}{g}=\frac{(19.6 \mathrm{~m} / \mathrm{s})^{2} \sin \left(90^{\circ}\right)}{9.80 \mathrm{~m} / \mathrm{s}^{2}}=39 \mathrm{~m}
$$

38. Choose the origin to be the point on the ground directly below the point where the baseball was hit. Choose upward to be the positive $y$ direction. Then $y_{0}=1.0 \mathrm{~m}, y=13.0 \mathrm{~m}$ at the end of the motion, $v_{y 0}=\left(27.0 \sin 45.0^{\circ}\right) \mathrm{m} / \mathrm{s}=19.09 \mathrm{~m} / \mathrm{s}$, and $a_{y}=-9.80 \mathrm{~m} / \mathrm{s}^{2}$. Use Eq. 2-12b to find the time of flight.

$$
\begin{aligned}
y & =y_{0}+v_{y 0} t+\frac{1}{2} a_{y} t^{2} \rightarrow \frac{1}{2} a_{y} t^{2}+v_{y 0} t+\left(y_{0}-y\right)=0 \rightarrow \\
t & =\frac{-v_{y 0} \pm \sqrt{v_{y 0}^{2}-4\left(\frac{1}{2} a_{y}\right)\left(y_{0}-y\right)}}{2 \frac{1}{2} a_{y}}=\frac{-19.09 \pm \sqrt{(19.09)^{2}-2(-9.80)(-12.0)}}{-9.80} \\
& =0.788 \mathrm{~s}, 3.108 \mathrm{~s}
\end{aligned}
$$

The smaller time is the time the baseball reached the building's height on the way up, and the larger time is the time the baseball reached the building's height on the way down. We must choose the larger result, because the baseball cannot land on the roof on the way up. Now calculate the horizontal distance traveled using the horizontal motion at constant velocity.

$$
\Delta x=v_{x} t=\left[\left(27.0 \cos 45.0^{\circ}\right) \mathrm{m} / \mathrm{s}\right](3.108 \mathrm{~s})=59.3 \mathrm{~m}
$$

39. We choose the origin at the same place. With the new definition of the coordinate axes, we have the following data: $y_{0}=0, y=+1.00 \mathrm{~m}, v_{y 0}=-12.0 \mathrm{~m} / \mathrm{s}, v_{x 0}=-16.0 \mathrm{~m} / \mathrm{s}, a=9.80 \mathrm{~m} / \mathrm{s}^{2}$.

$$
\begin{aligned}
& y=y_{0}+v_{y 0} t+\frac{1}{2} g t^{2} \rightarrow 1.00 \mathrm{~m}=0-(12.0 \mathrm{~m} / \mathrm{s}) t+\left(4.90 \mathrm{~m} / \mathrm{s}^{2}\right) t^{2} \rightarrow \\
& \left(4.90 \mathrm{~m} / \mathrm{s}^{2}\right) t^{2}-(12.0 \mathrm{~m} / \mathrm{s}) t-(1.00 \mathrm{~m})=0
\end{aligned}
$$

This is the same equation as in Example 3-11, and so we know the appropriate solution is $t=2.53 \mathrm{~s}$. We use that time to calculate the horizontal distance the ball travels.

$$
x=v_{x 0} t=(-16.0 \mathrm{~m} / \mathrm{s})(2.53 \mathrm{~s})=-40.5 \mathrm{~m}
$$

Since the x -direction is now positive to the left, the negative value means that the ball lands 40.5 m to the right of where it departed the punter's foot.
40. The horizontal range formula from Example 3-10 can be used to find the launching velocity of the grasshopper.

$$
R=\frac{v_{0}^{2} \sin 2 \theta_{0}}{g} \rightarrow v_{0}=\sqrt{\frac{R g}{\sin 2 \theta_{0}}}=\sqrt{\frac{(1.0 \mathrm{~m})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{\sin 90^{\circ}}}=3.13 \mathrm{~m} / \mathrm{s}
$$

Since there is no time between jumps, the horizontal velocity of the grasshopper is the horizontal component of the launching velocity.

$$
v_{x}=v_{0} \cos \theta_{0}=(3.13 \mathrm{~m} / \mathrm{s}) \cos 45^{\circ}=2.2 \mathrm{~m} / \mathrm{s}
$$

41. (a) Take the ground to be the $y=0$ level, with upward as the positive direction. Use Eq. 2-12b to solve for the time, with an initial vertical velocity of 0 .

$$
y=y_{0}+v_{0 y} t+\frac{1}{2} a_{y} t^{2} \rightarrow 150 \mathrm{~m}=910 \mathrm{~m}+\frac{1}{2}\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right) t^{2} \rightarrow
$$

$$
t=\sqrt{\frac{2(150-910)}{\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}}=12.45 \mathrm{~s} \approx 12 \mathrm{~s}
$$

(b) The horizontal motion is at a constant speed, since air resistance is being ignored.

$$
\Delta x=v_{x} t=(5.0 \mathrm{~m} / \mathrm{s})(12.45 \mathrm{~s})=62.25 \mathrm{~m} \approx 62 \mathrm{~m}
$$

42. Consider the downward vertical component of the motion, which will occur in half the total time.

Take the starting position to be $y=0$, and the positive direction to be downward. Use Eq. 2-12b with an initial vertical velocity of 0 .

$$
y=y_{0}+v_{0 y} t+\frac{1}{2} a_{y} t^{2} \rightarrow h=0+0+\frac{1}{2} g t_{\text {down }}^{2}=\frac{1}{2} g\left(\frac{t}{2}\right)^{2}=\frac{9.80}{8} t^{2}=1.225 t^{2} \approx 1.2 t^{2}
$$

43. Choose downward to be the positive $y$ direction. The origin is the point where the supplies are dropped. In the vertical direction, $v_{y 0}=0, a_{y}=9.80 \mathrm{~m} / \mathrm{s}^{2}, y_{0}=0$, and the final position is $y=150 \mathrm{~m}$. The time of flight is found from applying Eq. 2-12b to the vertical motion.

$$
\begin{aligned}
& y=y_{0}+v_{y 0} t+\frac{1}{2} a_{y} t^{2} \rightarrow 160 \mathrm{~m}=0+0+\frac{1}{2}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) t^{2} \quad \rightarrow \\
& t=\sqrt{\frac{2(150 \mathrm{~m})}{9.80 \mathrm{~m} / \mathrm{s}^{2}}}=5.5 \mathrm{~s}
\end{aligned}
$$

Note that the horizontal speed of the airplane does not enter into this calculation.
44. (a) Use the "level horizontal range" formula from Example 3-10 to find her takeoff speed.

$$
R=\frac{v_{0}^{2} \sin 2 \theta_{0}}{g} \rightarrow v_{0}=\sqrt{\frac{g R}{\sin 2 \theta_{0}}}=\sqrt{\frac{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(8.0 \mathrm{~m})}{\sin 90^{\circ}}}=8.854 \mathrm{~m} / \mathrm{s} \approx 8.9 \mathrm{~m} / \mathrm{s}
$$

(b) Let the launch point be at the $y=0$ level, and choose upward to be positive. Use Eq. 2-12b to solve for the time to fall to 2.5 meters below the starting height, and then calculate the horizontal distance traveled.

$$
\begin{aligned}
& y=y_{0}+v_{0 y} t+\frac{1}{2} a_{y} t^{2} \rightarrow-2.5 \mathrm{~m}=(8.854 \mathrm{~m} / \mathrm{s}) \sin 45^{\circ} t+\frac{1}{2}\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right) t^{2} \\
& 4.9 t^{2}-6.261 \mathrm{t}-2.5 \mathrm{~m}=0 \rightarrow \\
& t=\frac{6.261 \pm \sqrt{(6.261)^{2}-4(4.9)(-2.5)}}{2(4.9)}=\frac{6.261 \pm 9.391}{2(4.9)}=-0.319 \mathrm{~s}, 1.597 \mathrm{~s}
\end{aligned}
$$

Use the positive time to find the horizontal displacement during the jump.

$$
\Delta x=v_{0 x} t=v_{0} \cos 45^{\circ} t=(8.854 \mathrm{~m} / \mathrm{s}) \cos 45^{\circ}(1.597 \mathrm{~s})=10.0 \mathrm{~m}
$$

## She will land exactly on the opposite bank, neither long nor short.

45. Choose the origin to be the location at water level directly underneath the diver when she left the board. Choose upward as the positive $y$ direction. For the diver, $y_{0}=5.0 \mathrm{~m}$, the final $y$ position is $y=0.0 \mathrm{~m}$ (water level), $a_{y}=-g$, the time of flight is $t=1.3 \mathrm{~s}$, and the horizontal displacement is $\Delta x=3.0 \mathrm{~m}$.
(a) The horizontal velocity is determined from the horizontal motion at constant velocity.

$$
\Delta x=v_{x} t \quad \rightarrow \quad v_{x}=\frac{\Delta x}{t}=\frac{3.0 \mathrm{~m}}{1.3 \mathrm{~s}}=2.31 \mathrm{~m} / \mathrm{s}
$$

The initial $y$ velocity is found using Eq. 2-12b.

$$
\begin{aligned}
& y=y_{0}+v_{y 0} t+\frac{1}{2} a_{y} t^{2} \rightarrow 0 \mathrm{~m}=5.0 \mathrm{~m}+v_{y 0}(1.3 \mathrm{~s})+\frac{1}{2}\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(1.3 \mathrm{~s})^{2} \rightarrow \\
& v_{y 0}=2.52 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Thus the velocity in both vector and magnitude / direction format are as follows.

$$
\begin{aligned}
& \overrightarrow{\mathbf{v}}_{0}=(2.3 \hat{\mathbf{i}}+2.5 \hat{\mathbf{j}}) \mathrm{m} / \mathrm{s} \\
& \theta=\tan ^{-1} \frac{v_{y 0}}{v_{x}}=\tan ^{-1} \frac{2.52 \mathrm{~m} / \mathrm{s}}{2.31 \mathrm{~m} / \mathrm{s}}=48_{x}^{\circ} \text { above the horizontal }
\end{aligned}
$$

(b) The maximum height will be reached when the $y$ velocity is zero. Use Eq. 2-12c.

$$
\begin{aligned}
& v_{y}^{2}=v_{y 0}^{2}+2 a \Delta y \rightarrow 0=(2.52 \mathrm{~m} / \mathrm{s})^{2}+2\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(y_{\text {max }}-5.0 \mathrm{~m}\right) \rightarrow \\
& y_{\text {max }}=5.3 \mathrm{~m}
\end{aligned}
$$

(c) To find the velocity when she enters the water, the horizontal velocity is the (constant) value of $v_{x}=2.31 \mathrm{~m} / \mathrm{s}$. The vertical velocity is found from Eq. 2-12a.

$$
v_{y}=v_{y 0}+a t=2.52 \mathrm{~m} / \mathrm{s}+\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(1.3 \mathrm{~s})=-10.2 \mathrm{~m} / \mathrm{s}
$$

The velocity is as follows.

$$
\begin{aligned}
& \overrightarrow{\mathbf{v}}_{\mathrm{f}}=(2.3 \hat{\mathbf{i}}-10.2 \hat{\mathbf{j}}) \mathrm{m} / \mathrm{s} \\
& v_{\mathrm{f}}=\sqrt{v_{x}^{2}+v_{y}^{2}}=\sqrt{(2.31 \mathrm{~m} / \mathrm{s})^{2}+(-10.2 \mathrm{~m} / \mathrm{s})^{2}}=10.458 \mathrm{~m} / \mathrm{s} \approx 10 \mathrm{~m} / \mathrm{s} \\
& \theta_{\mathrm{f}}=\tan ^{-1} \frac{v_{\mathrm{fy}}}{v_{\mathrm{fx}}}=\tan ^{-1} \frac{-10.2 \mathrm{~m} / \mathrm{s}}{2.31 \mathrm{~m} / \mathrm{s}}=-77^{\circ}(\text { below the horizontal })
\end{aligned}
$$

46. Choose the origin to be at ground level, under the place where the projectile is launched, and upwards to be the positive $y$ direction. For the projectile, $v_{0}=65.0 \mathrm{~m} / \mathrm{s}, \theta_{0}=35.0^{\circ}, a_{y}=-g$, $y_{0}=115 \mathrm{~m}$, and $v_{y 0}=v_{0} \sin \theta_{0}$.
(a) The time taken to reach the ground is found from Eq. 2-12b, with a final height of 0 .

$$
\begin{aligned}
& y=y_{0}+v_{y 0} t+\frac{1}{2} a_{y} t^{2} \rightarrow 0=y_{0}+v_{0} \sin \theta_{0} t-\frac{1}{2} g t^{2} \rightarrow \\
& t=\frac{-v_{0} \sin \theta_{0} \pm \sqrt{v_{0}^{2} \sin ^{2} \theta_{0}-4\left(-\frac{1}{2} g\right) y_{0}}}{2\left(-\frac{1}{2} g\right)}=9.964 \mathrm{~s},-2.3655 \mathrm{~s}=9.96 \mathrm{~s}
\end{aligned}
$$

Choose the positive time since the projectile was launched at time $t=0$.
(b) The horizontal range is found from the horizontal motion at constant velocity.

$$
\Delta x=v_{x} t=\left(v_{0} \cos \theta_{0}\right) t=(65.0 \mathrm{~m} / \mathrm{s})\left(\cos 35.0^{\circ}\right)(9.964 \mathrm{~s})=531 \mathrm{~m}
$$

(c) At the instant just before the particle reaches the ground, the horizontal component of its velocity is the constant $v_{x}=v_{0} \cos \theta_{0}=(65.0 \mathrm{~m} / \mathrm{s}) \cos 35.0^{\circ}=53.2 \mathrm{~m} / \mathrm{s}$. The vertical component is found from Eq. 2-12a.

$$
\begin{aligned}
v_{y} & =v_{y 0}+a t=v_{0} \sin \theta_{0}-g t=(65.0 \mathrm{~m} / \mathrm{s}) \sin 35.0^{\circ}-\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(9.964 \mathrm{~s}) \\
& =-60.4 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(d) The magnitude of the velocity is found from the $x$ and $y$ components calculated in part (c) above.

$$
v=\sqrt{v_{x}^{2}+v_{y}^{2}}=\sqrt{(53.2 \mathrm{~m} / \mathrm{s})^{2}+(-60.4 \mathrm{~m} / \mathrm{s})^{2}}=80.5 \mathrm{~m} / \mathrm{s}
$$

(e) The direction of the velocity is $\theta=\tan ^{-1} \frac{v_{y}}{v_{x}}=\tan ^{-1} \frac{-60.4}{53.2}=-48.6^{\circ}$, and so the object is moving $48.6^{\circ}$ below the horizon.
$(f)$ The maximum height above the cliff top reached by the projectile will occur when the $y$ velocity is 0 , and is found from Eq. 2-12c.

$$
\begin{aligned}
& v_{y}^{2}=v_{y 0}^{2}+2 a_{y}\left(y-y_{0}\right) \rightarrow 0=v_{0}^{2} \sin ^{2} \theta_{0}-2 g y_{\max } \\
& y_{\max }=\frac{v_{0}^{2} \sin ^{2} \theta_{0}}{2 g}=\frac{(65.0 \mathrm{~m} / \mathrm{s})^{2} \sin ^{2} 35.0^{\circ}}{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=70.9 \mathrm{~m}
\end{aligned}
$$

47. Choose upward to be the positive $y$ direction. The origin is the point from which the football is kicked. The initial speed of the football is $v_{0}=20.0 \mathrm{~m} / \mathrm{s}$. We have $v_{y 0}=v_{0} \sin 37.0^{\circ}=12.04 \mathrm{~m} / \mathrm{s}$, $y_{0}=0$, and $a_{y}=-9.80 \mathrm{~m} / \mathrm{s}^{2}$. In the horizontal direction, $v_{x}=v_{0} \cos 37.0^{\circ}=15.97 \mathrm{~m} / \mathrm{s}$, and $\Delta x=36.0 \mathrm{~m}$. The time of flight to reach the goalposts is found from the horizontal motion at constant speed.

$$
\Delta x=v_{x} t \quad \rightarrow \quad t=\Delta x / v_{x}=36.0 \mathrm{~m} / 15.97 \mathrm{~m} / \mathrm{s}=2.254 \mathrm{~s}
$$

Now use this time with the vertical motion data and Eq. 2-12b to find the height of the football when it reaches the horizontal location of the goalposts.

$$
y=y_{0}+v_{y 0} t+\frac{1}{2} a_{y} t^{2}=0+(12.04 \mathrm{~m} / \mathrm{s})(2.254 \mathrm{~s})+\frac{1}{2}\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(2.254 \mathrm{~s})^{2}=2.24 \mathrm{~m}
$$

Since the ball's height is less than 3.00 m , the football does not clear the bar. It is 0.76 m too low when it reaches the horizontal location of the goalposts.

To find the distances from which a score can be made, redo the problem (with the same initial conditions) to find the times at which the ball is exactly 3.00 m above the ground. Those times would correspond with the maximum and minimum distances for making the score. Use Eq. 2-12b.

$$
\begin{aligned}
& y=y_{0}+v_{y 0} t+\frac{1}{2} a_{y} t^{2} \rightarrow 3.00=0+(12.04 \mathrm{~m} / \mathrm{s}) t+\frac{1}{2}\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right) t^{2} \rightarrow \\
& 4.90 t^{2}-12.04 t+3.00=0 \rightarrow t=\frac{12.04 \pm \sqrt{(12.04)^{2}-4(4.90)(3.00)}}{2(4.90)}=2.1757 \mathrm{~s}, 0.2814 \mathrm{~s} \\
& \Delta x_{1}=v_{x} t=15.97 \mathrm{~m} / \mathrm{s}(0.2814 \mathrm{~s})=4.49 \mathrm{~m} ; \Delta x_{1}=v_{x} t=15.97 \mathrm{~m} / \mathrm{s}(2.1757 \mathrm{~s})=34.746 \mathrm{~m}
\end{aligned}
$$

So the kick must be made in the range from 4.5 m to 34.7 m .
48. The constant acceleration of the projectile is given by $\overrightarrow{\mathbf{a}}=-9.80 \mathrm{~m} / \mathrm{s}^{2} \hat{\mathbf{j}}$. We use Eq. 3-13a with the given velocity, the acceleration, and the time to find the initial velocity.

$$
\overrightarrow{\mathbf{v}}=\overrightarrow{\mathbf{v}}_{0}+\overrightarrow{\mathbf{a}} t \rightarrow \overrightarrow{\mathbf{v}}_{0}=\overrightarrow{\mathbf{v}}-\overrightarrow{\mathbf{a}} t=(8.6 \hat{\mathbf{i}}+4.8 \hat{\mathbf{j}}) \mathrm{m} / \mathrm{s}-\left(-9.80 \mathrm{~m} / \mathrm{s}^{2} \hat{\mathbf{j}}\right)(3.0 \mathrm{~s})=(8.6 \hat{\mathbf{i}}+34.2 \hat{\mathbf{j}}) \mathrm{m} / \mathrm{s}
$$

The initial speed is $v_{0}=\sqrt{(8.6 \mathrm{~m} / \mathrm{s})^{2}+(34.2 \mathrm{~m} / \mathrm{s})^{2}}=35.26 \mathrm{~m} / \mathrm{s}$, and the original launch direction is given by $\theta_{0}=\tan ^{-1} \frac{34.2 \mathrm{~m} / \mathrm{s}}{8.6 \mathrm{~m} / \mathrm{s}}=75.88^{\circ}$. Use this information with the horizontal range formula from Example 3-10 to find the range.
(a)

$$
R=\frac{v_{0}^{2} \sin 2 \theta_{0}}{g}=\frac{(35.26 \mathrm{~m} / \mathrm{s})^{2}\left(\sin 151.76^{\circ}\right)}{9.80 \mathrm{~m} / \mathrm{s}^{2} g}=6.0 \times 10^{1} \mathrm{~m}
$$

(b) We use the vertical information to find the maximum height. The initial vertical velocity is $34.2 \mathrm{~m} / \mathrm{s}$, and the vertical acceleration is $-9.80 \mathrm{~m} / \mathrm{s}^{2}$. The vertical velocity at the maximum height is 0 , and the initial height is 0 . Use Eq. 2-12c.

$$
\begin{aligned}
& v_{y}^{2}=v_{y 0}^{2}+2 a_{y}\left(y_{\max }-y_{0}\right) \rightarrow \\
& y_{\max }=y_{0}+\frac{v_{y}^{2}-v_{y 0}^{2}}{2 a_{y}}=\frac{-v_{0 y}^{2}}{2 a_{y}}=\frac{-(34.2 \mathrm{~m} / \mathrm{s})^{2}}{2\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=59.68 \mathrm{~m} \approx 6.0 \times 10^{1} \mathrm{~m}
\end{aligned}
$$

(c) From the information above and the symmetry of projectile motion, we know that the final speed just before the projectile hits the ground is the same as the initial speed, and the angle is the same as the launching angle, but below the horizontal. So $v_{\text {final }}=35 \mathrm{~m} / \mathrm{s}$ and

$$
\theta_{\text {final }}=76^{\circ} \text { below the horizontal. }
$$

49. Choose the origin to be the location from which the balloon is fired, and choose upward as the positive $y$ direction. Assume the boy in the tree is a distance $H$ up from the point at which the balloon is fired, and that the tree is a distance $d$ horizontally from the point at which the balloon is fired. The equations of motion for the balloon and boy are as follows, using constant acceleration relationships.

$$
x_{\text {Balloon }}=v_{0} \cos \theta_{0} t \quad y_{\text {Balloon }}=0+v_{0} \sin \theta_{0} t-\frac{1}{2} g t^{2} \quad y_{\text {Boy }}=H-\frac{1}{2} g t^{2}
$$

Use the horizontal motion at constant velocity to find the elapsed time after the balloon has traveled $d$ to the right.

$$
d=v_{0} \cos \theta_{0} t_{D} \quad \rightarrow \quad t_{D}=\frac{d}{v_{0} \cos \theta_{0}}
$$

Where is the balloon vertically at that time?

$$
y_{\text {Balloon }}=v_{0} \sin \theta_{0} t_{D}-\frac{1}{2} g t_{D}^{2}=v_{0} \sin \theta_{0} \frac{d}{v_{0} \cos \theta_{0}}-\frac{1}{2} g\left(\frac{d}{v_{0} \cos \theta_{0}}\right)^{2}=d \tan \theta_{0}-\frac{1}{2} g\left(\frac{d}{v_{0} \cos \theta_{0}}\right)^{2}
$$

Where is the boy vertically at that time? Note that $H=d \tan \theta_{o}$.

$$
y_{\text {Boy }}=H-\frac{1}{2} g t_{D}^{2}=H-\frac{1}{2} g\left(\frac{d}{v_{0} \cos \theta_{0}}\right)^{2}=d \tan \theta_{0}-\frac{1}{2} g\left(\frac{d}{v_{0} \cos \theta_{0}}\right)^{2}
$$

Note that $y_{\text {Balloon }}=y_{\text {Boy }}$, and so the boy and the balloon are at the same height and the same horizontal location at the same time. Thus they collide!
50. (a) Choose the origin to be the location where the car leaves the ramp, and choose upward to be the positive $y$ direction. At the end of its flight over the 8 cars, the car must be at $y=-1.5 \mathrm{~m}$. Also for the car, $v_{y 0}=0, a_{y}=-g, v_{x}=v_{0}$, and $\Delta x=22 \mathrm{~m}$. The time of flight is found from the horizontal motion at constant velocity: $\Delta x=v_{x} t \rightarrow t=\Delta x / v_{0}$. That expression for the time is used in Eq. 2-12b for the vertical motion.

$$
y=y_{0}+v_{y 0} t+\frac{1}{2} a_{y} t^{2} \rightarrow y=0+0+\frac{1}{2}(-g)\left(\Delta x / v_{0}\right)^{2} \rightarrow
$$

$$
v_{0}=\sqrt{\frac{-g(\Delta x)^{2}}{2(y)}}=\sqrt{\frac{-\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(22 \mathrm{~m})^{2}}{2(-1.5 \mathrm{~m})}}=39.76 \mathrm{~m} / \mathrm{s} \approx 40 \mathrm{~m} / \mathrm{s}
$$

(b) Again choose the origin to be the location where the car leaves the ramp, and choose upward to be the positive $y$ direction. The $y$ displacement of the car at the end of its flight over the 8 cars must again be $y=-1.5 \mathrm{~m}$. For the car, $v_{y 0}=v_{0} \sin \theta_{0}, a_{y}=-g, v_{x}=v_{0} \cos \theta_{0}$, and $\Delta x=22 \mathrm{~m}$. The launch angle is $\theta_{0}=7.0^{\circ}$. The time of flight is found from the horizontal motion at constant velocity.

$$
\Delta x=v_{x} t \quad \rightarrow \quad t=\frac{\Delta x}{v_{0} \cos \theta_{0}}
$$

That expression for the time is used in Eq. 2-12b for the vertical motion.

$$
\begin{aligned}
& y=y_{0}+v_{y 0} t+\frac{1}{2} a_{y} t^{2} \rightarrow y=v_{0} \sin \theta_{0} \frac{\Delta x}{v_{0} \cos \theta_{0}}+\frac{1}{2}(-g)\left(\frac{\Delta x}{v_{0} \cos \theta_{0}}\right)^{2} \\
& v_{0}=\sqrt{\frac{g(\Delta x)^{2}}{2\left(\Delta x \tan \theta_{0}-y\right) \cos ^{2} \theta_{0}}}=\sqrt{\frac{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(22 \mathrm{~m})^{2}}{2\left((22 \mathrm{~m}) \tan 7.0^{\circ}+1.5 \mathrm{~m}\right) \cos ^{2} 7.0^{\circ}}}=24 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

51. The angle is in the direction of the velocity, so find the components of the velocity, and use them to define the angle. Let the positive $y$-direction be down.

$$
v_{x}=v_{0} \quad v_{y}=v_{y 0}+a_{y} t=g t \quad \theta=\tan ^{-1} \frac{v_{y}}{v_{x}}=\tan ^{-1} \frac{g t}{v_{0}}
$$

52. Choose the origin to be where the projectile is launched, and upwards to be the positive $y$ direction. The initial velocity of the projectile is $v_{0}$, the launching angle is $\theta_{0}, a_{y}=-g$, and $v_{y 0}=v_{0} \sin \theta_{0}$. The range of the projectile is given by the range formula from Example 3-10, $R=\frac{v_{0}^{2} \sin 2 \theta_{0}}{g}$. The maximum height of the projectile will occur when its vertical speed is 0 . Apply Eq. 2-12c.

$$
v_{y}^{2}=v_{y 0}^{2}+2 a_{y}\left(y-y_{0}\right) \quad \rightarrow 0=v_{0}^{2} \sin ^{2} \theta_{0}-2 g y_{\max } \quad \rightarrow \quad y_{\max }=\frac{v_{0}^{2} \sin ^{2} \theta_{0}}{2 g}
$$

Now find the angle for which $R=y_{\max }$.

$$
\begin{aligned}
& R=y_{\text {max }} \rightarrow \frac{v_{0}^{2} \sin 2 \theta_{0}}{g}=\frac{v_{0}^{2} \sin ^{2} \theta_{0}}{2 g} \rightarrow \sin 2 \theta_{0}=\frac{1}{2} \sin ^{2} \theta_{0} \rightarrow \\
& 2 \sin \theta_{0} \cos \theta_{0}=\frac{1}{2} \sin ^{2} \theta_{0} \rightarrow 4 \cos \theta_{0}=\sin \theta_{0} \rightarrow \tan \theta_{0}=4 \rightarrow \theta_{0}=\tan ^{-1} 4=76^{\circ}
\end{aligned}
$$

53. Choose the origin to be where the projectile is launched, and upwards to be the positive $y$ direction. The initial velocity of the projectile is $v_{0}$, the launching angle is $\theta_{0}, a_{y}=-g$, and $v_{y 0}=v_{0} \sin \theta_{0}$.
(a) The maximum height is found from Eq. 2-12c, $v_{y}^{2}=v_{y 0}^{2}+2 a_{y}\left(y-y_{0}\right)$, with $v_{y}=0$ at the maximum height.

$$
y_{\text {max }}=0+\frac{v_{y}^{2}-v_{y 0}^{2}}{2 a_{y}}=\frac{-v_{0}^{2} \sin ^{2} \theta_{0}}{-2 g}=\frac{v_{0}^{2} \sin ^{2} \theta_{0}}{2 g}=\frac{(46.6 \mathrm{~m} / \mathrm{s})^{2} \sin ^{2} 42.2^{\circ}}{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=50.0 \mathrm{~m}
$$

(b) The total time in the air is found from Eq. 2-12b, with a total vertical displacement of 0 for the ball to reach the ground.

$$
\begin{aligned}
& y=y_{0}+v_{y 0} t+\frac{1}{2} a_{y} t^{2} \rightarrow 0=v_{0} \sin \theta_{0} t-\frac{1}{2} g t^{2} \rightarrow \\
& t=\frac{2 v_{0} \sin \theta_{0}}{g}=\frac{2(46.6 \mathrm{~m} / \mathrm{s}) \sin 42.2^{\circ}}{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=6.39 \mathrm{~s} \text { and } t=0
\end{aligned}
$$

The time of 0 represents the launching of the ball.
(c) The total horizontal distance covered is found from the horizontal motion at constant velocity.

$$
\Delta x=v_{x} t=\left(v_{0} \cos \theta_{0}\right) t=(46.6 \mathrm{~m} / \mathrm{s})\left(\cos 42.2^{\circ}\right)(6.39 \mathrm{~s})=221 \mathrm{~m}
$$

(d) The velocity of the projectile 1.50 s after firing is found as the vector sum of the horizontal and vertical velocities at that time. The horizontal velocity is a constant $v_{0} \cos \theta_{0}=$ $(46.6 \mathrm{~m} / \mathrm{s})\left(\cos 42.2^{\circ}\right)=34.5 \mathrm{~m} / \mathrm{s}$. The vertical velocity is found from Eq. 2-12a.

$$
v_{y}=v_{y 0}+a t=v_{0} \sin \theta_{0}-g t=(46.6 \mathrm{~m} / \mathrm{s}) \sin 42.2^{\circ}-\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(1.50 \mathrm{~s})=16.6 \mathrm{~m} / \mathrm{s}
$$

Thus the speed of the projectile is $v=\sqrt{v_{x}^{2}+v_{y}^{2}}=\sqrt{34.5^{2}+16.6^{2}}=38.3 \mathrm{~m} / \mathrm{s}$.
The direction above the horizontal is given by $\theta=\tan ^{-1} \frac{v_{y}}{v_{x}}=\tan ^{-1} \frac{16.6}{34.5}=25.7^{\circ}$.
54. (a) Use the "level horizontal range" formula from Example 3-10.

$$
R=\frac{v_{0}^{2} \sin 2 \theta_{0}}{g} \rightarrow v_{0}=\sqrt{\frac{R g}{\sin 2 \theta_{0}}}=\sqrt{\frac{(7.80 \mathrm{~m})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{\sin 54.0^{\circ}}}=9.72 \mathrm{~m} / \mathrm{s}
$$

(b) Now increase the speed by $5.0 \%$ and calculate the new range. The new speed would be
$9.72 \mathrm{~m} / \mathrm{s}(1.05)=10.2 \mathrm{~m} / \mathrm{s}$ and the new range would be as follows.

$$
R=\frac{v_{0}^{2} \sin 2 \theta_{0}}{g}=\frac{(10.2 \mathrm{~m} / \mathrm{s})^{2} \sin 54^{\circ}}{9.80 \mathrm{~m} / \mathrm{s}^{2}}=8.59 \mathrm{~m}
$$

This is an increase of $0.79 \mathrm{~m}(10 \%$ increase $)$.
55. Choose the origin to be at the bottom of the hill, just where the incline starts. The equation of the line describing the hill is $y_{2}=x \tan \phi$. The equations of the motion of the object are $y_{1}=v_{0 y} t+\frac{1}{2} a_{y} t^{2}$ and $x=v_{0 x} t$, with $v_{0 x}=v_{0} \cos \theta$ and $v_{0 y}=v_{0} \sin \theta$. Solve the horizontal equation for the time of flight, and insert that into the vertical projectile motion equation.

$$
t=\frac{x}{v_{0 x}}=\frac{x}{v_{0} \cos \theta} \rightarrow y_{1}=v_{0} \sin \theta \frac{x}{v_{0} \cos \theta}-\frac{1}{2} g\left(\frac{x}{v_{0} \cos \theta}\right)^{2}=x \tan \theta-\frac{g x^{2}}{2 v_{0}^{2} \cos ^{2} \theta}
$$

Equate the $y$-expressions for the line and the parabola to find the location where the two $x$ coordinates intersect.

$$
\begin{aligned}
& x \tan \phi=x \tan \theta-\frac{g x^{2}}{2 v_{0}^{2} \cos ^{2} \theta} \rightarrow \tan \theta-\tan \phi=\frac{g x}{2 v_{0}^{2} \cos ^{2} \theta} \rightarrow \\
& x=\frac{(\tan \theta-\tan \phi)}{g} 2 v_{0}^{2} \cos ^{2} \theta
\end{aligned}
$$

This intersection $x$-coordinate is related to the desired quantity $d$ by $x=d \cos \phi$.

$$
d \cos \phi=(\tan \theta-\tan \phi) \frac{2 v_{0}^{2} \cos ^{2} \theta}{g} \rightarrow d=\frac{2 v_{0}^{2}}{g \cos \phi}\left(\sin \theta \cos \theta-\tan \phi \cos ^{2} \theta\right)
$$

To maximize the distance, set the derivative of $d$ with respect to $\theta$ equal to 0 , and solve for $\theta$.

$$
\begin{aligned}
\frac{d(d)}{d \theta} & =\frac{2 v_{0}^{2}}{g \cos \phi} \frac{d}{d \theta}\left(\sin \theta \cos \theta-\tan \phi \cos ^{2} \theta\right) \\
& =\frac{2 v_{0}^{2}}{g \cos \phi}[\sin \theta(-\sin \theta)+\cos \theta(\cos \theta)-\tan \phi(2) \cos \theta(-\sin \theta)] \\
& =\frac{2 v_{0}^{2}}{g \cos \phi}\left[-\sin ^{2} \theta+\cos ^{2} \theta+2 \tan \phi \cos \theta \sin \theta\right]=\frac{2 v_{0}^{2}}{g \cos \phi}[\cos 2 \theta+\sin 2 \theta \tan \phi]=0 \\
\cos 2 \theta & +\sin 2 \theta \tan \phi=0 \rightarrow \theta=\frac{1}{2} \tan ^{-1}\left(-\frac{1}{\tan \phi}\right)
\end{aligned}
$$

This expression can be confusing, because it would seem that a negative sign enters the solution. In order to get appropriate values, $180^{\circ}$ or $\pi$ radians must be added to the angle resulting from the inverse tangent operation, to have a positive angle. Thus a more appropriate expression would be the following:

$$
\begin{aligned}
& \theta=\frac{1}{2}\left[\pi+\tan ^{-1}\left(-\frac{1}{\tan \phi}\right)\right] . \text { This can be shown to be equivalent to } \theta=\frac{\phi}{2}+\frac{\pi}{4} \text {, because } \\
& \tan ^{-1}\left(-\frac{1}{\tan \phi}\right)=\tan ^{-1}(-\cot \phi)=\cot ^{-1} \cot \phi-\frac{\pi}{2}=\phi-\frac{\pi}{2} .
\end{aligned}
$$

56. See the diagram. Solve for $R$, the horizontal range, which is the horizontal speed times the time of flight.

$$
\begin{aligned}
& R=\left(v_{0} \cos \theta_{0}\right) t \rightarrow t=\frac{R}{v_{0} \cos \theta_{0}} \\
& h=\left(v_{0} \sin \theta_{0}\right) t-\frac{1}{2} g t^{2} \rightarrow \frac{1}{2} g t^{2}-\left(v_{0} \sin \theta_{0}\right) t+h=0 \rightarrow \\
& R^{2}-R \frac{2 v_{0}^{2} \cos ^{2} \theta_{0} \tan \theta}{g}+\frac{2 h v_{0}^{2} \cos ^{2} \theta_{0}}{g}=0 \\
& R=\frac{\frac{2 v_{0}^{2} \cos ^{2} \theta_{0} \tan \theta}{g} \pm \sqrt{\left(\frac{2 v_{0}^{2} \cos ^{2} \theta_{0} \tan \theta}{g}\right)^{2}-4 \frac{2 h v_{0}^{2} \cos ^{2} \theta_{0}}{g}}}{2} \\
&=\frac{v_{0} \cos \theta_{0}}{g}\left[v_{0} \sin \theta_{0} \pm \sqrt{v_{0}^{2} \sin ^{2} \theta_{0}-2 g h}\right]
\end{aligned}
$$

Which sign is to be used? We know the result if $h=0$ from Example 3-10. Substituting $h=0$ gives $R=\frac{v_{0} \cos \theta_{0}}{g}\left[v_{0} \sin \theta_{0} \pm v_{0} \sin \theta_{0}\right]$. To agree with Example 3-10, we must choose the $+\operatorname{sign}$, and so $R=\frac{v_{0} \cos \theta_{0}}{g}\left[v_{0} \sin \theta_{0}+\sqrt{v_{0}^{2} \sin ^{2} \theta_{0}-2 g h}\right]$. We see from this result that if $h>0$, the range will shorten, and if $h<0$, the range will lengthen.
57. Call the direction of the boat relative to the water the positive direction. For the jogger moving towards the bow, we have the following:

$$
\overrightarrow{\mathbf{v}}_{\substack{\text { joger } \\ \text { rel. water }}}=\overrightarrow{\mathbf{v}}_{\text {joger }}^{\text {rel. baat }}, ~+\overrightarrow{\mathbf{v}}_{\substack{\text { boat rel. } \\ \text { water }}}=2.0 \mathrm{~m} / \mathrm{s} \hat{\mathbf{i}}+8.5 \mathrm{~m} / \mathrm{s} \hat{\mathbf{i}}=10.5 \mathrm{~m} / \mathrm{s} \hat{\mathbf{i}} \text {. }
$$

For the jogger moving towards the stern, we have the following.

$$
\overrightarrow{\mathbf{v}}_{\substack{\text { jogger } \\ \text { rel. water }}}=\overrightarrow{\mathbf{v}}_{\substack{\text { oogger } \\ \text { rel. boat }}}+\underset{\overrightarrow{\mathbf{v}}_{\text {boat rel. }}^{\text {water }}}{ }=-2.0 \mathrm{~m} / \mathrm{s} \hat{\mathbf{i}}+8.5 \mathrm{~m} / \mathrm{s} \hat{\mathbf{i}}=6.5 \mathrm{~m} / \mathrm{s} \hat{\mathbf{i}}
$$

58. Call the direction of the flow of the river the $x$ direction, and the direction of Huck walking relative to the raft the $y$ direction.

$$
\begin{aligned}
& \begin{array}{c}
\overrightarrow{\mathbf{v}}_{\text {Huck }} \\
\text { rel. bank }
\end{array}=\underset{\substack{\text { Huck } \\
\text { rel. raft }}}{\overrightarrow{\mathbf{v}}_{\text {raft rel. }}^{\text {bank }}}=0.70 \hat{\mathbf{j}} \mathrm{~m} / \mathrm{s}+1.50 \hat{\mathbf{i}} \mathrm{~m} / \mathrm{s} \\
&=(1.50 \hat{\mathbf{i}}+0.70 \hat{\mathbf{j}}) \mathrm{m} / \mathrm{s} \\
& \text { Magnitude: } \\
& v_{\text {Huck }}^{\text {rel. bank }}=\sqrt{1.50^{2}+0.70^{2}}=1.66 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$



$$
\text { Direction: } \theta=\tan ^{-1} \frac{0.70}{1.50}=25^{\circ} \text { relative to river }
$$

Direction: $\theta=\tan ^{-1} \frac{0.70}{1.50}=25^{\circ}$ relative to river
59. From the diagram in Figure 3-33, it is seen that $v_{\substack{\text { boatrel. } \\ \text { shore }}}=v_{\substack{\text { boat rel. } \\ \text { water }}} \cos \theta=$ $(1.85 \mathrm{~m} / \mathrm{s}) \cos 40.4^{\circ}=1.41 \mathrm{~m} / \mathrm{s}$.

60. If each plane has a speed of $780 \mathrm{~km} / \mathrm{hr}$, then their relative speed of approach is $1560 \mathrm{~km} / \mathrm{hr}$. If the planes are 12.0 km apart, then the time for evasive action is found as follows.

$$
\Delta d=v t \rightarrow t=\frac{\Delta d}{v}=\left(\frac{12.0 \mathrm{~km}}{1560 \mathrm{~km} / \mathrm{hr}}\right)\left(\frac{3600 \mathrm{sec}}{1 \mathrm{hr}}\right)=27.7 \mathrm{~s}
$$

61. The lifeguard will be carried downstream at the same rate as the child. Thus only the horizontal motion need be considered. To cover 45 meters horizontally at a rate of $2 \mathrm{~m} / \mathrm{s}$ takes $\frac{45 \mathrm{~m}}{2 \mathrm{~m} / \mathrm{s}}=$ $22.5 \mathrm{~s} \approx 23 \mathrm{~s}$ for the lifeguard to reach the child. During this time they would both be moving downstream at $1.0 \mathrm{~m} / \mathrm{s}$, and so would travel $(1.0 \mathrm{~m} / \mathrm{s})(22.5 \mathrm{~s})=22.5 \mathrm{~m} \approx 23 \mathrm{~m}$ downstream.
62. Call the direction of the boat relative to the water the $x$ direction, and upward the $y$ direction. Also see the diagram.


$$
\begin{aligned}
& \overrightarrow{\mathbf{V}}_{\substack{\text { passenger } \\
\text { rel. water }}}=\overrightarrow{\mathbf{v}}_{\substack{\text { passenger } \\
\text { rel. boat }}}+\underset{\substack{\text { boat rel. } \\
\text { water }}}{\overrightarrow{\vec{y}^{\prime}}} \\
& =\left(0.60 \cos 45^{\circ} \hat{\mathbf{i}}+0.60 \sin 45^{\circ} \hat{\mathbf{j}}\right) \mathrm{m} / \mathrm{s}+1.70 \hat{\mathbf{i}} \mathrm{~m} / \mathrm{s} \\
& =(2.12 \hat{\mathbf{i}}+0.42 \hat{\mathbf{j}}) \mathrm{m} / \mathrm{s}
\end{aligned}
$$

63. (a) Call the upward direction positive for the vertical motion. Then the velocity of the ball relative to a person on the ground is the vector sum of the horizontal and vertical motions. The horizontal velocity is $v_{x}=10.0 \mathrm{~m} / \mathrm{s}$ and the vertical velocity is $v_{y}=5.0 \mathrm{~m} / \mathrm{s}$.

$$
\begin{aligned}
& \overrightarrow{\mathbf{v}}=10.0 \mathrm{~m} / \mathrm{s} \hat{\mathbf{i}}+5.0 \mathrm{~m} / \mathrm{s} \hat{\mathbf{j}} \rightarrow v=\sqrt{(10.0 \mathrm{~m} / \mathrm{s})^{2}+(5.0 \mathrm{~m} / \mathrm{s})^{2}}=11.2 \mathrm{~m} / \mathrm{s} \\
& \theta=\tan ^{-1} \frac{5.0 \mathrm{~m} / \mathrm{s}}{10.0 \mathrm{~m} / \mathrm{s}}=27^{\circ} \text { above the horizontal }
\end{aligned}
$$

(b) The only change is the initial vertical velocity, and so $v_{y}=-5.0 \mathrm{~m} / \mathrm{s}$.

$$
\begin{aligned}
& \overrightarrow{\mathbf{v}}=10.0 \mathrm{~m} / \mathrm{s} \hat{\mathbf{i}}-5.0 \mathrm{~m} / \mathrm{s} \hat{\mathbf{j}} \rightarrow v=\sqrt{(10.0 \mathrm{~m} / \mathrm{s})^{2}+(-5.0 \mathrm{~m} / \mathrm{s})^{2}}=11.2 \mathrm{~m} / \mathrm{s} \\
& \theta=\tan ^{-1} \frac{-5.0 \mathrm{~m} / \mathrm{s}}{10.0 \mathrm{~m} / \mathrm{s}}=27^{\circ} \text { below the horizontal }
\end{aligned}
$$

64. Call east the positive $x$ direction and north the positive $y$ direction. Then the following vector velocity relationship exists.
(a)

$$
\begin{aligned}
& \overrightarrow{\mathbf{v}}_{\text {plane rel. }}^{\text {ground }}= \\
&=\overrightarrow{\mathbf{v}}_{\text {plane }}+\overrightarrow{\mathbf{v}}_{\text {air rel. }}^{\text {rel. iir }} \text { ground } \\
&=-580 \hat{\mathbf{j}} \mathrm{~km} / \mathrm{h}+\left(90.0 \cos 45.0^{\circ} \hat{\mathbf{i}}+90.0 \sin 45.0^{\circ} \hat{\mathbf{j}}\right) \mathrm{km} / \mathrm{h} \\
&=(63.6 \hat{\mathbf{i}}-516 \hat{\mathbf{j}}) \mathrm{km} / \mathrm{h} \\
& \begin{array}{c}
v_{\text {plane rel. }} \\
\text { ground }
\end{array}=\sqrt{(63.6 \mathrm{~km} / \mathrm{h})^{2}+(-516 \mathrm{~km} / \mathrm{h})^{2}}=520 \mathrm{~km} / \mathrm{h} \\
& \theta=\tan ^{-1} \frac{63.6}{-516}=-7.0^{\circ}=7.0^{\circ} \text { east of south }
\end{aligned}
$$

(b) The plane is away from its intended position by the distance the air has caused it to move. The wind speed is $90.0 \mathrm{~km} / \mathrm{h}$, so after 11.0 min the plane is off course by the following amount.

$$
\Delta x=v_{x} t=(90.0 \mathrm{~km} / \mathrm{h})(11.0 \mathrm{~min})\left(\frac{1 \mathrm{~h}}{60 \mathrm{~min}}\right)=16.5 \mathrm{~km} .
$$

65. Call east the positive $x$ direction and north the positive $y$ direction. Then the following vector velocity relationship exists.

$$
\begin{aligned}
& -v_{\substack{\text { plane rel. } \\
\text { ground }}} \hat{\mathbf{j}}=(-580 \sin \theta \hat{\mathbf{i}}+580 \cos \theta \hat{\mathbf{j}}) \mathrm{km} / \mathrm{h} \\
& +\left(90.0 \cos 45.0^{\circ} \hat{\mathbf{i}}+90.0 \sin 45.0^{\circ} \hat{\mathbf{j}}\right) \mathrm{km} / \mathrm{h}
\end{aligned}
$$

Equate $x$ components in the above equation.

$$
\begin{aligned}
& 0=-580 \sin \theta+90.0 \cos 45.0^{\circ} \rightarrow \\
& \theta=\sin ^{-1} \frac{90.0 \cos 45.0^{\circ}}{580}=6.3^{\circ}, \text { west of south }
\end{aligned}
$$



66．Call east the positive $x$ direction and north the positive $y$ direction．From the first diagram，this relative velocity relationship is seen．


$$
\begin{aligned}
& \underset{\substack{\text { car 1 rel. } \\
\text { car 2 }}}{ }=\overrightarrow{\mathbf{v}}_{\substack{\text { car 1 rel. } \\
\text { street }}}-\overrightarrow{\mathbf{v}}_{\text {car 2 rel. }}^{\text {stret }}=35 \hat{\mathbf{j}} \mathrm{~km} / \mathrm{h}-45 \hat{\mathbf{i}} \mathrm{~km} / \mathrm{h}=(-45 \hat{\mathbf{i}}+35 \hat{\mathbf{j}}) \mathrm{km} / \mathrm{h}
\end{aligned}
$$

For the other relative velocity relationship：

$$
\begin{aligned}
& \underset{\substack{\text { car 2 rel. } \\
\text { street }}}{\overrightarrow{\mathbf{V}}_{\text {car 1 }}^{\text {car } 2 \text { rel. }}} \underset{\text { street }}{ }+\overrightarrow{\mathbf{V}}_{\text {car 1 rel. }}^{\text {str }} \quad \rightarrow \\
& \overrightarrow{\mathbf{v}}_{\text {car 2 rel. }}^{\text {car 1 }}=\underset{\substack{\text { car 2 rel. } \\
\text { street }}}{ }-\overrightarrow{\mathbf{v}}_{\text {car I rel. }}^{\text {street }}=45 \hat{\mathbf{i}} \mathrm{~km} / \mathrm{h}-35 \hat{\mathbf{j}} \mathrm{~km} / \mathrm{h}=(45 \hat{\mathbf{i}}-35 \hat{\mathbf{j}}) \mathrm{km} / \mathrm{h}
\end{aligned}
$$



Notice that the two relative velocities are opposites of each other： $\overrightarrow{\mathbf{v}}_{\substack{\text { car 2 rel．} \\ \text { car } 1}}=-\overrightarrow{\mathbf{v}}_{\substack{\text { car 1 rel } \\ \text { car } 2}}$ ．
67．Call the direction of the flow of the river the $x$ direction，and the direction straight across the river the $y$ direction．Call the location of the swimmer＇s starting point the origin．

$$
\overrightarrow{\mathbf{v}}_{\substack{\text { swimmer } \\ \text { rel. Shore }}}=\overrightarrow{\mathbf{v}}_{\text {swimmer }}^{\text {rel. water }} ⿺ ⿻ ⿻ 一 ㇂ ㇒ \mathbf{v}_{\text {water rel. }}^{\text {shore }}=0.60 \mathrm{~m} / \mathrm{s} \hat{\mathbf{j}}+0.50 \mathrm{~m} / \mathrm{s} \hat{\mathbf{i}}
$$

（a）Since the swimmer starts from the origin，the distances covered in the $x$ and $y$ directions will be exactly proportional to the speeds in
 those directions．

$$
\frac{\Delta x}{\Delta y}=\frac{v_{x} t}{v_{y} t}=\frac{v_{x}}{v_{y}} \rightarrow \frac{\Delta x}{55 \mathrm{~m}}=\frac{0.50 \mathrm{~m} / \mathrm{s}}{0.60 \mathrm{~m} / \mathrm{s}} \rightarrow \Delta x=46 \mathrm{~m}
$$

（b）The time is found from the constant velocity relationship for either the $x$ or $y$ directions．

$$
\Delta y=v_{y} t \quad \rightarrow \quad t=\frac{\Delta y}{v_{y}}=\frac{55 \mathrm{~m}}{0.60 \mathrm{~m} / \mathrm{s}}=92 \mathrm{~s}
$$

68．（a）Call the direction of the flow of the river the $x$ direction，and the direction straight across the river the $y$ direction．

$$
\sin \theta=\frac{v_{\text {water rel. }}}{\substack{\text { shor }}}=\frac{0.50 \mathrm{~m} / \mathrm{s}}{v_{\text {swimmer }}} \text { rel. water }-60 \mathrm{~m} / \mathrm{s} \rightarrow \theta=\sin ^{-1} \frac{0.50}{0.60}=56.44^{\circ} \approx 56^{\circ}
$$


（b）From the diagram her speed with respect to the shore is found as follows．

$$
v_{\substack{\text { swimmer } \\ \text { rel. shore }}}=v_{\substack{\text { swimmer } \\ \text { rel. water }}} \cos \theta=(0.60 \mathrm{~m} / \mathrm{s}) \cos 56.44^{\circ}=0.332 \mathrm{~m} / \mathrm{s}
$$

The time to cross the river can be found from the constant velocity relationship．

$$
\Delta x=v t \rightarrow t=\frac{\Delta x}{v}=\frac{55 \mathrm{~m}}{0.332 \mathrm{~m} / \mathrm{s}}=170 \mathrm{~s}=2.8 \mathrm{~min}
$$

69．The boat is traveling directly across the stream，with a heading of $\theta=19.5^{\circ}$ upstream，and speed of $v_{\substack{\text { boat el．} \\ \text { water }}}=3.40 \mathrm{~m} / \mathrm{s}$ ．

$$
\begin{aligned}
& \text { (a) } v_{\substack{\text { whatr rel. } \\
\text { shore }}}=v_{\substack{\text { boat rel. } \\
\text { water }}} \sin \theta=(3.40 \mathrm{~m} / \mathrm{s}) \sin 19.5^{\circ}=1.13 \mathrm{~m} / \mathrm{s} \\
& \text { (b) } v_{\text {boat rel. }}=v_{\text {boat rel. }} \cos \theta=(3.40 \mathrm{~m} / \mathrm{s}) \cos 19.5^{\circ}=3.20 \mathrm{~m} / \mathrm{s} \\
& \text { whater }
\end{aligned}
$$


70. Call the direction of the flow of the river the $x$ direction (to the left in the diagram), and the direction straight across the river the $y$ direction (to the top in the diagram). From the diagram, $\theta=\tan ^{-1} 120 \mathrm{~m} / 280 \mathrm{~m}$ $=23^{\circ}$. Equate the vertical components of the velocities to find the speed of the boat relative to the shore.

$$
\begin{aligned}
& v_{\text {boat ele }}^{\substack{\text { shore }}} \cos \theta=v_{\substack{\text { boat ele } \\
\text { water }}} \sin 45^{\circ} \rightarrow \\
& v_{\text {boat rel. }}=(2.70 \mathrm{~m} / \mathrm{s}) \frac{\sin 45^{\circ}}{\operatorname{shos} 23^{\circ}}=2.07 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$



Equate the horizontal components of the velocities.

$$
\begin{aligned}
& v_{\substack{\text { boat rel. } \\
\text { shore }}} \sin \theta=v_{\substack{\text { boat el. } \\
\text { water }}} \cos 45^{\circ}-v_{\substack{\text { water } \\
\text { rel. shore }}} \rightarrow \\
& v_{\substack{\text { water } \\
\text { rel shore }}}=v_{\substack{\text { boat rel } \\
\text { water }}} \cos 45^{\circ}-v_{\substack{\text { boat rel. } \\
\text { shore }}} \sin \theta \\
& =(2.70 \mathrm{~m} / \mathrm{s}) \cos 45^{\circ}-(2.07 \mathrm{~m} / \mathrm{s}) \sin 23^{\circ}=1.10 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

71. Call east the positive $x$ direction and north the positive $y$ direction. The following is seen from the diagram. Apply the law of sines to the triangle formed by the three vectors.

$$
\begin{aligned}
& \frac{v_{\text {plane }}}{\text { rel. air }}=\frac{v_{\text {air rel. }}}{\sin 128^{\circ}}=\frac{\sin \theta}{\sin \theta} \rightarrow \quad \sin \theta=\frac{v_{\text {air rel. }} \text { ground }}{v_{1}} \sin 128^{\circ} \quad \rightarrow \\
& \theta=\sin ^{-1}\left(\frac{v_{\text {air rel. }}}{\substack{\text { ground }}} \sin 128^{\circ}\right)=\sin ^{-1}\left(\frac{72}{580 \mathrm{~km} / \mathrm{h}} \sin 128^{\circ}\right)=5.6^{\circ}
\end{aligned}
$$



So the plane should head in a direction of $38.0^{\circ}+5.6^{\circ}=43.6^{\circ}$ north of east. .
72. (a) For the magnitudes to add linearly, the two vectors must be parallel. $\overrightarrow{\mathbf{V}}_{1} \| \overrightarrow{\mathbf{V}}_{2}$
(b) For the magnitudes to add according to the Pythagorean theorem, the two vectors must be at right angles to each other. $\overrightarrow{\mathbf{V}}_{1} \perp \overrightarrow{\mathbf{V}}_{2}$
(c) The magnitude of $\overrightarrow{\mathbf{V}}_{2}$ vector 2 must be $0 . \overrightarrow{\mathbf{V}}_{2}=0$
73. Let east be the positive $x$-direction, north be the positive $y$-direction, and up be the positive $z$-direction. Then the plumber's resultant displacement in component notation is $\overrightarrow{\mathbf{D}}=66 \mathrm{~m} \hat{\mathbf{i}}-35 \mathrm{~m} \hat{\mathbf{j}}-12 \mathrm{~m} \hat{\mathbf{k}}$. Since this is a $3-$ dimensional problem, it requires 2 angles to determine his location (similar
 to latitude and longitude on the surface of the Earth). For the $x-y$ (horizontal) plane, see the first figure.

$$
\begin{aligned}
& \phi=\tan ^{-1} \frac{D_{y}}{D_{x}}=\tan ^{-1} \frac{-35}{66}=-28^{\circ}=28^{\circ} \text { south of east } \\
& D_{x y}=\sqrt{D_{x}^{2}+D_{y}^{2}}=\sqrt{(66)^{2}+(-35)^{2}}=74.7 \mathrm{~m} \approx 75 \mathrm{~m}
\end{aligned}
$$

For the vertical motion, consider another right triangle, made up of $D_{x y}$ as one leg, and the vertical displacement $D_{z}$ as the other leg. See the second figure, and the following calculations.


$$
\begin{aligned}
& \theta_{2}=\tan ^{-1} \frac{D_{z}}{D_{x y}}=\tan ^{-1} \frac{-12 \mathrm{~m}}{74.7 \mathrm{~m}}=-9^{\circ}=9^{\circ} \text { below the horizontal } \\
& D=\sqrt{D_{x y}^{2}+D_{z}^{2}}=\sqrt{D_{x}^{2}+D_{y}^{2}+D_{z}^{2}}=\sqrt{(66)^{2}+(-35)^{2}+(-12)^{2}}=76 \mathrm{~m}
\end{aligned}
$$

The result is that the displacement is 76 m , at an angle of $28^{\circ}$ south of east, and

## $9^{\circ}$ below the horizontal.

74. The deceleration is along a straight line. The starting velocity is $110 \mathrm{~km} / \mathrm{h}\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)=30.6 \mathrm{~m} / \mathrm{s}$, and the ending velocity is $0 \mathrm{~m} / \mathrm{s}$. The acceleration is found from Eq. 2-12a.

$$
v=v_{0}+a t \rightarrow 0=30.6 \mathrm{~m} / \mathrm{s}+a(7.0 \mathrm{~s}) \rightarrow a=-\frac{30.6 \mathrm{~m} / \mathrm{s}}{7.0 \mathrm{~s}}=-4.37 \mathrm{~m} / \mathrm{s}^{2}
$$

The horizontal acceleration is $a_{\text {horiz }}=a \cos \theta=-4.37 \mathrm{~m} / \mathrm{s}^{2}\left(\cos 26^{\circ}\right)=-3.9 \mathrm{~m} / \mathrm{s}^{2}$.
The vertical acceleration is $a_{\text {vert }}=a \sin \theta=-4.37 \mathrm{~m} / \mathrm{s}^{2}\left(\sin 26^{\circ}\right)=-1.9 \mathrm{~m} / \mathrm{s}^{2}$.
The horizontal acceleration is to the left in Figure 3-54, and the vertical acceleration is down.
75. Call east the positive $x$ direction and north the positive $y$ direction. Then this relative velocity relationship follows (see the accompanying diagram).

$$
\overrightarrow{\mathbf{v}}_{\substack{\text { plane rel. } \\ \text { ground }}}=\overrightarrow{\mathbf{v}}_{\substack{\text { plane } \\ \text { rel. air }}}+\overrightarrow{\mathbf{v}}_{\text {air rel. }}^{\text {ground }}
$$

Equate the $x$ components of the velocity vectors. The magnitude of $\overrightarrow{\mathbf{v}}_{\substack{\text { plane rel. } \\ \text { ground }}}$
 is given as $135 \mathrm{~km} / \mathrm{h}$.

$$
(135 \mathrm{~km} / \mathrm{h}) \cos 45^{\circ}=0+v_{\operatorname{wind} x} \rightarrow v_{\operatorname{wind} x}=95.5 \mathrm{~km} / \mathrm{h} .
$$

From the $y$ components of the above equation, we find $v_{\text {wind } y}$.

$$
-135 \sin 45^{\circ}=-185+v_{\text {wind } y} \rightarrow v_{\text {wind } y}=185-135 \sin 45^{\circ}=89.5 \mathrm{~km} / \mathrm{h}
$$

The magnitude of the wind velocity is as follows.

$$
v_{\text {wind }}=\sqrt{v_{\text {wind } x}^{2}+v_{\text {wind } y}^{2}}=\sqrt{(95.5 \mathrm{~km} / \mathrm{h})^{2}+(89.5 \mathrm{~km} / \mathrm{h})^{2}}=131 \mathrm{~km} / \mathrm{h}
$$

The direction of the wind is $\theta=\tan ^{-1} \frac{v_{\text {wind-y }}}{v_{\text {wind-x }}}=\tan ^{-1} \frac{89.5}{95.5}=43.1^{\circ}$ north of east.
76. The time of flight is found from the constant velocity relationship for horizontal motion.

$$
\Delta x=v_{x} t=\rightarrow t=\Delta x / v_{x}=8.0 \mathrm{~m} / 9.1 \mathrm{~m} / \mathrm{s}=0.88 \mathrm{~s}
$$

The $y$ motion is symmetric in time - it takes half the time of flight to rise, and half to fall. Thus the time for the jumper to fall from his highest point to the ground is 0.44 sec . His vertical speed is zero at the highest point. From the time, the initial vertical speed, and the acceleration of gravity, the maximum height can be found. Call upward the positive $y$ direction. The point of maximum height
is the starting position $y_{0}$ ，the ending position is $y=0$ ，the starting vertical speed is 0 ，and $a=-g$ ． Use Eq．2－12b to find the height．

$$
y=y_{0}+v_{y 0} t+\frac{1}{2} a_{y} t^{2} \rightarrow 0=y_{0}+0-\frac{1}{2}\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)(0.44 \mathrm{~s})^{2} \rightarrow y_{0}=0.95 \mathrm{~m}
$$

77．Choose upward to be the positive $y$ direction．The origin is the point from which the pebbles are released．In the vertical direction，$a_{y}=-9.80 \mathrm{~m} / \mathrm{s}^{2}$ ，the velocity at the window is $v_{y}=0$ ，and the vertical displacement is 8.0 m ．The initial $y$ velocity is found from Eq．2－12c．

$$
\begin{aligned}
& v_{y}^{2}=v_{y 0}^{2}+2 a_{y}\left(y-y_{0}\right) \rightarrow \\
& v_{y 0}=\sqrt{v_{y}^{2}-2 a_{y}\left(y-y_{0}\right)}=\sqrt{0-2\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(8.0 \mathrm{~m})}=12.5 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Find the time for the pebbles to travel to the window from Eq．2－12a．

$$
v_{y}=v_{y 0}+a t \rightarrow t=\frac{v_{y}-v_{y 0}}{a}=\frac{0-12.5 \mathrm{~m} / \mathrm{s}}{-9.80 \mathrm{~m} / \mathrm{s}^{2}}=1.28 \mathrm{~s}
$$

Find the horizontal speed from the horizontal motion at constant velocity．

$$
\Delta x=v_{x} t \rightarrow v_{x}=\Delta x / t=9.0 \mathrm{~m} / 1.28 \mathrm{~s}=7.0 \mathrm{~m} / \mathrm{s}
$$

This is the speed of the pebbles when they hit the window．
78．Choose the $x$ direction to be the direction of train travel（the direction the passenger is facing）and choose the $y$ direction to be up．This relationship exists among the velocities： $\overrightarrow{\mathbf{v}}_{\text {rain rel．}}^{\text {ground }}, ~=\overrightarrow{\mathbf{v}}_{\text {rain rel．}}^{\text {train }}, ~+\overrightarrow{\mathbf{v}}_{\text {trair rel }}^{\text {ground }}$ ．From the diagram，find the expression for the speed of the raindrops．

$$
\tan \theta=\frac{\substack{v_{\text {trin rel. }} \\ \text { ground }}}{v_{\text {rain rel. }}^{\text {rand }} \text { ground }} ⿺ ⿻ ⿻ 一 ㇂ ㇒ 丶 𠃌 ⿴ 囗 十 .
$$



79．Assume that the golf ball takes off and lands at the same height，so that the range formula derived in Example 3－10 can be applied．The only variable is to be the acceleration due to gravity．

$$
\begin{aligned}
& R_{\text {Earth }}=v_{0}^{2} \sin 2 \theta_{0} / g_{\text {Earth }} \quad R_{\text {Moon }}=v_{0}^{2} \sin 2 \theta_{0} / g_{\text {Moon }} \\
& \frac{R_{\text {Earth }}}{R_{\text {Moon }}}=\frac{v_{0}^{2} \sin 2 \theta_{0} / g_{\text {Earth }}}{v_{0}^{2} \sin 2 \theta_{0} / g_{\text {Moon }}}=\frac{1 / g_{\text {Earth }}}{1 / g_{\text {Moon }}}=\frac{g_{\text {Moon }}}{g_{\text {Earth }}}=\frac{32 \mathrm{~m}}{180 \mathrm{~m}}=0.18 \rightarrow \\
& g_{\text {Moon }}=0.18 g_{\text {Earth }}=0.18\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \approx 1.8 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned} \rightarrow
$$

80．（a）Choose downward to be the positive $y$ direction．The origin is the point where the bullet leaves the gun．In the vertical direction，$v_{y 0}=0, y_{0}=0$ ，and $a_{y}=9.80 \mathrm{~m} / \mathrm{s}^{2}$ ．In the horizontal direction，$\Delta x=68.0 \mathrm{~m}$ and $v_{x}=175 \mathrm{~m} / \mathrm{s}$ ．The time of flight is found from the horizontal motion at constant velocity．

$$
\Delta x=v_{x} t \rightarrow t=\Delta x / v_{x}=68.0 \mathrm{~m} / 175 \mathrm{~m} / \mathrm{s}=0.3886 \mathrm{~s}
$$

This time can now be used in Eq．2－12b to find the vertical drop of the bullet．

$$
y=y_{0}+v_{y 0} t+\frac{1}{2} a_{y} t^{2} \rightarrow y=0+0+\frac{1}{2}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.3886 \mathrm{~s})^{2}=0.740 \mathrm{~m}
$$

(b) For the bullet to hit the target at the same level, the level horizontal range formula of Example $3-10$ applies. The range is 68.0 m , and the initial velocity is $175 \mathrm{~m} / \mathrm{s}$. Solving for the angle of launch results in the following.

$$
R=\frac{v_{0}^{2} \sin 2 \theta_{0}}{g} \rightarrow \sin 2 \theta_{0}=\frac{R g}{v_{0}^{2}} \rightarrow \theta_{0}=\frac{1}{2} \sin ^{-1} \frac{(68.0 \mathrm{~m})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{(175 \mathrm{~m} / \mathrm{s})^{2}}=0.623^{\circ}
$$

Because of the symmetry of the range formula, there is also an answer of the complement of the above answer, which would be $89.4^{\circ}$. That is an unreasonable answer from a practical physical viewpoint - it is pointing the gun almost straight up.
81. Choose downward to be the positive $y$ direction. The origin is at the point from which the divers push off the cliff. In the vertical direction, the initial velocity is $v_{y 0}=0$, the acceleration is $a_{y}=9.80 \mathrm{~m} / \mathrm{s}^{2}$, and the displacement is 35 m . The time of flight is found from Eq. 2-12b.

$$
y=y_{0}+v_{y 0} t+\frac{1}{2} a_{y} t^{2} \rightarrow 35 \mathrm{~m}=0+0+\frac{1}{2}\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right) t^{2} \quad \rightarrow \quad t=\sqrt{\frac{2(35 \mathrm{~m})}{9.8 \mathrm{~m} / \mathrm{s}^{2}}}=2.7 \mathrm{~s}
$$

The horizontal speed (which is the initial speed) is found from the horizontal motion at constant velocity.

$$
\Delta x=v_{x} t \rightarrow v_{x}=\Delta x / t=5.0 \mathrm{~m} / 2.7 \mathrm{~s}=1.9 \mathrm{~m} / \mathrm{s}
$$

82. The minimum speed will be that for which the ball just clears the fence; i.e., the ball has a height of 8.0 m when it is 98 m horizontally from home plate. The origin is at home plate, with upward as the positive $y$ direction. For the ball, $y_{0}=1.0 \mathrm{~m}$, $y=8.0 \mathrm{~m}, a_{y}=-g, v_{y 0}=v_{0} \sin \theta_{0}, v_{x}=v_{0} \cos \theta_{0}$, and $\theta_{0}=36^{\circ}$. See the diagram (not to scale). For the constant-velocity horizontal
 motion, $\Delta x=v_{x} t=v_{0} \cos \theta_{0} t$, and so $t=\frac{\Delta x}{v_{0} \cos \theta_{0}}$. For the vertical motion, apply Eq. 2-12b.

$$
y=y_{0}+v_{y 0} t+\frac{1}{2} a_{y} t^{2}=y_{0}+v_{0}\left(\sin \theta_{0}\right) t-\frac{1}{2} g t^{2}
$$

Substitute the value of the time of flight for the first occurrence only in the above equation, and then solve for the time.

$$
\begin{aligned}
& y=y_{0}+v_{0} t \sin \theta_{0}-\frac{1}{2} g t^{2} \rightarrow y=y_{0}+v_{0} \sin \theta_{0} \frac{\Delta x}{v_{0} \cos \theta_{0}}-\frac{1}{2} g t^{2} \rightarrow \\
& t=\sqrt{2\left(\frac{y_{0}-y+\Delta x \tan \theta_{0}}{g}\right)}=\sqrt{2\left(\frac{1.0 \mathrm{~m}-8.0 \mathrm{~m}+(98 \mathrm{~m}) \tan 36^{\circ}}{9.80 \mathrm{~m} / \mathrm{s}^{2}}\right)}=3.620 \mathrm{~s}
\end{aligned}
$$

Finally, use the time with the horizontal range to find the initial speed.

$$
\Delta x=v_{0} \cos \theta_{0} t \rightarrow v_{0}=\frac{\Delta x}{t \cos \theta_{0}}=\frac{98 \mathrm{~m}}{(3.620 \mathrm{~s}) \cos 36^{\circ}}=33 \mathrm{~m} / \mathrm{s}
$$

83. (a) For the upstream trip, the boat will cover a distance of $D / 2$ with a net speed of $v-u$, so the time is $t_{1}=\frac{D / 2}{v-u}=\frac{D}{2(v-u)}$. For the downstream trip, the boat will cover a distance of $D / 2$
with a net speed of $v+u$, so the time is $t_{2}=\frac{D / 2}{v+u}=\frac{D}{2(v+u)}$. Thus the total time for the
round trip will be $t=t_{1}+t_{2}=\frac{D}{2(v-u)}+\frac{D}{2(v+u)}=\frac{D v}{\left(v^{2}-u^{2}\right)}$.
(b) For the boat to go directly across the river, it must be angled against the current in such a way that the net velocity is straight across the river, as in the picture. This equation must be satisfied:

$$
\overrightarrow{\mathbf{v}}_{\text {boat rel. }}^{\text {shore }} \ll \underset{\substack{\text { boat rel. } \\ \text { water }}}{ }+\underset{\substack{\text { water rel. } \\ \text { shore }}}{ }=\overrightarrow{\mathbf{v}}+\overrightarrow{\mathbf{u}}
$$



Thus $v_{\substack{\text { boat rel. } \\ \text { shore }}}=\sqrt{v^{2}-u^{2}}$, and the time to go a distance $D / 2$ across
the river is $t_{1}=\frac{D / 2}{\sqrt{v^{2}-u^{2}}}=\frac{D}{2 \sqrt{v^{2}-u^{2}}}$. The same relationship would be in effect for crossing
back, so the time to come back is given by $t_{2}=t_{1}$ and the total time is $t=t_{1}+t_{2}=\frac{D}{\sqrt{v^{2}-u^{2}}}$.
The speed $v$ must be greater than the speed $u$. The velocity of the boat relative to the shore when going upstream is $v-u$. If $v<u$, the boat will not move upstream at all, and so the first part of the trip would be impossible. Also, in part (b), we see that $v$ is longer than $u$ in the triangle, since $v$ is the hypotenuse, and so we must have $v>u$.
84. Choose the origin to be the location on the ground directly underneath the ball when served, and choose upward as the positive $y$ direction. Then for the ball, $y_{0}=2.50 \mathrm{~m}, v_{y 0}=0, a_{y}=-g$, and the $y$ location when the ball just clears the net is $y=0.90 \mathrm{~m}$. The time for the ball to reach the net is calculated from Eq. 2-12b.

$$
\begin{aligned}
& y=y_{0}+v_{y 0} t+\frac{1}{2} a_{y} t^{2} \rightarrow 0.90 \mathrm{~m}=2.50 \mathrm{~m}+0+\frac{1}{2}\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right) t^{2} \rightarrow \\
& t_{\mathrm{to}}=\sqrt{\frac{2(-1.60 \mathrm{~m})}{-9.80 \mathrm{~m} / \mathrm{s}^{2}}}=0.57143 \mathrm{~s}
\end{aligned}
$$

The $x$ velocity is found from the horizontal motion at constant velocity.

$$
\Delta x=v_{x} t \quad \rightarrow \quad v_{x}=\frac{\Delta x}{t}=\frac{15.0 \mathrm{~m}}{0.57143 \mathrm{~s}}=26.25 \approx 26.3 \mathrm{~m} / \mathrm{s}
$$

This is the minimum speed required to clear the net.
To find the full time of flight of the ball, set the final $y$ location to be $y=0$, and again use Eq. 2-12b.

$$
\begin{aligned}
& y=y_{0}+v_{y 0} t+\frac{1}{2} a_{y} t^{2} \rightarrow 0.0 \mathrm{~m}=2.50 \mathrm{~m}+\frac{1}{2}\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right) t^{2} \rightarrow \\
& t_{\text {total }}=\sqrt{\frac{2(-2.50 \mathrm{~m})}{-9.80 \mathrm{~m} / \mathrm{s}^{2}}}=0.7143 \approx 0.714 \mathrm{~s}
\end{aligned}
$$

The horizontal position where the ball lands is found from the horizontal motion at constant velocity.

$$
\Delta x=v_{x} t=(26.25 \mathrm{~m} / \mathrm{s})(0.7143 \mathrm{~s})=18.75 \approx 18.8 \mathrm{~m}
$$

Since this is between 15.0 and 22.0 m , the ball lands in the "good" region.
85. Work in the frame of reference in which the car is at rest at ground level. In this reference frame, the helicopter is moving horizontally with a speed of $208 \mathrm{~km} / \mathrm{h}-156 \mathrm{~km} / \mathrm{h}=52 \mathrm{~km} / \mathrm{h}\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)$ $=14.44 \mathrm{~m} / \mathrm{s}$. For the vertical motion, choose the level of the helicopter to be the origin, and downward to be positive. Then the package's $y$ displacement is $y=78.0 \mathrm{~m}, v_{y 0}=0$, and $a_{y}=g$. The time for the package to fall is calculated from Eq. 2-12b.

$$
y=y_{0}+v_{y 0} t+\frac{1}{2} a_{y} t^{2} \rightarrow 78.0 \mathrm{~m}=\frac{1}{2}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) t^{2} \rightarrow t=\sqrt{\frac{2(78.0 \mathrm{~m})}{9.80 \mathrm{~m} / \mathrm{s}^{2}}}=3.99 \mathrm{sec}
$$

The horizontal distance that the package must move, relative to the "stationary" car, is found from the horizontal motion at constant velocity.

$$
\Delta x=v_{x} t=(14.44 \mathrm{~m} / \mathrm{s})(3.99 \mathrm{~s})=57.6 \mathrm{~m}
$$

Thus the angle under the horizontal for the package release will be as follows.

$$
\theta=\tan ^{-1}\left(\frac{\Delta y}{\Delta x}\right)=\tan ^{-1}\left(\frac{78.0 \mathrm{~m}}{57.6 \mathrm{~m}}\right)=53.6^{\circ} \approx 54^{\circ}
$$

86. The proper initial speeds will be those for which the ball has traveled a horizontal distance somewhere between 10.78 m and 11.22 m while it changes height from 2.10 m to 3.05 m with a shooting angle of $38.0^{\circ}$. Choose the origin to be at the shooting location of the basketball, with upward as the
 positive $y$ direction. Then the vertical displacement is $y=0.95 \mathrm{~m}, a_{y}=-9.80 \mathrm{~m} / \mathrm{s}^{2}, v_{y 0}=v_{0} \sin \theta_{0}$, and the (constant) $x$ velocity is $v_{x}=v_{0} \cos \theta_{0}$. See the diagram (not to scale). For the constant-velocity horizontal motion, $\Delta x=v_{x} t=v_{0} \cos \theta_{0} t$ and so $t=\frac{\Delta x}{v_{0} \cos \theta_{0}}$. For the vertical motion, apply Eq. 2-12b.

$$
y=y_{0}+v_{y 0} t+\frac{1}{2} a_{y} t^{2}=v_{0} \sin \theta t-\frac{1}{2} g t^{2}
$$

Substitute the expression for the time of flight and solve for the initial velocity.

$$
\begin{aligned}
& y=v_{0} \sin \theta t-\frac{1}{2} g t^{2}=v_{0} \sin \theta \frac{\Delta x}{v_{0} \cos \theta_{0}}-\frac{1}{2} g\left(\frac{\Delta x}{v_{0} \cos \theta_{0}}\right)^{2}=\Delta x \tan \theta-\frac{g(\Delta x)^{2}}{2 v_{0}^{2} \cos ^{2} \theta_{0}} \\
& v_{0}=\sqrt{\frac{g(\Delta x)^{2}}{2 \cos ^{2} \theta_{0}(-y+\Delta x \tan \theta)}}
\end{aligned}
$$

For $\Delta x=10.78 \mathrm{~m}$, the shortest shot:

$$
v_{0}=\sqrt{\frac{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(10.78 \mathrm{~m})^{2}}{2 \cos ^{2} 38.0^{\circ}\left[\left(-0.95 \mathrm{~m}+(10.78 \mathrm{~m}) \tan 38.0^{\circ}\right)\right]}}=11.1 \mathrm{~m} / \mathrm{s} .
$$

For $\Delta x=11.22 \mathrm{~m}$, the longest shot:

$$
v_{0}=\sqrt{\frac{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(11.22 \mathrm{~m})^{2}}{2 \cos ^{2} 38.0^{\circ}\left[\left(-0.95 \mathrm{~m}+(11.22 \mathrm{~m}) \tan 38.0^{\circ}\right)\right]}}=11.3 \mathrm{~m} / \mathrm{s} \text {. }
$$

87. The acceleration is the derivative of the velocity.

$$
\overrightarrow{\mathbf{a}}=\frac{d \overrightarrow{\mathbf{v}}}{d t}=3.5 \mathrm{~m} / \mathrm{s}^{2} \hat{\mathbf{j}}
$$

Since the acceleration is constant, we can use Eq. 3-13b.

$$
\begin{aligned}
\overrightarrow{\mathbf{r}} & =\overrightarrow{\mathbf{r}}_{0}+\overrightarrow{\mathbf{v}}_{0} t+\frac{1}{2} \overrightarrow{\mathbf{a}} t^{2}=(1.5 \hat{\mathbf{i}}-3.1 \hat{\mathbf{j}})+(-2.0 \hat{\mathbf{i}}) t+\frac{1}{2}(3.5 \hat{\mathbf{j}}) t^{2} \\
& =(1.5-2.0 t) \mathrm{m} \hat{\mathbf{i}}+\left(-3.1+1.75 t^{2}\right) \mathrm{m} \hat{\mathbf{j}}
\end{aligned}
$$

The shape is parabolic, with the parabola opening in the $y$-direction.
88. Choose the origin to be the point from which the projectile is launched, and choose upward as the positive $y$ direction. The $y$ displacement of the projectile is 135 m , and the horizontal range of the projectile is 195 m . The acceleration in the $y$ direction is $a_{y}=-g$, and the time of flight is 6.6 s . The horizontal velocity is found from the horizontal motion at constant velocity.

$$
\Delta x=v_{x} t \quad \rightarrow \quad v_{x}=\frac{\Delta x}{t}=\frac{195 \mathrm{~m}}{6.6 \mathrm{~s}}=29.55 \mathrm{~m} / \mathrm{s}
$$

Calculate the initial $y$ velocity from the given data and Eq. 2-12b.

$$
y=y_{0}+v_{y 0} t+\frac{1}{2} a_{y} t^{2} \rightarrow 135 \mathrm{~m}=v_{y 0}(6.6 \mathrm{~s})+\frac{1}{2}\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(6.6 \mathrm{~s})^{2} \rightarrow v_{y 0}=52.79 \mathrm{~m} / \mathrm{s}
$$

Thus the initial velocity and direction of the projectile are as follows.

$$
\begin{aligned}
& v_{0}=\sqrt{v_{x}^{2}+v_{y 0}^{2}}=\sqrt{(29.55 \mathrm{~m} / \mathrm{s})^{2}+(52.79 \mathrm{~m} / \mathrm{s})^{2}}=60 \mathrm{~m} / \mathrm{s} \\
& \theta=\tan ^{-1} \frac{v_{y 0}}{v_{x}}=\tan ^{-1} \frac{52.79 \mathrm{~m} / \mathrm{s}}{29.55 \mathrm{~m} / \mathrm{s}}=61^{\circ}
\end{aligned}
$$

89. We choose to initially point the boat downstream at an angle of $\phi$ relative to straight across the river, because then all horizontal velocity components are in the same direction, and the algebraic signs might be less confusing. If the boat should in reality be pointed upstream, the solution will give a negative angle. We use $v_{\mathrm{BW}}=1.60 \mathrm{~m} / \mathrm{s}$, the speed of the boat relative to the water (the rowing speed); $v_{\mathrm{ws}}=0.80 \mathrm{~m} / \mathrm{s}$, the speed of the water relative to the shore (the current); and $v_{\mathrm{R}}=3.00 \mathrm{~m} / \mathrm{s}$, his running speed. The width of the river is $w=1200 \mathrm{~m}$, and the length traveled along the bank is $\ell$. The time spent in the water is $t_{\mathrm{w}}$, and the time running is $t_{\mathrm{R}}$. The actual vector velocity of the boat is $\overrightarrow{\mathbf{v}}_{\text {BS }}=\overrightarrow{\mathbf{v}}_{\text {BW }}+\overrightarrow{\mathbf{v}}_{\text {WS }}$. That vector addition is illustrated on the diagram (not drawn to scale).


The distance straight across the river $(w)$ is the velocity component across the river, times the time in the water. The distance along the bank $(\ell)$ is the velocity component parallel to the river, times the time in the water. The distance along the bank is also his running speed times the time running. These three distances are expressed below.

$$
w=\left(v_{\mathrm{BW}} \cos \phi\right) t_{\mathrm{W}} ; \ell=\left(v_{\mathrm{BW}} \sin \phi+v_{W S}\right) t_{\mathrm{W}} ; \ell=v_{\mathrm{R}} t_{\mathrm{R}}
$$

The total time is $t=t_{\mathrm{w}}+t_{\mathrm{R}}$, and needs to be expressed as a function of $\phi$. Use the distance relations above to write this function.

$$
\begin{aligned}
t & =t_{\mathrm{W}}+t_{\mathrm{R}}=t_{\mathrm{W}}+\frac{\ell}{v_{\mathrm{R}}}=t_{\mathrm{W}}+\frac{\left(v_{\mathrm{BW}} \sin \phi+v_{W S}\right) t_{\mathrm{W}}}{v_{\mathrm{R}}}=t_{\mathrm{W}}\left[1+\frac{\left(v_{\mathrm{BW}} \sin \phi+v_{W S}\right)}{v_{\mathrm{R}}}\right] \\
& =\frac{w}{v_{\mathrm{BW}} v_{\mathrm{R}} \cos \phi}\left[v_{\mathrm{R}}+v_{W S}+v_{\mathrm{BW}} \sin \phi\right]=\frac{w}{v_{\mathrm{BW}} v_{\mathrm{R}}}\left[\left(v_{\mathrm{R}}+v_{\mathrm{WS}}\right) \sec \phi+v_{\mathrm{BW}} \tan \phi\right]
\end{aligned}
$$

To find the angle corresponding to the minimum time, we set $\frac{d t}{d \phi}=0$ and solve for the angle.

$$
\begin{aligned}
& \begin{aligned}
& \frac{d t}{d \phi}=\frac{d}{d \phi}\left\{\frac{w}{v_{\mathrm{BW}} v_{\mathrm{R}}}\left[\left(v_{\mathrm{R}}+v_{\mathrm{WS}}\right) \sec \phi+v_{\mathrm{BW}} \tan \phi\right]\right\} \\
&=\frac{w}{v_{\mathrm{BW}} v_{\mathrm{R}}}\left[\left(v_{\mathrm{R}}+v_{\mathrm{WS}}\right) \tan \phi \sec \phi+v_{\mathrm{BW}} \sec ^{2} \phi\right]=0 \rightarrow \\
& {\left[\left(v_{\mathrm{R}}+v_{\mathrm{WS}}\right) \tan \phi+v_{\mathrm{BW}} \sec \phi\right] \sec \phi=0 \rightarrow \sec \phi=0, \sin \phi=-\frac{v_{\mathrm{BW}}}{v_{\mathrm{R}}+v_{\mathrm{WS}}} }
\end{aligned}
\end{aligned}
$$

The first answer is impossible, and so we must use the second solution.

$$
\sin \phi=-\frac{v_{\mathrm{BW}}}{v_{\mathrm{R}}+v_{\mathrm{wS}}}=-\frac{1.60 \mathrm{~m} / \mathrm{s}}{3.00 \mathrm{~m} / \mathrm{s}+0.80 \mathrm{~m} / \mathrm{s}}=-0.421 \rightarrow \phi=\sin ^{-1}(-0.421)=-24.9^{\circ}
$$

To know that this is really a minimum and not a maximum, some argument must be made. The maximum time would be infinity, if he pointed his point either directly upstream or downstream.
Thus this angle should give a minimum. A second derivative test could be done, but that would be algebraically challenging. A graph of $t$ vs. $\phi$ could also be examined to see that the angle is a minimum. Here is a portion of such a graph, showing a minimum time of somewhat more than 800 seconds near $\phi=-25^{\circ}$. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH03.XLS," on
 tab "Problem 3.89."

The time he takes in getting to the final location can be calculated from the angle.

$$
\begin{aligned}
t_{\mathrm{w}} & =\frac{w}{v_{\mathrm{BW}} \cos \phi}=\frac{1200 \mathrm{~m}}{(1.60 \mathrm{~m} / \mathrm{s}) \cos \left(-24.9^{\circ}\right)}=826.86 \mathrm{~s} \\
\ell & =\left(v_{\mathrm{BW}} \sin \phi+v_{\text {WS }}\right) t_{\mathrm{w}}=\left[(1.60 \mathrm{~m} / \mathrm{s}) \sin \left(-24.9^{\circ}\right)+0.80 \mathrm{~m} / \mathrm{s}\right](826.86 \mathrm{~s})=104.47 \mathrm{~m} \\
t_{\mathrm{R}} & =\frac{\ell}{v_{\mathrm{R}}}=\frac{104.47 \mathrm{~m}}{3.00 \mathrm{~m} / \mathrm{s}}=34.82 \mathrm{~s} \quad t=t_{\mathrm{W}}+t_{\mathrm{R}}=826.86 \mathrm{~s}+34.82 \mathrm{~s}=862 \mathrm{~s}
\end{aligned}
$$

Thus he must point the boat $24.9^{\circ}$ upstream, taking 827 seconds to cross, and landing 104 m from the point directly across from his starting point. Then he runs the 104 m from his landing point to the point directly across from his starting point, in 35 seconds, for a total elapsed time of 862 seconds (about 14.4 minutes).
90. Call the direction of the flow of the river the $x$ direction, and the direction the boat is headed (which is different than the direction it is moving) the $y$ direction.
(a) $v_{\substack{\text { boat rel. } \\ \text { shore }}}=\sqrt{v_{\begin{array}{c}\text { water rel. } \\ \text { shore }\end{array}}^{2}+v_{\substack{\text { boat rel. } \\ \text { water }}}^{2}}=\sqrt{1.30^{2}+2.20^{2}}=2.56 \mathrm{~m} / \mathrm{s}$

$$
\theta=\tan ^{-1} \frac{1.30}{2.20}=30.6^{\circ}, \phi=90^{\circ}-\theta=59.4^{\circ} \text { relative to shore }
$$

(b) The position of the boat after 3.00 seconds is given by the following.

$$
\begin{aligned}
\Delta d & =v_{\substack{\text { boat rel. } \\
\text { shore }}} t=[(1.30 \hat{\mathbf{i}}+2.20 \overrightarrow{\mathbf{j}}) \mathrm{m} / \mathrm{s}](3.00 \mathrm{sec}) \\
& =(3.90 \mathrm{~m} \text { downstream }, 6.60 \mathrm{~m} \text { across the river })
\end{aligned}
$$



As a magnitude and direction, it would be 7.67 m away from the starting point, at an angle of $59.4^{\circ}$ relative to the shore.
91. First, we find the direction of the straight-line path that the boat must take to pass 150 m to the east of the buoy. See the first diagram (not to scale). We find the net displacement of the boat in the horizontal and vertical directions, and then calculate the angle.

$$
\begin{aligned}
& \Delta x=(3000 \mathrm{~m}) \sin 22.5^{\circ}+150 \mathrm{~m} \quad \Delta y=(3000 \mathrm{~m}) \cos 22.5^{\circ} \\
& \phi=\tan ^{-1} \frac{\Delta y}{\Delta x}=\frac{(3000 \mathrm{~m}) \cos 22.5^{\circ}}{(3000 \mathrm{~m}) \sin 22.5^{\circ}+150 \mathrm{~m}}=64.905^{\circ}
\end{aligned}
$$

This angle gives the direction that the boat must travel, so it is the direction of the velocity of the boat with respect to the shore, $\overrightarrow{\mathbf{v}}_{\substack{\text { boat rel. } \\ \text { shore }}}$. So $\overrightarrow{\mathbf{v}}_{\text {boat rel. }}^{\text {shore }} \begin{aligned} & =v_{\text {boat rel. }}^{\text {shore }}\end{aligned}(\cos \phi \hat{\mathbf{i}}+\sin \phi \hat{\mathbf{j}})$. Then, using the second diagram (also not to scale), we can write the relative velocity equation relating the boat's travel and the current. The relative velocity equation gives us the following. See the second diagram.

$$
\begin{aligned}
& v_{\substack{\text { boat rel. } \\
\text { shore }}}(\cos \phi \hat{\mathbf{i}}+\sin \phi \hat{\mathbf{j}})=2.1(\cos \theta \hat{\mathbf{i}}+\sin \theta \hat{\mathbf{j}})+0.2 \hat{\mathbf{i}} \rightarrow \\
& v_{\substack{\text { boat rel. } \\
\text { shore }}} \cos \phi=2.1 \cos \theta+0.2 ; v_{\substack{\text { boat rel. } \\
\text { shore }}} \sin \phi=2.1 \sin \theta
\end{aligned}
$$

These two component equations can then be solved for $v_{\substack{\text { boat rel. } \\ \text { shore }}}$ and $\theta$. One technique is to isolate the terms with $\theta$ in each equation, and then square those equations and add them. That gives a quadratic equation for $v_{\substack{\text { boat rel. } \\ \text { shore }}}$, which is solved by $v_{\substack{\text { boat rel. } \\ \text { shore }}}=2.177 \mathrm{~m} / \mathrm{s}$. Then the angle is found to be

$$
\theta=69.9^{\circ} \mathrm{N} \text { of } \mathrm{E} .
$$

92. See the sketch of the geometry. We assume that the hill is sloping downward to the right. Then if we take the point where the child jumps as the origin, with the $x$-direction positive to the right and the $y$-direction positive upwards, then the equation for the hill is given by $y=-x \tan 12^{\circ}$.


The path of the child (shown by the dashed line) is projectile motion. With the same origin and coordinate system, the horizontal motion of the child is given by $x=v_{0} \cos 15^{\circ}(t)$, and the vertical motion of the child will be given by Eq. 2-12b, $y=v_{0} \sin 15^{\circ} t-\frac{1}{2} g t^{2}$. The landing point of the child is given by $x_{\text {landing }}=1.4 \cos 12^{\circ}$ and $y_{\text {landing }}=-1.4 \sin 12^{\circ}$. Use the horizontal motion and landing point to find an expression for the time the child is in the air, and then use that time to find the initial speed.

$$
x=v_{0} \cos 15^{\circ}(t) \rightarrow t=\frac{x}{v_{0} \cos 15^{\circ}}, t_{\text {landing }}=\frac{1.4 \cos 12^{\circ}}{v_{0} \cos 15^{\circ}}
$$

Equate the $y$ expressions, and use the landing time. We also use the trigonometric identity that $\sin 12^{\circ} \cos 15^{\circ}+\sin 15^{\circ} \cos 12^{\circ}=\sin \left(12^{\circ}+15^{\circ}\right)$.

$$
\begin{aligned}
& y_{\text {landing }}=y_{\text {projectile }} \rightarrow-1.4 \sin 12^{\circ}=v_{0} \sin 15^{\circ} t_{\text {landing }}-\frac{1}{2} g t_{\text {landing }}^{2} \rightarrow \\
& -1.4 \sin 12^{\circ}=v_{0} \sin 15^{\circ} \frac{1.4 \cos 12^{\circ}}{v_{0} \cos 15^{\circ}}-\frac{1}{2} g\left(\frac{1.4 \cos 12^{\circ}}{v_{0} \cos 15^{\circ}}\right)^{2} \rightarrow \\
& v_{0}^{2}=\frac{1}{2} g \frac{\cos ^{2} 12^{\circ}}{\sin 27^{\circ}}\left(\frac{1.4}{\cos 15^{\circ}}\right) \rightarrow v_{0}=3.8687 \mathrm{~m} / \mathrm{s} \approx 3.9 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

93. Find the time of flight from the vertical data, using Eq. 2-12b. Call the floor the $y=0$ location, and choose upwards as positive.

$$
\begin{aligned}
& y=y_{0}+v_{0 y} t+\frac{1}{2} a_{y} t^{2} \rightarrow 3.05 \mathrm{~m}=2.4 \mathrm{~m}+(12 \mathrm{~m} / \mathrm{s}) \sin 35^{\circ} t+\frac{1}{2}\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right) t^{2} \\
& 4.90 t^{2}-6.883 t+0.65 \mathrm{~m}=0 \rightarrow \\
& t=\frac{6.883 \pm \sqrt{6.883^{2}-4(4.90)(0.65)}}{2(4.90)}=1.303 \mathrm{~s}, 0.102 \mathrm{~s}
\end{aligned}
$$

(a) Use the larger time for the time of flight. The shorter time is the time for the ball to rise to the basket height on the way up, while the longer time is the time for the ball to be at the basket height on the way down.

$$
x=v_{x} t=v_{0}\left(\cos 35^{\circ}\right) t=(12 \mathrm{~m} / \mathrm{s})\left(\cos 35^{\circ}\right)(1.303 \mathrm{~s})=12.81 \mathrm{~m} \approx 13 \mathrm{~m}
$$

(b) The angle to the horizontal is determined by the components of the velocity.

$$
\begin{aligned}
& v_{x}=v_{0} \cos \theta_{0}=12 \cos 35^{\circ}=9.830 \mathrm{~m} / \mathrm{s} \\
& v_{y}=v_{y 0}+a t=v_{0} \sin \theta_{0}-g t=12 \sin 35^{\circ}-9.80(1.303)=-5.886 \mathrm{~m} / \mathrm{s} \\
& \theta=\tan ^{-1} \frac{v_{y}}{v_{x}}=\tan ^{-1} \frac{-5.886}{9.830}=-30.9^{\circ} \approx-31^{\circ}
\end{aligned}
$$

The negative angle means it is below the horizontal.
94. We have $v_{\substack{\text { car rel. } \\ \text { ground }}}=25 \mathrm{~m} / \mathrm{s}$. Use the diagram, illustrating

$$
\begin{aligned}
& \overrightarrow{\mathbf{v}}_{\text {snow rel. }}=\overrightarrow{\mathbf{v}}_{\text {snow rel. }}+\underset{\substack{\text { crond } \\
\text { car }}}{ }+\overrightarrow{\mathbf{v}}_{\substack{\text { car rel. } \\
\text { ground }}} \text {, to calculate the other speeds. } \\
& \qquad \cos 37^{\circ}=\frac{v_{\text {car rel. }}^{\text {ground }}}{v_{\text {snow rel. }}} \rightarrow v_{\text {sonw rel. }}=\frac{25 \mathrm{~m} / \mathrm{s}}{\operatorname{car} 37^{\circ}}=31 \mathrm{~m} / \mathrm{s} \\
& \text { car }
\end{aligned}
$$



$$
\tan 37^{\circ}=\frac{v_{\text {snow rel. }}^{\text {ground }}}{v_{\substack{\text { car rel. } \\ \text { ground }}}} \rightarrow v_{\substack{\text { snow rel. } \\ \text { ground }}}=(25 \mathrm{~m} / \mathrm{s}) \tan 37^{\circ}=19 \mathrm{~m} / \mathrm{s}
$$

95. Let the launch point be the origin of coordinates, with right and upwards as the positive directions. The equation of the line representing the ground is $y_{\mathrm{gnd}}=-x$. The equations representing the motion of the rock are $x_{\text {rock }}=v_{0} t$ and $y_{\text {rock }}=-\frac{1}{2} g t^{2}$, which can be combined into $y_{\text {rock }}=-\frac{1}{2} \frac{g}{v_{0}^{2}} x_{\text {rock }}^{2}$. Find the intersection (the landing point of the rock) by equating the two expressions for $y$, and so finding where the rock meets the ground.

$$
y_{\text {rock }}=y_{\text {gnd }} \rightarrow-\frac{1}{2} \frac{g}{v_{0}^{2}} x^{2}=-x \rightarrow x=\frac{2 v_{0}^{2}}{g} \rightarrow t=\frac{x}{v_{0}}=\frac{2 v_{0}}{g}=\frac{2(25 \mathrm{~m} / \mathrm{s})}{9.80 \mathrm{~m} / \mathrm{s}^{2}}=5.1 \mathrm{~s}
$$

96. Choose the origin to be the point at ground level directly below where the ball was hit. Call upwards the positive $y$ direction. For the ball, we have $v_{0}=28 \mathrm{~m} / \mathrm{s}, \theta_{0}=61^{\circ}, a_{y}=-g, y_{0}=0.9 \mathrm{~m}$, and $y=0.0 \mathrm{~m}$.
(a) To find the horizontal displacement of the ball, the horizontal velocity and the time of flight are needed. The (constant) horizontal velocity is given by $v_{x}=v_{0} \cos \theta_{0}$. The time of flight is found from Eq. 2-12b.

$$
\begin{aligned}
y & =y_{0}+v_{y 0} t+\frac{1}{2} a_{y} t^{2} \rightarrow 0=y_{0}+v_{0} \sin \theta_{0} t-\frac{1}{2} g t^{2} \rightarrow \\
t & =\frac{-v_{0} \sin \theta_{0} \pm \sqrt{v_{0}^{2} \sin ^{2} \theta_{0}-4\left(-\frac{1}{2} g\right) y_{0}}}{2\left(-\frac{1}{2} g\right)} \\
& =\frac{-(28 \mathrm{~m} / \mathrm{s}) \sin 61^{\circ} \pm \sqrt{(28 \mathrm{~m} / \mathrm{s})^{2} \sin ^{2} 61^{\circ}-4\left(-\frac{1}{2}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.9 \mathrm{~m})}}{2\left(-\frac{1}{2}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)} \\
& =5.034 \mathrm{~s},-0.0365 \mathrm{~s}
\end{aligned}
$$

Choose the positive time, since the ball was hit at $t=0$. The horizontal displacement of the ball will be found by the constant velocity relationship for horizontal motion.

$$
\Delta x=v_{x} t=v_{0} \cos \theta_{0} t=(28 \mathrm{~m} / \mathrm{s})\left(\cos 61^{\circ}\right)(5.034 \mathrm{~s})=68.34 \mathrm{~m} \approx 68 \mathrm{~m}
$$

(b) The center fielder catches the ball right at ground level. He ran $105 \mathrm{~m}-68.34 \mathrm{~m}=36.66 \mathrm{~m}$ to catch the ball, so his average running speed would be as follows.

$$
v_{\text {avg }}=\frac{\Delta d}{t}=\frac{36.66 \mathrm{~m}}{5.034 \mathrm{~s}}=7.282 \mathrm{~m} / \mathrm{s} \approx 7.3 \mathrm{~m} / \mathrm{s}
$$

97. Choose the origin to be the point at the top of the building from which the ball is shot, and call upwards the positive $y$ direction. The initial velocity is $v_{0}=18 \mathrm{~m} / \mathrm{s}$ at an angle of $\theta_{0}=42^{\circ}$. The acceleration due to gravity is $a_{y}=-g$.
(a) $v_{x}=v_{0} \cos \theta_{0}=(18 \mathrm{~m} / \mathrm{s}) \cos 42^{\circ}=13.38 \approx 13 \mathrm{~m} / \mathrm{s}$
$v_{y 0}=v_{0} \sin \theta_{0}=(18 \mathrm{~m} / \mathrm{s}) \sin 42^{\circ}=12.04 \approx 12 \mathrm{~m} / \mathrm{s}$
(b) Since the horizontal velocity is known and the horizontal distance is known, the time of flight can be found from the constant velocity equation for horizontal motion.

$$
\Delta x=v_{x} t \quad \rightarrow \quad t=\frac{\Delta x}{v_{x}}=\frac{55 \mathrm{~m}}{13.38 \mathrm{~m} / \mathrm{s}}=4.111 \mathrm{~s}
$$

With that time of flight, calculate the vertical position of the ball using Eq. 2-12b.

$$
\begin{aligned}
y & =y_{0}+v_{y 0} t+\frac{1}{2} a_{y} t^{2}=(12.04 \mathrm{~m} / \mathrm{s})(4.111 \mathrm{~s})+\frac{1}{2}\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(4.111 \mathrm{~s})^{2} \\
& =-33.3=-33 \mathrm{~m}
\end{aligned}
$$

So the ball will strike 33 m below the top of the building.
98. Since the ball is being caught at the same height from which it was struck, use the range formula from Example 3-10 to find the horizontal distance the ball travels.

$$
R=\frac{v_{0}^{2} \sin 2 \theta_{0}}{g}=\frac{(28 \mathrm{~m} / \mathrm{s})^{2} \sin \left(2 \times 55^{\circ}\right)}{9.80 \mathrm{~m} / \mathrm{s}^{2}}=75.175 \mathrm{~m}
$$



The angle $\theta$ at which the outfielder should run is found from the law of sines.

$$
\frac{\sin 22^{\circ}}{32.048 \mathrm{~m}}=\frac{\sin \theta}{75.175 \mathrm{~m}} \quad \rightarrow \quad \theta=\sin ^{-1}\left(\frac{75.175}{32.048} \sin 22^{\circ}\right)=61.49^{\circ} \text { or } 118.51^{\circ}
$$

Since $75.175^{2}<85^{2}+32.048^{2}$, the angle must be acute, so we choose $\theta=61.49^{\circ}$.
Now assume that the outfielder's time for running is the same as the time of flight of the ball. The time of flight of the ball is found from the horizontal motion of the ball at constant velocity.

$$
R=v_{x} t=v_{0} \cos \theta_{0} t \quad \rightarrow \quad t=\frac{R}{v_{0} \cos \theta_{0}}=\frac{75.175 \mathrm{~m}}{(28 \mathrm{~m} / \mathrm{s}) \cos 55^{\circ}}=4.681 \mathrm{~s}
$$

Thus the average velocity of the outfielder must be $v_{\text {avg }}=\frac{\Delta d}{t}=\frac{32.048 \mathrm{~m}}{4.681 \mathrm{~s}}=6.8 \mathrm{~m} / \mathrm{s}$ at an angle of
$61^{\circ}$ relative to the outfielder's line of sight to home plate.
99. (a) To determine the best-fit straight line, the data was plotted in Excel and a linear trendline was added, giving the equation $x=(3.03 t-0.0265) \mathrm{m}$. The initial speed of the ball is the $x$ component of the velocity, which from the equation has the value of $3.03 \mathrm{~m} / \mathrm{s}$. The graph is below. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH03.XLS," on tab "Problem 3.99a."

(b) To determine the best-fit quadratic equation, the data was plotted in Excel and a quadratic trendline was added, giving the equation $y=\left(0.158-0.855 t+6.09 t^{2}\right) \mathrm{m}$. Since the quadratic term in this relationship is $\frac{1}{2} a t^{2}$, we have the acceleration as $12.2 \mathrm{~m} / \mathrm{s}^{2}$. The graph is below. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH03.XLS," on tab "Problem 3.99b."

100. Use the vertical motion to determine the time of flight. Let the ground be the $y=0$ level, and choose upwards to be the positive $y$-direction. Use Eq. 2-12b.

$$
\begin{aligned}
& y=y_{0}+v_{0 y} t+\frac{1}{2} a_{y} t^{2} \rightarrow 0=h+v_{0}\left(\sin \theta_{0}\right) t-\frac{1}{2} g t^{2} \rightarrow \frac{1}{2} g t^{2}-v_{0}\left(\sin \theta_{0}\right) t-h=0 \\
& t=\frac{v_{0} \sin \theta_{0} \pm \sqrt{v_{0}^{2} \sin ^{2} \theta_{0}-4\left(\frac{1}{2} g\right)(-h)}}{2\left(\frac{1}{2} g\right)}=\frac{v_{0} \sin \theta_{0} \pm \sqrt{v_{0}^{2} \sin ^{2} \theta_{0}+2 g h}}{g}
\end{aligned}
$$

To get a positive value for the time of flight, the positive sign must be taken.

$$
t=\frac{v_{0} \sin \theta_{0}+\sqrt{v_{0}^{2} \sin ^{2} \theta_{0}+2 g h}}{g}
$$

To find the horizontal range, multiply the horizontal velocity by the time of flight.

$$
\begin{aligned}
& R=v_{x} t=v_{0} \cos \theta_{0}\left[\frac{v_{0} \sin \theta_{0}+\sqrt{v_{0}^{2} \sin ^{2} \theta_{0}+2 g h}}{g}\right]=\frac{v_{0}^{2} \cos \theta_{0} \sin \theta_{0}}{g}\left[1+\sqrt{1+\frac{2 g h}{v_{0}^{2} \sin ^{2} \theta_{0}}}\right] \\
& R=\frac{v_{0}^{2} \sin 2 \theta_{0}}{2 g}\left[1+\sqrt{1+\frac{2 g h}{v_{0}^{2} \sin ^{2} \theta_{0}}}\right]
\end{aligned}
$$

As a check, if $h$ is set to 0 in the above equation, we get $R=\frac{\nu_{0}^{2} \sin 2 \theta_{0}}{g}$, the level horizontal range formula.
With the values given in the problem of $v_{0}=13.5 \mathrm{~m} / \mathrm{s}, h=2.1 \mathrm{~m}$, and $g=9.80 \mathrm{~m} / \mathrm{s}^{2}$, the following relationship is obtained.

$$
\begin{aligned}
R & =\frac{v_{0}^{2} \sin 2 \theta_{0}}{2 g}\left[1+\sqrt{1+\frac{2 g h}{v_{0}^{2} \sin ^{2} \theta_{0}}}\right]=\frac{(13.5)^{2} \sin 2 \theta_{0}}{2(9.80)}\left[1+\sqrt{1+\frac{2(9.80)(2.1)}{(13.5)^{2} \sin ^{2} \theta_{0}}}\right] \\
& =9.30 \sin 2 \theta_{0}\left[1+\sqrt{1+\frac{0.226}{\sin ^{2} \theta_{0}}}\right]
\end{aligned}
$$

Here is a plot of that relationship. The maximum is at approximately $42^{\circ}$. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH03.XLS," on tab "Problem 3.100."
As a further investigation, let us find $\frac{d R}{d \theta_{0}}$, set it equal to 0 , and
 solve for the angle.

$$
\begin{aligned}
& R=\frac{v_{0}^{2} \sin 2 \theta_{0}}{2 g}\left[1+\sqrt{1+\frac{2 g h}{v_{0}^{2} \sin ^{2} \theta_{0}}}\right] \\
& \frac{d R}{d \theta_{0}}=\frac{2 v_{0}^{2} \cos 2 \theta_{0}}{2 g}\left[1+\sqrt{1+\frac{2 g h}{v_{0}^{2} \sin ^{2} \theta_{0}}}\right]+\frac{v_{0}^{2} \sin 2 \theta_{0}}{2 g}\left[\frac{1}{2}\left(1+\frac{2 g h}{v_{0}^{2} \sin ^{2} \theta_{0}}\right)^{-1 / 2}\left(\frac{(-2) 2 g h \cos \theta_{0}}{v_{0}^{2} \sin ^{3} \theta_{0}}\right)\right] \\
& =\frac{v_{0}^{2}}{2 g}\left\{2 \cos 2 \theta_{0}\left[1+\sqrt{1+\frac{2 g h}{v_{0}^{2} \sin ^{2} \theta_{0}}}\right]-\sin 2 \theta_{0}\left[\left(1+\frac{2 g h}{v_{0}^{2} \sin ^{2} \theta_{0}}\right)^{-1 / 2}\left(\frac{2 g h \cos \theta_{0}}{v_{0}^{2} \sin ^{3} \theta_{0}}\right)\right]\right\}=0 \\
& 2 \cos 2 \theta_{0}\left[1+\sqrt{1+\frac{2 g h}{v_{0}^{2} \sin ^{2} \theta_{0}}}\right]=\sin 2 \theta_{0}\left[\left(1+\frac{2 g h}{v_{0}^{2} \sin ^{2} \theta_{0}}\right)^{-1 / 2}\left(\frac{2 g h \cos \theta_{0}}{v_{0}^{2} \sin ^{3} \theta_{0}}\right)\right]
\end{aligned}
$$

Calculate the two sides of the above equation and find where they are equal. This again happens at about $42.1^{\circ}$.

## CHAPTER 4: Dynamics: Newton's Laws of Motion

## Responses to Questions

When you give the wagon a sharp pull forward, the force of friction between the wagon and the child acts on the child to move her forward. But the force of friction acts at the contact point between the child and the wagon - either the feet, if the child is standing, or her bottom, if sitting. In either case, the lower part of the child begins to move forward, while the upper part, following Newton's first law (the law of inertia), remains almost stationary, making it seem as if the child falls backward.
2. (a) Andrea, standing on the ground beside the truck, will see the box remain motionless while the truck accelerates out from under it. Since there is no friction, there is no net force on the box and it will not speed up.
(b) Jim, riding on the truck, will see the box appear to accelerate backwards with respect to his frame of reference, which is not inertial. (Jim better hold on, though; if the truck bed is frictionless, he too will slide off if he is just standing!)
3. If the acceleration of an object is zero, the vector sum of the forces acting on the object is zero (Newton's second law), so there can be forces on an object that has no acceleration. For example, a book resting on a table is acted on by gravity and the normal force, but it has zero acceleration, because the forces are equal in magnitude and opposite in direction.
4. Yes, the net force can be zero on a moving object. If the net force is zero, then the object's acceleration is zero, but its velocity is not necessarily zero. [Instead of classifying objects as "moving" and "not moving," Newtonian dynamics classifies them as "accelerating" and "not accelerating." Both zero velocity and constant velocity fall in the "not accelerating" category.]
5. If only one force acts on an object, the object cannot have zero acceleration (Newton's second law). It is possible for the object to have zero velocity, but only for an instant. For example (if we neglect air resistance), a ball thrown up into the air has only the force of gravity acting on it. Its speed will decrease while it travels upward, stop, then begin to fall back to the ground. At the instant the ball is at its highest point, its velocity is zero.
6. (a) Yes, there must be a force on the golf ball (Newton's second law) to make it accelerate upward.
(b) The pavement exerts the force (just like a "normal force").

As you take a step on the log, your foot exerts a force on the $\log$ in the direction opposite to the direction in which you want to move, which pushes the log "backwards." (The log exerts an equal and opposite force forward on you, by Newton's third law.) If the log had been on the ground, friction between the ground and the log would have kept the log from moving. However, the log is floating in water, which offers little resistance to the movement of the log as you push it backwards.
8. When you kick a heavy desk or a wall, your foot exerts a force on the desk or wall. The desk or wall exerts a force equal in magnitude on your foot (Newton's third law). Ouch!
9. (a) The force that causes you to stop quickly is the force of friction between your shoes and the ground (plus the forces your muscles exert in moving your legs more slowly and bracing yourself).
(b) If we assume the top speed of a person to be around $6 \mathrm{~m} / \mathrm{s}$ (equivalent to about $12 \mathrm{mi} / \mathrm{h}$, or a 5minute mile), and if we assume that it take 2 s to stop, then the maximum rate of deceleration is about $3 \mathrm{~m} / \mathrm{s}^{2}$.
10. (a) When you first start riding a bicycle you need to exert a strong force to accelerate the bike and yourself. Once you are moving at a constant speed, you only need to exert a force to equal the opposite force of friction and air resistance.
(b) When the bike is moving at a constant speed, the net force on it is zero. Since friction and air resistance are present, you would slow down if you didn't pedal to keep the net force on the bike (and you) equal to zero.
11. The father and daughter will each have the same magnitude force acting on them as they push each other away (Newton's third law). If we assume the young daughter has less mass than the father, her acceleration should be greater $(a=F / m)$. Both forces, and therefore both accelerations, act over the same time interval (while the father and daughter are in contact), so the daughter's final speed will be greater than her dad's.
12. The carton would collapse (a). When you jump, you accelerate upward, so there must be a net upward force on you. This net upward force can only come from the normal force exerted by the carton on you and must be greater than your weight. How can you increase the normal force of a surface on you? According to Newton's third law, the carton pushes up on you just as hard as you push down on it. That means you push down with a force greater than your weight in order to accelerate upwards. If the carton can just barely support you, it will collapse when you exert this extra force.
13. If a person gives a sharp pull on the dangling thread, the thread is likely to break below the stone. In the short time interval of a sharp pull, the stone barely begins to accelerate because of its great mass (inertia), and so does not transmit the force to the upper string quickly. The stone will not move much before the lower thread breaks. If a person gives a slow and steady pull on the thread, the thread is most likely to break above the stone because the tension in the upper thread is the applied force plus the weight of the stone. Since the tension in the upper thread is greater, it is likely to break first.
14. The force of gravity on the $2-\mathrm{kg}$ rock is twice as great as the force on the $1-\mathrm{kg}$ rock, but the $2-\mathrm{kg}$ rock has twice the mass (and twice the inertia) of the $1-\mathrm{kg}$ rock. Acceleration is the ratio of force to mass ( $a=F / m$, Newton's second law), so the two rocks have the same acceleration.
15. A spring responds to force, and will correctly give the force or weight in pounds, even on the Moon. Objects weigh much less on the Moon, so a spring calibrated in kilograms will give incorrect results (by a factor of 6 or so).
16. The acceleration of the box will (c) decrease. Newton's second law is a vector equation. When you pull the box at an angle $\theta$, only the horizontal component of the force, $F \cos \theta$, will accelerate the box horizontally across the floor.
17. The Earth actually does move as seen from an inertial reference frame. But the mass of the Earth is so great, the acceleration is undetectable (Newton's second law).
18. Because the acceleration due to gravity on the Moon is less than it is on the Earth, an object with a mass of 10 kg will weigh less on the Moon than it does on the Earth. Therefore, it will be easier to lift on the Moon. (When you lift something, you exert a force to oppose its weight.) However, when throwing the object horizontally, the force needed to accelerate it to the desired horizontal speed is proportional to the object's mass, $F=m a$. Therefore, you would need to exert the same force to throw the $2-\mathrm{kg}$ object on the Moon as you would on Earth.
19. A weight of 1 N corresponds to 0.225 lb . That's about the weight of $(a)$ an apple.
20. Newton's third law involves forces on different objects, in this case, on the two different teams. Whether or not a team moves and in what direction is determined by Newton's second law and the net force on the team. The net force on one team is the vector sum of the pull of the other team and the friction force exerted by the ground on the team. The winning team is the one that pushes hardest against the ground (and so has a greater force on them exerted by the ground).
21. When you stand still on the ground, two forces act on you: your weight downward, and the normal force exerted upward by the ground. You are at rest, so Newton's second law tells you that the normal force must equal your weight, $m g$. You don't rise up off the ground because the force of gravity acts downward, opposing the normal force.
22. The victim's head is not really thrown backwards during the car crash. If the victim's car was initially at rest, or even moving forward, the impact from the rear suddenly pushes the car, the seat, and the person's body forward. The head, being attached by the somewhat flexible neck to the body, can momentarily remain where it was (inertia, Newton's first law), thus lagging behind the body.
23. (a) The reaction force has a magnitude of 40 N .
(b) It points downward.
(c) It is exerted on Mary's hands and arms.
(d) It is exerted by the bag of groceries.
24. No. In order to hold the backpack up, the rope must exert a vertical force equal to the backpack's weight, so that the net vertical force on the backpack is zero. The force, $F$, exerted by the rope on each side of the pack is always along the length of the rope. The vertical component of this force is $F \sin \theta$, where $\theta$ is the angle the rope makes with the horizontal. The higher the pack goes, the smaller $\theta$ becomes and the larger $F$ must be to hold the pack up there. No matter how hard you pull, the rope can never be horizontal because it must exert an upward (vertical) component of force to balance the pack's weight. See also Example 4-16 and Figure 4-26.

## Solutions to Problems

1. Use Newton's second law to calculate the force.

$$
\sum F=m a=(55 \mathrm{~kg})\left(1.4 \mathrm{~m} / \mathrm{s}^{2}\right)=77 \mathrm{~N}
$$

2. Use Newton's second law to calculate the mass.

$$
\sum F=m a \rightarrow m=\frac{\sum F}{a}=\frac{265 \mathrm{~N}}{2.30 \mathrm{~m} / \mathrm{s}^{2}}=115 \mathrm{~kg}
$$

3. In all cases, $W=m g$, where $g$ changes with location.
(a) $W_{\text {Earth }}=m g_{\text {Earth }}=(68 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=670 \mathrm{~N}$
(b) $W_{\text {Moon }}=m g_{\text {Moon }}=(68 \mathrm{~kg})\left(1.7 \mathrm{~m} / \mathrm{s}^{2}\right)=120 \mathrm{~N}$
(c) $W_{\text {Mars }}=m g_{\text {Mars }}=(68 \mathrm{~kg})\left(3.7 \mathrm{~m} / \mathrm{s}^{2}\right)=250 \mathrm{~N}$
(d) $W_{\text {Space }}=m g_{\text {Space }}=(68 \mathrm{~kg})\left(0 \mathrm{~m} / \mathrm{s}^{2}\right)=0 \mathrm{~N}$
4. Use Newton's second law to calculate the tension.

$$
\sum F=F_{\mathrm{T}}=m a=(1210 \mathrm{~kg})\left(1.20 \mathrm{~m} / \mathrm{s}^{2}\right)=1452 \mathrm{~N} \approx 1.45 \times 10^{3} \mathrm{~N}
$$

5. Find the average acceleration from Eq. 2-12c, and then find the force needed from Newton's second law. We assume the train is moving in the positive direction.

$$
\begin{aligned}
& v=0 \quad v_{0}=(120 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)=33.33 \mathrm{~m} / \mathrm{s} \quad a_{\text {avg }}=\frac{v^{2}-v_{0}^{2}}{2\left(x-x_{0}\right)} \\
& F_{\text {avg }}=m a_{\text {avg }}=m \frac{v^{2}-v_{0}^{2}}{2\left(x-x_{0}\right)}=\left(3.6 \times 10^{5} \mathrm{~kg}\right)\left[\frac{0-(33.33 \mathrm{~m} / \mathrm{s})^{2}}{2(150 \mathrm{~m})}\right]=-1.333 \times 10^{6} \mathrm{~N} \approx-1.3 \times 10^{6} \mathrm{~N}
\end{aligned}
$$

The negative sign indicates the direction of the force, in the opposite direction to the initial velocity. We compare the magnitude of this force to the weight of the train.

$$
\frac{F_{\text {avg }}}{m g}=\frac{1.333 \times 10^{6} \mathrm{~N}}{\left(3.6 \times 10^{5} \mathrm{~kg}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=0.3886
$$

Thus the force is $39 \%$ of the weight of the train.
By Newton's third law, the train exerts the same magnitude of force on Superman that Superman exerts on the train, but in the opposite direction. So the train exerts a force of $1.3 \times 10^{6} \mathrm{~N}$ in the forward direction on Superman.
6. Find the average acceleration from Eq. 2-5. The average force on the car is found from Newton's second law.

$$
\begin{aligned}
& v=0 \quad v_{0}=(95 \mathrm{~km} / \mathrm{h})\left(\frac{0.278 \mathrm{~m} / \mathrm{s}}{1 \mathrm{~km} / \mathrm{h}}\right)=26.4 \mathrm{~m} / \mathrm{s} \quad a_{a v g}=\frac{v-v_{0}}{t}=\frac{0-26.4 \mathrm{~m} / \mathrm{s}}{8.0 \mathrm{~s}}=-3.30 \mathrm{~m} / \mathrm{s}^{2} \\
& F_{a v g}=m a_{a v g}=(950 \mathrm{~kg})\left(-3.30 \mathrm{~m} / \mathrm{s}^{2}\right)=-3.1 \times 10^{3} \mathrm{~N}
\end{aligned}
$$

The negative sign indicates the direction of the force, in the opposite direction to the initial velocity.
7. Find the average acceleration from Eq. 2-12c, and then find the force needed from Newton's second law.

$$
\begin{aligned}
& a_{\text {avg }}=\frac{v^{2}-v_{0}^{2}}{2\left(x-x_{0}\right)} \rightarrow \\
& F_{\text {avg }}=m a_{\text {avg }}=m \frac{v^{2}-v_{0}^{2}}{2\left(x-x_{0}\right)}=(7.0 \mathrm{~kg})\left[\frac{(13 \mathrm{~m} / \mathrm{s})^{2}-0}{2(2.8 \mathrm{~m})}\right]=211.25 \mathrm{~N} \approx 210 \mathrm{~N}
\end{aligned}
$$

8. The problem asks for the average force on the glove, which in a direct calculation would require knowledge about the mass of the glove and the acceleration of the glove. But no information about the glove is given. By Newton's third law, the force exerted by the ball on the glove is equal and opposite to the force exerted by the glove on the ball. So calculate the average force on the ball, and then take the opposite of that result to find the average force on the glove. The average force on the ball is its mass times its average acceleration. Use Eq. 2-12c to find the acceleration of the ball, with $v=0, v_{0}=35.0 \mathrm{~m} / \mathrm{s}$, and $x-x_{0}=0.110 \mathrm{~m}$. The initial direction of the ball is the positive direction.

$$
a_{\text {avg }}=\frac{v^{2}-v_{0}^{2}}{2\left(x-x_{0}\right)}=\frac{0-(35.0 \mathrm{~m} / \mathrm{s})^{2}}{2(0.110 \mathrm{~m})}=-5568 \mathrm{~m} / \mathrm{s}^{2}
$$

$$
F_{\text {avg }}=m a_{\text {avg }}=(0.140 \mathrm{~kg})\left(-5568 \mathrm{~m} / \mathrm{s}^{2}\right)=-7.80 \times 10^{2} \mathrm{~N}
$$

Thus the average force on the glove was 780 N , in the direction of the initial velocity of the ball.
9. We assume that the fish line is pulling vertically on the fish, and that the fish is not jerking the line. A free-body diagram for the fish is shown. Write Newton's second law for the fish in the vertical direction, assuming that up is positive. The tension is at its maximum.

$$
\begin{aligned}
& \sum F=F_{\mathrm{T}}-m g=m a \rightarrow F_{\mathrm{T}}=m(g+a) \rightarrow \\
& m=\frac{F_{\mathrm{T}}}{g+a}=\frac{18 \mathrm{~N}}{9.80 \mathrm{~m} / \mathrm{s}^{2}+2.5 \mathrm{~m} / \mathrm{s}^{2}}=1.5 \mathrm{~kg}
\end{aligned}
$$



Thus a mass of 1.5 kg is the maximum that the fish line will support with the given acceleration. Since the line broke, the fish's mass is given by $m>1.5 \mathrm{~kg}$ (about 3 lbs ).
10. (a) The 20.0 kg box resting on the table has the free-body diagram shown. Its weight is $m g=(20.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=196 \mathrm{~N}$. Since the box is at rest, the net force on the box must be 0 , and so the normal force must also be 196 N .
(b) Free-body diagrams are shown for both boxes. $\quad \overrightarrow{\mathbf{F}}_{12}$ is the force on box 1 (the
 top box) due to box 2 (the bottom box), and is the normal force on box $1 . \overrightarrow{\mathbf{F}}_{21}$ is the force on box 2 due to box 1 , and has the same magnitude as $\overrightarrow{\mathbf{F}}_{12}$ by Newton's third law. $\overrightarrow{\mathbf{F}}_{\mathrm{N} 2}$ is the force of the table on box 2. That is the normal force on box 2. Since both boxes are at rest, the net force on each box must
 be 0 . Write Newton's second law in the vertical direction for each box, taking the upward direction to be positive.

$$
\begin{aligned}
& \sum F_{1}=F_{\mathrm{N} 1}-m_{1} g=0 \\
& F_{\mathrm{N} 1}=m_{1} g=(10.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=98.0 \mathrm{~N}=F_{12}=F_{21} \\
& \sum F_{2}=F_{\mathrm{N} 2}-F_{21}-m_{2} g=0 \\
& F_{\mathrm{N} 2}=F_{21}+m_{2} g=98.0 \mathrm{~N}+(20.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=294 \mathrm{~N}
\end{aligned}
$$


11. The average force on the pellet is its mass times its average acceleration. The average acceleration is found from Eq. 2-12c. For the pellet, $v_{0}=0, v=125 \mathrm{~m} / \mathrm{s}$, and $x-x_{0}=0.800 \mathrm{~m}$.

$$
\begin{aligned}
& a_{a v g}=\frac{v^{2}-v_{0}^{2}}{2\left(x-x_{0}\right)}=\frac{(125 \mathrm{~m} / \mathrm{s})^{2}-0}{2(0.800 \mathrm{~m})}=9766 \mathrm{~m} / \mathrm{s}^{2} \\
& F_{\text {avg }}=m a_{\text {avg }}=\left(9.20 \times 10^{-3} \mathrm{~kg}\right)\left(9766 \mathrm{~m} / \mathrm{s}^{2}\right)=89.8 \mathrm{~N}
\end{aligned}
$$

12. Choose up to be the positive direction. Write Newton's second law for the vertical direction, and solve for the tension force.

$$
\begin{aligned}
& \sum F=F_{\mathrm{T}}-m g=m a \rightarrow F_{\mathrm{T}}=m(g+a) \\
& F_{\mathrm{T}}=(1200 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}+0.70 \mathrm{~m} / \mathrm{s}^{2}\right)=1.3 \times 10^{4} \mathrm{~N}
\end{aligned}
$$


13. Choose up to be the positive direction. Write Newton's second law for the vertical direction, and solve for the acceleration.

$$
\begin{aligned}
& \sum F=F_{\mathrm{T}}-m g=m a \\
& a=\frac{F_{\mathrm{T}}-m g}{m}=\frac{163 \mathrm{~N}-(14.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{14.0 \mathrm{~kg}}=1.8 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$



Since the acceleration is positive, the bucket has an upward acceleration.
14. Use Eq. 2-12b with $v_{0}=0$ to find the acceleration.

$$
x-x_{0}=v_{0} t+\frac{1}{2} a t^{2} \rightarrow a=\frac{2\left(x-x_{0}\right)}{t^{2}}=\frac{2(402 \mathrm{~m})}{(6.40 \mathrm{~s})^{2}}=19.63 \mathrm{~m} / \mathrm{s}^{2}\left(\frac{1 " g "}{9.80 \mathrm{~m} / \mathrm{s}^{2}}\right)=2.00 \mathrm{~g}^{\prime} \mathrm{s}
$$

The accelerating force is found by Newton's second law.

$$
F=m a=(535 \mathrm{~kg})\left(19.63 \mathrm{~m} / \mathrm{s}^{2}\right)=1.05 \times 10^{4} \mathrm{~N}
$$

15. If the thief were to hang motionless on the sheets, or descend at a constant speed, the sheets would not support him, because they would have to support the full 75 kg . But if he descends with an acceleration, the sheets will not have to support the total mass. A freebody diagram of the thief in descent is shown. If the sheets can support a mass of 58 kg , then the tension force that the sheets can exert is $F_{\mathrm{T}}=(58 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=568 \mathrm{~N}$.
Assume that is the tension in the sheets. Then write Newton's second law for the thief, taking the upward direction to be positive.


$$
\sum F=F_{\mathrm{T}}-m g=m a \rightarrow a=\frac{F_{\mathrm{T}}-m g}{m}=\frac{568 \mathrm{~N}-(75 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{75 \mathrm{~kg}}=-2.2 \mathrm{~m} / \mathrm{s}^{2}
$$

The negative sign shows that the acceleration is downward.
If the thief descends with an acceleration of $2.2 \mathrm{~m} / \mathrm{s}^{2}$ or greater, the sheets will support his descent.
16. In both cases, a free-body diagram for the elevator would look like the adjacent diagram. Choose up to be the positive direction. To find the MAXIMUM tension, assume that the acceleration is up. Write Newton's second law for the elevator.

$$
\begin{aligned}
& \sum F=m a=F_{\mathrm{T}}-m g \rightarrow \\
& F_{\mathrm{T}}=m a+m g=m(a+g)=m(0.0680 g+g)=(4850 \mathrm{~kg})(1.0680)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \\
&=5.08 \times 10^{4} \mathrm{~N}
\end{aligned}
$$



To find the MINIMUM tension, assume that the acceleration is down. Then Newton's second law for the elevator becomes the following.

$$
\begin{aligned}
\sum F=m a=F_{\mathrm{T}}-m g \rightarrow F_{\mathrm{T}} & =m a+m g=m(a+g)=m(-0.0680 g+g) \\
& =(4850 \mathrm{~kg})(0.9320)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=4.43 \times 10^{4} \mathrm{~N}
\end{aligned}
$$

17. Use Eq. 2-12c to find the acceleration. The starting speed is $35 \mathrm{~km} / \mathrm{h}\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)=9.72 \mathrm{~m} / \mathrm{s}$.

$$
v^{2}=v_{0}^{2}+2 a\left(x-x_{0}\right) \rightarrow a=\frac{v^{2}-v_{0}^{2}}{2\left(x-x_{0}\right)}=\frac{0-(9.72 \mathrm{~m} / \mathrm{s})^{2}}{2(0.017 \mathrm{~m})}=-2779 \mathrm{~m} / \mathrm{s}^{2} \approx-2800 \mathrm{~m} / \mathrm{s}^{2}
$$

$$
2779 \mathrm{~m} / \mathrm{s}^{2}\left(\frac{1 g}{9.80 \mathrm{~m} / \mathrm{s}^{2}}\right)=284 g^{\prime} \mathrm{s} \approx 280 g^{\prime} \mathrm{s}
$$

The acceleration is negative because the car is slowing down. The required force is found by Newton's second law.

$$
F=m a=(68 \mathrm{~kg})\left(2779 \mathrm{~m} / \mathrm{s}^{2}\right)=1.9 \times 10^{5} \mathrm{~N}
$$

This huge acceleration would not be possible unless the car hit some very heavy, stable object.
18. There will be two forces on the person - their weight, and the normal force of the scales pushing up on the person. A free-body diagram for the person is shown. Choose up to be the positive direction, and use Newton's second law to find the acceleration.

$$
\begin{aligned}
& \sum F=F_{\mathrm{N}}-m g=m a \rightarrow 0.75 m g-m g=m a \rightarrow \\
& a=-0.25 g=-0.25\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)=-2.5 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$



Due to the sign of the result, the direction of the acceleration is down. Thus the elevator must have started to move down since it had been motionless.
19. (a) To calculate the time to accelerate from rest, use Eq. 2-12a.

$$
v=v_{0}+a t \rightarrow t=\frac{v-v_{0}}{a}=\frac{9.0 \mathrm{~m} / \mathrm{s}-0}{1.2 \mathrm{~m} / \mathrm{s}^{2}}=7.5 \mathrm{~s}
$$

The distance traveled during this acceleration is found from Eq. 2-12b.

$$
x-x_{0}=v_{0} t+\frac{1}{2} a t^{2}=\frac{1}{2}\left(1.2 \mathrm{~m} / \mathrm{s}^{2}\right)(7.5 \mathrm{~s})^{2}=33.75 \mathrm{~m}
$$

To calculate the time to decelerate to rest, use Eq. 2-12a.


$$
v=v_{0}+a t \rightarrow t=\frac{v-v_{0}}{a}=\frac{0-9.0 \mathrm{~m} / \mathrm{s}}{-1.2 \mathrm{~m} / \mathrm{s}^{2}}=7.5 \mathrm{~s}
$$

The distance traveled during this deceleration is found from Eq. 2-12b.

$$
x-x_{0}=v_{0} t+\frac{1}{2} a t^{2}=(9.0 \mathrm{~m} / \mathrm{s})(7.5 \mathrm{~s})+\frac{1}{2}\left(-1.2 \mathrm{~m} / \mathrm{s}^{2}\right)(7.5 \mathrm{~s})^{2}=33.75 \mathrm{~m}
$$

To distance traveled at constant velocity is $180 \mathrm{~m}-2(33.75 \mathrm{~m})=112.5 \mathrm{~m}$.
To calculate the time spent at constant velocity, use Eq. 2-8.

$$
x=x_{0}+\bar{v} t \rightarrow t=\frac{x-x_{0}}{\bar{v}}=\frac{112.5 \mathrm{~m} / \mathrm{s}}{9.0 \mathrm{~m} / \mathrm{s}}=12.5 \mathrm{~s} \approx 13 \mathrm{~s}
$$

Thus the times for each stage are:

## Accelerating: 7.5 s Constant Velocity: 13 s Decelerating: 7.5 s

(b) The normal force when at rest is $m g$. From the free-body diagram, if up is the positive direction, we have that $F_{\mathrm{N}}-m g=m a$. Thus the change in normal force is the difference in the normal force and the weight of the person, or $m a$.

Accelerating: $\frac{\Delta F_{N}}{F_{N}}=\frac{m a}{m g}=\frac{a}{g}=\frac{1.2 \mathrm{~m} / \mathrm{s}^{2}}{9.80 \mathrm{~m} / \mathrm{s}^{2}} \times 100=12 \%$
Constant velocity: $\frac{\Delta F_{N}}{F_{N}}=\frac{m a}{m g}=\frac{a}{g}=\frac{0}{9.80 \mathrm{~m} / \mathrm{s}^{2}} \times 100=0 \%$
Decelerating: $\frac{\Delta F_{N}}{F_{N}}=\frac{m a}{m g}=\frac{a}{g}=\frac{-1.2 \mathrm{~m} / \mathrm{s}^{2}}{9.80 \mathrm{~m} / \mathrm{s}^{2}} \times 100=-12 \%$
(c) The normal force is not equal to the weight during the accelerating and deceleration phases.

$$
\frac{7.5 \mathrm{~s}+7.5 \mathrm{~s}}{7.5 \mathrm{~s}+12.5 \mathrm{~s}+7.5 \mathrm{~s}}=55 \%
$$

20. The ratio of accelerations is the same as the ratio of the force.

$$
\begin{aligned}
\begin{aligned}
\frac{a_{\text {optics }}}{g} & =\frac{m a_{\text {optics }}}{m g}=\frac{F_{\text {optics }}}{m g}=\frac{F_{\text {optics }}}{\rho\left(\frac{4}{3} \pi r^{3}\right) g} \\
& =\frac{10 \times 10^{-12} \mathrm{~N}}{\left(\frac{1.0 \mathrm{~g}}{1.0 \mathrm{~cm}^{3}} \frac{1 \mathrm{~kg}}{1000 \mathrm{~g}} \frac{10^{6} \mathrm{~cm}^{3}}{1 \mathrm{~m}^{3}}\right) \frac{4}{3} \pi\left(.5 \times 10^{-6} \mathrm{~m}\right)^{3}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=1949 \rightarrow \\
a & \approx 2000 \mathrm{~g}^{\prime \mathrm{s}}
\end{aligned}
\end{aligned}
$$

21. (a) Since the rocket is exerting a downward force on the gases, the gases will exert an upward force on the rocket, typically called the thrust. The free-body diagram for the rocket shows two forces - the thrust and the weight. Newton's second law can be used to find the acceleration of the rocket.

$$
\begin{aligned}
& \sum F=F_{\mathrm{T}}-m g=m a \rightarrow \\
& a=\frac{F_{\mathrm{T}}-m g}{m}=\frac{3.55 \times 10^{7} \mathrm{~N}-\left(2.75 \times 10^{6} \mathrm{~kg}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{\left(2.75 \times 10^{6} \mathrm{~kg}\right)}=3.109 \mathrm{~m} / \mathrm{s}^{2} \approx 3.1 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

(b) The velocity can be found from Eq. 2-12a.

$$
v=v_{0}+a t=0+\left(3.109 \mathrm{~m} / \mathrm{s}^{2}\right)(8.0 \mathrm{~s})=24.872 \mathrm{~m} / \mathrm{s} \approx 25 \mathrm{~m} / \mathrm{s}
$$

(c) The time to reach a displacement of 9500 m can be found from Eq. 2-12b.

$$
x-x_{0}=v_{0} t+\frac{1}{2} a t^{2} \rightarrow t=\sqrt{\frac{2\left(x-x_{0}\right)}{a}}=\sqrt{\frac{2(9500 \mathrm{~m})}{\left(3.109 \mathrm{~m} / \mathrm{s}^{2}\right)}}=78 \mathrm{~s}
$$


22. (a) There will be two forces on the skydivers - their combined weight, and the upward force of air resistance, $\overrightarrow{\mathbf{F}}_{\mathrm{A}}$. Choose up to be the positive direction. Write Newton's second law for the skydivers.

$$
\begin{aligned}
& \sum F=F_{\mathrm{A}}-m g=m a \rightarrow 0.25 m g-m g=m a \rightarrow \\
& a=-0.75 g=-0.75\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=-7.35 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

Due to the sign of the result, the direction of the acceleration is down.
(b) If they are descending at constant speed, then the net force on them must
 be zero, and so the force of air resistance must be equal to their weight.

$$
F_{\mathrm{A}}=m g=(132 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=1.29 \times 10^{3} \mathrm{~N}
$$

23. The velocity that the person must have when losing contact with the ground is found from Eq. 2-12c, using the acceleration due to gravity, with the condition that their speed at the top of the jump is 0 . We choose up to be the positive direction.

$$
\begin{aligned}
& v^{2}=v_{0}^{2}+2 a\left(x-x_{0}\right) \rightarrow \\
& v_{0}=\sqrt{v^{2}-2 a\left(x-x_{0}\right)}=\sqrt{0-2\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.80 \mathrm{~m})}=3.960 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$



This velocity is the velocity that the jumper must have as a result of pushing with their legs. Use that velocity with Eq. 2-12c again to find what acceleration the jumper must have during their push on the floor, given that their starting speed is 0 .

$$
v^{2}=v_{0}^{2}+2 a\left(x-x_{0}\right) \rightarrow a=\frac{v^{2}-v_{0}^{2}}{2\left(x-x_{0}\right)}=\frac{(3.960 \mathrm{~m} / \mathrm{s})^{2}-0}{2(0.20 \mathrm{~m})}=39.20 \mathrm{~m} / \mathrm{s}^{2}
$$

Finally, use this acceleration to find the pushing force against the ground.

$$
\begin{aligned}
& \sum F=F_{\mathrm{P}}-m g=m a \\
& F_{\mathrm{P}}=m(g+a)=(68 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}+39.20 \mathrm{~m} / \mathrm{s}^{2}\right)=3300 \mathrm{~N}
\end{aligned}
$$

24. Choose UP to be the positive direction. Write Newton's second law for the elevator.

$$
\begin{aligned}
& \sum F=F_{\mathrm{T}}-m g=m a \rightarrow \\
& a=\frac{F_{\mathrm{T}}-m g}{m}=\frac{21,750 \mathrm{~N}-(2125 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{2125 \mathrm{~kg}}=0.4353 \mathrm{~m} / \mathrm{s}^{2} \approx 0.44 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$


25. We break the race up into two portions. For the acceleration phase, we call the distance $d_{1}$ and the time $t_{1}$. For the constant speed phase, we call the distance $d_{2}$ and the time $t_{2}$. We know that $d_{1}=45 \mathrm{~m}, d_{2}=55 \mathrm{~m}$, and $t_{2}=10.0 \mathrm{~s}-t_{1}$. Eq. $2-12 \mathrm{~b}$ is used for the acceleration phase and Eq. 2-2 is used for the constant speed phase. The speed during the constant speed phase is the final speed of the acceleration phase, found from Eq. 2-12a.

$$
x-x_{0}=v_{0} t+\frac{1}{2} a t^{2} \rightarrow d_{1}=\frac{1}{2} a t_{1}^{2} ; \Delta x=v t \rightarrow d_{2}=v t_{2}=v\left(10.0 \mathrm{~s}-t_{1}\right) ; v=v_{0}+a t_{1}
$$

This set of equations can be solved for the acceleration and the velocity.

$$
\begin{aligned}
& d_{1}=\frac{1}{2} a t_{1}^{2} ; d_{2}=v\left(10.0 \mathrm{~s}-t_{1}\right) ; v=a t_{1} \rightarrow 2 d_{1}=a t_{1}^{2} ; d_{2}=a t_{1}\left(10.0-t_{1}\right) \rightarrow \\
& a=\frac{2 d_{1}}{t_{1}^{2}} ; d_{2}=\frac{2 d_{1}}{t_{1}^{2}} t_{1}\left(10.0-t_{1}\right)=\frac{2 d_{1}}{t_{1}}\left(10.0-t_{1}\right) \rightarrow d_{2} t_{1}=2 d_{1}\left(10.0-t_{1}\right) \rightarrow \\
& t_{1}=\frac{20.0 d_{1}}{\left(d_{2}+2 d_{1}\right)} \rightarrow a=\frac{2 d_{1}}{t_{1}^{2}}=\frac{2 d_{1}}{\left[\frac{20.0 d_{1}}{\left(d_{2}+2 d_{1}\right)}\right]^{2}}=\frac{\left(d_{2}+2 d_{1}\right)^{2}}{\left(200 \mathrm{~s}^{2}\right) d_{1}} \\
& v=a t_{1}=\frac{\left(d_{2}+2 d_{1}\right)^{2}}{200 d_{1}} \frac{20.0 d_{1}}{\left(d_{2}+2 d_{1}\right)}=\frac{\left(d_{2}+2 d_{1}\right)}{10.0 \mathrm{~s}}
\end{aligned}
$$

(a) The horizontal force is the mass of the sprinter times their acceleration.

$$
F=m a=m \frac{\left(d_{2}+2 d_{1}\right)^{2}}{\left(200 \mathrm{~s}^{2}\right) d_{1}}=(66 \mathrm{~kg}) \frac{(145 \mathrm{~m})^{2}}{\left(200 \mathrm{~s}^{2}\right)(45 \mathrm{~m})}=154 \mathrm{~N} \approx 150 \mathrm{~N}
$$

(b) The velocity for the second portion of the race was found above.

$$
v=\frac{\left(d_{2}+2 d_{1}\right)}{10.0 \mathrm{~s}}=\frac{145 \mathrm{~m}}{10.0 \mathrm{~s}}=14.5 \mathrm{~m} / \mathrm{s}
$$

26. (a) Use Eq. 2-12c to find the speed of the person just before striking the ground. Take down to be the positive direction. For the person, $v_{0}=0, y-y_{0}=3.9 \mathrm{~m}$, and $a=9.80 \mathrm{~m} / \mathrm{s}^{2}$.

$$
v^{2}-v_{0}^{2}=2 a\left(y-y_{0}\right) \rightarrow v=\sqrt{2 a\left(y-y_{0}\right)}=\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(3.9 \mathrm{~m})}=8.743=8.7 \mathrm{~m} / \mathrm{s}
$$

(b) For the deceleration, use Eq. 2-12c to find the average deceleration, choosing down to be positive.

$$
\begin{aligned}
& v_{0}=8.743 \mathrm{~m} / \mathrm{s} \quad v=0 \quad y-y_{0}=0.70 \mathrm{~m} \quad v^{2}-v_{0}^{2}=2 a\left(y-y_{0}\right) \quad \rightarrow \\
& a=\frac{-v_{0}^{2}}{2 \Delta y}=\frac{-(8.743 \mathrm{~m} / \mathrm{s})^{2}}{2(0.70 \mathrm{~m})}=-54.6 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

The average force on the torso $\left(F_{\mathrm{T}}\right)$ due to the legs is found from Newton's second law. See the free-body diagram. Down is positive.


$$
\begin{aligned}
& F_{\text {net }}=m g-F_{\mathrm{T}}=m a \rightarrow \\
& F_{\mathrm{T}}=m g-m a=m(g-a)=(42 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}--54.6 \mathrm{~m} / \mathrm{s}^{2}\right)=2.7 \times 10^{3} \mathrm{~N}
\end{aligned}
$$

The force is upward.
27. Free-body diagrams for the box and the weight are shown below. The tension exerts the same magnitude of force on both objects.
(a) If the weight of the hanging weight is less than the weight of the box, the objects will not move, and the tension will be the same as the weight of the hanging weight. The acceleration of the box will also be zero, and so the sum of the forces on it will be zero. For the box,


$$
F_{\mathrm{N}}+F_{\mathrm{T}}-m_{1} g=0 \rightarrow F_{\mathrm{N}}=m_{1} g-F_{\mathrm{T}}=m_{1} g-m_{2} g=77.0 \mathrm{~N}-30.0 \mathrm{~N}=47.0 \mathrm{~N}
$$

(b) The same analysis as for part (a) applies here.

$$
F_{\mathrm{N}}=m_{1} g-m_{2} g=77.0 \mathrm{~N}-60.0 \mathrm{~N}=17.0 \mathrm{~N}
$$

(c) Since the hanging weight has more weight than the box on the table, the box on the table will be lifted up off the table, and normal force of the table on the box will be 0 N .
28. (a) Just before the player leaves the ground, the forces on the player are his weight and the floor pushing up on the player. If the player jumps straight up, then the force of the floor will be straight up - a normal force. See the first diagram. In this case, while touching the floor, $F_{\mathrm{N}}>m g$.
(b) While the player is in the air, the only force on the player is their weight. See the second diagram.

29. (a) Just as the ball is being hit, ignoring air resistance, there are two main forces on the ball: the weight of the ball, and the force of the bat on the ball.
(b) As the ball flies toward the outfield, the only force on it is its weight, if air resistance is ignored.

30. The two forces must be oriented so that the northerly component of the first force is exactly equal to the southerly component of the second force. Thus the second force must act southwesterly. See the diagram.

31. (a) We draw a free-body diagram for the piece of the rope that is directly above the person. That piece of rope should be in equilibrium. The person's weight will be pulling down on that spot, and the rope tension will be pulling away from that spot towards the points of attachment. Write Newton's
 second law for that small piece of the rope.

$$
\begin{aligned}
& \sum F_{y}=2 F_{\mathrm{T}} \sin \theta-m g=0 \rightarrow \theta=\sin ^{-1} \frac{m g}{2 F_{\mathrm{T}}}=\sin ^{-1} \frac{(72.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{2(2900 \mathrm{~N})}=6.988^{\circ} \\
& \tan \theta=\frac{x}{12.5 \mathrm{~m}} \rightarrow x=(12.5 \mathrm{~m}) \tan 6.988^{\circ}=1.532 \mathrm{~m} \approx 1.5 \mathrm{~m}
\end{aligned}
$$

(b) Use the same equation to solve for the tension force with a sag of only $1 / 4$ that found above.

$$
\begin{aligned}
& x=\frac{1}{4}(1.532 \mathrm{~m})=0.383 \mathrm{~m} ; \theta=\tan ^{-1} \frac{0.383 \mathrm{~m}}{12.5 \mathrm{~m}}=1.755^{\circ} \\
& F_{\mathrm{T}}=\frac{m g}{2 \sin \theta}=\frac{(72.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{2\left(\sin 1.755^{\circ}\right)}=11.5 \mathrm{kN}
\end{aligned}
$$

The rope will not break, but it exceeds the recommended tension by a factor of about 4 .
32. The window washer pulls down on the rope with her hands with a tension force $F_{\mathrm{T}}$, so the rope pulls up on her hands with a tension force $F_{\mathrm{T}}$. The tension in the rope is also applied at the other end of the rope, where it attaches to the bucket. Thus there is another force $F_{\mathrm{T}}$ pulling up on the bucket. The bucket-washer combination thus has a net force of $2 F_{\mathrm{T}}$ upwards. See the adjacent free-body diagram, showing only forces on the bucket-washer combination, not forces exerted by the combination (the pull down on the rope by the person) or internal forces (normal force of bucket on person).
(a) Write Newton's second law in the vertical direction, with up as positive. The net force must be zero if the bucket and washer have a constant speed.

$$
\begin{aligned}
& \sum F=F_{\mathrm{T}}+F_{\mathrm{T}}-m g=0 \rightarrow 2 F_{\mathrm{T}}=m g \rightarrow \\
& F_{\mathrm{T}}=\frac{1}{2} m g=\frac{1}{2}(72 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=352.8 \mathrm{~N} \approx 350 \mathrm{~N}
\end{aligned}
$$


(b) Now the force is increased by $15 \%$, so $F_{\mathrm{T}}=358.2 \mathrm{~N}(1.15)=405.72 \mathrm{~N}$. Again write Newton's second law, but with a non-zero acceleration.

$$
\begin{aligned}
& \sum F=F_{\mathrm{T}}+F_{\mathrm{T}}-m g=m a \rightarrow \\
& a=\frac{2 F_{\mathrm{T}}-m g}{m}=\frac{2(405.72 \mathrm{~N})-(72 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{72 \mathrm{~kg}}=1.47 \mathrm{~m} / \mathrm{s}^{2} \approx 1.5 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

33. We draw free-body diagrams for each bucket.
(a) Since the buckets are at rest, their acceleration is 0 . Write Newton's second law for each bucket, calling UP the positive direction.

$$
\begin{aligned}
& \sum F_{1}=F_{\mathrm{T} 1}-m g=0 \rightarrow \\
& F_{\mathrm{T} 1}=m g=(3.2 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=31 \mathrm{~N}
\end{aligned}
$$



Top (\# 2)

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$$
\begin{aligned}
& \sum F_{2}=F_{\mathrm{T} 2}-F_{\mathrm{T} 1}-m g=0 \rightarrow \\
& F_{\mathrm{T} 2}=F_{\mathrm{T} 1}+m g=2 m g=2(3.2 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=63 \mathrm{~N}
\end{aligned}
$$

(b) Now repeat the analysis, but with a non-zero acceleration. The free-body diagrams are unchanged.

$$
\begin{aligned}
& \sum F_{1}=F_{\mathrm{T} 1}-m g=m a \rightarrow \\
& F_{\mathrm{T} 1}=m g+m a=(3.2 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}+1.25 \mathrm{~m} / \mathrm{s}^{2}\right)=35.36 \mathrm{~N} \approx 35 \mathrm{~N} \\
& \sum F_{2}=F_{\mathrm{T} 2}-F_{\mathrm{T} 1}-m g=m a \rightarrow F_{\mathrm{T} 2}=F_{\mathrm{T} 1}+m g+m a=2 F_{\mathrm{T} 1}=71 \mathrm{~N}
\end{aligned}
$$

34. See the free-body diagram for the bottom bucket, and write Newton's second law to find the tension. Take the upward direction as positive.

$$
\begin{aligned}
& \sum F=\underset{\substack{\text { bottom }}}{F_{\text {bucket }} g=m_{\text {bucket }} a \rightarrow \infty, ~} \\
& \underset{\substack{\mathrm{~T} 1 \\
\text { bottom }}}{F_{\text {bucket }}}(g+a)=(3.2 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}+1.25 \mathrm{~m} / \mathrm{s}^{2}\right)=35.36 \mathrm{~N} \approx 35 \mathrm{~N}
\end{aligned}
$$



Next, see the free-body for the rope between the buckets. The mass of the cord is given by

$$
\begin{aligned}
& m_{\text {cord }}=\frac{W_{\text {cord }}}{g} \\
& \quad \sum F=\underset{\substack{\mathrm{T} 1 \\
\text { top }}}{ }-m_{\text {cord }} g-F_{\mathrm{T} 1}^{\text {bottom }}=m_{\text {cord }} a \rightarrow \\
& F_{\substack{\mathrm{T} 1 \\
\text { top }}}=\underset{\substack{\mathrm{T} 1 \\
\text { bottom }}}{F_{\text {cord }}}+m_{\text {cord }}(g+a)=m_{\text {bucket }}(g+a)+m_{\text {cord }}(g+a) \\
& \quad=\left(m_{\text {bucket }}+\frac{W_{\text {cord }}}{g}\right)(g+a)=\left(3.2 \mathrm{~kg}+\frac{2.0 \mathrm{~N}}{9.80 \mathrm{~m} / \mathrm{s}^{2}}\right)\left(11.05 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& \quad=37.615 \mathrm{~N} \approx 38 \mathrm{~N}
\end{aligned}
$$



Note that this is the same as saying that the tension at the top is accelerating the bucket and cord together.

Now use the free-body diagram for the top bucket to find the tension at the bottom of the second cord.

$$
\begin{aligned}
\sum & F=F_{\mathrm{T} 2}-F_{\mathrm{T} 1}-m_{\text {bucket }} g=m_{\text {bucket }} a \rightarrow \\
F_{\mathrm{T} 2} & =F_{\mathrm{T} 1}+m_{\text {bucket }}(g+a)=m_{\text {bucket }}(g+a)+m_{\text {cord }}(g+a)+m_{\text {bucket }}(g+a) \\
& =\left(2 m_{\text {bucket }}+m_{\text {cord }}\right)(g+a)=\left(2 m_{\text {bucket }}+\frac{W_{\text {cord }}}{g}\right)(g+a) \\
& =\left(2(3.2 \mathrm{~kg})+\frac{2.0 \mathrm{~N}}{9.80 \mathrm{~m} / \mathrm{s}^{2}}\right)\left(11.05 \mathrm{~m} / \mathrm{s}^{2}\right)=72.98 \mathrm{~N} \approx 73 \mathrm{~N}
\end{aligned}
$$

Note that this is the same as saying that the tension in the top cord is accelerating the two buckets and the connecting cord.
35. Choose the $y$ direction to be the "forward" direction for the motion of the snowcats, and the $x$ direction to be to the right on the diagram in the textbook. Since the housing unit moves in the forward direction on a straight line, there is no acceleration in the $x$ direction, and so the net force in the $x$ direction must be 0 . Write Newton's second law for the $x$ direction.

$$
\begin{aligned}
& \sum F_{x}=F_{\mathrm{Ax}}+F_{\mathrm{Bx}}=0 \rightarrow-F_{\mathrm{A}} \sin 48^{\circ}+F_{\mathrm{B}} \sin 32^{\circ}=0 \rightarrow \\
& F_{\mathrm{B}}=\frac{F_{\mathrm{A}} \sin 48^{\circ}}{\sin 32^{\circ}}=\frac{(4500 \mathrm{~N}) \sin 48^{\circ}}{\sin 32^{\circ}}=6311 \mathrm{~N} \approx 6300 \mathrm{~N}
\end{aligned}
$$

Since the $x$ components add to 0 , the magnitude of the vector sum of the two forces will just be the sum of their $y$ components.

$$
\begin{aligned}
\sum F_{y} & =F_{\mathrm{A} y}+F_{\mathrm{B} y}=F_{\mathrm{A}} \cos 48^{\circ}+F_{\mathrm{B}} \cos 32^{\circ}=(4500 \mathrm{~N}) \cos 48^{\circ}+(6311 \mathrm{~N}) \cos 32^{\circ} \\
& =8363 \mathrm{~N} \approx 8400 \mathrm{~N}
\end{aligned}
$$

36. Since all forces of interest in this problem are horizontal, draw the free-body diagram showing only the horizontal forces. $\overrightarrow{\mathbf{F}}_{\mathrm{T} 1}$ is the tension in the coupling between the locomotive and the first car, and it pulls to the right on the first car. $\overrightarrow{\mathbf{F}}_{\mathrm{T} 2}$ is the tension in the coupling between the first car an the second car. It pulls to the right on car 2, labeled $\overrightarrow{\mathbf{F}}_{\mathrm{T} 2 \mathrm{R}}$ and to the left on car 1, labeled $\overrightarrow{\mathbf{F}}_{\mathrm{T} 2 L}$. Both cars have the same mass $m$ and the same acceleration $a$. Note that $\left|\overrightarrow{\mathbf{F}}_{\mathrm{T} 2 \mathrm{R}}\right|=\left|\overrightarrow{\mathbf{F}}_{\mathrm{T} 22}\right|=F_{T 2}$ by Newton's third law.


Write a Newton's second law expression for each car.

$$
\sum F_{1}=F_{T 1}-F_{T 2}=m a \quad \sum F_{2}=F_{T 2}=m a
$$

Substitute the expression for $m a$ from the second expression into the first one.

$$
F_{T 1}-F_{T 2}=m a=F_{T 2} \rightarrow F_{\mathrm{T} 1}=2 F_{\mathrm{T} 2} \rightarrow F_{\mathrm{T} 1} / F_{\mathrm{T} 2}=2
$$

This can also be discussed in the sense that the tension between the locomotive and the first car is pulling 2 cars, while the tension between the cars is only pulling one car.
37. The net force in each case is found by vector addition with components.
(a) $F_{\text {Netx }}=-F_{1}=-10.2 \mathrm{~N} \quad F_{\text {Net } y}=-F_{2}=-16.0 \mathrm{~N}$

$$
F_{\text {Net }}=\sqrt{(-10.2)^{2}+(-16.0)^{2}}=19.0 \mathrm{~N} \quad \theta=\tan ^{-1} \frac{-16.0}{-10.2}=57.48^{\circ}
$$

The actual angle from the $x$-axis is then $237.48^{\circ}$. Thus the net force is
$F_{\text {Net }}=19.0 \mathrm{~N}$ at $237.5^{\circ}$

$a=\frac{F_{\text {Net }}}{m}=\frac{19.0 \mathrm{~N}}{18.5 \mathrm{~kg}}=1.03 \mathrm{~m} / \mathrm{s}^{2}$ at $237.5^{\circ}$
(b)

$$
\begin{aligned}
& F_{\text {Net } X}=F_{1} \cos 30^{\circ}=8.833 \mathrm{~N} \quad F_{\text {Net } y}=F_{2}-F_{1} \sin 30^{\circ}=10.9 \mathrm{~N} \\
& F_{\text {Net }}=\sqrt{(8.833 \mathrm{~N})^{2}+(10.9 \mathrm{~N})^{2}}=14.03 \mathrm{~N} \approx 14.0 \mathrm{~N} \\
& \theta=\tan ^{-1} \frac{10.9}{8.833}=51.0^{\circ} \quad a=\frac{F_{\text {Net }}}{m}=\frac{14.03 \mathrm{~N}}{18.5 \mathrm{~kg}}=0.758 \mathrm{~m} / \mathrm{s}^{2} \text { at } 51.0^{\circ}
\end{aligned}
$$


38. Since the sprinter exerts a force of 720 N on the ground at an angle of $22^{\circ}$ below the horizontal, by Newton's third law the ground will exert a force of 720 N on the sprinter at an angle of $22^{\circ}$ above the horizontal. A free-body diagram for the sprinter is shown.
(a) The horizontal acceleration will be found from the net horizontal force. Using Newton's second law, we have the following.

$$
\begin{aligned}
\sum F_{x}=F_{\mathrm{P}} \cos 22^{\circ}=m a_{x} \rightarrow a_{x} & =\frac{F_{\mathrm{P}} \cos 22^{\circ}}{m}=\frac{(720 \mathrm{~N}) \cos 22^{\circ}}{65 \mathrm{~kg}} \\
& =10.27 \mathrm{~m} / \mathrm{s}^{2} \approx 1.0 \times 10^{1} \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$


(b) Eq. 2-12a is used to find the final speed. The starting speed is 0 .

$$
v=v_{0}+a t \rightarrow v=0+a t=\left(10.27 \mathrm{~m} / \mathrm{s}^{2}\right)(0.32 \mathrm{~s})=3.286 \mathrm{~m} / \mathrm{s} \approx 3.3 \mathrm{~m} / \mathrm{s}
$$

39. During the time while the force is $F_{0}$, the acceleration is $a=\frac{F_{0}}{m}$. Thus the distance traveled would be given by Eq. 2-12b, with a 0 starting velocity, $x-x_{0}=v_{0} t+\frac{1}{2} a t^{2}=\frac{1}{2} \frac{F_{0}}{m} t_{0}^{2}$. The velocity at the end of that time is given by Eq. 2-12a, $v=v_{0}+a t=0+\left(\frac{F_{0}}{m}\right) t_{0}$. During the time while the force is $2 F_{0}$, the acceleration is $a=\frac{2 F_{0}}{m}$. The distance traveled during this time interval would again be given by Eq. 2-12b, with a starting velocity of $\left(\frac{F_{0}}{m}\right) t_{0}$.

$$
x-x_{0}=v_{0} t+\frac{1}{2} a t^{2}=\left[\left(\frac{F_{0}}{m}\right) t_{0}\right] t_{0}+\frac{1}{2}\left(\frac{2 F_{0}}{m}\right) t_{0}^{2}=2 \frac{F_{0}}{m} t_{0}^{2}
$$

The total distance traveled is $\frac{1}{2} \frac{F_{0}}{m} t_{0}^{2}+2 \frac{F_{0}}{m} t_{0}^{2}=\frac{5}{2} \frac{F_{0}}{m} t_{0}^{2}$.
40. Find the net force by adding the force vectors. Divide that net force by the mass to find the acceleration, and then use Eq. 3-13a to find the velocity at the given time.

$$
\begin{aligned}
& \sum \overrightarrow{\mathbf{F}}=(16 \hat{\mathbf{i}}+12 \hat{\mathbf{j}}) \mathrm{N}+(-10 \hat{\mathbf{i}}+22 \hat{\mathbf{j}}) \mathrm{N}=(6 \hat{\mathbf{i}}+34 \hat{\mathbf{j}}) \mathrm{N}=m \overrightarrow{\mathbf{a}}=(3.0 \mathrm{~kg}) \overrightarrow{\mathbf{a}} \rightarrow \\
& \overrightarrow{\mathbf{a}}=\frac{(6 \hat{\mathbf{i}}+34 \hat{\mathbf{j}}) \mathrm{N}}{3.0 \mathrm{~kg}} \quad \overrightarrow{\mathbf{v}}=\overrightarrow{\mathbf{v}}_{0}+\overrightarrow{\mathbf{a}} t=0+\frac{(6 \hat{\mathbf{i}}+34 \hat{\mathbf{j}}) \mathrm{N}}{3.0 \mathrm{~kg}}(3.0 \mathrm{~s})=(6 \hat{\mathbf{i}}+34 \hat{\mathbf{j}}) \mathrm{m} / \mathrm{s}
\end{aligned}
$$

In magnitude and direction, the velocity is $35 \mathrm{~m} / \mathrm{s}$ at an angle of $80^{\circ}$.
41. For a simple ramp, the decelerating force is the component of gravity along the ramp. See the free-body diagram, and use Eq. 2-12c to calculate the distance.

$$
\begin{aligned}
& \sum F_{x}=-m g \sin \theta=m a \rightarrow a=-g \sin \theta \\
& x-x_{0}=\frac{v^{2}-v_{0}^{2}}{2 a}=\frac{0-v_{0}^{2}}{2(-g \sin \theta)}=\frac{v_{0}^{2}}{2 g \sin \theta}
\end{aligned}
$$



$$
=\frac{\left[(140 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)\right]^{2}}{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \sin 11^{\circ}}=4.0 \times 10^{2} \mathrm{~m}
$$

42. The average force can be found from the average acceleration. Use Eq. 2-12c to find the acceleration.

$$
\begin{aligned}
& v^{2}=v_{0}^{2}+2 a\left(x-x_{0}\right) \rightarrow a=\frac{v^{2}-v_{0}^{2}}{2\left(x-x_{0}\right)} \\
& F=m a=m \frac{v^{2}-v_{0}^{2}}{2\left(x-x_{0}\right)}=(60.0 \mathrm{~kg}) \frac{0-(10.0 \mathrm{~m} / \mathrm{s})^{2}}{2(25.0 \mathrm{~m})}=-120 \mathrm{~N}
\end{aligned}
$$

The average retarding force is $1.20 \times 10^{2} \mathrm{~N}$, in the direction opposite to the child's velocity.
43. From the free-body diagram, the net force along the plane on the skater is $m g \sin \theta$, and so the acceleration along the plane is $g \sin \theta$. We use the kinematical data and Eq. 2-12b to write an equation for the acceleration, and then solve for the angle.

$$
\begin{aligned}
& x-x_{0}=v_{0} t+\frac{1}{2} a t^{2}=v_{0} t+\frac{1}{2} g t^{2} \sin \theta \rightarrow \\
& \theta=\sin ^{-1}\left(\frac{2 \Delta x-v_{0} t}{g t^{2}}\right)=\sin ^{-1}\left(\frac{2(18 \mathrm{~m})-2(2.0 \mathrm{~m} / \mathrm{s})(3.3 \mathrm{~s})}{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(3.3 \mathrm{~s})^{2}}\right)=12^{\circ}
\end{aligned}
$$


44. For each object, we have the free-body diagram shown, assuming that the string doesn't break. Newton's second law is used to get an expression for the tension. Since the string broke for the 2.10 kg mass, we know that the required tension to accelerate that mass was more than 22.2 N . Likewise, since the string didn't break for the 2.05 kg mass, we know that the required tension to accelerate that mass was less than 22.2 N . These relationships can be used to get the range of accelerations.


$$
\begin{aligned}
& \sum F=F_{\mathrm{T}}-m g=m a \rightarrow F_{\mathrm{T}}=m(a+g) \\
& F_{\mathrm{T}}<m_{2.10}(a+g) ; \underset{\max }{F_{\mathrm{T}}}>m_{2.05}(a+g) \rightarrow \frac{F_{\mathrm{T}}}{\mathrm{~m}_{\text {max }}}-g<a ; \frac{F_{\mathrm{T}}}{m_{2.10}} \underset{m_{2.05}}{m_{2}}-g>a \rightarrow \\
& \frac{F_{\mathrm{T}}}{m_{\text {max }}}-g<a<\frac{F_{\mathrm{T}}}{m_{2.10}}-g \rightarrow \frac{22.2 \mathrm{~N}}{2.10 \mathrm{~kg}}-9.80 \mathrm{~m} / \mathrm{s}^{2}<a<\frac{22.2 \mathrm{~N}}{2.05 \mathrm{~kg}}-9.80 \mathrm{~m} / \mathrm{s}^{2} \rightarrow \\
& 0.77 \mathrm{~m} / \mathrm{s}^{2}<a<1.03 \mathrm{~m} / \mathrm{s}^{2} \rightarrow 0.8 \mathrm{~m} / \mathrm{s}^{2}<a<1.0 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

45. We use the free-body diagram with Newton's first law for the stationary lamp to find the forces in question. The angle is found from the horizontal displacement and the length of the wire.

$$
\begin{aligned}
& \text { (a) } \theta=\sin ^{-1} \frac{0.15 \mathrm{~m}}{4.0 \mathrm{~m}}=2.15^{\circ} \\
& F_{\mathrm{net}}=F_{\mathrm{T}} \sin \theta-F_{\mathrm{H}}=0 \rightarrow F_{\mathrm{H}}=F_{\mathrm{T}} \sin \theta
\end{aligned}
$$



$$
\begin{aligned}
& F_{\substack{\mathrm{n} t}}=F_{\mathrm{T}} \cos \theta-m g=0 \rightarrow F_{\mathrm{T}}=\frac{m g}{\cos \theta} \rightarrow \\
& F_{\mathrm{H}}=\frac{m g}{\cos \theta} \sin \theta=m g \tan \theta=(27 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \tan 2.15^{\circ}=9.9 \mathrm{~N} \\
& \text { (b) } \quad F_{\mathrm{T}}=\frac{m g}{\cos \theta}=\frac{(27 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{\cos 2.15^{\circ}}=260 \mathrm{~N}
\end{aligned}
$$

46. (a) In the free-body diagrams below, $\overrightarrow{\mathbf{F}}_{\mathrm{AB}}=$ force on block A exerted by block $\mathrm{B}, \overrightarrow{\mathbf{F}}_{\mathrm{BA}}=$ force on block B exerted by block $\mathrm{A}, \overrightarrow{\mathbf{F}}_{\mathrm{BC}}$ = force on block B exerted by block C , and $\overrightarrow{\mathbf{F}}_{\mathrm{CB}}$ = force on block $C$ exerted by block $B$. The magnitudes of $\overrightarrow{\mathbf{F}}_{\mathrm{BA}}$ and $\overrightarrow{\mathbf{F}}_{\mathrm{AB}}$ are equal, and the magnitudes of $\overrightarrow{\mathbf{F}}_{\mathrm{BC}}$ and $\overrightarrow{\mathbf{F}}_{\mathrm{CB}}$ are equal, by Newton's third law.

(b) All of the vertical forces on each block add up to zero, since there is no acceleration in the vertical direction. Thus for each block, $F_{N}=m g$. For the horizontal direction, we have the following.

$$
\sum F=F-F_{\mathrm{AB}}+F_{\mathrm{BA}}-F_{\mathrm{BC}}+F_{\mathrm{CB}}=F=\left(m_{\mathrm{A}}+m_{\mathrm{B}}+m_{\mathrm{C}}\right) a \rightarrow a=\frac{F}{m_{\mathrm{A}}+m_{\mathrm{B}}+m_{\mathrm{C}}}
$$

(c) For each block, the net force must be $m a$ by Newton's second law. Each block has the same acceleration since they are in contact with each other.

$$
F_{\mathrm{A} \text { net }}=F \frac{m_{\mathrm{A}}}{m_{\mathrm{A}}+m_{\mathrm{B}}+m_{\mathrm{C}}} \quad F_{\mathrm{B} \text { net }}=F \frac{m_{\mathrm{B}}}{m_{\mathrm{A}}+m_{\mathrm{B}}+m_{\mathrm{C}}} \quad F_{3 n e t}=F \frac{m_{\mathrm{C}}}{m_{\mathrm{A}}+m_{\mathrm{B}}+m_{\mathrm{C}}}
$$

(d) From the free-body diagram, we see that for $m_{\mathrm{C}}, F_{\mathrm{CB}}=F_{\mathrm{C} \text { net }}=F \frac{m_{\mathrm{C}}}{m_{\mathrm{A}}+m_{\mathrm{B}}+m_{\mathrm{C}}}$. And by

Newton's third law, $F_{\mathrm{BC}}=F_{\mathrm{CB}}=F \frac{m_{\mathrm{C}}}{m_{\mathrm{A}}+m_{\mathrm{B}}+m_{\mathrm{C}}}$. Of course, $\overrightarrow{\mathbf{F}}_{23}$ and $\overrightarrow{\mathbf{F}}_{32}$ are in opposite directions. Also from the free-body diagram, we use the net force on $m_{\mathrm{A}}$.

$$
\begin{aligned}
& F-F_{\mathrm{AB}}=F_{\mathrm{Anet}}=F \frac{m_{\mathrm{A}}}{m_{\mathrm{A}}+m_{\mathrm{B}}+m_{\mathrm{C}}} \rightarrow F_{\mathrm{AB}}=F-F \frac{m_{\mathrm{A}}}{m_{\mathrm{A}}+m_{\mathrm{B}}+m_{\mathrm{C}}} \rightarrow \\
& F_{\mathrm{AB}}=F \frac{m_{\mathrm{B}}+m_{\mathrm{C}}}{m_{\mathrm{A}}+m_{\mathrm{B}}+m_{\mathrm{C}}}
\end{aligned}
$$

By Newton's third law, $F_{\mathrm{BC}}=F_{\mathrm{AB}}=F \frac{m_{2}+m_{3}}{m_{1}+m_{2}+m_{3}}$.
(e) Using the given values, $a=\frac{F}{m_{1}+m_{2}+m_{3}}=\frac{96.0 \mathrm{~N}}{30.0 \mathrm{~kg}}=3.20 \mathrm{~m} / \mathrm{s}^{2}$. Since all three masses are the same value, the net force on each mass is $F_{\text {net }}=m a=(10.0 \mathrm{~kg})\left(3.20 \mathrm{~m} / \mathrm{s}^{2}\right)=32.0 \mathrm{~N}$. This is also the value of $F_{\mathrm{CB}}$ and $F_{\mathrm{BC}}$. The value of $F_{\mathrm{AB}}$ and $F_{\mathrm{BA}}$ is found as follows.

$$
F_{\mathrm{AB}}=F_{\mathrm{BA}}=\left(m_{2}+m_{3}\right) a=(20.0 \mathrm{~kg})\left(3.20 \mathrm{~m} / \mathrm{s}^{2}\right)=64.0 \mathrm{~N}
$$

To summarize:

$$
F_{\mathrm{A} \text { net }}=F_{\mathrm{B} \text { net }}=F_{\mathrm{C} \text { net }}=32.0 \mathrm{~N} \quad F_{\mathrm{AB}}=F_{\mathrm{BA}}=64.0 \mathrm{~N} \quad F_{\mathrm{BC}}=F_{\mathrm{CB}}=32.0 \mathrm{~N}
$$

The values make sense in that in order of magnitude, we should have $F>F_{\mathrm{BA}}>F_{\mathrm{CB}}$, since $F$ is the net force pushing the entire set of blocks, $F_{\mathrm{AB}}$ is the net force pushing the right two blocks, and $F_{\mathrm{BC}}$ is the net force pushing the right block only.
47. (a) Refer to the free-body diagrams shown. With the stipulation that the direction of the acceleration be in the direction of motion for both objects, we have $a_{\mathrm{C}}=a_{\mathrm{E}}=a$.

$$
m_{\mathrm{E}} g-F_{\mathrm{T}}=m_{\mathrm{E}} a ; F_{\mathrm{T}}-m_{\mathrm{C}} g=m_{\mathrm{C}} a
$$

(b) Add the equations together to solve them.

$$
\begin{aligned}
& \left(m_{\mathrm{E}} g-F_{\mathrm{T}}\right)+\left(F_{\mathrm{T}}-m_{\mathrm{C}} g\right)=m_{\mathrm{E}} a+m_{\mathrm{C}} a \rightarrow \\
& m_{\mathrm{E}} g-m_{\mathrm{C}} g=m_{\mathrm{E}} a+m_{\mathrm{C}} a \rightarrow \\
& \begin{array}{l}
a=\frac{m_{\mathrm{E}}-m_{\mathrm{C}}}{m_{\mathrm{E}}+m_{\mathrm{C}}} g=\frac{1150 \mathrm{~kg}-1000 \mathrm{~kg}}{1150 \mathrm{~kg}+1000 \mathrm{~kg}}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=0.68 \mathrm{~m} / \mathrm{s}^{2} \\
F_{\mathrm{T}}=m_{\mathrm{C}}(g+a)=m_{\mathrm{C}}\left(g+\frac{m_{\mathrm{E}}-m_{\mathrm{C}}}{m_{\mathrm{E}}+m_{\mathrm{C}}} g\right)=\frac{2 m_{\mathrm{C}} m_{\mathrm{E}}}{m_{\mathrm{E}}+m_{\mathrm{C}}} g=\frac{2(1000 \mathrm{~kg})(1150 \mathrm{~kg})}{1150 \mathrm{~kg}+1000 \mathrm{~kg}}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \\
\quad=10,483 \mathrm{~N} \approx 10,500 \mathrm{~N}
\end{array}
\end{aligned}
$$

48. (a) Consider the free-body diagram for the block on the frictionless
surface. There is no acceleration in the $y$ direction. Use Newton's second law for the $x$ direction to find the acceleration.

$$
\begin{aligned}
& \sum F_{x}=m g \sin \theta=m a \rightarrow \\
& a=g \sin \theta=\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \sin 22.0^{\circ}=3.67 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$


(b) Use Eq. 2-12c with $v_{0}=0$ to find the final speed.


$$
v^{2}-v_{0}^{2}=2 a\left(x-x_{0}\right) \rightarrow v=\sqrt{2 a\left(x-x_{0}\right)}=\sqrt{2\left(3.67 \mathrm{~m} / \mathrm{s}^{2}\right)(12.0 \mathrm{~m})}=9.39 \mathrm{~m} / \mathrm{s}
$$

49. (a) Consider the free-body diagram for the block on the frictionless surface. There is no acceleration in the $y$ direction. Write Newton's second law for the $x$ direction.

$$
\sum F_{x}=m g \sin \theta=m a \rightarrow a=g \sin \theta
$$

Use Eq. 2-12c with $v_{0}=-4.5 \mathrm{~m} / \mathrm{s}$ and $v=0 \mathrm{~m} / \mathrm{s}$ to find the distance that it slides before stopping.


$$
\begin{aligned}
& v^{2}-v_{0}^{2}=2 a\left(x-x_{0}\right) \rightarrow \\
& \left(x-x_{0}\right)=\frac{v^{2}-v_{0}^{2}}{2 a}=\frac{0-(-4.5 \mathrm{~m} / \mathrm{s})^{2}}{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \sin 22.0^{\circ}}=-2.758 \mathrm{~m} \approx 2.8 \mathrm{~m} \text { up the plane }
\end{aligned}
$$

(b) The time for a round trip can be found from Eq. 2-12a. The free-body diagram (and thus the acceleration) is the same whether the block is rising or falling. For the entire trip, $v_{0}=-4.5 \mathrm{~m} / \mathrm{s}$ and $v=+4.5 \mathrm{~m} / \mathrm{s}$.

$$
v=v_{0}+a t \rightarrow t=\frac{v-v_{0}}{a}=\frac{(4.5 \mathrm{~m} / \mathrm{s})-(-4.5 \mathrm{~m} / \mathrm{s})}{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \sin 22^{\circ}}=2.452 \mathrm{~s} \approx 2.5 \mathrm{~s}
$$

50. Consider a free-body diagram of the object. The car is moving to the right. The acceleration of the dice is found from Eq. 2-12a.

$$
v=v_{0}+=a_{x} t \quad \rightarrow \quad a_{x}=\frac{v-v_{0}}{t}=\frac{28 \mathrm{~m} / \mathrm{s}-0}{6.0 \mathrm{~s}}=4.67 \mathrm{~m} / \mathrm{s}^{2}
$$

Now write Newton's second law for both the vertical $(y)$ and horizontal ( $x$ ) directions.


$$
\sum F_{y}=F_{\mathrm{T}} \cos \theta-m g=0 \rightarrow F_{\mathrm{T}}=\frac{m g}{\cos \theta} \quad \sum F_{x}=F_{\mathrm{T}} \sin \theta=m a_{x}
$$

Substitute the expression for the tension from the $y$ equation into the $x$ equation.

$$
\begin{aligned}
& m a_{x}=F_{\mathrm{T}} \sin \theta=\frac{m g}{\cos \theta} \sin \theta=m g \tan \theta \rightarrow a_{x}=g \tan \theta \\
& \theta=\tan ^{-1} \frac{a_{x}}{g}=\tan ^{-1} \frac{4.67 \mathrm{~m} / \mathrm{s}^{2}}{9.80 \mathrm{~m} / \mathrm{s}^{2}}=25.48^{\circ} \approx 25^{\circ}
\end{aligned}
$$

51. (a) See the free-body diagrams included.
(b) For block A, since there is no motion in the vertical direction, we have $F_{\mathrm{NA}}=m_{\mathrm{A}} g$. We write Newton's second law for the $x$ direction: $\sum F_{\mathrm{A} x}=F_{\mathrm{T}}=m_{\mathrm{A}} a_{\mathrm{A} x}$. For block B , we only need to consider vertical forces: $\sum F_{\mathrm{B} y}=m_{\mathrm{B}} g-F_{\mathrm{T}}=m_{\mathrm{B}} a_{\mathrm{B} y}$. Since the two blocks are connected, the magnitudes of their accelerations
 will be the same, and so let $a_{\mathrm{A} x}=a_{\mathrm{B} y}=a$. Combine the two force equations from above, and solve for $a$ by substitution.

$$
\begin{aligned}
& F_{\mathrm{T}}=m_{\mathrm{A}} a \quad m_{\mathrm{B}} g-F_{\mathrm{T}}=m_{\mathrm{B}} a \rightarrow m_{\mathrm{B}} g-m_{\mathrm{A}} a=m_{\mathrm{B}} a \rightarrow \\
& m_{\mathrm{A}} a+m_{\mathrm{B}} a=m_{\mathrm{B}} g \rightarrow a=g \frac{m_{\mathrm{B}}}{m_{\mathrm{A}}+m_{\mathrm{B}}} \quad F_{\mathrm{T}}=m_{\mathrm{A}} a=g \frac{m_{\mathrm{A}} m_{\mathrm{B}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}
\end{aligned}
$$

52. (a) From Problem 51, we have the acceleration of each block. Both blocks have the same acceleration.

$$
a=g \frac{m_{\mathrm{B}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}=\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \frac{5.0 \mathrm{~kg}}{(5.0 \mathrm{~kg}+13.0 \mathrm{~kg})}=2.722 \mathrm{~m} / \mathrm{s}^{2} \approx 2.7 \mathrm{~m} / \mathrm{s}^{2}
$$

(b) Use Eq. 2-12b to find the time.

$$
x-x_{0}=v_{0} t+\frac{1}{2} a t^{2} \rightarrow t=\sqrt{\frac{2\left(x-x_{0}\right)}{a}}=\sqrt{\frac{2(1.250 \mathrm{~m})}{\left(2.722 \mathrm{~m} / \mathrm{s}^{2}\right)}}=0.96 \mathrm{~s}
$$

(c) Again use the acceleration from Problem 51.

$$
a=g \frac{m_{\mathrm{B}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}=\frac{1}{100} g \rightarrow \frac{m_{\mathrm{B}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}=\frac{1}{100} \rightarrow m_{\mathrm{A}}=99 m_{B}=99 \mathrm{~kg}
$$

53. This problem can be solved in the same way as problem 51, with the modification that we increase mass $m_{\mathrm{A}}$ by the mass of $\ell_{\mathrm{A}}$ and we increase mass $m_{\mathrm{B}}$ by the mass of $\ell_{\mathrm{B}}$. We take the result from problem 51 for the acceleration and make these modifications. We assume that the cord is uniform, and so the mass of any segment is directly proportional to the length of that segment.

$$
a=g \frac{m_{\mathrm{B}}}{m_{\mathrm{A}}+m_{\mathrm{B}}} \rightarrow a=g \frac{m_{\mathrm{B}}+\frac{\ell_{\mathrm{B}}}{\ell_{\mathrm{A}}+\ell_{\mathrm{B}}} m_{C}}{\left(m_{\mathrm{A}}+\frac{\ell_{\mathrm{A}}}{\ell_{\mathrm{A}}+\ell_{\mathrm{B}}} m_{C}\right)+\left(m_{\mathrm{B}}+\frac{\ell_{\mathrm{B}}}{\ell_{\mathrm{A}}+\ell_{\mathrm{B}}} m_{C}\right)}=g \frac{m_{\mathrm{B}}+\frac{\ell_{\mathrm{B}}}{\ell_{\mathrm{A}}+\ell_{\mathrm{B}}} m_{C}}{m_{\mathrm{A}}+m_{\mathrm{B}}+m_{\mathrm{C}}}
$$

Note that this acceleration is NOT constant, because the lengths $\ell_{\mathrm{A}}$ and $\ell_{\mathrm{B}}$ are functions of time. Thus constant acceleration kinematics would not apply to this system.
54. We draw a free-body diagram for each mass. We choose UP to be the positive direction. The tension force in the cord is found from analyzing the two hanging masses. Notice that the same tension force is applied to each mass. Write Newton's second law for each of the masses.

$$
F_{\mathrm{T}}-m_{1} g=m_{1} a_{1} \quad F_{\mathrm{T}}-m_{2} g=m_{2} a_{2}
$$

Since the masses are joined together by the cord, their accelerations will have the same magnitude but opposite directions. Thus $a_{1}=-a_{2}$.


Substitute this into the force expressions and solve for the tension force.

$$
\begin{aligned}
& F_{\mathrm{T}}-m_{1} g=-m_{1} a_{2} \rightarrow F_{\mathrm{T}}=m_{1} g-m_{1} a_{2} \rightarrow a_{2}=\frac{m_{1} g-F_{\mathrm{T}}}{m_{1}} \\
& F_{\mathrm{T}}-m_{2} g=m_{2} a_{2}=m_{2}\left(\frac{m_{1} g-F_{\mathrm{T}}}{m_{1}}\right) \rightarrow F_{\mathrm{T}}=\frac{2 m_{1} m_{2} g}{m_{1}+m_{2}}
\end{aligned}
$$

Apply Newton's second law to the stationary pulley.

$$
F_{\mathrm{C}}-2 F_{T}=0 \rightarrow F_{\mathrm{C}}=2 F_{T}=\frac{4 m_{1} m_{2} g}{m_{1}+m_{2}}=\frac{4(3.2 \mathrm{~kg})(1.2 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{4.4 \mathrm{~kg}}=34 \mathrm{~N}
$$

55. If $m$ doesn't move on the incline, it doesn't move in the vertical direction, and so has no vertical component of acceleration. This suggests that we analyze the forces parallel and perpendicular to the floor. See the force diagram for the small block, and use Newton's second law to find the acceleration of the small block.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{N}} \cos \theta-m g=0 \rightarrow F_{\mathrm{N}}=\frac{m g}{\cos \theta} \\
& \sum F_{x}=F_{\mathrm{N}} \sin \theta=m a \rightarrow a=\frac{F_{\mathrm{N}} \sin \theta}{m}=\frac{m g \sin \theta}{m \cos \theta}=g \tan \theta
\end{aligned}
$$



Since the small block doesn't move on the incline, the combination of both masses has the same horizontal acceleration of $g \tan \theta$. That can be used to find the applied force.

$$
F_{\mathrm{applied}}=(m+M) a=(m+M) g \tan \theta
$$

Note that this gives the correct answer for the case of $\theta=0$, , where it would take no applied force to keep $m$ stationary. It also gives a reasonable answer for the limiting case of $\theta \rightarrow 90^{\circ}$, where no force would be large enough to keep the block from falling, since there would be no upward force to counteract the force of gravity.
56. Because the pulleys are massless, the net force on them must be 0 . Because the cords are massless, the tension will be the same at both ends of the cords. Use the free-body diagrams to write Newton's second law for each mass. We are using the same approach taken in problem 47 , where we take the direction of acceleration to be positive in the direction of motion of the object. We assume that $m_{\mathrm{C}}$ is falling, $m_{\mathrm{B}}$ is falling relative to its pulley, and $m_{\mathrm{A}}$ is rising relative to its pulley. Also note that if the acceleration of $m_{\mathrm{A}}$ relative to the pulley above it is $a_{\mathrm{R}}$, then $a_{\mathrm{A}}=a_{\mathrm{R}}+a_{\mathrm{C}}$. Then, the acceleration of $m_{\mathrm{B}}$ is $a_{\mathrm{B}}=a_{\mathrm{R}}-a_{\mathrm{C}}$, since $a_{\mathrm{C}}$ is in the opposite
 direction of $a_{\mathrm{B}}$.

$$
\begin{aligned}
& m_{\mathrm{A}}: \sum F=F_{\mathrm{TA}}-m_{\mathrm{A}} g=m_{\mathrm{A}} a_{\mathrm{A}}=m_{\mathrm{A}}\left(a_{\mathrm{R}}+a_{\mathrm{C}}\right) \\
& m_{\mathrm{B}}: \sum F=m_{\mathrm{B}} g-F_{\mathrm{TA}}=m_{\mathrm{B}} a_{\mathrm{B}}=m_{\mathrm{B}}\left(a_{\mathrm{R}}-a_{\mathrm{C}}\right) \\
& m_{\mathrm{C}}: \sum F=m_{\mathrm{C}} g-F_{\mathrm{TC}}=m_{\mathrm{C}} a_{\mathrm{C}} \\
& \text { pulley: } \sum F=F_{\mathrm{TC}}-2 F_{\mathrm{TA}}=0 \rightarrow F_{\mathrm{TC}}=2 F_{\mathrm{TA}}
\end{aligned}
$$

Re-write this system as three equations in three unknowns $F_{\mathrm{TA}}, a_{\mathrm{R}}, a_{\mathrm{C}}$.

$$
\begin{array}{lll}
F_{\mathrm{TA}}-m_{\mathrm{A}} g=m_{\mathrm{A}}\left(a_{\mathrm{R}}+a_{\mathrm{C}}\right) & \rightarrow & F_{\mathrm{TA}}-m_{\mathrm{A}} a_{\mathrm{C}}-m_{\mathrm{A}} a_{\mathrm{R}}=m_{\mathrm{A}} g \\
m_{\mathrm{B}} g-F_{\mathrm{TA}}=m_{\mathrm{B}}\left(a_{\mathrm{R}}-a_{\mathrm{C}}\right) & \rightarrow & F_{\mathrm{TA}}-m_{\mathrm{B}} a_{\mathrm{C}}+m_{\mathrm{B}} a_{\mathrm{R}}=m_{\mathrm{B}} g \\
m_{\mathrm{C}} g-2 F_{\mathrm{TA}}=m_{\mathrm{C}} a_{\mathrm{C}} & \rightarrow & 2 F_{\mathrm{TA}}+m_{\mathrm{C}} a_{\mathrm{C}} \quad=m_{\mathrm{C}} g
\end{array}
$$

This system now needs to be solved. One method to solve a system of linear equations is by determinants. We show that for $a_{C}$.

$$
\begin{aligned}
a_{\mathrm{C}} & =\frac{\left|\begin{array}{ccc}
1 & m_{\mathrm{A}} & -m_{\mathrm{A}} \\
1 & m_{\mathrm{B}} & m_{\mathrm{B}} \\
2 & m_{\mathrm{C}} & 0
\end{array}\right|}{\left|\begin{array}{ccc}
1 & -m_{\mathrm{A}} & -m_{\mathrm{A}} \\
1 & -m_{\mathrm{B}} & m_{\mathrm{B}} \\
2 & m_{\mathrm{C}} & 0
\end{array}\right|} g=\frac{-m_{\mathrm{B}} m_{\mathrm{C}}+m_{\mathrm{A}}\left(2 m_{\mathrm{B}}\right)-m_{\mathrm{A}}\left(m_{\mathrm{C}}-2 m_{\mathrm{B}}\right)}{-m_{\mathrm{B}} m_{\mathrm{C}}-m_{\mathrm{A}}\left(2 m_{\mathrm{B}}\right)-m_{\mathrm{A}}\left(m_{\mathrm{C}}+2 m_{\mathrm{B}}\right)} g \\
& =\frac{4 m_{\mathrm{A}} m_{\mathrm{B}}-m_{\mathrm{A}} m_{\mathrm{C}}-m_{\mathrm{B}} m_{\mathrm{C}}}{-4 m_{\mathrm{A}} m_{\mathrm{B}}-m_{\mathrm{A}} m_{\mathrm{C}}-m_{\mathrm{B}} m_{\mathrm{C}}} g=\frac{m_{\mathrm{A}} m_{\mathrm{C}}+m_{\mathrm{B}} m_{\mathrm{C}}-4 m_{\mathrm{A}} m_{\mathrm{B}}}{4 m_{\mathrm{A}} m_{\mathrm{B}}+m_{\mathrm{A}} m_{\mathrm{C}}+m_{\mathrm{B}} m_{\mathrm{C}}} g
\end{aligned}
$$

Similar manipulations give the following results.

$$
a_{\mathrm{R}}=\frac{2\left(m_{\mathrm{A}} m_{\mathrm{C}}-m_{\mathrm{B}} m_{\mathrm{C}}\right)}{4 m_{\mathrm{A}} m_{\mathrm{B}}+m_{\mathrm{A}} m_{\mathrm{C}}+m_{\mathrm{B}} m_{\mathrm{C}}} g ; F_{\mathrm{TA}}=\frac{4 m_{\mathrm{A}} m_{\mathrm{B}} m_{\mathrm{C}}}{4 m_{\mathrm{A}} m_{\mathrm{B}}+m_{\mathrm{A}} m_{\mathrm{C}}+m_{\mathrm{B}} m_{\mathrm{C}}} g
$$

(a) The accelerations of the three masses are found below.

$$
\begin{aligned}
a_{\mathrm{A}} & =a_{\mathrm{R}}+a_{\mathrm{C}}=\frac{2\left(m_{\mathrm{A}} m_{\mathrm{C}}-m_{\mathrm{B}} m_{\mathrm{C}}\right)}{4 m_{\mathrm{A}} m_{\mathrm{B}}+m_{\mathrm{A}} m_{\mathrm{C}}+m_{\mathrm{B}} m_{\mathrm{C}}} g+\frac{m_{\mathrm{A}} m_{\mathrm{C}}+m_{\mathrm{B}} m_{\mathrm{C}}-4 m_{\mathrm{A}} m_{\mathrm{B}}}{4 m_{\mathrm{A}} m_{\mathrm{B}}+m_{\mathrm{A}} m_{\mathrm{C}}+m_{\mathrm{B}} m_{\mathrm{C}}} g \\
& =\frac{3 m_{\mathrm{A}} m_{\mathrm{C}}-m_{\mathrm{B}} m_{\mathrm{C}}-4 m_{\mathrm{A}} m_{\mathrm{B}}}{4 m_{\mathrm{A}} m_{\mathrm{B}}+m_{\mathrm{A}} m_{\mathrm{C}}+m_{\mathrm{B}} m_{\mathrm{C}}} g \\
a_{\mathrm{B}} & =a_{\mathrm{R}}-a_{\mathrm{C}}=\frac{2\left(m_{\mathrm{A}} m_{\mathrm{C}}-m_{\mathrm{B}} m_{\mathrm{C}}\right)}{4 m_{\mathrm{A}} m_{\mathrm{B}}+m_{\mathrm{A}} m_{\mathrm{C}}+m_{\mathrm{B}} m_{\mathrm{C}}} g-\frac{m_{\mathrm{A}} m_{\mathrm{C}}+m_{\mathrm{B}} m_{\mathrm{C}}-4 m_{\mathrm{A}} m_{\mathrm{B}}}{4 m_{\mathrm{A}} m_{\mathrm{B}}+m_{\mathrm{A}} m_{\mathrm{C}}+m_{\mathrm{B}} m_{\mathrm{C}}} g \\
& =\frac{m_{\mathrm{A}} m_{\mathrm{C}}-3 m_{\mathrm{B}} m_{\mathrm{C}}+4 m_{\mathrm{A}} m_{\mathrm{B}}}{4 m_{\mathrm{A}} m_{\mathrm{B}}+m_{\mathrm{A}} m_{\mathrm{C}}+m_{\mathrm{B}} m_{\mathrm{C}}} g \\
a_{\mathrm{C}} & =\frac{m_{\mathrm{A}} m_{\mathrm{C}}+m_{\mathrm{B}} m_{\mathrm{C}}-4 m_{\mathrm{A}} m_{\mathrm{B}}}{4 m_{\mathrm{A}} m_{\mathrm{B}}+m_{\mathrm{A}} m_{\mathrm{C}}+m_{\mathrm{B}} m_{\mathrm{C}}} g
\end{aligned}
$$

(b) The tensions are shown below.

$$
F_{\mathrm{TA}}=\frac{4 m_{\mathrm{A}} m_{\mathrm{B}} m_{\mathrm{C}}}{4 m_{\mathrm{A}} m_{\mathrm{B}}+m_{\mathrm{A}} m_{\mathrm{C}}+m_{\mathrm{B}} m_{\mathrm{C}}} g ; F_{\mathrm{TC}}=2 F_{\mathrm{TA}}=\frac{8 m_{\mathrm{A}} m_{\mathrm{B}} m_{\mathrm{C}}}{4 m_{\mathrm{A}} m_{\mathrm{B}}+m_{\mathrm{A}} m_{\mathrm{C}}+m_{\mathrm{B}} m_{\mathrm{C}}} g
$$

57. Please refer to the free-body diagrams given in the textbook for this problem. Initially, treat the two boxes and the rope as a single system. Then the only accelerating force on the system is $\overrightarrow{\mathbf{F}}_{\mathrm{p}}$. The mass of the system is 23.0 kg , and so using Newton's second law, the acceleration of the system is $a=\frac{F_{\mathrm{P}}}{m}=\frac{35.0 \mathrm{~N}}{23.0 \mathrm{~kg}}=1.522 \mathrm{~m} / \mathrm{s}^{2} \approx 1.52 \mathrm{~m} / \mathrm{s}^{2}$. This is the acceleration of each part of the system.

Now consider $m_{\mathrm{B}}$ alone. The only force on it is $\overrightarrow{\mathbf{F}}_{\mathrm{BT}}$, and it has the acceleration found above. Thus $F_{\text {BT }}$ can be found from Newton's second law.

$$
F_{\mathrm{BT}}=m_{\mathrm{B}} a=(12.0 \mathrm{~kg})\left(1.522 \mathrm{~m} / \mathrm{s}^{2}\right)=18.26 \mathrm{~N} \approx 18.3 \mathrm{~N}
$$

Now consider the rope alone. The net force on it is $\overrightarrow{\mathbf{F}}_{\mathrm{TA}}-\overrightarrow{\mathbf{F}}_{\mathrm{TB}}$, and it also has the acceleration found above. Thus $F_{\mathrm{TA}}$ can be found from Newton's second law.

$$
F_{\mathrm{TA}}-F_{\mathrm{TB}}=m_{C} a \rightarrow F_{\mathrm{TA}}=F_{\mathrm{TB}}+m_{\mathrm{C}} a=18.26 \mathrm{~N}+(1.0 \mathrm{~kg})\left(1.522 \mathrm{~m} / \mathrm{s}^{2}\right)=19.8 \mathrm{~N}
$$

58. First, draw a free-body diagram for each mass. Notice that the same tension force is applied to each mass. Choose UP to be the positive direction. Write Newton's second law for each of the masses.

$$
F_{\mathrm{T}}-m_{2} g=m_{2} a_{2} \quad F_{\mathrm{T}}-m_{1} g=m_{1} a_{1}
$$

Since the masses are joined together by the cord, their accelerations will have the same magnitude but opposite directions. Thus $a_{1}=-a_{2}$. Substitute this into the force expressions and solve for the acceleration by
 subtracting the second equation from the first.

$$
\begin{aligned}
& F_{\mathrm{T}}-m_{1} g=-m_{1} a_{2} \rightarrow F_{\mathrm{T}}=m_{1} g-m_{1} a_{2} \\
& F_{\mathrm{T}}-m_{2} g=m_{2} a_{2} \rightarrow m_{1} g-m_{1} a_{2}-m_{2} g=m_{2} a_{2} \rightarrow m_{1} g-m_{2} g=m_{1} a_{2}+m_{2} a_{2} \\
& a_{2}=\frac{m_{1}-m_{2}}{m_{1}+m_{2}} g=\frac{3.6 \mathrm{~kg}-2.2 \mathrm{~kg}}{3.6 \mathrm{~kg}+2.2 \mathrm{~kg}}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=2.366 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

The lighter block starts with a speed of 0 , and moves a distance of 1.8 meters with the acceleration found above. Using Eq. 2-12c, the velocity of the lighter block at the end of this accelerated motion can be found.

$$
v^{2}-v_{0}^{2}=2 a\left(y-y_{0}\right) \rightarrow v=\sqrt{v_{0}^{2}+2 a\left(y-y_{0}\right)}=\sqrt{0+2\left(2.366 \mathrm{~m} / \mathrm{s}^{2}\right)(1.8 \mathrm{~m})}=2.918 \mathrm{~m} / \mathrm{s}
$$

Now the lighter block has different conditions of motion. Once the heavier block hits the ground, the tension force disappears, and the lighter block is in free fall. It has an initial speed of $2.918 \mathrm{~m} / \mathrm{s}$ upward as found above, with an acceleration of $-9.80 \mathrm{~m} / \mathrm{s}^{2}$ due to gravity. At its highest point, its speed will be 0 . Eq. 2-12c can again be used to find the height to which it rises.

$$
v^{2}-v_{0}^{2}=2 a\left(y-y_{0}\right) \rightarrow\left(y-y_{0}\right)=\frac{v^{2}-v_{0}^{2}}{2 a}=\frac{0-(2.918 \mathrm{~m} / \mathrm{s})^{2}}{2\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=0.434 \mathrm{~m}
$$

Thus the total height above the ground is $1.8 \mathrm{~m}+1.8 \mathrm{~m}+0.43 \mathrm{~m}=4.0 \mathrm{~m}$.
59. The force $\overrightarrow{\mathbf{F}}$ is accelerating the total mass, since it is the only force external to the system. If mass $m_{\mathrm{A}}$ does not move relative to $m_{\mathrm{C}}$, then all the blocks have the same horizontal acceleration, and none of the blocks have vertical acceleration. We solve for the acceleration of the system and then find the magnitude of $\overrightarrow{\mathbf{F}}$ from Newton's second law. Start with free-body diagrams for $m_{\mathrm{A}}$ and $m_{\mathrm{B}}$.


$$
\begin{aligned}
m_{\mathrm{B}}: & \sum F_{x}=F_{\mathrm{T}} \sin \theta=m_{\mathrm{B}} a ; \\
& \sum F_{y}=F_{\mathrm{T}} \cos \theta-m_{\mathrm{B}} g=0 \rightarrow F_{\mathrm{T}} \cos \theta=m_{\mathrm{B}} g
\end{aligned}
$$

Square these two expressions and add them, to get a relationship between $F_{\mathrm{T}}$ and $a$.

$$
\begin{aligned}
& F_{\mathrm{T}}^{2} \sin ^{2} \theta=m_{\mathrm{B}}^{2} a^{2} ; F_{\mathrm{T}}^{2} \cos ^{2} \theta=m_{\mathrm{B}}^{2} g^{2} \rightarrow \\
& F_{\mathrm{T}}^{2}\left(\sin ^{2} \theta+\cos ^{2} \theta\right)=m_{\mathrm{B}}^{2}\left(g^{2}+a^{2}\right) \rightarrow F_{\mathrm{T}}^{2}=m_{\mathrm{B}}^{2}\left(g^{2}+a^{2}\right)
\end{aligned}
$$



Now analyze $m_{\mathrm{A}}$.

$$
m_{\mathrm{A}}: \quad \sum F_{x}=F_{\mathrm{T}}=m_{\mathrm{A}} a \rightarrow F_{\mathrm{T}}^{2}=m_{\mathrm{A}}^{2} a^{2} ; \sum F_{y}=F_{\mathrm{N}}-m_{\mathrm{A}} g=0
$$

Equate the two expressions for $F_{\mathrm{T}}^{2}$, solve for the acceleration and then finally the magnitude of the applied force.

$$
\begin{aligned}
& F_{\mathrm{T}}^{2}=m_{\mathrm{B}}^{2}\left(g^{2}+a^{2}\right)=m_{\mathrm{A}}^{2} a^{2} \rightarrow a^{2}=\frac{m_{\mathrm{B}}^{2} g^{2}}{\left(m_{\mathrm{A}}^{2}-m_{\mathrm{B}}^{2}\right)} \rightarrow a=\frac{m_{\mathrm{B}} g}{\sqrt{\left(m_{\mathrm{A}}^{2}-m_{\mathrm{B}}^{2}\right)}} \rightarrow \\
& F=\left(m_{\mathrm{A}}+m_{\mathrm{B}}+m_{\mathrm{C}}\right) a=\frac{\left(m_{\mathrm{A}}+m_{\mathrm{B}}+m_{\mathrm{C}}\right) m_{\mathrm{B}}}{\sqrt{\left(m_{\mathrm{A}}^{2}-m_{\mathrm{B}}^{2}\right)}} g
\end{aligned}
$$

60. The velocity can be found by integrating the acceleration function, and the position can be found by integrating the position function.

$$
\begin{aligned}
& F=m a=C t^{2} \rightarrow a=\frac{C}{m} t^{2}=\frac{d v}{d t} \rightarrow d v=\frac{C}{m} t^{2} d t \rightarrow \int_{0}^{v} d v=\int_{0}^{t} \frac{C}{m} t^{2} d t \rightarrow v=\frac{C}{3 m} t^{3} \\
& v=\frac{C}{3 m} t^{3}=\frac{d x}{d t} \rightarrow d x=\frac{C}{3 m} t^{3} d t \rightarrow \int_{0}^{x} d x=\int_{0}^{t} \frac{C}{3 m} t^{3} d t \rightarrow x=\frac{C}{12 m} t^{4}
\end{aligned}
$$

61. We assume that the pulley is small enough that the part of the cable that is touching the surface of the pulley is negligible, and so we ignore any force on the cable due to the pulley itself. We also assume that the cable is uniform, so that the mass of a portion of the cable is proportional to the length of that portion. We then treat the cable as two masses, one on each side of the pulley. The masses are given by
 $m_{1}=\frac{y}{\ell} M$ and $m_{2}=\frac{\ell-y}{\ell} M$. Free-body diagrams for the masses are shown.
(a) We take downward motion of $m_{1}$ to be the positive direction for $m_{1}$, and upward motion of $m_{2}$ to be the positive direction for $m_{2}$. Newton's second law for the masses gives the following.

$$
\begin{aligned}
& F_{\text {net } 1}=m_{1} g-F_{\mathrm{T}}=m_{1} a ; F_{\mathrm{net} 2}=F_{\mathrm{T}}-m_{2} g \rightarrow a=\frac{m_{1}-m_{2}}{m_{1}+m_{2}} g \\
& a=\frac{\frac{y}{\ell} M-\frac{\ell-y}{\ell} M}{\frac{y}{\ell} M+\frac{\ell-y}{\ell} M} g=\frac{y-(\ell-y)}{y+(\ell-y)} g=\frac{2 y-\ell}{\ell} g=\left(\frac{2 y}{\ell}-1\right) g
\end{aligned}
$$


(b) Use the hint supplied with the problem to set up the equation for the velocity. The cable starts with a length $y_{0}$ (assuming $y_{0}>\frac{1}{2} \ell$ ) on the right side of the pulley, and finishes with a length $\ell$ on the right side of the pulley.

$$
\begin{gathered}
a=\left(\frac{2 y}{\ell}-1\right) g=\frac{d v}{d t}=\frac{d v}{d y} \frac{d y}{d t}=v \frac{d v}{d y} \rightarrow\left(\frac{2 y}{\ell}-1\right) g d y=v d v \rightarrow \\
\int_{y_{0}}^{\ell}\left(\frac{2 y}{\ell}-1\right) g d y=\int_{0}^{v_{f}} v d v \rightarrow g\left(\frac{y^{2}}{\ell}-y\right)_{y_{0}}^{\ell}=\left(\frac{1}{2} v^{2}\right)_{0}^{v_{f}} \rightarrow g y_{0}\left(1-\frac{y_{0}}{\ell}\right)=\frac{1}{2} v_{f}^{2} \rightarrow \\
v_{f}=\sqrt{2 g y_{0}\left(1-\frac{y_{0}}{\ell}\right)} \\
\text { (c) For } y_{0}=\frac{2}{3} \ell, \text { we have } v_{f}=\sqrt{2 g y_{0}\left(1-\frac{y_{0}}{\ell}\right)}=\sqrt{2 g\left(\frac{2}{3}\right) \ell\left(1-\frac{\frac{2}{3} \ell}{\ell}\right)}=\frac{2}{3} \sqrt{g \ell} .
\end{gathered}
$$

62. The acceleration of a person having a 30 " $g$ " deceleration is $a=(30 " g ")\left(\frac{9.80 \mathrm{~m} / \mathrm{s}^{2}}{" g "}\right)=294 \mathrm{~m} / \mathrm{s}^{2}$. The average force causing that acceleration is $F=m a=(65 \mathrm{~kg})\left(294 \mathrm{~m} / \mathrm{s}^{2}\right)=1.9 \times 10^{4} \mathrm{~N}$. Since the person is undergoing a deceleration, the acceleration and force would both be directed opposite to the direction of motion. Use Eq. 2-12c to find the distance traveled during the deceleration. Take
the initial velocity to be in the positive direction, so that the acceleration will have a negative value, and the final velocity will be 0 .

$$
\begin{aligned}
& v_{0}=(95 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)=26.4 \mathrm{~m} / \mathrm{s} \\
& v^{2}-v_{0}^{2}=2 a\left(x-x_{0}\right) \rightarrow\left(x-x_{0}\right)=\frac{v^{2}-v_{0}^{2}}{2 a}=\frac{0-(26.4 \mathrm{~m} / \mathrm{s})^{2}}{2\left(-294 \mathrm{~m} / \mathrm{s}^{2}\right)}=1.2 \mathrm{~m}
\end{aligned}
$$

63. See the free-body diagram for the falling purse. Assume that down is the positive direction, and that the air resistance force $\overrightarrow{\mathbf{F}}_{\mathrm{fr}}$ is constant. Write Newton's second law for the vertical direction.

$$
\sum F=m g-F_{\mathrm{fr}}=m a \rightarrow F_{\mathrm{fr}}=m(g-a)
$$

Now obtain an expression for the acceleration from Eq. 2-12c with $v_{0}=0$, and substitute back into the friction force.

$$
\begin{aligned}
& v^{2}-v_{0}^{2}=2 a\left(x-x_{0}\right) \rightarrow a=\frac{v^{2}}{2\left(x-x_{0}\right)} \\
& F_{f}=m\left(g-\frac{v^{2}}{2\left(x-x_{0}\right)}\right)=(2.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}-\frac{(27 \mathrm{~m} / \mathrm{s})^{2}}{2(55 \mathrm{~m})}\right)=6.3 \mathrm{~N}
\end{aligned}
$$

64. Each rope must support $1 / 6$ of Tom's weight, and so must have a vertical component of tension given by $T_{\text {vert }}=\frac{1}{6} m g$. For the vertical ropes, their entire tension is vertical.

$$
T_{1}=\frac{1}{6} m g=\frac{1}{6}(74.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=120.9 \mathrm{~N} \approx 1.21 \times 10^{2} \mathrm{~N}
$$

For the ropes displaced $30^{\circ}$ from the vertical, see the first diagram.

$$
T_{2 \text { vert }}=T_{2} \cos 30^{\circ}=\frac{1}{6} m g \rightarrow T_{2}=\frac{m g}{6 \cos 30^{\circ}}=\frac{120.9 \mathrm{~N}}{\cos 30^{\circ}}=1.40 \times 10^{2} \mathrm{~N}
$$

For the ropes displaced $60^{\circ}$ from the vertical, see the second diagram.

$$
T_{3 \text { vert }}=T_{3} \cos 60^{\circ}=\frac{1}{6} m g \rightarrow T_{3}=\frac{m g}{6 \cos 60^{\circ}}=\frac{120.9 \mathrm{~N}}{\cos 60^{\circ}}=2.42 \times 10^{2} \mathrm{~N}
$$

The corresponding ropes on the other side of the glider will also have the same
 tensions as found here.
65. Consider the free-body diagram for the soap block on the frictionless surface. There is no acceleration in the $y$ direction. Write Newton's second law for the $x$ direction.

$$
\sum F_{x}=m g \sin \theta=m a \rightarrow a=g \sin \theta
$$

Use Eq. $2-12 \mathrm{~b}$ with $v_{0}=0$ to find the time of travel.

$$
\begin{aligned}
& x-x_{0}=v_{0} t+\frac{1}{2} a t^{2} \rightarrow \\
& t=\sqrt{\frac{2\left(x-x_{0}\right)}{a}}=\sqrt{\frac{2\left(x-x_{0}\right)}{g \sin \theta}}=\sqrt{\frac{2(3.0 \mathrm{~m})}{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \sin \left(8.5^{\circ}\right)}}=2.0 \mathrm{~s}
\end{aligned}
$$

Since the mass does not enter into the calculation, the time would be the same for the heavier bar of soap.
66. See the free-body diagram for the load. The vertical component of the tension force must be equal to the weight of the load, and the horizontal component of the tension accelerates the load. The angle is exaggerated in the picture.

$$
\begin{aligned}
& F_{\text {net }}=F_{\mathrm{T}} \sin \theta=m a \rightarrow a=\frac{F_{\mathrm{T}} \sin \theta}{m} ; F_{\substack{\text { net } \\
\mathrm{x}}}=F_{\mathrm{T}} \cos \theta-m g=0 \rightarrow \\
& F_{\mathrm{T}}=\frac{m g}{\cos \theta} \rightarrow a_{\mathrm{H}}=\frac{m g}{\cos \theta} \frac{\sin \theta}{m}=g \tan \theta=\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \tan 5.0^{\circ}=0.86 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$


67.
(a) Draw a free-body diagram for each block. Write Newton's second law for each block. Notice that the acceleration of block A in the $y_{\mathrm{A}}$ direction will be zero, since it has no motion in the $y_{\mathrm{A}}$ direction.

$$
\begin{aligned}
& \sum F_{y \mathrm{~A}}=F_{\mathrm{N}}-m_{\mathrm{A}} g \cos \theta=0 \rightarrow F_{\mathrm{N}}=m_{\mathrm{A}} g \cos \theta \\
& \sum F_{x \mathrm{~A}}=m_{\mathrm{A}} g \sin \theta-F_{\mathrm{T}}=m_{\mathrm{A}} a_{x \mathrm{~A}} \\
& \sum F_{y \mathrm{~B}}=F_{\mathrm{T}}-m_{\mathrm{B}} g=m_{\mathrm{B}} a_{y \mathrm{~B}} \rightarrow F_{\mathrm{T}}=m_{\mathrm{B}}\left(g+a_{y \mathrm{~B}}\right)
\end{aligned}
$$



Since the blocks are connected by the cord, $a_{y \mathrm{~B}}=a_{x \mathrm{~A}}=a$. Substitute the expression for the tension force from the last equation into the $x$ direction equation for block 1 , and solve for the acceleration.

$$
\begin{aligned}
& m_{\mathrm{A}} g \sin \theta-m_{\mathrm{B}}(g+a)=m_{\mathrm{A}} a \rightarrow m_{\mathrm{A}} g \sin \theta-m_{\mathrm{B}} g=m_{\mathrm{A}} a+m_{\mathrm{B}} a \\
& a=g \frac{\left(m_{\mathrm{A}} \sin \theta-m_{\mathrm{B}}\right)}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)}
\end{aligned}
$$

(b) If the acceleration is to be down the plane, it must be positive. That will happen if $m_{\mathrm{A}} \sin \theta>m_{\mathrm{B}}$ (down the plane). The acceleration will be up the plane (negative) if $m_{\mathrm{A}} \sin \theta<m_{\mathrm{B}}$ (up the plane). If $m_{\mathrm{A}} \sin \theta=m_{\mathrm{B}}$, then the system will not accelerate. It will move with a constant speed if set in motion by a push.
68. (a) From problem 67, we have an expression for the acceleration.

$$
\begin{aligned}
a & =g \frac{\left(m_{\mathrm{A}} \sin \theta-m_{\mathrm{B}}\right)}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)}=\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \frac{\left[(1.00 \mathrm{~kg}) \sin 33.0^{\circ}-1.00 \mathrm{~kg}\right]}{2.00 \mathrm{~kg}}=-2.23 \mathrm{~m} / \mathrm{s}^{2} \\
& \approx-2.2 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

The negative sign means that $m_{\mathrm{A}}$ will be accelerating UP the plane.
(b) If the system is at rest, then the acceleration will be 0 .

$$
a=g \frac{\left(m_{\mathrm{A}} \sin \theta-m_{\mathrm{B}}\right)}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)}=0 \rightarrow m_{\mathrm{B}}=m_{\mathrm{A}} \sin \theta=(1.00 \mathrm{~kg}) \sin 33.0^{\circ}=0.5446 \mathrm{~kg} \approx 0.545 \mathrm{~kg}
$$

(c) Again from problem 68, we have $F_{\mathrm{T}}=m_{\mathrm{B}}(g+a)$.

Case $(a): F_{\mathrm{T}}=m_{\mathrm{B}}(g+a)=(1.00 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}-2.23 \mathrm{~m} / \mathrm{s}^{2}\right)=7.57 \mathrm{~N} \approx 7.6 \mathrm{~N}$
Case $(b): F_{\mathrm{T}}=m_{\mathrm{B}}(g+a)=(0.5446 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}+0\right)=5.337 \mathrm{~N} \approx 5.34 \mathrm{~N}$
69. (a) A free-body diagram is shown for each block. We define the positive $x$-direction for $m_{\mathrm{A}}$ to be up its incline, and the positive $x$-direction for $m_{\mathrm{B}}$ to be down its incline. With that definition the masses will both have the same acceleration. Write Newton's second law for each body in the $x$ direction, and combine those equations to find the acceleration.


$$
\begin{aligned}
& m_{\mathrm{A}}: \sum F_{x}=F_{\mathrm{T}}-m_{\mathrm{A}} g \sin \theta_{\mathrm{A}}=m_{\mathrm{A}} a \\
& m_{\mathrm{B}}: \sum F_{x}=m_{\mathrm{B}} g \sin \theta_{\mathrm{B}}-F_{\mathrm{T}}=m_{\mathrm{B}} a \quad \text { add these two equations } \\
& \left(F_{\mathrm{T}}-m_{\mathrm{A}} g \sin \theta_{\mathrm{A}}\right)+\left(m_{\mathrm{B}} g \sin \theta_{\mathrm{B}}-F_{\mathrm{T}}\right)=m_{\mathrm{A}} a+m_{\mathrm{B}} a \rightarrow a=\frac{m_{\mathrm{B}} \sin \theta_{\mathrm{B}}-m_{\mathrm{A}} \sin \theta_{\mathrm{A}}}{m_{\mathrm{A}}+m_{\mathrm{B}}} g
\end{aligned}
$$

(b) For the system to be at rest, the acceleration must be 0 .

$$
\begin{aligned}
& a=\frac{m_{\mathrm{B}} \sin \theta_{\mathrm{B}}-m_{\mathrm{A}} \sin \theta_{\mathrm{A}}}{m_{\mathrm{A}}+m_{\mathrm{B}}} g=0 \rightarrow m_{\mathrm{B}} \sin \theta_{\mathrm{B}}-m_{\mathrm{A}} \sin \theta_{\mathrm{A}} \rightarrow \\
& m_{\mathrm{B}}=m_{\mathrm{A}} \frac{\sin \theta_{\mathrm{A}}}{\sin \theta_{\mathrm{B}}}=(5.0 \mathrm{~kg}) \frac{\sin 32^{\circ}}{\sin 23^{\circ}}=6.8 \mathrm{~kg}
\end{aligned}
$$

The tension can be found from one of the Newton's second law expression from part (a).

$$
m_{\mathrm{A}}: F_{\mathrm{T}}-m_{\mathrm{A}} g \sin \theta_{\mathrm{A}}=0 \rightarrow F_{\mathrm{T}}=m_{\mathrm{A}} g \sin \theta_{\mathrm{A}}=(5.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \sin 32^{\circ}=26 \mathrm{~N}
$$

(c) As in part (b), the acceleration will be 0 for constant velocity in either direction.

$$
\begin{aligned}
& a=\frac{m_{\mathrm{B}} \sin \theta_{\mathrm{B}}-m_{\mathrm{A}} \sin \theta_{\mathrm{A}}}{m_{\mathrm{A}}+m_{\mathrm{B}}} g=0 \rightarrow m_{\mathrm{B}} \sin \theta_{\mathrm{B}}-m_{\mathrm{A}} \sin \theta_{\mathrm{A}} \rightarrow \\
& \frac{m_{\mathrm{A}}}{m_{\mathrm{B}}}=\frac{\sin \theta_{\mathrm{B}}}{\sin \theta_{\mathrm{A}}}=\frac{\sin 23^{\circ}}{\sin 32^{\circ}}=0.74
\end{aligned}
$$

70. A free-body diagram for the person in the elevator is shown. The scale reading is the magnitude of the normal force. Choosing up to be the positive direction, Newton's second law for the person says that $\sum F=F_{\mathrm{N}}-m g=m a \rightarrow F_{\mathrm{N}}=m(g+a)$. The kg reading of the scale is the apparent weight, $F_{\mathrm{N}}$, divided by $g$, which gives
$F_{\mathrm{N}-\mathrm{kg}}=\frac{F_{\mathrm{N}}}{g}=\frac{m(g+a)}{g}$.
(a) $a=0 \rightarrow F_{\mathrm{N}}=m g=(75.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=7.35 \times 10^{2} \mathrm{~N}$

$$
F_{\mathrm{N} \mathrm{~kg}}=\frac{m g}{g}=m=75.0 \mathrm{~kg}
$$

(b) $a=0 \rightarrow F_{\mathrm{N}}=7.35 \times 10^{2} \mathrm{~N}, F_{\mathrm{N} \mathrm{kg}}=75.0 \mathrm{~kg}$
(c) $a=0 \rightarrow F_{\mathrm{N}}=7.35 \times 10^{2} \mathrm{~N}, F_{\mathrm{N}-\mathrm{kg}}=75.0 \mathrm{~kg}$
(d) $F_{\mathrm{N}}=m(g+a)=(75.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}+3.0 \mathrm{~m} / \mathrm{s}^{2}\right) a=9.60 \times 10^{2} \mathrm{~N}$

$$
F_{\mathrm{N} \cdot \mathrm{~kg}}=\frac{F_{\mathrm{N}}}{g}=\frac{960 \mathrm{~N}}{9.80 \mathrm{~m} / \mathrm{s}^{2}}=98.0 \mathrm{~kg}
$$

(e) $F_{\mathrm{N}}=m(g+a)=(75.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}-3.0 \mathrm{~m} / \mathrm{s}^{2}\right) a=5.1 \times 10^{2} \mathrm{~N}$

$$
F_{\mathrm{N} \mathrm{~kg}}=\frac{F_{\mathrm{N}}}{g}=\frac{510 \mathrm{~N}}{9.80 \mathrm{~m} / \mathrm{s}^{2}}=52 \mathrm{~kg}
$$

71. The given data can be used to calculate the force with which the road pushes against the car, which in turn is equal in magnitude to the force the car pushes against the road. The acceleration of the car on level ground is found from Eq. 2-12a.

$$
v-v_{0}=a t \rightarrow a=\frac{v-v_{0}}{t}=\frac{21 \mathrm{~m} / \mathrm{s}-0}{12.5 \mathrm{~s}}=1.68 \mathrm{~m} / \mathrm{s}^{2}
$$

The force pushing the car in order to have this acceleration is found from Newton's second law.


$$
F_{\mathrm{P}}=m a=(920 \mathrm{~kg})\left(1.68 \mathrm{~m} / \mathrm{s}^{2}\right)=1546 \mathrm{~N}
$$

We assume that this is the force pushing the car on the incline as well. Consider a free-body diagram for the car climbing the hill. We assume that the car will have a constant speed on the maximum incline. Write Newton's second law for the $x$ direction, with a net force of zero since the car is not accelerating.

$$
\begin{aligned}
& \sum F_{x}=F_{\mathrm{P}}-m g \sin \theta=0 \rightarrow \sin \theta=\frac{F_{\mathrm{P}}}{m g} \\
& \theta=\sin ^{-1} \frac{F_{\mathrm{P}}}{m g}=\sin ^{-1} \frac{1546 \mathrm{~N}}{(920 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=9.9^{\circ}
\end{aligned}
$$

72. Consider a free-body diagram for the cyclist coasting downhill at a constant speed. Since there is no acceleration, the net force in each direction must be zero. Write Newton's second law for the $x$ direction (down the plane).

$$
\sum F_{x}=m g \sin \theta-F_{\mathrm{ff}}=0 \rightarrow F_{\mathrm{fr}}=m g \sin \theta
$$

This establishes the size of the air friction force at $6.0 \mathrm{~km} / \mathrm{h}$, and so can be used in the next part.


Now consider a free-body diagram for the cyclist climbing the hill. $F_{\mathrm{P}}$ is the force pushing the cyclist uphill. Again, write Newton's second law for the $x$ direction, with a net force of 0 .

$$
\begin{aligned}
& \sum F_{x}=F_{\mathrm{fr}}+m g \sin \theta-F_{\mathrm{P}}=0 \rightarrow \\
& F_{\mathrm{P}}=F_{\mathrm{fr}}+m g \sin \theta=2 m g \sin \theta \\
& \quad=2(65 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(\sin 6.5^{\circ}\right)=1.4 \times 10^{2} \mathrm{~N}
\end{aligned}
$$


73. (a) The value of the constant $c$ can be found from the free-body diagram, knowing that the net force is 0 when coasting downhill at the specified speed.

$$
\sum F_{x}=m g \sin \theta-F_{\text {air }}=0 \rightarrow F_{\text {air }}=m g \sin \theta=c v \rightarrow
$$



$$
c=\frac{m g \sin \theta}{v}=\frac{(80.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \sin 5.0^{\circ}}{(6.0 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)}=40.998 \frac{\mathrm{~N}}{\mathrm{~m} / \mathrm{s}} \approx 41 \frac{\mathrm{~N}}{\mathrm{~m} / \mathrm{s}}
$$

(b) Now consider the cyclist with an added pushing force $\overrightarrow{\mathbf{F}}_{\mathrm{P}}$ directed along the plane. The free-body diagram changes to reflect the additional force the cyclist must exert. The same axes definitions are used as in part (a).

$$
\begin{aligned}
& \sum F_{x}=F_{\mathrm{p}}+m g \sin \theta-F_{\text {air }}=0 \rightarrow \\
& F_{\mathrm{p}}=F_{\text {air }}-m g \sin \theta=c v-m g \sin \theta \\
& =\left(40.998 \frac{\mathrm{~N}}{\mathrm{~m} / \mathrm{s}}\right)\left((18.0 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)\right) \\
& \quad-(80.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \sin 5.0^{\circ}=136.7 \mathrm{~N} \approx 140 \mathrm{~N}
\end{aligned}
$$

74. Consider the free-body diagram for the watch. Write Newton's second law for both the $x$ and $y$ directions. Note that the net force in the $y$ direction is 0 because there is no acceleration in the $y$ direction.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{T}} \cos \theta-m g=0 \rightarrow F_{\mathrm{T}}=\frac{m g}{\cos \theta} \\
& \sum F_{x}=F_{\mathrm{T}} \sin \theta=m a \rightarrow \frac{m g}{\cos \theta} \sin \theta=m a \\
& \quad a=g \tan \theta=\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \tan 25^{\circ}=4.57 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$



Use Eq. 2-12a with $v_{0}=0$ to find the final velocity (takeoff speed).

$$
v-v_{0}=a t \rightarrow v=v_{0}+a t=0+\left(4.57 \mathrm{~m} / \mathrm{s}^{2}\right)(16 \mathrm{~s})=73 \mathrm{~m} / \mathrm{s}
$$

75. (a) To find the minimum force, assume that the piano is moving with a constant velocity. Since the piano is not accelerating, $F_{\mathrm{T} 4}=M g$. For the lower pulley, since the tension in a rope is the same throughout, and since the pulley is not accelerating, it is seen that $F_{\mathrm{T} 1}+F_{\mathrm{T} 2}=2 F_{\mathrm{T} 1}=M g \rightarrow F_{\mathrm{T} 1}=F_{\mathrm{T} 2}=M g / 2$.
It also can be seen that since $F=F_{\mathrm{T} 2}$, that $F=M g / 2$.
(b) Draw a free-body diagram for the upper pulley. From that
 diagram, we see that $F_{\mathrm{T} 3}=F_{\mathrm{T} 1}+F_{\mathrm{T} 2}+F=\frac{3 M g}{2}$.
To summarize:

$$
F_{\mathrm{T} 1}=F_{\mathrm{T} 2}=M g / 2 \quad F_{\mathrm{T} 3}=3 M g / 2 \quad F_{\mathrm{T} 4}=M g
$$


76. Consider a free-body diagram for a grocery cart being pushed up an incline. Assuming that the cart is not accelerating, we write Newton's second law for the $x$ direction.

$$
\begin{aligned}
& \sum F_{x}=F_{P}-m g \sin \theta=0 \rightarrow \sin \theta=\frac{F_{P}}{m g} \\
& \theta=\sin ^{-1} \frac{F_{P}}{m g}=\sin ^{-1} \frac{18 \mathrm{~N}}{(25 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=4.2^{\circ}
\end{aligned}
$$


77. The acceleration of the pilot will be the same as that of the plane, since the pilot is at rest with respect to the plane. Consider first a free-body diagram of the pilot, showing only the net force. By Newton's second law, the net force MUST point in the direction of the acceleration, and its magnitude is $m a$. That net force is the sum of ALL forces on the pilot. If we assume that the force of gravity and the force of the cockpit seat on the pilot are the only forces on the pilot, then in terms of vectors, $\overrightarrow{\mathbf{F}}_{\text {net }}=m \overrightarrow{\mathbf{g}}+\overrightarrow{\mathbf{F}}_{\text {seat }}=m \overrightarrow{\mathbf{a}}$. Solve this equation for the force of the seat to find $\overrightarrow{\mathbf{F}}_{\text {seat }}=\overrightarrow{\mathbf{F}}_{\text {net }}-m \overrightarrow{\mathbf{g}}=m \overrightarrow{\mathbf{a}}-m \overrightarrow{\mathbf{g}}$. A vector diagram of that equation is shown. Solve for the force of the seat on the pilot using components.


$$
\begin{aligned}
F_{x \text { seat }} & =F_{x \text { net }}=m a \cos 18^{\circ}=(75 \mathrm{~kg})\left(3.8 \mathrm{~m} / \mathrm{s}^{2}\right) \cos 18^{\circ}=271.1 \mathrm{~N} \\
F_{y \text { seat }} & =m g+F_{y \text { net }}=m g+m a \sin 18^{\circ} \\
& =(75 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)+(75 \mathrm{~kg})\left(3.8 \mathrm{~m} / \mathrm{s}^{2}\right) \sin 18^{\circ}=823.2 \mathrm{~N}
\end{aligned}
$$

The magnitude of the cockpit seat force is as follows.

$$
F=\sqrt{F_{x \text { seat }}^{2}+F_{y \text { seat }}^{2}}=\sqrt{(271.1 \mathrm{~N})^{2}+(823.2 \mathrm{~N})^{2}}=866.7 \mathrm{~N} \approx 870 \mathrm{~N}
$$

The angle of the cockpit seat force is as follows.

$$
\theta=\tan ^{-1} \frac{F_{y \text { seat }}}{F_{x \text { seat }}}=\tan ^{-1} \frac{823.2 \mathrm{~N}}{271.1 \mathrm{~N}}=72^{\circ} \text { above the horizontal }
$$

78. (a) The helicopter and frame will both have the same acceleration, and so can be treated as one object if no information about internal forces (like the cable tension) is needed. A free-body diagram for the helicopter-frame combination is shown. Write Newton's second law for the combination, calling UP the positive direction.

$$
\begin{aligned}
\sum F & =F_{\text {lift }}-\left(m_{\mathrm{H}}+m_{\mathrm{F}}\right) g=\left(m_{\mathrm{H}}+m_{\mathrm{F}}\right) a \rightarrow \\
F_{\text {lift }} & =\left(m_{\mathrm{H}}+m_{\mathrm{F}}\right)(g+a)=(7650 \mathrm{~kg}+1250 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}+0.80 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =9.43 \times 10^{4} \mathrm{~N}
\end{aligned}
$$


(b) Now draw a free-body diagram for the frame alone, in order to find the tension in the cable. Again use Newton's second law.

$$
\begin{aligned}
& \sum F=F_{\mathrm{T}}-m_{\mathrm{F}} g=m_{\mathrm{F}} a \rightarrow \\
& F_{\mathrm{T}}=m_{\mathrm{F}}(g+a)=(1250 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}+0.80 \mathrm{~m} / \mathrm{s}^{2}\right)=1.33 \times 10^{4} \mathrm{~N}
\end{aligned}
$$

(c) The tension in the cable is the same at both ends, and so the cable exerts a
 force of $1.33 \times 10^{4} \mathrm{~N}$ downward on the helicopter.
79. (a) We assume that the maximum horizontal force occurs when the train is moving very slowly, and so the air resistance is negligible. Thus the maximum acceleration is given by the following.

$$
a_{\text {max }}=\frac{F_{\text {max }}}{m}=\frac{4 \times 10^{5} \mathrm{~N}}{6.4 \times 10^{5} \mathrm{~kg}}=0.625 \mathrm{~m} / \mathrm{s}^{2} \approx 0.6 \mathrm{~m} / \mathrm{s}^{2}
$$

(b) At top speed, we assume that the train is moving at constant velocity. Therefore the net force on the train is 0 , and so the air resistance and friction forces together must be of the same magnitude as the horizontal pushing force, which is $1.5 \times 10^{5} \mathrm{~N}$.
80. See the free-body diagram for the fish being pulled upward vertically. From Newton's second law, calling the upward direction positive, we have this relationship.

$$
\sum F_{y}=F_{\mathrm{T}}-m g=m a \rightarrow F_{\mathrm{T}}=m(g+a)
$$

(a) If the fish has a constant speed, then its acceleration is zero, and so $F_{\mathrm{T}}=m g$. Thus the heaviest fish that could be pulled from the water in this case is $45 \mathrm{~N}(10 \mathrm{lb})$.

(b) If the fish has an acceleration of $2.0 \mathrm{~m} / \mathrm{s}^{2}$, and $F_{\mathrm{T}}$ is at its maximum of 45 N , then solve the equation for the mass of the fish.

$$
\begin{aligned}
& m=\frac{F_{\mathrm{T}}}{g+a}=\frac{45 \mathrm{~N}}{9.8 \mathrm{~m} / \mathrm{s}^{2}+2.0 \mathrm{~m} / \mathrm{s}^{2}}=3.8 \mathrm{~kg} \rightarrow \\
& m g=(3.8 \mathrm{~kg})\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)=37 \mathrm{~N}(\approx 8.4 \mathrm{lb})
\end{aligned}
$$

(c) It is not possible to land a $15-\mathrm{lb}$ fish using $10-\mathrm{lb}$ line, if you have to lift the fish vertically. If the fish were reeled in while still in the water, and then a net used to remove the fish from the water, it might still be caught with the $10-\mathrm{lb}$ line.
81. Choose downward to be positive. The elevator's acceleration is calculated by Eq. 2-12c.

$$
v^{2}-v_{0}^{2}=2 a\left(y-y_{0}\right) \rightarrow a=\frac{v^{2}-v_{0}^{2}}{2\left(y-y_{0}\right)}=\frac{0-(3.5 \mathrm{~m} / \mathrm{s})^{2}}{2(2.6 \mathrm{~m})}=-2.356 \mathrm{~m} / \mathrm{s}^{2}
$$

See the free-body diagram of the elevator/occupant combination. Write Newton's second law for the elevator.

$$
\begin{aligned}
& \sum F_{y}=m g-F_{\mathrm{T}}=m a \\
& F_{\mathrm{T}}=m(g-a)=(1450 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}--2.356 \mathrm{~m} / \mathrm{s}^{2}\right)=1.76 \times 10^{4} \mathrm{~N}
\end{aligned}
$$


82. (a) First calculate Karen's speed from falling. Let the downward direction be positive, and use Eq. $2-12 \mathrm{c}$ with $v_{0}=0$.

$$
v^{2}-v_{0}^{2}=2 a\left(y-y_{0}\right) \rightarrow v=\sqrt{0+2 a\left(y-y_{0}\right)}=\sqrt{2\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)(2.0 \mathrm{~m})}=6.26 \mathrm{~m} / \mathrm{s}
$$

Now calculate the average acceleration as the rope stops Karen, again using Eq. 2-12c, with down as positive.

$$
v^{2}-v_{0}^{2}=2 a\left(y-y_{0}\right) \rightarrow a=\frac{v^{2}-v_{0}^{2}}{2\left(y-y_{0}\right)}=\frac{0-(6.26 \mathrm{~m} / \mathrm{s})^{2}}{2(1.0 \mathrm{~m})}=-19.6 \mathrm{~m} / \mathrm{s}^{2}
$$

The negative sign indicates that the acceleration is upward. Since this is her acceleration, the net force on Karen is given by Newton's second law, $F_{\text {net }}=m a$. That net force will also be upward. Now consider the free-body diagram of Karen as

she decelerates. Call DOWN the positive direction. Newton's second law says that $F_{\text {net }}=m a=m g-F_{\text {rope }} \rightarrow F_{\text {rope }}=m g-m a$. The ratio of this force to Karen's weight is $\frac{F_{\text {rope }}}{m g}=\frac{m g-m a}{g}=1.0-\frac{a}{g}=1.0-\frac{-19.6 \mathrm{~m} / \mathrm{s}^{2}}{9.8 \mathrm{~m} / \mathrm{s}^{2}}=3.0$. Thus the rope pulls upward on Karen with an average force of 3.0 times her weight.
(b) A completely analogous calculation for Bill gives the same speed after the 2.0 m fall, but since he stops over a distance of 0.30 m , his acceleration is $-65 \mathrm{~m} / \mathrm{s}^{2}$, and the rope pulls upward on Bill with an average force of $\square$ 7.7 times his weight. Thus, Bill is more likely to get hurt.
83. Since the climbers are on ice, the frictional force for the lower two climbers is negligible. Consider the freebody diagram as shown. Note that all the masses are the same. Write Newton's second law in the $x$ direction for the lowest climber, assuming he is at rest.

$$
\begin{aligned}
& \sum F_{x}=F_{\mathrm{T} 2}-m g \sin \theta=0 \\
& F_{\mathrm{T} 2}=m g \sin \theta=(75 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \sin 31.0^{\circ} \\
& \quad=380 \mathrm{~N}
\end{aligned}
$$

Write Newton's second law in the $x$ direction for the
 middle climber, assuming he is at rest.

$$
\sum F_{x}=F_{\mathrm{T} 1}-F_{\mathrm{T} 2}-m g \sin \theta=0 \rightarrow F_{\mathrm{T} 1}=F_{\mathrm{T} 2}+m g \sin \theta=2 F_{\mathrm{T} 2} g \sin \theta=760 \mathrm{~N}
$$

84. Use Newton's second law.

$$
F=m a=m \frac{\Delta v}{\Delta t} \rightarrow \Delta t=\frac{m \Delta v}{F}=\frac{\left(1.0 \times 10^{10} \mathrm{~kg}\right)\left(2.0 \times 10^{-3} \mathrm{~m} / \mathrm{s}\right)}{(2.5 \mathrm{~N})}=8.0 \times 10^{6} \mathrm{~s}=93 \mathrm{~d}
$$

85. Use the free-body diagram to find the net force in the $x$ direction, and then find the acceleration. Then Eq. 2-12c can be used to find the final speed at

$$
\begin{aligned}
& \text { Use the free-body diagram to find the net force in the } x \text { direction, and then } \\
& \text { find the acceleration. Then Eq. 2-12c can be used to find the final speed at } \\
& \text { the bottom of the ramp. } \\
& \qquad \begin{array}{l}
\sum F_{x}=m g \sin \theta-F_{\mathrm{P}}=m a \rightarrow \\
a=\frac{m g \sin \theta-F_{\mathrm{P}}}{m}=\frac{(450 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \sin 22^{\circ}-1420 \mathrm{~N}}{450 \mathrm{~kg}} \\
=0.516 \mathrm{~m} / \mathrm{s}^{2} \\
v^{2}=v_{0}^{2}+2 a\left(x-x_{0}\right) \rightarrow v=\sqrt{2 a\left(x-x_{0}\right)}=\sqrt{2\left(0.516 \mathrm{~m} / \mathrm{s}^{2}\right)(11.5 \mathrm{~m})}=3.4 \mathrm{~m} / \mathrm{s}
\end{array}
\end{aligned}
$$

86. (a) We use the free-body diagram to find the force needed to pull the masses at a constant velocity. We choose the "up the plane" direction as the positive direction for both masses. Then they both have the same acceleration even if it is non-zero.

$$
\begin{aligned}
& m_{\mathrm{A}}: \sum F_{x}=F_{\mathrm{T}}-m_{\mathrm{A}} g \sin \theta_{\mathrm{A}}=m_{\mathrm{A}} a=0 \\
& m_{\mathrm{b}}: \sum F_{x}=F-F_{\mathrm{T}}-m_{\mathrm{B}} g \sin \theta_{\mathrm{B}}=m_{\mathrm{B}} a=0
\end{aligned}
$$

Add the equations to eliminate the tension force and solve for $F$.

$$
\begin{aligned}
& \left(F_{\mathrm{T}}-m_{\mathrm{A}} g \sin \theta_{\mathrm{A}}\right)+\left(F-F_{\mathrm{T}}-m_{\mathrm{B}} g \sin \theta_{\mathrm{B}}\right)=0 \rightarrow \\
& F=g\left(m_{\mathrm{A}} \sin \theta_{\mathrm{A}}+m_{\mathrm{B}} \sin \theta_{\mathrm{B}}\right) \\
& \quad=\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left[(9.5 \mathrm{~kg}) \sin 59^{\circ}+(11.5 \mathrm{~kg}) \sin 32^{\circ}\right]=1.40 \times 10^{2} \mathrm{~N}
\end{aligned}
$$

(b) Since $\theta_{\mathrm{A}}>\theta_{\mathrm{B}}$, if there were no connecting string, $m_{\mathrm{A}}$ would have a larger acceleration than $m_{\mathrm{B}}$. If $\theta_{\mathrm{A}}<\theta_{\mathrm{B}}$, there would be no tension. But, since there is a connecting string, there will be tension in the string. Use the free-body diagram from above but ignore the applied force $\overrightarrow{\mathbf{F}}$.

$$
m_{\mathrm{A}}: \sum F_{x}=F_{\mathrm{T}}-m_{\mathrm{A}} g \sin \theta_{\mathrm{A}}=m_{\mathrm{A}} a ; m_{\mathrm{b}}: \sum F_{x}=-F_{\mathrm{T}}-m_{\mathrm{B}} g \sin \theta_{\mathrm{B}}=m_{\mathrm{B}} a
$$

Again add the two equations to eliminate the tension force.

$$
\begin{aligned}
&\left(F_{\mathrm{T}}-m_{\mathrm{A}} g \sin \theta_{\mathrm{A}}\right)+\left(-F_{\mathrm{T}}-m_{\mathrm{B}} g \sin \theta_{\mathrm{B}}\right)=m_{\mathrm{A}} a+m_{\mathrm{B}} a \rightarrow \\
& a=-g \frac{m_{\mathrm{A}} \sin \theta_{\mathrm{A}}+m_{\mathrm{B}} \sin \theta_{\mathrm{B}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}=-\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \frac{(9.5 \mathrm{~kg}) \sin 59^{\circ}+(11.5 \mathrm{~kg}) \sin 32^{\circ}}{21.0 \mathrm{~kg}} \\
&=-6.644 \mathrm{~m} / \mathrm{s}^{2} \approx 6.64 \mathrm{~m} / \mathrm{s}^{2}, \text { down the planes }
\end{aligned}
$$

(c) Use one of the Newton's second law expressions from part (b) to find the string tension. It must be positive if there is a tension.

$$
\begin{aligned}
& F_{\mathrm{T}}-m_{\mathrm{A}} g \sin \theta_{\mathrm{A}}=m_{\mathrm{A}} a \rightarrow \\
& F_{\mathrm{T}}=m_{\mathrm{A}}\left(g \sin \theta_{\mathrm{A}}+a\right)=(9.5 \mathrm{~kg})\left[\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(\sin 59^{\circ}\right)-6.644 \mathrm{~m} / \mathrm{s}^{2}\right]=17 \mathrm{~N}
\end{aligned}
$$

87. (a) If the 2-block system is taken as a whole system, then the net force on the system is just the force $\overrightarrow{\mathbf{F}}$, accelerating the total mass. Use Newton's second law to find the force from the mass and acceleration. Take the direction of motion caused by the force (left for the bottom block, right for the top block) as the positive direction. Then both blocks have the same acceleration.

$$
\sum F_{x}=F=\left(m_{\text {top }}+m_{\text {botom }}\right) a=(9.0 \mathrm{~kg})\left(2.5 \mathrm{~m} / \mathrm{s}^{2}\right)=22.5 \mathrm{~N} \approx 23 \mathrm{~N}
$$

(b) The tension in the connecting cord is the only force acting on the top block, and so must be causing its acceleration. Again use Newton's second law.

$$
\sum F_{x}=F_{\mathrm{T}}=m_{\mathrm{top}} a=(1.5 \mathrm{~kg})\left(2.5 \mathrm{~m} / \mathrm{s}^{2}\right)=3.75 \mathrm{~N} \approx 3.8 \mathrm{~N}
$$

This could be checked by using the bottom block.

$$
\sum F_{x}=F-F_{\mathrm{T}}=m_{\text {botom }} a \rightarrow F_{\mathrm{T}}=F-m_{\text {botom }} a=22.5 \mathrm{~N}-(7.5 \mathrm{~kg})\left(2.5 \mathrm{~m} / \mathrm{s}^{2}\right)=3.75 \mathrm{~N}
$$

88. (a) For this scenario, find your location at a time of 4.0 sec , using Eq. 2-12b. The acceleration is found from Newton's second law.

$$
a=\frac{F_{\text {forvard }}}{m}=\frac{1200 \mathrm{~N}}{750 \mathrm{~kg}} \rightarrow
$$

$$
x-x_{0}=v_{0} t+\frac{1}{2} a t^{2}=(15 \mathrm{~m} / \mathrm{s})(4.0 \mathrm{~s})+\frac{1}{2} \frac{1200 \mathrm{~N}}{750 \mathrm{~kg}}(4.0 \mathrm{~s})^{2}=72.8 \mathrm{~m}>65 \mathrm{~m}
$$

Yes, you will make it through the intersection before the light turns red.
(b) For this scenario, find your location when the car has been fully stopped, using Eq. 2-12c. The acceleration is found from Newton's second law.

$$
\begin{aligned}
& a=\frac{F_{\text {braking }}}{m}=-\frac{1800 \mathrm{~N}}{750 \mathrm{~kg}} \rightarrow v^{2}=v_{0}^{2}+2 a\left(x-x_{0}\right) \rightarrow \\
& x-x_{0}=\frac{v^{2}-v_{0}^{2}}{2 a}=\frac{0-(15 \mathrm{~m} / \mathrm{s})^{2}}{2\left(-\frac{1800 \mathrm{~N}}{750 \mathrm{~kg}}\right)}=46.9 \mathrm{~m}>45 \mathrm{~m}
\end{aligned}
$$

No, you will not stop before entering the intersection.
89. We take the mass of the crate as $m$ until we insert values. A free-body diagram is shown.
(a) (i) Use Newton's second law to find the acceleration.

$$
\sum F_{x}=m g \sin \theta=m a \rightarrow a=g \sin \theta
$$

(ii) Use Eq. 2-12b to find the time for a displacement of $\ell$.

$$
x-x_{0}=v_{0} t+\frac{1}{2} a t^{2} \quad \rightarrow \quad \ell=\frac{1}{2} g(\sin \theta) t^{2} \rightarrow
$$



$$
t=\sqrt{\frac{2 \ell}{g \sin \theta}}
$$

(iii) Use Eq. 2-12a to find the final velocity.

$$
v=v_{0}+a t=g \sin \theta\left[\sqrt{\frac{2 \ell}{g \sin \theta}}\right]=\sqrt{2 \ell g \sin \theta}
$$

(iv) Use Newton's second law to find the normal force.

$$
\sum F_{y}=F_{\mathrm{N}}-m g \cos \theta=0 \rightarrow F_{\mathrm{N}}=m g \cos \theta
$$

(b) Using the values of $m=1500 \mathrm{~kg}, g=9.80 \mathrm{~m} / \mathrm{s}^{2}$, and $\ell=100 \mathrm{~m}$, the requested quantities become as follows.

$$
\begin{aligned}
& a=(9.80 \sin \theta) \mathrm{m} / \mathrm{s}^{2} ; t=\sqrt{\frac{2(100)}{9.80 \sin \theta}} \mathrm{~s} ; \\
& v=\sqrt{2(100)(9.80) \sin \theta} \mathrm{m} / \mathrm{s} ; F_{\mathrm{N}}=(1500)(9.80) \cos \theta
\end{aligned}
$$

Graphs of these quantities as a function of $\theta$ are given here.


We consider the limiting cases: at an angle of $0^{\circ}$, the crate does not move, and so the acceleration and final velocity would be 0 . The time to travel 100 m would be infinite, and the normal force would be equal to the weight of

$$
\begin{aligned}
W & =m g \\
& =(1500 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =1.47 \times 10^{4} \mathrm{~N} .
\end{aligned}
$$



The graphs are all consistent with those results.

For an angle of $90^{\circ}$, we would expect free-fall motion. The acceleration should be $9.80 \mathrm{~m} / \mathrm{s}^{2}$. The normal force would be 0 . The free-fall time for an object dropped from rest a distance of 100 m and the final velocity after that distance are calculated below.


$$
\begin{aligned}
& x-x_{0}=v_{0} t+\frac{1}{2} a t^{2} \rightarrow \\
& \ell
\end{aligned}=\frac{1}{2} g t^{2} \rightarrow 7 \text { 有 } \begin{aligned}
\frac{2 \ell}{g} & =\sqrt{\frac{2(100 \mathrm{~m})}{9.80 \mathrm{~m} / \mathrm{s}^{2}}}=4.5 \mathrm{~s} \\
t & =\sqrt{v^{2}}=v_{0}^{2}+2 a\left(x-x_{0}\right) \rightarrow \\
v & =\sqrt{2 g\left(x-x_{0}\right)} \\
& =\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(100 \mathrm{~m})} \\
& =44 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$



Yes, the graphs agree with these results for the limiting cases.
The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH04.XLS," on tab "Problem 4.89b."

## CHAPTER 5: Using Newton's Laws: Friction, Circular Motion, Drag Forces

## Responses to Questions

1. Static friction between the crate and the truck bed causes the crate to accelerate.
2. The kinetic friction force is parallel to the ramp and the block's weight has a component parallel to the ramp. The parallel component of the block's weight is directed down the ramp whether the block is sliding up or down. However, the frictional force is always in the direction opposite the block's motion, so it will be down the ramp while the block is sliding up, but up the ramp while the block is sliding down. When the block is sliding up the ramp, the two forces acting on it parallel to the ramp are both acting in the same direction, and the magnitude of the net force is the sum of their magnitudes. But when the block is sliding down the ramp, the friction and the parallel component of the weight act in opposite directions, resulting in a smaller magnitude net force. A smaller net force yields a smaller (magnitude) acceleration.
3. Because the train has a larger mass. If the stopping forces on the truck and train are equal, the (negative) acceleration of the train will be much smaller than that of the truck, since acceleration is inversely proportional to mass $(\overrightarrow{\mathbf{a}}=\overrightarrow{\mathbf{F}} / m)$. The train will take longer to stop, as it has a smaller acceleration, and will travel a greater distance before stopping. The stopping force on the train may actually be greater than the stopping force on the truck, but not enough greater to compensate for the much greater mass of the train.
4. Yes. Refer to Table 5-1. The coefficient of static friction between rubber and many solid surfaces is typically between 1 and 4 . The coefficient of static friction can also be greater than one if either of the surfaces is sticky.
5. When a skier is in motion, a small coefficient of kinetic friction lets the skis move easily on the snow with minimum effort. A large coefficient of static friction lets the skier rest on a slope without slipping and keeps the skier from sliding backward when going uphill.
6. When the wheels of a car are rolling without slipping, the force between each tire and the road is static friction, whereas when the wheels lock, the force is kinetic friction. The coefficient of static friction is greater than the coefficient of kinetic friction for a set of surfaces, so the force of friction between the tires and the road will be greater if the tires are rolling. Once the wheels lock, you also have no steering control over the car. It is better to apply the brakes slowly and use the friction between the brake mechanism and the wheel to stop the car while maintaining control. If the road is slick, the coefficients of friction between the road and the tires are reduced, and it is even more important to apply the brakes slowly to stay in control.
7. (b). If the car comes to a stop without skidding, the force that stops the car is the force of kinetic friction between the brake mechanism and the wheels. This force is designed to be large. If you slam on the brakes and skid to a stop, the force that stops the car will be the force of kinetic friction between the tires and the road. Even with a dry road, this force is likely to be less that the force of kinetic friction between the brake mechanism and the wheels. The car will come to a stop more quickly if the tires continue to roll, rather than skid. In addition, once the wheels lock, you have no steering control over the car.
8. The forces in $(a),(b)$, and (d) are all equal to 400 N in magnitude.
(a) You exert a force of 400 N on the car; by Newton's third law the force exerted by the car on you also has a magnitude of 400 N .
(b) Since the car doesn't move, the friction force exerted by the road on the car must equal 400 N , too. Then, by Newton's third law, the friction force exerted by the car on the road is also 400 N .
(c) The normal force exerted by the road on you will be equal in magnitude to your weight (assuming you are standing vertically and have no vertical acceleration). This force is not required to be 400 N .
(d) The car is exerting a 400 N horizontal force on you, and since you are not accelerating, the ground must be exerting an equal and opposite horizontal force. Therefore, the magnitude of the friction force exerted by the road on you is 400 N .
9. On an icy surface, you need to put your foot straight down onto the sidewalk, with no component of velocity parallel to the surface. If you can do that, the interaction between you and the ice is through the static frictional force. If your foot has a component of velocity parallel to the surface of the ice, any resistance to motion will be caused by the kinetic frictional force, which is much smaller. You will be much more likely to slip.
10. Yes, the centripetal acceleration will be greater when the speed is greater since centripetal acceleration is proportional to the square of the speed. An object in uniform circular motion has an acceleration, since the direction of the velocity vector is changing even though the speed is constant.
11. No. The centripetal acceleration depends on $1 / \mathrm{r}$, so a sharp curve, with a smaller radius, will generate a larger centripetal acceleration than a gentle curve, with a larger radius. (Note that the centripetal force in this case is provided by the static frictional force between the car and the road.)
12. The three main forces on the child are the downward force of gravity (weight), the normal force up on the child from the horse, and the static frictional force on the child from the surface of the horse. The frictional force provides the centripetal acceleration. If there are other forces, such as contact forces between the child's hands or legs and the horse, which have a radial component, they will contribute to the centripetal acceleration.
13. As the child and sled come over the crest of the hill, they are moving in an arc. There must be a centripetal force, pointing inward toward the center of the arc. The combination of gravity (down) and the normal force (up) provides this centripetal force, which must be greater than or equal to zero. (At the top of the arc, $F_{y}=m g-N=m v^{2} / r \geq 0$.) The normal force must therefore be less than the child's weight.
14. No. The barrel of the dryer provides a centripetal force on the clothes to keep them moving in a circular path. A water droplet on the solid surface of the drum will also experience this centripetal force and move in a circle. However, as soon as the water droplet is at the location of a hole in the drum there will be no centripetal force on it and it will therefore continue moving in a path in the direction of its tangential velocity, which will take it out of the drum. There is no centrifugal force throwing the water outward; there is rather a lack of centripetal force to keep the water moving in a circular path.
15. When describing a centrifuge experiment, the force acting on the object in the centrifuge should be specified. Stating the rpm will let you calculate the speed of the object in the centrifuge. However, to find the force on an object, you will also need the distance from the axis of rotation.
16. She should let go of the string at the moment that the tangential velocity vector is directed exactly at the target.
17. The acceleration of the ball is inward, directly toward the pole, and is provided by the horizontal component of the tension in the string.
18. For objects (including astronauts) on the inner surface of the cylinder, the normal force provides a centripetal force which points inward toward the center of the cylinder. This normal force simulates the normal force we feel when on the surface of Earth.
(a) Falling objects are not in contact with the floor, so when released they will continue to move with constant velocity until the floor reaches them. From the frame of reference of the astronaut inside the cylinder, it will appear that the object falls in a curve, rather than straight down.
(b) The magnitude of the normal force on the astronaut's feet will depend on the radius and speed of the cylinder. If these are such that $v^{2} / r=g$ (so that $m v^{2} / r=m g$ for all objects), then the normal force will feel just like it does on the surface of Earth.
(c) Because of the large size of Earth compared to humans, we cannot tell any difference between the gravitational force at our heads and at our feet. In a rotating space colony, the difference in the simulated gravity at different distances from the axis of rotation would be significant.
19. At the top of bucket's arc, the gravitational force and normal forces from the bucket provide the centripetal force needed to keep the water moving in a circle. (If we ignore the normal forces, $m g=$ $m v^{2} / r$, so the bucket must be moving with speed $v \geq \sqrt{g r}$ or the water will spill out of the bucket.)
At the top of the arc, the water has a horizontal velocity. As the bucket passes the top of the arc, the velocity of the water develops a vertical component. But the bucket is traveling with the water, with the same velocity, and contains the water as it falls through the rest of its path.
20. (a) The normal force on the car is largest at point $C$. In this case, the centripetal force keeping the car in a circular path of radius $R$ is directed upward, so the normal force must be greater than the weight to provide this net upward force.
(b) The normal force is smallest at point A, the crest of the hill. At this point the centripetal force must be downward (towards the center of the circle) so the normal force must be less than the weight. (Notice that the normal force is equal to the weight at point B.)
(c) The driver will feel heaviest where the normal force is greatest, or at point C .
(d) The driver will feel lightest at point A, where the normal force is the least.
(e) At point A, the centripetal force is weight minus normal force, or $m g-N=m v^{2} / r$. The point at which the car just loses contact with the road corresponds to a normal force of zero. Setting $N=0$ gives $m g=m v^{2} / r$ or $v=\sqrt{g r}$.
21. Leaning in when rounding a curve on a bicycle puts the bicycle tire at an angle with respect to the ground. This increases the component of the (static) frictional force on the tire due to the road. This force component points inward toward the center of the curve, thereby increasing the centripetal force on the bicycle and making it easier to turn.
22. When an airplane is in level flight, the downward force of gravity is counteracted by the upward lift force, analogous to the upward normal force on a car driving on a level road. The lift on an airplane is perpendicular to the plane of the airplane's wings, so when the airplane banks, the lift vector has both vertical and horizontal components (similar to the vertical and horizontal components of the normal force on a car on a banked turn). The vertical component of the lift balances the weight and the horizontal component of the lift provides the centripetal force. If $L=$ the total lift and $\varphi=$ the banking angle, measured from the vertical, then $L \cos \varphi=m g$ and $L \sin \varphi=m v^{2} / r$ so
$\varphi=\tan ^{-1}\left(v^{2} / g r\right)$.
23. If we solve for $b$, we have $b=-F / v$. The units for $b$ are $\mathrm{N} \cdot \mathrm{s} / \mathrm{m}=\mathrm{kg} \cdot \mathrm{m} \cdot \mathrm{s} /\left(\mathrm{m} \cdot \mathrm{s}^{2}\right)=\mathrm{kg} / \mathrm{s}$.
24. The force proportional to $v^{2}$ will dominate at high speed.

## Solutions to Problems

1. A free-body diagram for the crate is shown. The crate does not accelerate vertically, and so $F_{\mathrm{N}}=m g$. The crate does not accelerate horizontally, and so $F_{\mathrm{P}}=F_{\mathrm{fr}}$.

$$
F_{\mathrm{P}}=F_{\mathrm{fr}}=\mu_{k} F_{\mathrm{N}}=\mu_{k} m g=(0.30)(22 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=65 \mathrm{~N}
$$



If the coefficient of kinetic friction is zero, then the horizontal force required is 0 N , since there is no friction to counteract. Of course, it would take a force to START the crate moving, but once it was moving, no further horizontal force would be necessary to maintain the motion.
2. A free-body diagram for the box is shown. Since the box does not accelerate vertically, $F_{\mathrm{N}}=m g$.
(a) To start the box moving, the pulling force must just overcome the force of static friction, and that means the force of static friction will reach its maximum value of $F_{\mathrm{fr}}=\mu_{s} F_{\mathrm{N}}$. Thus we have for the starting
 motion,

$$
\begin{aligned}
& \sum F_{x}=F_{\mathrm{P}}-F_{\mathrm{fr}}=0 \rightarrow \\
& F_{\mathrm{P}}=F_{\mathrm{fr}}=\mu_{s} F_{\mathrm{N}}=\mu_{s} m g \rightarrow \mu_{s}=\frac{F_{\mathrm{P}}}{m g}=\frac{35.0 \mathrm{~N}}{(6.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=0.60
\end{aligned}
$$

(b) The same force diagram applies, but now the friction is kinetic friction, and the pulling force is NOT equal to the frictional force, since the box is accelerating to the right.

$$
\begin{aligned}
& \sum F=F_{\mathrm{P}}-F_{\mathrm{fr}}=m a \rightarrow F_{\mathrm{P}}-\mu_{k} F_{\mathrm{N}}=m a \rightarrow F_{\mathrm{P}}-\mu_{k} m g=m a \rightarrow \\
& \mu_{k}=\frac{F_{\mathrm{P}}-m a}{m g}=\frac{35.0 \mathrm{~N}-(6.0 \mathrm{~kg})\left(0.60 \mathrm{~m} / \mathrm{s}^{2}\right)}{(6.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=0.53
\end{aligned}
$$

3. A free-body diagram for you as you stand on the train is shown. You do not accelerate vertically, and so $F_{\mathrm{N}}=m g$. The maximum static frictional force is $\mu_{s} F_{N}$, and that must be greater than or equal to the force needed to accelerate you in order for you not to slip.

$$
F_{\mathrm{fr}} \geq m a \rightarrow \mu_{s} F_{\mathrm{N}} \geq m a \rightarrow \mu_{s} m g \geq m a \rightarrow \mu_{s} \geq a / g=0.20 g / g=0.20
$$



The static coefficient of friction must be at least 0.20 for you to not slide.
4. See the included free-body diagram. To find the maximum angle, assume that the car is just ready to slide, so that the force of static friction is a maximum. Write Newton's second law for both directions. Note that for both directions, the net force must be zero since the car is not accelerating.

$$
\sum F_{y}=F_{\mathrm{N}}-m g \cos \theta=0 \rightarrow F_{\mathrm{N}}=m g \cos \theta
$$



$$
\begin{aligned}
& \sum F_{x}=m g \sin \theta-F_{\mathrm{fr}}=0 \rightarrow m g \sin \theta=F_{\mathrm{fr}}=\mu_{s} F_{\mathrm{N}}=\mu_{s} m g \cos \theta \\
& \mu_{s}=\frac{m g \sin \theta}{m g \cos \theta}=\tan \theta=0.90 \rightarrow \theta=\tan ^{-1} 0.90^{\circ}=42^{\circ}
\end{aligned}
$$

5. A free-body diagram for the accelerating car is shown. The car does not accelerate vertically, and so $F_{\mathrm{N}}=m g$. The static frictional force is the accelerating force, and so $F_{\mathrm{fr}}=m a$. If we assume the maximum acceleration, then we need the maximum force, and so the static frictional force would be its maximum value of $\mu_{s} F_{\mathrm{N}}$. Thus we have


$$
\begin{aligned}
& F_{\mathrm{fr}}=m a \rightarrow \mu_{s} F_{\mathrm{N}}=m a \rightarrow \mu_{s} m g=m a \rightarrow \\
& a=\mu_{s} g=0.90\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=8.8 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

6. (a) Here is a free-body diagram for the box at rest on the plane. The force of friction is a STATIC frictional force, since the box is at rest.
(b) If the box were sliding down the plane, the only change is that the force of friction would be a KINETIC frictional force.
(c) If the box were sliding up the plane, the force of friction would be a KINETIC frictional force, and it would point down the plane, in the opposite direction to that shown in the diagram.


Notice that the angle is not used in this solution.
7. Start with a free-body diagram. Write Newton's second law for each direction.

$$
\begin{aligned}
& \sum F_{x}=m g \sin \theta-F_{\mathrm{fr}}=m a_{x} \\
& \sum F_{y}=F_{\mathrm{N}}-m g \cos \theta=m a_{y}=0
\end{aligned}
$$

Notice that the sum in the $y$ direction is 0 , since there is no motion (and hence no acceleration) in the $y$ direction. Solve for the force of friction.


$$
\begin{aligned}
& m g \sin \theta-F_{\mathrm{fr}}=m a_{x} \rightarrow \\
& F_{\mathrm{fr}}=m g \sin \theta-m a_{x}=(25.0 \mathrm{~kg})\left[\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(\sin 27^{\circ}\right)-0.30 \mathrm{~m} / \mathrm{s}^{2}\right]=103.7 \mathrm{~N} \approx 1.0 \times 10^{2} \mathrm{~N}
\end{aligned}
$$

Now solve for the coefficient of kinetic friction. Note that the expression for the normal force comes from the $y$ direction force equation above.

$$
F_{\mathrm{fr}}=\mu_{k} F_{\mathrm{N}}=\mu_{k} m g \cos \theta \rightarrow \mu_{k}=\frac{F_{\mathrm{fr}}}{m g \cos \theta}=\frac{103.7 \mathrm{~N}}{(25.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(\cos 27^{\circ}\right)}=0.48
$$

8. The direction of travel for the car is to the right, and that is also the positive horizontal direction. Using the free-body diagram, write Newton's second law in the $x$ direction for the car on the level road. We assume that the car is just on the verge of skidding, so that the magnitude of the friction force is $F_{\mathrm{fr}}=\mu_{s} F_{\mathrm{N}}$.

$$
\sum F_{x}=-F_{\mathrm{fr}}=m a \quad F_{\mathrm{fr}}=-m a=-\mu_{s} m g \rightarrow \mu_{\mathrm{s}}=\frac{a}{g}=\frac{3.80 \mathrm{~m} / \mathrm{s}^{2}}{9.80 \mathrm{~m} / \mathrm{s}^{2}}=0.3878
$$



Now put the car on an inclined plane. Newton's second law in the $x$-direction for the car on the plane is used to find the acceleration. We again assume the car is on the verge of slipping, so the static frictional force is at its maximum.

$$
\begin{aligned}
& \sum F_{x}=-F_{\mathrm{fr}}-m g \sin \theta=m a \rightarrow \\
& a=\frac{-F_{\mathrm{fr}}-m g \sin \theta}{m}=\frac{-\mu_{\mathrm{s}} m g \cos \theta-m g \sin \theta}{m}=-g\left(\mu_{\mathrm{s}} \cos \theta+\sin \theta\right) \\
& \quad=-\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(0.3878 \cos 9.3^{\circ}+\sin 9.3^{\circ}\right)=-5.3 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$


9. Since the skier is moving at a constant speed, the net force on the skier must be 0 . See the free-body diagram, and write Newton's second law for both the $x$ and $y$ directions.

$$
\begin{aligned}
& m g \sin \theta=F_{\mathrm{fr}}=\mu_{s} F_{\mathrm{N}}=\mu_{s} m g \cos \theta \rightarrow \\
& \mu_{s}=\tan \theta=\tan 27^{\circ}=0.51
\end{aligned}
$$


10. A free-body diagram for the bar of soap is shown. There is no motion in the $y$ direction and thus no acceleration in the $y$ direction. Write Newton's second law for both directions, and use those expressions to find the acceleration of the soap.

$$
\begin{aligned}
& \sum F_{x}=F_{\mathrm{N}}-m g \cos \theta=0 \rightarrow F_{\mathrm{N}}=m g \cos \theta \\
& \sum F_{x}=m g \sin \theta-F_{\mathrm{ff}}=m a \\
& m a=m g \sin \theta-\mu_{k} F_{\mathrm{N}}=m g \sin \theta-\mu_{k} m g \cos \theta \\
& a=g\left(\sin \theta-\mu_{k} \cos \theta\right)
\end{aligned}
$$



Now use Eq. 2-12b, with an initial velocity of 0 , to find the final velocity.

$$
\begin{aligned}
& x=x_{0}+v_{0} t+\frac{1}{2} a t^{2} \rightarrow \\
& t=\sqrt{\frac{2 x}{a}}=\sqrt{\frac{2 x}{g\left(\sin \theta-\mu_{k} \cos \theta\right)}}=\sqrt{\frac{2(9.0 \mathrm{~m})}{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(\sin 8.0^{\circ}-(0.060) \cos 8.0^{\circ}\right)}}=4.8 \mathrm{~s}
\end{aligned}
$$

11. A free-body diagram for the box is shown, assuming that it is moving to the right. The "push" is not shown on the free-body diagram because as soon as the box moves away from the source of the pushing force, the push is no longer applied to the box. It is apparent from the diagram that $F_{\mathrm{N}}=m g$ for the vertical direction. We write Newton's second law for the horizontal direction, with
 positive to the right, to find the acceleration of the box.

$$
\begin{aligned}
& \sum F_{x}=-F_{\mathrm{fr}}=m a \rightarrow m a=-\mu_{k} F_{\mathrm{N}}=-\mu_{k} m g \rightarrow \\
& a=-\mu_{k} g=-0.15\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=-1.47 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

Eq. 2-12c can be used to find the distance that the box moves before stopping. The initial speed is $4.0 \mathrm{~m} / \mathrm{s}$, and the final speed will be 0 .

$$
v^{2}-v_{0}^{2}=2 a\left(x-x_{0}\right) \rightarrow x-x_{0}=\frac{v^{2}-v_{0}^{2}}{2 a}=\frac{0-(3.5 \mathrm{~m} / \mathrm{s})^{2}}{2\left(-1.47 \mathrm{~m} / \mathrm{s}^{2}\right)}=4.17 \mathrm{~m} \approx 4.2 \mathrm{~m}
$$

12. (a) A free-body diagram for the car is shown, assuming that it is moving to the right. It is apparent from the diagram that $F_{\mathrm{N}}=m g$ for the vertical direction. Write Newton's second law for the horizontal direction, with positive to the right, to find the acceleration of the car. Since the car is assumed to NOT be sliding, use the maximum force of static friction.


$$
\sum F_{x}=-F_{\mathrm{fr}}=m a \rightarrow m a=-\mu_{s} F_{\mathrm{N}}=-\mu_{s} m g \rightarrow a=-\mu_{s} g
$$

Eq. 2-12c can be used to find the distance that the car moves before stopping. The initial speed is given as $v$, and the final speed will be 0 .

$$
v^{2}-v_{0}^{2}=2 a\left(x-x_{0}\right) \rightarrow\left(x-x_{0}\right)=\frac{v^{2}-v_{0}^{2}}{2 a}=\frac{0-v^{2}}{2\left(-\mu_{s} g\right)}=\frac{v^{2}}{2 \mu_{s} g}
$$

(b) Using the given values:

$$
v=(95 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)=26.38 \mathrm{~m} / \mathrm{s} \quad\left(x-x_{0}\right)=\frac{v^{2}}{2 \mu_{s} g}=\frac{(26.38 \mathrm{~m} / \mathrm{s})^{2}}{2(0.65)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=55 \mathrm{~m}
$$

(c) From part (a), we see that the distance is inversely proportional to $g$, and so if $g$ is reduced by a factor of 6 , the distance is increased by a factor of 6 to 330 m .
13. We draw three free-body diagrams - one for the car, one for the trailer, and then "add" them for the combination of car and trailer. Note that since the car pushes against the ground, the ground will push against the car with an equal but oppositely directed force. $\overrightarrow{\mathbf{F}}_{\mathrm{CG}}$ is the force on the car due to the ground, $\overrightarrow{\mathbf{F}}_{\text {TC }}$ is the force on the trailer due to the car, and $\overrightarrow{\mathbf{F}}_{\text {CT }}$ is the force on
 the car due to the trailer. Note that by Newton's rhird law, $\left|\overrightarrow{\mathbf{F}}_{\mathrm{CT}}\right|=\left|\overrightarrow{\mathbf{F}}_{\mathrm{TC}}\right|$.

From consideration of the vertical forces in the individual free-body diagrams, it is apparent that the normal force on each object is equal to its
 weight. This leads to the conclusion that $F_{\mathrm{ff}}=\mu_{k} F_{\mathrm{NT}}=\mu_{k} m_{\mathrm{T}} g=$ $(0.15)(350 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=514.5 \mathrm{~N}$.
Now consider the combined free-body diagram. Write Newton's second law for the horizontal direction, This allows the calculation of the acceleration of the system.

$$
\begin{aligned}
& \sum F=F_{\mathrm{CG}}-F_{\mathrm{fr}}=\left(m_{\mathrm{C}}+m_{\mathrm{T}}\right) a \rightarrow \\
& a=\frac{F_{\mathrm{CG}}-F_{\mathrm{fr}}}{m_{\mathrm{C}}+m_{\mathrm{T}}}=\frac{3600 \mathrm{~N}-514.5 \mathrm{~N}}{1630 \mathrm{~kg}}=1.893 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$



Finally, consider the free-body diagram for the trailer alone. Again write Newton's second law for the horizontal direction, and solve for $F_{\mathrm{TC}}$.

$$
\begin{aligned}
& \sum F=F_{\mathrm{TC}}-F_{\mathrm{fr}}=m_{\mathrm{T}} a \rightarrow \\
& F_{\mathrm{TC}}=F_{\mathrm{fr}}+m_{\mathrm{T}} a=514.5 \mathrm{~N}+(350 \mathrm{~kg})\left(1.893 \mathrm{~m} / \mathrm{s}^{2}\right)=1177 \mathrm{~N} \approx 1200 \mathrm{~N}
\end{aligned}
$$

14. Assume that kinetic friction is the net force causing the deceleration. See the free-body diagram for the car, assuming that the right is the positive direction, and the direction of motion of the skidding car. There is no acceleration in the vertical direction, and so $F_{\mathrm{N}}=m g$. Applying Newton's second law to the $x$

direction gives the following.

$$
\sum F=-F_{f}=m a \rightarrow-\mu_{k} F_{N}=-\mu_{k} m g=m a \rightarrow a=-\mu_{k} g
$$

Use Eq. 2-12c to determine the initial speed of the car, with the final speed of the car being zero.

$$
\begin{aligned}
& v^{2}-v_{0}^{2}=2 a\left(x-x_{0}\right) \rightarrow \\
& v_{0}=\sqrt{v^{2}-2 a\left(x-x_{0}\right)}=\sqrt{0-2\left(-\mu_{k} g\right)\left(x-x_{0}\right)}=\sqrt{2(0.80)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(72 \mathrm{~m})}=34 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

15. (a) Consider the free-body diagram for the snow on the roof. If the snow is just ready to slip, then the static frictional force is at its maximum value, $F_{\mathrm{fr}}=\mu_{s} F_{\mathrm{N}}$. Write Newton's second law in both directions, with the net force equal to zero since the snow is not accelerating.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{N}}-m g \cos \theta=0 \rightarrow F_{\mathrm{N}}=m g \cos \theta \\
& \sum F_{x}=m g \sin \theta-F_{\mathrm{fr}}=0 \rightarrow
\end{aligned}
$$



$$
m g \sin \theta=F_{\mathrm{fr}}=\mu_{s} F_{\mathrm{N}}=\mu_{s} m g \cos \theta \rightarrow \mu_{s}=\tan \theta=\tan 34^{\circ}=0.67
$$

If $\mu_{s}>0.67$, then the snow would not be on the verge of slipping.
(b) The same free-body diagram applies for the sliding snow. But now the force of friction is kinetic, so $F_{\mathrm{fr}}=\mu_{k} F_{\mathrm{N}}$, and the net force in the $x$ direction is not zero. Write Newton's second law for the $x$ direction again, and solve for the acceleration.

$$
\begin{aligned}
& \sum F_{x}=m g \sin \theta-F_{\mathrm{fr}}=m a \\
& a=\frac{m g \sin \theta-F_{\mathrm{ff}}}{m}=\frac{m g \sin \theta-\mu_{k} m g \cos \theta}{m}=g\left(\sin \theta-\mu_{k} \cos \theta\right)
\end{aligned}
$$

Use Eq. 2-12c with $v_{i}=0$ to find the speed at the end of the roof.

$$
\begin{aligned}
v^{2} & -v_{0}^{2}=2 a\left(x-x_{0}\right) \\
v & =\sqrt{v_{0}+2 a\left(x-x_{0}\right)}=\sqrt{2 g\left(\sin \theta-\mu_{k} \cos \theta\right)\left(x-x_{0}\right)} \\
& =\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(\sin 34^{\circ}-(0.20) \cos 34^{\circ}\right)(6.0 \mathrm{~m})}=6.802 \mathrm{~m} / \mathrm{s} \approx 6.8 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(c) Now the problem becomes a projectile motion problem. The projectile has an initial speed of $6.802 \mathrm{~m} / \mathrm{s}$, directed at an angle of $34^{\circ}$ below the horizontal. The horizontal component of the speed, $(6.802 \mathrm{~m} / \mathrm{s}) \cos 34^{\circ}$ $=5.64 \mathrm{~m} / \mathrm{s}$, will stay constant. The vertical component will change due to gravity. Define the positive direction to be downward. Then the starting vertical velocity is $(6.802 \mathrm{~m} / \mathrm{s}) \sin 34^{\circ}=3.804 \mathrm{~m} / \mathrm{s}$, the vertical acceleration is $9.80 \mathrm{~m} / \mathrm{s}^{2}$, and the vertical displacement is 10.0 m . Use Eq. 2-12c to find the final vertical speed.

$$
\begin{aligned}
& v_{y}^{2}-v_{y 0 y}^{2}=2 a\left(y-y_{0}\right) \\
& v_{y}=\sqrt{v_{y 0}^{2}+2 a\left(y-y_{0}\right)}=\sqrt{(3.804 \mathrm{~m} / \mathrm{s})^{2}+2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(10.0 \mathrm{~m})}=14.5 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

To find the speed when it hits the ground, the horizontal and vertical components of velocity must again be combined, according to the Pythagorean theorem.

$$
v=\sqrt{v_{x}^{2}+v_{y}^{2}}=\sqrt{(5.64 \mathrm{~m} / \mathrm{s})^{2}+(14.5 \mathrm{~m} / \mathrm{s})^{2}}=15.6 \mathrm{~m} / \mathrm{s} \approx 16 \mathrm{~m} / \mathrm{s}
$$

16. Consider a free-body diagram for the box, showing force on the box. When $F_{\mathrm{P}}=23 \mathrm{~N}$, the block does not move. Thus in that case, the force of friction is static friction, and must be at its maximum value, given by $F_{\mathrm{fr}}=\mu_{s} F_{\mathrm{N}}$. Write Newton's second law in both the $x$ and $y$ directions. The net force in each case must be 0 , since the block is at rest.


$$
\begin{aligned}
& \sum F_{x}=F_{\mathrm{P}} \cos \theta-F_{\mathrm{N}}=0 \rightarrow F_{\mathrm{N}}=F_{\mathrm{P}} \cos \theta \\
& \sum F_{y}=F_{\mathrm{fr}}+F_{\mathrm{P}} \sin \theta-m g=0 \rightarrow F_{\mathrm{fr}}+F_{\mathrm{P}} \sin \theta=m g \\
& \mu_{s} F_{\mathrm{N}}+F_{\mathrm{P}} \sin \theta=m g \rightarrow \mu_{s} F_{\mathrm{P}} \cos \theta+F_{\mathrm{P}} \sin \theta=m g \\
& m=\frac{F_{\mathrm{P}}}{g}\left(\mu_{s} \cos \theta+\sin \theta\right)=\frac{23 \mathrm{~N}}{9.80 \mathrm{~m} / \mathrm{s}^{2}}\left(0.40 \cos 28^{\circ}+\sin 28^{\circ}\right)=1.9 \mathrm{~kg}
\end{aligned}
$$

17. (a) Since the two blocks are in contact, they can be treated as a single object as long as no information is needed about internal forces (like the force of one block pushing on the other block). Since there is no motion in the vertical direction, it is apparent that $F_{\mathrm{N}}=\left(m_{1}+m_{2}\right) g$, and so $F_{\mathrm{fr}}=\mu_{k} F_{\mathrm{N}}=\mu_{k}\left(m_{1}+m_{2}\right) g$. Write


Newton's second law for the horizontal direction.

$$
\begin{aligned}
& \sum F_{x}=F_{\mathrm{P}}-F_{\mathrm{fr}}=\left(m_{1}+m_{2}\right) a \rightarrow \\
& a=\frac{F_{\mathrm{p}}-F_{\mathrm{fr}}}{m_{1}+m_{2}}=\frac{F_{\mathrm{P}}-\mu_{k}\left(m_{1}+m_{2}\right) g}{m_{1}+m_{2}}=\frac{650 \mathrm{~N}-(0.18)(190 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{190 \mathrm{~kg}} \\
& \quad=1.657 \mathrm{~m} / \mathrm{s}^{2} \approx 1.7 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

(b) To solve for the contact forces between the blocks, an individual block must be analyzed. Look at the free-body diagram for the second block. $\overrightarrow{\mathbf{F}}_{21}$ is the force of the first block pushing on the second block. Again, it is apparent that $F_{\mathrm{N} 2}=m_{2} g$ and so $F_{\mathrm{f} 2}=\mu_{k} F_{\mathrm{N} 2}=\mu_{k} m_{2} g$. Write Newton's second law for the horizontal direction.


$$
\begin{aligned}
& \sum F_{x}=F_{21}-F_{\text {fi2 }}=m_{2} a \rightarrow \\
& F_{21}=\mu_{k} m_{2} g+m_{2} a=(0.18)(125 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)+(125 \mathrm{~kg})\left(1.657 \mathrm{~m} / \mathrm{s}^{2}\right)=430 \mathrm{~N}
\end{aligned}
$$

By Newton's third law, there will also be a 430 N force to the left on block \# 1 due to block \# 2 .
(c) If the crates are reversed, the acceleration of the system will remain the same - the analysis from part (a) still applies. We can also repeat the analysis from part ( $b$ ) to find the force of one block on the other, if we simply change $m_{1}$ to $m_{2}$ in the free-body diagram and the resulting equations.

$$
\begin{aligned}
& a=1.7 \mathrm{~m} / \mathrm{s}^{2} ; \sum F_{x}=F_{12}-F_{\mathrm{fr} 1}=m_{1} a \rightarrow \\
& F_{12}=\mu_{k} m_{1} g+m_{1} a=(0.18)(65 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)+(65 \mathrm{~kg})\left(1.657 \mathrm{~m} / \mathrm{s}^{2}\right)=220 \mathrm{~N}
\end{aligned}
$$

18. (a) Consider the free-body diagram for the crate on the surface. There is no motion in the $y$ direction and thus no acceleration in the $y$ direction. Write Newton's second law for both directions.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{N}}-m g \cos \theta=0 \rightarrow F_{\mathrm{N}}=m g \cos \theta \\
& \sum F_{x}=m g \sin \theta-F_{\mathrm{fr}}=m a \\
& m a=m g \sin \theta-\mu_{k} F_{\mathrm{N}}=m g \sin \theta-\mu_{k} m g \cos \theta \\
& a=g\left(\sin \theta-\mu_{k} \cos \theta\right) \\
& =\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(\sin 25.0^{\circ}-0.19 \cos 25.0^{\circ}\right)=2.454 \mathrm{~m} / \mathrm{s}^{2} \approx 2.5 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$


(b) Now use Eq. 2-12c, with an initial velocity of 0 , to find the final velocity.

$$
v^{2}-v_{0}^{2}=2 a\left(x-x_{0}\right) \rightarrow v=\sqrt{2 a\left(x-x_{0}\right)}=\sqrt{2\left(2.454 \mathrm{~m} / \mathrm{s}^{2}\right)(8.15 \mathrm{~m})}=6.3 \mathrm{~m} / \mathrm{s}
$$

19. (a) Consider the free-body diagram for the crate on the surface. There is no motion in the $y$ direction and thus no acceleration in the $y$ direction. Write Newton's second law for both directions, and find the acceleration.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{N}}-m g \cos \theta=0 \rightarrow F_{\mathrm{N}}=m g \cos \theta \\
& \sum F_{x}=m g \sin \theta+F_{\mathrm{fr}}=m a \\
& m a=m g \sin \theta+\mu_{k} F_{\mathrm{N}}=m g \sin \theta+\mu_{k} m g \cos \theta \\
& a=g\left(\sin \theta+\mu_{k} \cos \theta\right)
\end{aligned}
$$



Now use Eq. 2-12c, with an initial velocity of $-3.0 \mathrm{~m} / \mathrm{s}$ and a final velocity of 0 to find the distance the crate travels up the plane.

$$
\begin{aligned}
& v^{2}-v_{0}^{2}=2 a\left(x-x_{0}\right) \rightarrow \\
& x-x_{0}=\frac{-v_{0}^{2}}{2 a}=\frac{-(-3.0 \mathrm{~m} / \mathrm{s})^{2}}{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(\sin 25.0^{\circ}+0.17 \cos 25.0^{\circ}\right)}=-0.796 \mathrm{~m}
\end{aligned}
$$

The crate travels 0.80 m up the plane.
(b) We use the acceleration found above with the initial velocity in Eq. 2-12a to find the time for the crate to travel up the plane.

$$
v=v_{0}+a t \rightarrow t_{\mathrm{up}}=-\frac{v_{0}}{a_{u p}}=-\frac{(-3.0 \mathrm{~m} / \mathrm{s})}{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(\sin 25.0^{\circ}+0.17 \cos 25.0^{\circ}\right)}=0.5308 \mathrm{~s}
$$

The total time is NOT just twice the time to travel up the plane, because the acceleration of the block is different for the two parts of the motion. The second free-body diagram applies to the block sliding down the plane. A similar analysis will give the acceleration, and then Eq. 2-12b with an initial velocity of 0 is used to find the time to move down the plane.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{N}}-m g \cos \theta=0 \rightarrow F_{\mathrm{N}}=m g \cos \theta \\
& \sum F_{x}=m g \sin \theta-F_{\mathrm{ff}}=m a \\
& m a=m g \sin \theta-\mu_{k} F_{\mathrm{N}}=m g \sin \theta-\mu_{k} m g \cos \theta \\
& a=g\left(\sin \theta-\mu_{k} \cos \theta\right)
\end{aligned}
$$

$$
\begin{aligned}
& x-x_{0}=v_{0} t+\frac{1}{2} a t^{2} \\
& t_{\text {down }}=\sqrt{\frac{2\left(x-x_{0}\right)}{a_{\text {down }}}}=\sqrt{\frac{2(0.796 \mathrm{~m})}{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(\sin 25.0^{\circ}-0.17 \cos 25.0^{\circ}\right)}}=0.7778 \mathrm{~s} \\
& t=t_{\text {up }}+t_{\text {down }}=0.5308 \mathrm{~s}+0.7778 \mathrm{~s}=1.3 \mathrm{~s}
\end{aligned}
$$

It is worth noting that the final speed is about $2.0 \mathrm{~m} / \mathrm{s}$, significantly less than the $3.0 \mathrm{~m} / \mathrm{s}$ original speed.
20. Since the upper block has a higher coefficient of friction, that block will "drag behind" the lower block. Thus there will be tension in the cord, and the blocks will have the same acceleration. From the free-body diagrams for each block, we write Newton's second law for both the $x$ and $y$ directions for each block, and then combine those equations to find the acceleration and tension.
(a) Block A:

$$
\begin{aligned}
& \sum F_{y \mathrm{~A}}=F_{\mathrm{NA}}-m_{\mathrm{A}} g \cos \theta=0 \rightarrow F_{\mathrm{NA}}=m_{\mathrm{A}} g \cos \theta \\
& \sum F_{x \mathrm{~A}}=m_{\mathrm{A}} g \sin \theta-F_{\mathrm{frA}}-F_{\mathrm{T}}=m_{\mathrm{A}} a \\
& m_{\mathrm{A}} a=m_{\mathrm{A}} g \sin \theta-\mu_{\mathrm{A}} F_{\mathrm{NA}}-F_{\mathrm{T}}=m_{\mathrm{A}} g \sin \theta-\mu_{\mathrm{A}} m_{\mathrm{A}} g \cos \theta-F_{\mathrm{T}}
\end{aligned}
$$



Block B:

$$
\begin{aligned}
& \sum F_{y B}=F_{\mathrm{NB}}-m_{\mathrm{B}} g \cos \theta=0 \rightarrow F_{\mathrm{NB}}=m_{\mathrm{B}} g \cos \theta \\
& \sum F_{x B}=m_{\mathrm{A}} g \sin \theta-F_{\mathrm{frA}}+F_{\mathrm{T}}=m_{\mathrm{B}} a \\
& m_{\mathrm{B}} a=m_{\mathrm{B}} g \sin \theta-\mu_{\mathrm{B}} F_{\mathrm{NB}}+F_{\mathrm{T}}=m_{\mathrm{B}} g \sin \theta-\mu_{\mathrm{B}} m_{\mathrm{B}} g \cos \theta+F_{\mathrm{T}}
\end{aligned}
$$

Add the final equations together from both analyses and solve for the acceleration.

$$
\begin{aligned}
& m_{\mathrm{A}} a=m_{\mathrm{A}} g \sin \theta-\mu_{\mathrm{A}} m_{\mathrm{A}} g \cos \theta-F_{\mathrm{T}} ; m_{\mathrm{B}} a=m_{\mathrm{B}} g \sin \theta-\mu_{\mathrm{B}} m_{\mathrm{B}} g \cos \theta+F_{\mathrm{T}} \\
& m_{\mathrm{A}} a+m_{\mathrm{B}} a=m_{\mathrm{A}} g \sin \theta-\mu_{\mathrm{A}} m_{\mathrm{A}} g \cos \theta-F_{\mathrm{T}}+m_{\mathrm{B}} g \sin \theta-\mu_{\mathrm{B}} m_{\mathrm{B}} g \cos \theta+F_{\mathrm{T}} \rightarrow \\
& a=g\left[\frac{m_{\mathrm{A}}\left(\sin \theta-\mu_{\mathrm{A}} \cos \theta\right)+m_{\mathrm{B}}\left(\sin \theta-\mu_{\mathrm{B}} \cos \theta\right)}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)}\right] \\
& =\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left[\frac{(5.0 \mathrm{~kg})\left(\sin 32^{\circ}-0.20 \cos 32^{\circ}\right)+(5.0 \mathrm{~kg})\left(\sin 32^{\circ}-0.30 \cos 32^{\circ}\right)}{(10.0 \mathrm{~kg})}\right] \\
& \quad=3.1155 \mathrm{~m} / \mathrm{s}^{2} \approx 3.1 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

(b) Solve one of the equations for the tension force.

$$
\begin{aligned}
& m_{\mathrm{A}} a=m_{\mathrm{A}} g \sin \theta-\mu_{\mathrm{A}} m_{\mathrm{A}} g \cos \theta-F_{\mathrm{T}} \rightarrow \\
& F_{\mathrm{T}}=m_{\mathrm{A}}\left(g \sin \theta-\mu_{\mathrm{A}} g \cos \theta-a\right) \\
& \quad=(5.0 \mathrm{~kg})\left[\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(\sin 32^{\circ}-0.20 \cos 32^{\circ}\right)-3.1155 \mathrm{~m} / \mathrm{s}^{2}\right]=2.1 \mathrm{~N}
\end{aligned}
$$

21. (a) If $\mu_{\mathrm{A}}<\mu_{\mathrm{B}}$, the untethered acceleration of $m_{\mathrm{A}}$ would be greater than that of $m_{\mathrm{B}}$. If there were no cord connecting the masses, $m_{\mathrm{A}}$ would "run away" from $m_{\mathrm{B}}$. So if they are joined together, $m_{\mathrm{A}}$ would be restrained by the tension in the cord, $m_{\mathrm{B}}$ would be pulled forward by the tension in the cord, and the two masses would have the same acceleration. This is exactly the situation for Problem 20.
(b) If $\mu_{\mathrm{A}}>\mu_{\mathrm{B}}$, the untethered acceleration of $m_{\mathrm{A}}$ would be less than that of $m_{\mathrm{B}}$. So even if there is a cord between them, $m_{\mathrm{B}}$ will move ever closer to $m_{\mathrm{A}}$, and there will be no tension in the cord. If the incline were long enough, eventually $m_{\mathrm{B}}$ would catch up to $m_{\mathrm{A}}$ and begin to push it down the plane.
(c) For $\mu_{\mathrm{A}}<\mu_{\mathrm{B}}$, the analysis will be exactly like Problem 20. Refer to that free-body diagram and analysis. The acceleration and tension are as follows, taken from the Problem 20 analysis.

$$
\begin{aligned}
a & =g\left[\frac{m_{\mathrm{A}}\left(\sin \theta-\mu_{\mathrm{A}} \cos \theta\right)+m_{\mathrm{B}}\left(\sin \theta-\mu_{\mathrm{B}} \cos \theta\right)}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)}\right] \\
m_{\mathrm{A}} a & =m_{\mathrm{A}} g \sin \theta-\mu_{\mathrm{A}} m_{\mathrm{A}} g \cos \theta-F_{\mathrm{T}} \rightarrow \\
F_{\mathrm{T}} & =m_{\mathrm{A}} g \sin \theta-\mu_{\mathrm{A}} m_{\mathrm{A}} g \cos \theta-m_{\mathrm{A}} a \\
& =m_{\mathrm{A}} g \sin \theta-\mu_{\mathrm{A}} m_{\mathrm{A}} g \cos \theta-m_{\mathrm{A}} g\left[\frac{m_{\mathrm{A}}\left(\sin \theta-\mu_{\mathrm{A}} \cos \theta\right)+m_{\mathrm{B}}\left(\sin \theta-\mu_{\mathrm{B}} \cos \theta\right)}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)}\right] \\
& =\frac{m_{\mathrm{A}} m_{\mathrm{B}} g \cos \theta}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)}\left(\mu_{\mathrm{B}}-\mu_{\mathrm{A}}\right)
\end{aligned}
$$

For $\mu_{\mathrm{A}}>\mu_{\mathrm{B}}$, we can follow the analysis of Problem 20 but not include the tension forces. Each block will have its own acceleration. Refer to the free-body diagram for Problem 20. Block A:

$$
\begin{aligned}
& \sum F_{y \mathrm{~A}}=F_{\mathrm{NA}}-m_{\mathrm{A}} g \cos \theta=0 \rightarrow F_{\mathrm{NA}}=m_{\mathrm{A}} g \cos \theta \\
& \sum F_{x \mathrm{~A}}=m_{\mathrm{A}} g \sin \theta-F_{\mathrm{ff}}=m_{\mathrm{A}} a_{\mathrm{A}} \\
& m_{\mathrm{A}} a_{\mathrm{A}}=m_{\mathrm{A}} g \sin \theta-\mu_{\mathrm{A}} F_{\mathrm{NA}}=m_{\mathrm{A}} g \sin \theta-\mu_{\mathrm{A}} m_{\mathrm{A}} g \cos \theta \rightarrow \\
& a_{\mathrm{A}}=g\left(\sin \theta-\mu_{\mathrm{A}} \cos \theta\right) \\
& \mathrm{k} \mathrm{~B}: \\
& \sum F_{y B}=F_{\mathrm{NB}}-m_{\mathrm{B}} g \cos \theta=0 \rightarrow F_{\mathrm{NB}}=m_{\mathrm{B}} g \cos \theta \\
& \sum F_{x B}=m_{\mathrm{A}} g \sin \theta-F_{\mathrm{frA}}=m_{\mathrm{B}} a_{\mathrm{B}} \\
& m_{\mathrm{B}} a_{\mathrm{B}}=m_{\mathrm{B}} g \sin \theta-\mu_{\mathrm{B}} F_{\mathrm{NB}}=m_{\mathrm{B}} g \sin \theta-\mu_{\mathrm{B}} m_{\mathrm{B}} g \cos \theta \rightarrow \\
& a_{\mathrm{B}}=g\left(\sin \theta-\mu_{\mathrm{B}} \cos \theta\right)
\end{aligned}
$$

Block B:

Note that since $\mu_{\mathrm{A}}>\mu_{\mathrm{B}}, a_{\mathrm{A}}>a_{\mathrm{B}}$ as mentioned above. And $F_{\mathrm{T}}=0$.
22. The force of static friction is what decelerates the crate if it is not sliding on the truck bed. If the crate is not to slide, but the maximum deceleration is desired, then the maximum static frictional force must be exerted, and so $F_{\mathrm{fr}}=\mu_{s} F_{\mathrm{N}}$. The direction of travel is to the right. It is apparent that $F_{\mathrm{N}}=m g$ since there is
 no acceleration in the $y$ direction. Write Newton's second law for the truck in
the horizontal direction.

$$
\sum F_{x}=-F_{\mathrm{fr}}=m a \rightarrow-\mu_{s} m g=m a \rightarrow a=-\mu_{s} g=-(0.75)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=-7.4 \mathrm{~m} / \mathrm{s}^{2}
$$

The negative sign indicates the direction of the acceleration - opposite to the direction of motion.
23. (a) For $m_{\mathrm{B}}$ to not move, the tension must be equal to $m_{\mathrm{B}} g$, and so $m_{\mathrm{B}} g=F_{\mathrm{T}}$. For $m_{\mathrm{A}}$ to not move, the tension must be equal to the force of static friction, and so $F_{\mathrm{S}}=F_{\mathrm{T}}$. Note that the normal force on $m_{\mathrm{A}}$ is equal to its weight. Use these relationships to solve for $m_{\mathrm{A}}$.

$$
m_{\mathrm{B}} g=F_{\mathrm{T}}=F_{\mathrm{s}} \leq \mu_{s} m_{\mathrm{A}} g \rightarrow m_{\mathrm{A}} \geq \frac{m_{\mathrm{B}}}{\mu_{s}}=\frac{2.0 \mathrm{~kg}}{0.40}=5.0 \mathrm{~kg} \rightarrow m_{\mathrm{A}} \geq 5.0 \mathrm{~kg}
$$

(b) For $m_{\mathrm{B}}$ to move with constant velocity, the tension must be equal to $m_{\mathrm{B}} g$. For $m_{\mathrm{A}}$ to move with constant velocity, the tension must be equal to the force of kinetic friction. Note that the normal force on $m_{\mathrm{A}}$ is equal to its weight. Use these relationships to solve for $m_{\mathrm{A}}$.

$$
m_{\mathrm{B}} g=F_{\mathrm{k}}=\mu_{k} m_{\mathrm{A}} g \rightarrow m_{\mathrm{A}}=\frac{m_{\mathrm{B}}}{\mu_{\mathrm{k}}}=\frac{2.0 \mathrm{~kg}}{0.30}=6.7 \mathrm{~kg}
$$

24. We define $f$ to be the fraction of the cord that is handing down, between $m_{\mathrm{B}}$ and the pulley. Thus the mass of that piece of cord is $f m_{C}$.
We assume that the system is moving to the right as well. We take the tension in the cord to be $F_{\mathrm{T}}$ at the pulley. We treat the hanging mass and hanging fraction of the cord as one mass, and the sliding mass and horizontal part of the cord as another mass. See the free-body
 diagrams. We write Newton's second law for each object.

$$
\begin{aligned}
& \sum F_{y \mathrm{~A}}=F_{\mathrm{N}}-\left(m_{\mathrm{A}}+(1-f) m_{\mathrm{C}}\right) g=0 \\
& \sum F_{x \mathrm{~A}}=F_{\mathrm{T}}-F_{\mathrm{fr}}=F_{\mathrm{T}}-\mu_{\mathrm{k}} F_{\mathrm{N}}=\left(m_{\mathrm{A}}+(1-f) m_{\mathrm{C}}\right) a \\
& \sum F_{x \mathrm{~B}}=\left(m_{\mathrm{B}}+f m_{\mathrm{C}}\right) g-F_{\mathrm{T}}=\left(m_{\mathrm{B}}+f m_{\mathrm{C}}\right) a
\end{aligned}
$$

Combine the relationships to solve for the acceleration. In particular, add the two equations for the $x$-direction, and then substitute the normal force.

$$
a=\left[\frac{m_{\mathrm{B}}+f m_{\mathrm{C}}-\mu_{\mathrm{k}}\left(m_{\mathrm{A}}+(1-f) m_{\mathrm{C}}\right)}{m_{\mathrm{A}}+m_{\mathrm{B}}+m_{\mathrm{C}}}\right] g
$$

25. (a) Consider the free-body diagram for the block on the surface. There is no motion in the $y$ direction and thus no acceleration in the $y$ direction. Write Newton's second law for both directions, and find the acceleration.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{N}}-m g \cos \theta=0 \rightarrow F_{\mathrm{N}}=m g \cos \theta \\
& \sum F_{x}=m g \sin \theta+F_{\mathrm{ff}}=m a \\
& m a=m g \sin \theta+\mu_{k} F_{\mathrm{N}}=m g \sin \theta+\mu_{k} m g \cos \theta \\
& a=g\left(\sin \theta+\mu_{k} \cos \theta\right)
\end{aligned}
$$



Now use Eq. 2-12c, with an initial velocity of $v_{0}$, a final velocity of 0 , and a displacement of $-d$ to find the coefficient of kinetic friction.

$$
\begin{aligned}
& v^{2}-v_{0}^{2}=2 a\left(x-x_{0}\right) \rightarrow 0-v_{0}^{2}=2 g\left(\sin \theta+\mu_{k} \cos \theta\right)(-d) \rightarrow \\
& \mu_{k}=\frac{v_{0}^{2}}{2 g d \cos \theta}-\tan \theta
\end{aligned}
$$

(b) Now consider the free-body diagram for the block at the top of its motion. We use a similar force analysis, but now the magnitude of the friction force is given by $F_{\mathrm{fr}} \leq \mu_{\mathrm{s}} F_{\mathrm{N}}$, and the acceleration is 0 .

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{N}}-m g \cos \theta=0 \rightarrow F_{\mathrm{N}}=m g \cos \theta \\
& \sum F_{x}=m g \sin \theta-F_{\mathrm{fr}}=m a=0 \rightarrow F_{\mathrm{fr}}=m g \sin \theta \\
& F_{\mathrm{fr}} \leq \mu_{\mathrm{s}} F_{\mathrm{N}} \rightarrow m g \sin \theta \leq \mu_{\mathrm{s}} m g \cos \theta \rightarrow \mu_{\mathrm{s}} \geq \tan \theta
\end{aligned}
$$


26. First consider the free-body diagram for the snowboarder on the incline.

Write Newton's second law for both directions, and find the acceleration.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{N}}-m g \cos \theta=0 \rightarrow F_{\mathrm{N}}=m g \cos \theta \\
& \sum F_{x}=m g \sin \theta-F_{\mathrm{fr}}=m a \\
& m a=m g \sin \theta-\mu_{k 1} F_{\mathrm{N}}=m g \sin \theta-\mu_{k 1} m g \cos \theta \\
& a_{\text {slope }}=g\left(\sin \theta-\mu_{k 1} \cos \theta\right)=\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(\sin 28^{\circ}-0.18 \cos 28^{\circ}\right) \\
& \quad=3.043 \mathrm{~m} / \mathrm{s}^{2} \approx 3.0 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$



Now consider the free-body diagram for the snowboarder on the flat surface.
Again use Newton's second law to find the acceleration. Note that the normal force and the frictional force are different in this part of the problem, even though the same symbol is used.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{N}}-m g=0 \rightarrow F_{\mathrm{N}}=m g \quad \sum F_{x}=-F_{\mathrm{fr}}=m a \\
& m a_{\mathrm{flat}}=-F_{\mathrm{fr}}=-\mu_{k 2} F_{\mathrm{N}}=-\mu_{k 1} m g \rightarrow \\
& a_{\mathrm{flat}}=-\mu_{k 2} g=-(0.15)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=-1.47 \mathrm{~m} / \mathrm{s}^{2} \approx-1.5 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$



Use Eq. 2-12c to find the speed at the bottom of the slope. This is the speed at the start of the flat section. Eq. 2-12c can be used again to find the distance $x$.

$$
\begin{aligned}
& v^{2}-v_{0}^{2}=2 a\left(x-x_{0}\right) \rightarrow \\
& v_{\text {end of }}=\sqrt{v_{0}^{2}+2 a_{\text {slope }}\left(x-x_{0}\right)}=\sqrt{(5.0 \mathrm{~m} / \mathrm{s})^{2}+2\left(3.043 \mathrm{~m} / \mathrm{s}^{2}\right)(110 \mathrm{~m})}=26.35 \mathrm{~m} / \mathrm{s} \\
& v^{2}-v_{0}^{2}=2 a\left(x-x_{0}\right) \rightarrow \\
& \left(x-x_{0}\right)=\frac{v^{2}-v_{0}^{2}}{2 a_{\text {flat }}}=\frac{0-(26.35 \mathrm{~m} / \mathrm{s})^{2}}{2\left(-1.47 \mathrm{~m} / \mathrm{s}^{2}\right)}=236 \mathrm{~m} \approx 240 \mathrm{~m}
\end{aligned}
$$

27. The belt is sliding underneath the box (to the right), so there will be a force of kinetic friction on the box, until the box reaches a speed of $1.5 \mathrm{~m} / \mathrm{s}$. Use the freebody diagram to calculate the acceleration of the box.
(a) $\sum F_{x}=F_{\mathrm{fr}}=m a=\mu_{\mathrm{k}} F_{\mathrm{N}}=\mu_{\mathrm{k}} m g \rightarrow a=\mu_{\mathrm{k}} g$


$$
\begin{align*}
& \sum F_{x}=F_{\mathrm{fr}}=m a=\mu_{\mathrm{k}} F_{\mathrm{N}}=\mu_{\mathrm{k}} m g \rightarrow a=\mu_{\mathrm{k}} g \\
& v=v_{0}+a t \rightarrow t=\frac{v-v_{0}}{a}=\frac{v-0}{\mu_{\mathrm{k}} g}=\frac{1.5 \mathrm{~m} / \mathrm{s}}{(0.70)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=0.22 \mathrm{~s} \\
& x-x_{0}=\frac{v^{2}-v_{0}^{2}}{2 a}=\frac{v^{2}}{2 \mu_{\mathrm{k}} g}=\frac{(1.5 \mathrm{~m} / \mathrm{s})^{2}}{2(0.70)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=0.16 \mathrm{~m} \tag{b}
\end{align*}
$$

28. We define the positive $x$ direction to be the direction of motion for each block. See the free-body diagrams. Write Newton's second law in both dimensions for both objects. Add the two $x$-equations to find the acceleration.
Block A:

$$
\begin{aligned}
& \sum F_{y \mathrm{~A}}=F_{\mathrm{NA}}-m_{\mathrm{A}} g \cos \theta_{\mathrm{A}}=0 \rightarrow F_{\mathrm{NA}}=m_{\mathrm{A}} g \cos \theta_{\mathrm{A}} \\
& \sum F_{x \mathrm{~A}}=F_{\mathrm{T}}-m_{\mathrm{A}} g \sin \theta-F_{\mathrm{frA}}=m_{\mathrm{A}} a
\end{aligned}
$$

Block B:

$$
\begin{aligned}
& \sum F_{y B}=F_{\mathrm{NB}}-m_{\mathrm{B}} g \cos \theta_{\mathrm{B}}=0 \rightarrow F_{\mathrm{NB}}=m_{\mathrm{B}} g \cos \theta_{\mathrm{B}} \\
& \sum F_{x B}=m_{\mathrm{B}} g \sin \theta-F_{\mathrm{frB}}-F_{\mathrm{T}}=m_{\mathrm{B}} a
\end{aligned}
$$

Add the final equations together from both analyses and solve for the acceleration, noting that in both cases the friction force is found as $F_{\mathrm{fr}}=\mu F_{\mathrm{N}}$.

$$
\begin{aligned}
& m_{\mathrm{A}} a=F_{\mathrm{T}}-m_{\mathrm{A}} g \sin \theta_{\mathrm{A}}-\mu_{\mathrm{A}} m_{\mathrm{A}} g \cos \theta_{\mathrm{A}} \quad ; m_{\mathrm{B}} a=m_{\mathrm{B}} g \sin \theta_{\mathrm{B}}-\mu_{\mathrm{B}} m_{\mathrm{B}} g \cos \theta_{\mathrm{B}}-F_{\mathrm{T}} \\
& m_{\mathrm{A}} a+m_{\mathrm{B}} a=F_{\mathrm{T}}-m_{\mathrm{A}} g \sin \theta_{\mathrm{A}}-\mu_{\mathrm{A}} m_{\mathrm{A}} g \cos \theta_{\mathrm{A}}+m_{\mathrm{B}} g \sin \theta_{\mathrm{B}}-\mu_{\mathrm{B}} m_{\mathrm{B}} g \cos \theta_{\mathrm{B}}-F_{\mathrm{T}} \rightarrow \\
& a=g\left[\frac{-m_{\mathrm{A}}\left(\sin \theta_{\mathrm{A}}+\mu_{\mathrm{A}} \cos \theta_{\mathrm{A}}\right)+m_{\mathrm{B}}\left(\sin \theta-\mu_{\mathrm{B}} \cos \theta\right)}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)}\right] \\
& \\
& =\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left[\frac{-(2.0 \mathrm{~kg})\left(\sin 51^{\circ}+0.30 \cos 51^{\circ}\right)+(5.0 \mathrm{~kg})\left(\sin 21^{\circ}-0.30 \cos 21^{\circ}\right)}{(7.0 \mathrm{~kg})}\right] \\
& \\
& =-2.2 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

29. We assume that the child starts from rest at the top of the slide, and then slides a distance $x-x_{0}$ along the slide. A force diagram is shown for the child on the slide. First, ignore the frictional force and so consider the no-friction case. All of the motion is in the $x$ direction, so we will only consider Newton's second law for the $x$ direction.

$$
\sum F_{x}=m g \sin \theta=m a \rightarrow a=g \sin \theta
$$



Use Eq. $2-12 \mathrm{c}$ to calculate the speed at the bottom of the slide.

$$
v^{2}-v_{0}^{2}=2 a\left(x-x_{0}\right) \rightarrow v_{(\text {No friction })}=\sqrt{v_{0}^{2}+2 a\left(x-x_{0}\right)}=\sqrt{2 g \sin \theta\left(x-x_{0}\right)}
$$

Now include kinetic friction. We must consider Newton's second law in both the $x$ and $y$ directions now. The net force in the $y$ direction must be 0 since there is no acceleration in the $y$ direction.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{N}}-m g \cos \theta=0 \rightarrow F_{\mathrm{N}}=m g \cos \theta \\
& \sum F_{x}=m a=m g \sin \theta-F_{\mathrm{fr}}=m g \sin \theta-\mu_{k} F_{\mathrm{N}}=m g \sin \theta-\mu_{k} m g \cos \theta
\end{aligned}
$$

$$
a=\frac{m g \sin \theta-\mu_{k} m g \cos \theta}{m}=g\left(\sin \theta-\mu_{k} \cos \theta\right)
$$

With this acceleration, we can again use Eq. 2-12c to find the speed after sliding a certain distance.

$$
v^{2}-v_{0}^{2}=2 a\left(x-x_{0}\right) \rightarrow v_{(\text {friction })}=\sqrt{v_{0}^{2}+2 a\left(x-x_{0}\right)}=\sqrt{2 g\left(\sin \theta-\mu_{k} \cos \theta\right)\left(x-x_{0}\right)}
$$

Now let the speed with friction be half the speed without friction, and solve for the coefficient of friction. Square the resulting equation and divide by $g \cos \theta$ to get the result.

$$
\begin{aligned}
& v_{\text {(friction) }}=\frac{1}{2} v_{(\text {No friction) }} \rightarrow \sqrt{2 g\left(\sin \theta-\mu_{k} \cos \theta\right)\left(x-x_{0}\right)}=\frac{1}{2} \sqrt{2 g(\sin \theta)\left(x-x_{0}\right)} \\
& 2 g\left(\sin \theta-\mu_{k} \cos \theta\right)\left(x-x_{0}\right)=\frac{1}{4} 2 g(\sin \theta)\left(x-x_{0}\right) \\
& \mu_{k}=\frac{3}{4} \tan \theta=\frac{3}{4} \tan 34^{\circ}=0.51
\end{aligned}
$$

30. (a) Given that $m_{\mathrm{B}}$ is moving down, $m_{\mathrm{A}}$ must be moving up the incline, and so the force of kinetic friction on $m_{\mathrm{A}}$ will be directed down the incline. Since the blocks are tied together, they will both have the same acceleration, and so $a_{y \mathrm{~B}}=a_{x \mathrm{~A}}=a$. Write
Newton's second law for each mass.

$$
\begin{aligned}
& \sum F_{y \mathrm{~B}}=m_{\mathrm{B}} g-F_{\mathrm{T}}=m_{\mathrm{B}} a \rightarrow F_{\mathrm{T}}=m_{\mathrm{B}} g-m_{\mathrm{B}} a \\
& \sum F_{x \mathrm{~A}}=F_{\mathrm{T}}-m_{\mathrm{A}} g \sin \theta-F_{\mathrm{fr}}=m_{\mathrm{A}} a \\
& \sum F_{y \mathrm{~A}}=F_{\mathrm{N}}-m_{\mathrm{A}} g \cos \theta=0 \rightarrow F_{\mathrm{N}}=m_{\mathrm{A}} g \cos \theta
\end{aligned}
$$

 acceleration.

$$
\begin{aligned}
& m_{\mathrm{B}} g-m_{\mathrm{B}} a-m_{\mathrm{A}} g \sin \theta-\mu_{k} m_{\mathrm{A}} g \cos \theta=m_{\mathrm{A}} a \rightarrow \\
& a=\frac{m_{\mathrm{B}} g-m_{\mathrm{A}} g \sin \theta-m_{\mathrm{A}} g \mu_{k} g \cos \theta}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)}=\frac{1}{2} g\left(1-\sin \theta-\mu_{k} g \cos \theta\right) \\
& \\
& =\frac{1}{2}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(1-\sin 34^{\circ}-0.15 \cos 34^{\circ}\right)=1.6 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

(b) To have an acceleration of zero, the expression for the acceleration must be zero.

$$
\begin{aligned}
& a=\frac{1}{2} g\left(1-\sin \theta-\mu_{k} \cos \theta\right)=0 \rightarrow 1-\sin \theta-\mu_{k} \cos \theta=0 \rightarrow \\
& \mu_{k}=\frac{1-\sin \theta}{\cos \theta}=\frac{1-\sin 34^{\circ}}{\cos 34^{\circ}}=0.53
\end{aligned}
$$

31. Draw a free-body diagram for each block.


Block A (top)

$\overrightarrow{\mathbf{F}}_{\mathrm{fr} \text { AB }}$ is the force of friction between the two blocks, $\overrightarrow{\mathbf{F}}_{\mathrm{NA}}$ is the normal force of contact between the two blocks, $\overrightarrow{\mathbf{F}}_{\mathrm{fr}}$ is the force of friction between the bottom block and the floor, and $\overrightarrow{\mathbf{F}}_{\mathrm{NB}}$ is the normal force of contact between the bottom block and the floor.

Neither block is accelerating vertically, and so the net vertical force on each block is zero.
top: $\quad F_{\mathrm{NA}}-m_{\mathrm{A}} g=0 \quad \rightarrow \quad F_{\mathrm{NA}}=m_{\mathrm{A}} g$
bottom: $F_{\mathrm{NB}}-F_{\mathrm{NA}}-m_{\mathrm{B}} g=0 \rightarrow F_{\mathrm{NB}}=F_{\mathrm{NA}}+m_{\mathrm{B}} g=\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) g$
Take the positive horizontal direction to be the direction of motion of each block. Thus for the bottom block, positive is to the right, and for the top block, positive is to the left. Then, since the blocks are constrained to move together by the connecting string, both blocks will have the same acceleration. Write Newton's second law for the horizontal direction for each block.
top: $F_{\mathrm{T}}-F_{\mathrm{fr} \mathrm{AB}}=m_{\mathrm{A}} a \quad$ bottom: $F-F_{\mathrm{T}}-F_{\mathrm{fr} \mathrm{AB}}-F_{\mathrm{fr} \mathrm{B}}=m_{\mathrm{B}} a$
(a) If the two blocks are just to move, then the force of static friction will be at its maximum, and so the frictions forces are as follows.

$$
F_{\mathrm{fr} \mathrm{AB}}=\mu_{\mathrm{s}} F_{\mathrm{NA}}=\mu_{\mathrm{s}} m_{\mathrm{A}} g \quad ; \quad F_{\mathrm{fr} \mathrm{~B}}=\mu_{\mathrm{s}} F_{\mathrm{NB}}=\mu_{\mathrm{s}}\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) g
$$

Substitute into Newton's second law for the horizontal direction with $a=0$ and solve for $F$.

$$
\begin{aligned}
& \text { top: } F_{\mathrm{T}}-\mu_{\mathrm{s}} m_{\mathrm{A}} g=0 \rightarrow F_{\mathrm{T}}=\mu_{\mathrm{s}} m_{\mathrm{A}} g \\
& \text { bottom: } F-F_{\mathrm{T}}-\mu_{\mathrm{s}} m_{\mathrm{A}} g-\mu_{\mathrm{s}}\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) g=0 \rightarrow \\
& \qquad \begin{aligned}
F & =F_{\mathrm{T}}+\mu_{\mathrm{s}} m_{\mathrm{A}} g+\mu_{\mathrm{s}}\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) g=\mu_{\mathrm{s}} m_{\mathrm{A}} g+\mu_{\mathrm{s}} m_{\mathrm{A}} g+\mu_{\mathrm{s}}\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) g \\
& =\mu_{\mathrm{s}}\left(3 m_{\mathrm{A}}+m_{\mathrm{B}}\right) g=(0.60)(14 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=82.32 \mathrm{~N} \approx 82 \mathrm{~N}
\end{aligned}
\end{aligned}
$$

(b) Multiply the force by 1.1 so that $F=1.1(82.32 \mathrm{~N})=90.55 \mathrm{~N}$. Again use Newton's second law for the horizontal direction, but with $a \neq 0$ and using the coefficient of kinetic friction.

$$
\begin{array}{ll}
\text { top: } & F_{\mathrm{T}}-\mu_{\mathrm{k}} m_{\mathrm{A}} g=m_{\mathrm{A}} a \\
\text { bottom: } & F-F_{\mathrm{T}}-\mu_{\mathrm{k}} m_{\mathrm{A}} g-\mu_{\mathrm{k}}\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) g=m_{\mathrm{B}} a \\
\text { sum: } & F-\mu_{\mathrm{k}} m_{\mathrm{A}} g-\mu_{\mathrm{k}} m_{\mathrm{A}} g-\mu_{\mathrm{k}}\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) g=\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) a \rightarrow \\
& a=\frac{F-\mu_{\mathrm{k}} m_{\mathrm{A}} g-\mu_{\mathrm{k}} m_{\mathrm{A}} g-\mu_{\mathrm{k}}\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) g}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)}=\frac{F-\mu_{\mathrm{k}}\left(3 m_{\mathrm{A}}+m_{\mathrm{B}}\right) g}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)} \\
& =\frac{90.55 \mathrm{~N}-(0.40)(14.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{(8.0 \mathrm{~kg})}=4.459 \mathrm{~m} / \mathrm{s}^{2} \approx 4.5 \mathrm{~m} / \mathrm{s}^{2}
\end{array}
$$

32. Free-body diagrams are shown for both blocks. There is a force of friction between the two blocks, which acts to the right on the top block, and to the left on the bottom block. They are a Newton's third law pair of forces.
(a) If the 4.0 kg block does not slide off, then it must have the same acceleration as the 12.0 kg block. That acceleration is caused by the force of static friction between the two blocks. To find the minimum coefficient,
 we use the maximum force of static friction.

$$
\underset{\substack{\mathrm{fr} \\ \text { top }}}{ }=m_{\text {top }} a=\mu \underset{\substack{\mathrm{N} \\ \text { top }}}{\mu F_{\text {top }} g \rightarrow \mu=\frac{a}{g}=\frac{5.2 \mathrm{~m} / \mathrm{s}^{2}}{9.80 \mathrm{~m} / \mathrm{s}^{2}}=0.5306 \approx 0.53}
$$

(b) If the coefficient of friction only has half the value, then the blocks will be sliding with respect to one another, and so the friction will be kinetic.

$$
\mu=\frac{1}{2}(0.5306)=0.2653 ; \underset{\substack{\text { fop } \\ \text { top }}}{F_{\text {top }}}=m_{\text {top }} a=\underset{\text { top }}{\mu F_{\mathrm{N}}}=\mu m_{\text {top }} g \rightarrow
$$



$$
a=\mu g=(0.2653)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=2.6 \mathrm{~m} / \mathrm{s}^{2}
$$

(c) The bottom block is still accelerating to the right at $5.2 \mathrm{~m} / \mathrm{s}^{2}$. Since the top block has a smaller acceleration than that, it has a negative acceleration relative to the bottom block.

The top block has an acceleration of $2.6 \mathrm{~m} / \mathrm{s}^{2}$ to the left relative to the bottom block.
(d) No sliding:

$$
\begin{aligned}
\begin{array}{c}
F_{x} \\
\text { botom } \\
\text { net }
\end{array} & =F_{\mathrm{P}}-F_{\mathrm{fr}}=m_{\text {bototom }} a_{\text {botom }} \rightarrow \\
F_{\mathrm{P} \text { potom }} & =F_{\substack{\mathrm{f} \\
\text { botom }}}+m_{\text {bottom }} a_{\text {bototom }}=F_{\mathrm{fr}}+m_{\text {botom }}^{\text {top }} \\
& a_{\text {botoom }}=m_{\text {top }} a_{\text {top }}+m_{\text {botom }} a_{\text {botom }}=\left(m_{\text {top }}+m_{\text {botom }}\right) a \\
& =(16.0 \mathrm{~kg})\left(5.2 \mathrm{~m} / \mathrm{s}^{2}\right)=83 \mathrm{~N}
\end{aligned}
$$

This is the same as simply assuming that the external force is accelerating the total mass. The internal friction need not be considered if the blocks are not moving relative to each other.

Sliding:

$$
\begin{aligned}
& F_{\mathrm{P}}=\underset{\text { bottom }}{F_{\mathrm{fr}}}+m_{\mathrm{bottom}} a_{\mathrm{b} \text { bottom }}=\underset{\text { top }}{F_{\mathrm{fr}}}+m_{\mathrm{bottom}} a_{\mathrm{bottom}}=m_{\mathrm{top}} a_{\mathrm{top}}+m_{\mathrm{bottom}} a_{\mathrm{bottom}} \\
& =(4.0 \mathrm{~kg})\left(2.6 \mathrm{~m} / \mathrm{s}^{2}\right)+(12.0 \mathrm{~kg})\left(5.2 \mathrm{~m} / \mathrm{s}^{2}\right)=73 \mathrm{~N}
\end{aligned}
$$

Again this can be interpreted as the external force providing the acceleration for each block. The internal friction need not be considered.
33. To find the limiting value, we assume that the blocks are NOT slipping, but that the force of static friction on the smaller block is at its maximum value, so that $F_{\mathrm{fr}}=\mu F_{\mathrm{N}}$. For the two-block system, there is no friction on the system, and so $F=(M+m) a$ describes the horizontal motion of the system. Thus the upper block has a vertical acceleration of 0 and a horizontal acceleration of $\frac{F}{(M+m)}$. Write


Newton's second law for the upper block, using the force diagram, and solve for the applied force $F$. Note that the static friction force will be DOWN the plane, since the block is on the verge of sliding UP the plane.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{N}} \cos \theta-F_{\mathrm{fr}} \sin \theta-m g=F_{\mathrm{N}}(\cos \theta-\mu \sin \theta)-m g=0 \rightarrow F_{\mathrm{N}}=\frac{m g}{(\cos \theta-\mu \sin \theta)} \\
& \sum F_{x}=F_{\mathrm{N}} \sin \theta+F_{\mathrm{fr}} \cos \theta=F_{\mathrm{N}}(\sin \theta+\mu \cos \theta)=m a=m \frac{F}{M+m} \rightarrow \\
& F=F_{\mathrm{N}}(\sin \theta+\mu \cos \theta) \frac{M+m}{m}=\frac{m g}{(\cos \theta-\mu \sin \theta)}(\sin \theta+\mu \cos \theta) \frac{M+m}{m} \\
& \quad=(M+m) g \frac{(\sin \theta+\mu \cos \theta)}{(\cos \theta-\mu \sin \theta)}
\end{aligned}
$$

34. A free-body diagram for the car at one instant of time is shown. In the diagram, the car is coming out of the paper at the reader, and the center of the circular path is to the right of the car, in the plane of the paper. If the car has its maximum speed, it would be on the verge of slipping, and the force of static friction would be at its maximum value. The vertical forces (gravity and normal force) are of the same magnitude, because the car is not accelerating vertically. We assume that the force
 of friction is the force causing the circular motion.

$$
\begin{aligned}
& F_{\mathrm{R}}=F_{\mathrm{fr}} \rightarrow m v^{2} / r=\mu_{s} F_{\mathrm{N}}=\mu_{s} m g \rightarrow \\
& v=\sqrt{\mu_{s} r g}=\sqrt{(0.65)(80.0 \mathrm{~m})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=22.57 \mathrm{~m} / \mathrm{s} \approx 23 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Notice that the result is independent of the car's mass.
35. (a) Find the centripetal acceleration from Eq. 5-1.

$$
a_{\mathrm{R}}=v^{2} / r=(1.30 \mathrm{~m} / \mathrm{s})^{2} / 1.20 \mathrm{~m}=1.408 \mathrm{~m} / \mathrm{s}^{2} \approx 1.41 \mathrm{~m} / \mathrm{s}^{2}
$$

(b) The net horizontal force is causing the centripetal motion, and so will be the centripetal force.

$$
F_{\mathrm{R}}=m a_{\mathrm{R}}=(22.5 \mathrm{~kg})\left(1.408 \mathrm{~m} / \mathrm{s}^{2}\right)=31.68 \mathrm{~N} \approx 31.7 \mathrm{~N}
$$

36. Find the centripetal acceleration from Eq. 5-1.

$$
a_{R}=v^{2} / r=\frac{(525 \mathrm{~m} / \mathrm{s})^{2}}{4.80 \times 10^{3} \mathrm{~m}}=\left(57.42 \mathrm{~m} / \mathrm{s}^{2}\right)\left(\frac{1 \mathrm{~g}}{9.80 \mathrm{~m} / \mathrm{s}^{2}}\right)=5.86 \mathrm{~g}^{\prime} \mathrm{s}
$$

37. We assume the water is rotating in a vertical circle of radius $r$. When the bucket is at the top of its motion, there would be two forces on the water (considering the water as a single mass). The weight of the water would be directed down, and the normal force of the bottom of the bucket pushing on the water would
 also be down. See the free-body diagram. If the water is moving in a circle, then the net downward force would be a centripetal force.

$$
\sum F=F_{\mathrm{N}}+m g=m a=m v^{2} / r \quad \rightarrow \quad F_{\mathrm{N}}=m\left(v^{2} / r-g\right)
$$

The limiting condition of the water falling out of the bucket means that the water loses contact with the bucket, and so the normal force becomes 0 .

$$
F_{\mathrm{N}}=m\left(v^{2} / r-g\right) \rightarrow m\left(v_{\text {critical }}^{2} / r-g\right)=0 \rightarrow v_{\text {critical }}=\sqrt{r g}
$$

From this, we see that yes, it is possible to whirl the bucket of water fast enough. The minimum speed is $\sqrt{r g}$.
38. The centripetal acceleration of a rotating object is given by $a_{\mathrm{R}}=v^{2} / r$.

$$
\begin{aligned}
& v=\sqrt{a_{\mathrm{R}} r}=\sqrt{\left(1.25 \times 10^{5} g\right) r}=\sqrt{\left(1.25 \times 10^{5}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(8.00 \times 10^{-2} \mathrm{~m}\right)}=3.13 \times 10^{2} \mathrm{~m} / \mathrm{s} \\
& \left(3.13 \times 10^{2} \mathrm{~m} / \mathrm{s}\right)\left(\frac{1 \mathrm{rev}}{2 \pi\left(8.00 \times 10^{-2} \mathrm{~m}\right)}\right)\left(\frac{60 \mathrm{~s}}{1 \mathrm{~min}}\right)=3.74 \times 10^{4} \mathrm{rpm}
\end{aligned}
$$

39. For an unbanked curve, the centripetal force to move the car in a circular path must be provided by the static frictional force. Also, since the roadway is level, the normal force on the car is equal to its weight. Assume the static frictional force is at its maximum value, and use the force relationships to calculate the radius of the

curve. See the free-body diagram, which assumes the center of the curve is to the right in the diagram.

$$
\begin{aligned}
& F_{\mathrm{R}}=F_{\mathrm{fr}} \rightarrow m v^{2} / r=\mu_{s} F_{\mathrm{N}}=\mu_{s} m g \rightarrow \\
& r=v^{2} / \mu_{s} g=\frac{\left[(30 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)\right]^{2}}{(0.7)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=28 \mathrm{~m} \approx 30 \mathrm{~m}
\end{aligned}
$$

40. At the top of a circle, a free-body diagram for the passengers would be as shown, assuming the passengers are upside down. Then the car's normal force would be pushing DOWN on the passengers, as shown in the diagram. We assume no safety devices are present. Choose the positive direction to be down, and write Newton's second law for the passengers.

$$
\sum F=F_{\mathrm{N}}+m g=m a=m v^{2} / r \rightarrow F_{\mathrm{N}}=m\left(v^{2} / r-g\right)
$$



We see from this expression that for a high speed, the normal force is positive, meaning the passengers are in contact with the car. But as the speed decreases, the normal force also decreases. If the normal force becomes 0 , the passengers are no longer in contact with the car - they are in free fall. The limiting condition is as follows.

$$
v_{\min }^{2} / r-g=0 \rightarrow v_{\min }=\sqrt{r g}=\sqrt{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(7.6 \mathrm{~m})}=8.6 \mathrm{~m} / \mathrm{s}
$$

41. A free-body diagram for the car is shown. Write Newton's second law for the car in the vertical direction, assuming that up is positive. The normal force is twice the weight.

$$
\begin{aligned}
& \sum F=F_{\mathrm{N}}-m g=m a \rightarrow 2 m g-m g=m v^{2} / r \rightarrow \\
& v=\sqrt{r g}=\sqrt{(95 \mathrm{~m})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=30.51 \mathrm{~m} / \mathrm{s} \approx 31 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$


42. In the free-body diagram, the car is coming out of the paper at the reader, and the center of the circular path is to the right of the car, in the plane of the paper. The vertical forces (gravity and normal force) are of the same magnitude, because the car is not accelerating vertically. We assume that the force of friction is the force causing the circular motion. If the car has its maximum speed, it would be on the
 verge of slipping, and the force of static friction would be at its maximum value.

$$
F_{R}=F_{\mathrm{fr}} \rightarrow m v^{2} / r=\mu_{s} F_{\mathrm{N}}=\mu_{s} m g \rightarrow \mu_{s}=\frac{v^{2}}{r g}=\frac{\left[(95 \mathrm{~km} / \mathrm{hr})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{hr}}\right)\right]^{2}}{(85 \mathrm{~m})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=0.84
$$

Notice that the result is independent of the car's mass.
43. The orbit radius will be the sum of the Earth's radius plus the 400 km orbit height. The orbital period is about 90 minutes. Find the centripetal acceleration from these data.

$$
\begin{aligned}
& r=6380 \mathrm{~km}+400 \mathrm{~km}=6780 \mathrm{~km}=6.78 \times 10^{6} \mathrm{~m} \quad T=90 \mathrm{~min}\left(\frac{60 \mathrm{sec}}{1 \mathrm{~min}}\right)=5400 \mathrm{sec} \\
& a_{\mathrm{R}}=\frac{4 \pi^{2} r}{T^{2}}=\frac{4 \pi^{2}\left(6.78 \times 10^{6} \mathrm{~m}\right)}{(5400 \mathrm{sec})^{2}}=\left(9.18 \mathrm{~m} / \mathrm{s}^{2}\right)\left(\frac{1 \mathrm{~g}}{9.80 \mathrm{~m} / \mathrm{s}^{2}}\right)=0.937 \approx 0.9 \mathrm{~g}^{\prime} \mathrm{s}
\end{aligned}
$$

Notice how close this is to $g$, because the shuttle is not very far above the surface of the Earth, relative to the radius of the Earth.
44. (a) At the bottom of the motion, a free-body diagram of the bucket would be as shown. Since the bucket is moving in a circle, there must be a net force on it towards the center of the circle, and a centripetal acceleration. Write Newton's second law for the bucket, with up as the positive direction.

$$
\begin{aligned}
& \sum F_{\mathrm{R}}=F_{\mathrm{T}}-m g=m a=m v^{2} / r \rightarrow \\
& v=\sqrt{\frac{r\left(F_{\mathrm{T}}-m g\right)}{m}}=\sqrt{\frac{(1.10 \mathrm{~m})\left[25.0 \mathrm{~N}-(2.00 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\right]}{2.00 \mathrm{~kg}}}=1.723 \approx 1.7 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(b) A free-body diagram of the bucket at the top of the motion is shown. Since the bucket is moving in a circle, there must be a net force on it towards the center of the circle, and a centripetal acceleration. Write Newton's second law for the bucket, with down as the positive direction.

$$
\sum F_{\mathrm{R}}=F_{\mathrm{T}}+m g=m a=m v^{2} / r \rightarrow v=\sqrt{\frac{r\left(F_{\mathrm{T}}+m g\right)}{m}}
$$



If the tension is to be zero, then

$$
v=\sqrt{\frac{r(0+m g)}{m}}=\sqrt{r g}=\sqrt{(1.10 \mathrm{~m})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=3.28 \mathrm{~m} / \mathrm{s}
$$

The bucket must move faster than $3.28 \mathrm{~m} / \mathrm{s}$ in order for the rope not to go slack.
45. The free-body diagram for passengers at the top of a Ferris wheel is as shown. $F_{\mathrm{N}}$ is the normal force of the seat pushing up on the passenger. The sum of the forces on the passenger is producing the centripetal motion, and so must be a centripetal force. Call the downward direction positive, and write Newton's second law for the passenger.


$$
\sum F_{\mathrm{R}}=m g-F_{\mathrm{N}}=m a=m v^{2} / r
$$

Since the passenger is to feel "weightless," they must lose contact with their seat, and so the normal force will be 0 . The diameter is 22 m , so the radius is 11 m .

$$
\begin{aligned}
& m g=m v^{2} / r \rightarrow v=\sqrt{g r}=\sqrt{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(11 \mathrm{~m})}=10.38 \mathrm{~m} / \mathrm{s} \\
& (10.38 \mathrm{~m} / \mathrm{s})\left(\frac{1 \mathrm{rev}}{2 \pi(11 \mathrm{~m})}\right)\left(\frac{60 \mathrm{~s}}{1 \mathrm{~min}}\right)=9.0 \mathrm{rpm}
\end{aligned}
$$

46. To describe the motion in a circle, two independent quantities are needed. The radius of the circle and the speed of the object are independent of each other, so we choose those two quantities. The radius has dimensions of [L] and the speed has dimensions of $[\mathrm{L} / \mathrm{T}]$. These two dimensions need to be combined to get dimensions of $\left[\mathrm{L} / \mathrm{T}^{2}\right]$. The speed must be squared, which gives $\left[\mathrm{L}^{2} / \mathrm{T}^{2}\right]$, and then dividing by the radius gives $\left[\mathrm{L} / \mathrm{T}^{2}\right]$. So $a_{\mathrm{R}}=v^{2} / r$ is a possible form for the centripetal acceleration. Note that we are unable to get numerical factors, like $\pi$ or $\frac{1}{2}$, from dimensional analysis.
47. (a) See the free-body diagram for the pilot in the jet at the bottom of the loop. We have $a_{\mathrm{R}}=v^{2} / r=6 g$.

$$
v^{2} / r=6.0 g \rightarrow r=\frac{v^{2}}{6.0 g}=\frac{\left[(1200 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)\right]^{2}}{6.0\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=1900 \mathrm{~m}
$$


(b) The net force must be centripetal, to make the pilot go in a circle. Write Newton's second law for the vertical direction, with up as positive. The normal force is the apparent weight.

$$
\sum F_{\mathrm{R}}=F_{\mathrm{N}}-m g=m v^{2} / r
$$

The centripetal acceleration is to be $v^{2} / r=6.0 g$.

$$
F_{\mathrm{N}}=m g+m v^{2} / r=7 m g=7(78 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=5350 \mathrm{~N}=5400 \mathrm{~N}
$$

(c) See the free-body diagram for the pilot at the top of the loop. Notice that the normal force is down, because the pilot is upside down. Write Newton's second law in the vertical direction, with down as positive.

$$
\sum F_{\mathrm{R}}=F_{\mathrm{N}}+m g=m v^{2} / r=6 m g \rightarrow F_{\mathrm{N}}=5 m g=3800 \mathrm{~N}
$$


48. To experience a gravity-type force, objects must be on the inside of the outer wall of the tube, so that there can be a centripetal force to move the objects in a circle. See the free-body diagram for an object on the inside of the outer wall, and a portion of the tube. The normal force of contact between the object and the wall must be maintaining the circular motion. Write Newton's second law for the radial direction.

$$
\sum F_{\mathrm{R}}=F_{\mathrm{N}}=m a=m v^{2} / r
$$

If this is to have the same effect as Earth gravity, then we must also have that
 $F_{\mathrm{N}}=m g$. Equate the two expressions for normal force and solve for the speed.

$$
\begin{aligned}
& F_{\mathrm{N}}=m v^{2} / r=m g \rightarrow v=\sqrt{g r}=\sqrt{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(550 \mathrm{~m})}=73.42 \mathrm{~m} / \mathrm{s} \\
& (73.42 \mathrm{~m} / \mathrm{s})\left(\frac{1 \mathrm{rev}}{2 \pi(550 \mathrm{~m})}\right)\left(\frac{86,400 \mathrm{~s}}{1 \mathrm{~d}}\right)=1836 \mathrm{rev} / \mathrm{d} \approx 1.8 \times 10^{3} \mathrm{rev} / \mathrm{d}
\end{aligned}
$$

49. The radius of either skater's motion is 0.80 m , and the period is 2.5 sec . Thus their speed is given by $v=2 \pi r / T=\frac{2 \pi(0.80 \mathrm{~m})}{2.5 \mathrm{~s}}=2.0 \mathrm{~m} / \mathrm{s}$. Since each skater is moving in a circle, the net radial force on each one is given by Eq. 5-3.

$$
F_{\mathrm{R}}=m v^{2} / r=\frac{(60.0 \mathrm{~kg})(2.0 \mathrm{~m} / \mathrm{s})^{2}}{0.80 \mathrm{~m}}=3.0 \times 10^{2} \mathrm{~N} .
$$

50. A free-body diagram for the ball is shown. The tension in the suspending cord must not only hold the ball up, but also provide the centripetal force needed to make the ball move in a circle. Write Newton's second law for the vertical direction, noting that the ball is not accelerating vertically.

$$
\sum F_{y}=F_{\mathrm{T}} \sin \theta-m g=0 \rightarrow F_{\mathrm{T}}=\frac{m g}{\sin \theta}
$$

The force moving the ball in a circle is the horizontal portion of the tension. Write Newton's second law for that radial motion.

$$
\sum F_{\mathrm{R}}=F_{\mathrm{T}} \cos \theta=m a_{\mathrm{R}}=m v^{2} / r
$$

Substitute the expression for the tension from the first equation into the second equation, and solve for the angle. Also substitute in the fact that for a rotating object, $v=2 \pi r / T$. Finally we recognize that if the string is of length $\ell$, then the radius of the circle is $r=\ell \cos \theta$.

$$
\begin{aligned}
& F_{\mathrm{T}} \cos \theta=\frac{m g}{\sin \theta} \cos \theta=\frac{m v^{2}}{r}=\frac{4 \pi^{2} m r}{T^{2}}=\frac{4 \pi^{2} m \ell \cos \theta}{T^{2}} \rightarrow \\
& \sin \theta=\frac{g T^{2}}{4 \pi^{2} \ell} \rightarrow \theta=\sin ^{-1} \frac{g T^{2}}{4 \pi^{2} \ell}=\sin ^{-1} \frac{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.500 \mathrm{~s})^{2}}{4 \pi^{2}(0.600 \mathrm{~m})}=5.94^{\circ}
\end{aligned}
$$

The tension is then given by $F_{\mathrm{T}}=\frac{m g}{\sin \theta}=\frac{(0.150 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{\sin 5.94^{\circ}}=14.2 \mathrm{~N}$
51. The force of static friction is causing the circular motion - it is the centripetal force. The coin slides off when the static frictional force is not large enough to move the coin in a circle. The maximum static frictional force is the coefficient of static friction times the normal force, and the normal force is equal to the weight of the coin as seen in the free-body diagram, since there is no vertical
 acceleration. In the free-body diagram, the coin is coming out of the paper and the center of the circle is to the right of the coin, in the plane of the paper.
The rotational speed must be changed into a linear speed.

$$
\begin{aligned}
& v=\left(35.0 \frac{\mathrm{rev}}{\min }\right)\left(\frac{1 \mathrm{~min}}{60 \mathrm{~s}}\right)\left(\frac{2 \pi(0.120 \mathrm{~m})}{1 \mathrm{rev}}\right)=0.4398 \mathrm{~m} / \mathrm{s} \\
& F_{\mathrm{R}}=F_{\mathrm{fr}} \rightarrow m v^{2} / r=\mu_{s} F_{\mathrm{N}}=\mu_{s} m g \rightarrow \mu_{s}=\frac{v^{2}}{r g}=\frac{(0.4398 \mathrm{~m} / \mathrm{s})^{2}}{(0.120 \mathrm{~m})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=0.164
\end{aligned}
$$

52. For the car to stay on the road, the normal force must be greater than 0 . See the free-body diagram, write the net radial force, and solve for the radius.

$$
F_{\mathrm{R}}=m g \cos \theta-F_{\mathrm{N}}=\frac{m v^{2}}{r} \rightarrow r=\frac{m v^{2}}{m g \cos \theta-F_{\mathrm{N}}}
$$

For the car to be on the verge of leaving the road, the normal force would be 0 , and so $r_{\text {critical }}=\frac{m v^{2}}{m g \cos \theta}=\frac{v^{2}}{g \cos \theta}$. This expression gets larger as the angle increases, and so we must evaluate at the
 largest angle to find a radius that is good for all angles in the range.

$$
r_{\substack{\text { critical } \\ \text { maximum }}}=\frac{v^{2}}{g \cos \theta_{\max }}=\frac{\left[95 \mathrm{~km} / \mathrm{h}\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)\right]^{2}}{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \cos 22^{\circ}}=77 \mathrm{~m}
$$

53. (a) A free-body diagram of the car at the instant it is on the top of the hill is shown. Since the car is moving in a circular path, there must be a net centripetal force downward. Write Newton's second law for the car, with down as the positive direction.


$$
\begin{aligned}
& \sum F_{\mathrm{R}}=m g-F_{\mathrm{N}}=m a=m v^{2} / r \rightarrow \\
& F_{\mathrm{N}}=m\left(g-v^{2} / r\right)=(975 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}-\frac{(12.0 \mathrm{~m} / \mathrm{s})^{2}}{88.0 \mathrm{~m}}\right)=7960 \mathrm{~N}
\end{aligned}
$$

(b) The free-body diagram for the passengers would be the same as the one for the car, leading to the same equation for the normal force on the passengers.

$$
F_{\mathrm{N}}=m\left(g-v^{2} / r\right)=(72.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}-\frac{(12.0 \mathrm{~m} / \mathrm{s})^{2}}{88.0 \mathrm{~m}}\right)=588 \mathrm{~N}
$$

Notice that this is significantly less than the $700-\mathrm{N}$ weight of the passenger. Thus the passenger will feel "light" as they drive over the hill.
(c) For the normal force to be zero, we must have the following.

$$
F_{\mathrm{N}}=m\left(g-v^{2} / r\right)=0 \rightarrow g=v^{2} / r \rightarrow v=\sqrt{g r}=\sqrt{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(88.0 \mathrm{~m})}=29.4 \mathrm{~m} / \mathrm{s}
$$

54. If the masses are in line and both have the same frequency of rotation, then they will always stay in line. Consider a freebody diagram for both masses, from a side view, at the instant that they are to the left of the post. Note that the same tension that pulls inward on mass 2 pulls outward on mass 1 , by Newton's third law. Also notice that since there is no vertical acceleration, the normal force on each mass is equal
 to its weight. Write Newton's second law for the horizontal direction for both masses, noting that they are in uniform circular motion.

$$
\sum F_{\mathrm{RA}}=F_{\mathrm{TA}}-F_{\mathrm{TB}}=m_{\mathrm{A}} a_{\mathrm{A}}=m_{\mathrm{A}} v_{\mathrm{A}}^{2} / r_{\mathrm{A}} \quad \sum F_{\mathrm{RB}}=F_{\mathrm{TB}}=m_{\mathrm{B}} a_{\mathrm{B}}=m_{\mathrm{B}} v_{\mathrm{B}}^{2} / r_{\mathrm{B}}
$$

The speeds can be expressed in terms of the frequency as follows: $v=\left(f \frac{\mathrm{rev}}{\mathrm{sec}}\right)\left(\frac{2 \pi r}{1 \mathrm{rev}}\right)=2 \pi r f$.

$$
\begin{aligned}
& F_{\text {TB }}=m_{\mathrm{B}} v_{\mathrm{B}}^{2} / r_{\mathrm{B}}=m_{\mathrm{B}}\left(2 \pi r_{\mathrm{B}} f\right)^{2} / r_{\mathrm{B}}=4 \pi^{2} m_{\mathrm{B}} r_{\mathrm{B}} f^{2} \\
& F_{\text {TA }}=F_{\text {TB }}+m_{\mathrm{A}} v_{\mathrm{A}}^{2} / r_{\mathrm{A}}=4 \pi m_{\mathrm{B}} r_{\mathrm{B}} f^{2}+m_{\mathrm{A}}\left(2 \pi r_{\mathrm{A}} f\right)^{2} / r_{\mathrm{A}}=4 \pi^{2} f^{2}\left(m_{\mathrm{A}} r_{\mathrm{A}}+m_{\mathrm{B}} r_{\mathrm{B}}\right)
\end{aligned}
$$

55. A free-body diagram of Tarzan at the bottom of his swing is shown. The upward tension force is created by his pulling down on the vine. Write Newton's second law in the vertical direction. Since he is moving in a circle, his acceleration will be centripetal, and points upward when he is at the bottom.

$$
\sum F=F_{\mathrm{T}}-m g=m a=m v^{2} / r \rightarrow v=\sqrt{\frac{\left(F_{\mathrm{T}}-m g\right) r}{m}}
$$



The maximum speed will be obtained with the maximum tension.

$$
v_{\max }=\sqrt{\frac{\left(\overrightarrow{\mathbf{F}}_{\mathrm{T} \max }-m g\right) r}{m}}=\sqrt{\frac{\left(1350 \mathrm{~N}-(78 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\right) 5.2 \mathrm{~m}}{78 \mathrm{~kg}}}=6.2 \mathrm{~m} / \mathrm{s}
$$

56. The fact that the pilot can withstand 9.0 g 's without blacking out, along with the speed of the aircraft, will determine the radius of the circle that he must fly as he pulls out of the dive. To just avoid crashing into the sea, he must begin to form that circle (pull out of the dive) at a height equal to the radius of that circle.


$$
a_{\mathrm{R}}=v^{2} / r=9.0 g \rightarrow r=\frac{v^{2}}{9.0 g}=\frac{(310 \mathrm{~m} / \mathrm{s})^{2}}{9.0\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=1.1 \times 10^{3} \mathrm{~m}
$$

57. (a) We are given that $x=(2.0 \mathrm{~m}) \cos (3.0 \mathrm{rad} / \mathrm{s} t)$ and $y=(2.0 \mathrm{~m}) \sin (3.0 \mathrm{rad} / \mathrm{s} t)$. Square both components and add them together.

$$
\begin{aligned}
x^{2}+y^{2} & =[(2.0 \mathrm{~m}) \cos (3.0 \mathrm{rad} / \mathrm{s} t)]^{2}+[(2.0 \mathrm{~m}) \sin (3.0 \mathrm{rad} / \mathrm{s} t)]^{2} \\
& =(2.0 \mathrm{~m})^{2}\left[\cos ^{2}(3.0 \mathrm{rad} / \mathrm{s} t)+\sin ^{2}(3.0 \mathrm{rad} / \mathrm{s} t)\right]=(2.0 \mathrm{~m})^{2}
\end{aligned}
$$

This is the equation of a circle, $x^{2}+y^{2}=r^{2}$, with a radius of 2.0 m .
(b)

$$
\begin{aligned}
& \overrightarrow{\mathbf{v}}=(-6.0 \mathrm{~m} / \mathrm{s}) \sin (3.0 \mathrm{rad} / \mathrm{s} t) \hat{\mathbf{i}}+(6.0 \mathrm{~m} / \mathrm{s}) \cos (3.0 \mathrm{rad} / \mathrm{s} t) \hat{\mathbf{j}} \\
& \overrightarrow{\mathbf{a}}=\left(-18 \mathrm{~m} / \mathrm{s}^{2}\right) \cos (3.0 \mathrm{rad} / \mathrm{s} t) \hat{\mathbf{i}}+\left(-18 \mathrm{~m} / \mathrm{s}^{2}\right) \sin (3.0 \mathrm{rad} / \mathrm{s} t) \hat{\mathbf{j}}
\end{aligned}
$$

(c) $v=\sqrt{v_{x}^{2}+v_{y}^{2}}=\sqrt{[(-6.0 \mathrm{~m} / \mathrm{s}) \sin (3.0 \mathrm{rad} / \mathrm{s} t)]^{2}+[(6.0 \mathrm{~m} / \mathrm{s}) \cos (3.0 \mathrm{rad} / \mathrm{s} t)]^{2}}=6.0 \mathrm{~m} / \mathrm{s}$

$$
a=\sqrt{a_{x}^{2}+a_{y}^{2}}=\sqrt{\left[\left(-18 \mathrm{~m} / \mathrm{s}^{2}\right) \cos (3.0 \mathrm{rad} / \mathrm{s} t)\right]^{2}+\left[\left(-18 \mathrm{~m} / \mathrm{s}^{2}\right) \sin (3.0 \mathrm{rad} / \mathrm{s} t)\right]^{2}}=18 \mathrm{~m} / \mathrm{s}^{2}
$$

(d) $\frac{v^{2}}{r}=\frac{(6.0 \mathrm{~m} / \mathrm{s})^{2}}{2.0 \mathrm{~m}}=18 \mathrm{~m} / \mathrm{s}^{2}=a$
(e) $\quad \overrightarrow{\mathbf{a}}=\left(-18 \mathrm{~m} / \mathrm{s}^{2}\right) \cos (3.0 \mathrm{rad} / \mathrm{s} t) \hat{\mathbf{i}}+\left(-18 \mathrm{~m} / \mathrm{s}^{2}\right) \sin (3.0 \mathrm{rad} / \mathrm{s} t) \hat{\mathbf{j}}$

$$
=\left(-9.0 / \mathrm{s}^{2}\right)[2.0 \mathrm{~m} \cos (3.0 \mathrm{rad} / \mathrm{s} t) \hat{\mathbf{i}}+2.0 \mathrm{~m} \sin (3.0 \mathrm{rad} / \mathrm{s} t) \hat{\mathbf{j}}]=\left(9.0 / \mathrm{s}^{2}\right)(-\overrightarrow{\mathbf{r}})
$$

We see that the acceleration vector is directed oppositely of the position vector. Since the position vector points outward from the center of the circle, the acceleration vector points toward the center of the circle.
58. Since the curve is designed for $65 \mathrm{~km} / \mathrm{h}$, traveling at a higher speed with the same radius means that more centripetal force will be required. That extra centripetal force will be supplied by a force of static friction, downward along the incline. See the free-body diagram for the car on the incline. Note that from Example 5-15 in the textbook, the no-friction banking angle is given by the following.

$$
\theta=\tan ^{-1} \frac{v^{2}}{r g}=\tan ^{-1} \frac{\left[(65 \mathrm{~km} / \mathrm{h})\left(\frac{1.0 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)\right]^{2}}{(85 \mathrm{~m})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=21.4^{\circ}
$$

Write Newton's second law in both the $x$ and $y$ directions. The car will have no acceleration in the $y$ direction, and centripetal acceleration in the $x$ direction. We also assume that the car is on the verge of skidding, so that the static frictional force has its maximum value of $F_{\mathrm{fr}}=\mu_{s} F_{\mathrm{N}}$. Solve each equation for the normal force.

$$
\sum F_{y}=F_{\mathrm{N}} \cos \theta-m g-F_{\mathrm{fr}} \sin \theta=0 \rightarrow F_{\mathrm{N}} \cos \theta-\mu_{s} F_{\mathrm{N}} \sin \theta=m g \quad \rightarrow
$$

$$
\begin{aligned}
& F_{\mathrm{N}}=\frac{m g}{\left(\cos \theta-\mu_{s} \sin \theta\right)} \\
& \sum F_{x}=F_{\mathrm{N}} \sin \theta+F_{\mathrm{fr}} \cos \theta=F_{\mathrm{R}}=m v^{2} / r \rightarrow F_{\mathrm{N}} \sin \theta+\mu_{s} F_{\mathrm{N}} \cos \theta=m v^{2} / r \rightarrow \\
& F_{\mathrm{N}}=\frac{m v^{2} / r}{\left(\sin \theta+\mu_{s} \cos \theta\right)}
\end{aligned}
$$

Equate the two expressions for $F_{\mathrm{N}}$, and solve for the coefficient of friction. The speed of rounding the curve is given by $v=(95 \mathrm{~km} / \mathrm{h})\left(\frac{1.0 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)=26.39 \mathrm{~m} / \mathrm{s}$.

$$
\frac{m g}{\left(\cos \theta-\mu_{s} \sin \theta\right)}=\frac{m v^{2} / r}{\left(\sin \theta+\mu_{s} \cos \theta\right)} \rightarrow
$$

$$
\mu_{s}=\frac{\left(\frac{v^{2}}{r} \cos \theta-g \sin \theta\right)}{\left(g \cos \theta+\frac{v^{2}}{r} \sin \theta\right)}=\frac{\left(\frac{v^{2}}{r}-g \tan \theta\right)}{\left(g+\frac{v^{2}}{r} \tan \theta\right)}=\frac{\left(\frac{(26.39 \mathrm{~m} / \mathrm{s})^{2}}{85 \mathrm{~m}}-\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \tan 21.4^{\circ}\right)}{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}+\frac{(26.39 \mathrm{~m} / \mathrm{s})^{2}}{85 \mathrm{~m}} \tan 21.4^{\circ}\right)}=0.33
$$

59. Since the curve is designed for a speed of $85 \mathrm{~km} / \mathrm{h}$, traveling at that speed would mean no friction is needed to round the curve. From Example 515 in the textbook, the no-friction banking angle is given by

$$
\theta=\tan ^{-1} \frac{v^{2}}{r g}=\tan ^{-1} \frac{\left[(85 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)\right]^{2}}{(68 \mathrm{~m})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=39.91^{\circ}
$$



Driving at a higher speed with the same radius means that more centripetal force will be required than is present by the normal force alone. That extra centripetal force will be supplied by a force of static friction, downward along the incline, as shown in the first free-body diagram for the car on the incline. Write Newton's second law in both the $x$ and $y$ directions. The car will have no acceleration in the $y$ direction, and centripetal acceleration in the $x$ direction. We also assume that the car is on the verge of skidding, so that the static frictional force has its maximum value of $F_{\mathrm{fr}}=\mu_{s} F_{\mathrm{N}}$.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{N}} \cos \theta-m g-F_{\mathrm{fr}} \sin \theta=0 \rightarrow F_{\mathrm{N}} \cos \theta-\mu_{s} F_{\mathrm{N}} \sin \theta=m g \rightarrow \\
& F_{\mathrm{N}}=\frac{m g}{\left(\cos \theta-\mu_{s} \sin \theta\right)} \\
& \sum F_{x}=F_{\mathrm{N}} \sin \theta+F_{\mathrm{fr}} \cos \theta=m v^{2} / r \rightarrow F_{\mathrm{N}} \sin \theta+\mu_{s} F_{\mathrm{N}} \cos \theta=m v^{2} / r \rightarrow \\
& F_{\mathrm{N}}=\frac{m v^{2} / r}{\left(\sin \theta+\mu_{s} \cos \theta\right)}
\end{aligned}
$$

Equate the two expressions for the normal force, and solve for the speed.

$$
\begin{aligned}
& \frac{m v^{2} / r}{\left(\sin \theta+\mu_{s} \cos \theta\right)}=\frac{m g}{\left(\cos \theta-\mu_{s} \sin \theta\right)} \rightarrow \\
& v=\sqrt{r g \frac{\left(\sin \theta+\mu_{s} \cos \theta\right)}{\left(\cos \theta-\mu_{s} \sin \theta\right)}}=\sqrt{(68 \mathrm{~m})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \frac{\left(\sin 39.91^{\circ}+0.30 \cos 39.91^{\circ}\right)}{\left(\cos 39.91^{\circ}-0.30 \sin 39.91^{\circ}\right)}}=32 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Now for the slowest possible speed. Driving at a slower speed with the same radius means that less centripetal force will be required than that supplied by the normal force. That decline in centripetal force will be supplied by a force of static friction, upward along the incline, as shown in the second free-body diagram for the car on the incline. Write Newton's second law in both the $x$ and $y$ directions. The car will have no acceleration in the $y$ direction, and centripetal
 acceleration in the $x$ direction. We also assume that the car is on the verge of skidding, so that the static frictional force has its maximum value of $F_{\mathrm{fr}}=\mu_{s} F_{\mathrm{N}}$.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{N}} \cos \theta-m g+F_{\mathrm{fr}} \sin \theta=0 \rightarrow \\
& F_{\mathrm{N}} \cos \theta+\mu_{s} F_{\mathrm{N}} \sin \theta=m g \rightarrow \quad F_{\mathrm{N}}=\frac{m g}{\left(\cos \theta+\mu_{s} \sin \theta\right)} \\
& \sum F_{x}=F_{\mathrm{N}} \sin \theta-F_{\mathrm{fr}} \cos \theta=m v^{2} / r \rightarrow F_{\mathrm{N}} \sin \theta-\mu_{s} F_{\mathrm{N}} \cos \theta=m v^{2} / r \rightarrow \\
& F_{\mathrm{N}}=\frac{m v^{2} / r}{\left(\sin \theta-\mu_{s} \cos \theta\right)}
\end{aligned}
$$

Equate the two expressions for the normal force, and solve for the speed.

$$
\begin{aligned}
& \frac{m v^{2} / r}{\left(\sin \theta-\mu_{s} \cos \theta\right)}=\frac{m g}{\left(\cos \theta+\mu_{s} \sin \theta\right)} \rightarrow \\
& v=\sqrt{r g \frac{\left(\sin \theta-\mu_{s} \cos \theta\right)}{\left(\cos \theta+\mu_{s} \sin \theta\right)}}=\sqrt{(68 \mathrm{~m})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \frac{\left(\sin 39.91^{\circ}-0.30 \cos 39.91^{\circ}\right)}{\left(\cos 39.91^{\circ}+0.30 \sin 39.91^{\circ}\right)}}=17 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Thus the range is $17 \mathrm{~m} / \mathrm{s} \leq v \leq 32 \mathrm{~m} / \mathrm{s}$, which is $61 \mathrm{~km} / \mathrm{h} \leq v \leq 115 \mathrm{~km} / \mathrm{h}$.
60. (a) The object has a uniformly increasing speed, which means the tangential acceleration is constant, and so constant acceleration relationships can be used for the tangential motion. The object is moving in a circle of radius 2.0 meters.

$$
\Delta x_{\tan }=\frac{v_{\tan }+v_{0}}{2} t \rightarrow v_{\tan }=\frac{2 \Delta x_{\tan }}{t}-v_{0}=\frac{2\left[\frac{1}{4}(2 \pi r)\right]}{t}=\frac{\pi(2.0 \mathrm{~m})}{2.0 \mathrm{~s}}=\pi \mathrm{m} / \mathrm{s}
$$

(b) The initial location of the object is at $2.0 \mathrm{~m} \hat{\mathbf{j}}$, and the final location is $2.0 \mathrm{~m} \hat{\mathbf{i}}$.

$$
\overrightarrow{\mathbf{v}}_{\mathrm{avg}}=\frac{\overrightarrow{\mathbf{r}}-\overrightarrow{\mathbf{r}}_{0}}{t}=\frac{2.0 \mathrm{~m} \hat{\mathbf{i}}-2.0 \mathrm{~m} \hat{\mathbf{j}}}{2.0 \mathrm{~s}}=1.0 \mathrm{~m} / \mathrm{s}(\hat{\mathbf{i}}-\hat{\mathbf{j}})
$$

(c) The velocity at the end of the 2.0 seconds is pointing in the $-\hat{\mathbf{j}}$ direction.

$$
\overrightarrow{\mathbf{a}}_{\mathrm{arg}}=\frac{\overrightarrow{\mathbf{v}}-\overrightarrow{\mathbf{v}}_{0}}{t}=\frac{-(\pi \mathrm{m} / \mathrm{s}) \hat{\mathbf{j}}}{2.0 \mathrm{~s}}=\left(-\pi / 2 \mathrm{~m} / \mathrm{s}^{2}\right) \hat{\mathbf{j}}
$$

61. Apply uniform acceleration relationships to the tangential motion to find the tangential acceleration. Use Eq. 2-12b.

$$
\Delta x_{\mathrm{tan}}=\underset{\tan }{v_{0}} t+\frac{1}{2} a_{\mathrm{tan}} t^{2} \rightarrow a_{\mathrm{tan}}=\frac{2 \Delta x_{\mathrm{tan}}}{t^{2}}=\frac{2\left[\frac{1}{4}(2 \pi r)\right]}{t^{2}}=\frac{\pi(2.0 \mathrm{~m})}{(2.0 \mathrm{~s})^{2}}=(\pi / 2) \mathrm{m} / \mathrm{s}^{2}
$$

The tangential acceleration is constant. The radial acceleration is found from $a_{\mathrm{rad}}=\frac{v_{\tan }^{2}}{r}=\frac{\left(a_{\tan } t\right)^{2}}{r}$.
(a) $a_{\mathrm{tan}}=(\pi / 2) \mathrm{m} / \mathrm{s}^{2}, a_{\mathrm{rad}}=\frac{\left(a_{\mathrm{tan}} t\right)^{2}}{r}=\frac{\left[(\pi / 2) \mathrm{m} / \mathrm{s}^{2}(0 \mathrm{~s})\right]^{2}}{2.0 \mathrm{~m}}=0$
(b) $a_{\text {tan }}=(\pi / 2) \mathrm{m} / \mathrm{s}^{2}, a_{\mathrm{rad}}=\frac{\left(a_{\mathrm{tan}} t\right)^{2}}{r}=\frac{\left[(\pi / 2) \mathrm{m} / \mathrm{s}^{2}(1.0 \mathrm{~s})\right]^{2}}{2.0 \mathrm{~m}}=\left(\pi^{2} / 8\right) \mathrm{m} / \mathrm{s}^{2}$
(c) $a_{\text {tan }}=(\pi / 2) \mathrm{m} / \mathrm{s}^{2}, a_{\mathrm{rad}}=\frac{\left(a_{\tan } t\right)^{2}}{r}=\frac{\left[(\pi / 2) \mathrm{m} / \mathrm{s}^{2}(2.0 \mathrm{~s})\right]^{2}}{2.0 \mathrm{~m}}=\left(\pi^{2} / 2\right) \mathrm{m} / \mathrm{s}^{2}$
62. (a) The tangential acceleration is the time derivative of the speed.

$$
a_{\mathrm{tan}}=\frac{d v_{\mathrm{tan}}}{d t}=\frac{d\left(3.6+1.5 t^{2}\right)}{d t}=3.0 t \rightarrow a_{\mathrm{tan}}(3.0 \mathrm{~s})=3.0(3.0)=9.0 \mathrm{~m} / \mathrm{s}^{2}
$$

(b) The radial acceleration is given by Eq. 5-1.

$$
a_{\mathrm{rad}}=\frac{v_{\mathrm{tan}}^{2}}{r}=\frac{\left(3.6+1.5 t^{2}\right)^{2}}{r} \rightarrow a_{\mathrm{rad}}(3.0 \mathrm{~s})=\frac{\left(3.6+1.5(3.0)^{2}\right)^{2}}{22 \mathrm{~m}}=13 \mathrm{~m} / \mathrm{s}^{2}
$$

63. We show a top view of the particle in circular motion, traveling clockwise. Because the particle is in circular motion, there must be a radially-inward component of the acceleration.
(a) $a_{\mathrm{R}}=a \sin \theta=v^{2} / r \rightarrow$

$$
v=\sqrt{a r \sin \theta}=\sqrt{\left(1.15 \mathrm{~m} / \mathrm{s}^{2}\right)(3.80 \mathrm{~m}) \sin 38.0^{\circ}}=1.64 \mathrm{~m} / \mathrm{s}
$$

(b) The particle's speed change comes from the tangential acceleration,
 which is given by $a_{\mathrm{tan}}=a \cos \theta$. If the tangential acceleration is constant, then using Eq. 2-12a,

$$
\begin{aligned}
& v_{\tan }-v_{0 \tan }=a_{\tan } t \rightarrow \\
& v_{\tan }=v_{0 \tan }+a_{\tan } t=1.64 \mathrm{~m} / \mathrm{s}+\left(1.15 \mathrm{~m} / \mathrm{s}^{2}\right)\left(\cos 38.0^{\circ}\right)(2.00 \mathrm{~s})=3.45 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

64. The tangential force is simply the mass times the tangential acceleration.

$$
a_{T}=b+c t^{2} \rightarrow F_{T}=m a_{T}=m\left(b+c t^{2}\right)
$$

To find the radial force, we need the tangential velocity, which is the anti-derivative of the tangential acceleration. We evaluate the constant of integration so that $v=v_{0}$ at $t=0$.

$$
\begin{aligned}
& a_{T}=b+c t^{2} \rightarrow v_{T}=c+b t+\frac{1}{3} c t^{3} \rightarrow v(0)=c=v_{0} \rightarrow v_{T}=v_{0}+b t+\frac{1}{3} c t^{3} \\
& F_{R}=\frac{m v_{T}^{2}}{r}=\frac{m}{r}\left(v_{0}+b t+\frac{1}{3} c t^{3}\right)^{2}
\end{aligned}
$$

65. The time constant $\tau$ must have dimensions of [T]. The units of $m$ are [M]. Since the expression $b v$ is a force, we must have the dimensions of $b$ as force units divided by speed units. So the dimensions of $b$ are as follows: $\frac{\text { Force units }}{\text { speed units }}=\frac{[\mathrm{M}]\left[\mathrm{L} / \mathrm{T}^{2}\right]}{[\mathrm{L} / \mathrm{T}]}=\left[\frac{\mathrm{M}}{\mathrm{T}}\right]$. Thus to get dimensions of [T], we must have $\tau=m / b$.
66. (a) The terminal velocity is given by Eq. 5-9. This can be used to find the value of $b$.

$$
v_{\mathrm{T}}=\frac{m g}{b} \rightarrow b=\frac{m g}{v_{\mathrm{T}}}=\frac{\left(3 \times 10^{-5} \mathrm{~kg}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{(9 \mathrm{~m} / \mathrm{s})}=3.27 \times 10^{-5} \mathrm{~kg} / \mathrm{s} \approx 3 \times 10^{-5} \mathrm{~kg} / \mathrm{s}
$$

(b) From Example 5-17, the time required for the velocity to reach $63 \%$ of terminal velocity is the time constant, $\tau=m / b$.

$$
\tau=\frac{m}{b}=\frac{3 \times 10^{-5} \mathrm{~kg}}{3.27 \times 10^{-5} \mathrm{~kg} / \mathrm{s}}=0.917 \mathrm{~s} \approx 1 \mathrm{~s}
$$

67. (a) We choose downward as the positive direction. Then the force of gravity is in the positive direction, and the resistive force is upwards. We follow the analysis given in Example 5-17.

$$
\begin{aligned}
& F_{\text {net }}=m g-b v=m a \rightarrow a=\frac{d v}{d t}=g-\frac{b}{m} v=-\frac{b}{m}\left(v-\frac{m g}{b}\right) \rightarrow \\
& \frac{d v}{v-\frac{m g}{b}}=-\frac{b}{m} d t \rightarrow \int_{v_{0}}^{v} \frac{d v}{v-\frac{m g}{b}}=-\frac{b}{m} \int_{0}^{t} d t \rightarrow \ln \left[v-\frac{m g}{b}\right]_{v_{0}}^{v}=-\frac{b}{m} t \rightarrow \\
& \ln \left[\frac{v-\frac{m g}{b}}{v_{0}-\frac{m g}{b}}\right]=-\frac{b}{m} t \rightarrow \frac{v-\frac{m g}{b}}{v_{0}-\frac{m g}{b}}=e^{-\frac{b}{m} t} \rightarrow v=\frac{m g}{b}\left(1-e^{-\frac{b}{m} t}\right)+v_{0} e^{-\frac{b}{m} t}
\end{aligned}
$$

Note that this motion has a terminal velocity of $v_{\text {terminal }}=m g / b$.
(b) We choose upwards as the positive direction. Then both the force of gravity and the resistive force are in the negative direction.

$$
\begin{aligned}
& F_{\text {net }}=-m g-b v=m a \rightarrow a=\frac{d v}{d t}=-g-\frac{b}{m} v=-\frac{b}{m}\left(v+\frac{m g}{b}\right) \rightarrow \\
& \frac{d v}{v+\frac{m g}{b}}=-\frac{b}{m} d t \rightarrow \int_{v_{0}}^{v} \frac{d v}{v+\frac{m g}{b}}=-\frac{b}{m} \int_{0}^{t} d t \rightarrow \ln \left[v+\frac{m g}{b}\right]_{v_{0}}^{v}=-\frac{b}{m} t \rightarrow \\
& \ln \left[\frac{v+\frac{m g}{b}}{v_{0}+\frac{m g}{b}}\right]=-\frac{b}{m} t \rightarrow \frac{v+\frac{m g}{b}}{v_{0}+\frac{m g}{b}}=e^{-\frac{b}{m} t} \rightarrow v=\frac{m g}{b}\left(e^{-\frac{b}{m} t}-1\right)+v_{0} e^{-\frac{b}{m} t}
\end{aligned}
$$

After the object reaches its maximum height $\left[t_{\text {rise }}=\frac{m}{b} \ln \left(1+\frac{b v_{0}}{m g}\right)\right]$, at which point the speed will be 0 , it will then start to fall. The equation from part (a) will then describe its falling motion.
68. The net force on the falling object, taking downward as positive, will be $\sum F=m g-b v^{2}=m a$.
(a) The terminal velocity occurs when the acceleration is 0 .
$m g-b v^{2}=m a \rightarrow m g-b v_{\mathrm{T}}^{2}=0 \rightarrow v_{\mathrm{T}}=\sqrt{m g / b}$
(b) $v_{\mathrm{T}}=\sqrt{\frac{m g}{b}} \rightarrow b=\frac{m g}{v_{\mathrm{T}}^{2}}=\frac{(75 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{(60 \mathrm{~m} / \mathrm{s})^{2}}=0.2 \mathrm{~kg} / \mathrm{m}$
(c) The curve would be qualitatively like Fig. 5-27, because the speed would increase from 0 to the terminal velocity, asymptotically. But this curve would be ABOVE the one in Fig. 5-27, because the friction force increases more rapidly. For Fig. 5-27, if the speed doubles, the friction force doubles. But in this case, if the speed doubles, the friction force would increase by a factor of 4, bringing the friction force closer to the weight of the object in a shorter period of time.
69. (a) See the free-body diagram for the coasting. Since the bicyclist has a constant velocity, the net force on the bicycle must be 0 . Use this to find the value of the constant $c$.

$$
\begin{aligned}
& \sum F_{x}=m g \sin \theta-F_{\mathrm{D}}=m g \sin \theta-c v^{2}=0 \rightarrow \\
& c=\frac{m g \sin \theta}{v^{2}}=\frac{(80.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \sin 7.0^{\circ}}{\left[9.5 \mathrm{~km} / \mathrm{h}\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)\right]^{2}}=13.72 \mathrm{~kg} / \mathrm{m} \\
& \approx 14 \mathrm{~kg} / \mathrm{m}
\end{aligned}
$$

(b) Now another force, $\overrightarrow{\mathbf{F}}_{\mathrm{P}}$, must be added down the plane to represent the additional force needed to descend at the higher speed. The velocity is still constant. See the new free-body diagram.

$$
\begin{aligned}
& \sum F_{x}=m g \sin \theta+F_{\mathrm{P}}-F_{\mathrm{D}}=m g \sin \theta+F_{\mathrm{P}}-c v^{2}=0 \rightarrow \\
& F_{\mathrm{P}}=c v^{2}-m g \sin \theta \\
& \quad=(13.72 \mathrm{~kg} / \mathrm{m})\left[25 \mathrm{~km} / \mathrm{h}\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)\right]^{2}-(80.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \sin 7.0^{\circ}=570 \mathrm{~N}
\end{aligned}
$$

70. (a) The rolling drag force is given as $F_{\mathrm{D} 1} \approx 4.0 \mathrm{~N}$. The air resistance drag force is proportional to $v^{2}$, and so $F_{\mathrm{D} 2}=b v^{2}$. Use the data to find the proportionality constant, and then sum the two drag forces to find the total drag force.

$$
\begin{aligned}
& F_{\mathrm{D} 2}=b v^{2} \rightarrow 1.0 \mathrm{~N}=b(2.2 \mathrm{~m} / \mathrm{s})^{2} \rightarrow b=\frac{1.0 \mathrm{~N}}{(2.2 \mathrm{~m} / \mathrm{s})^{2}}=0.2066 \mathrm{~kg} / \mathrm{m} \\
& F_{\mathrm{D}}=F_{\mathrm{D} 1}+F_{\mathrm{D} 2}=\left(4.0+0.21 v^{2}\right) \mathrm{N}
\end{aligned}
$$

(b) See the free-body diagram for the coasting bicycle and rider. Take the positive direction to be down the plane, parallel to the plane.
The net force in that direction must be 0 for the bicycle to coast at a constant speed.

$$
\begin{aligned}
& \sum F_{x}=m g \sin \theta-F_{\mathrm{D}}=0 \rightarrow m g \sin \theta=F_{\mathrm{D}} \rightarrow \\
& \theta=\sin ^{-1} \frac{F_{\mathrm{D}}}{m g}=\sin ^{-1} \frac{\left(4.0+0.2066 v^{2}\right)}{m g} \\
& =\sin ^{-1} \frac{\left(4.0 \mathrm{~N}+(0.2066 \mathrm{~kg} / \mathrm{m})(8.0 \mathrm{~m} / \mathrm{s})^{2}\right)}{(78 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=1.3^{\circ}
\end{aligned}
$$


71. From Example 5-17, we have that $v=\frac{m g}{b}\left(1-e^{-\frac{b}{m} t}\right)$. We use this expression to find the position and acceleration expressions.

$$
\begin{aligned}
& a=\frac{d v}{d t}=\frac{m g}{b}\left(-e^{-\frac{b}{m} t}\right)\left(-\frac{b}{m}\right)=g e^{-\frac{b}{m} t} \\
& v=\frac{d x}{d t} \rightarrow d x=v d t \rightarrow \int_{0}^{x} d x=\int_{0}^{t} v d t=\int_{0}^{t} \frac{m g}{b}\left(1-e^{-\frac{b}{m} t}\right) d t \rightarrow \\
& x=\left[\frac{m g}{b} t+\frac{m g}{b} \frac{m}{b} e^{-\frac{b}{m} t}\right]_{0}^{t}=\frac{m g}{b} t+\frac{m^{2} g}{b^{2}}\left(e^{-\frac{b}{m} t}-1\right)
\end{aligned}
$$

72. We solve this problem by integrating the acceleration to find the velocity, and integrating the velocity to find the position.

$$
\begin{aligned}
F_{\text {net }} & =-b v^{\frac{1}{2}}=m a=m \frac{d v}{d t} \rightarrow \frac{d v}{d t}=-\frac{b}{m} v^{\frac{1}{2}} \rightarrow \frac{d v}{v^{\frac{1}{2}}}=-\frac{b}{m} d t \rightarrow \\
\frac{d x}{d t} & =\left(v_{v_{0}}^{v} \frac{d v}{v^{\frac{1}{2}}}=-\frac{b}{m} \int_{0}^{t} d t \rightarrow 2 v^{\frac{1}{2}}-2 v_{0}^{\frac{1}{2}}=-\frac{b}{m} t \rightarrow v=\left(v_{0}^{2}-\frac{b t}{2 m}\right)^{2}\right. \\
x & \left.=-\frac{2 m}{3 b}\left[\left(v_{0}^{\frac{1}{2}}-\frac{b t}{2 m}\right)^{3}-v_{0}^{\frac{3}{2}}\right]=\frac{2 m}{3 b}\left[v_{0}^{\frac{1}{2}}-\frac{b t}{2 m}\right)^{2} d t \rightarrow \int_{0}^{\frac{3}{2}}-\left(v_{0}^{\frac{1}{2}}-\frac{b t}{2 m}\right)^{3}\right] \int_{0}^{t}\left(v_{0}^{\frac{1}{2}}-\frac{b t}{2 m}\right)^{2} d t \rightarrow \\
& =\frac{2 m}{3 b}\left(v_{0}^{\frac{3}{2}}-\left(v_{0}^{\frac{3}{2}}-3 v_{0} \frac{b t}{2 m}+3 v_{0}^{\frac{1}{2}} \frac{b^{2} t^{2}}{4 m^{2}}-\frac{b^{3} t^{3}}{8 m^{3}}\right)\right)=\frac{2 m}{3 b}\left(v_{0}^{\frac{3}{2}}-v_{0}^{\frac{3}{2}}+3 v_{0} \frac{b t}{2 m}-3 v_{0}^{\frac{1}{2}} \frac{b^{2} t^{2}}{4 m^{2}}+\frac{b^{3} t^{3}}{8 m^{3}}\right) \\
& =\left(v_{0} t-\frac{v_{0}^{\frac{1}{2}} b}{2 m} t^{2}+\frac{b^{2}}{12 m^{2}} t^{3}\right)
\end{aligned}
$$

73. From problem 72 , we have that $v=\left(v_{0}^{\frac{1}{2}}-\frac{b t}{2 m}\right)^{2}$ and $x=\left(v_{0} t-\frac{v_{0}^{\frac{1}{2}} b}{2 m} t^{2}+\frac{b^{2}}{12 m^{2}} t^{3}\right)$. The maximum distance will occur at the time when the velocity is 0 . From the equation for the velocity, we see that happens at $t_{\max }=\frac{2 m v_{0}^{\frac{1}{2}}}{b}$. Use this time in the expression for distance to find the maximum distance.

$$
x\left(t=t_{\max }\right)=v_{0} \frac{2 m v_{0}^{\frac{1}{2}}}{b}-\frac{v_{0}^{\frac{1}{2}}}{2 m}\left(\frac{2 m v_{0}^{\frac{1}{2}}}{b}\right)^{2}+\frac{b^{2}}{12 m^{2}}\left(\frac{2 m v_{0}^{\frac{1}{2}}}{b}\right)^{3}=\frac{2 m v_{0}^{\frac{3}{2}}}{b}-\frac{2 m v_{0}^{\frac{3}{2}}}{b}+\frac{2 m v_{0}^{\frac{3}{2}}}{3 b}=\frac{2 m v_{0}^{\frac{3}{2}}}{3 b}
$$

74. The net force is the force of gravity downward, and the drag force upwards. Let the downward direction be positive. Represent the value of $1.00 \times 10^{4} \mathrm{~kg} / \mathrm{s}$ by the symbol $b$, as in Eq. 5-6.

$$
\sum F=m g-F_{\mathrm{d}}=m g-b v=m a=m \frac{d v}{d t} \rightarrow \frac{d v}{d t}=g-\frac{b}{m} v \rightarrow
$$

$$
\frac{d v}{v-\frac{m g}{b}}=-\frac{b}{m} d t \rightarrow \int_{v_{0}}^{v} \frac{d v}{v-\frac{m g}{b}}=-\frac{b}{m} \int_{0}^{t} d t \rightarrow \ln \left(v-\frac{m g}{b}\right)-\ln \left(v_{0}-\frac{m g}{b}\right)=-\frac{b}{m} t
$$

Solve for $t$, and evaluate at $v=0.02 v_{0}$.

$$
\begin{aligned}
t & =\frac{\ln \left(0.02 v_{0}-\frac{m g}{b}\right)-\ln \left(v_{0}-\frac{m g}{b}\right)}{-b / m} \\
& =\frac{\ln \left(0.02(5.0 \mathrm{~m} / \mathrm{s})-\frac{(75 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{1.00 \times 10^{4} \mathrm{~kg} / \mathrm{s}}\right)-\ln \left((5.0 \mathrm{~m} / \mathrm{s})-\frac{(75 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{1.00 \times 10^{4} \mathrm{~kg} / \mathrm{s}}\right)}{-\left(1.00 \times 10^{4} \mathrm{~kg} / \mathrm{s}\right) /(75 \mathrm{~kg})} \\
& =3.919 \times 10^{-2} \mathrm{~s} \approx 3.9 \times 10^{-2} \mathrm{~s}
\end{aligned}
$$

75. The only force accelerating the boat is the drag force, and so Newton's second law becomes $\sum F=-b v=m a$. Use this to solve for the velocity and position expressions, and then find the distance traveled under the given conditions.

$$
\begin{aligned}
& \sum F=-b v=m a=m \frac{d v}{d t} \rightarrow \frac{d v}{d t}=-\frac{b}{m} v \rightarrow \int_{v_{0}}^{v} \frac{d v}{v}=-\frac{b}{m} \int_{0}^{t} d t \rightarrow \ln \frac{v}{v_{0}}=-\frac{b}{m} t \rightarrow \\
& v=v_{0} e^{-\frac{b}{m} t}
\end{aligned}
$$

Note that this velocity never changes sign. It asymptotically approaches 0 as time approaches infinity. Apply the condition that at $t=3.0 \mathrm{~s}$ the speed is $v=\frac{1}{2} v_{0}$.

$$
v(t=3.0)=v_{0} e^{-\frac{b}{m}(3.0)}=\frac{1}{2} v_{0} \rightarrow \frac{b}{m}=\frac{\ln 2}{3.0 \mathrm{~s}}
$$

Now solve for the position expression. The object will reach its maximum position when it stops, which is after an infinite time.

$$
\begin{aligned}
& v=\frac{d x}{d t}=v_{0} e^{-\frac{b}{m} t} \rightarrow d x=v_{0} e^{-\frac{b}{m} t} d t \rightarrow \int_{0}^{x} d x=\int_{0}^{t} v_{0} e^{-\frac{b}{m} t} d t \rightarrow \\
& x=-v_{0} \frac{m}{b}\left(e^{-\frac{b}{m} t}-1\right)=v_{0} \frac{m}{b}\left(1-e^{-\frac{b}{m} t}\right) \rightarrow x(t=\infty)=v_{0} \frac{m}{b}=(2.4 \mathrm{~m} / \mathrm{s}) \frac{3.0 \mathrm{~s}}{\ln 2}=10.39 \mathrm{~m} \approx 10 \mathrm{~m}
\end{aligned}
$$

76. A free-body diagram for the coffee cup is shown. Assume that the car is moving to the right, and so the acceleration of the car (and cup) will be to the left. The deceleration of the cup is caused by friction between the cup and the dashboard. For the cup to not slide on the dash, and to have the minimum deceleration time means the largest possible static frictional force is acting, so $F_{\mathrm{fr}}=\mu_{s} F_{\mathrm{N}}$. The normal force
 on the cup is equal to its weight, since there is no vertical acceleration. The horizontal acceleration of the cup is found from Eq. 2-12a, with a final velocity of zero.

$$
\begin{aligned}
& v_{0}=(45 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)=12.5 \mathrm{~m} / \mathrm{s} \\
& v-v_{0}=a t \rightarrow a=\frac{v-v_{0}}{t}=\frac{0-12.5 \mathrm{~m} / \mathrm{s}}{3.5 \mathrm{~s}}=-3.57 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

Write Newton's second law for the horizontal forces, considering to the right to be positive.

$$
\sum F_{x}=-F_{\mathrm{fr}}=m a \rightarrow m a=-\mu_{s} F_{\mathrm{N}}=-\mu_{s} m g \rightarrow \mu_{s}=-\frac{a}{g}=-\frac{\left(-3.57 \mathrm{~m} / \mathrm{s}^{2}\right)}{9.80 \mathrm{~m} / \mathrm{s}^{2}}=0.36
$$

77. Since the drawer moves with the applied force of 9.0 N , we assume that the maximum static frictional force is essentially 9.0 N . This force is equal to the coefficient of static friction times the normal force. The normal force is assumed to be equal to the weight, since the drawer is horizontal.

$$
F_{\mathrm{fr}}=\mu_{s} F_{\mathrm{N}}=\mu_{s} m g \rightarrow \mu_{s}=\frac{F_{\mathrm{fr}}}{m g}=\frac{9.0 \mathrm{~N}}{(2.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=0.46
$$

78. See the free-body diagram for the descending roller coaster. It starts its descent with $v_{0}=(6.0 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)=1.667 \mathrm{~m} / \mathrm{s}$. The total displacement in the $x$ direction is $x-x_{0}=45.0 \mathrm{~m}$. Write Newton's second law for both the $x$ and $y$ directions.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{N}}-m g \cos \theta=0 \rightarrow F_{\mathrm{N}}=m g \cos \theta \\
& \sum F_{x}=m a=m g \sin \theta-F_{\mathrm{fr}}=m g \sin \theta-\mu_{k} F_{\mathrm{N}}=m g \sin \theta-\mu_{k} m g \cos \theta \\
& \quad a=\frac{m g \sin \theta-\mu_{k} m g \cos \theta}{m}=g\left(\sin \theta-\mu_{k} \cos \theta\right)
\end{aligned}
$$

Now use Eq. 2-12c to solve for the final velocity.

$$
\begin{aligned}
v^{2} & -v_{0}^{2}=2 a\left(x-x_{0}\right) \rightarrow \\
v & =\sqrt{v_{0}^{2}+2 a\left(x-x_{0}\right)}=\sqrt{v_{0}^{2}+2 g\left(\sin \theta-\mu_{k} \cos \theta\right)\left(x-x_{0}\right)} \\
& =\sqrt{(1.667 \mathrm{~m} / \mathrm{s})^{2}+2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left[\sin 45^{\circ}-(0.12) \cos 45^{\circ}\right](45.0 \mathrm{~m})} \\
& =23.49 \mathrm{~m} / \mathrm{s} \approx 23 \mathrm{~m} / \mathrm{s} \approx 85 \mathrm{~km} / \mathrm{h}
\end{aligned}
$$

79. Consider a free-body diagram of the box. Write Newton's second law for both directions. The net force in the $y$ direction is 0 because there is no acceleration in the $y$ direction.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{N}}-m g \cos \theta=0 \rightarrow F_{\mathrm{N}}=m g \cos \theta \\
& \sum F_{x}=m g \sin \theta-F_{\mathrm{fr}}=m a
\end{aligned}
$$

Now solve for the force of friction and the coefficient of friction.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{N}}-m g \cos \theta=0 \rightarrow F_{\mathrm{N}}=m g \cos \theta \\
& \sum F_{x}=m g \sin \theta-F_{\mathrm{fr}}=m a \\
& F_{\mathrm{fr}}=m g \sin \theta-m a=m(g \sin \theta-a)=(18.0 \mathrm{~kg})\left[\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(\sin 37.0^{\circ}\right)-0.220 \mathrm{~m} / \mathrm{s}^{2}\right] \\
& \quad=102.2 \mathrm{~N} \approx 102 \mathrm{~N} \\
& F_{\mathrm{fr}}=\mu_{k} F_{\mathrm{N}}=\mu_{k} m g \cos \theta \rightarrow \mu_{k}=\frac{F_{\mathrm{fr}}}{m g \cos \theta}=\frac{102.2 \mathrm{~N}}{(18.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \cos 37.0^{\circ}}=0.725
\end{aligned}
$$

80. Since mass $m$ is dangling, the tension in the cord must be equal to the weight of mass $m$, and so $F_{\mathrm{T}}=m g$. That same tension is in the other end of the cord, maintaining the circular motion of mass $M$, and so $F_{\mathrm{T}}=F_{\mathrm{R}}=M a_{\mathrm{R}}=M v^{2} / r$. Equate the expressions for tension and solve for the velocity.

$$
M v^{2} / r=m g \rightarrow v=\sqrt{m g R / M}
$$

81. Consider the free-body diagram for the cyclist in the sand, assuming that the cyclist is traveling to the right. It is apparent that $F_{\mathrm{N}}=m g$ since there is no vertical acceleration. Write Newton's second law for the horizontal direction, positive to the right.

$$
\sum F_{x}=-F_{\mathrm{fr}}=m a \rightarrow-\mu_{k} m g=m a \rightarrow a=-\mu_{k} g
$$

Use Eq. 2-12c to determine the distance the cyclist could travel in the sand before coming to rest.


$$
v^{2}-v_{0}^{2}=2 a\left(x-x_{0}\right) \rightarrow\left(x-x_{0}\right)=\frac{v^{2}-v_{0}^{2}}{2 a}=\frac{-v_{0}^{2}}{-2 \mu_{k} g}=\frac{(20.0 \mathrm{~m} / \mathrm{s})^{2}}{2(0.70)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=29 \mathrm{~m}
$$

Since there is only 15 m of sand, the cyclist will emerge from the sand. The speed upon emerging is found from Eq. 2-12c.

$$
\begin{aligned}
v^{2} & -v_{0}^{2}=2 a\left(x-x_{0}\right) \rightarrow \\
v & =\sqrt{v_{0}^{2}+2 a\left(x-x_{0}\right)}=\sqrt{v_{i}^{2}-2 \mu_{k} g\left(x-x_{0}\right)}=\sqrt{(20.0 \mathrm{~m} / \mathrm{s})^{2}-2(0.70)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(15 \mathrm{~m})} \\
& =14 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

82. Consider the free-body diagram for a person in the "Rotor-ride." $\overrightarrow{\mathbf{F}}_{\mathrm{N}}$ is the normal force of contact between the rider and the wall, and $\overrightarrow{\mathbf{F}}_{\mathrm{fr}}$ is the static frictional force between the back of the rider and the wall. Write Newton's second law for the vertical forces, noting that there is no vertical acceleration.

$$
\sum F_{y}=F_{\mathrm{ff}}-m g=0 \rightarrow F_{\mathrm{fr}}=m g
$$

If we assume that the static friction force is a maximum, then

$$
F_{\mathrm{fr}}=\mu_{s} F_{\mathrm{N}}=m g \rightarrow F_{\mathrm{N}}=m g / \mu_{s} .
$$



But the normal force must be the force causing the centripetal motion - it is the only force pointing to the center of rotation. Thus $F_{\mathrm{R}}=F_{\mathrm{N}}=m v^{2} / r$. Using $v=2 \pi r / T$, we have $F_{N}=\frac{4 \pi^{2} m r}{T^{2}}$. Equate the two expressions for the normal force and solve for the coefficient of friction. Note that since there are 0.50 rev per sec, the period is 2.0 sec .

$$
F_{N}=\frac{4 \pi^{2} m r}{T^{2}}=\frac{m g}{\mu_{s}} \rightarrow \mu_{s}=\frac{g T^{2}}{4 \pi^{2} r}=\frac{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(2.0 \mathrm{~s})^{2}}{4 \pi^{2}(5.5 \mathrm{~m})}=0.18
$$

Any larger value of the coefficient of friction would mean that the normal force could be smaller to achieve the same frictional force, and so the period could be longer or the cylinder radius smaller.

There is no force pushing outward on the riders. Rather, the wall pushes against the riders, so by Newton's third law, the riders push against the wall. This gives the sensation of being pressed into the wall.
83. The force is a centripetal force, and is of magnitude 7.45 mg . Use Eq. $5-3$ for centripetal force.

$$
\begin{aligned}
& F=m \frac{v^{2}}{r}=7.45 \mathrm{mg} \rightarrow v=\sqrt{7.45 r g}=\sqrt{7.45(11.0 \mathrm{~m})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=28.34 \mathrm{~m} / \mathrm{s} \approx 28.3 \mathrm{~m} / \mathrm{s} \\
& (28.34 \mathrm{~m} / \mathrm{s}) \times \frac{1 \mathrm{rev}}{2 \pi(11.0 \mathrm{~m})}=0.410 \mathrm{rev} / \mathrm{s}
\end{aligned}
$$

84. The car moves in a horizontal circle, and so there must be a net horizontal centripetal force. The car is not accelerating vertically. Write Newton's second law for both the $x$ and $y$ directions.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{N}} \cos \theta-m g=0 \rightarrow F_{\mathrm{N}}=\frac{m g}{\cos \theta} \\
& \sum F_{x}=\sum F_{\mathrm{R}}=F_{\mathrm{N}} \sin \theta=m a_{x}
\end{aligned}
$$



The amount of centripetal force needed for the car to round the curve is as follows.

$$
F_{\mathrm{R}}=m v^{2} / r=(1250 \mathrm{~kg}) \frac{\left[(85 \mathrm{~km} / \mathrm{h})\left(\frac{1.0 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)\right]^{2}}{72 \mathrm{~m}}=9.679 \times 10^{3} \mathrm{~N}
$$

The actual horizontal force available from the normal force is as follows.

$$
F_{\mathrm{N}} \sin \theta=\frac{m g}{\cos \theta} \sin \theta=m g \tan \theta=(1250 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \tan 14^{\circ}=3.054 \times 10^{3} \mathrm{~N}
$$

Thus more force is necessary for the car to round the curve than can be supplied by the normal force. That extra force will have to have a horizontal component to the right in order to provide the extra centripetal force. Accordingly, we add a frictional force pointed down the plane. That corresponds to the car not being able to make the curve without friction.

Again write Newton's second law for both directions, and again the $y$ acceleration is zero.


$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{N}} \cos \theta-m g-F_{\mathrm{fr}} \sin \theta=0 \rightarrow F_{\mathrm{N}}=\frac{m g+F_{\mathrm{fr}} \sin \theta}{\cos \theta} \\
& \sum F_{x}=F_{\mathrm{N}} \sin \theta+F_{\mathrm{fr}} \cos \theta=m v^{2} / r
\end{aligned}
$$

Substitute the expression for the normal force from the $y$ equation into the $x$ equation, and solve for the friction force.

$$
\begin{aligned}
& \frac{m g+F_{\mathrm{fr}} \sin \theta}{\cos \theta} \sin \theta+F_{\mathrm{fr}} \cos \theta=m v^{2} / r \rightarrow\left(m g+F_{\mathrm{fr}} \sin \theta\right) \sin \theta+F_{\mathrm{fr}} \cos ^{2} \theta=m \frac{v^{2}}{r} \cos \theta \\
& F_{\mathrm{fr}}=m \frac{v^{2}}{r} \cos \theta-m g \sin \theta=\left(9.679 \times 10^{3} \mathrm{~N}\right) \cos 14^{\circ}-(1250 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \sin 14^{\circ} \\
& \quad=6.428 \times 10^{3} \mathrm{~N}
\end{aligned}
$$

So a frictional force of $6.4 \times 10^{3} \mathrm{~N}$ down the plane is needed to provide the necessary centripetal force to round the curve at the specified speed.
85. The radial force is given by Eq. 5-3.

$$
F_{\mathrm{R}}=m \frac{v^{2}}{r}=(1150 \mathrm{~kg}) \frac{(27 \mathrm{~m} / \mathrm{s})^{2}}{450 \mathrm{~m} / \mathrm{s}}=1863 \mathrm{~N} \approx 1900 \mathrm{~N}
$$

The tangential force is the mass times the tangential acceleration. The tangential acceleration is the change in tangential speed divided by the elapsed time.

$$
F_{\mathrm{T}}=m a_{\mathrm{T}}=m \frac{\Delta v_{\mathrm{T}}}{\Delta t}=(1150 \mathrm{~kg}) \frac{(27 \mathrm{~m} / \mathrm{s})}{(9.0 \mathrm{~s})}=3450 \mathrm{~N} \approx 3500 \mathrm{~N}
$$

86. Since the walls are vertical, the normal forces are horizontal, away from the wall faces. We assume that the frictional forces are at their maximum values, so $F_{\mathrm{fr}}=\mu_{s} F_{\mathrm{N}}$ applies at each wall. We assume that the rope in the diagram is not under any tension and so does not exert any forces. Consider the free-body diagram for the climber. $F_{\mathrm{NR}}$ is the normal force on the climber from the right
 wall, and $F_{\mathrm{NL}}$ is the normal force on the climber from the left wall. The static frictional forces are $F_{\mathrm{frL}}=\mu_{s \mathrm{~L}} F_{\mathrm{NL}}$ and $F_{\mathrm{frR}}=\mu_{s \mathrm{R}} F_{\mathrm{NR}}$. Write Newton's second law for both the $x$ and $y$ directions. The net force in each direction must be zero if the climber is stationary.

$$
\sum F_{x}=F_{\mathrm{NL}}-F_{\mathrm{NR}}=0 \rightarrow F_{\mathrm{NL}}=F_{\mathrm{NR}} \quad \sum F_{y}=F_{\mathrm{frL}}+F_{\mathrm{frR}}-m g=0
$$

Substitute the information from the $x$ equation into the $y$ equation.

$$
\begin{aligned}
& F_{\mathrm{frL}}+F_{\mathrm{frR}}=m g \rightarrow \mu_{s \mathrm{~L}} F_{\mathrm{NL}}+\mu_{s \mathrm{R}} F_{\mathrm{NR}}=m g \rightarrow\left(\mu_{s \mathrm{~L}}+\mu_{\mathrm{sR}}\right) F_{\mathrm{NL}}=m g \\
& F_{\mathrm{NL}}=\frac{m g}{\left(\mu_{s \mathrm{~L}}+\mu_{s \mathrm{R}}\right)}=\frac{(70.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{1.40}=4.90 \times 10^{2} \mathrm{~N}
\end{aligned}
$$

And so $F_{\mathrm{NL}}=F_{\mathrm{NR}}=4.90 \times 10^{2} \mathrm{~N}$. These normal forces arise as Newton's third law reaction forces to the climber pushing on the walls. Thus the climber must exert a force of at least 490 N against each wall.
87. The mass would start sliding when the static frictional force was not large enough to counteract the component of gravity that will be pulling the mass along the curved surface. See the free-body diagram, and assume that the static frictional force is a maximum. We also assume the block has no speed, so the radial force must be 0 .

$$
\begin{aligned}
& \sum F_{\text {radial }}=F_{\mathrm{N}}-m g \cos \phi \rightarrow F_{\mathrm{N}}=m g \cos \phi \\
& \sum F_{\text {tangential }}=m g \sin \phi-F_{\mathrm{fr}} \rightarrow F_{\mathrm{fr}}=m g \sin \phi \\
& F_{\mathrm{fr}}=\mu_{\mathrm{s}} F_{\mathrm{N}}=\mu_{\mathrm{s}} m g \cos \phi=m g \sin \phi \rightarrow \mu_{\mathrm{s}}=\tan \phi \rightarrow \\
& \phi=\tan ^{-1} \mu_{s}=\tan ^{-1} 0.70=35^{\circ}
\end{aligned}
$$


88. (a) Consider the free-body diagrams for both objects, initially stationary. As sand is added, the tension will increase, and the force of static friction on the block will increase until it reaches its maximum of $F_{\mathrm{fr}}=\mu_{\mathrm{s}} F_{\mathrm{N}}$. Then the system will start to move. Write Newton's second law for each object, when the static frictional force is at its maximum, but the objects are still stationary.

$$
\begin{aligned}
& \sum F_{y \text { bucket }}=m_{1} g-F_{\mathrm{T}}=0 \rightarrow F_{\mathrm{T}}=m_{1} g \\
& \sum F_{y \text { block }}=F_{\mathrm{N}}-m_{2} g=0 \rightarrow F_{\mathrm{N}}=m_{2} g \\
& \sum F_{x \text { block }}=F_{\mathrm{T}}-F_{\mathrm{fr}}=0 \rightarrow F_{\mathrm{T}}=F_{\mathrm{fr}}
\end{aligned}
$$

Equate the two expressions for tension, and substitute in the expression for the normal force to find the masses.

$$
\begin{aligned}
& m_{1} g=F_{\mathrm{fr}} \rightarrow m_{1} g=\mu_{s} F_{\mathrm{N}}=\mu_{s} m_{2} g \rightarrow \\
& m_{1}=\mu_{s} m_{2}=(0.45)(28.0 \mathrm{~kg})=12.6 \mathrm{~kg}
\end{aligned}
$$

Thus $12.6 \mathrm{~kg}-2.00 \mathrm{~kg}=10.6 \mathrm{~kg} \approx 11 \mathrm{~kg}$ of sand was added.
(b) The same free-body diagrams can be used, but now the objects will accelerate. Since they are tied together, $a_{y 1}=a_{x 2}=a$. The frictional force is now kinetic friction, given by $F_{\mathrm{fr}}=\mu_{k} F_{\mathrm{N}}=\mu_{k} m_{2} g$. Write Newton's second
 laws for the objects in the direction of their acceleration.

$$
\begin{aligned}
& \sum F_{y \text { bucket }}=m_{1} g-F_{\mathrm{T}}=m_{1} a \rightarrow F_{\mathrm{T}}=m_{1} g-m_{1} a \\
& \sum F_{x \text { block }}=F_{\mathrm{T}}-F_{\mathrm{fr}}=m_{2} a \rightarrow F_{\mathrm{T}}=F_{\mathrm{fr}}+m_{2} a
\end{aligned}
$$

Equate the two expressions for tension, and solve for the acceleration.

$$
\begin{aligned}
& m_{1} g-m_{1} a=\mu_{k} m_{2} g+m_{2} a \rightarrow \\
& a=g \frac{\left(m_{1}-\mu_{k} m_{2}\right)}{\left(m_{1}+m_{2}\right)}=\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \frac{(12.6 \mathrm{~kg}-(0.32)(28.0 \mathrm{~kg}))}{(12.6 \mathrm{~kg}+28.0 \mathrm{~kg})}=0.88 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

89. The acceleration that static friction can provide can be found from the minimum stopping distance, assuming that the car is just on the verge of sliding. Use Eq. 2-12c. Then, assuming an unbanked curve, the same static frictional force is used to provide the centripetal acceleration needed to make the curve. The acceleration from the stopping distance is negative, and so the centripetal acceleration is the opposite of that expression.

$$
v^{2}-v_{0}^{2}=2 a\left(x-x_{0}\right) \rightarrow a_{\text {stopping }}=\frac{v^{2}-v_{0}^{2}}{2\left(x-x_{0}\right)}=\frac{-v_{0}^{2}}{2\left(x-x_{0}\right)} \rightarrow a_{\mathrm{R}}=\frac{v_{0}^{2}}{2\left(x-x_{0}\right)}
$$

Equate the above expression to the typical expression for centripetal acceleration.

$$
a_{\mathrm{R}}=\frac{v^{2}}{r}=\frac{v_{0}^{2}}{2\left(x-x_{0}\right)} \rightarrow r=2\left(x-x_{0}\right)=132 \mathrm{~m}
$$

Notice that we didn't need to know the mass of the car, the initial speed, or the coefficient of friction.
90. The radial acceleration is given by $a_{R}=v^{2} / r$. Substitute in the speed of the tip of the sweep hand, given by $v=2 \pi r / T$, to get $a_{R}=\frac{4 \pi^{2} r}{T^{2}}$. For the tip of the sweep hand, $r=0.015 \mathrm{~m}$, and $T=60 \mathrm{sec}$.

$$
a_{R}=\frac{4 \pi^{2} r}{T^{2}}=\frac{4 \pi^{2}(0.015 \mathrm{~m})}{(60 \mathrm{~s})^{2}}=1.6 \times 10^{-4} \mathrm{~m} / \mathrm{s}^{2}
$$

91. (a) The horizontal component of the lift force will produce a centripetal acceleration. Write Newton's second law for both the horizontal and vertical directions, and combine those equations to solve for the time needed to reverse course (a half-period of the circular motion). Note that

$$
\begin{aligned}
T= & \frac{2 \pi r}{v} . \\
& \sum F_{\text {vericial }}=F_{\text {lift }} \cos \theta=m g ; \sum F_{\text {horizontal }}=F_{\text {lifi }} \sin \theta=m \frac{v^{2}}{r}
\end{aligned}
$$



Divide these two equations.

$$
\begin{aligned}
& \frac{F_{\text {lift }} \sin \theta}{F_{\text {lift }} \cos \theta}=\frac{m v^{2}}{r m g} \rightarrow \tan \theta=\frac{v^{2}}{r g}=\frac{v^{2}}{\frac{T v}{2 \pi} g}=\frac{2 \pi v}{g T} \rightarrow \\
& \frac{T}{2}=\frac{\pi v}{g \tan \theta}=\frac{\pi\left[(480 \mathrm{~km} / \mathrm{h})\left(\frac{1.0 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)\right]}{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \tan 38^{\circ}}=55 \mathrm{~s}
\end{aligned}
$$

(b) The passengers will feel a change in the normal force that their seat exerts on them. Prior to the banking, the normal force was equal to their weight. During banking, the normal force will increase, so that $F_{\substack{\text { normal } \\ \text { banking }}}=\frac{m g}{\cos \theta}=1.27 \mathrm{mg}$. Thus they will feel "pressed down" into their seats, with about a $25 \%$ increase in their apparent weight. If the plane is banking to the left, they will feel pushed to the right by that extra $25 \%$ in their apparent weight.
92. From Example 5-15 in the textbook, the no-friction banking angle is given by $\theta=\tan ^{-1} \frac{v_{0}^{2}}{R g}$. The centripetal force in this case is provided by a component of the normal force. Driving at a higher speed with the same radius requires more centripetal force than that provided by the normal force alone. The additional centripetal force is supplied by a force of static friction, downward along the incline. See the free-body diagram for the car on the incline. The center of the circle of the car's motion is to the right of the car in the diagram. Write Newton's second law in both the $x$ and $y$
 directions. The car will have no acceleration in the $y$ direction, and centripetal acceleration in the $x$ direction. Assume that the car is on the verge of skidding, so that the static frictional force has its maximum value of $F_{\mathrm{fr}}=\mu_{s} F_{\mathrm{N}}$.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{N}} \cos \theta-m g-F_{\mathrm{fr}} \sin \theta=0 \rightarrow F_{\mathrm{N}} \cos \theta-\mu_{s} F_{\mathrm{N}} \sin \theta=m g \rightarrow \\
& F_{\mathrm{N}}=\frac{m g}{\left(\cos \theta-\mu_{s} \sin \theta\right)} \\
& \sum F_{x}=F_{\mathrm{R}}=F_{\mathrm{N}} \sin \theta+F_{\mathrm{fr}} \cos \theta=m v^{2} / R \rightarrow F_{\mathrm{N}} \sin \theta+\mu_{s} F_{\mathrm{N}} \cos \theta=m v^{2} / R \rightarrow \\
& F_{\mathrm{N}}=\frac{m v^{2} / R}{\left(\sin \theta+\mu_{s} \cos \theta\right)}
\end{aligned}
$$

Equate the two expressions for the normal force, and solve for the speed, which is the maximum speed that the car can have.

$$
\begin{aligned}
& \frac{m v^{2} / R}{\left(\sin \theta+\mu_{s} \cos \theta\right)}=\frac{m g}{\left(\cos \theta-\mu_{s} \sin \theta\right)} \rightarrow \\
& v_{\max }=\sqrt{R g \frac{\sin \theta}{\cos \theta} \frac{\left(1+\mu_{s} / \tan \theta\right)}{\left(1-\mu_{s} \tan \theta\right)}}=v_{0} \sqrt{\frac{\left(1+R g \mu_{s} / v_{0}^{2}\right)}{\left(1-\mu_{s} v_{0}^{2} / R g\right)}}
\end{aligned}
$$

Driving at a slower speed with the same radius requires less centripetal force than that provided by the normal force alone. The decrease in centripetal force is supplied by a force of static friction, upward along the incline. See the free-body diagram for the car on the incline. Write Newton's second law in both the $x$ and $y$ directions. The car will have no acceleration in the $y$ direction, and centripetal acceleration in the $x$ direction. Assume that the car is on the verge of
 skidding, so that the static frictional force is given by $F_{\mathrm{fr}}=\mu_{s} F_{\mathrm{N}}$.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{N}} \cos \theta-m g+F_{\mathrm{fr}} \sin \theta=0 \rightarrow \\
& F_{\mathrm{N}} \cos \theta+\mu_{s} F_{\mathrm{N}} \sin \theta=m g \rightarrow F_{\mathrm{N}}=\frac{m g}{\left(\cos \theta+\mu_{s} \sin \theta\right)} \\
& \sum F_{x}=F_{\mathrm{R}}=F_{\mathrm{N}} \sin \theta-F_{\mathrm{fr}} \cos \theta=m v^{2} / R \rightarrow F_{\mathrm{N}} \sin \theta-\mu_{s} F_{\mathrm{N}} \cos \theta=m v^{2} / R \rightarrow \\
& F_{\mathrm{N}}=\frac{m v^{2} / R}{\left(\sin \theta-\mu_{s} \cos \theta\right)}
\end{aligned}
$$

Equate the two expressions for the normal force, and solve for the speed.

$$
\frac{m v^{2} / R}{\left(\sin \theta-\mu_{s} \cos \theta\right)}=\frac{m g}{\left(\cos \theta+\mu_{s} \sin \theta\right)} \rightarrow
$$

$$
v_{\min }=\sqrt{R g \frac{\sin \theta}{\cos \theta} \frac{\left(1-\mu_{s} / \tan \theta\right)}{\left(1+\mu_{s} \tan \theta\right)}}=v_{0} \sqrt{\frac{\left(1-\mu_{s} R g / v_{0}^{2}\right)}{\left(1+\mu_{s} v_{0}^{2} / R g\right)}}
$$

Thus $v_{\text {min }}=v_{0} \sqrt{\frac{\left(1-\mu_{s} R g / v_{0}^{2}\right)}{\left(1+\mu_{s} v_{0}^{2} / R g\right)}}$ and $v_{\max }=v_{0} \sqrt{\frac{\left(1+R g \mu_{s} / v_{0}^{2}\right)}{\left(1-\mu_{s} v_{0}^{2} / R g\right)}}$.
93. (a) Because there is no friction between the bead and the hoop, the hoop can only exert a normal force on the bead. See the free-body diagram for the bead at the instant shown in the textbook figure. Note that the bead moves in a horizontal circle, parallel to the floor. Thus the centripetal force is horizontal, and the net vertical force must be 0 . Write Newton's second law for both the horizontal and vertical directions, and use those equations to determine the angle $\theta$. We also use the fact that the speed and the frequency are related to each other, by $v=2 \pi f r \sin \theta$.


$$
\begin{aligned}
& \sum F_{\text {vertical }}=F_{\mathrm{N}} \cos \theta-m g=0 \rightarrow F_{\mathrm{N}}=\frac{m g}{\cos \theta} \\
& \sum F_{\text {radial }}=F_{\mathrm{N}} \sin \theta=m \frac{v^{2}}{r \sin \theta}=m \frac{4 \pi^{2} f^{2} r^{2} \sin ^{2} \theta}{r \sin \theta}
\end{aligned}
$$

$$
F_{\mathrm{N}} \sin \theta=\frac{m g}{\cos \theta} \sin \theta=m \frac{4 \pi^{2} f^{2} r^{2} \sin ^{2} \theta}{r \sin \theta} \rightarrow \theta=\cos ^{-1} \frac{g}{4 \pi^{2} f^{2} r}
$$

(b)

$$
\theta=\cos ^{-1} \frac{g}{4 \pi^{2} f^{2} r}=\cos ^{-1} \frac{9.80 \mathrm{~m} / \mathrm{s}^{2}}{4 \pi^{2}(2.00 \mathrm{~Hz})^{2}(0.220 \mathrm{~m})}=73.6^{\circ}
$$

(c) No, the bead cannot ride as high as the center of the circle. If the bead were located there, the normal force of the wire on the bead would point horizontally. There would be no force to counteract the bead's weight, and so it would have to slip back down below the horizontal to balance the force of gravity. From a mathematical standpoint, the expression $\frac{g}{4 \pi^{2} f^{2} r}$ would have to be equal to 0 and that could only happen if the frequency or the radius were infinitely large.
94. An object at the Earth's equator is rotating in a circle with a radius equal to the radius of the Earth, and a period equal to one day. Use that data to find the centripetal acceleration and then compare it to $g$.

$$
a_{\mathrm{R}}=\frac{v^{2}}{r}=\frac{\left(\frac{2 \pi r}{T}\right)^{2}}{r}=\frac{4 \pi^{2} r}{T^{2}} \rightarrow \frac{a_{\mathrm{R}}}{g}=\frac{\frac{4 \pi^{2}\left(6.38 \times 10^{6} \mathrm{~m}\right)}{(86,400 \mathrm{~s})^{2}}}{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=0.00344 \approx \frac{3}{1000}
$$

So, for example, if we were to calculate the normal force on an object at the Earth's equator, we could not say $\sum F=F_{\mathrm{N}}-m g=0$. Instead, we would have the following.

$$
\sum F=F_{\mathrm{N}}-m g=-m \frac{v^{2}}{r} \rightarrow F_{\mathrm{N}}=m g-m \frac{v^{2}}{r}
$$

If we then assumed that $F_{\mathrm{N}}=m g_{\text {eff }}=m g-m \frac{v^{2}}{r}$, then we see that the effective value of $g$ is $g_{\text {eff }}=g-\frac{v^{2}}{r}=g-0.003 g=0.997 g$.
95. A free-body diagram for the sinker weight is shown. $L$ is the length of the string actually swinging the sinker. The radius of the circle of motion is moving is $r=L \sin \theta$. Write Newton's second law for the vertical direction, noting that the sinker is not accelerating vertically. Take up to be positive.

$$
\sum F_{y}=F_{\mathrm{T}} \cos \theta-m g=0 \rightarrow F_{\mathrm{T}}=\frac{m g}{\cos \theta}
$$



The radial force is the horizontal portion of the tension. Write Newton's second law for the radial motion.

$$
\sum F_{\mathrm{R}}=F_{\mathrm{T}} \sin \theta=m a_{\mathrm{R}}=m v^{2} / r
$$

Substitute the tension from the vertical equation, and the relationships $r=L \sin \theta$ and $v=2 \pi r / T$.

$$
\begin{aligned}
& F_{\mathrm{T}} \sin \theta=m v^{2} / r \rightarrow \frac{m g}{\cos \theta} \sin \theta=\frac{4 \pi^{2} m L \sin \theta}{T^{2}} \rightarrow \cos \theta=\frac{g T^{2}}{4 \pi^{2} L} \\
& \theta=\cos ^{-1} \frac{g T^{2}}{4 \pi^{2} L}=\cos ^{-1} \frac{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.50 \mathrm{~s})^{2}}{4 \pi^{2}(0.45 \mathrm{~m})}=82^{\circ}
\end{aligned}
$$

96. The speed of the train is $(160 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)=44.44 \mathrm{~m} / \mathrm{s}$.
(a) If there is no tilt, then the friction force must supply the entire centripetal force on the passenger.

$$
F_{\mathrm{R}}=m v^{2} / R=\frac{(75 \mathrm{~kg})(44.44 \mathrm{~m} / \mathrm{s})^{2}}{(570 \mathrm{~m})}=259.9 \mathrm{~N} \approx 2.6 \times 10^{2} \mathrm{~N}
$$

(b) For the banked case, the normal force will contribute to the radial force needed. Write Newton's second law for both the $x$ and $y$ directions. The $y$ acceleration is zero, and the $x$ acceleration is radial.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{N}} \cos \theta-m g-F_{\mathrm{fr}} \sin \theta=0 \rightarrow F_{\mathrm{N}}=\frac{m g+F_{\mathrm{fr}} \sin \theta}{\cos \theta} \\
& \sum F_{x}=F_{\mathrm{N}} \sin \theta+F_{\mathrm{fr}} \cos \theta=m v^{2} / r
\end{aligned}
$$



Substitute the expression for the normal force from the $y$ equation into the $x$ equation, and solve for the friction force.

$$
\begin{aligned}
& \frac{m g+F_{\mathrm{ff}} \sin \theta}{\cos \theta} \sin \theta+F_{\mathrm{fr}} \cos \theta=m v^{2} / r \rightarrow \\
& \left(m g+F_{\mathrm{fr}} \sin \theta\right) \sin \theta+F_{\mathrm{fr}} \cos ^{2} \theta=m \frac{v^{2}}{r} \cos \theta \rightarrow \\
& F_{\mathrm{fr}}=m\left(\frac{v^{2}}{r} \cos \theta-g \sin \theta\right) \\
& \quad=(75 \mathrm{~kg})\left[\frac{(44.44 \mathrm{~m} / \mathrm{s})^{2}}{570 \mathrm{~m}} \cos 8.0^{\circ}-\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \sin 8.0^{\circ}\right]=155 \mathrm{~N} \approx 1.6 \times 10^{2} \mathrm{~N}
\end{aligned}
$$

97. We include friction from the start, and then for the no-friction result, set the coefficient of friction equal to 0 . Consider a free-body diagram for the car on the hill. Write Newton's second law for both directions. Note that the net force on the $y$ direction will be zero, since there is no acceleration in the $y$ direction.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{N}}-m g \cos \theta=0 \rightarrow F_{\mathrm{N}}=m g \cos \theta \\
& \sum F_{x}=m g \sin \theta-F_{\mathrm{fr}}=m a \rightarrow \\
& a=g \sin \theta-\frac{F_{\mathrm{ff}}}{m}=g \sin \theta-\frac{\mu_{k} m g \cos \theta}{m}=g\left(\sin \theta-\mu_{k} \cos \theta\right)
\end{aligned}
$$



Use Eq. 2-12c to determine the final velocity, assuming that the car starts from rest.

$$
v^{2}-v_{0}^{2}=2 a\left(x-x_{0}\right) \rightarrow v=\sqrt{0+2 a\left(x-x_{0}\right)}=\sqrt{2 g\left(x-x_{0}\right)\left(\sin \theta-\mu_{k} \cos \theta\right)}
$$

The angle is given by $\sin \theta=1 / 4 \rightarrow \theta=\sin ^{-1} 0.25=14.5^{\circ}$
(a) $\mu_{k}=0 \rightarrow v=\sqrt{2 g\left(x-x_{0}\right) x \sin \theta}=\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(55 \mathrm{~m}) \sin 14.5^{\circ}}=16 \mathrm{~m} / \mathrm{s}$
(b) $\mu_{k}=0.10 \rightarrow v=\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(55 \mathrm{~m})\left(\sin 14.5^{\circ}-0.10 \cos 14.5^{\circ}\right)}=13 \mathrm{~m} / \mathrm{s}$
98. The two positions on the cone correspond to two opposite directions of the force of static friction. In one case, the frictional force points UP the cone's surface, and in the other case, it points DOWN the cone's surface. In each case the net vertical force is 0 , and force of static friction is assumed to be its maximum value. The net horizontal force is producing centripetal motion.

$$
\begin{aligned}
& \sum F_{\text {vericial }}=F_{\mathrm{N}} \sin \phi-F_{\mathrm{fr}} \cos \phi-m g=F_{\mathrm{N}} \sin \phi-\mu_{\mathrm{s}} F_{\mathrm{N}} \cos \phi-m g=0 \rightarrow \\
& F_{\mathrm{N}}=\frac{m g}{\sin \phi-\mu_{\mathrm{s}} \cos \phi} \\
& \begin{aligned}
\sum F_{\text {horizontal }}= & F_{\mathrm{N}} \cos \phi+F_{\mathrm{fr}} \sin \phi=F_{\mathrm{N}} \cos \phi+\mu_{\mathrm{s}} F_{\mathrm{N}} \sin \phi
\end{aligned} \\
& \quad=F_{\mathrm{N}}\left(\cos \phi+\mu_{\mathrm{s}} \sin \phi\right)=m \frac{v^{2}}{r}=m \frac{(2 \pi r f)^{2}}{r}=4 \pi^{2} r m f^{2} \rightarrow \\
& F_{\mathrm{N}}=\frac{4 \pi^{2} r m f^{2}}{\left(\cos \phi+\mu_{\mathrm{s}} \sin \phi\right)}
\end{aligned}
$$



Equate the two expressions for the normal force, and solve for the radius.

$$
F_{\mathrm{N}}=\frac{m g}{\sin \phi-\mu_{\mathrm{s}} \cos \phi}=\frac{4 \pi^{2} r m f^{2}}{\left(\cos \phi+\mu_{\mathrm{s}} \sin \phi\right)} \rightarrow r_{\max }=\frac{g\left(\cos \phi+\mu_{\mathrm{s}} \sin \phi\right)}{4 \pi^{2} f^{2}\left(\sin \phi-\mu_{\mathrm{s}} \cos \phi\right)}
$$

$$
\begin{aligned}
& \text { A similar analysis will lead to the minimum radius. } \\
& \begin{array}{l}
\sum F_{\text {vertical }}=F_{\mathrm{N}} \sin \phi+F_{\mathrm{fr}} \cos \phi-m g=F_{\mathrm{N}} \sin \phi+\mu_{\mathrm{s}} F_{\mathrm{N}} \cos \phi-m g=0 \rightarrow \\
F_{\mathrm{N}}=\frac{m g}{\sin \phi+\mu_{\mathrm{s}} \cos \phi} \\
\sum F_{\text {horizonal }}=F_{\mathrm{N}} \cos \phi-F_{\mathrm{fr}} \sin \phi=F_{\mathrm{N}} \cos \phi-\mu_{\mathrm{s}} F_{\mathrm{N}} \sin \phi \\
=F_{\mathrm{N}}\left(\cos \phi-\mu_{\mathrm{s}} \sin \phi\right)=m \frac{v^{2}}{r}=m \frac{(2 \pi r f)^{2}}{r}=4 \pi^{2} r m f^{2} \rightarrow \\
F_{\mathrm{N}}=\frac{4 \pi^{2} r m f^{2}}{\left(\cos \phi-\mu_{\mathrm{s}} \sin \phi\right)} \\
F_{\mathrm{N}}=\frac{m g}{\sin \phi+\mu_{\mathrm{s}} \cos \phi}=\frac{4 \pi^{2} r m f^{2}}{\left(\cos \phi-\mu_{\mathrm{s}} \sin \phi\right)} \rightarrow \vec{r}_{\min }=\frac{g\left(\cos \phi-\mu_{\mathrm{s}} \sin \phi\right)}{4 \pi^{2} f^{2}\left(\sin \phi+\mu_{\mathrm{s}} \cos \phi\right)}
\end{array}
\end{aligned}
$$

99. (a) See the free-body diagram for the skier when the tow rope is horizontal. Use Newton's second law for both the vertical and horizontal directions in order to find the acceleration.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{N}}-m g=0 \rightarrow F_{\mathrm{N}}=m g \\
& \sum F_{x}=F_{\mathrm{T}}-F_{\mathrm{fr}}=F_{\mathrm{T}}-\mu_{\mathrm{k}} F_{\mathrm{N}}=F_{\mathrm{T}}-\mu_{\mathrm{k}} m g=m a \\
& a=\frac{F_{\mathrm{T}}-\mu_{\mathrm{k}} m g}{m}=\frac{(240 \mathrm{~N})-0.25(72 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{(72 \mathrm{~kg})}=0.88 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

(b) Now see the free-body diagram for the skier when the tow rope has an upward component.


$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{N}}+F_{\mathrm{T}} \sin \theta-m g=0 \rightarrow F_{\mathrm{N}}=m g-F_{\mathrm{T}} \sin \theta \\
& \begin{aligned}
\sum F_{x}=F_{\mathrm{T}} \cos \theta-F_{\mathrm{fr}}=F_{\mathrm{T}} \cos \theta-\mu_{\mathrm{k}} F_{\mathrm{N}} \\
\quad=F_{\mathrm{T}} \cos \theta-\mu_{\mathrm{k}}\left(m g-F_{\mathrm{T}} \sin \theta\right)=m a
\end{aligned} \\
& a=\frac{F_{\mathrm{T}}\left(\cos \theta+\mu_{\mathrm{k}} \sin \theta\right)-\mu_{\mathrm{k}} m g}{m} \\
& =\frac{(240 \mathrm{~N})\left(\cos 12^{\circ}+0.25 \sin 12^{\circ}\right)-0.25(72 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{(72 \mathrm{~kg})}=0.98 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

(c) The acceleration is greater in part (b) because the upward tilt of the tow rope reduces the normal force, which then reduces the friction. The reduction in friction is greater than the reduction in horizontal applied force, and so the horizontal acceleration increases.
100. The radial acceleration is $a_{\mathrm{R}}=\frac{v^{2}}{r}$, and so $a_{\mathrm{R}}=\frac{v^{2}}{r}=\frac{(6.0 \mathrm{~m} / \mathrm{s})^{2}}{0.80 \mathrm{~m}}=45 \mathrm{~m} / \mathrm{s}^{2}$.

The tension force has no tangential component, and so the tangential force is seen from the diagram to be $F_{\text {tang }}=m g \cos \theta$.

$$
F_{\text {tang }}=m g \cos \theta=m a_{\text {tang }} \rightarrow a_{\text {tang }}=g \cos \theta=\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \cos 30^{\circ}=8.5 \mathrm{~m} / \mathrm{s}^{2}
$$

The tension force can be found from the net radial force.

$$
\begin{aligned}
& F_{\mathrm{R}}=F_{\mathrm{T}}-m g \sin \theta=m \frac{v^{2}}{r} \rightarrow \\
& F_{\mathrm{T}}=m\left(g \sin \theta+\frac{v^{2}}{r}\right)=(1.0 \mathrm{~kg})\left(\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \sin 30^{\circ}+45 \mathrm{~m} / \mathrm{s}^{2}\right)=50 \mathrm{~N}
\end{aligned}
$$

Note that the answer has 2 significant figures.
101. (a) The acceleration has a magnitude given by $a=v^{2} / r$.

$$
\begin{aligned}
& a=\sqrt{\left(-15.7 \mathrm{~m} / \mathrm{s}^{2}\right)^{2}+\left(-23.2 \mathrm{~m} / \mathrm{s}^{2}\right)^{2}}=28.01 \mathrm{~m} / \mathrm{s}^{2}=\frac{v^{2}}{63.5 \mathrm{~m}} \rightarrow \\
& v=\sqrt{\left(28.01 \mathrm{~m} / \mathrm{s}^{2}\right)(63.5 \mathrm{~m})}=42.17 \mathrm{~m} / \mathrm{s} \approx 42.2 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(b) Since the acceleration points radially in and the position vector points radially out, the components of the position vector are in the same proportion as the components of the acceleration vector, but of opposite sign.

$$
x=r \frac{\left|a_{x}\right|}{a}=(63.5 \mathrm{~m}) \frac{15.7 \mathrm{~m} / \mathrm{s}^{2}}{28.01 \mathrm{~m} / \mathrm{s}^{2}}=35.6 \mathrm{~m} \quad y=r \frac{\left|a_{y}\right|}{a}=(63.5 \mathrm{~m}) \frac{23.2 \mathrm{~m} / \mathrm{s}^{2}}{28.01 \mathrm{~m} / \mathrm{s}^{2}}=52.6 \mathrm{~m}
$$

102. (a) We find the acceleration as a function of velocity, and then use numeric integration with a constant acceleration approximation to estimate the speed and position of the rocket at later times. We take the downward direction to be positive, and the starting position to be $y=0$.

$$
F=m g-k v^{2}=m a \rightarrow a=g-\frac{k}{m} v^{2}
$$

For $t=0, y(0)=y_{0}=0, v(0)=v_{0}=0$, and $a(0)=a_{0}=g-\frac{k}{m} v^{2}=9.80 \mathrm{~m} / \mathrm{s}^{2}$. Assume this
acceleration is constant over the next interval, and so $y_{1}=y_{0}+v_{0} \Delta t+\frac{1}{2} a_{0}(\Delta t)^{2}, v_{1}=v_{0}+a_{0} \Delta t$, and $a_{1}=-g-\frac{k}{m} v_{1}^{2}$. This continues for each successive interval. We apply this method first for a time interval of 1 s , and get the speed and position at $t=15.0 \mathrm{~s}$. Then we reduce the interval to 0.5 s and again find the speed and position at $t=15.0 \mathrm{~s}$. We compare the results from the smaller time interval with those of the larger time interval to see if they agree within $2 \%$. If not, a smaller interval is used, and the process repeated. For this problem, the results for position and velocity for time intervals of 1.0 s and 0.5 s agree to within $2 \%$, but to get two successive acceleration values to agree to $2 \%$, intervals of 0.05 s and 0.02 s are used. Here are the results for various intervals.

$$
\begin{array}{llll}
\Delta t=1 \mathrm{~s}: & x(15 \mathrm{~s})=648 \mathrm{~m} & v(15 \mathrm{~s})=57.5 \mathrm{~m} / \mathrm{s} & a(15 \mathrm{~s})=0.109 \mathrm{~m} / \mathrm{s}^{2} \\
\Delta t=0.5 \mathrm{~s}: & x(15 \mathrm{~s})=641 \mathrm{~m} & v(15 \mathrm{~s})=57.3 \mathrm{~m} / \mathrm{s} & a(15 \mathrm{~s})=0.169 \mathrm{~m} / \mathrm{s}^{2} \\
\Delta t=0.2 \mathrm{~s}: & x(15 \mathrm{~s})=636 \mathrm{~m} & v(15 \mathrm{~s})=57.2 \mathrm{~m} / \mathrm{s} & a(15 \mathrm{~s})=0.210 \mathrm{~m} / \mathrm{s}^{2} \\
\Delta t=0.1 \mathrm{~s}: & x(15 \mathrm{~s})=634.4 \mathrm{~m} & v(15 \mathrm{~s})=57.13 \mathrm{~m} / \mathrm{s} & a(15 \mathrm{~s})=0.225 \mathrm{~m} / \mathrm{s}^{2} \\
\Delta t=0.05 \mathrm{~s}: & x(15 \mathrm{~s})=633.6 \mathrm{~m} & v(15 \mathrm{~s})=57.11 \mathrm{~m} / \mathrm{s} & a(15 \mathrm{~s})=0.232 \mathrm{~m} / \mathrm{s}^{2} \\
\Delta t=0.02 \mathrm{~s}: & x(15 \mathrm{~s})=633.1 \mathrm{~m} & v(15 \mathrm{~s})=57.10 \mathrm{~m} / \mathrm{s} & a(15 \mathrm{~s})=0.236 \mathrm{~m} / \mathrm{s}^{2}
\end{array}
$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH05.XLS," on tab "Problem 102a."
(b) The terminal velocity is the velocity that produces an acceleration of 0 . Use the acceleration equation from above.

$$
a=g-\frac{k}{m} v^{2} \rightarrow v_{\text {terminal }}=\sqrt{\frac{m g}{k}}=\sqrt{\frac{(75 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{0.22 \mathrm{~kg} / \mathrm{m} k}}=58 \mathrm{~m} / \mathrm{s}
$$

At this velocity, the drag force is equal in magnitude to the force of gravity, so the skydiver no longer accelerates, and thus the velocity stays constant.
(c) From the spreadsheet, it is seen that it takes 17.6 s to reach $99.5 \%$ of terminal velocity.
103. Use the free body diagram to write Newton's second law for the block, and solve for the acceleration.

$$
\begin{aligned}
& F=m a=F_{\mathrm{P}}-F_{\mathrm{fr}}=F_{\mathrm{P}}-\mu_{\mathrm{k}} F_{\mathrm{N}}=F_{\mathrm{P}}-\mu_{\mathrm{k}} m g \rightarrow \\
& a=\frac{F_{\mathrm{P}}}{m}-\mu_{\mathrm{k}} g=\frac{41 \mathrm{~N}}{8.0 \mathrm{~kg}}-\frac{0.20\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{\left(1+0.0020 v^{2}\right)^{2}}=\left(5.125-\frac{1.96}{\left(1+0.0020 v^{2}\right)^{2}}\right) \mathrm{m} / \mathrm{s}^{2}
\end{aligned}
$$



For $t=0, x(0)=x_{0}=0, v(0)=v_{0}=0$, and $a(0)=a_{0}=3.165 \mathrm{~m} / \mathrm{s}^{2}$. Assume this acceleration is constant over the next time interval, and so $x_{1}=x_{0}+v_{0} \Delta t+\frac{1}{2} a_{0}(\Delta t)^{2}, v_{1}=v_{0}+a_{0} \Delta t$, and $a_{1}=\left(5.125-\frac{1.96}{\left(1+0.0020 v_{1}^{2}\right)^{2}}\right) \mathrm{m} / \mathrm{s}^{2}$. This continues for each successive interval. We apply this method first for a time interval of 1 second, and get the speed and position at $t=5.0 \mathrm{~s}$. Then we reduce the interval to 0.5 s and again find the speed and position at $t=5.0 \mathrm{~s}$. We compare the results from the smaller time interval with those of the larger time interval to see if they agree within $2 \%$. If not, a smaller interval is used, and the process repeated. For this problem, the results for position and velocity for time intervals of 1.0 s and 0.5 s agree to within $2 \%$.
(a) The speed at 5.0 s , from the numeric integration, is $18.0 \mathrm{~m} / \mathrm{s}$. The velocity-time graph is shown, along with a graph for a constant coefficient of friction, $\mu_{\mathrm{k}}=0.20$. The varying (decreasing) friction gives a higher speed than the constant friction. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH05.XLS," on tab "Problem 5.103."

(b) The position at 5.0 s , from the numeric integration, is 42.4 m . The position-time graph is shown, along with a graph for a constant coefficient of friction, $\mu_{\mathrm{k}}=0.20$. The varying (decreasing) friction gives a larger distance than the constant friction. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH05.XLS," on tab "Problem 5.103."
(c) If the coefficient of friction is constant, then $a=3.165 \mathrm{~m} / \mathrm{s}^{2}$. Constant acceleration relationships can find the speed and position at $t=5.0 \mathrm{~s}$.

$$
\begin{aligned}
& v=v_{0}+a t=0+a t \rightarrow v_{\text {final }}=\left(3.165 \mathrm{~m} / \mathrm{s}^{2}\right)(5.0 \mathrm{~s})=15.8 \mathrm{~m} / \mathrm{s} \\
& x=x_{0}+v_{0} t+\frac{1}{2} a t^{2}=0+0+\frac{1}{2} a t^{2} \rightarrow x_{\text {final }}=\frac{1}{2}\left(3.165 \mathrm{~m} / \mathrm{s}^{2}\right)(5.0 \mathrm{~s})^{2}=39.6 \mathrm{~m}
\end{aligned}
$$

We compare the variable friction results to the constant friction results.

$$
\begin{aligned}
& v: \quad \% \text { diff }=\frac{v_{\mu \text { constant }}-v_{\mu \text { variable }}}{v_{\mu \text { variable }}}=\frac{15.8 \mathrm{~m} / \mathrm{s}-18.0 \mathrm{~m} / \mathrm{s}}{18.0 \mathrm{~m} / \mathrm{s}}=-12 \% \\
& x: \quad \% \operatorname{diff}=\frac{x_{\mu \text { constant }}-x_{\mu \text { variable }}}{x_{\mu \text { variable }}}=\frac{39.6 \mathrm{~m} / \mathrm{s}-42.4 \mathrm{~m} / \mathrm{s}}{42.4 \mathrm{~m} / \mathrm{s}}=-6.6 \%
\end{aligned}
$$

104. We find the acceleration as a function of velocity, and then use numeric integration with a constant acceleration approximation to estimate the speed and position of the rocket at later times.

$$
F=-m g-k v^{2}=m a \rightarrow a=-g-\frac{k}{m} v^{2}
$$

For $t=0, y(0)=0, v(0)=v_{0}=120 \mathrm{~m} / \mathrm{s}$, and

| $t(\mathrm{~s})$ | $y(\mathrm{~m})$ | $v(\mathrm{~m} / \mathrm{s})$ | $a\left(\mathrm{~m} / \mathrm{s}^{2}\right)$ |
| :---: | ---: | ---: | ---: |
| 0 | 0 | 120.0 | -47.2 |
| 1 | 96 | 72.8 | -23.6 |
| 2 | 157 | 49.2 | -16.1 |
| 3 | 199 | 33.1 | -12.6 |
| 4 | 225 | 20.5 | -10.9 |
| 5 | 240 | 9.6 | -10.0 |
| 6 | 245 | -0.5 | -9.8 | $a(0)=a_{0}=-g-\frac{k}{m} v^{2}=-9.80 \mathrm{~m} / \mathrm{s}^{2}$. Assume this

acceleration is constant over the next time interval, and so $y_{1}=y_{0}+v_{0} \Delta t+\frac{1}{2} a_{0}(\Delta t)^{2}, v_{1}=v_{0}+a_{0} \Delta t$,
and $a_{1}=-g-\frac{k}{m} v_{1}^{2}$. This continues for each successive interval. Applying this method gives the results shown in the table. We estimate the maximum height reached as $y_{\max }=245 \mathrm{~m}$.
If air resistance is totally ignored, then the acceleration is a constant $-g$ and Eq. 2-12c may be used to find the maximum height.

$$
\begin{aligned}
& v^{2}-v_{0}^{2}=2 a\left(y-y_{0}\right) \rightarrow \\
& y-y_{0}=\frac{v^{2}-v_{0}^{2}}{2 a}=\frac{-v_{0}^{2}}{-2 g}=\frac{(120 \mathrm{~m} / \mathrm{s})^{2}}{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=730 \mathrm{~m}
\end{aligned}
$$

Thus the air resistance reduces the maximum height to about $1 / 3$ of the no-resistance value. A more detailed analysis (with smaller time intervals) gives 302 m for the maximum height, which is also the answer obtained from an analytical solution.

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH05.XLS," on tab "Problem 5.104."

## CHAPTER 6: Gravitation and Newton's Synthesis

## Responses to Questions

Whether the apple is attached to a tree or falling, it exerts a gravitational force on the Earth equal to the force the Earth exerts on it, which is the weight of the apple (Newton's third law).
2. The tides are caused by the difference in gravitational pull on two opposite sides of the Earth. The gravitational pull from the Sun on the side of the Earth closest to it depends on the distance from the Sun to the close side of the Earth. The pull from the Sun on the far side of the Earth depends on this distance plus the diameter of the Earth. The diameter of the Earth is a very small fraction of the total Earth-Sun distance, so these two forces, although large, are nearly equal. The diameter of the Earth is a larger fraction of the Earth-Moon distance, and so the difference in gravitational force from the Moon to the two opposite sides of the Earth will be greater.
3. The object will weigh more at the poles. The value of $r^{2}$ at the equator is greater, both from the Earth's center and from the bulging mass on the opposite side of the Earth. Also, the object has centripetal acceleration at the equator. The two effects do not oppose each other.
4. Since the Earth's mass is greater than the Moon's, the point at which the net gravitational pull on the spaceship is zero is closer to the Moon. A spaceship traveling from the Earth towards the Moon must therefore use fuel to overcome the net pull backwards for over half the distance of the trip. However, when the spaceship is returning to the Earth, it reaches the zero point at less than half the trip distance, and so spends more of the trip "helped" by the net gravitational pull in the direction of travel.
5. The gravitational force from the Sun provides the centripetal force to keep the Moon and the Earth going around the Sun. Since the Moon and Earth are at the same average distance from the Sun, they travel together, and the Moon is not pulled away from the Earth.
6. As the Moon revolves around the Earth, its position relative to the distant background stars changes. This phenomenon is known as "parallax." As a demonstration, hold your finger at arm's length and look at it with one eye at a time. Notice that it "lines up" with different objects on the far wall depending on which eye is open. If you bring your finger closer to your face, the shift in its position against the background increases. Similarly, the Moon's position against the background stars will shift as we view it in different places in its orbit. The distance to the Moon can be calculated by the amount of shift.

At the very center of the Earth, all of the gravitational forces would cancel, and the net force on the object would be zero.
8. A satellite in a geosynchronous orbit stays over the same spot on the Earth at all times. The satellite travels in an orbit about the Earth's axis of rotation. The needed centripetal force is supplied by the component of the gravitational force perpendicular to the axis of rotation. A satellite directly over the North Pole would lie on the axis of rotation of the Earth. The gravitational force on the satellite in this case would be parallel to the axis of rotation, with no component to supply the centripetal force needed to keep the satellite in orbit.
9. According to Newton's third law, the force the Earth exerts on the Moon has the same magnitude as the force the Moon exerts on the Earth. The Moon has a larger acceleration, since it has a smaller mass (Newton's second law, $F=m a$ ).
10. The satellite needs a certain speed with respect to the center of the Earth to achieve orbit. The Earth rotates towards the east so it would require less speed (with respect to the Earth's surface) to launch a satellite towards the east $(a)$. Before launch, the satellite is moving with the surface of the Earth so already has a "boost" in the right direction.
11. If the antenna becomes detached from a satellite in orbit, the antenna will continue in orbit around the Earth with the satellite. If the antenna were given a component of velocity toward the Earth (even a very small one), it would eventually spiral in and hit the Earth.
12. Ore normally has a greater density than the surrounding rock. A large ore deposit will have a larger mass than an equal amount of rock. The greater the mass of ore, the greater the acceleration due to gravity will be in its vicinity. Careful measurements of this slight increase in $g$ can therefore be used to estimate the mass of ore present.
13. Yes. At noon, the gravitational force on a person due to the Sun and the gravitational force due to the Earth are in the opposite directions. At midnight, the two forces point in the same direction. Therefore, your apparent weight at midnight is greater than your apparent weight at noon.
14. Your apparent weight will be greatest in case (b), when the elevator is accelerating upward. The scale reading (your apparent weight) indicates your force on the scale, which, by Newton's third law, is the same as the normal force of the scale on you. If the elevator is accelerating upward, then the net force must be upward, so the normal force (up) must be greater than your actual weight (down). When in an elevator accelerating upward, you "feel heavy."

Your apparent weight will be least in case $(c)$, when the elevator is in free fall. In this situation your apparent weight is zero since you and the elevator are both accelerating downward at the same rate and the normal force is zero.

Your apparent weight will be the same as when you are on the ground in case ( $d$ ), when the elevator is moving upward at a constant speed. If the velocity is constant, acceleration is zero and $N=m g$. (Note that it doesn't matter if the elevator is moving up or down or even at rest, as long as the velocity is constant.)
15. If the Earth's mass were double what it is, the radius of the Moon's orbit would have to double (if the Moon's speed remained constant), or the Moon's speed in orbit would have to increase by a factor of the square root of 2 (if the radius remained constant). If both the radius and orbital speed were free to change, then the product $r v^{2}$ would have to double.
16. If the Earth were a perfect, nonrotating sphere, then the gravitational force on each droplet of water in the Mississippi would be the same at the headwaters and at the outlet, and the river wouldn't flow. Since the Earth is rotating, the droplets of water experience a centripetal force provided by a part of the component of the gravitational force perpendicular to the Earth's axis of rotation. The centripetal force is smaller for the headwaters, which are closer to the North pole, than for the outlet, which is closer to the equator. Since the centripetal force is equal to $m g-N$ (apparent weight) for each droplet, $N$ is smaller at the outlet, and the river will flow. This effect is large enough to overcome smaller effects on the flow of water due to the bulge of the Earth near the equator.
17. The satellite remains in orbit because it has a velocity. The instantaneous velocity of the satellite is tangent to the orbit. The gravitational force provides the centripetal force needed to keep the satellite in orbit, acting like the tension in a string when twirling a rock on a string. A force is not needed to keep the satellite "up"; a force is needed to bend the velocity vector around in a circle.
18. Between steps, the runner is not touching the ground. Therefore there is no normal force up on the runner and so she has no apparent weight. She is momentarily in free fall since the only force is the force of gravity pulling her back toward the ground.
19. If you were in a satellite orbiting the Earth, you would have no apparent weight (no normal force). Walking, which depends on the normal force, would not be possible. Drinking would be possible, but only from a tube or pouch, from which liquid could be sucked. Scissors would not sit on a table (no apparent weight $=$ no normal force).
20. The centripetal acceleration of Mars in its orbit around the Sun is smaller than that of the Earth. For both planets, the centripetal force is provided by gravity, so the centripetal acceleration is inversely proportional to the square of the distance from the planet to the Sun:

$$
\frac{m_{p} v^{2}}{r}=\frac{G m_{s} m_{p}}{r^{2}} \quad \text { so } \quad \frac{v^{2}}{r}=\frac{G m_{s}}{r^{2}}
$$

Since Mars is at a greater distance from the Sun than Earth, it has a smaller centripetal acceleration. Note that the mass of the planet does not appear in the equation for the centripetal acceleration.
21. For Pluto's moon, we can equate the gravitational force from Pluto on the moon to the centripetal force needed to keep the moon in orbit:

$$
\frac{m_{m} v^{2}}{r}=\frac{G m_{p} m_{m}}{r^{2}}
$$

This allows us to solve for the mass of Pluto $\left(m_{p}\right)$ if we know $G$, the radius of the moon's orbit, and the velocity of the moon, which can be determined from the period and orbital radius. Note that the mass of the moon cancels out.
22. The Earth is closer to the Sun in January. The gravitational force between the Earth and the Sun is a centripetal force. When the distance decreases, the speed increases. (Imagine whirling a rock around your head in a horizontal circle. If you pull the string through your hand to shorten the distance between your hand and the rock, the rock speeds up.)

$$
\frac{m_{E} v^{2}}{r}=\frac{G m_{S} m_{E}}{r^{2}} \quad \text { so } \quad v=\sqrt{\frac{G m_{S}}{r}}
$$

Since the speed is greater in January, the distance must be less. This agrees with Kepler's second law.
23. The Earth's orbit is an ellipse, not a circle. Therefore, the force of gravity on the Earth from the Sun is not perfectly perpendicular to the Earth's velocity at all points. A component of the force will be parallel to the velocity vector and will cause the planet to speed up or slow down.
24. Standing at rest, you feel an upward force on your feet. In free fall, you don't feel that force. You would, however, be aware of the acceleration during free fall, possibly due to your inner ear.
25. If we treat $\overrightarrow{\mathbf{g}}$ as the acceleration due to gravity, it is the result of a force from one mass acting on another mass and causing it to accelerate. This implies action at a distance, since the two masses do not have to be in contact. If we view $\overrightarrow{\mathbf{g}}$ as a gravitational field, then we say that the presence of a mass changes the characteristics of the space around it by setting up a field, and the field then interacts with other masses that enter the space in which the field exists. Since the field is in contact with the mass, this conceptualization does not imply action at a distance.

## Solutions to Problems

1. The spacecraft is at 3.00 Earth radii from the center of the Earth, or three times as far from the Earth's center as when at the surface of the Earth. Therefore, since the force of gravity decreases as the square of the distance, the force of gravity on the spacecraft will be one-ninth of its weight at the Earth's surface.

$$
F_{G}=\frac{1}{9} m g_{\substack{\text { Earlhs } \\ \text { surface }}}=\frac{(1480 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{9}=1610 \mathrm{~N}
$$

This could also have been found using Eq. 6-1, Newton's law of universal gravitation.
2. The force of gravity on an object at the surface of a planet is given by Newton's law of universal gravitation, Eq. 6-1, using the mass and radius of the planet. If that is the only force on an object, then the acceleration of a freely falling object is acceleration due to gravity.

$$
\begin{aligned}
& F_{G}=G \frac{M_{\text {Moon }} m}{r_{\text {Moon }}^{2}}=m g_{\text {Moon }} \rightarrow \\
& g_{\text {Moon }}=G \frac{M_{\text {Moon }}}{r_{\text {Moon }}^{2}}=\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right) \frac{\left(7.35 \times 10^{22} \mathrm{~kg}\right)}{\left(1.74 \times 10^{6} \mathrm{~m}\right)^{2}}=1.62 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

3. The acceleration due to gravity at any location on or above the surface of a planet is given by $g_{\text {planet }}=G M_{\text {planet }} / r^{2}$, where $r$ is the distance from the center of the planet to the location in question.

$$
g_{\text {planet }}=G \frac{M_{\text {Planet }}}{r^{2}}=G \frac{M_{\text {Earth }}}{\left(2.3 R_{\text {Earhh }}\right)^{2}}=\frac{1}{2.3^{2}} G \frac{M_{\text {Earth }}}{R_{\text {Earth }}^{2}}=\frac{1}{2.3^{2}} g_{\text {Earth }}=\frac{9.80 \mathrm{~m} / \mathrm{s}^{2}}{2.3^{2}}=1.9 \mathrm{~m} / \mathrm{s}^{2}
$$

4. The acceleration due to gravity at any location at or above the surface of a planet is given by $g_{\text {planet }}=G M_{\text {Planet }} / r^{2}$, where $r$ is the distance from the center of the planet to the location in question.

$$
g_{\text {planet }}=G \frac{M_{\text {Planet }}}{r^{2}}=G \frac{1.80 M_{\text {Earth }}}{R_{\text {Earth }}^{2}}=1.80\left(G \frac{M_{\text {Earth }}}{R_{\text {Earth }}^{2}}\right)=1.80 g_{\text {Earth }}=1.80\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=17.6 \mathrm{~m} / \mathrm{s}^{2}
$$

5. The acceleration due to gravity is determined by the mass of the Earth and the radius of the Earth.

$$
g_{0}=\frac{G M_{0}}{r_{0}^{2}} \quad g_{\text {new }}=\frac{G M_{\text {new }}}{r_{\text {new }}^{2}}=\frac{G 2 M_{0}}{\left(3 r_{0}\right)^{2}}=\frac{2}{9} \frac{G M_{0}}{r_{0}^{2}}=\frac{2}{9} g_{0}
$$

So $g$ is multiplied by a factor of $2 / 9$.
6. The acceleration due to gravity at any location at or above the surface of a planet is given by $g_{\text {planet }}=G M_{\text {Planet }} / r^{2}$, where $r$ is the distance from the center of the planet to the location in question. For this problem, $M_{\text {Planet }}=M_{\text {Earth }}=5.97 \times 10^{24} \mathrm{~kg}$.
(a) $r=R_{\text {Earth }}+6400 \mathrm{~m}=6.38 \times 10^{6} \mathrm{~m}+6400 \mathrm{~m}$

$$
g=G \frac{M_{\text {Earth }}}{r^{2}}=\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right) \frac{\left(5.98 \times 10^{24} \mathrm{~kg}\right)}{\left(6.38 \times 10^{6} \mathrm{~m}+6400 \mathrm{~m}\right)^{2}}=9.78 \mathrm{~m} / \mathrm{s}^{2}
$$

(b) $r=R_{\text {Earth }}+6400 \mathrm{~km}=6.38 \times 10^{6} \mathrm{~m}+6.4 \times 10^{6} \mathrm{~m}=12.78 \times 10^{6} \mathrm{~m}(3 \mathrm{sig}$ fig $)$

$$
g=G \frac{M_{\text {Earth }}}{r^{2}}=\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right) \frac{\left(5.98 \times 10^{24} \mathrm{~kg}\right)}{\left(12.78 \times 10^{6} \mathrm{~m}\right)^{2}}=2.44 \mathrm{~m} / \mathrm{s}^{2}
$$

7. The distance from the Earth's center is $r=R_{\text {Earth }}+300 \mathrm{~km}=6.38 \times 10^{6} \mathrm{~m}+3 \times 10^{5} \mathrm{~m}=$ $6.68 \times 10^{6} \mathrm{~m}(2 \mathrm{sig}$ fig $)$. Calculate the acceleration due to gravity at that location.

$$
\begin{aligned}
g & =G \frac{M_{\text {Earth }}}{r^{2}}=G \frac{M_{\text {Earth }}}{r^{2}}=\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right) \frac{5.97 \times 10^{24} \mathrm{~kg}}{\left(6.68 \times 10^{6} \mathrm{~m}\right)^{2}}=8.924 \mathrm{~m} / \mathrm{s}^{2} \\
& =8.924 \mathrm{~m} / \mathrm{s}^{2}\left(\frac{1 " g}{9.80 \mathrm{~m} / \mathrm{s}^{2}}\right)=0.91 g^{\prime} \mathrm{s}
\end{aligned}
$$

This is only about a $9 \%$ reduction from the value of $g$ at the surface of the Earth.
8. We are to calculate the force on Earth, so we need the distance of each planet from Earth.

$$
\begin{aligned}
& r_{\text {Earth }}=(150-108) \times 10^{6} \mathrm{~km}=4.2 \times 10^{10} \mathrm{~m} \quad \substack{\text { Earth } \\
\text { Venus }}^{r_{\text {Jupiter }}}=(778-150) \times 10^{6} \mathrm{~km}=6.28 \times 10^{11} \mathrm{~m} \\
& r_{\text {Earth }}=(1430-150) \times 10^{6} \mathrm{~km}=1.28 \times 10^{12} \mathrm{~m} \\
& \text { Sauturn }
\end{aligned},
$$

Jupiter and Saturn will exert a rightward force, while Venus will exert a leftward force. Take the right direction as positive.

$$
\begin{aligned}
F_{\substack{\text { Earhh } \\
\text { planets }}} & =G \frac{M_{\text {Earh }} M_{\text {Jupiter }}}{\substack{\text { Earth } \\
\text { Juppiter }}}+G \frac{M_{\text {Earth }} M_{\text {Satum }}}{r_{\text {Earth }}^{2}}-G \frac{M_{\text {Earth }} M_{\text {Venus }}}{r_{\text {Saturn }}^{2}} \begin{array}{l}
\text { Eerth } \\
\text { Venus }
\end{array} \\
& =G M_{\text {Earth }}^{2}\left(\frac{318}{\left(6.28 \times 10^{11} \mathrm{~m}\right)^{2}}+\frac{95.1}{\left(1.28 \times 10^{12} \mathrm{~m}\right)^{2}}-\frac{0.815}{\left(4.2 \times 10^{10} \mathrm{~m}\right)^{2}}\right) \\
& =\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(5.97 \times 10^{24} \mathrm{~kg}\right)^{2}\left(4.02 \times 10^{-22} \mathrm{~m}^{-2}\right)=9.56 \times 10^{17} \mathrm{~N} \approx 9.6 \times 10^{17} \mathrm{~N}
\end{aligned}
$$

The force of the Sun on the Earth is as follows.

$$
F_{\text {Earth- }}^{\text {Sun }}<=G \frac{M_{\text {Earth }} M_{\text {Sun }}}{\substack{r_{\text {Earth }}^{2} \\ \text { Sunh }}}=\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right) \frac{\left(5.97 \times 10^{24} \mathrm{~kg}\right)\left(1.99 \times 10^{30} \mathrm{~kg}\right)}{\left(1.50 \times 10^{11} \mathrm{~m}\right)^{2}}=3.52 \times 10^{22} \mathrm{~N}
$$

And so the ratio is $F_{\substack{\text { Earth } \\ \text { planest }}} / F_{\substack{\text { Earth. } \\ \text { Sun }}}=9.56 \times 10^{17} \mathrm{~N} / 3.52 \times 10^{22} \mathrm{~N}=2.7 \times 10^{-5}$, which is 27 millionths.
9. Calculate the force on the sphere in the lower left corner, using the freebody diagram shown. From the symmetry of the problem, the net forces in the $x$ and $y$ directions will be the same. Note $\theta=45^{\circ}$.

$$
F_{x}=F_{\text {right }}+F_{\text {dia }} \cos \theta=G \frac{m^{2}}{d^{2}}+G \frac{m^{2}}{(\sqrt{2} d)^{2}} \frac{1}{\sqrt{2}}=G \frac{m^{2}}{d^{2}}\left(1+\frac{1}{2 \sqrt{2}}\right)
$$

Thus $F_{y}=F_{x}=G \frac{m^{2}}{d^{2}}\left(1+\frac{1}{2 \sqrt{2}}\right)$. The net force can be found by the


Pythagorean combination of the two component forces. Due to the symmetry of the arrangement, the net force will be along the diagonal of the square.

$$
\begin{aligned}
F & =\sqrt{F_{x}^{2}+F_{y}^{2}}=\sqrt{2 F_{x}^{2}}=F_{x} \sqrt{2}=G \frac{m^{2}}{d^{2}}\left(1+\frac{1}{2 \sqrt{2}}\right) \sqrt{2}=G \frac{m^{2}}{d^{2}}\left(\sqrt{2}+\frac{1}{2}\right) \\
& =\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right) \frac{(8.5 \mathrm{~kg})^{2}}{(0.80 \mathrm{~m})^{2}}\left(\sqrt{2}+\frac{1}{2}\right)=1.4 \times 10^{-8} \mathrm{~N} \text { at } 45^{\circ}
\end{aligned}
$$

The force points towards the center of the square.
10. Assume that the two objects can be treated as point masses, with $m_{1}=m$ and $m_{2}=4.00 \mathrm{~kg}-m$. The gravitational force between the two masses is given by the following.

$$
F=G \frac{m_{1} m_{2}}{r^{2}}=G \frac{m(4.00-m)}{r^{2}}=\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right) \frac{4.00 \mathrm{~m}-m^{2}}{(0.25 \mathrm{~m})^{2}}=2.5 \times 10^{-10} \mathrm{~N}
$$

This can be rearranged into a quadratic form of $m^{2}-4.00 m+0.234=0$. Use the quadratic formula to solve for $m$, resulting in two values which are the two masses.

$$
m_{1}=3.94 \mathrm{~kg}, m_{2}=0.06 \mathrm{~kg}
$$

11. The force on $m$ due to $2 m$ points in the $\hat{\mathbf{i}}$ direction. The force on $m$ due to $4 m$ points in the $\hat{\mathbf{j}}$ direction. The force on $m$ due to $3 m$ points in the direction given by $\theta=\tan ^{-1} \frac{y_{0}}{x_{0}}$. Add the force vectors together to find the net force.

$$
\begin{aligned}
\overrightarrow{\mathbf{F}} & =G \frac{(2 m) m}{x_{0}^{2}} \hat{\mathbf{i}}+G \frac{(4 m) m}{y_{0}^{2}} \hat{\mathbf{j}}+G \frac{(3 m) m}{x_{0}^{2}+y_{0}^{2}} \cos \theta \hat{\mathbf{i}}+G \frac{(3 m) m}{x_{0}^{2}+y_{0}^{2}} \sin \theta \hat{\mathbf{j}} \\
& =G \frac{2 m^{2}}{x_{0}^{2}} \hat{\mathbf{i}}+G \frac{4 m^{2}}{y_{0}^{2}} \hat{\mathbf{j}}+G \frac{3 m^{2}}{x_{0}^{2}+y_{0}^{2}} \frac{x_{0}}{\sqrt{x_{0}^{2}+y_{0}^{2}}} \hat{\mathbf{i}}+G \frac{(3 m) m}{x_{0}^{2}+y_{0}^{2}} \frac{y_{0}}{\sqrt{x_{0}^{2}+y_{0}^{2}}} \hat{\mathbf{j}} \\
& =G m^{2}\left[\left(\frac{2}{x_{0}^{2}}+\frac{3 x_{0}}{\left(x_{0}^{2}+y_{0}^{2}\right)^{3 / 2}}\right) \hat{\mathbf{i}}+\left(\frac{4}{y_{0}^{2}}+\frac{3 y_{0}}{\left(x_{0}^{2}+y_{0}^{2}\right)^{3 / 2}}\right) \hat{\mathbf{j}}\right]
\end{aligned}
$$

12. With the assumption that the density of Europa is the same as Earth's, the radius of Europa can be calculated.

$$
\begin{aligned}
& \rho_{\text {Europa }}=\rho_{\text {Earth }} \rightarrow \frac{M_{\text {Europa }}}{\frac{4}{3} \pi r_{\text {Europa }}^{3}}=\frac{M_{\text {Earth }}}{\frac{4}{3} \pi r_{\text {Earth }}^{3}} \rightarrow r_{\text {Europa }}=r_{\text {Earth }}\left(\frac{M_{\text {Europa }}}{M_{\text {Earth }}}\right)^{1 / 3} \\
& g_{\text {Europa }}=\frac{G M_{\text {Europa }}}{r_{\text {Europa }}^{2}}=\frac{G M_{\text {Europa }}}{\left(r_{\text {Earth }}\left(\frac{M_{\text {Europa }}}{M_{\text {Earh }}}\right)^{1 / 3}\right)^{2}}=\frac{G M_{\text {Europa }}^{1 / 3} M_{\text {Earth }}^{2 / 3}}{r_{\text {Earth }}^{2}}=\frac{G M_{\text {Earth }}^{2}}{r_{\text {Earth }}^{2}} \frac{M_{\text {Eurpa }}^{1 / 3}}{M_{\text {Earth }}^{1 / 3}}=g_{\text {Earth }}\left(\frac{M_{\text {Europa }}}{M_{\text {Earth }}}\right)^{1 / 3} \\
& =\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(\frac{4.9 \times 10^{22} \mathrm{~kg}}{5.98 \times 10^{24} \mathrm{~kg}}\right)^{1 / 3}=1.98 \mathrm{~m} / \mathrm{s}^{2} \approx 2.0 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

13. To find the new weight of objects at the Earth's surface, the new value of $g$ at the Earth's surface needs to be calculated. Since the spherical shape is being maintained, the Earth can be treated as a point mass. Find the density of the Earth using the actual values, and use that density to find $g$ under the revised conditions.

$$
\begin{aligned}
& g_{\text {orignal }}=G \frac{m_{\mathrm{E}}}{r_{\mathrm{E}}^{2}} ; \rho=\frac{m_{\mathrm{E}}}{\frac{4}{3} \pi r_{\mathrm{E}}^{3}}=\frac{3 m_{\mathrm{E}}}{4 \pi r_{\mathrm{E}}^{3}} \rightarrow r_{\mathrm{E}}=\left(\frac{3 m_{\mathrm{E}}}{4 \pi \rho}\right)^{1 / 3} \rightarrow \\
& g_{\text {orignal }}=G \frac{m_{\mathrm{E}}}{\left(\frac{3 m_{\mathrm{E}}}{4 \pi \rho}\right)^{2 / 3}}=G \frac{\left(m_{\mathrm{E}}\right)^{1 / 3}}{\left(\frac{3}{4 \pi \rho}\right)^{2 / 3}} ; g_{\text {new }}=G \frac{\left(2 m_{\mathrm{E}}\right)^{1 / 3}}{\left(\frac{3}{4 \pi \rho}\right)^{2 / 3}}=2^{1 / 3} G \frac{\left(m_{\mathrm{E}}\right)^{1 / 3}}{\left(\frac{3}{4 \pi \rho}\right)^{2 / 3}}=2^{1 / 3} g
\end{aligned}
$$

Thus $g$ is multiplied by $2^{1 / 3}$, and so the weight would be multiplied by $2^{1 / 3}$.
14. The expression for the acceleration due to gravity at the surface of a body is $g_{\text {body }}=G \frac{M_{\text {body }}}{R_{\text {body }}^{2}}$, where $R_{\text {body }}$ is the radius of the body. For Mars, $g_{\text {Mars }}=0.38 g_{\text {Earth }}$.

$$
\begin{aligned}
& G \frac{M_{\text {Mars }}}{R_{\text {Mars }}^{2}}=0.38 G \frac{M_{\text {Earth }}}{R_{\text {Earth }}^{2}} \rightarrow \\
& M_{\text {Mars }}=0.38 M_{\text {Earth }}\left(\frac{R_{\text {Mars }}}{R_{\text {Earth }}}\right)^{2}=0.38\left(5.98 \times 10^{24} \mathrm{~kg}\right)\left(\frac{3400 \mathrm{~km}}{6380 \mathrm{~km}}\right)^{2}=6.5 \times 10^{23} \mathrm{~kg}
\end{aligned}
$$

15. For the net force to be zero means that the gravitational force on the spacecraft due to the Earth must be the same as that due to the Moon. Write the gravitational forces on the spacecraft, equate them, and solve
 for the distance $x$. We measure from the center of the bodies.

$$
\begin{aligned}
& F_{\substack{\text { Earth- } \\
\text { spacecrat }}}=G \frac{M_{\text {Earth }} m_{\text {spacecraft }}}{x^{2}} ; \quad F_{\text {Moon }}=G \frac{M_{\text {Moon }} m_{\text {spaceceraft }}}{(d-x)^{2}} \\
& G \frac{M_{\text {Earth }} m_{\text {spacecraft }}}{x^{2}}=G \frac{M_{\text {Moon }} m_{\text {spacecraft }}}{(d-x)^{2}} \rightarrow \frac{x^{2}}{M_{\text {Earth }}}=\frac{(d-x)^{2}}{M_{\text {Moon }}} \rightarrow \frac{x}{\sqrt{M_{\text {Earth }}}}=\frac{d-x}{\sqrt{M_{\text {Moon }}}} \\
& x=d \frac{\sqrt{M_{\text {Earth }}}}{\left(\sqrt{M_{\text {Moon }}}+\sqrt{M_{\text {Earth }}}\right)}=\left(3.84 \times 10^{8} \mathrm{~m}\right) \frac{\sqrt{5.97 \times 10^{24} \mathrm{~kg}}}{\left(\sqrt{7.35 \times 10^{22} \mathrm{~kg}}+\sqrt{5.97 \times 10^{24} \mathrm{~kg}}\right)}=3.46 \times 10^{8} \mathrm{~m}
\end{aligned}
$$

This is only about 22 Moon radii away from the Moon. Or, it is about $90 \%$ of the distance from the center of the Earth to the center of the Moon.
16. The speed of an object in an orbit of radius $r$ around the Sun is given by $v=\sqrt{G M_{\text {Sun }} / r}$, and is also given by $v=2 \pi r / T$, where $T$ is the period of the object in orbit. Equate the two expressions for the speed and solve for $M_{\text {Sun }}$, using data for the Earth.

$$
\sqrt{G \frac{M_{\text {Sun }}}{r}}=\frac{2 \pi r}{T} \rightarrow M_{\text {Sun }}=\frac{4 \pi^{2} r^{3}}{G T^{2}}=\frac{4 \pi^{2}\left(1.50 \times 10^{11} \mathrm{~m}\right)^{3}}{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(3.15 \times 10^{7} \mathrm{sec}\right)^{2}}=2.01 \times 10^{30} \mathrm{~kg}
$$

This is the same result obtained in Example 6-9 using Kepler's third law.
17. Each mass $M$ will exert a gravitational force on mass $m$. The vertical components of the two forces will sum to be 0 , and so the net force on $m$ is directed horizontally. That net force will be twice the horizontal component of either force.

$$
\begin{aligned}
& F_{M m}=\frac{G M m}{\left(x^{2}+R^{2}\right)} \rightarrow \\
& F_{M m x}=\frac{G M m}{\left(x^{2}+R^{2}\right)} \cos \theta=\frac{G M m}{\left(x^{2}+R^{2}\right)} \frac{x}{\sqrt{x^{2}+R^{2}}}=\frac{G M m x}{\left(x^{2}+R^{2}\right)^{3 / 2}} \\
& F_{\text {netx }}=2 F_{M m x}=\frac{2 G M m x}{\left(x^{2}+R^{2}\right)^{3 / 2}}
\end{aligned}
$$

18. From the symmetry of the problem, we can examine diametrically opposite infinitesimal masses and see that only the horizontal components of the force will be left. Any off-axis components of force will add to zero. The infinitesimal horizontal force on $m$ due to an infinitesimal mass $d M$ is $d F_{d N m}=\frac{G m}{\left(x^{2}+r^{2}\right)} d M$.


The horizontal component of that force is given by the following.

$$
\left(d F_{d N m}\right)_{x}=\frac{G m}{\left(x^{2}+r^{2}\right)} \cos \theta d M=\frac{G m}{\left(x^{2}+r^{2}\right)} \frac{x}{\sqrt{\left(x^{2}+r^{2}\right)}} d M=\frac{G m x}{\left(x^{2}+r^{2}\right)^{3 / 2}} d M
$$

The total force is then found by integration.

$$
d F_{x}=\frac{G m x d M}{\left(x^{2}+r^{2}\right)^{3 / 2}} \rightarrow \int d F_{x}=\int \frac{G m x d M}{\left(x^{2}+r^{2}\right)^{3 / 2}} \rightarrow F_{x}=\frac{G M m x}{\left(x^{2}+r^{2}\right)^{3 / 2}}
$$

From the diagram we see that it points inward towards the center of the ring.
19. The expression for $g$ at the surface of the Earth is $g=G \frac{m_{\mathrm{E}}}{r_{\mathrm{E}}^{2}}$. Let $g+\Delta g$ be the value at a distance of $r_{\mathrm{E}}+\Delta r$ from the center of Earth, which is $\Delta r$ above the surface.
(a) $g=G \frac{m_{\mathrm{E}}}{r_{\mathrm{E}}^{2}} \rightarrow g+\Delta g=G \frac{m_{\mathrm{E}}}{\left(r_{\mathrm{E}}+\Delta r\right)^{2}}=G \frac{m_{\mathrm{E}}}{r_{\mathrm{E}}^{2}\left(1+\frac{\Delta r}{r_{\mathrm{E}}}\right)^{2}}=G \frac{m_{\mathrm{E}}}{r_{\mathrm{E}}^{2}}\left(1+\frac{\Delta r}{r_{\mathrm{E}}}\right)^{-2} \approx g\left(1-2 \frac{\Delta r}{r_{\mathrm{E}}}\right) \rightarrow$

$$
\Delta g \approx-2 g \frac{\Delta r}{r_{\mathrm{E}}}
$$

(b) The minus sign indicated that the change in $g$ is in the opposite direction as the change in $r$. So, if $r$ increases, $g$ decreases, and vice-versa.
(c) Using this result:

$$
\Delta g \approx-2 g \frac{\Delta r}{r_{\mathrm{E}}}=-2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \frac{1.25 \times 10^{5} \mathrm{~m}}{6.38 \times 10^{6} \mathrm{~m}}=-0.384 \mathrm{~m} / \mathrm{s}^{2} \rightarrow g=9.42 \mathrm{~m} / \mathrm{s}^{2}
$$

Direct calculation:

$$
g=G \frac{m_{\mathrm{E}}}{r^{2}}=\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right) \frac{\left(5.98 \times 10^{24} \mathrm{~kg}\right)}{\left(6.38 \times 10^{6} \mathrm{~m}+1.25 \times 10^{5} \mathrm{~m}\right)^{2}}=9.43 \mathrm{~m} / \mathrm{s}^{2}
$$

The difference is only about $0.1 \%$.
20. We can find the actual $g$ by taking $g$ due to the uniform Earth, subtracting away $g$ due to the bubble as if it contained uniform Earth matter, and adding in $g$ due to the oil-filled bubble. In the diagram, $r=1000 \mathrm{~m}$ (the diameter of the bubble, and the distance from the surface to the center of the bubble). The mass of matter in the bubble is found by taking the density of the matter times the volume of the bubble.

$$
\begin{aligned}
& g_{\substack{\text { oil } \\
\text { present }}}=g_{\substack{\text { uniform } \\
\text { Earth }}}-\underset{\substack{\text { bubble } \\
\text { (aath } \\
\text { matter) }}}{g_{\text {prest }}}+g_{\substack{\text { bubble } \\
\text { (oil) }}} \rightarrow \\
& \Delta g=g_{\substack{\text { oil } \\
\text { present }}}-g_{\substack{\text { uniform } \\
\text { Earth }}}=g_{\substack{\text { bubble } \\
\text { (oil) }}}-g_{\substack{\text { bubble } \\
\text { (Earth } \\
\text { matter) }}}
\end{aligned}
$$



$$
=\frac{G M_{\text {bubble }}}{\text { (oil) }^{2}}-\frac{\begin{array}{c}
G M_{\text {bubble }} \\
r^{2} \\
\text { (Earth } \\
\text { matter) }
\end{array}}{r^{2}}=\frac{G}{r^{2}}\left(M_{\substack{\text { bubble } \\
\text { (oil) }}}-M_{\substack{\text { bubble } \\
\text { (Earth } \\
\text { matter) }}}\right)=\frac{G}{r^{2}}\left(\rho_{\text {oil }}-\rho_{\text {Earth }}^{\text {matter }}\right) ~ \frac{4}{3} \pi r_{\text {bubble }}^{3}
$$

The density of oil is given, but we must calculate the density of a uniform Earth.

$$
\left.\begin{array}{l}
\rho_{\text {Earth }}=\frac{m_{\mathrm{E}}}{\text { mater }}=\frac{5.98 \times 10^{24} \mathrm{~kg}}{\frac{4}{3} \pi r_{\mathrm{E}}^{3}}=5.50 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3} \\
\left.\begin{array}{rl}
\Delta g & =\frac{G}{r^{2}}\left(6.38 \times 10^{6} \mathrm{~m}\right)^{3}
\end{array} \rho_{\text {oil }}-\rho_{\text {Earth }}\right) \frac{4}{3} \pi r_{\text {buble }}^{3} \\
\text { matter }
\end{array}\right)
$$

Finally we calculate the percentage difference.

$$
\frac{\Delta g}{g}(\%)=\frac{-1.6414 \times 10^{-4} \mathrm{~m} / \mathrm{s}^{2}}{9.80 \mathrm{~m} / \mathrm{s}^{2}} \times 100=-1.7 \times 10^{-3} \%
$$

The negative sign means that the value of $g$ would decrease from the uniform Earth value.
21. For an object "at rest" on the surface of the rotating Earth, there are two force vectors that add together to form the net force: $\overrightarrow{\mathbf{F}}_{\text {grav }}$, the force of gravity, directed towards the center of the Earth; and $\overrightarrow{\mathbf{F}}_{\mathrm{N}}$, the normal force, which is given by $\overrightarrow{\mathbf{F}}_{\mathrm{N}}=-m \overrightarrow{\mathbf{g}}_{\text {eff }}$. The sum of these two forces must produce the centripetal force that acts on the object, causing centripetal motion. See the diagram. Notice that the component axes are parallel and perpendicular to the surface of the Earth. Write Newton's second law in vector component form for the object, and solve for $\overrightarrow{\mathbf{g}}_{\text {eff }}$. The radius of the circular motion of the object is $r=r_{\mathrm{E}} \cos \theta$, and the speed of the circular motion is $v=\frac{2 \pi r}{T}$, where $T$ is the period of the rotation, one day.

$$
\begin{aligned}
\overrightarrow{\mathbf{F}}_{\text {grav }} & +\overrightarrow{\mathbf{F}}_{\mathrm{N}}=\overrightarrow{\mathbf{F}}_{\text {net }} \rightarrow-G \frac{m_{\mathrm{E}} m}{r_{\mathrm{E}}^{2}} \hat{\mathbf{j}}+\overrightarrow{\mathbf{F}}_{\mathrm{N}}=\frac{m v^{2}}{r} \sin \theta \hat{\mathbf{i}}-\frac{m v^{2}}{r} \cos \theta \hat{\mathbf{j}} \rightarrow \\
\overrightarrow{\mathbf{F}}_{\mathrm{N}} & =\frac{m v^{2}}{r} \sin \theta \hat{\mathbf{i}}+\left(G \frac{m_{\mathrm{E}} m}{r_{\mathrm{E}}^{2}}-\frac{m v^{2}}{r} \cos \theta\right) \hat{\mathbf{j}}=m\left[\frac{4 \pi^{2} r}{T^{2}} \sin \theta \hat{\mathbf{i}}+\left(G \frac{m_{\mathrm{E}}}{r_{\mathrm{E}}^{2}}-\frac{4 \pi^{2} r}{T^{2}} \cos \theta\right) \hat{\mathbf{j}}\right] \\
& =m\left[\frac{4 \pi^{2} r_{\mathrm{E}} \cos \theta}{T^{2}} \sin \theta \hat{\mathbf{i}}+\left(G \frac{m_{\mathrm{E}}}{r_{\mathrm{E}}^{2}}-\frac{4 \pi^{2} r_{\mathrm{E}} \cos \theta}{T^{2}} \cos \theta\right) \hat{\mathbf{j}}\right] \\
& =m\left[\frac{4 \pi^{2}\left(6.38 \times 10^{6} \mathrm{~m}\right)}{(86,400 \mathrm{~s})^{2}} \frac{1}{2} \hat{\mathbf{i}}+\left(9.80 \mathrm{~m} / \mathrm{s}^{2}-\frac{4 \pi^{2}\left(6.38 \times 10^{6} \mathrm{~m}\right)}{(86,400 \mathrm{~s})^{2}} \frac{1}{2}\right) \hat{\mathbf{j}}\right] \\
& =m\left[\left(1.687 \times 10^{-2} \mathrm{~m} / \mathrm{s}^{2}\right) \hat{\mathbf{i}}+\left(9.783 \mathrm{~m} / \mathrm{s}^{2}\right) \hat{\mathbf{j}}\right]
\end{aligned}
$$

From this calculation we see that $\overrightarrow{\mathbf{F}}_{\mathrm{N}}$ points at an angle of $\phi=\tan ^{-1} \frac{\left(1.687 \times 10^{-2} \mathrm{~m} / \mathrm{s}^{2}\right)}{\left(9.783 \mathrm{~m} / \mathrm{s}^{2}\right)}=0.0988^{\circ}$ north of local "upwards" direction. Now solve $\overrightarrow{\mathbf{F}}_{\mathrm{N}}=-m \overrightarrow{\mathbf{g}}_{\text {eff }}$ for $\overrightarrow{\mathbf{g}}_{\text {eff }}$.

$$
\begin{aligned}
& \overrightarrow{\mathbf{F}}_{\mathrm{N}}=m\left[\left(1.687 \times 10^{-2} \mathrm{~m} / \mathrm{s}^{2}\right) \hat{\mathbf{i}}+\left(9.783 \mathrm{~m} / \mathrm{s}^{2}\right) \hat{\mathbf{j}}\right]=-m \overrightarrow{\mathbf{g}}_{\text {eff }} \rightarrow \\
& \overrightarrow{\mathbf{g}}_{\text {eff }}=-\left[\left(1.687 \times 10^{-2} \mathrm{~m} / \mathrm{s}^{2}\right) \hat{\mathbf{i}}+\left(9.783 \mathrm{~m} / \mathrm{s}^{2}\right) \hat{\mathbf{j}}\right] \rightarrow \\
& g_{\text {eff }}=\sqrt{\left(1.687 \times 10^{-2} \mathrm{~m} / \mathrm{s}^{2}\right)+\left(9.783 \mathrm{~m} / \mathrm{s}^{2}\right)}=9.78 \mathrm{~m} / \mathrm{s}^{2} \\
& \overrightarrow{\mathbf{g}}_{\text {eff }} \text { points } 0.099^{\circ} \text { south of radially inward }
\end{aligned}
$$

22. Consider a distance $r$ from the center of the Earth that satisfies $r<R_{\text {Earth }}$. Calculate the force due to the mass inside the radius $r$.

$$
\begin{aligned}
& M_{\substack{\text { closer to } \\
\text { center }}}(r)=\rho V=\rho \frac{4}{3} \pi r^{3}=\frac{M_{\text {Earth }}}{\frac{4}{3} \pi R_{\text {Earth }}^{3} \pi r^{3}=\frac{M_{\text {Earth }}}{R_{\text {Earth }}^{3}} r^{3}} \\
& F_{\text {gravity }}=G \frac{M_{\text {cosest to }} m}{\text { center }^{2}}=G \frac{\left(\frac{M_{\text {Earth }}^{3}}{R_{\text {Earth }}^{3}}\right) m}{r^{2}}=G \frac{M_{\text {Earth }}}{R_{\text {Earth }}^{2}} m\left(\frac{r}{R_{\text {Earth }}}\right)=m g_{\text {surface }}\left(\frac{r}{R_{\text {Earth }}}\right)
\end{aligned}
$$

Thus for $F_{\text {gravity }}=0.95 \mathrm{mg}$, we must have $r=0.95 R_{\text {Earth }}$, and so we must drill down a distance equal to $5 \%$ of the Earth's radius.

$$
0.05 R_{\text {Earth }}=0.05\left(6.38 \times 10^{6} \mathrm{~m}\right)=3.19 \times 10^{5} \mathrm{~m} \approx 320 \mathrm{~km}
$$

23. The shuttle must be moving at "orbit speed" in order for the satellite to remain in the orbit when released. The speed of a satellite in circular orbit around the Earth is shown in Example 6-6 to be

$$
\begin{aligned}
v_{\text {orbit }} & =\sqrt{G \frac{M_{\text {Earth }}}{r}} . \\
v & =\sqrt{G \frac{M_{\text {Earth }}}{r}}=\sqrt{G \frac{M_{\text {Earth }}}{\left(R_{\text {Earth }}+680 \mathrm{~km}\right)}}=\sqrt{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right) \frac{\left(5.98 \times 10^{24} \mathrm{~kg}\right)}{\left(6.38 \times 10^{6} \mathrm{~m}+6.8 \times 10^{5} \mathrm{~m}\right)}} \\
& =7.52 \times 10^{3} \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

24. The speed of a satellite in a circular orbit around a body is shown in Example 6-6 to be $v_{\text {orbit }}=\sqrt{G M_{\text {body }} / r}$, where $r$ is the distance from the satellite to the center of the body.

$$
\begin{aligned}
v & =\sqrt{G \frac{M_{\text {body }}}{r}}=\sqrt{G \frac{M_{\text {Earh }}}{R_{\text {Earth }}+5.8 \times 10^{6} \mathrm{~m}}}=\sqrt{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right) \frac{\left(5.98 \times 10^{24} \mathrm{~kg}\right)}{\left(12.18 \times 10^{6} \mathrm{~m}\right)}} \\
& =5.72 \times 10^{3} \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

25. Consider a free-body diagram of yourself in the elevator. $\overrightarrow{\mathbf{F}}_{\mathrm{N}}$ is the force of the scale pushing up on you, and reads the normal force. Since the scale reads 76 kg , if it were calibrated in Newtons, the normal force would be $F_{\mathrm{N}}=(76 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=744.8 \mathrm{~N}$. Write Newton's second law in the vertical direction, with upward as positive.


$$
\sum F=F_{\mathrm{N}}-m g=m a \rightarrow a=\frac{F_{\mathrm{N}}-m g}{m}=\frac{744.8 \mathrm{~N}-(65 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{65 \mathrm{~kg}}=1.7 \mathrm{~m} / \mathrm{s}^{2} \text { upward }
$$

Since the acceleration is positive, the acceleration is upward.
26. Draw a free-body diagram of the monkey. Then write Newton's second law for the vertical direction, with up as positive.

$$
\sum F=F_{\mathrm{T}}-m g=m a \rightarrow a=\frac{F_{\mathrm{T}}-m g}{m}
$$

For the maximum tension of 185 N ,


$$
a=\frac{185 \mathrm{~N}-(13.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{(13.0 \mathrm{~kg})}=4.43 \mathrm{~m} / \mathrm{s}^{2} \approx 4.4 \mathrm{~m} / \mathrm{s}^{2}
$$

Thus the elevator must have an upward acceleration greater than $a=4.4 \mathrm{~m} / \mathrm{s}^{2}$ for the cord to break. Any downward acceleration would result in a tension less than the monkey's weight.
27. The speed of an object in a circular orbit of radius $r$ around mass $M$ is given in Example 6-6 by $v=\sqrt{G M / r}$, and is also given by $v=2 \pi r / T$, where $T$ is the period of the orbiting object. Equate the two expressions for the speed and solve for $T$.

$$
\begin{aligned}
& \sqrt{G \frac{M}{r}}=\frac{2 \pi r}{T} \rightarrow \\
& T=2 \pi \sqrt{\frac{r^{3}}{G M}}=2 \pi \sqrt{\frac{\left(1.86 \times 10^{6} \mathrm{~m}\right)^{3}}{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(7.35 \times 10^{22} \mathrm{~m}\right)}}=7.20 \times 10^{3} \mathrm{~s} \approx 120 \mathrm{~min}
\end{aligned}
$$

28. The speed of a satellite in circular orbit around the Earth is shown in Example 6-6 to be $v_{\text {orbit }}=\sqrt{G \frac{M_{\text {Earth }}}{r}}$. Thus the velocity is inversely related to the radius, and so the closer satellite will be orbiting faster.

$$
\frac{v_{\text {close }}}{v_{\text {far }}}=\frac{\sqrt{\frac{G M_{\text {Earth }}}{r_{\text {close }}}}}{\sqrt{\frac{G M_{\text {Earth }}}{r_{\text {far }}}}}=\sqrt{\frac{r_{\text {far }}}{r_{\text {close }}}}=\sqrt{\frac{R_{\text {Earth }}+1.5 \times 10^{7} \mathrm{~m}}{R_{\text {Earth }}+5 \times 10^{6} \mathrm{~m}}}=\sqrt{\frac{6.38 \times 10^{6} \mathrm{~m}+1.5 \times 10^{7} \mathrm{~m}}{6.38 \times 10^{6} \mathrm{~m}+5 \times 10^{6} \mathrm{~m}}}=1.37
$$

And so the close satellite is moving 1.4 times faster than the far satellite.
29. Consider a free-body diagram for the woman in the elevator. $\overrightarrow{\mathbf{F}}_{\mathrm{N}}$ is the upwards force the spring scale exerts, providing a normal force. Write Newton's second law for the vertical direction, with up as positive.

$$
\sum F=F_{\mathrm{N}}-m g=m a \rightarrow F_{\mathrm{N}}=m(g+a)
$$

$(a, b)$ For constant speed motion in a straight line, the acceleration is 0 , and so the normal force is equal to the weight.


$$
F_{\mathrm{N}}=m g=(53 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=520 \mathrm{~N}
$$

(c) Here $a=+0.33 g$ and so $F_{\mathrm{N}}=1.33 \mathrm{mg}=1.33(53 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=690 \mathrm{~N}$.
(d) Here $a=-0.33 g$ and so $F_{\mathrm{N}}=0.67 \mathrm{mg}=0.67(53 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=350 \mathrm{~N}$.
(e) Here $a=-g$ and so $F_{\mathrm{N}}=0 \mathrm{~N}$.
30. The speed of an object in an orbit of radius $r$ around the Earth is given in Example 6-6 by $v=\sqrt{G M_{\text {Earth }} / r}$, and is also given by $v=2 \pi r / T$, where $T$ is the period of the object in orbit. Equate the two expressions for the speed and solve for $T$. Also, for a "near-Earth" orbit, $r=R_{\text {Earth }}$.

$$
\begin{aligned}
& \sqrt{G \frac{M_{\text {Earth }}}{r}}=\frac{2 \pi r}{T} \rightarrow T=2 \pi \sqrt{\frac{r^{3}}{G M_{\mathrm{Earth}}}} \\
& T=2 \pi \sqrt{\frac{R_{\text {Earth }}^{3}}{G M_{\text {Earth }}}}=2 \pi \sqrt{\frac{\left(6.38 \times 10^{6} \mathrm{~m}\right)^{3}}{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(5.98 \times 10^{24} \mathrm{~m}\right)}}=5070 \mathrm{~s}=84.5 \mathrm{~min}
\end{aligned}
$$

No, the result does not depend on the mass of the satellite.
31. Consider the free-body diagram for the astronaut in the space vehicle. The Moon is below the astronaut in the figure. We assume that the astronaut is touching the inside of the space vehicle, or in a seat, or strapped in somehow, and so a force will be exerted on the astronaut by the spacecraft. That force has been labeled $\overrightarrow{\mathbf{F}}_{\mathrm{N}}$. The magnitude of
 that force is the apparent weight of the astronaut. Take down as the positive direction.
(a) If the spacecraft is moving with a constant velocity, then the acceleration of the astronaut must be 0 , and so the net force on the astronaut is 0 .

$$
\begin{aligned}
& \sum F=m g-F_{N}=0 \rightarrow \\
& F_{N}=m g=G \frac{m M_{\text {Moon }}}{r^{2}}=\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right) \frac{(75 \mathrm{~kg})\left(7.4 \times 10^{22} \mathrm{~kg}\right)}{\left(2.5 \times 10^{6} \mathrm{~m}\right)^{2}}=59.23 \mathrm{~N}
\end{aligned}
$$

Since the value here is positive, the normal force points in the original direction as shown on the free-body diagram. The astronaut will be pushed "upward" by the floor or the seat. Thus the astronaut will perceive that he has a "weight" of 59 N , towards the Moon.
(b) Now the astronaut has an acceleration towards the Moon. Write Newton's second law for the astronaut, with down as the positive direction.

$$
\sum F=m g-F_{N}=m a \rightarrow F_{N}=m g-m a=59.23 \mathrm{~N}-(75 \mathrm{~kg})\left(2.3 \mathrm{~m} / \mathrm{s}^{2}\right)=-113.3 \mathrm{~N}
$$

Because of the negative value, the normal force points in the opposite direction from what is shown on the free-body diagram - it is pointing towards the Moon. So perhaps the astronaut is pinned against the "ceiling" of the spacecraft, or safety belts are pulling down on the astronaut. The astronaut will perceive being "pushed downwards," and so has an upward apparent weight of 110 N , away from the Moon.
32. The apparent weight is the normal force on the passenger. For a person at rest, the normal force is equal to the actual weight. If there is acceleration in the vertical direction, either up or down, then the normal force (and hence the apparent weight) will be different than the actual weight. The speed of the Ferris wheel is $v=2 \pi r / T=2 \pi(11.0 \mathrm{~m}) / 12.5 \mathrm{~s}=5.529 \mathrm{~m} / \mathrm{s}$.
(a) See the free-body diagram for the highest point of the motion. We assume the passengers are right-side up, so that the normal force of the Ferris wheel seat is upward. The net force must point to the center of the circle, so write Newton's second law with downward as the positive direction. The acceleration is centripetal since the passengers are moving in a circle.


$$
\sum F=F_{\mathrm{R}}=m g-F_{\mathrm{N}}=m a=m v^{2} / r \rightarrow F_{\mathrm{N}}=m g-m v^{2} / r
$$

The ratio of apparent weight to real weight is given by the following.

$$
\frac{m g-m v^{2} / r}{m g}=\frac{g-v^{2} / r}{g}=1-\frac{v^{2}}{r g}=1-\frac{(5.529 \mathrm{~m} / \mathrm{s})^{2}}{(11.0 \mathrm{~m})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=0.716
$$

(b) At the bottom, consider the free-body diagram shown. We assume the passengers are right-side up, so that the normal force of the Ferris wheel seat is upward. The net force must point to the center of the circle, so write Newton's second law with upward as the positive direction. The acceleration is centripetal since the passengers are moving in a circle.


$$
\sum F=F_{\mathrm{R}}=F_{\mathrm{N}}-m g=m a=m v^{2} / r \quad \rightarrow \quad F_{\mathrm{N}}=m g+m v^{2} / r
$$

The ratio of apparent weight to real weight is given by the following.

$$
\frac{m g+m v^{2} / r}{m g}=1+\frac{v^{2}}{r g}=1+\frac{(5.529 \mathrm{~m} / \mathrm{s})^{2}}{(11.0 \mathrm{~m})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=1.284
$$

33. See the diagram for the two stars.
(a) The two stars don't crash into each other because of their circular motion. The force on them is centripetal, and maintains their circular motion. Another way to consider it is that the stars have a velocity, and the gravity force causes CHANGE in velocity, not actual
 velocity. If the stars were somehow brought to rest and then released under the influence of their mutual gravity, they would crash into each other.
(b) Set the gravity force on one of the stars equal to the centripetal force, using the relationship that $v=2 \pi r / T=\pi d / T$, and solve for the mass.

$$
\begin{aligned}
& F_{G}=G \frac{M^{2}}{d^{2}}=F_{\mathrm{R}}=M \frac{v^{2}}{d / 2}=M \frac{2(\pi d / T)^{2}}{d}=\frac{2 \pi^{2} M d}{T^{2}} \rightarrow G \frac{M^{2}}{d^{2}}=\frac{2 \pi^{2} M d}{T^{2}} \rightarrow \\
& M=\frac{2 \pi^{2} d^{3}}{G T^{2}}=\frac{2 \pi^{2}\left(8.0 \times 10^{11} \mathrm{~m}\right)^{3}}{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(12.6 \mathrm{y} \times \frac{3.15 \times 10^{7} \mathrm{~s}}{1 \mathrm{y}}\right)^{2}}=9.6 \times 10^{29} \mathrm{~kg}
\end{aligned}
$$

34. (a) The speed of an object in near-surface orbit around a planet is given in Example 6-6 to be $v=\sqrt{G M / R}$, where $M$ is the planet mass and $R$ is the planet radius. The speed is also given by $v=2 \pi R / T$, where $T$ is the period of the object in orbit. Equate the two expressions for the speed.

$$
\sqrt{G \frac{M}{R}}=\frac{2 \pi R}{T} \rightarrow G \frac{M}{R}=\frac{4 \pi^{2} R^{2}}{T^{2}} \rightarrow \frac{M}{R^{3}}=\frac{4 \pi^{2}}{G T^{2}}
$$

The density of a uniform spherical planet is given by $\rho=\frac{M}{\text { Volume }}=\frac{M}{\frac{4}{3} \pi R^{3}}$. Thus

$$
\rho=\frac{3 M}{4 \pi R^{3}}=\frac{3}{4 \pi} \frac{4 \pi^{2}}{G T^{2}}=\frac{3 \pi}{G T^{2}}
$$

(b) For Earth, we have the following.

$$
\rho=\frac{3 \pi}{G T^{2}}=\frac{3 \pi}{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)[(85 \mathrm{~min})(60 \mathrm{~s} / \mathrm{min})]^{2}}=5.4 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}
$$

35. Consider the lower left mass in the diagram. The center of the orbits is the intersection of the three dashed lines in the diagram. The net force on the lower left mass is the vector sum of the forces from the other two masses, and points to the center of the orbits. To find that net force, project each force to find the component that lies along the line towards the center. The angle is $\theta=30^{\circ}$.

$$
F=G \frac{M^{2}}{\ell^{2}} \rightarrow F_{\substack{\text { component } \\ \text { teward } \\ \text { center }}}=F \cos \theta=G \frac{M^{2}}{\ell^{2}} \frac{\sqrt{3}}{2} \rightarrow
$$



$$
F_{\text {net }}=2 G \frac{M^{2}}{\ell^{2}} \frac{\sqrt{3}}{2}=\sqrt{3} G \frac{M^{2}}{\ell^{2}}
$$

The net force is causing centripetal motion, and so is of the form $M v^{2} / r$. Note that $r \cos \theta=\ell / 2$.

$$
\begin{aligned}
& F_{\text {net }}=2 G \frac{M^{2}}{\ell^{2}} \frac{\sqrt{3}}{2}=\sqrt{3} G \frac{M^{2}}{\ell^{2}}=\frac{M v^{2}}{r}=\frac{M v^{2}}{\ell /(2 \cos \theta)}=\frac{M v^{2}}{\ell / \sqrt{3}} \rightarrow \sqrt{3} G \frac{M^{2}}{\ell^{2}}=\frac{M v^{2}}{\ell / \sqrt{3}} \rightarrow \\
& v=\sqrt{\frac{G M}{\ell}}
\end{aligned}
$$

36. The effective value of the acceleration due to gravity in the elevator is $g_{\text {eff }}=g+a_{\text {elevator }}$. We take the upwards direction to be positive. The acceleration relative to the plane is along the plane, as shown in the freebody diagram.
(a) The elevator acceleration is $a_{\text {elvator }}=+0.50 g$.

$$
\begin{aligned}
& g_{\text {eff }}=g+0.50 g=1.50 g \rightarrow \\
& a_{\text {rel }}=g_{\text {eff }} \sin \theta=1.50 g \sin 32^{\circ}=7.79 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$


(b) The elevator acceleration is $a_{\text {elevator }}=-0.50 \mathrm{~g}$.

$$
g_{\text {eff }}=g-0.50 g=0.50 g \rightarrow a_{\text {rel }}=g_{\text {eff }} \sin \theta=0.50 g \sin 32^{\circ}=2.60 \mathrm{~m} / \mathrm{s}^{2}
$$

(c) The elevator acceleration is $a_{\text {elevator }}=-g$.

$$
g_{\text {eff }}=g-g=0 \rightarrow a_{\text {rel }}=g_{\text {eff }} \sin \theta=0 \sin 32^{\circ}=0 \mathrm{~m} / \mathrm{s}^{2}
$$

(d) The elevator acceleration is 0 .

$$
g_{\text {eff }}=g-0=g \rightarrow a_{\mathrm{rel}}=g_{\text {eff }} \sin \theta=5.19 \mathrm{~m} / \mathrm{s}^{2}
$$

37. Use Kepler's third law for objects orbiting the Earth. The following are given.

$$
\begin{aligned}
& T_{2}=\text { period of Moon }=(27.4 \text { day })\left(\frac{86,400 \mathrm{~s}}{1 \text { day }}\right)=2.367 \times 10^{6} \mathrm{sec} \\
& r_{2}=\text { radius of Moon's orbit }=3.84 \times 10^{8} \mathrm{~m} \\
& r_{1}=\text { radius of near-Earth orbit }=R_{\text {Earth }}=6.38 \times 10^{6} \mathrm{~m}
\end{aligned}
$$

$$
\begin{aligned}
& \left(T_{1} / T_{2}\right)^{2}=\left(r_{1} / r_{2}\right)^{3} \rightarrow \\
& T_{1}=T_{2}\left(r_{1} / r_{2}\right)^{3 / 2}=\left(2.367 \times 10^{6} \mathrm{sec}\right)\left(\frac{6.38 \times 10^{6} \mathrm{~m}}{3.84 \times 10^{8} \mathrm{~m}}\right)^{3 / 2}=5.07 \times 10^{3} \mathrm{sec}(=84.5 \mathrm{~min})
\end{aligned}
$$

38. Knowing the period of the Moon and the distance to the Moon, we can calculate the speed of the Moon by $v=2 \pi r / T$. But the speed can also be calculated for any Earth satellite by $v=\sqrt{G M_{\text {Earth }} / r}$, as derived in Example 6-6. Equate the two expressions for the speed, and solve for the mass of the Earth.

$$
\begin{aligned}
& \sqrt{G M_{\text {Earth }} / r}=2 \pi r / T \rightarrow \\
& M_{\text {Earth }}=\frac{4 \pi^{2} r^{3}}{G T^{2}}=\frac{4 \pi^{2}\left(3.84 \times 10^{8} \mathrm{~m}\right)^{3}}{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)[(27.4 \mathrm{~d})(86,400 \mathrm{~s} / \mathrm{d})]^{2}}=5.98 \times 10^{24} \mathrm{~kg}
\end{aligned}
$$

39. Use Kepler's third law for objects orbiting the Sun.

$$
\begin{aligned}
& \left(T_{\text {Neptune }} / T_{\text {Earth }}\right)^{2}=\left(r_{\text {Neptune }} / r_{\text {Earth }}\right)^{3} \rightarrow \\
& T_{\text {Neptune }}=T_{\text {Earth }}\left(\frac{r_{\text {Neptune }}}{r_{\text {Earth }}}\right)^{3 / 2}=(1 \text { year })\left(\frac{4.5 \times 10^{9} \mathrm{~km}}{1.50 \times 10^{8} \mathrm{~km}}\right)^{3 / 2}=160 \text { years }
\end{aligned}
$$

40. As found in Example 6-6, the speed for an object orbiting a distance $r$ around a mass $M$ is given by $v=\sqrt{G M / r}$.

$$
\frac{v_{\mathrm{A}}}{v_{\mathrm{B}}}=\frac{\sqrt{\frac{G M_{\text {star }}}{r_{\mathrm{A}}}}}{\sqrt{\frac{G M_{\text {star }}}{r_{\mathrm{B}}}}}=\sqrt{\frac{r_{\mathrm{B}}}{r_{\mathrm{A}}}}=\sqrt{\frac{1}{9}}=\frac{1}{3}
$$

41. There are two expressions for the velocity of an object in circular motion around a mass $M$ : $v=\sqrt{G M / r}$ and $v=2 \pi r / T$. Equate the two expressions and solve for $T$.

$$
\begin{aligned}
& \sqrt{G M / r}=2 \pi r / T \rightarrow \\
& T=2 \pi \sqrt{\frac{r^{3}}{G M}}=2 \pi \sqrt{\frac{\left(\left(3 \times 10^{4} \mathrm{ly}\right) \frac{\left(3 \times 10^{8} \mathrm{~m} / \mathrm{s}\right)\left(3.16 \times 10^{7} \mathrm{sec}\right)}{\frac{1 \mathrm{y}}{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(4 \times 10^{41} \mathrm{~kg}\right)}}\right)^{3}}{\left(2.8 .8 \times 10^{15} \mathrm{~s}=1.8 \times 10^{8} \mathrm{y}\right.}} \\
& \approx 2 \times 10^{8} \mathrm{y}
\end{aligned}
$$

42. (a) The relationship between satellite period $T$, mean satellite distance $r$, and planet mass $M$ can be derived from the two expressions for satellite speed: $v=\sqrt{G M / r}$ and $v=2 \pi r / T$. Equate the two expressions and solve for $M$.

$$
\sqrt{G M / r}=2 \pi r / T \quad \rightarrow M=\frac{4 \pi^{2} r^{3}}{G T^{2}}
$$

Substitute the values for Io to get the mass of Jupiter.

$$
M_{\substack{\text { Jupier- } \\ \text { lo }}}=\frac{4 \pi^{2}\left(4.22 \times 10^{8} \mathrm{~m}\right)^{3}}{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(1.77 \mathrm{~d} \times \frac{24 \mathrm{~h}}{1 \mathrm{~d}} \times \frac{3600 \mathrm{~s}}{1 \mathrm{~h}}\right)^{2}}=1.90 \times 10^{27} \mathrm{~kg}
$$

(b) For the other moons, we have the following.

$$
M_{\substack{\text { Iupiter } \\ \text { Europa }}}=\frac{4 \pi^{2}\left(6.71 \times 10^{8} \mathrm{~m}\right)^{3}}{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)(3.55 \times 24 \times 3600 \mathrm{~s})^{2}}=1.90 \times 10^{27} \mathrm{~kg}
$$

$$
\begin{aligned}
& M_{\substack{\text { Jupiter- } \\
\text { Canymede }}}=\frac{4 \pi^{2}\left(1.07 \times 10^{9} \mathrm{~m}\right)^{3}}{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)(7.16 \times 24 \times 3600 \mathrm{~s})^{2}}=1.89 \times 10^{27} \mathrm{~kg} \\
& M_{\substack{\text { Jupiter- } \\
\text { Calliso }}}=\frac{4 \pi^{2}\left(1.883 \times 10^{9} \mathrm{~m}\right)^{3}}{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)(16.7 \times 24 \times 3600 \mathrm{~s})^{2}}=1.90 \times 10^{27} \mathrm{~kg}
\end{aligned}
$$

Yes, the results are consistent - only about $0.5 \%$ difference between them.
43. Use Kepler's third law to find the radius of each moon of Jupiter, using Io's data for $r_{2}$ and $T_{2}$.
$\left(r_{1} / r_{2}\right)^{3}=\left(T_{1} / T_{2}\right)^{2} \rightarrow r_{1}=r_{2}\left(T_{1} / T_{2}\right)^{2 / 3}$
$r_{\text {Europa }}=r_{\text {lo }}\left(T_{\text {Europa }} / T_{\text {lo }}\right)^{2 / 3}=\left(422 \times 10^{3} \mathrm{~km}\right)(3.55 \mathrm{~d} / 1.77 \mathrm{~d})^{2 / 3}=671 \times 10^{3} \mathrm{~km}$
$r_{\text {Ganymede }}=\left(422 \times 10^{3} \mathrm{~km}\right)(7.16 \mathrm{~d} / 1.77 \mathrm{~d})^{2 / 3}=1070 \times 10^{3} \mathrm{~km}$
$r_{\text {Callisto }}=\left(422 \times 10^{3} \mathrm{~km}\right)(16.7 \mathrm{~d} / 1.77 \mathrm{~d})^{2 / 3}=1880 \times 10^{3} \mathrm{~km}$
The agreement with the data in the table is excellent.
44. (a) Use Kepler's third law to relate the Earth and the hypothetical planet in their orbits around the Sun.

$$
\begin{aligned}
& \left(T_{\text {planet }} / T_{\text {Earth }}\right)^{2}=\left(r_{\text {planet }} / r_{\text {Earth }}\right)^{3} \rightarrow \\
& T_{\text {planet }}=T_{\text {Earth }}\left(r_{\text {planet }} / r_{\text {Earth }}\right)^{3 / 2}=(1 \mathrm{y})(3 / 1)^{3 / 2}=5.20 \mathrm{y} \approx 5 \mathrm{y}
\end{aligned}
$$

(b) No mass data can be calculated from this relationship, because the relationship is massindependent. Any object at the orbit radius of 3 times the Earth's orbit radius would have a period of 5.2 years, regardless of its mass.
45. (a) Use Kepler's third law to relate the orbits of the Earth and the comet around the Sun.

$$
\begin{aligned}
& \left(\frac{r_{\text {comet }}}{r_{\text {Earth }}}\right)^{3}=\left(\frac{T_{\text {comet }}}{T_{\text {Earth }}}\right)^{2} \rightarrow \\
& r_{\text {comet }}=r_{\text {Earth }}\left(\frac{T_{\text {comet }}}{T_{\text {Earth }}}\right)^{2 / 3}=(1 \mathrm{AU})\left(\frac{2400 \mathrm{y}}{1 \mathrm{y}}\right)^{2 / 3}=179.3 \mathrm{AU} \approx 180 \mathrm{AU}
\end{aligned}
$$

(b) The mean distance is the numeric average of the closest and farthest distances.

$$
\text { 179.3 } \mathrm{AU}=\frac{1.00 \mathrm{AU}+r_{\max }}{2} \rightarrow r_{\max }=357.6 \mathrm{AU} \approx 360 \mathrm{AU}
$$

(c) Refer to Figure 6-17, which illustrates Kepler's second law. If the time for each shaded region is made much shorter, then the area of each region can be approximated as a triangle. The area of each triangle is half the "base" (speed of comet multiplied by the amount of time) times the "height" (distance from Sun). So we have the following.

$$
\begin{aligned}
& \text { Area }_{\text {min }}=\text { Area }_{\text {max }} \rightarrow \frac{1}{2}\left(v_{\text {min }} t\right) r_{\text {min }}=\frac{1}{2}\left(v_{\text {max }} t\right) r_{\text {max }} \rightarrow \\
& v_{\text {min }} / v_{\text {max }}=r_{\text {max }} / r_{\text {min }}=360 / 1
\end{aligned}
$$

46. (a) In a short time $\Delta t$, the planet will travel a distance $v \Delta t$ along its orbit. That distance is essentially a straight line segment for a short time duration. The time (and distance moved) during $\Delta t$ have been greatly exaggerated on the diagram. Kepler's second law states that the area swept out by a line from the Sun to the planet during the planet's motion for the $\Delta t$ is
 the same anywhere on the orbit. Take the areas swept out at the near and far points, as shown on the diagram, and approximate them as triangles (which will be reasonable for short $\Delta t$ ).

$$
(\text { Area })_{\mathrm{N}}=(\text { Area })_{\mathrm{F}} \rightarrow \frac{1}{2}\left(v_{\mathrm{N}} \Delta t\right) d_{\mathrm{N}}=\frac{1}{2}\left(v_{\mathrm{F}} \Delta t\right) d_{\mathrm{F}} \rightarrow v_{\mathrm{N}} / v_{\mathrm{F}}=d_{\mathrm{F}} / d_{\mathrm{N}}
$$

(b) Since the orbit is almost circular, an average velocity can be found by assuming a circular orbit with a radius equal to the average distance.

$$
v_{\text {avg }}=\frac{2 \pi r}{T}=\frac{2 \pi \frac{1}{2}\left(d_{\mathrm{N}}+d_{\mathrm{F}}\right)}{T}=\frac{2 \pi \frac{1}{2}\left(1.47 \times 10^{11} \mathrm{~m}+1.52 \times 10^{11} \mathrm{~m}\right)}{3.16 \times 10^{7} \mathrm{~s}}=2.973 \times 10^{4} \mathrm{~m} / \mathrm{s}
$$

From part ( $a$ ) we find the ratio of near and far velocities.

$$
v_{\mathrm{N}} / v_{\mathrm{F}}=d_{\mathrm{F}} / d_{\mathrm{N}}=1.52 / 1.47=1.034
$$

For this small change in velocities ( $3.4 \%$ increase from smallest to largest), we assume that the minimum velocity is $1.7 \%$ lower than the average velocity and the maximum velocity is $1.7 \%$ higher than the average velocity.

$$
\begin{aligned}
& v_{\mathrm{N}}=v_{\text {avg }}(1+0.017)=2.973 \times 10^{4} \mathrm{~m} / \mathrm{s}(1.017)=3.02 \times 10^{4} \mathrm{~m} / \mathrm{s} \\
& v_{\mathrm{F}}=v_{\text {avg }}(1-0.017)=2.973 \times 10^{4} \mathrm{~m} / \mathrm{s}(0.983)=2.92 \times 10^{4} \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

47. (a) Take the logarithm of both sides of the Kepler's third law expression.

$$
\begin{aligned}
& T^{2}=\left(\frac{4 \pi^{2}}{G m_{\mathrm{J}}}\right) r^{3} \rightarrow \log T^{2}=\log \left(\frac{4 \pi^{2}}{G m_{\mathrm{J}}}\right) r^{3} \rightarrow 2 \log T=\log \left(\frac{4 \pi^{2}}{G m_{\mathrm{J}}}\right)+3 \log r \rightarrow \\
& \log T=\frac{3}{2} \log r+\frac{1}{2} \log \left(\frac{4 \pi^{2}}{G m_{\mathrm{J}}}\right)
\end{aligned}
$$

This predicts a straight line graph for $\log (T)$ vs. $\log (r)$, with a slope of $3 / 2$ and a
$y$-intercept of $\frac{1}{2} \log \left(\frac{4 \pi^{2}}{G m_{\mathrm{J}}}\right)$.
(b) The data is taken from Table 6-3, and the graph is shown here, with a straightline fit to the data. The data need to be converted to seconds and meters before the logarithms are calculated.

From the graph, the slope is 1.50 (as expected), and the $y$-intercept is -7.76 .


$$
\frac{1}{2} \log \left(\frac{4 \pi^{2}}{G m_{\mathrm{J}}}\right)=b \rightarrow m_{\mathrm{J}}=\frac{4 \pi^{2}}{G\left(10^{2 b}\right)}=\frac{4 \pi^{2}}{\left(6.67 \times 10^{-11}\right)\left(10^{-15.52}\right)}=1.97 \times 10^{27} \mathrm{~kg}
$$

The actual mass of Jupiter is given in problem 8 as 318 times the mass of the Earth, which is $1.90 \times 10^{27} \mathrm{~kg}$. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH06.XLS," on tab "Problem 6.47b."
48. We choose the line joining the Earth and Moon centers to be the $x$-axis. The field of the Earth will point towards the Earth, and the field of the Moon will point towards the Moon.


$$
\begin{aligned}
\overrightarrow{\mathbf{g}} & =\frac{G M_{\text {Earth }}}{\left.(-\hat{\mathbf{i}})+\frac{G M_{\text {Moon }}}{\left(\frac{1}{2} r_{\text {Earth- }}\right)^{2}}(\hat{\mathbf{i}})=\frac{G\left(M_{\text {Moon }}-M_{\text {Earth }}\right)}{\binom{\left.\frac{1}{2} r_{\text {Earth- }}\right)^{2}}{\text { Moon }}^{2}} \hat{\left(\frac{1}{2} r_{\text {Earth- }}\right)^{2}} \begin{array}{l}
\text { Moon }
\end{array}\right)^{2}} \\
& =\frac{\left(6.67 \times 10^{-11} \mathrm{~N} \bullet \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(7.35 \times 10^{22} \mathrm{~kg}-5.97 \times 10^{24} \mathrm{~kg}\right)}{\left(\frac{1}{2}\left(384 \times 10^{6} \mathrm{~m}\right)\right)^{2}} \hat{\mathbf{i}}=-1.07 \times 10^{-2} \mathrm{~m} / \mathrm{s}^{2} \hat{\mathbf{i}}
\end{aligned}
$$

So the magnitude is $1.07 \times 10^{-2} \mathrm{~m} / \mathrm{s}^{2}$ and the direction is towards the center of the Earth.
49. (a) The gravitational field due to a spherical mass $M$, at a distance $r$ from the center of the mass, is $g=G M / r^{2}$.

$$
g_{\substack{\text { Sun at } \\ \text { Earth }}}=\frac{G M_{\text {Sun }}}{r_{\text {sunto }}^{2}}=\frac{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(1.99 \times 10^{30} \mathrm{~kg}\right)}{\left(1.496 \times 10^{11} \mathrm{~m}\right)^{2}}=5.93 \times 10^{-3} \mathrm{~m} / \mathrm{s}^{2}
$$

(b) Compare this to the field caused by the Earth at the surface of the Earth.

$$
\frac{g_{\text {Sun at }}}{\text { Earth }}=\frac{5.93 \times 10^{-3} \mathrm{~m} / \mathrm{s}^{2}}{g_{\text {Earth }}}=6.85 \times 10^{-4}
$$

No, this is not going to affect your weight significantly. The effect is less than $0.1 \%$.
50. (a) From the symmetry of the situation, the net force on the object will be down. However, we will show that explicitly by writing the field in vector component notation.


$$
\begin{aligned}
\overrightarrow{\mathbf{g}}=\overrightarrow{\mathbf{g}}_{\text {left }}+\overrightarrow{\mathbf{g}}_{\text {right }} & =\left[\left(-G \frac{m}{x_{0}^{2}+y^{2}} \sin \theta\right) \hat{\mathbf{i}}+\left(-G \frac{m}{x_{0}^{2}+y^{2}} \cos \theta\right) \hat{\mathbf{j}}\right] \\
& +\left[\left(G \frac{m}{x_{0}^{2}+y^{2}} \sin \theta\right) \hat{\mathbf{i}}+\left(-G \frac{m}{x_{0}^{2}+y^{2}} \cos \theta\right) \hat{\mathbf{j}}\right] \\
= & \left(-2 G \frac{m}{x_{0}^{2}+y^{2}} \cos \theta\right) \hat{\mathbf{j}}=\left(-2 G \frac{m}{x_{0}^{2}+y^{2}} \frac{y}{\sqrt{x_{0}^{2}+y^{2}}}\right) \hat{\mathbf{j}}=\left(-2 G m \frac{y}{\left(x_{0}^{2}+y^{2}\right)^{3 / 2}}\right) \hat{\mathbf{j}}
\end{aligned}
$$

(b) If we keep $y$ as a positive quantity, then the magnitude of the field is $g=2 G m \frac{y}{\left(x_{0}^{2}+y^{2}\right)^{3 / 2}}$.

We find locations of the maximum magnitude by setting the first derivative equal to 0 . Since the expression is never negative, any extrema will be maxima.

$$
\begin{aligned}
& g=2 G m \frac{y}{\left(x_{0}^{2}+y^{2}\right)^{3 / 2}} \rightarrow \frac{d g}{d t}=2 G m\left[\frac{\left(x_{0}^{2}+y^{2}\right)^{3 / 2}-y^{\frac{3}{2}}\left(x_{0}^{2}+y^{2}\right)^{1 / 2} 2 y}{\left(x_{0}^{2}+y^{2}\right)^{3}}\right]=0 \rightarrow \\
& \left(x_{0}^{2}+y^{2}\right)^{3 / 2}-y^{3}\left(x_{0}^{2}+y^{2}\right)^{1 / 2} 2 y=0 \rightarrow y_{\max }=\frac{x_{0}}{\sqrt{2}} \approx 0.71 x_{0} \\
& g_{\max }=g\left(y=\frac{x_{0}}{\sqrt{2}}\right)=2 G m \frac{\frac{x_{0}}{\sqrt{2}}}{\left(x_{0}^{2}+\left(\frac{x_{0}}{\sqrt{2}}\right)^{2}\right)^{3 / 2}}=\frac{4 G m}{3 \sqrt{3} x_{0}^{2}} \approx 0.77 \frac{G m}{x_{0}^{2}}
\end{aligned}
$$

There would also be a maximum at $y=-x_{0} / \sqrt{2}$.
51. The acceleration due to the Earth's gravity at a location at or above the surface is given by $g=G M_{\text {Earth }} / r^{2}$, where $r$ is the distance from the center of the Earth to the location in question.
Find the location where $g=\frac{1}{2} g_{\text {surface }}$.

$$
\frac{G M_{\text {Earth }}}{r^{2}}=\frac{1}{2} \frac{G M_{\text {Earth }}}{R_{\text {Earth }}^{2}} \rightarrow r^{2}=2 R_{\text {Earth }}^{2} \rightarrow r=\sqrt{2} R_{\text {Earth }}
$$

The distance above the Earth's surface is as follows.

$$
r-R_{\text {Earth }}=(\sqrt{2}-1) R_{\text {Earth }}=(\sqrt{2}-1)\left(6.38 \times 10^{6} \mathrm{~m}\right)=2.64 \times 10^{6} \mathrm{~m}
$$

52. (a) Mass is independent of location and so the mass of the ball is 13.0 kg on both the Earth and the planet.
(b) The weight is found by $W=m g$.

$$
\begin{aligned}
& W_{\text {Earth }}=m g_{\text {Earth }}=(13.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=127 \mathrm{~N} \\
& W_{\text {Planet }}=m g_{\text {Planet }}=(13.0 \mathrm{~kg})\left(12.0 \mathrm{~m} / \mathrm{s}^{2}\right)=156 \mathrm{~N}
\end{aligned}
$$

53. (a) The acceleration due to gravity at any location at or above the surface of a star is given by $g_{\text {star }}=G M_{\text {star }} / r^{2}$, where $r$ is the distance from the center of the star to the location in question.

$$
g_{\text {star }}=G \frac{M_{\text {sun }}}{R_{\text {Moon }}^{2}}=\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right) \frac{\left(1.99 \times 10^{30} \mathrm{~kg}\right)}{\left(1.74 \times 10^{6} \mathrm{~m}\right)^{2}}=4.38 \times 10^{7} \mathrm{~m} / \mathrm{s}^{2}
$$

(b) $W=m g_{\text {star }}=(65 \mathrm{~kg})\left(4.38 \times 10^{7} \mathrm{~m} / \mathrm{s}^{2}\right)=2.8 \times 10^{9} \mathrm{~N}$
(c) Use Eq. 2-12c, with an initial velocity of 0 .

$$
\begin{aligned}
& v^{2}=v_{0}^{2}+2 a\left(x-x_{0}\right) \rightarrow \\
& v=\sqrt{2 a\left(x-x_{0}\right)}=\sqrt{2\left(4.38 \times 10^{7} \mathrm{~m} / \mathrm{s}^{2}\right)(1.0 \mathrm{~m})}=9.4 \times 10^{3} \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

54. In general, the acceleration due to gravity of the Earth is given by $g=G M_{\text {Earth }} / r^{2}$, where $r$ is the distance from the center of the Earth to the location in question. So for the location in question, we have the following.

$$
\begin{aligned}
& g=\frac{1}{10} g_{\text {surface }} \rightarrow G \frac{M_{\text {Earth }}}{r^{2}}=\frac{1}{10} G \frac{M_{\text {Earth }}}{R_{\text {Earth }}^{2}} \rightarrow r^{2}=10 R_{\text {Earth }}^{2} \\
& r=\sqrt{10} R_{\text {Earth }}=\sqrt{10}\left(6.38 \times 10^{6} \mathrm{~m}\right)=2.02 \times 10^{7} \mathrm{~m}
\end{aligned}
$$

55. The speed of an object in an orbit of radius $r$ around a planet is given in Example 6-6 as $v=\sqrt{G M_{\text {planet }} / r}$, and is also given by $v=2 \pi r / T$, where $T$ is the period of the object in orbit. Equate the two expressions for the speed and solve for $T$.

$$
\sqrt{G \frac{M_{\text {Planet }}}{r}}=\frac{2 \pi r}{T} \rightarrow T=2 \pi \sqrt{\frac{r^{3}}{G M_{\text {Planet }}}}
$$

For this problem, the inner orbit has radius $r_{\text {inner }}=7.3 \times 10^{7} \mathrm{~m}$, and the outer orbit has radius $r_{\text {outer }}=1.7 \times 10^{8} \mathrm{~m}$. Use these values to calculate the periods.

$$
\begin{aligned}
& T_{\text {inner }}=2 \pi \sqrt{\frac{\left(7.3 \times 10^{7} \mathrm{~m}\right)^{3}}{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(5.7 \times 10^{26} \mathrm{~kg}\right)}}=2.0 \times 10^{4} \mathrm{~s} \\
& T_{\text {outer }}=2 \pi \sqrt{\frac{\left(1.7 \times 10^{8} \mathrm{~m}\right)^{3}}{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(5.7 \times 10^{26} \mathrm{~kg}\right)}}=7.1 \times 10^{4} \mathrm{~s}
\end{aligned}
$$

Saturn's rotation period (day) is 10 hr 39 min , which is about $3.8 \times 10^{4} \mathrm{sec}$. Thus the inner ring will appear to move across the sky "faster" than the Sun (about twice per Saturn day), while the outer ring will appear to move across the sky "slower" than the Sun (about once every two Saturn days).
56. The speed of an object in an orbit of radius $r$ around the Moon is given by $v=\sqrt{G M_{\text {Moon }} / r}$, and is also given by $v=2 \pi r / T$, where $T$ is the period of the object in orbit. Equate the two expressions for the speed and solve for $T$.

$$
\begin{aligned}
& \sqrt{G M_{\text {Moon }} / r}=2 \pi r / T \rightarrow \\
& T=2 \pi \sqrt{\frac{r^{3}}{G M_{\text {Moon }}}}=2 \pi \sqrt{\frac{\left(R_{\text {Moon }}+100 \mathrm{~km}\right)^{3}}{G M_{\text {Moon }}}}=2 \pi \sqrt{\frac{\left(1.74 \times 10^{6} \mathrm{~m}+1 \times 10^{5} \mathrm{~m}\right)^{3}}{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(7.35 \times 10^{22} \mathrm{~kg}\right)}} \\
&=7.1 \times 10^{3} \mathrm{~s}(\sim 2.0 \mathrm{~h})
\end{aligned}
$$

57. Use Kepler's third law to relate the orbits of Earth and Halley's comet around the Sun.

$$
\begin{aligned}
& \left(r_{\text {Halley }} / r_{\text {Earth }}\right)^{3}=\left(T_{\text {Halley }} / T_{\text {Earth }}\right)^{2} \rightarrow \\
& r_{\text {Halley }}=r_{\text {Earth }}\left(T_{\text {Halley }} / T_{\text {Earth }}\right)^{2 / 3}=\left(150 \times 10^{6} \mathrm{~km}\right)(76 \mathrm{y} / 1 \mathrm{y})^{2 / 3}=2690 \times 10^{6} \mathrm{~km}
\end{aligned}
$$

This value is half the sum of the nearest and farthest distances of Halley's comet from the Sun. Since the nearest distance is very close to the Sun, we will approximate that nearest distance as 0 . Then the
farthest distance is twice the value above, or $5380 \times 10^{6} \mathrm{~km}=5.4 \times 10^{12} \mathrm{~m}$. This distance approaches the mean orbit distance of Pluto, which is $5.9 \times 10^{12} \mathrm{~m}$. It is still in the solar system, nearest to Pluto's orbit.
58. (a) The speed of a satellite orbiting the Earth is given by $v=\sqrt{G M_{\text {Earth }} / r}$. For the GPS satellites,

$$
\begin{aligned}
& r=R_{\text {Earth }}+(11,000)(1.852 \mathrm{~km})=2.68 \times 10^{7} \mathrm{~m} . \\
& \\
& \quad v=\sqrt{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right) \frac{\left(5.97 \times 10^{24} \mathrm{~kg}\right)}{2.68 \times 10^{7} \mathrm{~m}}}=3.86 \times 10^{3} \mathrm{~m} / \mathrm{s} \approx 3.9 \times 10^{3} \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(b) The period can be found from the speed and the radius.

$$
v=2 \pi r / T \rightarrow T=\frac{2 \pi r}{v}=\frac{2 \pi\left(2.68 \times 10^{7} \mathrm{~m}\right)}{3.86 \times 10^{3} \mathrm{~m} / \mathrm{s}}=4.4 \times 10^{4} \mathrm{sec} \sim 12 \mathrm{~h}
$$

59. For a body on the equator, the net motion is circular. Consider the freebody diagram as shown. $\mathrm{F}_{\mathrm{N}}$ is the normal force, which is the apparent weight. The net force must point to the center of the circle for the object to be moving in a circular path at constant speed. Write Newton's second law with the inward direction as positive.

$$
\begin{aligned}
& \sum F_{\mathrm{R}}=m g_{\text {Jupiter }}-F_{\mathrm{N}}=m v^{2} / R_{\text {Jupiter }} \rightarrow \\
& F_{\mathrm{N}}=m\left(g_{\text {Jupiter }}-v^{2} / R_{\text {Jupiter }}\right)=m\left(G \frac{M_{\text {Jupiter }}}{R_{\text {Jupiter }}^{2}}-\frac{v^{2}}{R_{\text {Jupiter }}}\right)
\end{aligned}
$$



Use the fact that for a rotating object, $v=2 \pi r / T$.

$$
F_{\mathrm{N}}=m\left(G \frac{M_{\text {Jupiter }}}{R_{\text {Jupiter }}^{2}}-\frac{4 \pi^{2} R_{\text {Jupiter }}}{T_{\text {Jupiter }}^{2}}\right)=m g_{\text {perceived }}
$$

Thus the perceived acceleration due to gravity of the object on the surface of Jupiter is as follows.

$$
\begin{aligned}
g_{\text {perceived }} & =G \frac{M_{\text {Jupiter }}}{R_{\text {Jupiter }}^{2}}-\frac{4 \pi^{2} R_{\text {Jupiter }}}{T_{\text {Jupiter }}^{2}} \\
& =\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right) \frac{\left(1.9 \times 10^{27} \mathrm{~kg}\right)}{\left(7.1 \times 10^{7} \mathrm{~m}\right)^{2}}-\frac{4 \pi^{2}\left(7.1 \times 10^{7} \mathrm{~m}\right)}{\left[(595 \mathrm{~min})\left(\frac{60 \mathrm{~s}}{1 \mathrm{~min}}\right)\right]^{2}} \\
& =22.94 \mathrm{~m} / \mathrm{s}^{2}\left(\frac{1 \mathrm{~g}}{9.8 \mathrm{~m} / \mathrm{s}^{2}}\right)=2.3 \mathrm{~g}^{\prime \mathrm{s}}
\end{aligned}
$$

Based on this result, you would not be crushed at all. You would feel "heavy," but not at all crushed.
60. The speed of rotation of the Sun about the galactic center, under the assumptions made, is given by $v=\sqrt{G \frac{M_{\text {galaxy }}}{r_{\text {Sun orbit }}}}$ and so $M_{\text {galaxy }}=\frac{r_{\text {Sun orbit }} v^{2}}{G}$. Substitute in the relationship that $v=2 \pi r_{\text {Sun orbit }} / T$.

$$
\begin{aligned}
M_{\text {galaxy }} & =\frac{4 \pi^{2}\left(r_{\text {Sun orbit }}\right)^{3}}{G T^{2}}=\frac{4 \pi^{2}\left[(30,000)\left(9.5 \times 10^{15} \mathrm{~m}\right)\right]^{3}}{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left[\left(200 \times 10^{6} \mathrm{y}\right)\left(\frac{3.15 \times 10^{7} \mathrm{~s}}{1 \mathrm{y}}\right)\right]^{2}} \\
& =3.452 \times 10^{41} \mathrm{~kg} \approx 3 \times 10^{41} \mathrm{~kg}
\end{aligned}
$$

The number of solar masses is found by dividing the result by the solar mass.

$$
\# \text { stars }=\frac{M_{\text {galaxy }}}{M_{\text {Sun }}}=\frac{3.452 \times 10^{41} \mathrm{~kg}}{2.0 \times 10^{30} \mathrm{~kg}}=1.726 \times 10^{11} \approx 2 \times 10^{11} \text { stars }
$$

61. In the text, it says that Eq. 6-6 is valid if the radius $r$ is replaced with the semi-major axis $s$. From Fig. 6-16, the distance of closest approach $r_{\min }$ is seen to be $r_{\text {min }}=s-e s=s(1-e)$, and so the semi-major axis is given by $s=\frac{r_{\text {min }}}{1-e}$.

$$
\frac{T^{2}}{s^{3}}=\frac{4 \pi^{2}}{G M_{\mathrm{SgrA}}} \rightarrow
$$

$$
M_{\mathrm{SgrA}}=\frac{4 \pi^{2} s^{3}}{G T^{2}}=\frac{4 \pi^{2}\left(\frac{r_{\min }}{1-e}\right)^{3}}{G T^{2}}=\frac{4 \pi^{2}\left(\frac{1 \mathrm{AU}}{1-0.87}\right)}{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(15.2 \mathrm{y} \times \frac{3.156 \times 10^{7} \mathrm{~s}}{1 \mathrm{y}}\right)^{2}}
$$

$$
=7.352 \times 10^{36} \mathrm{~kg} \approx 7.4 \times 10^{36} \mathrm{~kg}
$$

$\frac{M_{\text {SgrA }}}{M_{\text {Sun }}}=\frac{7.352 \times 10^{36} \mathrm{~kg}}{1.99 \times 10^{30} \mathrm{~kg}}=3.7 \times 10^{6}$ and so SgrA is almost 4 million times more massive than our Sun.
62. (a) The gravitational force on the satellite is given by $F_{\text {grav }}=G \frac{M_{\text {Earth }} m}{r^{2}}$, where $r$ is the distance of the satellite from the center of the Earth. Since the satellite is moving in circular motion, then the net force on the satellite can be written as $F_{\text {net }}=m v^{2} / r$. By substituting $v=2 \pi r / T$ for a circular orbit, we have $F_{\text {net }}=\frac{4 \pi^{2} m r}{T^{2}}$. Then, since gravity is the only force on the satellite, the two expressions for force can be equated, and solved for the orbit radius.

$$
\begin{aligned}
& G \frac{M_{\text {Earth }} m}{r^{2}}=\frac{4 \pi^{2} m r}{T^{2}} \rightarrow \\
& r=\left(\frac{G M_{\text {Earth }} T^{2}}{4 \pi^{2}}\right)^{1 / 3}=\left[\frac{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(6.0 \times 10^{24} \mathrm{~kg}\right)(6200 \mathrm{~s})^{2}}{4 \pi^{2}}\right]^{1 / 3} \\
& =7.304 \times 10^{6} \mathrm{~m} \approx 7.3 \times 10^{6} \mathrm{~m}
\end{aligned}
$$

(b) From this value the gravitational force on the satellite can be calculated.

$$
\begin{aligned}
F_{\text {grav }} & =G \frac{M_{\text {Earth }} m}{r^{2}}=\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right) \frac{\left(6.0 \times 10^{24} \mathrm{~kg}\right)(5500 \mathrm{~kg})}{\left(7.304 \times 10^{6} \mathrm{~m}\right)^{2}}=4.126 \times 10^{4} \mathrm{~N} \\
& \approx 4.1 \times 10^{4} \mathrm{~N}
\end{aligned}
$$

(c) The altitude of the satellite above the Earth's surface is given by the following.

$$
r-R_{\text {Earth }}=7.304 \times 10^{6} \mathrm{~m}-6.38 \times 10^{6} \mathrm{~m}=9.2 \times 10^{5} \mathrm{~m}
$$

63. Your weight is given by the law of universal gravitation. The derivative of the weight with respect to time is found by taking the derivative of the weight with respect to distance from the Earth's center, and using the chain rule.

$$
W=G \frac{m_{\mathrm{E}} m}{r^{2}} \rightarrow \frac{d W}{d t}=\frac{d W}{d r} \frac{d r}{d t}=-2 G \frac{m_{\mathrm{E}} m}{r^{3}} v
$$

64. The speed of an orbiting object is given in Example 6-6 as $v=\sqrt{G M / r}$, where $r$ is the radius of the orbit, and $M$ is the mass around which the object is orbiting. Solve the equation for $M$.

$$
v=\sqrt{G M / r} \rightarrow M=\frac{r v^{2}}{G}=\frac{\left(5.7 \times 10^{17} \mathrm{~m}\right)\left(7.8 \times 10^{5} \mathrm{~m} / \mathrm{s}\right)^{2}}{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)}=5.2 \times 10^{39} \mathrm{~kg}
$$

The number of solar masses is found by dividing the result by the solar mass.

$$
\text { \# solar masses }=\frac{M_{\text {galaxy }}}{M_{\text {Sun }}}=\frac{5.2 \times 10^{39} \mathrm{~kg}}{2 \times 10^{30} \mathrm{~kg}}=2.6 \times 10^{9} \text { solar masses }
$$

65. Find the "new" Earth radius by setting the acceleration due to gravity at the Sun's surface equal to the acceleration due to gravity at the "new" Earth's surface.

$$
\begin{aligned}
g_{\substack{\text { Earth } \\
\text { new }}}=g_{\text {Sun }} \rightarrow \frac{G M_{\text {Earth }}}{r_{\text {Earth }}^{2}}=\frac{G M_{\text {Sun }}}{r_{\text {Suw }}^{2}} & \rightarrow r_{\substack{\text { Earth } \\
\text { new }}}=r_{\text {Sun }} \sqrt{\frac{M_{\text {Earth }}}{M_{\text {Sun }}}}=\left(6.96 \times 10^{8} \mathrm{~m}\right) \sqrt{\frac{5.98 \times 10^{24} \mathrm{~kg}}{1.99 \times 10^{30} \mathrm{~kg}}} \\
& =1.21 \times 10^{6} \mathrm{~m}, \text { about } \frac{1}{5} \text { the actual Earth radius. }
\end{aligned}
$$

66. (a) See the free-body diagram for the plumb bob. The attractive gravitational force on the plumb bob is $F_{\mathrm{M}}=G \frac{m m_{\mathrm{M}}}{D_{M}^{2}}$. Since the bob is not accelerating, the net force in any direction will be zero. Write the net force for both vertical and
 horizontal directions. Use $g=G \frac{M_{\text {Earth }}}{R_{\text {Earth }}^{2}}$.

$$
\begin{aligned}
& \sum F_{\text {vericical }}=F_{\mathrm{T}} \cos \theta-m g=0 \rightarrow F_{\mathrm{T}}=\frac{m g}{\cos \theta} \\
& \sum F_{\text {horizontal }}=F_{\mathrm{M}}-F_{\mathrm{T}} \sin \theta=0 \rightarrow F_{\mathrm{M}}=F_{\mathrm{T}} \sin \theta=m g \tan \theta \\
& G \frac{m m_{\mathrm{M}}}{D_{M}^{2}}=m g \tan \theta \rightarrow \theta=\tan ^{-1} G \frac{m_{\mathrm{M}}}{g D_{M}^{2}}=\tan ^{-1} \frac{m_{\mathrm{M}} R_{\mathrm{Earth}}^{2}}{M_{\text {Earth }} D_{M}^{2}}
\end{aligned}
$$

(b) We estimate the mass of Mt. Everest by taking its volume times its mass density. If we approximate Mt. Everest as a cone with the same size diameter as height, then its volume is $V=\frac{1}{3} \pi r^{2} h=\frac{1}{3} \pi(2000 \mathrm{~m})^{2}(4000 \mathrm{~m})=1.7 \times 10^{10} \mathrm{~m}^{3}$. The density is $\rho=3 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}$. Find the mass by multiplying the volume times the density.

$$
M=\rho V=\left(3 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)\left(1.7 \times 10^{10} \mathrm{~m}^{3}\right)=5 \times 10^{13} \mathrm{~kg}
$$

(c) With $D=5000 \mathrm{~m}$, use the relationship derived in part (a).

$$
\theta=\tan ^{-1} \frac{M_{\mathrm{M}} R_{\text {Earth }}^{2}}{M_{\text {Earth }} D_{\mathrm{M}}^{2}}=\tan ^{-1} \frac{\left(5 \times 10^{13} \mathrm{~kg}\right)\left(6.38 \times 10^{6} \mathrm{~m}\right)^{2}}{\left(5.97 \times 10^{24} \mathrm{~kg}\right)(5000 \mathrm{~m})^{2}}=8 \times 10^{-4} \text { degrees }
$$

67. Since all of the masses (or mass holes) are spherical, and $g$ is being measured outside of their boundaries, we can use the simple Newtonian gravitation expression. In the diagram, the distance $r=$ 2000 m . The radius of the deposit is unknown.

$$
\begin{aligned}
& g_{\text {actual }}=\underset{\substack{\text { full } \\
\text { Earth }}}{g_{\text {mirs mass }}}-g_{\substack{\text { missing } \\
\text { dit }}}+g_{\text {oil }}=\underset{\substack{\text { full } \\
\text { Earth }}}{r^{2}} \frac{G M_{\text {missing }}^{\text {dirt }}}{r^{2}} \\
& =g_{\substack{\text { full } \\
\text { Earth }}}-\frac{G\left(M_{\begin{array}{c}
\text { missing } \\
\text { dirt }
\end{array}}-M_{\text {oil }}\right)}{r^{2}} \\
& \Delta g=g_{\substack{\text { full } \\
\text { Earth }}}-g_{\text {actual }}=\frac{G\left(M_{\begin{array}{c}
\text { missing } \\
\text { dirt }
\end{array}}-M_{\text {oil }}\right)}{r^{2}}=\frac{G}{r^{2}}\left(V_{\substack{\text { missing } \\
\text { dirt }}} \rho_{\text {missing }}-V_{\text {oil }} \rho_{\text {oil }}\right)=\frac{G V_{\text {oil }}}{r^{2}}\left(\rho_{\begin{array}{c}
\text { missing } \\
\text { dirt }
\end{array}}-\rho_{\text {oil }}\right)=\frac{2}{10^{7}} g \\
& V_{\text {oil }}=\frac{2}{10^{7}} g \frac{r^{2}}{G} \frac{1}{\left(\rho_{\text {missing }}-\rho_{\text {oil }}\right)}=\frac{2}{10^{7}}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \frac{(2000 \mathrm{~m})^{2}}{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)} \frac{1}{(3000-800) \mathrm{kg} / \mathrm{m}^{2}} \\
& =5.34 \times 10^{7} \mathrm{~m}^{3} \approx 5 \times 10^{7} \mathrm{~m}^{3} \\
& r_{\text {deposit }}=\left(\frac{3 V_{\text {oil }}}{4 \pi}\right)^{1 / 3}=234 \mathrm{~m} \approx 200 \mathrm{~m} ; m_{\text {deposit }}=V_{\text {oil }} \rho_{\text {oil }}=4.27 \times 10^{10} \mathrm{~kg} \approx 4 \times 10^{10} \mathrm{~kg}
\end{aligned}
$$

68. The relationship between orbital speed and orbital radius for objects in orbit around the Earth is given in Example 6-6 as $v=\sqrt{G M_{\text {Earth }} / r}$. There are two orbital speeds involved - the one at the original radius, $v_{0}=\sqrt{G M_{\text {Earth }} / r_{0}}$, and the faster speed at the reduced radius, $v=\sqrt{G M_{\text {Earth }} /\left(r_{0}-\Delta r\right)}$.
(a) At the faster speed, 25,000 more meters will be traveled during the "catch-up" time, $t$. Note that

$$
r_{0}=6.38 \times 10^{6} \mathrm{~m}+4 \times 10^{5} \mathrm{~m}=6.78 \times 10^{6} \mathrm{~m} .
$$

$$
v t=v_{0} t+2.5 \times 10^{4} \mathrm{~m} \rightarrow\left(\sqrt{G \frac{M_{\text {Earth }}}{r_{0}-\Delta r}}\right) t=\left(\sqrt{G \frac{M_{\text {Earth }}}{r_{0}}}\right) t+2.5 \times 10^{4} \mathrm{~m} \rightarrow
$$

$$
\begin{aligned}
t & =\frac{2.5 \times 10^{4} \mathrm{~m}}{\sqrt{G M_{\text {Earth }}}}\left(\frac{1}{\sqrt{r_{0}-\Delta r}}-\frac{1}{\sqrt{r_{0}}}\right)^{-1} \\
& =\frac{2.5 \times 10^{4} \mathrm{~m}}{\sqrt{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(5.97 \times 10^{24} \mathrm{~kg}\right)}}\left(\frac{1}{\sqrt{6.78 \times 10^{6} \mathrm{~m}-1 \times 10^{3} \mathrm{~m}}}-\frac{1}{\sqrt{6.78 \times 10^{6} \mathrm{~m}}}\right)^{-1} \\
& =4.42 \times 10^{4} \mathrm{~s} \approx 12 \mathrm{~h}
\end{aligned}
$$

(b) Again, 25,000 more meters must be traveled at the faster speed in order to catch up to the satellite.

$$
\begin{aligned}
& v t=v_{0} t+2.5 \times 10^{4} \mathrm{~m} \rightarrow\left(\sqrt{G \frac{M_{\text {Earth }}}{r_{0}-\Delta r}}\right) t=\left(\sqrt{G \frac{M_{\text {Earth }}}{r_{0}}}\right) t+2.5 \times 10^{4} \mathrm{~m} \rightarrow \\
& \sqrt{\frac{1}{r_{0}-\Delta r}}=\frac{1}{\sqrt{r_{0}}}+\frac{2.5 \times 10^{4} \mathrm{~m}}{t \sqrt{G M_{\text {Earth }}}} \rightarrow \sqrt{r_{0}-\Delta r}=\left[\frac{1}{\sqrt{r_{0}}}+\frac{2.5 \times 10^{4} \mathrm{~m}}{t \sqrt{G M_{\text {Earth }}}}\right]^{-1} \rightarrow \\
& \Delta r=r_{0}-\left[\frac{1}{\sqrt{r_{0}}}+\frac{2.5 \times 10^{4} \mathrm{~m}}{t \sqrt{G M_{\text {Earth }}}}\right]^{-2}=\left(6.78 \times 10^{6} \mathrm{~m}\right) \\
& \quad-\left[\frac{1}{\sqrt{\left(6.78 \times 10^{6} \mathrm{~m}\right)}}+\frac{2.5 \times 10^{4} \mathrm{~m}}{(25200 \mathrm{~s}) \sqrt{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(5.97 \times 10^{24} \mathrm{~kg}\right)}}\right]^{-2} \\
& \\
& =1755 \mathrm{~m} \approx 1.8 \times 10^{3} \mathrm{~m}
\end{aligned}
$$

69. If the ring is to produce an apparent gravity equivalent to that of Earth, then the normal force of the ring on objects must be given by $F_{\mathrm{N}}=m g$. The Sun will also exert a force on objects on the ring. See the free-body diagram. Write Newton's second law for the object, with the fact that the acceleration is centripetal.

$$
\sum F=F_{\mathrm{R}}=F_{\mathrm{Sun}}+F_{\mathrm{N}}=m v^{2} / r
$$



Substitute in the relationships that $v=2 \pi r / T, F_{\mathrm{N}}=m g$, and $F_{\mathrm{Sun}}=G \frac{M_{\mathrm{Sun}} m}{r^{2}}$, and solve for the period of the rotation.

$$
\begin{aligned}
& F_{\text {Sun }}+F_{N}=m v^{2} / r \rightarrow G \frac{M_{\text {Sun }} m}{r^{2}}+m g=\frac{4 \pi^{2} m r}{T^{2}} \rightarrow G \frac{M_{\text {Sun }}}{r^{2}}+g=\frac{4 \pi^{2} r}{T^{2}} \\
& T=\sqrt{\frac{4 \pi^{2}\left(1.50 \times 10^{11} \mathrm{~m}\right)}{G \frac{4 \pi^{2} r}{M_{\text {Sun }}}+g}}=\sqrt{\frac{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right) \frac{\left(1.99 \times 10^{30} \mathrm{~kg}\right)}{\left(1.50 \times 10^{11} \mathrm{~m}\right)^{2}}+9.80 \mathrm{~m} / \mathrm{s}^{2}}{\left(.77 \times 10^{5} \mathrm{~s}\right.}=8.99 \mathrm{~d}} \\
& =7 .
\end{aligned}
$$

The force of the Sun is only about $1 / 1600$ the size of the normal force. The force of the Sun could have been ignored in the calculation with no significant change in the result given above.
70. For an object to be apparently weightless would mean that the object would have a centripetal acceleration equal to $g$. This is really the same as asking what the orbital period would be for an object orbiting the Earth with an orbital radius equal to the Earth's radius. To calculate, use $g=a_{C}=v^{2} / R_{\text {Earth }}$, along with $v=2 \pi R_{\text {Earth }} / T$, and solve for $T$.

$$
g=\frac{v^{2}}{R_{\text {Earth }}}=\frac{4 \pi^{2} R_{\text {Earth }}}{T^{2}} \rightarrow T=2 \pi \sqrt{\frac{R_{\text {Earth }}}{g}}=2 \pi \sqrt{\frac{6.38 \times 10^{6} \mathrm{~m}}{9.80 \mathrm{~m} / \mathrm{s}^{2}}}=5.07 \times 10^{3} \mathrm{~s}(\sim 84.5 \mathrm{~min})
$$

71. The speed of an object orbiting a mass is given in Example 6-6 as $v=\sqrt{\frac{G M_{\text {Sun }}}{r}}$.

$$
\begin{aligned}
& v_{\text {new }}=1.5 v \text { and } v_{\text {new }}=\sqrt{\frac{G M_{\mathrm{Sun}}}{r_{\text {new }}}} \rightarrow 1.5 v=\sqrt{\frac{G M_{\mathrm{Sun}}}{r_{\text {new }}}} \rightarrow 1.5 \sqrt{\frac{G M_{\mathrm{Sun}}}{r}}=\sqrt{\frac{G M_{\mathrm{Sun}}}{r_{\text {new }}}} \rightarrow \\
& r_{\text {new }}=\frac{r}{1.5^{2}}=0.44 r
\end{aligned}
$$

72. From the Venus data, the mass of the Sun can be determined by the following. Set the gravitational force on Venus equal to the centripetal force acting on Venus to make it orbit.

Then likewise, for Callisto orbiting Jupiter, $M_{\text {Jupiter }}=\frac{4 \pi^{2} r_{\text {Callisto }}^{3} \text { orbit }}{G T_{\text {Callisto }}^{3}}$, and for the Moon orbiting the Earth, $M_{\text {Earth }}=\frac{4 \pi^{2} r_{\text {Moon }}^{3} \text { orbit }}{G T_{\text {Moon }}^{2}}$. To find the density ratios, take the mass ratios with the mass expressed as density times volume, and expressed as found above.

And likewise for the Earth-Sun combination:

$$
\frac{\rho_{\text {Earth }}}{\rho_{\text {Sun }}}=\frac{r_{\text {Moon }}^{3} \text { orbit }}{T_{\text {Moon }}^{2}} \frac{T_{\text {Venus }}^{2}}{r_{\text {Venus }}^{3}} \frac{r_{\text {Sun }}^{3}}{r_{\text {Earth }}^{3}}=\frac{(0.003069)^{3}}{(27.32)^{2}} \frac{(224.7)^{2}}{(0.724)^{3}} \frac{1}{(0.0109)^{3}}=3.98
$$

73. The initial force of 120 N can be represented as $F_{\text {grav }}=\frac{G M_{\text {planet }}}{r^{2}}=120 \mathrm{~N}$.
(a) The new radius is 1.5 times the original radius.

$$
F_{\substack{\text { new } \\ \text { radius }}}=\frac{G M_{\text {planet }}}{r_{\text {new }}^{2}}=\frac{G M_{\text {planet }}}{(1.5 r)^{2}}=\frac{G M_{\text {planet }}}{2.25 r^{2}}=\frac{1}{2.25}(120 \mathrm{~N})=53 \mathrm{~N}
$$

(b) With the larger radius, the period is $T=7200$ seconds. As found in Example 6-6, orbit speed can be calculated by $v=\sqrt{\frac{G M}{r}}$.

$$
v=\sqrt{\frac{G M}{r}}=\frac{2 \pi r}{T} \rightarrow M=\frac{4 \pi^{2} r^{3}}{G T^{2}}=\frac{4 \pi^{2}\left(3.0 \times 10^{7} \mathrm{~m}\right)^{3}}{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)(7200 \mathrm{~s})^{2}}=3.1 \times 10^{26} \mathrm{~kg}
$$

74. The density of the sphere is uniform, and is given by $\rho=\frac{M}{\frac{4}{3} \pi r^{3}}$. The mass that was removed to make the cavity is $M_{\text {cavity }}=\pi V_{\text {cavity }}=\frac{M}{\frac{4}{3} \pi r^{3}}\left(\frac{4}{3} \pi(r / 2)^{3}\right)=\frac{1}{8} M$. The net force on the point mass can be found by finding the force due to the entire sphere, and then subtracting the force caused by the cavity alone.

$$
\begin{aligned}
F_{\text {net }} & =F_{\text {sphere }}-F_{\text {caxity }}=\frac{G M m}{d^{2}}-\frac{G\left(\frac{1}{8} M\right) m}{(d-r / 2)^{2}}=G M m\left(\frac{1}{d^{2}}-\frac{1}{8(d-r / 2)^{2}}\right) \\
& =\frac{G M m}{d^{2}}\left(1-\frac{1}{8(1-r / 2 d)^{2}}\right)
\end{aligned}
$$

75. (a) We use the law of universal gravitation to express the force for each mass $m$. One mass is "near" the Moon, and so the distance from that mass to the center of the Moon is $R_{\mathrm{EM}}-R_{\mathrm{E}}$. The other mass is "far" from the Moon, and so the distance from that mass to the center of the Moon is $R_{\mathrm{EM}}+R_{\mathrm{E}}$.

$$
\begin{aligned}
& F_{\text {near }}^{\text {Moon }}
\end{aligned}=\frac{G M_{\text {Moon }} m}{\left(R_{\mathrm{EM}}-R_{\mathrm{E}}\right)^{2}} \quad F_{\text {far }}=\frac{G M_{\text {Moon }} m}{\left(R_{\mathrm{EM}}+R_{\mathrm{E}}\right)^{2}}
$$

(b) We use a similar analysis to part (a).

$$
\begin{aligned}
& F_{\text {near }}=\frac{G M_{\text {Sun }} m}{\left(r_{\mathrm{ES}}-r_{\mathrm{E}}\right)^{2}} \quad F_{\text {far }}=\frac{G M_{\text {Suu }} m}{\left(r_{\text {Moon }}\right.}\left(r_{\mathrm{ES}}+r_{\mathrm{E}}\right)^{2} \\
& \left(\frac{F_{\text {near }}}{F_{\text {far }}}\right)_{\text {Sun }}=\frac{\frac{G M_{\text {Sun }} m}{\left(r_{\mathrm{ES}}-r_{\mathrm{E}}\right)^{2}}}{\frac{G M_{\text {Sun }} m}{\left(r_{\mathrm{ES}}+r_{\mathrm{E}}\right)^{2}}}=\left(\frac{r_{\mathrm{EM}}+r_{\mathrm{E}}}{r_{\mathrm{EM}}-r_{\mathrm{E}}}\right)^{2}=\left(\frac{1.496 \times 10^{11} \mathrm{~m}+6.38 \times 10^{6} \mathrm{~m}}{1.496 \times 10^{11} \mathrm{~m}-6.38 \times 10^{6} \mathrm{~m}}\right)^{2}=1.000171
\end{aligned}
$$

(c) For the average gravitational force on the large masses, we use the distance between their centers.

$$
\begin{aligned}
& F_{\mathrm{Sun}}=\frac{G M_{\mathrm{Sun}} M_{\mathrm{Earth}}}{r_{\mathrm{ES}}^{2}} \quad F_{\mathrm{Moon}}=\frac{G M_{\mathrm{Moon}} M_{\mathrm{Earrh}}}{r_{\mathrm{EM}}^{2}} \\
& \frac{F_{\mathrm{Sun}}}{F_{\mathrm{Moon}}}=\frac{\frac{G M_{\mathrm{Sun}} M_{\mathrm{Earth}}}{\frac{r_{\mathrm{ES}}^{2}}{G M_{\mathrm{Moon}} M_{\mathrm{Earth}}}} r_{\mathrm{EM}}^{2}}{r_{\mathrm{EM}}}=\frac{M_{\mathrm{Sun}}}{r_{\mathrm{ES}}^{2}} \frac{r_{\mathrm{EM}}^{2}}{M_{\mathrm{Moon}}}=\frac{\left(1.99 \times 10^{30} \mathrm{~kg}\right)}{\left(1.496 \times 10^{11} \mathrm{~m}\right)^{2}} \frac{\left(3.84 \times 10^{8} \mathrm{~m}\right)^{2}}{\left(7.35 \times 10^{22} \mathrm{~kg}\right)}=178
\end{aligned}
$$

(d) Apply the expression for $\Delta F$ as given in the statement of the problem.

$$
\frac{\Delta F_{\text {Moon }}}{\Delta F_{\mathrm{Sun}}}=\frac{F_{\mathrm{Moon}}\left(\frac{F_{\text {near }}}{F_{\mathrm{far}}}-1\right)_{\mathrm{Moon}}}{F_{\mathrm{Sun}}\left(\frac{F_{\text {near }}}{F_{\mathrm{far}}}-1\right)_{\mathrm{Sun}}}=\frac{F_{\mathrm{Moon}}}{F_{\mathrm{Sun}}} \frac{\left(\frac{F_{\text {near }}}{F_{\mathrm{far}}}-1\right)_{\mathrm{Moon}}}{\left(\frac{F_{\mathrm{near}}}{F_{\mathrm{far}}}-1\right)_{\mathrm{Sun}}}=\frac{1}{178} \frac{(1.0687-1)}{(1.000171-1)}=2.3
$$

76. The acceleration is found from the law of universal gravitation. Using the chain rule, a relationship between the acceleration expression and the velocity can be found which is integrated to find the velocity as a function of distance. The outward radial direction is taken to be positive, so the acceleration is manifestly negative.

$$
\begin{aligned}
& F=m a=-G \frac{m_{\mathrm{E}} m}{r^{2}} \rightarrow a=-\frac{G m_{\mathrm{E}}}{r^{2}}=\frac{d v}{d t}=\frac{d v}{d r} \frac{d r}{d t}=v \frac{d v}{d r} \rightarrow-\frac{G m_{\mathrm{E}}}{r^{2}}=v \frac{d v}{d r} \rightarrow \\
& -G m_{\mathrm{E}} \frac{d r}{r^{2}}=v d v \rightarrow-G m_{\mathrm{E}} \int_{2 r_{\mathrm{E}}}^{r_{\mathrm{E}}} \frac{d r}{r^{2}}=\int_{0}^{v_{f}} v d v \rightarrow\left[\frac{G m_{\mathrm{E}}}{r}\right]_{2 r_{\mathrm{E}}}^{r_{\mathrm{E}}}=\frac{1}{2} v_{f}^{2} \rightarrow \\
& \frac{G m_{\mathrm{E}}}{r_{\mathrm{E}}}-\frac{G m_{\mathrm{E}}}{2 r_{\mathrm{E}}}=\frac{1}{2} v_{f}^{2} \rightarrow v_{f}= \pm \sqrt{\frac{G m_{\mathrm{E}}}{r_{\mathrm{E}}}} \rightarrow v_{f}=-\sqrt{\frac{G m_{\mathrm{E}}}{r_{\mathrm{E}}}}
\end{aligned}
$$

The negative sign is chosen because the object is moving towards the center of the Earth, and the outward radial direction is positive.
77. Equate the force of gravity on a mass $m$ at the surface of the Earth as expressed by the acceleration due to gravity to that as expressed by Newton's law of universal gravitation.

$$
\begin{aligned}
m g=\frac{G M_{\text {Earth }} m}{R_{\text {Earth }}^{2}} \rightarrow G & =\frac{g R_{\text {Earth }}^{2}}{M_{\text {Earth }}}=\frac{g R_{\text {Earth }}^{2}}{\rho_{\text {Earth }} \frac{4}{3} \pi R_{\text {Earth }}^{3}}=\frac{3 g}{4 \pi \rho R_{\text {Earth }}}=\frac{3 g}{4 \pi \rho \frac{C_{\text {Earth }}}{2 \pi}}=\frac{3 g}{2 \rho C_{\text {Earth }}} \\
& =\frac{3\left(10 \mathrm{~m} / \mathrm{s}^{2}\right)}{2\left(3000 \mathrm{~kg} / \mathrm{m}^{3}\right)\left(4 \times 10^{7} \mathrm{~m}\right)}=1.25 \times 10^{-10} \frac{\mathrm{~m}^{3}}{\mathrm{~kg} \cdot \mathrm{~s}^{2}} \approx 1 \times 10^{-10} \frac{\mathrm{~m}^{3}}{\mathrm{~kg} \cdot \mathrm{~s}^{2}}
\end{aligned}
$$

This is roughly twice the size of the accepted value of $G$.
78. (a) From Example 6-6, the speed of an object in a circular orbit of radius $r$ about mass M is

$$
\begin{gathered}
v=\sqrt{\frac{G M}{r}} . \text { Use that relationship along with the definition of density to find the speed. } \\
v=\sqrt{\frac{G M}{r}} \rightarrow v^{2}=\frac{G M}{r}=\frac{G \rho \frac{4}{3} \pi r^{3}}{r} \rightarrow
\end{gathered}
$$

$$
r=\sqrt{\frac{3 v^{2}}{4 \pi G \rho}}=\sqrt{\frac{3(22 \mathrm{~m} / \mathrm{s})^{2}}{4 \pi\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(2700 \mathrm{~kg} / \mathrm{m}^{3}\right)}}=25330 \mathrm{~m} \approx 2.5 \times 10^{4} \mathrm{~m}
$$

(b) $v=\frac{2 \pi r}{T} \rightarrow T=\frac{2 \pi r}{v}=\frac{2 \pi(25330 \mathrm{~m})}{22 \mathrm{~m} / \mathrm{s}}=7234 \mathrm{~s} \approx 2.0 \mathrm{~h}$
79. (a) The graph is shown.
(b) From the graph, we get this equation.

$$
\begin{aligned}
& T^{2}=0.9999 r^{3}+0.3412 \\
& r=\left(\frac{T^{2}-0.3412}{0.9999}\right)^{1 / 3}
\end{aligned}
$$



$$
r(T=247.7 \mathrm{y})=\left(\frac{247.7^{2}-0.3412}{0.9999}\right)^{1 / 3}=39.44 \mathrm{AU}
$$

A quoted value for the means distance of Pluto is 39.47 AU . The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH06.XLS," on tab "Problem 6.79."

## CHAPTER 7: Work and Energy

## Responses to Questions

1. "Work" as used in everyday language generally means "energy expended," which is similar to the way "work" is defined in physics. However, in everyday language, "work" can involve mental or physical energy expended, and is not necessarily connected with displacement, as it is in physics. So a student could say she "worked" hard carrying boxes up the stairs to her dorm room (similar in meaning to the physics usage), or that she "worked" hard on a problem set (different in meaning from the physics usage).
2. Yes, she is doing work. The work done by her and the work done on her by the river are opposite in sign, so they cancel and she does not move with respect to the shore. When she stops swimming, the river continues to do work on her, so she floats downstream.
3. No, not if the object is moving in a circle. Work is the product of force and the displacement in the direction of the force. Therefore, a centripetal force, which is perpendicular to the direction of motion, cannot do work on an object moving in a circle.
4. You are doing no work on the wall. Your muscles are using energy generated by the cells in your body and producing byproducts which make you feel fatigued.
5. No. The magnitudes of the vectors and the angle between them are the relevant quantities, and these do not depend on the choice of coordinate system.
6. Yes. A dot product can be negative if corresponding components of the vectors involved point in opposite directions. For example, if one vector points along the positive $x$-axis, and the other along the negative $x$-axis, the angle between the vectors is $180^{\circ} . \operatorname{Cos} 180^{\circ}=-1$, and so the dot product of the two vectors will be negative.
7. No. For instance, imagine $\overrightarrow{\mathbf{C}}$ as a vector along the $+x$ axis. $\overrightarrow{\mathbf{A}}$ and $\overrightarrow{\mathbf{B}}$ could be two vectors with the same magnitude and the same $x$-component but with $y$-components in opposite directions, so that one is in quadrant I and the other in quadrant IV. Then $\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{C}}=\overrightarrow{\mathbf{B}} \cdot \overrightarrow{\mathbf{C}}$ even though $\overrightarrow{\mathbf{A}}$ and $\overrightarrow{\mathbf{B}}$ are different vectors.
8. No. The dot product of two vectors is always a scalar, with only a magnitude.
9. Yes. The normal force is the force perpendicular to the surface an object is resting on. If the object moves with a component of its displacement perpendicular to this surface, the normal force will do work. For instance, when you jump, the normal force does work on you in accelerating you vertically.
10. (a) If the force is the same, then $F=k_{1} x_{1}=k_{2} x_{2}$, so $x_{2}=k_{1} x_{1} / k_{2}$. The work done on spring 1 will be $W_{1}=\frac{1}{2} k_{1} x_{1}^{2}$. The work done on spring 2 will be $W_{2}=\frac{1}{2} k_{2} x_{2}^{2}=\frac{1}{2} k_{2}\left(k_{1}^{2} x_{1}^{2} / k_{2}^{2}\right)=W_{1}\left(k_{1} / k_{2}\right)$. Since $k_{1}>k_{2}, W_{2}>W_{1}$, so more work is done on spring 2.
(b) If the displacement is the same, then $W_{1}=\frac{1}{2} k_{1} x^{2}$ and $W_{2}=\frac{1}{2} k_{2} x^{2}$. Since $k_{1}>k_{2}, W_{1}>W_{2}$, so more work is done on spring 1.
11. The kinetic energy increases by a factor of 9 , since the kinetic energy is proportional to the square of the speed.
12. Until the $x=0$ point, the spring has a positive acceleration and is accelerating the block, and therefore will remain in contact with it. After the $x=0$ point, the spring begins to slow down, but (in the absence of friction), the block will continue to move with its maximum speed and will therefore move faster than the spring and will separate from it.
13. The bullet with the smaller mass has a speed which is greater by a factor of $\sqrt{2} \approx 1.4$. Since their kinetic energies are equal, then $\frac{1}{2} m_{1} v_{1}^{2}=\frac{1}{2} m_{2} v_{2}^{2}$. If $m_{2}=2 m_{1}$, then $\frac{1}{2} m_{1} v_{1}^{2}=\frac{1}{2} \cdot 2 m_{1} v_{2}^{2}$, so $v_{1}=\sqrt{2} v_{2}$. They can both do the same amount of work, however, since their kinetic energies are the same. (See the work-energy principle.)
14. The net work done on a particle and the change in the kinetic energy are independent of the choice of reference frames only if the reference frames are at rest with respect to each other. The workenergy principle is also independent of the choice of reference frames if the frames are at rest with respect to each other.

If the reference frames are in relative motion, the net work done on a particle, the kinetic energy, and the change in the kinetic energy all will be different in different frames. The work-energy theorem will still be true.
15. The speed at point $C$ will be less than twice the speed at point $B$. The force is constant and the displacements are the same, so the same work is done on the block from A to B as from B to C. Since there is no friction, the same work results in the same change in kinetic energy. But kinetic energy depends on the square of the speed, so the speed at point C will be greater than the speed at point $B$ by a factor of $\sqrt{2}$, not a factor of 2 .

## Solutions to Problems

1. The force and the displacement are both downwards, so the angle between them is $0^{\circ}$. Use Eq. 7-1.

$$
W_{\mathrm{G}}=m g d \cos \theta=(280 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(2.80 \mathrm{~m}) \cos 0^{\circ}=7.7 \times 10^{3} \mathrm{~J}
$$

2. The rock will rise until gravity does -80.0 J of work on the rock. The displacement is upwards, but the force is downwards, so the angle between them is $180^{\circ}$. Use Eq. 7-1.

$$
W_{\mathrm{G}}=m g d \cos \theta \rightarrow d=\frac{W_{\mathrm{G}}}{\mathrm{mg} \cos \theta}=\frac{-80.0 \mathrm{~J}}{(1.85 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(-1)}=4.41 \mathrm{~m}
$$

3. The minimum force required to lift the firefighter is equal to his weight. The force and the displacement are both upwards, so the angle between them is $0^{\circ}$. Use Eq. 7-1.

$$
W_{\text {climb }}=F_{\text {climb }} d \cos \theta=m g d \cos \theta=(75.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(20.0 \mathrm{~m}) \cos 0^{\circ}=1.47 \times 10^{4} \mathrm{~J}
$$

4. The maximum amount of work would be the work done by gravity. Both the force and the displacement are downwards, so the angle between them is $0^{\circ}$. Use Eq. 7-1.

$$
W_{\mathrm{G}}=m g d \cos \theta=(2.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.50 \mathrm{~m}) \cos 0^{\circ}=9.8 \mathrm{~J}
$$

This is a small amount of energy. If the person adds a larger force to the hammer during the fall, then the hammer will have a larger amount of energy to give to the nail.
5. The distance over which the force acts is the area to be mowed divided by the width of the mower. The force is parallel to the displacement, so the angle between them is $0^{\circ}$. Use Eq. 7-1.

$$
W=F d \cos \theta=F \frac{A}{w} \cos \theta=(15 \mathrm{~N}) \frac{200 \mathrm{~m}^{2}}{0.50 \mathrm{~m}}=6000 \mathrm{~J}
$$

6. Consider the diagram shown. If we assume that the man pushes straight down on the end of the lever, then the work done by the man (the "input" work) is given by $W_{\mathrm{I}}=F_{\mathrm{I}} h_{\mathrm{I}}$. The object moves a shorter distance, as seen from the diagram, and so $W_{\mathrm{o}}=F_{\mathrm{o}} h_{\mathrm{o}}$. Equate the two amounts of work.

$$
W_{\mathrm{O}}=W_{\mathrm{I}} \rightarrow F_{\mathrm{o}} h_{\mathrm{O}}=F_{\mathrm{I}} h_{\mathrm{I}} \rightarrow \frac{F_{\mathrm{o}}}{F_{\mathrm{I}}}=\frac{h_{\mathrm{I}}}{h_{\mathrm{O}}}
$$

But by similar triangles, we see that $\frac{h_{\mathrm{I}}}{h_{\mathrm{O}}}=\frac{\ell_{\mathrm{I}}}{\ell_{\mathrm{O}}}$, and so $\frac{F_{\mathrm{O}}}{F_{\mathrm{I}}}=\frac{\ell_{\mathrm{I}}}{\ell_{\mathrm{O}}}$.

7. Draw a free-body diagram of the car on the incline. The minimum work will occur when the car is moved at a constant velocity. Write Newton's second law in the $x$ direction, noting that the car is unaccelerated. Only the forces parallel to the plane do work.

$$
\sum F_{x}=F_{\mathrm{P}}-m g \sin \theta=0 \rightarrow F_{\mathrm{P}}=m g \sin \theta
$$

The work done by $\overrightarrow{\mathbf{F}}_{\mathrm{p}}$ in moving the car a distance $d$ along the plane
 (parallel to $\overrightarrow{\mathbf{F}}_{\mathrm{p}}$ ) is given by Eq. 7-1.

$$
W_{\mathrm{P}}=F_{\mathrm{P}} d \cos 0^{\circ}=m g d \sin \theta=(950 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(310 \mathrm{~m}) \sin 9.0^{\circ}=4.5 \times 10^{5} \mathrm{~J}
$$

8. The first book is already in position, so no work is required to position it. The second book must be moved upwards by a distance $d$, by a force equal to its weight, $m g$. The force and the displacement are in the same direction, so the work is $m g d$. The third book will need to be moved a distance of $2 d$ by the same size force, so the work is $2 m g d$. This continues through all seven books, with each needing to be raised by an additional amount of $d$ by a force of $m g$. The total work done is

$$
\begin{aligned}
W & =m g d+2 m g d+3 m g d+4 m g d+5 m g d+6 m g d+7 m g d \\
& =28 m g d=28(1.8 \mathrm{~kg})\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)(0.040 \mathrm{~m})=2.0 \times 10^{1} \mathrm{~J}
\end{aligned}
$$

9. Since the acceleration of the box is constant, use Eq. 2-12b to find the distance moved. Assume that the box starts from rest.

$$
d=x-x_{0}=v_{0} t+\frac{1}{2} a t^{2}=0+\frac{1}{2}\left(2.0 \mathrm{~m} / \mathrm{s}^{2}\right)(7.0 \mathrm{~s})^{2}=49 \mathrm{~m}
$$

Then the work done in moving the crate is found using Eq. 7-1.

$$
W=F d \cos 0^{\circ}=m a d=(6.0 \mathrm{~kg})\left(2.0 \mathrm{~m} / \mathrm{s}^{2}\right)(49 \mathrm{~m})=590 \mathrm{~J}
$$

10. (a) Write Newton's second law for the vertical direction, with up as positive.

$$
\sum F_{y}=F_{\mathrm{L}}-M g=M a=M(0.10 g) \rightarrow F_{\mathrm{L}}=1.10 M g
$$

(b) The work done by the lifting force in lifting the helicopter a vertical distance $h$ is given by Eq. 7-1. The lifting force and the displacement are in the same direction.


$$
W_{\mathrm{L}}=F_{\mathrm{L}} h \cos 0^{\circ}=1.10 M g h
$$

11. The piano is moving with a constant velocity down the plane. $\overrightarrow{\mathbf{F}}_{\mathrm{p}}$ is the force of the man pushing on the piano.
(a) Write Newton's second law on each direction for the piano, with an acceleration of 0 .

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{N}}-m g \cos \theta=0 \rightarrow F_{\mathrm{N}}=m g \cos \theta \\
& \sum F_{x}=m g \sin \theta-F_{\mathrm{P}}=0 \rightarrow \\
& F_{\mathrm{P}}=m g \sin \theta=m g \sin \theta \\
& \quad=(380 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(\sin 27^{\circ}\right)=1691 \mathrm{~N} \approx 1700 \mathrm{~N}
\end{aligned}
$$


(b) The work done by the man is the work done by $\overrightarrow{\mathbf{F}}_{\mathrm{p}}$. The angle between $\overrightarrow{\mathbf{F}}_{\mathrm{p}}$ and the direction of motion is $180^{\circ}$. Use Eq. 7-1.

$$
W_{\mathrm{P}}=F_{\mathrm{P}} d \cos 180^{\circ}=-(1691 \mathrm{~N})(3.9 \mathrm{~m})=-6595 \mathrm{~J} \approx-6600 \mathrm{~J} .
$$

(c) The angle between the force of gravity and the direction of motion is $63^{\circ}$. Calculate the work done by gravity.

$$
\begin{aligned}
W_{G} & =F_{G} d \cos 63^{\circ}=m g d \cos 63^{\circ}=(380 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(3.9 \mathrm{~m}) \cos 63^{\circ} \\
& =6594 \mathrm{~N} \approx 6600 \mathrm{~J}
\end{aligned}
$$

(d) Since the piano is not accelerating, the net force on the piano is 0 , and so the net work done on the piano is also 0 . This can also be seen by adding the two work amounts calculated.

$$
W_{\text {net }}=W_{\mathrm{P}}+W_{\mathrm{G}}=-6.6 \times 10^{3} \mathrm{~J}+6.6 \times 10^{3} \mathrm{~J}=0 \mathrm{~J}
$$

12. (a) The motor must exert a force equal and opposite to the force of gravity on the gondola and passengers in order to lift it. The force is in the same direction as the displacement. Use Eq. 7-1 to calculate the work.

$$
W_{\text {motor }}=F_{\text {motor }} d \cos 0^{\circ}=m g d=(2250 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(3345 \mathrm{~m}-2150 \mathrm{~m})=2.63 \times 10^{7} \mathrm{~J}
$$

(b) Gravity would do the exact opposite amount of work as the motor, because the force and displacement are of the same magnitude, but the angle between the gravity force and the displacement is $180^{\circ}$.

$$
W_{\mathrm{G}}=F_{\mathrm{G}} d \cos 180^{\circ}=-m g d=-(2250 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(3345 \mathrm{~m}-2150 \mathrm{~m})=-2.63 \times 10^{7} \mathrm{~J}
$$

(c) If the motor is generating $10 \%$ more work, than it must be able to exert a force that is $10 \%$ larger than the force of gravity. The net force then would be as follows, with up the positive direction.

$$
F_{\mathrm{net}}=F_{\text {motor }}-F_{\mathrm{G}}=1.1 m g-m g=0.1 m g=m a \rightarrow a=0.1 g=0.98 \mathrm{~m} / \mathrm{s}^{2}
$$

13. (a) The gases exert a force on the jet in the same direction as the displacement of the jet. From the graph we see the displacement of the jet during launch is 85 m . Use Eq. 7-1 to find the work.

$$
W_{\mathrm{gas}}=F_{\mathrm{gas}} d \cos 0^{\circ}=\left(130 \times 10^{3} \mathrm{~N}\right)(85 \mathrm{~m})=1.1 \times 10^{7} \mathrm{~J}
$$

(b) The work done by catapult is the area underneath the graph in Figure 7-22. That area is a trapezoid.

$$
W_{\text {cataput }}=\frac{1}{2}\left(1100 \times 10^{3} \mathrm{~N}+65 \times 10^{3} \mathrm{~N}\right)(85 \mathrm{~m})=5.0 \times 10^{7} \mathrm{~J}
$$

14. (a) See the free-body diagram for the crate as it is being pulled. Since the crate is not accelerating horizontally, $F_{\mathrm{p}}=F_{\mathrm{fr}}=230 \mathrm{~N}$. The work done to move it across the floor is the work done by the pulling force. The angle between the pulling force and the direction of motion is $0^{\circ}$. Use Eq. 7-1.

$$
W_{\mathrm{P}}=F_{\mathrm{P}} d \cos 0^{\circ}=(230 \mathrm{~N})(4.0 \mathrm{~m})(1)=920 \mathrm{~J}
$$


(b) See the free-body diagram for the crate as it is being lifted. Since the crate is not accelerating vertically, the pulling force is the same magnitude as the weight. The angle between the pulling force and the direction of motion is $0^{\circ}$.

$$
W_{\mathrm{P}}=F_{\mathrm{P}} d \cos 0^{\circ}=m g d=(2200 \mathrm{~N})(4.0 \mathrm{~m})=8800 \mathrm{~J}
$$


15. Consider a free-body diagram for the grocery cart being pushed up the ramp. If the cart is not accelerating, then the net force is 0 in all directions. This can be used to find the size of the pushing force. The angles are $\phi=17^{\circ}$ and $\theta=12^{\circ}$. The displacement is in the $x$-direction. The work done by the normal force is 0 since the normal force is perpendicular to the displacement. The angle between the force of gravity and the displacement is $90^{\circ}+\theta=102^{\circ}$. The angle between the normal force and the displacement is $90^{\circ}$. The angle between the
 pushing force and the displacement is total work done is $\phi+\theta=29^{\circ}$.

$$
\begin{aligned}
& \sum F_{x}=F_{P} \cos (\phi+\theta)-m g \sin \theta=0 \rightarrow F_{P}=\frac{m g \sin \theta}{\cos (\phi+\theta)} \\
& W_{m g}=m g d \cos 112^{\circ}=(16 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(15 \mathrm{~m}) \cos 102^{\circ}=-490 \mathrm{~J} \\
& W_{\text {normal }}=F_{\mathrm{N}} d \cos 90^{\circ}=0 \\
& W_{P}=F_{\mathrm{p}} d \cos 29^{\circ}=\left(\frac{m g \sin 12^{\circ}}{\cos 29^{\circ}}\right) d \cos 29^{\circ}=m g d \sin 12^{\circ} \\
& \quad=(16 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(15 \mathrm{~m}) \sin 12^{\circ}=490 \mathrm{~J}
\end{aligned}
$$

16. Use Eq. 7.4 to calculate the dot product.

$$
\begin{aligned}
\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{B}} & =A_{x} B_{x}+A_{y} B_{y}+A_{z} B_{z}=\left(2.0 x^{2}\right)(11.0)+(-4.0 x)(2.5 x)+(5.0)(0)=22 x^{2}-10 x^{2} \\
& =12 x^{2}
\end{aligned}
$$

17. Use Eq. 7.4 to calculate the dot product. Note that $\hat{\mathbf{i}}=1 \hat{\mathbf{i}}+0 \hat{\mathbf{j}}+0 \hat{\mathbf{k}}, \hat{\mathbf{j}}=0 \hat{\mathbf{i}}+1 \hat{\mathbf{j}}+0 \hat{\mathbf{k}}$, and $\hat{\mathbf{k}}=0 \hat{\mathbf{i}}+0 \hat{\mathbf{j}}+1 \hat{\mathbf{k}}$.

$$
\begin{array}{ll}
\hat{\mathbf{i}} \cdot \overrightarrow{\mathbf{V}}=(1) V_{x}+(0) V_{y}+(0) V_{z}=V_{x} & \hat{\mathbf{j}} \cdot \overrightarrow{\mathbf{V}}=(0) V_{x}+(1) V_{y}+(0) V_{z}=V_{y} \\
\hat{\mathbf{k}} \cdot \overrightarrow{\mathbf{V}}=(0) V_{x}+(0) V_{y}+(1) V_{z}=V_{z} &
\end{array}
$$

18. Use Eq. 7.4 and Eq. 7.2 to calculate the dot product, and then solve for the angle.

$$
\begin{aligned}
& \overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{B}}=A_{x} B_{x}+A_{y} B_{y}+A_{z} B_{z}=(6.8)(8.2)+(-3.4)(2.3)+(-6.2)(-7.0)=91.34 \\
& A=\sqrt{\left(6.8^{2}\right)+(-3.4)^{2}+(-6.2)^{2}}=9.81 \quad B=\sqrt{\left(8.2^{2}\right)+(2.3)^{2}+(-7.0)^{2}}=11.0 \\
& \overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{B}}=A B \cos \theta \rightarrow \theta=\cos ^{-1} \frac{\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{B}}}{A B}=\cos ^{-1} \frac{91.34}{(9.81)(11.0)}=32^{\circ}
\end{aligned}
$$

19. We utilize the fact that if $\overrightarrow{\mathbf{B}}=B_{x} \hat{\mathbf{i}}+B_{y} \hat{\mathbf{j}}+B_{z} \hat{\mathbf{k}}$, then $-\overrightarrow{\mathbf{B}}=\left(-B_{x}\right) \hat{\mathbf{i}}+\left(-B_{y}\right) \hat{\mathbf{j}}+\left(-B_{z}\right) \hat{\mathbf{k}}$.

$$
\begin{aligned}
\overrightarrow{\mathbf{A}} \cdot(-\overrightarrow{\mathbf{B}}) & =A_{x}\left(-B_{x}\right)+A_{y}\left(-B_{y}\right)+A_{z}\left(-B_{z}\right) \\
& =\left(-A_{x}\right)\left(B_{x}\right)+\left(-A_{y}\right)\left(B_{y}\right)+\left(-A_{z}\right)\left(B_{z}\right)=-\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{B}}
\end{aligned}
$$

20. See the diagram to visualize the geometric relationship between the two vectors. The angle between the two vectors is $138^{\circ}$.

$$
\overrightarrow{\mathbf{V}}_{1} \cdot \overrightarrow{\mathbf{V}}_{2}=V_{1} V_{2} \cos \theta=(75)(58) \cos 138^{\circ}=-3200
$$

21. If $\overrightarrow{\mathbf{A}}$ is perpendicular to $\overrightarrow{\mathbf{B}}$, then $\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{B}}=0$. Use this to find $\overrightarrow{\mathbf{B}}$.


$$
\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{B}}=A_{x} B_{x}+A_{y} B_{y}=(3.0) B_{x}+(1.5) B_{y}=0 \rightarrow B_{y}=-2.0 B_{x}
$$

Any vector $\overrightarrow{\mathbf{B}}$ that satisfies $B_{y}=-2.0 B_{x}$ will be perpendicular to $\overrightarrow{\mathbf{A}}$. For example, $\overrightarrow{\mathbf{B}}=1.5 \hat{\mathbf{i}}-3.0 \hat{\mathbf{j}}$.
22. Both vectors are in the first quadrant, so to find the angle between them, we can simply subtract the angles of each of them.

$$
\begin{aligned}
& \overrightarrow{\mathbf{F}}=(2.0 \hat{\mathbf{i}}+4.0 \hat{\mathbf{j}}) \mathrm{N} \rightarrow F=\sqrt{(2.0 \mathrm{~N})^{2}+(4.0 \mathrm{~N})^{2}}=(\sqrt{20}) \mathrm{N} ; \phi_{F}=\tan ^{-1} \frac{4.0}{2.0}=\tan ^{-1} 2.0 \\
& \overrightarrow{\mathbf{d}}=(1.0 \hat{\mathbf{i}}+5.0 \hat{\mathbf{j}}) \mathrm{m} \rightarrow d=\sqrt{(1.0 \mathrm{~m})^{2}+(5.0 \mathrm{~m})^{2}}=(\sqrt{26}) \mathrm{m} ; \phi_{d}=\tan ^{-1} \frac{5.0}{1.0}=\tan ^{-1} 5.0
\end{aligned}
$$

(a) $W=F d \cos \theta=[(\sqrt{20}) \mathrm{N}][(\sqrt{26}) \mathrm{m}] \cos \left[\tan ^{-1} 5.0-\tan ^{-1} 2.0\right]=22 \mathrm{~J}$
(b) $W=F_{x} d_{x}+F_{y} d_{y}=(2.0 \mathrm{~N})(1.0 \mathrm{~m})+(4.0 \mathrm{~N})(5.0 \mathrm{~m})=22 \mathrm{~J}$
23. (a) $\overrightarrow{\mathbf{A}} \cdot(\overrightarrow{\mathbf{B}}+\overrightarrow{\mathbf{C}})=(9.0 \hat{\mathbf{i}}-8.5 \hat{\mathbf{j}}) \cdot[(-8.0 \hat{\mathbf{i}}+7.1 \hat{\mathbf{j}}+4.2 \hat{\mathbf{k}})+(6.8 \hat{\mathbf{i}}-9.2 \hat{\mathbf{j}})]$

$$
=(9.0 \hat{\mathbf{i}}-8.5 \hat{\mathbf{j}}) \cdot(-1.2 \hat{\mathbf{i}}-2.1 \hat{\mathbf{j}}+4.2 \hat{\mathbf{k}})=(9.0)(-1.2)+(-8.5)(-2.1)+(0)(4.2)=7.05 \approx 7.1
$$

(b) $(\overrightarrow{\mathbf{A}}+\overrightarrow{\mathbf{C}}) \cdot \overrightarrow{\mathbf{B}}=[(9.0 \hat{\mathbf{i}}-8.5 \hat{\mathbf{j}})+(6.8 \hat{\mathbf{i}}-9.2 \hat{\mathbf{j}})] \cdot(-8.0 \hat{\mathbf{i}}+7.1 \hat{\mathbf{j}}+4.2 \hat{\mathbf{k}})$

$$
\begin{aligned}
& =(15.8 \hat{\mathbf{i}}-17.7 \hat{\mathbf{j}}) \cdot(-8.0 \hat{\mathbf{i}}+7.1 \hat{\mathbf{j}}+4.2 \hat{\mathbf{k}})=(15.8)(-8.0)+(-17.7)(7.1)+(0)(4.2) \\
& =-252 \approx-250 \\
(\overrightarrow{\mathbf{B}}+\overrightarrow{\mathbf{A}}) \cdot \overrightarrow{\mathbf{C}} & =[(-8.0 \hat{\mathbf{i}}+7.1 \hat{\mathbf{j}}+4.2 \hat{\mathbf{k}})+(9.0 \hat{\mathbf{i}}-8.5 \hat{\mathbf{j}})] \cdot(6.8 \hat{\mathbf{i}}-9.2 \hat{\mathbf{j}}) \\
& =(1.0 \hat{\mathbf{i}}-1.4 \hat{\mathbf{j}}+4.2 \hat{\mathbf{k}}) \cdot(6.8 \hat{\mathbf{i}}-9.2 \hat{\mathbf{j}})=(1.0)(6.8)+(-1.4)(-9.2)+(4.2)(0) \\
& =19.68 \approx 20
\end{aligned}
$$

(c)
24. We assume that the dot product of two vectors is given by Eq. 7-2. Note that for two unit vectors, this gives the following.

$$
\hat{\mathbf{i}} \cdot \hat{\mathbf{i}}=(1)(1) \cos 0^{\circ}=1=\hat{\mathbf{j}} \cdot \hat{\mathbf{j}}=\hat{\mathbf{k}} \cdot \hat{\mathbf{k}} \text { and } \hat{\mathbf{i}} \cdot \hat{\mathbf{j}}=(1)(1) \cos 90^{\circ}=0=\hat{\mathbf{i}} \cdot \hat{\mathbf{k}}=\hat{\mathbf{j}} \cdot \hat{\mathbf{i}}=\hat{\mathbf{j}} \cdot \hat{\mathbf{k}}=\hat{\mathbf{k}} \cdot \hat{\mathbf{i}}=\hat{\mathbf{k}} \cdot \hat{\mathbf{j}}
$$

Apply these results to $\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{B}}$.

$$
\begin{aligned}
\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{B}}= & \left(A_{x} \hat{\mathbf{i}}+A_{y} \hat{\mathbf{j}}+A_{z} \hat{\mathbf{k}}\right) \cdot\left(B_{x} \hat{\mathbf{i}}+B_{y} \hat{\mathbf{j}}+B_{z} \hat{\mathbf{k}}\right) \\
= & A_{x} B_{x} \hat{\mathbf{i}} \cdot \hat{\mathbf{i}}+A_{x} B_{y} \hat{\mathbf{i}} \cdot \hat{\mathbf{j}}+A_{x} B_{z} \hat{\mathbf{i}} \cdot \hat{\mathbf{k}}+A_{y} B_{x} \hat{\mathbf{j}} \hat{\mathbf{i}}+A_{y} B_{y} \hat{\mathbf{j}} \hat{\mathbf{j}}+A_{y} B_{z} \hat{\mathbf{j}} \cdot \hat{\mathbf{k}}+A_{z} B_{x} \hat{\mathbf{k}} \cdot \hat{\mathbf{i}}+A_{z} B_{y} \hat{\mathbf{k}} \cdot \hat{\mathbf{j}}+A_{z} B_{z} \hat{\mathbf{k}} \cdot \hat{\mathbf{k}} \\
= & A_{x} B_{x}(1)+A_{x} B_{y}(0)+A_{x} B_{z}(0)+A_{y} B_{x}(0)+A_{y} B_{y}(1) \\
& +A_{y} B_{z}(0)+A_{z} B_{x}(0)+A_{z} B_{y}(0)+A_{z} B_{z}(1) \\
= & A_{x} B_{x}+A_{y} B_{y}+A_{z} B_{z}
\end{aligned}
$$

25. If $\overrightarrow{\mathbf{C}}$ is perpendicular to $\overrightarrow{\mathbf{B}}$, then $\overrightarrow{\mathbf{C}} \cdot \overrightarrow{\mathbf{B}}=0$. Use this along with the value of $\overrightarrow{\mathbf{C}} \cdot \overrightarrow{\mathbf{A}}$ to find $\overrightarrow{\mathbf{C}}$. We also know that $\overrightarrow{\mathbf{C}}$ has no $z$-component.

$$
\begin{aligned}
& \overrightarrow{\mathbf{C}}=C_{x} \hat{\mathbf{i}}+C_{y} \hat{\mathbf{j}} ; \overrightarrow{\mathbf{C}} \cdot \overrightarrow{\mathbf{B}}=C_{x} B_{x}+C_{y} B_{y}=0 ; \overrightarrow{\mathbf{C}} \cdot \overrightarrow{\mathbf{A}}=C_{x} A_{x}+C_{y} A_{y}=20.0 \rightarrow \\
& 9.6 C_{x}+6.7 C_{y}=0 ;-4.8 C_{x}+6.8 C_{y}=20.0
\end{aligned}
$$

This set of two equations in two unknowns can be solved for the components of $\overrightarrow{\mathbf{C}}$.

$$
\begin{aligned}
& 9.6 C_{x}+6.7 C_{y}=0 ;-4.8 C_{x}+6.8 C_{y}=20.0 \rightarrow C_{x}=-1.4, C_{y}=2.0 \rightarrow \\
& \overrightarrow{\mathbf{C}}=-1.4 \hat{\mathbf{i}}+2.0 \hat{\mathbf{j}}
\end{aligned}
$$

26. We are given that the magnitudes of the two vectors are the same, so $A_{x}^{2}+A_{y}^{2}+A_{z}^{2}=B_{x}^{2}+B_{y}^{2}+B_{z}^{2}$. If the sum and difference vectors are perpendicular, their dot product must be zero.

$$
\begin{aligned}
& \overrightarrow{\mathbf{A}}+\overrightarrow{\mathbf{B}}=\left(A_{x}+B_{x}\right) \hat{\mathbf{i}}+\left(A_{y}+B_{y}\right) \hat{\mathbf{j}}+\left(A_{z}+B_{z}\right) \hat{\mathbf{k}} \\
& \overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{B}}=\left(A_{x}-B_{x}\right) \hat{\mathbf{i}}+\left(A_{y}-B_{y}\right) \hat{\mathbf{j}}+\left(A_{z}-B_{z}\right) \hat{\mathbf{k}} \\
& (\overrightarrow{\mathbf{A}}+\overrightarrow{\mathbf{B}}) \cdot(\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{B}})=\left(A_{x}+B_{x}\right)\left(A_{x}-B_{x}\right)+\left(A_{y}+B_{y}\right)\left(A_{y}-B_{y}\right)+\left(A_{z}+B_{z}\right)\left(A_{z}-B_{z}\right) \\
& =A_{x}^{2}-B_{x}^{2}+A_{y}^{2}-B_{y}^{2}+A_{z}^{2}-B_{z}^{2}=\left(A_{x}^{2}+A_{y}^{2}+A_{z}^{2}\right)-\left(B_{x}^{2}+B_{y}^{2}+B_{z}^{2}\right)=0
\end{aligned}
$$

27. Note that by Eq. 7-2, the dot product of a vector $\overrightarrow{\mathbf{A}}$ with a unit vector $\overrightarrow{\mathbf{B}}$ would give the magnitude of $\overrightarrow{\mathbf{A}}$ times the cosine of the angle between the unit vector and $\overrightarrow{\mathbf{A}}$. Thus if the unit vector lies along one of the coordinate axes, we can find the angle between the vector and the coordinate axis. We also use Eq. 7-4 to give a second evaluation of the dot product.

$$
\overrightarrow{\mathbf{V}} \cdot \hat{\mathbf{i}}=V \cos \theta_{x}=V_{x} \rightarrow
$$

$$
\begin{aligned}
& \theta_{x}=\cos ^{-1} \frac{V_{x}}{V}=\cos ^{-1} \frac{V_{x}}{\sqrt{V_{x}^{2}+V_{y}^{2}+V_{z}^{2}}}=\cos ^{-1} \frac{20.0}{\sqrt{(20.0)^{2}+(22.0)^{2}+(-14.0)^{2}}}=52.5^{\circ} \\
& \theta_{y}=\cos ^{-1} \frac{V_{y}}{V}=\cos ^{-1} \frac{22.0}{\sqrt{(20.0)^{2}+(22.0)^{2}+(-14.0)^{2}}}=48.0^{\circ} \\
& \theta_{z}=\cos ^{-1} \frac{V_{z}}{V}=\cos ^{-1} \frac{-14.0}{\sqrt{(20.0)^{2}+(22.0)^{2}+(-14.0)^{2}}}=115^{\circ}
\end{aligned}
$$

28. For the diagram shown, $\overrightarrow{\mathbf{B}}+\overrightarrow{\mathbf{C}}=\overrightarrow{\mathbf{A}}$, or $\overrightarrow{\mathbf{C}}=\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{B}}$. Let the magnitude of each vector be represented by the corresponding lowercase letter, so $|\overrightarrow{\mathbf{C}}|=c$, for example. The angle between $\overrightarrow{\mathbf{A}}$ and $\overrightarrow{\mathbf{B}}$ is $\theta$. Take the dot product $\overrightarrow{\mathbf{C}} \cdot \overrightarrow{\mathbf{C}}$.

$$
\overrightarrow{\mathbf{C}} \cdot \overrightarrow{\mathbf{C}}=(\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{B}}) \cdot(\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{B}})=\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{A}}+\overrightarrow{\mathbf{B}} \cdot \overrightarrow{\mathbf{B}}-2 \overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{B}} \quad \rightarrow c^{2}=a^{2}+b^{2}-2 a b \cos \theta
$$


29. The scalar product is positive, so the angle between $\overrightarrow{\mathbf{A}}$ and $\overrightarrow{\mathbf{B}}$ must be acute. But the direction of the angle from $\overrightarrow{\mathbf{A}}$ to $\overrightarrow{\mathbf{B}}$ could be either counterclockwise or clockwise.

$$
\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{B}}=A B \cos \theta=(12.0)(24.0) \cos \theta=20.0 \rightarrow \theta=\cos ^{-1} \frac{20.0}{(12.0)(24.0)}=86.0^{\circ}
$$

So this angle could be either added or subtracted to the angle of $\overrightarrow{\mathbf{A}}$ to find the angle of $\overrightarrow{\mathbf{B}}$.

$$
\theta_{B}=\theta_{A} \pm \theta=27.4^{\circ} \pm 86.0^{\circ}=113.4^{\circ} \text { or }-58.6^{\circ}\left(301.4^{\circ}\right)
$$

30. We can represent the vectors as $\overrightarrow{\mathbf{A}}=A_{x} \hat{\mathbf{i}}+A_{y} \hat{\mathbf{j}}=A \cos \alpha \hat{\mathbf{i}}+A \sin \alpha \hat{\mathbf{j}}$ and $\overrightarrow{\mathbf{B}}=B_{x} \hat{\mathbf{i}}+B_{y} \hat{\mathbf{j}}$ $=B \cos \beta \hat{\mathbf{i}}+B \sin \beta \hat{\mathbf{j}}$. The angle between the two vectors is $\alpha-\beta$. Use Eqs. 7-2 and 7-4 to express the dot product.

$$
\begin{aligned}
& \overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{B}}=A B \cos (\alpha-\beta)=A_{x} B_{x}+A_{y} B_{y}=A \cos \alpha B \cos \beta+A \sin \alpha B \sin \beta \rightarrow \\
& A B \cos (\alpha-\beta)=A B \cos \alpha \cos \beta+A B \sin \alpha \sin \beta \rightarrow \cos (\alpha-\beta)=\cos \alpha \cos \beta+\sin \alpha \sin \beta
\end{aligned}
$$

31. (a) Use the two expressions for dot product, Eqs. 7-2 and 7-4, to find the angle between the two vectors.

$$
\begin{aligned}
& \overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{B}}=A B \cos \theta=A_{x} B_{x}+A_{y} B_{y}+A_{z} B_{z} \rightarrow \\
& \theta
\end{aligned}=\cos ^{-1} \frac{A_{x} B_{x}+A_{y} B_{y}+A_{z} B_{z}}{A B}
$$

(b) The negative sign in the argument of the inverse cosine means that the angle between the two vectors is obtuse.
32. To be perpendicular to the given vector means that the dot product will be 0 . Let the unknown vector be given as $\hat{\mathbf{u}}=u_{x} \hat{\mathbf{i}}+u_{y} \hat{\mathbf{j}}$.

$$
\begin{aligned}
& \hat{\mathbf{u}} \cdot(3.0 \hat{\mathbf{i}}+4.0 \hat{\mathbf{j}})=3.0 u_{x}+4.0 u_{y} \rightarrow u_{y}=-0.75 u_{x} ; \text { unit length } \rightarrow u_{x}^{2}+u_{y}^{2}=1 \rightarrow \\
& u_{x}^{2}+u_{y}^{2}=u_{x}^{2}+\left(-0.75 u_{x}\right)^{2}=1.5625 u_{x}^{2}=1 \rightarrow u_{x}= \pm \frac{1}{\sqrt{1.5625}}= \pm 0.8, u_{y}=\mp 0.6
\end{aligned}
$$

So the two possible vectors are $\hat{\mathbf{u}}=0.8 \hat{\mathbf{i}}-0.6 \hat{\mathbf{j}}$ and $\hat{\mathbf{u}}=-0.8 \hat{\mathbf{i}}+0.6 \hat{\mathbf{j}}$.
Note that it is very easy to get a non-unit vector perpendicular to another vector in two dimensions, simply by interchanging the coordinates and negating one of them. So a non-unit vector perpendicular to $(3.0 \hat{\mathbf{i}}+4.0 \hat{\mathbf{j}})$ could be either $(4.0 \hat{\mathbf{i}}-3.0 \hat{\mathbf{j}})$ or $(-4.0 \hat{\mathbf{i}}+3.0 \hat{\mathbf{j}})$. Then divide each of those vectors by its magnitude (5.0) to get the possible unit vectors.
33. From Figure 7-6, we see a graphical interpretation of the scalar product as the magnitude of one vector times the projection of the other vector onto the first vector. So to show that $\overrightarrow{\mathbf{A}} \cdot(\overrightarrow{\mathbf{B}}+\overrightarrow{\mathbf{C}})=\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{B}}+\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{C}}$ is the same as showing that $A(\overrightarrow{\mathbf{B}}+\overrightarrow{\mathbf{C}})_{\|}=A(\overrightarrow{\mathbf{B}})_{\|}+A(\overrightarrow{\mathbf{C}})_{\|}$, where the subscript is implying the component of the vector that is parallel to vector $\overrightarrow{\mathbf{A}}$. From the diagram, we see that $(\overrightarrow{\mathbf{B}}+\overrightarrow{\mathbf{C}})_{\|}=(\overrightarrow{\mathbf{B}})_{\|}+(\overrightarrow{\mathbf{C}})_{\|}$. Multiplying this equation by the magnitude of vector $\overrightarrow{\mathbf{A}}$ gives $A(\overrightarrow{\mathbf{B}}+\overrightarrow{\mathbf{C}})_{\|}=A(\overrightarrow{\mathbf{B}})_{\|}+A(\overrightarrow{\mathbf{C}})_{\|}$. But from Figure 7-6, this is the same as

34. The downward force is 450 N , and the downward displacement would be a diameter of the pedal circle. Use Eq. 7-1.

$$
W=F d \cos \theta=(450 \mathrm{~N})(0.36 \mathrm{~m}) \cos 0^{\circ}=160 \mathrm{~J}
$$

35. The force exerted to stretch a spring is given by $F_{\text {stretch }}=k x$ (the opposite of the force exerted by the spring, which is given by $F=-k x$. A graph of $F_{\text {stretch }}$ vs. $x$ will be a straight line of slope $k$ through the origin. The stretch from $x_{1}$ to $x_{2}$, as shown on the graph, outlines a trapezoidal area. This area represents the work.

$$
\begin{aligned}
W & =\frac{1}{2}\left(k x_{1}+k x_{2}\right)\left(x_{2}-x_{1}\right)=\frac{1}{2} k\left(x_{1}+x_{2}\right)\left(x_{2}-x_{1}\right) \\
& =\frac{1}{2}(65 \mathrm{~N} / \mathrm{m})(0.095 \mathrm{~m})(0.035 \mathrm{~m})=0.11 \mathrm{~J}
\end{aligned}
$$


36. For a non-linear path, the work is found by considering the path to be an infinite number of infinitesimal (or differential) steps, each of which can be considered to be in a specific direction, namely, the direction tangential to the path. From the diagram, for each step we have $d W=\overrightarrow{\mathbf{F}} \cdot d \vec{\ell}=F d \ell \cos \theta$. But $d \ell \cos \theta=-d y$, the projection of the path in the direction of the force, and $F=m g$, the force of

gravity. Find the work done by gravity.

$$
W_{\mathrm{g}}=\int \overrightarrow{\mathbf{F}} \cdot d \overrightarrow{\boldsymbol{\ell}}=\int m g \cos \theta d \boldsymbol{\ell}=m g \int(-d y)=-m g h
$$

This argument could even be extended to going part way up the hill, and then part way back down, and following any kind of path. The work done by gravity will only depend on the height of the path.
37. See the graph of force vs. distance. The work done is the area under the graph. It can be found from the formula for a trapezoid.

$$
\begin{aligned}
W & =\frac{1}{2}(12.0 \mathrm{~m}+4.0 \mathrm{~m})(380 \mathrm{~N}) \\
& =3040 \mathrm{~J} \approx 3.0 \times 10^{3} \mathrm{~J}
\end{aligned}
$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH07.XLS," on tab "Problem 7.37."

38. The work required to stretch a spring from equilibrium is proportional to the length of stretch, squared. So if we stretch the spring to 3 times its original distance, a total of 9 times as much work is required for the total stretch. Thus it would take 45.0 J to stretch the spring to a total of 6.0 cm . Since 5.0 J of work was done to stretch the first $2.0 \mathrm{~cm}, 40.0 \mathrm{~J}$ of work is required to stretch it the additional 4.0 cm .

This could also be done by calculating the spring constant from the data for the 2.0 cm stretch, and then using that spring constant to find the work done in stretching the extra distance.
39. The $x$-axis is portioned into 7 segments, so each segment is $1 / 7$ of the full $20.0-\mathrm{m}$ width. The force on each segment can be approximated by the force at the middle of the segment. Thus we are performing a simple Riemann sum to find the area under the curve. The value of the mass does not come into the calculation.

$$
\begin{aligned}
& W=\sum_{i=1}^{7} F_{i} \Delta x_{i}=\Delta x \sum_{i=1}^{7} F_{i}=\frac{1}{7}(20.0 \mathrm{~m})(180 \mathrm{~N}+200 \mathrm{~N}+175 \mathrm{~N}+125 \mathrm{~N}+110 \mathrm{~N}+100 \mathrm{~N}+95 \mathrm{~N}) \\
& =\frac{1}{7}(20.0 \mathrm{~m})(985 \mathrm{~N}) \approx 2800 \mathrm{~J}
\end{aligned}
$$

Another method is to treat the area as a trapezoid, with sides of 180 N and 100 N , and a base of 20.0 m . Then the work is $W=\frac{1}{2}(20.0 \mathrm{~m})(180 \mathrm{~N}+100 \mathrm{~N}) \approx 2800 \mathrm{~J}$.
40. The work done will be the area under the $F_{x}$ vs. $x$ graph.
(a) From $x=0.0$ to $x=10.0 \mathrm{~m}$, the shape under the graph is trapezoidal. The area is

$$
W_{a}=(400 \mathrm{~N}) \frac{1}{2}(10 \mathrm{~m}+4 \mathrm{~m})=2800 \mathrm{~J} .
$$

(b) From $x=10.0 \mathrm{~m}$ to $x=15.0 \mathrm{~m}$, the force is in the opposite direction from the direction of motion, and so the work will be negative. Again, since the shape is trapezoidal, we find

$$
W_{a}=(-200 \mathrm{~N}) \frac{1}{2}(5 \mathrm{~m}+2 \mathrm{~m})=-700 \mathrm{~J}
$$

Thus the total work from $x=0.0$ to $x=15.0 \mathrm{~m}$ is $2800 \mathrm{~J}-700 \mathrm{~J}=2100 \mathrm{~J}$.
41. Apply Eq. 7-1 to each segment of the motion.

$$
\begin{aligned}
W & =W_{1}+W_{2}+W_{3}=F_{1} d_{1} \cos \theta_{1}+F_{2} d_{2} \cos \theta_{2}+F_{3} d_{3} \cos \theta_{3} \\
& =(22 \mathrm{~N})(9.0 \mathrm{~m}) \cos 0^{\circ}+(38 \mathrm{~N})(5.0 \mathrm{~m}) \cos 12^{\circ}+(22 \mathrm{~N})(13.0 \mathrm{~m}) \cos 0^{\circ}=670 \mathrm{~J}
\end{aligned}
$$

42. Since the force only has an $x$-component, only the $x$-displacement is relevant. The object moves from $x=0$ to $x=d$.

$$
W=\int_{0}^{d} F_{x} d x=\int_{0}^{d} k x^{4} d x=\frac{1}{5} k d^{5}
$$

43. Since we are compressing the spring, the force and the displacement are in the same direction.

$$
W=\int_{0}^{X} F_{x} d x=\int_{0}^{X}\left(k x+a x^{3}+b x^{4}\right) d x=\frac{1}{2} k X^{2}+\frac{1}{4} a X^{4}+\frac{1}{5} b X^{5}
$$

44. Integrate the force over the distance the force acts to find the work. We assume the displacement is all in the $x$-direction.

$$
W=\int_{x_{i}}^{x_{f}} F(x) d x=\int_{0}^{0.20 \mathrm{~m}}\left(150 x-190 x^{2}\right) d x=\left(75 x^{2}-\frac{190}{3} x^{3}\right)_{0}^{0.20 \mathrm{~m}}=2.49 \mathrm{~J}
$$

45. Integrate the force over the distance the force acts to find the work.

$$
W=\int_{0}^{1.0 \mathrm{~m}} F_{1} d x=\int_{0}^{1.0 \mathrm{~m}} \frac{A}{\sqrt{x}} d x=\left.2 A \sqrt{x}\right|_{0} ^{1.0 \mathrm{~m}}=2\left(2.0 \mathrm{~N} \cdot \mathrm{~m}^{1 / 2}\right)(1.0 \mathrm{~m})^{1 / 2}=4.0 \mathrm{~J}
$$

Note that the work done is finite.
46. Because the object moves along a straight line, we know that the $x$-coordinate increases linearly from 0 to 10.0 m , and the $y$-coordinate increases linearly from 0 to 20.0 m . Use the relationship developed at the top of page 170 .

$$
\begin{aligned}
W & =\int_{x_{\mathrm{a}}}^{x_{\mathrm{b}}} F_{x} d x+\int_{y_{\mathrm{a}}}^{y_{\mathrm{b}}} F_{y} d y=\int_{0}^{10.0 \mathrm{~m}} 3.0 x d x+\int_{0}^{20.0 \mathrm{~m}} 4.0 y d y=\frac{1}{2}\left(3.0 x^{2}\right)_{0}^{10.0}+\frac{1}{2}\left(4.0 y^{2}\right)_{0}^{20.0}=150 \mathrm{~J}+800 \mathrm{~J} \\
& =950 \mathrm{~J}
\end{aligned}
$$

47. Since the force is of constant magnitude and always directed at $30^{\circ}$ to the displacement, we have a simple expression for the work done as the object moves.

$$
W=\int_{\text {start }}^{\text {finish }} \overrightarrow{\mathbf{F}} \cdot d \overrightarrow{\boldsymbol{\ell}}=\int_{\text {start }}^{\text {finish }} F \cos 30^{\circ} d \boldsymbol{\ell}=F \cos 30^{\circ} \int_{\text {start }}^{\text {finish }} d \boldsymbol{\ell}=F \cos 30^{\circ} \pi R=\frac{\sqrt{3} \pi F R}{2}
$$

48. The force on the object is given by Newton's law of universal gravitation, $F=G \frac{m m_{\mathrm{E}}}{r^{2}}$. The force is a function of distance, so to find the work, we must integrate. The directions are tricky. To use Eq. 7-7, we have $\overrightarrow{\mathbf{F}}=-G \frac{m m_{\mathrm{E}}}{r^{2}} \hat{\mathbf{r}}$ and $d \overrightarrow{\boldsymbol{\ell}}=d r \hat{\mathbf{r}}$. It is tempting to put a negative sign with the $d \overrightarrow{\boldsymbol{\ell}}$ relationship since the object moves inward, but since $r$ is measured outward away from the center of the Earth, we must not include that negative sign. Note that we move from a large radius to a small radius.

$$
\begin{aligned}
W & =\int \overrightarrow{\mathbf{F}} \cdot d \vec{\ell}=\int_{\text {far }}^{\text {near }}-G \frac{m m_{\mathrm{E}}}{r^{2}} \hat{\mathbf{r}} \cdot(d r \hat{\mathbf{r}})=-\int_{r_{\mathrm{E}}+3300 \mathrm{~km}}^{r_{\mathrm{E}}} G \frac{m m_{\mathrm{E}}}{r^{2}} d r=\left.G \frac{m m_{\mathrm{E}}}{r}\right|_{r_{\mathrm{E}}+3300 \mathrm{~km}} ^{r_{\mathrm{E}}} \\
& =G m m_{\mathrm{E}}\left(\frac{1}{r_{\mathrm{E}}}-\frac{1}{r_{\mathrm{E}}+3300 \mathrm{~km}}\right) \\
& =\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)(2800 \mathrm{~kg})\left(5.97 \times 10^{24} \mathrm{~kg}\right)\left(\frac{1}{6.38 \times 10^{6} \mathrm{~m}}-\frac{1}{(6.38+3.30) \times 10^{6} \mathrm{~m}}\right) \\
& =6.0 \times 10^{10} \mathrm{~J}
\end{aligned}
$$

49. Let $y$ represent the length of chain hanging over the table, and let $\lambda$ represent the weight per unit length of the chain. Then the force of gravity (weight) of the hanging chain is $F_{\mathrm{G}}=\lambda y$. As the next small length of chain $d y$ comes over the table edge, gravity does an infinitesimal amount of work on the hanging chain given by the force times the distance, $F_{\mathrm{G}} d y=\lambda y d y$. To find the total amount of work that gravity does on the chain, integrate that work expression, with the limits of integration representing the amount of chain hanging over the table.

$$
W=\int_{y_{\text {mitial }}}^{y_{\mathrm{G}} \mathrm{ma}} F_{\mathrm{G}} d y=\int_{1.0 \mathrm{~m}}^{3.0 \mathrm{~m}} \lambda y d y=\left.\frac{1}{2} \lambda y^{2}\right|_{1.0 \mathrm{~m}} ^{3.0 \mathrm{~m}}=\frac{1}{2}(18 \mathrm{~N} / \mathrm{m})\left(9.0 \mathrm{~m}^{2}-1.0 \mathrm{~m}^{2}\right)=72 \mathrm{~J}
$$

50. Find the velocity from the kinetic energy, using Eq. 7-10.

$$
K=\frac{1}{2} m v^{2} \rightarrow v=\sqrt{\frac{2 K}{m}}=\sqrt{\frac{2\left(6.21 \times 10^{-21} \mathrm{~J}\right)}{5.31 \times 10^{-26}}}=484 \mathrm{~m} / \mathrm{s}
$$

51. (a) Since $K=\frac{1}{2} m v^{2}$, then $v=\sqrt{2 K / m}$ and so $v \propto \sqrt{K}$. Thus if the kinetic energy is tripled, the speed will be multiplied by a factor of $\sqrt{3}$.
(b) Since $K=\frac{1}{2} m v^{2}$, then $K \propto v^{2}$. Thus if the speed is halved, the kinetic energy will be multiplied by a factor of $1 / 4$.
52. The work done on the electron is equal to the change in its kinetic energy.

$$
W=\Delta K=\frac{1}{2} m v_{2}^{2}-\frac{1}{2} m v_{1}^{2}=0-\frac{1}{2}\left(9.11 \times 10^{-31} \mathrm{~kg}\right)\left(1.40 \times 10^{6} \mathrm{~m} / \mathrm{s}\right)^{2}=-8.93 \times 10^{-19} \mathrm{~J}
$$

Note that the work is negative since the electron is slowing down.
53. The work done on the car is equal to the change in its kinetic energy.

$$
W=\Delta K=\frac{1}{2} m v_{2}^{2}-\frac{1}{2} m v_{1}^{2}=0-\frac{1}{2}(1300 \mathrm{~kg})\left[(95 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)\right]^{2}=-4.5 \times 10^{5} \mathrm{~J}
$$

Note that the work is negative since the car is slowing down.
54. We assume the train is moving $20 \mathrm{~m} / \mathrm{s}$ (which is about 45 miles per hour), and that the distance of "a few city blocks" is perhaps a half-mile, which is about 800 meters. First find the kinetic energy of the train, and then find out how much work the web must do to stop the train. Note that the web does negative work, since the force is in the OPPOSITE direction of the displacement.

$$
\begin{aligned}
& W_{\substack{\text { to stop } \\
\text { train }}}=\Delta K=\frac{1}{2} m v_{2}^{2}-\frac{1}{2} m v_{1}^{2}=0-\frac{1}{2}\left(10^{4} \mathrm{~kg}\right)(20 \mathrm{~m} / \mathrm{s})^{2}=-2 \times 10^{6} \mathrm{~J} \\
& W_{\mathrm{web}}=-\frac{1}{2} k x^{2}=-2 \times 10^{6} \mathrm{~J} \rightarrow k=\frac{2\left(2 \times 10^{6} \mathrm{~J}\right)}{\left(800 \mathrm{~m}^{2}\right)}=6 \mathrm{~N} / \mathrm{m}
\end{aligned}
$$

Note that this is not a very stiff "spring," but it does stretch a long distance.
55. The force of the ball on the glove will be the opposite of the force of the glove on the ball, by Newton's third law. Both objects have the same displacement, and so the work done on the glove is opposite the work done on the ball. The work done on the ball is equal to the change in the kinetic energy of the ball.

$$
W_{\text {on ball }}=\left(K_{2}-K_{1}\right)_{\text {ball }}=\frac{1}{2} m v_{2}^{2}-\frac{1}{2} m v_{1}^{2}=0-\frac{1}{2}(0.145 \mathrm{~kg})(32 \mathrm{~m} / \mathrm{s})^{2}=-74.24 \mathrm{~J}
$$

So $W_{\text {on glove }}=74.24 \mathrm{~J}$. But $W_{\text {on glove }}=F_{\text {on glove }} d \cos 0^{\circ}$, because the force on the glove is in the same direction as the motion of the glove.
$74.24 \mathrm{~J}=F_{\text {on glove }}(0.25 \mathrm{~m}) \rightarrow F_{\text {on glove }}=\frac{74.24 \mathrm{~J}}{0.25 \mathrm{~m}}=3.0 \times 10^{2} \mathrm{~N}$, in the direction of the original velocity of the ball.
56. The force exerted by the bow on the arrow is in the same direction as the displacement of the arrow. Thus $W=F d \cos 0^{\circ}=F d=(105 \mathrm{~N})(0.75 \mathrm{~m})=78.75 \mathrm{~J}$. But that work changes the kinetic energy of the arrow, by the work-energy theorem. Thus

$$
F d=W=K_{2}-K_{1}=\frac{1}{2} m v_{2}^{2}-\frac{1}{2} m v_{1}^{2} \rightarrow v_{2}=\sqrt{\frac{2 F d}{m}+v_{1}^{2}}=\sqrt{\frac{2(78.75 \mathrm{~J})}{0.085 \mathrm{~kg}}+0}=43 \mathrm{~m} / \mathrm{s}
$$

57. (a) The spring constant is found by the magnitudes of the initial force and displacement, and so $k=F / x$. As the spring compresses, it will do the same amount of work on the block as was done on the spring to stretch it. The work done is positive because the force of the spring is parallel to the displacement of the block. Use the work-energy theorem to determine the speed of the block.

$$
W_{\substack{\text { on block } \\ \text { during } \\ \text { compression }}}=\Delta K_{\text {block }}=W_{\substack{\text { on spring } \\ \text { during } \\ \text { stretching }}} \rightarrow \frac{1}{2} m v_{f}^{2}=\frac{1}{2} k x^{2}=\frac{1}{2} \frac{F}{x} x^{2} \quad \rightarrow \quad v_{f}=\sqrt{\frac{F x}{m}}
$$

(b) Now we must find how much work was done on the spring to stretch it from $x / 2$ to $x$. This will be the work done on the block as the spring pulls it back from $x$ to $x / 2$.

$$
\begin{aligned}
& W_{\substack{\text { on spring } \\
\text { during } \\
\text { stretching }}}=\int_{x / 2}^{x} F d x=\int_{x / 2}^{x} k x d x=\left.\frac{1}{2} k x^{2}\right|_{x / 2} ^{x}=\frac{1}{2} k x^{2}-\frac{1}{2} k(x / 2)^{2}=\frac{3}{8} k x^{2} \\
& \frac{1}{2} m v_{f}^{2}=\frac{3}{8} k x^{2} \rightarrow v_{f}=\sqrt{\frac{3 F x}{4 m}}
\end{aligned}
$$

58. The work needed to stop the car is equal to the change in the car's kinetic energy. That work comes from the force of friction on the car. Assume the maximum possible frictional force, which results in the minimum braking

distance. Thus $F_{\mathrm{fr}}=\mu_{s} F_{\mathrm{N}}$. The normal force is equal to the car's weight if it is on a level surface, and so $F_{\mathrm{fr}}=\mu_{s} m g$. In the diagram, the car is traveling to the right.

$$
W=\Delta K \rightarrow F_{\mathrm{fr}} d \cos 180^{\circ}=\frac{1}{2} m v_{2}^{2}-\frac{1}{2} m v_{1}^{2} \rightarrow-\mu_{s} m g d=-\frac{1}{2} m v_{1}^{2} \rightarrow d=\frac{v_{1}^{2}}{2 g \mu_{s}}
$$

Since $d \propto v_{1}^{2}$, if $v_{1}$ increases by $50 \%$, or is multiplied by 1.5 , then $d$ will be multiplied by a factor of $(1.5)^{2}$, or 2.25 .
59. The net work done on the car must be its change in kinetic energy. By applying Newton's third law, the negative work done on the car by the spring must be the opposite of the work done in compressing the spring.

$$
\begin{aligned}
& W=\Delta K=-W_{\text {sping }} \rightarrow \frac{1}{2} m v_{2}^{2}-\frac{1}{2} m v_{1}^{2}=-\frac{1}{2} k x^{2} \rightarrow \\
& k=m \frac{v_{1}^{2}}{x^{2}}=(1200 \mathrm{~kg}) \frac{\left[66 \mathrm{~km} / \mathrm{k}\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)\right]^{2}}{(2.2 \mathrm{~m})^{2}}=8.3 \times 10^{4} \mathrm{~N} / \mathrm{m}
\end{aligned}
$$

60. The first car mentioned will be called car 1. So we have these statements:

$$
K_{1}=\frac{1}{2} K_{2} \rightarrow \frac{1}{2} m_{1} v_{1}^{2}=\frac{1}{2}\left(\frac{1}{2} m_{2} v_{2}^{2}\right) ; K_{1, \text { fast }}=K_{2, \text { fast }} \rightarrow \frac{1}{2} m_{1}\left(v_{1}+7.0\right)^{2}=\frac{1}{2} m_{2}\left(v_{2}+7.0\right)^{2}
$$

Now use the mass information, that $m_{1}=2 m_{2}$.

$$
\begin{aligned}
& \frac{1}{2} 2 m_{2} v_{1}^{2}=\frac{1}{2}\left(\frac{1}{2} m_{2} v_{2}^{2}\right) ; \frac{1}{2} 2 m_{2}\left(v_{1}+7.0\right)^{2}=\frac{1}{2} m_{2}\left(v_{2}+7.0\right)^{2} \rightarrow \\
& 2 v_{1}=v_{2} ; 2\left(v_{1}+7.0\right)^{2}=\left(v_{2}+7.0\right)^{2} \rightarrow 2\left(v_{1}+7.0\right)^{2}=\left(2 v_{1}+7.0\right)^{2} \rightarrow \\
& \sqrt{2}\left(v_{1}+7.0\right)=\left(2 v_{1}+7.0\right) \rightarrow v_{1}=\frac{7.0}{\sqrt{2}}=4.9497 \mathrm{~m} / \mathrm{s} ; v_{2}=2 v_{1}=9.8994 \mathrm{~m} / \mathrm{s} \\
& v_{1}=4.9 \mathrm{~m} / \mathrm{s} ; v_{2}=9.9 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

61. The work done by the net force is the change in kinetic energy.

$$
\begin{aligned}
W & =\Delta K=\frac{1}{2} m v_{2}^{2}-\frac{1}{2} m v_{1}^{2} \\
& =\frac{1}{2}(4.5 \mathrm{~kg})\left[(15.0 \mathrm{~m} / \mathrm{s})^{2}+(30.0 \mathrm{~m} / \mathrm{s})^{2}\right]-\frac{1}{2}(4.5 \mathrm{~kg})\left[(10.0 \mathrm{~m} / \mathrm{s})^{2}+(20.0 \mathrm{~m} / \mathrm{s})^{2}\right]=1400 \mathrm{~J}
\end{aligned}
$$

62. (a) From the free-body diagram for the load being lifted, write Newton's second law for the vertical direction, with up being positive.

$$
\begin{aligned}
& \sum F=F_{\mathrm{T}}-m g=m a=0.150 \mathrm{mg} \rightarrow \\
& F_{\mathrm{T}}=1.150 \mathrm{mg}=1.150(265 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=2.99 \times 10^{3} \mathrm{~N}
\end{aligned}
$$

(b) The net work done on the load is found from the net force.


$$
\begin{aligned}
W_{\text {net }} & =F_{\text {net }} d \cos 0^{\circ}=(0.150 \mathrm{mg}) d=0.150(265 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(23.0 \mathrm{~m}) \\
& =8.96 \times 10^{3} \mathrm{~J}
\end{aligned}
$$

(c) The work done by the cable on the load is as follows.

$$
W_{\text {cable }}=F_{\mathrm{T}} d \cos 0^{\circ}=(1.150 \mathrm{mg}) d=1.15(265 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(23.0 \mathrm{~m})=6.87 \times 10^{4} \mathrm{~J}
$$

(d) The work done by gravity on the load is as follows.

$$
W_{\mathrm{G}}=m g d \cos 180^{\circ}=-m g d=-(265 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(23.0 \mathrm{~m})=-5.97 \times 10^{4} \mathrm{~J}
$$

(e) Use the work-energy theorem to find the final speed, with an initial speed of 0 .

$$
W_{\text {net }}=K_{2}-K_{1}=\frac{1}{2} m v_{2}^{2}-\frac{1}{2} m v_{1}^{2} \rightarrow v_{2}=\sqrt{\frac{2 W_{n e t}}{m}+v_{1}^{2}}=\sqrt{\frac{2\left(8.96 \times 10^{3} \mathrm{~J}\right)}{265 \mathrm{~kg}}+0}=8.22 \mathrm{~m} / \mathrm{s}
$$

63. (a) The angle between the pushing force and the displacement is $32^{\circ}$.

$$
W_{\mathrm{P}}=F_{\mathrm{P}} d \cos \theta=(150 \mathrm{~N})(5.0 \mathrm{~m}) \cos 32^{\circ}=636.0 \mathrm{~J} \approx 640 \mathrm{~J}
$$

(b) The angle between the force of gravity and the displacement is $122^{\circ}$.

$$
W_{\mathrm{G}}=F_{\mathrm{G}} d \cos \theta=m g d \cos \theta=(18 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(5.0 \mathrm{~m}) \cos 122^{\circ}=-467.4 \mathrm{~J} \approx-470 \mathrm{~J}
$$

(c) Because the normal force is perpendicular to the displacement, the work done by the normal force is 0 .
(d) The net work done is the change in kinetic energy.

$$
\begin{aligned}
& W=W_{\mathrm{p}}+W_{\mathrm{g}}+W_{\mathrm{N}}=\Delta K=\frac{1}{2} m v_{f}^{2}-\frac{1}{2} m v_{i}^{2} \rightarrow \\
& v_{f}=\sqrt{\frac{2 W}{m}}=\sqrt{\frac{2(636.0 \mathrm{~J}-467.4 \mathrm{~J})}{(18 \mathrm{~kg})}}=4.3 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

64. See the free-body diagram help in the determination of the frictional force.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{N}}-F_{\mathrm{P}} \sin \phi-m g \cos \phi=0 \rightarrow F_{\mathrm{N}}=F_{\mathrm{P}} \sin \phi+m g \cos \phi \\
& F_{\mathrm{f}}=\mu_{\mathrm{k}} F_{\mathrm{N}}=\mu_{\mathrm{k}}\left(F_{\mathrm{P}} \sin \phi+m g \cos \phi\right)
\end{aligned}
$$

(a) The angle between the pushing force and the displacement is $32^{\circ}$.

$$
W_{\mathrm{P}}=F_{\mathrm{P}} d \cos \theta=(150 \mathrm{~N})(5.0 \mathrm{~m}) \cos 32^{\circ}=636.0 \mathrm{~J} \approx 640 \mathrm{~J}
$$

(b) The angle between the force of gravity and the displacement is $122^{\circ}$.


$$
W_{\mathrm{G}}=F_{\mathrm{G}} d \cos \theta=m g d \cos \theta=(18 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(5.0 \mathrm{~m}) \cos 122^{\circ}=-467.4 \mathrm{~J} \approx-470 \mathrm{~J}
$$

(c) Because the normal force is perpendicular to the displacement, the work done by the normal force is 0 .
(d) To find the net work, we need the work done by the friction force. The angle between the friction force and the displacement is $180^{\circ}$.

$$
\begin{aligned}
W_{\mathrm{f}} & =F_{\mathrm{f}} d \cos \theta=\mu_{\mathrm{k}}\left(F_{\mathrm{p}} \sin \phi+m g \cos \phi\right) d \cos \theta \\
& =(0.10)\left[(150 \mathrm{~N}) \sin 32^{\circ}+(18 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \cos 32^{\circ}\right](5.0 \mathrm{~m}) \cos 180^{\circ}=-114.5 \mathrm{~J} \\
W & =W_{\mathrm{P}}+W_{\mathrm{g}}+W_{\mathrm{N}}+W_{\mathrm{f}}=\Delta K=\frac{1}{2} m v_{f}^{2}-\frac{1}{2} m v_{i}^{2} \rightarrow \\
v_{f} & =\sqrt{\frac{2 W}{m}}=\sqrt{\frac{2(636.0 \mathrm{~J}-467.4 \mathrm{~J}-114.5 \mathrm{~J})}{(18 \mathrm{~kg})}}=2.5 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

65. The work needed to stop the car is equal to the change in the car's kinetic energy. That work comes from the force of friction on the car, which is assumed to be static friction since the driver locked the brakes. Thus $F_{\mathrm{fr}}=\mu_{k} F_{\mathrm{N}}$. Since the car is on a level surface, the normal force is equal to the

car's weight, and so $F_{\mathrm{fr}}=\mu_{k} m g$ if it is on a level surface. See the diagram for the car. The car is traveling to the right.

$$
\begin{aligned}
& W=\Delta K \rightarrow F_{\mathrm{fr}} d \cos 180^{\circ}=\frac{1}{2} m v_{2}^{2}-\frac{1}{2} m v_{1}^{2} \rightarrow-\mu_{k} m g d=0-\frac{1}{2} m v_{1}^{2} \rightarrow \\
& v_{1}=\sqrt{2 \mu_{k} g d}=\sqrt{2(0.38)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(98 \mathrm{~m})}=27 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

The mass does not affect the problem, since both the change in kinetic energy and the work done by friction are proportional to the mass. The mass cancels out of the equation.
66. For the first part of the motion, the net force doing work is the 225 N force. For the second part of the motion, both the 225 N force and the force of friction do work. The friction force is the coefficient of friction times the normal force, and the normal force is equal to the weight. The workenergy theorem is then used to find the final speed.

$$
\begin{aligned}
W_{\text {total }} & =W_{1}+W_{2}=F_{\text {pull }} d_{1} \cos 0^{\circ}+F_{\text {pull }} d_{2} \cos 0^{\circ}+F_{\mathrm{f}} d_{2} \cos 180^{\circ}=\Delta K=\frac{1}{2} m\left(v_{f}^{2}-v_{i}^{2}\right) \rightarrow \\
v_{f} & =\sqrt{\frac{2\left[F_{\text {pull }}\left(d_{1}+d_{2}\right)-\mu_{\mathrm{k}} m g d_{2}\right]}{m}} \\
& =\sqrt{\frac{2\left[(225 \mathrm{~N})(21.0 \mathrm{~m})-(0.20)(46.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(10.0 \mathrm{~m})\right]}{(46.0 \mathrm{~kg})}}=13 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

67. (a) In the Earth frame of reference, the ball changes from a speed of $v_{1}$ to a speed of $v_{1}+v_{2}$.

$$
\begin{aligned}
\Delta K_{\text {Earth }} & =\frac{1}{2} m\left(v_{1}+v_{2}\right)^{2}-\frac{1}{2} m v_{1}^{2}=\frac{1}{2} m\left(v_{1}^{2}+2 v_{1} v_{2}+v_{2}^{2}\right)-\frac{1}{2} m v_{1}^{2}=m v_{1} v_{2}+\frac{1}{2} m v_{2}^{2} \\
& =\frac{1}{2} m v_{2}^{2}\left(1+2 \frac{v_{1}}{v_{2}}\right)
\end{aligned}
$$

(b) In the train frame of reference, the ball changes from a speed of 0 to a speed of $v_{2}$.

$$
\Delta K_{\text {train }}=\frac{1}{2} m v_{2}^{2}-0=\frac{1}{2} m v_{2}^{2}
$$

(c) The work done is the change of kinetic energy, in each case.

$$
W_{\text {Earth }}=\frac{1}{2} m v_{2}^{2}\left(1+2 \frac{v_{1}}{v_{2}}\right) ; W_{\text {train }}=\frac{1}{2} m v_{2}^{2}
$$

(d) The difference can be seen as due to the definition of work as force exerted through a distance. In both cases, the force on the ball is the same, but relative to the Earth, the ball moves further during the throwing process than it does relative to the train. Thus more work is done in the Earth frame of reference. Another way to say it is that kinetic energy is very dependent on reference frame, and so since work is the change in kinetic energy, the amount of work done will be very dependent on reference frame as well.
68. The kinetic energy of the spring would be found by adding together the kinetic energy of each infinitesimal part of the spring. The mass of an infinitesimal part is given by $d m=\frac{M_{\mathrm{s}}}{D} d x$, and the speed of an infinitesimal part is $v=\frac{x}{D} v_{0}$. Calculate the kinetic energy of the mass + spring.

$$
K_{\text {speed }}^{v_{0}}=K_{\text {mass }}+K_{\text {spring }}=\frac{1}{2} m v_{0}^{2}+\frac{1}{2} \int_{\text {mass }} v^{2} d m=\frac{1}{2} m v_{0}^{2}+\frac{1}{2} \int_{0}^{D}\left(v_{0} \frac{x}{D}\right)^{2} \frac{M_{\mathrm{S}}}{D} d x=\frac{1}{2} m v_{0}^{2}+\frac{v_{0}^{2} M_{\mathrm{S}}}{D^{3}} \frac{1}{2} \int_{0}^{D} x^{2} d x
$$

$$
=\frac{1}{2} m v_{0}^{2}+\frac{1}{2} \frac{v_{0}^{2} M_{\mathrm{S}}}{D^{3}} \frac{D^{3}}{3}=\frac{1}{2} v_{0}^{2}\left(m+\frac{1}{3} M_{\mathrm{S}}\right)
$$

So for a generic speed $v$, we have $K_{\text {speed }}=\frac{1}{2}\left(m+\frac{1}{3} M_{\mathrm{s}}\right) v^{2}$.
69. (a) The work done by gravity as the elevator falls is the weight times the displacement. They are in the same direction.

$$
W_{\mathrm{G}}=m g d \cos 0^{\circ}=(925 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(22.5 \mathrm{~m})=2.0396 \times 10^{5} \mathrm{~J} \approx 2.04 \times 10^{5} \mathrm{~J}
$$

(b) The work done by gravity on the elevator is the net work done on the elevator while falling, and so the work done by gravity is equal to the change in kinetic energy.

$$
W_{\mathrm{G}}=\Delta K=\frac{1}{2} m v^{2}-0 \rightarrow v=\sqrt{\frac{2 W_{\mathrm{G}}}{m}}=\sqrt{\frac{2\left(2.0396 \times 10^{5} \mathrm{~J}\right)}{(925 \mathrm{~kg})}}=21.0 \mathrm{~m} / \mathrm{s}
$$

(c) The elevator starts and ends at rest. Therefore, by the work-energy theorem, the net work done must be 0 . Gravity does positive work as it falls a distance of $(22.5+x) \mathrm{m}$, and the spring will do negative work at the spring is compressed. The work done on the spring is $\frac{1}{2} k x^{2}$, and so the work done by the spring is $-\frac{1}{2} k x^{2}$.

$$
\begin{aligned}
& W=W_{\mathrm{G}}+W_{\text {spring }}=m g(d+x)-\frac{1}{2} k x^{2}=0 \rightarrow \frac{1}{2} k x^{2}-m g x-m g d=0 \rightarrow \\
& x=\frac{m g \pm \sqrt{m^{2} g^{2}-4\left(\frac{1}{2} k\right)(-m g d)}}{2\left(\frac{1}{2} k\right)}
\end{aligned}
$$

The positive root must be taken since we have assumed $x>0$ in calculating the work done by gravity. Using the values given in the problem gives $x=2.37 \mathrm{~m}$.
70. (a) $K=\frac{1}{2} m v^{2}=\frac{1}{2}\left(3.0 \times 10^{-3} \mathrm{~kg}\right)(3.0 \mathrm{~m} / \mathrm{s})^{2}=1.35 \times 10^{-2} \mathrm{~J} \approx 1.4 \times 10^{-2} \mathrm{~J}$
(b) $\quad K_{\text {actual }}=0.35 E_{\text {required }} \rightarrow E_{\text {required }}=\frac{K_{\text {actual }}}{0.35}=\frac{1.35 \times 10^{-2} \mathrm{~J}}{0.35}=3.9 \times 10^{-2} \mathrm{~J}$
71. The minimum work required to shelve a book is equal to the weight of the book times the vertical distance the book is moved. See the diagram. Each book that is placed on the lowest shelf has its center moved upwards by 23.0 cm (the height of the bottom of the first shelf, plus half the height of a book). So the work to move 28 books to the lowest shelf is $W_{1}=28 m g(0.230 \mathrm{~m})$. Each book that is placed on the second shelf has its center of mass moved upwards by $56.0 \mathrm{~cm}(23.0 \mathrm{~cm}+33.0 \mathrm{~cm})$, so the work to
 move 28 books to the second shelf is $W_{2}=28 m g(0.560 \mathrm{~m})$.
Similarly, $W_{3}=28 m g(0.890 \mathrm{~m}), W_{4}=28 m g(1.220 \mathrm{~m})$, and $W_{5}=28 m g(1.550 \mathrm{~m})$. The total work done is the sum of the five work expressions.

$$
\begin{aligned}
W & =28 m g(0.230 \mathrm{~m}+.560 \mathrm{~m}+.890 \mathrm{~m}+1.220 \mathrm{~m}+1.550 \mathrm{~m}) \\
& =28(1.40 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(4.450 \mathrm{~m})=1710 \mathrm{~J}
\end{aligned}
$$

72. There are two forces on the meteorite - gravity and the force from the mud. Take down to be the positive direction, and then the net force is $F_{\text {net }}=m g-640 x^{3}$. Use this (variable) force to find the work done on the meteorite as it moves in the mud, and then use the work-energy theorem to find the initial velocity of the meteorite.

$$
\begin{aligned}
W & =\int_{x=0}^{x=5.0}\left(m g-640 x^{3}\right) d x=\left(m g x-160 x^{4}\right)_{x=0}^{x=5.0}=(75 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(5.0 \mathrm{~m})-160(5.0 \mathrm{~m})^{4} \\
& =-9.625 \times 10^{4} \mathrm{~J}
\end{aligned}
$$

$$
W=\Delta K=\frac{1}{2} m\left(v_{f}^{2}-v_{i}^{2}\right) \rightarrow v_{i}=\sqrt{\frac{-2 W}{m}}=\sqrt{\frac{-2\left(-9.625 \times 10^{4} \mathrm{~J}\right)}{(75 \mathrm{~kg})}}=51 \mathrm{~m} / \mathrm{s}
$$

73. Consider the free-body diagram for the block as it moves up the plane.
(a) $K_{1}=\frac{1}{2} m v_{1}^{2}=\frac{1}{2}(6.10 \mathrm{~kg})(3.25 \mathrm{~m} / \mathrm{s})^{2}=32.22 \mathrm{~J} \approx 32.2 \mathrm{~J}$
(b) $W_{\mathrm{P}}=F_{\mathrm{P}} d \cos 37^{\circ}=(75.0 \mathrm{~N})(9.25 \mathrm{~m}) \cos 37.0^{\circ}=554.05 \mathrm{~J}$

$$
\approx 554 \mathrm{~J}
$$

(c) $W_{G}=m g d \cos 127.0^{\circ}=(6.10 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(9.25 \mathrm{~m}) \cos 127.0^{\circ}$

$$
=-332.78 \mathrm{~J} \approx-333 \mathrm{~J}
$$


(d) $W_{\mathrm{N}}=F_{\mathrm{N}} d \cos 90^{\circ}=0 \mathrm{~J}$
(e) Apply the work-energy theorem.

$$
\begin{aligned}
& W_{\text {total }}=K_{2}-K_{1} \rightarrow \\
& K E_{2}=W_{\text {total }}+K_{1}=W_{\mathrm{P}}+W_{\mathrm{G}}+W_{\mathrm{N}}+K_{1}=(554.05-332.78+0+32.22) \mathrm{J} \approx 253 \mathrm{~J}
\end{aligned}
$$

74. The dot product can be used to find the angle between the vectors.

$$
\begin{aligned}
& \overrightarrow{\mathbf{d}}_{1-2}=\left[(0.230 \hat{\mathbf{i}}+0.133 \hat{\mathbf{j}}) \times 10^{-9} \mathrm{~m}\right] ; \overrightarrow{\mathbf{d}}_{1-3}=\left[(0.077 \hat{\mathbf{i}}+0.133 \hat{\mathbf{j}}+0.247 \hat{\mathbf{k}}) \times 10^{-9} \mathrm{~m}\right] \\
& \begin{aligned}
\overrightarrow{\mathbf{d}}_{1-2} \cdot \overrightarrow{\mathbf{d}}_{1-3} & =\left[(0.230 \hat{\mathbf{i}}+0.133 \hat{\mathbf{j}}) \times 10^{-9} \mathrm{~m}\right] \cdot\left[(0.077 \hat{\mathbf{i}}+0.133 \hat{\mathbf{j}}+0.247 \hat{\mathbf{k}}) \times 10^{-9} \mathrm{~m}\right] \\
& =\left[3.540 \times 10^{-2}\right] \times 10^{-18} \mathrm{~m}^{2}
\end{aligned} \\
& \begin{array}{l}
d_{1-2}=\sqrt{(0.230)^{2}+(0.133)^{2}} \times 10^{-9} \mathrm{~m}=0.2657 \times 10^{-9} \mathrm{~m} \\
d_{1-3}=\sqrt{(0.077)^{2}+(0.133)^{2}+(0.247)^{2}} \times 10^{-9} \mathrm{~m}=0.2909 \times 10^{-9} \mathrm{~m} \\
\overrightarrow{\mathbf{d}}_{1-2} \cdot \overrightarrow{\mathbf{d}}_{1-3}=d_{1-2} d_{1-3} \cos \theta \rightarrow \\
\theta=\cos ^{-1} \frac{\overrightarrow{\mathbf{d}}_{1-2} \cdot \overrightarrow{\mathbf{d}}_{1-3}}{d_{1-2} d_{1-3}}=\cos ^{-1} \frac{\left[3.540 \times 10^{-2}\right] \times 10^{-18} \mathrm{~m}^{2}}{\left(0.2657 \times 10^{-9} \mathrm{~m}\right)\left(0.2909 \times 10^{-9} \mathrm{~m}\right)}=62.7^{\circ}
\end{array}
\end{aligned}
$$

75. Since the forces are constant, we may use Eq. 7-3 to calculate the work done.

$$
\begin{aligned}
W_{\text {net }} & =\left(\overrightarrow{\mathbf{F}}_{1}+\overrightarrow{\mathbf{F}}_{2}\right) \cdot \overrightarrow{\mathbf{d}}=[(1.50 \hat{\mathbf{i}}-0.80 \hat{\mathbf{j}}+0.70 \hat{\mathbf{k}}) \mathrm{N}+(-0.70 \hat{\mathbf{i}}+1.20 \hat{\mathbf{j}}) \mathrm{N}] \cdot[(8.0 \hat{\mathbf{i}}+6.0 \hat{\mathbf{j}}+5.0 \hat{\mathbf{k}}) \mathrm{m}] \\
& =[(0.80 \hat{\mathbf{i}}+0.40 \hat{\mathbf{j}}+0.70 \hat{\mathbf{k}}) \mathrm{N}] \cdot[(8.0 \hat{\mathbf{i}}+6.0 \hat{\mathbf{j}}+5.0 \hat{\mathbf{k}}) \mathrm{m}]=(6.4+2.4+3.5) \mathrm{J}=12.3 \mathrm{~J}
\end{aligned}
$$

76. The work done by the explosive force is equal to the change in kinetic energy of the shells. The starting speed is 0 . The force is in the same direction as the displacement of the shell.

$$
\begin{aligned}
& W=\Delta K=\frac{1}{2} m v_{f}^{2}-\frac{1}{2} m v_{i}^{2}=\frac{1}{2} m v_{f}^{2} ; W=F d \cos \theta \rightarrow \frac{1}{2} m v_{f}^{2}=F d \cos \theta \rightarrow \\
& F=\frac{m v_{f}^{2}}{2 d \cos \theta}=\frac{(1250 \mathrm{~kg})(750 \mathrm{~m} / \mathrm{s})^{2}}{2(15 \mathrm{~m})}=2.344 \times 10^{7} \mathrm{~N} \approx 2.3 \times 10^{7} \mathrm{~N} \\
& 2.344 \times 10^{7} \mathrm{~N}\left(\frac{1 \mathrm{~b}}{4.45 \mathrm{~N}}\right)=5.3 \times 10^{6} \mathrm{lbs}
\end{aligned}
$$

77. We assume the force is in the $x$-direction, so that the angle between the force and the displacement is 0 . The work is found from Eq. 7-7.

$$
W=\int_{x=0.10 \mathrm{~m}}^{x=\infty} A e^{-k x} d x=-\left.\frac{A}{k} e^{-k x}\right|_{x=0.10} ^{x=\infty}=\frac{A}{k} e^{-0.10 k}
$$

78. The force exerted by the spring will be the same magnitude as the force to compress the spring. The spring will do positive work on the ball by exerting a force in the direction of the displacement. This work is equal to the change in kinetic energy of the ball. The initial speed of the ball is 0 .

$$
\begin{aligned}
& W=\Delta K=\frac{1}{2} m v_{f}^{2}-\frac{1}{2} m v_{i}^{2}=\frac{1}{2} m v_{f}^{2} ; W=\int_{x=0}^{x=2.0 \mathrm{~m}}\left(150 x+12 x^{3}\right) d x=\left(75 x^{2}+3 x^{4}\right)_{x=0}^{x=2.0}=348 \mathrm{~J} \\
& v_{f}=\sqrt{\frac{2 W}{m}}=\sqrt{\frac{2(348 \mathrm{~J})}{3.0 \mathrm{~kg}}}=15 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

79. The force is constant, and so we may calculate the force by Eq. 7-3. We may also use that to calculate the angle between the two vectors.

$$
\begin{aligned}
& W=\overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{d}}=[(10.0 \hat{\mathbf{i}}+9.0 \hat{\mathbf{j}}+12.0 \hat{\mathbf{k}}) \mathrm{kN}] \cdot[(5.0 \hat{\mathbf{i}}+4.0 \hat{\mathbf{j}}) \mathrm{m}]=86 \mathrm{~kJ} \\
& F=\left[(10.0)^{2}+(9.0)^{2}+(12.0)^{2}\right]^{1 / 2} \mathrm{kN}=18.0 \mathrm{kN} ; d=\left[(5.0)^{2}+(4.0)^{2}\right]^{1 / 2} \mathrm{~m}=6.40 \mathrm{~m} \\
& W=F d \cos \theta \rightarrow \theta=\cos ^{-1} \frac{W}{F d}=\cos ^{-1} \frac{8.6 \times 10^{4} \mathrm{~J}}{\left(1.80 \times 10^{4} \mathrm{~N}\right)(6.40 \mathrm{~m})}=42^{\circ}
\end{aligned}
$$

80. (a) The force and displacement are in the same direction.

$$
\begin{aligned}
& W=F d \cos \theta ; W=\Delta K \rightarrow \\
& F=\frac{\Delta K}{d}=\frac{\frac{1}{2} m\left(v_{f}^{2}-v_{i}^{2}\right)}{d}=\frac{\frac{1}{2}(0.033 \mathrm{~kg})(85 \mathrm{~m} / \mathrm{s})^{2}}{0.32 \mathrm{~m}}=372.5 \mathrm{~N} \approx 370 \mathrm{~N}
\end{aligned}
$$

(b) Combine Newton's second law with Eq. 2-12c for constant acceleration.

$$
F=m a=\frac{m\left(v_{f}^{2}-v_{i}^{2}\right)}{2 \Delta x}=\frac{(0.033 \mathrm{~kg})(85 \mathrm{~m} / \mathrm{s})^{2}}{2(0.32 \mathrm{~m})}=372.5 \mathrm{~N} \approx 370 \mathrm{~N}
$$

81. The original speed of the softball is $(110 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)=30.56 \mathrm{~m} / \mathrm{s}$. The final speed is $90 \%$ of this, or $27.50 \mathrm{~m} / \mathrm{s}$. The work done by air friction causes a change in the kinetic energy of the ball, and thus the speed change. In calculating the work, notice that the force of friction is directed oppositely to the direction of motion of the ball.

$$
\begin{aligned}
& W_{\mathrm{fr}}=F_{\mathrm{fr}} d \cos 180^{\circ}=K_{2}-K_{1}=\frac{1}{2} m\left(v_{2}^{2}-v_{1}^{2}\right) \rightarrow \\
& F_{\mathrm{fr}}=\frac{m\left(v_{2}^{2}-v_{1}^{2}\right)}{-2 d}=\frac{m v_{1}^{2}\left(0.9^{2}-1\right)}{-2 d}=\frac{(0.25 \mathrm{~kg})(30.56 \mathrm{~m} / \mathrm{s})^{2}\left(0.9^{2}-1\right)}{-2(15 \mathrm{~m})}=1.5 \mathrm{~N}
\end{aligned}
$$

82. (a) The pilot's initial speed when he hit the snow was $45 \mathrm{~m} / \mathrm{s}$. The work done on him as he fell the 1.1 m into the snow changed his kinetic energy. Both gravity and the snow did work on the pilot during that 1.1-meter motion. Gravity did positive work (the force was in the same direction as the displacement), and the snow did negative work (the force was in the opposite direction as the displacement).

$$
\begin{aligned}
& W_{\text {gravity }}+W_{\text {snow }}=\Delta K \rightarrow m g d+W_{\text {snow }}=-\frac{1}{2} m v_{i}^{2} \rightarrow \\
& \begin{aligned}
W_{\text {snow }} & =-\frac{1}{2} m v_{i}^{2}-m g d=-m\left(\frac{1}{2} v_{i}^{2}+g d\right)=-(88 \mathrm{~kg})\left[\frac{1}{2}(45 \mathrm{~m} / \mathrm{s})^{2}+\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(1.1 \mathrm{~m})\right] \\
& =-9.005 \times 10^{4} \mathrm{~J} \approx-9.0 \times 10^{4} \mathrm{~J}
\end{aligned}
\end{aligned}
$$

(b) The work done by the snowbank is done by an upward force, while the pilot moves down.

$$
\begin{aligned}
& W_{\text {snow }}=F_{\text {snow }} d \cos 180^{\circ}=-F_{\text {snow }} d \rightarrow \\
& F_{\text {snow }}=-\frac{W_{\text {snow }}}{d}=-\frac{-9.005 \times 10^{4} \mathrm{~J}}{1.1 \mathrm{~m}}=8.186 \times 10^{4} \mathrm{~N} \approx 8.2 \times 10^{4} \mathrm{~N}
\end{aligned}
$$

(c) During the pilot's fall in the air, positive work was done by gravity, and negative work by air resistance. The net work was equal to his change in kinetic energy while falling. We assume he started from rest when he jumped from the aircraft.

$$
\begin{aligned}
& W_{\text {gravity }}+W_{\text {air }}=\Delta K \rightarrow m g h+W_{\text {air }}=\frac{1}{2} m v_{f}^{2}-0 \rightarrow \\
& W_{\text {air }} \\
& =\frac{1}{2} m v_{f}^{2}-m g h=m\left(\frac{1}{2} v_{f}^{2}-g h\right)=(88 \mathrm{~kg})\left[\frac{1}{2}(45 \mathrm{~m} / \mathrm{s})^{2}-\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(370 \mathrm{~m})\right] \\
& \\
& \quad=-2.3 \times 10^{5} \mathrm{~J}
\end{aligned}
$$

83. The (negative) work done by the bumper on the rest of the car must equal the change in the car's kinetic energy. The work is negative because the force on the car is in the opposite direction to the car's displacement.

$$
\begin{aligned}
& W_{\text {bumper }}=\Delta K=\rightarrow-\frac{1}{2} k x^{2}=0-\frac{1}{2} m v_{i}^{2} \rightarrow \\
& k=m \frac{v_{i}^{2}}{x^{2}}=(1050 \mathrm{~kg}) \frac{\left[(8 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)\right]^{2}}{(0.015 \mathrm{~m})^{2}}=2 \times 10^{7} \mathrm{~N} / \mathrm{m}
\end{aligned}
$$

84. The spring must be compressed a distance such that the work done by the spring is equal to the change in kinetic energy of the car. The distance of compression can then be used to find the spring constant. Note that the work done by the spring will be negative, since the force exerted by the spring is in the opposite direction to the displacement of the spring.

$$
\begin{aligned}
& W_{\text {spring }}=\Delta K=\rightarrow-\frac{1}{2} k x^{2}=0-\frac{1}{2} m v_{i}^{2} \rightarrow x=v_{i} \sqrt{\frac{m}{k}} \\
& F=m a=-k x \rightarrow m(-5.0 g)=-k v_{i} \sqrt{\frac{m}{k}} \rightarrow
\end{aligned}
$$

$$
k=m\left(\frac{5.0 g}{v_{i}}\right)^{2}=(1300 \mathrm{~kg})(25) \frac{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)^{2}}{\left[90 \mathrm{~km} / \mathrm{h}\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)\right]^{2}}=5.0 \times 10^{3} \mathrm{~N} / \mathrm{m}
$$

85. If the rider is riding at a constant speed, then the positive work input by the rider to the (bicycle + rider) combination must be equal to the negative work done by gravity as he moves up the incline. The net work must be 0 if there is no change in kinetic energy.
(a) If the rider's force is directed downwards, then the rider will do an amount of work equal to the force times the distance parallel to the force. The distance parallel to the downward force would be the diameter of the circle in which the pedals move. Then consider that by using 2 feet, the rider does twice that amount of work when the pedals make one complete revolution. So in one revolution of the pedals, the rider does the work calculated below.

$$
W_{\text {rider }}=2\left(0.90 m_{\text {rider }} g\right) d_{\substack{\text { pedal } \\ \text { moioin }}}
$$

In one revolution of the front sprocket, the rear sprocket will make 42/19 revolutions, and so the back wheel (and the entire bicycle and rider as well) will move a distance of $(42 / 19)\left(2 \pi r_{\text {wheel }}\right)$. That is a distance along the plane, and so the height that the bicycle and rider will move is $h=(42 / 19)\left(2 \pi r_{\text {wheel }}\right) \sin \theta$. Finally, the work done by gravity in moving that height is calculated.

$$
W_{\mathrm{G}}=\left(m_{\text {rider }}+m_{\text {bike }}\right) g h \cos 180^{\circ}=-\left(m_{\text {rider }}+m_{\text {bike }}\right) g h=-\left(m_{\text {rider }}+m_{\text {bike }}\right) g(42 / 19)\left(2 \pi r_{\text {wheel }}\right) \sin \theta
$$

Set the total work equal to 0 , and solve for the angle of the incline.

$$
\begin{aligned}
& W_{\text {rider }}+W_{\mathrm{G}}=0 \rightarrow 2\left[0.90 m_{\text {rider }} g\right] d_{\substack{\text { pedal } \\
\text { notion }}}-\left(m_{\text {rider }}+m_{\text {bike }}\right) g(42 / 19)\left(2 \pi r_{\text {wheel }}\right) \sin \theta=0 \rightarrow \\
& \theta=\sin ^{-1} \frac{\left(0.90 m_{\text {rider }}\right) d_{\text {pedal }}}{\left(m_{\text {ridior }}+m_{\text {bike }}\right)(42 / 19)\left(\pi r_{\text {wheel }}\right)}=\sin ^{-1} \frac{0.90(65 \mathrm{~kg})(0.36 \mathrm{~m})}{(77 \mathrm{~kg})(42 / 19) \pi(0.34 \mathrm{~m})}=6.7^{\circ}
\end{aligned}
$$

(b) If the force is tangential to the pedal motion, then the distance that one foot moves while exerting a force is now half of the circumference of the circle in which the pedals move. The rest of the analysis is the same.

$$
\begin{aligned}
& W_{\text {rider }}=2\left(0.90 m_{\text {rider }} g\right)\left(\pi r_{\substack{\text { pedal } \\
\text { motion }}}\right) ; W_{\text {rider }}+W_{\mathrm{G}}=0 \rightarrow \\
& \theta=\sin ^{-1} \frac{\left(0.90 m_{\text {rider }}\right) \pi r_{\text {pedal }}}{\text { motion }} \\
& \left(m_{\text {rider }}+m_{\text {bike }}\right)(42 / 19)\left(\pi r_{\text {wheel }}\right)
\end{aligned} \sin ^{-1} \frac{0.90(65 \mathrm{~kg})(0.18 \mathrm{~m})}{(77 \mathrm{~kg})(42 / 19)(0.34 \mathrm{~m})}=10.5^{\circ} \approx 10^{\circ} .
$$

86. Because the acceleration is essentially 0 , the net force on the mass is 0 . The magnitude of $\overrightarrow{\mathbf{F}}$ is found with the help of the free-body diagram in the textbook.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{T}} \cos \theta-m g=0 \rightarrow F_{\mathrm{T}}=\frac{m g}{\cos \theta} \\
& \sum F_{x}=F-F_{\mathrm{T}} \sin \theta=0 \rightarrow F=F_{\mathrm{T}} \sin \theta=\frac{m g}{\cos \theta} \sin \theta=m g \tan \theta
\end{aligned}
$$

(a) A small displacement of the object along the circular path is given by $d r=\ell d \theta$, based on the definition of radian measure. The force $\overrightarrow{\mathbf{F}}$ is at an angle $\theta$ to the direction of motion. We use the symbol $d \overrightarrow{\mathbf{r}}$ for the infinitesimal displacement, since the symbol $\ell$ is already in use as the length of the pendulum.

$$
\begin{aligned}
W_{\mathrm{F}} & =\int \overrightarrow{\mathbf{F}} \cdot d \overrightarrow{\mathbf{r}}=\int_{\theta=0}^{\theta=\theta_{0}} F \cos \theta \ell d \theta=\ell \int_{\theta=0}^{\theta=\theta_{0}}(m g \tan \theta) \cos \theta d \theta=m g \ell \int_{\theta=0}^{\theta=\theta_{0}} \sin \theta d \theta \\
& =-\left.m g \ell \cos \theta\right|_{0} ^{\theta_{0}}=m g \ell\left(1-\cos \theta_{0}\right)
\end{aligned}
$$

(b) The angle between $m \overrightarrow{\mathbf{g}}$ and the direction of motion is $(90+\theta)$.

$$
\begin{aligned}
W_{\mathrm{G}} & =\int m \overrightarrow{\mathbf{g}} \cdot d \overrightarrow{\mathbf{r}}=m g \ell \int_{\theta=0}^{\theta=\theta_{0}} \cos \left(90^{\circ}+\theta\right) d \theta=-m g \ell \int_{\theta=0}^{\theta=\theta_{0}} \sin \theta d \theta \\
& =\left.m g \ell \cos \theta\right|_{0} ^{\theta_{0}}=m g \ell\left(\cos \theta_{0}-1\right)
\end{aligned}
$$

Alternatively, it is proven in problem 36 that the shape of the path does not determine the work done by gravity - only the height change. Since this object is rising, gravity will do negative work.

$$
\begin{aligned}
W_{\mathrm{G}} & =m g d \cos \phi=m g(\text { height }) \cos 180^{\circ}=-m g y_{\text {final }}=-m g\left(\ell-\ell \cos \theta_{0}\right) \\
& =m g \ell\left(\cos \theta_{0}-1\right)
\end{aligned}
$$

Since $\overrightarrow{\mathbf{F}}_{\mathrm{T}}$ is perpendicular to the direction of motion, it does 0 work on the bob.
Note that the total work done is 0 , since the object's kinetic energy does not change.
87. (a) The work done by the arms of the parent will change the kinetic energy of the child. The force is in the opposite direction of the displacement.

$$
\begin{aligned}
& W_{\text {parent }}=\Delta K_{\text {child }}=K_{\mathrm{f}}-K_{\mathrm{i}}=0-\frac{1}{2} m v_{\mathrm{i}}^{2} ; W_{\text {parent }}=F_{\mathrm{parent}} d \cos 180^{\circ} \rightarrow \\
& -\frac{1}{2} m v_{\mathrm{i}}^{2}=-F_{\text {parent }} d \rightarrow F_{\text {parent }}=\frac{m v_{\mathrm{i}}^{2}}{2 d}=\frac{(18 \mathrm{~kg})(25 \mathrm{~m} / \mathrm{s})^{2}}{2(45 \mathrm{~m})}=125 \mathrm{~N} \approx 130 \mathrm{~N} \approx 28 \mathrm{lbs}
\end{aligned}
$$

This force is achievable by an average parent.
(b) The same relationship may be used for the shorter distance.

$$
F_{\mathrm{parent}}=\frac{m v_{\mathrm{i}}^{2}}{2 d}=\frac{(18 \mathrm{~kg})(25 \mathrm{~m} / \mathrm{s})^{2}}{2(12 \mathrm{~m})}=469 \mathrm{~N} \approx 470 \mathrm{~N} \approx 110 \mathrm{lbs}
$$

This force may not be achievable by an average parent. Many people might have difficulty with a 110-pound bench press exercise, for example.
88. (a) From the graph, the shape of the force function is roughly that of a triangle. The work can be estimated using the formula for the area of a triangle of base 20 m and height 100 N .

$$
\begin{aligned}
W & \approx \frac{1}{2} " b^{\prime \prime} " h^{\prime \prime}=\frac{1}{2}(20.0 \mathrm{~m})(100 \mathrm{~N}) \\
& =1000 \mathrm{~J}
\end{aligned}
$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH07.XLS," on tab "Problem 7.88a."
(b) Integrate the force function to find the exact work done.

$$
W=\int_{x_{i}}^{x_{f}} F d x=\int_{0.0 \mathrm{~m}}^{20.0 \mathrm{~m}}\left[100-(x-10)^{2}\right] d x
$$

$$
=\int_{0.0 \mathrm{~m}}^{20.0 \mathrm{~m}}\left(20 x-x^{2}\right) d x=\left[10 x^{2}-\frac{1}{3} x^{3}\right]_{0.0 \mathrm{~m}}^{20.0 \mathrm{~m}}=1333 \mathrm{~J} \approx 1330 \mathrm{~J}
$$

89. (a) The work done by gravity is given by Eq. 7-1.

$$
\begin{aligned}
W_{\mathrm{G}} & =m g d \cos (90-\theta)=(85 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(250 \mathrm{~m}) \cos 86.0^{\circ} \\
& =1.453 \times 10^{4} \mathrm{~J} \approx 1.5 \times 10^{4} \mathrm{~J}
\end{aligned}
$$

(b) The work is the change in kinetic energy. The initial kinetic energy


$$
W_{\mathrm{G}}=\Delta K=K_{\mathrm{f}}-K_{\mathrm{i}}=\frac{1}{2} m v_{\mathrm{f}}^{2} \rightarrow v_{f}=\sqrt{\frac{2 W_{\mathrm{G}}}{m}}=\sqrt{\frac{2\left(1.453 \times 10^{4} \mathrm{~J}\right)}{85 \mathrm{~kg}}}=18 \mathrm{~m} / \mathrm{s}
$$

90. (a) The work-energy principle says the net work done is the change in kinetic energy. The climber both begins and ends the fall at rest, so the change in kinetic energy is 0 . Thus the total work done (by gravity and by the rope) must be 0 . This is used to find $x$. Note that the force of gravity is parallel to the displacement, so the work done by gravity is positive, but the force exerted by the rope is in the opposite direction to the displacement, so the work done by the rope is negative.

$$
\begin{aligned}
& W_{\text {net }}=W_{\text {grav }}+W_{\text {rope }}=m g(2 \ell+x)-\frac{1}{2} k x^{2}=0 \rightarrow \frac{1}{2} k x^{2}-m g x-2 \ell m g=0 \rightarrow \\
& x=\frac{m g \pm \sqrt{m^{2} g^{2}-4\left(\frac{1}{2} k\right)(-2 \ell m g)}}{2\left(\frac{1}{2} k\right)}=\frac{m g \pm \sqrt{m^{2} g^{2}+4 k \ell m g}}{k}=\frac{m g}{k}\left(1 \pm \sqrt{1+\frac{4 k \ell}{m g}}\right)
\end{aligned}
$$

We have assumed that $x$ is positive in the expression for the work done by gravity, and so the "plus" sign must be taken in the above expression.
Thus $x=\frac{m g}{k}\left(1+\sqrt{1+\frac{4 k l}{m g}}\right)$.
(b) Use the values given to calculate $\frac{x}{\ell}$ and $\frac{k x}{m g}$.

$$
\begin{aligned}
& x=\frac{m g}{k}\left(1+\sqrt{1+\frac{4 k \ell}{m g}}\right)=\frac{(85 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{(850 \mathrm{~N} / \mathrm{m})}\left(1+\sqrt{1+\frac{4(850 \mathrm{~N} / \mathrm{m})(8.0 \mathrm{~m})}{(85 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}}\right)=6.665 \mathrm{~m} \\
& \frac{x}{\ell}=\frac{6.665 \mathrm{~m}}{8.0 \mathrm{~m}}=0.83 ; \frac{k x}{m g}=\frac{(850 \mathrm{~N} / \mathrm{m})(6.665 \mathrm{~m})}{(85 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=6.8
\end{aligned}
$$

91. Refer to the free body diagram. The coordinates are defined simply to help analyze the components of the force. At any angle $\theta$, since the mass is not accelerating, we have the following.

$$
\sum F_{x}=F-m g \sin \theta=0 \rightarrow F=m g \sin \theta
$$

Find the work done in moving the mass from $\theta=0$ to $\theta=\theta_{0}$.


$$
\begin{aligned}
W_{\mathrm{F}} & =\int \overrightarrow{\mathbf{F}} \cdot d \overrightarrow{\mathbf{s}}=\int_{\theta=0}^{\theta=\theta_{0}} F \cos 0^{\circ} \ell d \theta=m g \ell \int_{\theta=0}^{\theta=\theta_{0}} \sin \theta d \theta \\
& =-\left.m g \ell \cos \theta\right|_{0} ^{\theta_{0}}=m g \ell\left(1-\cos \theta_{0}\right)
\end{aligned}
$$

See the second diagram to find the height that the mass has risen. We see that $h=\ell-\ell \cos \theta_{0}=\ell\left(1-\cos \theta_{0}\right)$, and so

$$
W_{\mathrm{F}}=m g \ell\left(1-\cos \theta_{0}\right)=m g h .
$$


92. For each interval, the average force for that interval was calculated as the numeric average of the forces at the beginning and end of the interval. Then this force was multiplied by $10.0 \mathrm{~cm}(0.0100$ m ) to find the work done on that interval. The total work is the sum of those work amounts. That process is expressed in a formula below. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH07.XLS," on tab "Problem 7.92."

$$
W_{\text {applied }}=\sum_{i=1}^{n-1} \frac{1}{2}\left(F_{i}+F_{i+1}\right) \Delta x=102.03 \mathrm{~J} \approx 102 \mathrm{~J}
$$

93. (a) See the adjacent graph. The bestfit straight line is as follows.

$$
F_{\text {applied }}=(10.0 \mathrm{~N} / \mathrm{m}) x
$$

The spreadsheet used for this problem can be found on the Media Manager, with filename
"PSE4_ISM_CH07.XLS," on tab "Problem 7.93a."

(b) Since $F_{\text {applied }}=k x$ for the stretched spring, the slope is the spring constant.

$$
k=10.0 \mathrm{~N} / \mathrm{m}
$$

(c) Use the best-fit equation from the graph.

$$
F=k x=(10.0 \mathrm{~N} / \mathrm{m})(0.200 \mathrm{~m})=2.00 \mathrm{~N}
$$

## CHAPTER 8: Conservation of Energy

## Responses to Questions

Friction is not conservative; it dissipates energy in the form of heat, sound, and light. Air resistance is not conservative; it dissipates energy in the form of heat and the kinetic energy of fluids. "Human" forces, for example, the forces produced by your muscles, are also not conservative. They dissipate energy in the form of heat and also through chemical processes.
2. The two forces on the book are the applied force upward (nonconservative) and the downward force of gravity (conservative). If air resistance is non-negligible, it is nonconservative.
3. (a) If the net force is conservative, the change in the potential energy is equal to the negative of the change in the kinetic energy, so $\Delta U=-300 \mathrm{~J}$.
(b) If the force is conservative, the total mechanical energy is conserved, so $\Delta E=0$.
4. No. The maximum height on the rebound cannot be greater than the initial height if the ball is dropped. Initially, the dropped ball's total energy is gravitational potential energy. This energy is changed to other forms (kinetic as it drops, and elastic potential during the collision with the floor) and eventually back into gravitational potential energy as the ball rises back up. The final energy cannot be greater than the initial (unless there is an outside energy source) so the final height cannot be greater than the initial height. Note that if you throw the ball down, it initially has kinetic energy as well as potential so it may rebound to a greater height.
5. (a) No. If there is no friction, then gravity is the only force doing work on the sled, and the system is conservative. All of the gravitational potential energy of the sled at the top of the hill will be converted into kinetic energy. The speed at the bottom of the hill depends only on the initial height $h$, and not on the angle of the hill. $K_{\mathrm{f}}=\frac{1}{2} m v^{2}=m g h$, and $v=(2 g h)^{1 / 2}$.
(b) Yes. If friction is present, then the net force doing work on the sled is not conservative. Only part of the gravitational potential energy of the sled at the top of the hill will be converted into kinetic energy; the rest will be dissipated by the frictional force. The frictional force is proportional to the normal force on the sled, which will depend on the angle $\theta$ of the hill. $K_{\mathrm{f}}=\frac{1}{2} m \nu^{2}=m g h-f x=m g h-\mu m g h \cos \theta / \sin \theta=m g h(1-\mu / \tan \theta)$, and $v=[2 g h(1-\mu / \tan \theta)]^{1 / 2}$, which does depend on the angle of the hill and will be smaller for smaller angles.
6. No work is done on the wall (since the wall does not undergo displacement) but internally your muscles are converting chemical energy to other forms of energy, which makes you tired.
7. At the top of the pendulum's swing, all of its energy is gravitational potential energy; at the bottom of the swing, all of the energy is kinetic.
(a) If we can ignore friction, then energy is transformed back and forth between potential and kinetic as the pendulum swings.
(b) If friction is present, then during each swing energy is lost to friction at the pivot point and also to air resistance. During each swing, the kinetic energy and the potential energy decrease, and the pendulum's amplitude decreases. When a grandfather clock is wound up, the energy lost to friction and air resistance is replaced by energy stored as potential energy (either elastic or gravitational, depending on the clock mechanism).
8. The drawing shows water falling over a waterfall and then flowing back to the top of the waterfall. The top of the waterfall is above the bottom, with greater gravitational potential energy. The optical illusion of the diagram implies that water is flowing freely from the bottom of the waterfall back to the top. Since water won't move uphill unless work is done on it to increase its gravitational potential energy (for example, work done by a pump), the water from the bottom of the waterfall would NOT be able to make it back to the top.
9. For each of the water balloons, the initial energy (kinetic plus potential) will equal the final energy (all kinetic). Since the initial energy depends only on the speed and not on the direction of the initial velocity, and all balloons have the same initial speed and height, the final speeds will all be the same. $\left[E_{i}=\frac{1}{2} m v_{i}^{2}+m g h=E_{f}=\frac{1}{2} m v_{f}^{2}\right]$
10. Yes, the spring can leave the table. When you push down on the spring, you do work on it and it gains elastic potential energy, and loses a little gravitational potential energy, since the center of mass of the spring is lowered. When you remove your hand, the spring expands, and the elastic potential energy is converted into kinetic energy and into gravitational potential energy. If enough elastic potential energy was stored, the center of mass of the spring will rise above its original position, and the spring will leave the table.
11. The initial potential energy of the water is converted first into the kinetic energy of the water as it falls. When the falling water hits the pool, it does work on the water already in the pool, creating splashes and waves. Additionally, some energy is converted into heat and sound.
12. Stepping on top of a log and jumping down the other side requires you to raise your center of mass farther than just stepping over a log does. Raising your center of mass farther requires you to do more work, or use more energy.
13. (a) As a car accelerates uniformly from rest, the potential energy stored in the fuel is converted into kinetic energy in the engine and transmitted through the transmission into the turning of the wheels, which causes the car to accelerate (if friction is present between the road and the tires).
(b) If there is a friction force present between the road and the tires, then when the wheels turn, the car moves forward and gains kinetic energy. If the static friction force is large enough, then the point of contact between the tire and the road is instantaneously at rest - it serves as an instantaneous axis of rotation. If the static friction force is not large enough, the tire will begin to slip, or skid, and the wheel will turn without the car moving forward as fast. If the static friction force is very small, the wheel may spin without moving the car forward at all, and the car will not gain any kinetic energy (except the kinetic energy of the spinning tires).
14. The gravitational potential energy is the greatest when the Earth is farthest from the Sun, or when the Northern Hemisphere has summer. (Note that the Earth moves fastest in its orbit, and therefore has the greatest kinetic energy, when it is closest to the Sun.)
15. Yes. If the potential energy $U$ is negative (which it can be defined to be), and the absolute value of the potential energy is greater than the kinetic energy $K$, then the total mechanical energy $E$ will be negative.
16. In order to escape the Earth's gravitational field, the rocket needs a certain minimum speed with respect to the center of the Earth. If you launch the rocket from any location except the poles, then the rocket will have a tangential velocity due to the rotation of the Earth. This velocity is towards the east and is greatest at the equator, where the surface of the Earth is farthest from the axis of rotation. In order to use the minimum amount of fuel, you need to maximize the contribution of this tangential
velocity to the needed escape velocity, so launch the rocket towards the east from a point as close as possible to the equator. (As an added bonus, the weight of the rocket will be slightly less at the equator because the Earth is not a perfect sphere and the surface is farthest from the center at the equator.)
17. For every meter the load is raised, two meters of rope must be pulled up. The work done on the piano must be equal to the work done by you. Since you are pulling with half the force (the tension in the rope is equal to half of the weight of the piano), you must pull through twice the distance to do the same amount of work.
18. The faster arrow has the same mass and twice the speed of the slower arrow, so will have four times the kinetic energy $\left(K=\frac{1}{2} m \nu^{2}\right)$. Therefore, four times as much work must be done on the faster arrow to bring it to rest. If the force on the arrows is constant, the faster arrow will travel four times the distance of the slower arrow in the hay.
19. When the ball is released, its potential energy will be converted into kinetic energy and then back into potential energy as the ball swings. If the ball is not pushed, it will lose a little energy to friction and air resistance, and so will return almost to the initial position, but will not hit the instructor. If the ball is pushed, it will have an initial kinetic energy, and will, when it returns, still have some kinetic energy when it reaches the initial position, so it will hit the instructor in the nose. (Ouch!)
20. Neglecting any air resistance or friction in the pivot, the pendulum bob will have the same speed at the lowest point for both launches. In both cases, the initial energy is equal to potential energy mgh plus kinetic energy $\frac{1}{2} m v^{2}$, with $v=3.0 \mathrm{~m} / \mathrm{s}$. (Notice that the direction of the velocity doesn't matter.) Since the total energy at any point in the swing is constant, the pendulum will have the same energy at the lowest point, and therefore the same speed, for both launches.
21. When a child hops around on a pogo stick, gravitational potential energy (at the top of the hop) is transformed into kinetic energy as the child moves downward, and then stored as spring potential energy as the spring in the pogo stick compresses. As the spring begins to expand, the energy is converted back to kinetic and gravitational potential energy, and the cycle repeats. Since energy is lost due to friction, the child must add energy to the system by pushing down on the pogo stick while it is on the ground to get a greater spring compression.
22. At the top of the hill, the skier has gravitational potential energy. If the friction between her skis and the snow is negligible, the gravitational potential energy is changed into kinetic energy as she glides down the hill and she gains speed as she loses elevation. When she runs into the snow bank, work is done by the friction between her skis and the snow and the energy changes from kinetic energy of the skier to kinetic energy of the snow as it moves and to thermal energy.
23. The work done on the suitcase depends only on (c) the height of the table and (d) the weight of the suitcase.
24. Power is the rate of doing work. Both $(c)$ and (d) will affect the total amount of work needed, and hence the power. (b), the time the lifting takes, will also affect the power. The length of the path (a) will only affect the power if different paths take different times to traverse.
25. When you climb a mountain by going straight up, the force needed is large (and the distance traveled is small), and the power needed (work per unit time) is also large. If you take a zigzag trail, you will use a smaller force (over a longer distance, so that the work done is the same) and less power, since
the time to climb the mountain will be longer. A smaller force and smaller power output make the climb seem easier.
26. (a) The force is proportional to the negative of the slope of the potential energy curve, so the magnitude of the force will be greatest where the curve is steepest, at point C .
(b) The force acts to the left at points $\mathrm{A}, \mathrm{E}$, and F , to the right at point C , and is zero at points $\mathrm{B}, \mathrm{D}$, and G .
(c) Equilibrium exists at points B, D, and G. B is a point of neutral equilibrium, D is a point of stable equilibrium, and G is a point of unstable equilibrium.
27. (a) If the particle has $E_{3}$ at $x_{6}$, then it has both potential and kinetic energy at that point. As the particle moves toward $x_{0}$, it gains kinetic energy as its speed increases. Its speed will be a maximum at $x_{0}$. As the particle moves to $x_{4}$, its speed will decrease, but will be larger than its initial speed. As the particle moves to $x_{5}$, its speed will increase, then decrease to zero. The process is reversed on the way back to $x_{6}$. At each point on the return trip the speed of the particle is the same as it was on the forward trip, but the direction of the velocity is opposite.
(b) The kinetic energy is greatest at point $x_{0}$, and least at $x_{5}$.
28. A is a point of unstable equilibrium, B is a point of stable equilibrium, and C is a point of neutral equilibrium.

## Solutions to Problems

1. The potential energy of the spring is given by $U_{\mathrm{el}}=\frac{1}{2} k x^{2}$ where $x$ is the distance of stretching or compressing of the spring from its natural length.

$$
x=\sqrt{\frac{2 U_{\mathrm{el}}}{k}}=\sqrt{\frac{2(35.0 \mathrm{~J})}{82.0 \mathrm{~N} / \mathrm{m}}}=0.924 \mathrm{~m}
$$

2. Subtract the initial gravitational potential energy from the final gravitational potential energy.

$$
\Delta U_{\text {grav }}=m g y_{2}-m g y_{1}=m g\left(y_{2}-y_{1}\right)=(6.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(1.3 \mathrm{~m})=76 \mathrm{~J}
$$

3. The spring will stretch enough to hold up the mass. The force exerted by the spring will be equal to the weight of the mass.

$$
m g=k(\Delta x) \rightarrow \Delta x=\frac{m g}{k}=\frac{(2.5 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{63 \mathrm{~N} / \mathrm{m}}=0.39 \mathrm{~m}
$$

Thus the ruler reading will be $39 \mathrm{~cm}+15 \mathrm{~cm}=54 \mathrm{~cm}$.
4. (a) The change in gravitational potential energy is given by the following.

$$
\Delta U_{\text {grav }}=m g\left(y_{2}-y_{1}\right)=(56.5 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(2660 \mathrm{~m}-1270 \mathrm{~m})=7.7 \times 10^{5} \mathrm{~J}
$$

(b) The minimum work required by the hiker would equal the change in potential energy, which is $7.7 \times 10^{5} \mathrm{~J}$.
(c) Yes. The actual work may be more than this, because the climber almost certainly had to overcome some dissipative forces such as air friction. Also, as the person steps up and down, they do not get the full amount of work back from each up-down event. For example, there will be friction in their joints and muscles.
5. (a) Relative to the ground, the potential energy is given by the following.

$$
U_{\text {grav }}=m g\left(y_{\text {book }}-y_{\text {ground }}\right)=(1.95 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(2.20 \mathrm{~m})=42.0 \mathrm{~J}
$$

(b) Relative to the top of the person's head, the potential energy is given by the following.

$$
U_{\text {grav }}=m g\left(y_{\text {book }}-y_{\text {head }}\right)=(1.95 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(2.20 \mathrm{~m}-1.60 \mathrm{~m})=11.47 \mathrm{~J} \approx 11 \mathrm{~J}
$$

(c) The work done by the person in lifting the book from the ground to the final height is the same as the answer to part $(a), 42.0 \mathrm{~J}$. In part $(a)$, the potential energy is calculated relative to the starting location of the application of the force on the book. The work done by the person is not related to the answer to part $(b)$, because the potential energy is not calculated relative to the starting location of the application of the force on the book.
6. Assume that all of the kinetic energy of the car becomes potential energy of the compressed spring.

$$
\frac{1}{2} m v_{0}^{2}=\frac{1}{2} k x_{\text {final }}^{2} \rightarrow k=\frac{m v_{0}^{2}}{x_{\text {final }}^{2}}=\frac{(1200 \mathrm{~kg})\left[(75 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)\right]^{2}}{(2.2 \mathrm{~m})^{2}}=1.1 \times 10^{5} \mathrm{~N} / \mathrm{m}
$$

7. (a) This force is conservative, because the work done by the force on an object moving from an initial position $\left(x_{1}\right)$ to a final position $\left(x_{2}\right)$ depends only on the endpoints.

$$
\begin{aligned}
W & =\int_{x_{1}}^{x_{2}} \overrightarrow{\mathbf{F}} \cdot d \overrightarrow{\mathbf{l}}=\int_{x_{1}}^{x_{2}} F_{x} d x=\int_{x_{1}}^{x_{2}}\left(-k x+a x^{3}+b x^{4}\right) d x=\left(-\frac{1}{2} k x^{2}+\frac{1}{4} a x^{4}+\frac{1}{5} b x^{5}\right)_{x_{1}}^{x_{2}} \\
& =\left(-\frac{1}{2} k x_{2}^{2}+\frac{1}{4} a x_{2}^{4}+\frac{1}{5} b x_{2}^{5}\right)-\left(-\frac{1}{2} k x_{1}^{2}+\frac{1}{4} a x_{1}^{4}+\frac{1}{5} b x_{1}^{5}\right)
\end{aligned}
$$

The expression for the work only depends on the endpoints.
(b) Since the force is conservative, there is a potential energy function $U$ such that $F_{x}=-\frac{\partial U}{\partial x}$.

$$
F_{x}=\left(-k x+a x^{3}+b x^{4}\right)=-\frac{\partial U}{\partial x} \rightarrow U(x)=\frac{1}{2} k x^{2}-\frac{1}{4} a x^{4}-\frac{1}{5} b x^{5}+C
$$

8. The force is found from the relations on page 189.

$$
\begin{aligned}
& F_{x}=-\frac{\partial U}{\partial x}=-(6 x+2 y) \quad F_{y}=-\frac{\partial U}{\partial y}=-(2 x+8 y z) \quad F_{z}=-\frac{\partial U}{\partial z}=-4 y^{2} \\
& \overrightarrow{\mathbf{F}}=\hat{\mathbf{i}}(-6 x-2 y)+\hat{\mathbf{j}}(-2 x-8 y z)+\overrightarrow{\mathbf{k}}\left(-4 y^{2}\right)
\end{aligned}
$$

9. Use Eq. 8-6 to find the potential energy function.

$$
\begin{aligned}
& U(x)=-\int F(x) d x+C=-\int-\frac{k}{x^{3}} d x+C=-\frac{k}{2 x^{2}}+C \\
& U(2.0 \mathrm{~m})=-\frac{k}{2(2.0 \mathrm{~m})^{2}}+C=0 \rightarrow C=\frac{k}{8 \mathrm{~m}^{2}} \rightarrow U(x)=-\frac{k}{2 x^{2}}+\frac{k}{8 \mathrm{~m}^{2}}
\end{aligned}
$$

10. Use Eq. 8-6 to find the potential energy function.

$$
U(x)=-\int F(x) d x+C=-\int A \sin (k x) d x+C=\frac{A}{k} \cos (k x)+C
$$

$$
U(0)=\frac{A}{k}+C=0 \rightarrow C=-\frac{A}{k} \rightarrow U(x)=\frac{A}{k}[\cos (k x)-1]
$$

11. The forces on the skier are gravity and the normal force. The normal force is perpendicular to the direction of motion, and so does no work. Thus the skier's mechanical energy is conserved. Subscript 1 represents the skier at the top of the hill, and subscript 2 represents the skier at the bottom of the hill. The ground is the zero location for gravitational potential energy $(y=0)$. We have
 $v_{1}=0, y_{1}=125 \mathrm{~m}$, and $y_{2}=0$ (bottom of the hill). Solve for $v_{2}$, the speed at the bottom.

$$
\begin{aligned}
& \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2} \rightarrow 0+m g y_{1}=\frac{1}{2} m v_{2}^{2}+0 \rightarrow \\
& v_{2}=\sqrt{2 g y_{1}}=\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(125 \mathrm{~m})}=49 \mathrm{~m} / \mathrm{s}(\approx 110 \mathrm{mi} / \mathrm{h})
\end{aligned}
$$

12. The only forces acting on Jane are gravity and the vine tension. The tension pulls in a centripetal direction, and so can do no work - the tension force is perpendicular at all times to her motion. So Jane's mechanical energy is conserved. Subscript 1 represents Jane at the point where she grabs the vine, and subscript 2 represents Jane at the highest point of her swing. The ground is the zero location for gravitational potential energy $(y=0)$. We have $v_{1}=5.0 \mathrm{~m} / \mathrm{s}$,
 $y_{1}=0$, and $v_{2}=0$ (top of swing). Solve for $y_{2}$, the height of her swing.

$$
\begin{aligned}
& \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+0=0+m g y_{2} \rightarrow \\
& y_{2}=\frac{v_{1}^{2}}{2 g}=\frac{(5.0 \mathrm{~m} / \mathrm{s})^{2}}{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=1.276 \mathrm{~m} \approx 1.3 \mathrm{~m}
\end{aligned}
$$

No , the length of the vine does not enter into the calculation, unless the vine is less than 0.65 m long. If that were the case, she could not rise 1.3 m high.
13. We assume that all the forces on the jumper are conservative, so that the mechanical energy of the jumper is conserved. Subscript 1 represents the jumper at the bottom of the jump, and subscript 2 represents the jumper at the top of the jump. Call the ground the zero location for gravitational potential energy $(y=0)$. We have $y_{1}=0, v_{2}=0.70 \mathrm{~m} / \mathrm{s}$, and $y_{2}=2.10 \mathrm{~m}$. Solve for $v_{1}$, the speed at the bottom.

$$
\begin{aligned}
& \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+0=\frac{1}{2} m v_{2}^{2}+m g y_{2} \rightarrow \\
& v_{1}=\sqrt{v_{2}^{2}+2 g y_{2}}=\sqrt{(0.70 \mathrm{~m} / \mathrm{s})^{2}+2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(2.10 \mathrm{~m})}=6.454 \mathrm{~m} / \mathrm{s} \approx 6.5 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

14. The forces on the sled are gravity and the normal force. The normal force is perpendicular to the direction of motion, and so does no work. Thus the sled's mechanical energy is conserved. Subscript 1 represents the sled at the bottom of the hill, and subscript 2 represents the sled at the top of the hill. The ground is the zero location for gravitational potential energy $(y=0)$. We have $y_{1}=0$,
 $v_{2}=0$, and $y_{2}=1.12 \mathrm{~m}$. Solve for $v_{1}$, the speed at the bottom. Note that the angle is not used.

$$
\begin{aligned}
& \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+0=0+m g y_{2} \rightarrow \\
& v_{1}=\sqrt{2 g y_{2}}=\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(1.12 \mathrm{~m})}=4.69 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

15. Consider this diagram for the jumper's fall.
(a) The mechanical energy of the jumper is conserved. Use $y$ for the distance from the 0 of gravitational potential energy and $x$ for the amount of bungee cord "stretch" from its unstretched length. Subscript 1 represents the jumper at the start of the fall, and subscript 2 represents the jumper at the lowest point of the fall. The bottom of the fall is the zero location for gravitational potential energy $(y=0)$, and the location where the bungee cord just starts to be stretched is the zero location for elastic potential energy $(x=0)$. We have $v_{1}=0, y_{1}=31 \mathrm{~m}, x_{1}=0, v_{2}=0$,


19 m

$y_{2}=0$, and $x_{2}=19 \mathrm{~m}$. Apply conservation of energy.

$$
\begin{aligned}
& E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}+\frac{1}{2} k x_{1}^{2}=\frac{1}{2} m v_{2}^{2}+m g y_{2}+\frac{1}{2} k x_{2}^{2} \rightarrow m g y_{1}=\frac{1}{2} k x_{2}^{2} \rightarrow \\
& k=\frac{2 m g y_{1}}{x_{2}^{2}}=\frac{2(55 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(31 \mathrm{~m})}{(19 \mathrm{~m})^{2}}=92.57 \mathrm{~N} / \mathrm{m} \approx 93 \mathrm{~N} / \mathrm{m}
\end{aligned}
$$

(b) The maximum acceleration occurs at the location of the maximum force, which occurs when the bungee cord has its maximum stretch, at the bottom of the fall. Write Newton's second law for the force on the jumper, with upward as positive.

$$
\begin{aligned}
& F_{\text {net }}=F_{\text {cord }}-m g=k x_{2}-m g=m a \rightarrow \\
& a=\frac{k x_{2}}{m}-g=\frac{(92.57 \mathrm{~N} / \mathrm{m})(19 \mathrm{~m})}{(55 \mathrm{~kg})}-9.80 \mathrm{~m} / \mathrm{s}^{2}=22.2 \mathrm{~m} / \mathrm{s}^{2} \approx 22 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$


16. (a) Since there are no dissipative forces present, the mechanical energy of the person-trampolineEarth combination will be conserved. We take the level of the unstretched trampoline as the zero level for both elastic and gravitational potential energy. Call up the positive direction. Subscript 1 represents the jumper at the start of the jump, and subscript 2 represents the jumper upon arriving at the trampoline. There is no elastic potential energy involved in this part of the problem. We have $v_{1}=4.5 \mathrm{~m} / \mathrm{s}, y_{1}=2.0 \mathrm{~m}$, and $y_{2}=0$. Solve for $v_{2}$, the speed upon arriving at the trampoline.

$$
\begin{aligned}
& E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+0 \rightarrow \\
& v_{2}= \pm \sqrt{v_{1}^{2}+2 g y_{1}}= \pm \sqrt{(4.5 \mathrm{~m} / \mathrm{s})^{2}+2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(2.0 \mathrm{~m})}= \pm 7.710 \mathrm{~m} / \mathrm{s} \approx 7.7 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

The speed is the absolute value of $v_{2}$.
(b) Now let subscript 3 represent the jumper at the maximum stretch of the trampoline, and $x$ represent the amount of stretch of the trampoline. We have $v_{2}=-7.710 \mathrm{~m} / \mathrm{s}, y_{2}=0, x_{2}=0$, $v_{3}=0$, and $x_{3}=y_{3}$. There is no elastic energy at position 2, but there is elastic energy at position 3. Also, the gravitational potential energy at position 3 is negative, and so $y_{3}<0$. A quadratic relationship results from the conservation of energy condition.

$$
\begin{aligned}
& E_{2}=E_{3} \rightarrow \frac{1}{2} m v_{2}^{2}+m g y_{2}+\frac{1}{2} k x_{2}^{2}=\frac{1}{2} m v_{3}^{2}+m g y_{3}+\frac{1}{2} k x_{3}^{2} \rightarrow \\
& \frac{1}{2} m v_{2}^{2}+0+0=0+m g y_{3}+\frac{1}{2} k y_{3}^{2} \rightarrow \frac{1}{2} k y_{3}^{2}+m g y_{3}-\frac{1}{2} m v_{2}^{2}=0 \rightarrow \\
& y_{3}=\frac{-m g \pm \sqrt{m^{2} g^{2}-4\left(\frac{1}{2} k\right)\left(-\frac{1}{2} m v_{2}^{2}\right)}}{2\left(\frac{1}{2} k\right)}=\frac{-m g \pm \sqrt{m^{2} g^{2}+k m v_{2}^{2}}}{k}
\end{aligned}
$$

$$
\begin{aligned}
& =\frac{-(72 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \pm \sqrt{(72 \mathrm{~kg})^{2}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)^{2}+\left(5.8 \times 10^{4} \mathrm{~N} / \mathrm{m}\right)(72 \mathrm{~kg})(7.71 \mathrm{~m} / \mathrm{s})^{2}}}{\left(5.8 \times 10^{4} \mathrm{~N} / \mathrm{m}\right)} \\
& =-0.284 \mathrm{~m}, 0.260 \mathrm{~m}
\end{aligned}
$$

Since $y_{3}<0, y_{3}=-0.28 \mathrm{~m}$.
The first term under the quadratic is about 500 times smaller than the second term, indicating that the problem could have been approximated by not even including gravitational potential energy for the final position. If that approximation were made, the result would have been found by taking the negative result from the following solution.

$$
E_{2}=E_{3} \rightarrow \frac{1}{2} m v_{2}^{2}=\frac{1}{2} k y_{3}^{2} \rightarrow y_{3}=v_{2} \sqrt{\frac{m}{k}}=(7.71 \mathrm{~m} / \mathrm{s}) \sqrt{\frac{72 \mathrm{~kg}}{5.8 \times 10^{4} \mathrm{~N} / \mathrm{m}}}= \pm 0.27 \mathrm{~m}
$$

17. Take specific derivatives with respect to position, and note that $E$ is constant.

$$
E=\frac{1}{2} m v^{2}+U \rightarrow \frac{d E}{d x}=\frac{1}{2} m\left(2 v \frac{d v}{d x}\right)+\frac{d U}{d x}=m v \frac{d v}{d x}+\frac{d U}{d x}=0
$$

Use the chain rule to change $v \frac{d v}{d x}$ to $\frac{d x}{d t} \frac{d v}{d x}=\frac{d v}{d t}$.

$$
m v \frac{d v}{d x}+\frac{d U}{d x}=0 \rightarrow m \frac{d v}{d t}=-\frac{d U}{d x} \rightarrow m a=F
$$

The last statement is Newton's second law.
18. (a) See the diagram for the thrown ball. The speed at the top of the path will be the horizontal component of the original velocity.

$$
v_{\text {top }}=v_{0} \cos \theta=(8.5 \mathrm{~m} / \mathrm{s}) \cos 36^{\circ}=6.9 \mathrm{~m} / \mathrm{s}
$$

(b) Since there are no dissipative forces in the problem, the mechanical energy of the ball is conserved. Subscript 1 represents the ball at the release point, and subscript 2 represents the ball at the top of the path. The ball's release point is the zero location for gravitational potential energy $(y=0)$. We have $v_{1}=8.5 \mathrm{~m} / \mathrm{s}, y_{1}=0$, and $v_{2}=v_{1} \cos \theta$. Solve for $y_{2}$.

$$
\begin{aligned}
& E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+0=\frac{1}{2} m v_{1}^{2} \cos ^{2} \theta+m g y_{2} \rightarrow \\
& y_{2}=\frac{v_{1}^{2}\left(1-\cos ^{2} \theta\right)}{2 g}=\frac{(8.5 \mathrm{~m} / \mathrm{s})^{2}\left(1-\cos ^{2} 36^{\circ}\right)}{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=1.3 \mathrm{~m}
\end{aligned}
$$

This is the height above its throwing level.
19. Use conservation of energy. The level of the ball on the uncompressed spring is taken as the zero location for both gravitational potential energy $(y=0)$ and elastic potential energy $(x=0)$. It is diagram 2 in the figure.
Take "up" to be positive for both $x$ and $y$.
(a) Subscript 1 represents the ball at the launch point, and subscript 2 represents the ball at the location where it just leaves the spring, at the uncompressed length. We have $v_{1}=0, x_{1}=y_{1}=-0.160 \mathrm{~m}$, and $x_{2}=y_{2}=0$. Solve for $v_{2}$.


$$
\begin{aligned}
& E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}+\frac{1}{2} k x_{1}^{2}=\frac{1}{2} m v_{2}^{2}+m g y_{2}+\frac{1}{2} k x_{2}^{2} \rightarrow \\
& 0+m g y_{1}+\frac{1}{2} k x_{1}^{2}=\frac{1}{2} m v_{2}^{2}+0+0 \rightarrow v_{2}=\sqrt{\frac{k x_{1}^{2}+2 m g y_{1}}{m}} \\
& v_{2}=\sqrt{\frac{(875 \mathrm{~N} / \mathrm{m})(0.160 \mathrm{~m})^{2}+2(0.380 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(-0.160 \mathrm{~m})}{(0.380 \mathrm{~kg})}}=7.47 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(b) Subscript 3 represents the ball at its highest point. We have $v_{1}=0, x_{1}=y_{1}=-0.160 \mathrm{~m}, v_{3}=0$, and $x_{3}=0$. Solve for $y_{3}$.

$$
\begin{aligned}
& E_{1}=E_{3} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}+\frac{1}{2} k x_{1}^{2}=\frac{1}{2} m v_{3}^{2}+m g y_{3}+\frac{1}{2} k x_{3}^{2} \rightarrow \\
& 0+m g y_{1}+\frac{1}{2} k x_{1}^{2}=0+m g y_{2}+0 \rightarrow y_{2}-y_{1}=\frac{k x_{1}^{2}}{2 m g}=\frac{(875 \mathrm{~N} / \mathrm{m})(0.160 \mathrm{~m})^{2}}{2(0.380 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=3.01 \mathrm{~m}
\end{aligned}
$$

20. Since there are no dissipative forces present, the mechanical energy of the roller coaster will be conserved. Subscript 1 represents the coaster at point 1 , etc. The height of point 2 is the zero location for gravitational potential energy. We have $v_{1}=0$ and $y_{1}=32 \mathrm{~m}$.
Point 2: $\frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2} \quad ; y_{2}=0 \rightarrow m g y_{1}=\frac{1}{2} m v_{2}^{2} \rightarrow$

$$
v_{2}=\sqrt{2 g y_{1}}=\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(32 \mathrm{~m})}=25 \mathrm{~m} / \mathrm{s}
$$

Point 3: $\frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{3}^{2}+m g y_{3} \quad ; y_{3}=26 \mathrm{~m} \rightarrow m g y_{1}=\frac{1}{2} m v_{3}^{2}+m g y_{3} \quad \rightarrow$

$$
v_{3}=\sqrt{2 g\left(y_{1}-y_{3}\right)}=\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(6 \mathrm{~m})}=11 \mathrm{~m} / \mathrm{s}
$$

Point 4: $\frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{4}^{2}+m g y_{4} \quad ; y_{4}=14 \mathrm{~m} \rightarrow m g y_{1}=\frac{1}{2} m v_{4}^{2}+m g y_{1} \quad \rightarrow$

$$
v_{4}=\sqrt{2 g\left(y_{1}-y_{4}\right)}=\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(18 \mathrm{~m})}=19 \mathrm{~m} / \mathrm{s}
$$

21. With the mass at rest on the spring, the upward force due to the spring must be the same as the weight of the mass.

$$
k d=m g \quad \rightarrow \quad d=\frac{m g}{k}
$$

The distance $D$ is found using conservation of energy. Subscript 1 represents the mass at the top of the uncompressed spring, and subscript 2 represents the mass at the bottom of its motion, where the spring is compressed by $D$. Take the top of the uncompressed spring to be the zero location for both gravitational and elastic potential energy $(y=0)$. Choose up to be the positive direction. We have $v_{1}=v_{2}=0, \quad y_{1}=0$, and $y_{2}=-D$. Solve for $D$.

$$
\begin{aligned}
& E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}+\frac{1}{2} k y_{1}^{2}=\frac{1}{2} m v_{2}^{2}+m g y_{2}+\frac{1}{2} k y_{2}^{2} \rightarrow \\
& 0+0+0=0-m g D+\frac{1}{2} k D^{2} \rightarrow D=\frac{2 m g}{k}
\end{aligned}
$$

We see that $D=2 d$, and so $D \neq d$. The reason that the two distances are not equal is that putting the mass at rest at the compressed position requires that other work be done in addition to the work done by gravity and the spring. That other work is not done by a conservative force, but done instead by an external agent such as your hand.
22. (a) Draw a free-body diagram for each block. Write Newton's second law for each block. Notice that the acceleration of block A in the $y_{\mathrm{A}}$ is 0 zero.

$$
\begin{aligned}
& \sum F_{y 1}=F_{\mathrm{N}}-m_{\mathrm{A}} g \cos \theta=0 \rightarrow F_{\mathrm{N}}=m_{\mathrm{A}} g \cos \theta \\
& \sum F_{x 1}=F_{\mathrm{T}}-m_{\mathrm{A}} g \sin \theta=m_{\mathrm{A}} a_{x \mathrm{~A}} \\
& \sum F_{y 2}=m_{\mathrm{B}} g-F_{\mathrm{T}}=m_{\mathrm{B}} a_{y \mathrm{~B}} \rightarrow F_{\mathrm{T}}=m_{\mathrm{B}}\left(g+a_{y \mathrm{~B}}\right)
\end{aligned}
$$

Since the blocks are connected by the cord,
 $a_{y \mathrm{~B}}=a_{x \mathrm{~A}}=a$. Substitute the expression for the tension force from the last equation into the $x$ direction equation for block 1 , and solve for the acceleration.

$$
\begin{aligned}
& m_{\mathrm{B}}(g+a)-m_{\mathrm{A}} g \sin \theta=m_{\mathrm{A}} a \rightarrow m_{\mathrm{B}} g-m_{\mathrm{A}} g \sin \theta=m_{\mathrm{A}} a+m_{\mathrm{B}} a \\
& a=g \frac{\left(m_{\mathrm{B}}-m_{\mathrm{A}} \sin \theta\right)}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)}=\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \frac{\left(5.0 \mathrm{~kg}-4.0 \mathrm{~kg} \sin 32^{\circ}\right)}{9.0 \mathrm{~kg}}=3.1 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

(b) Find the final speed of $m_{\mathrm{B}}$ (which is also the final speed of $m_{\mathrm{A}}$ ) using constant acceleration relationships.

$$
\begin{aligned}
& v_{f}^{2}=v_{0}^{2}+2 a \Delta y \rightarrow v_{f}^{2}=2 g \frac{\left(m_{\mathrm{B}}-m_{\mathrm{A}} \sin \theta\right)}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)} h \rightarrow \\
& v_{f}=\sqrt{2 g h \frac{\left(m_{\mathrm{B}}-m_{\mathrm{A}} \sin \theta\right)}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)}}=\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.75 \mathrm{~m}) \frac{\left(5.0 \mathrm{~kg}-4.0 \mathrm{~kg} \sin 32^{\circ}\right)}{9.0 \mathrm{~kg}}}=2.2 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(c) Since there are no dissipative forces in the problem, the mechanical energy of the system is conserved. Subscript 1 represents the blocks at the release point, and subscript 2 represents the blocks when $m_{\mathrm{B}}$ reaches the floor. The ground is the zero location for gravitational potential energy for $m_{\mathrm{B}}$, and the starting location for $m_{\mathrm{A}}$ is its zero location for gravitational potential energy. Since $m_{\mathrm{B}}$ falls a distance $h, m_{\mathrm{A}}$ moves a distance $h$ along the plane, and so rises a distance $h \sin \theta$. The starting speed is 0 .

$$
\begin{aligned}
& E_{1}=E_{2} \rightarrow 0+m_{\mathrm{A}} g h=\frac{1}{2}\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) v_{2}^{2}+m_{\mathrm{B}} g h \sin \theta \rightarrow \\
& v_{2}=\sqrt{2 g h\left(\frac{m_{\mathrm{A}}-m_{\mathrm{B}} \sin \theta}{m_{\mathrm{A}}+m_{\mathrm{B}}}\right)}
\end{aligned}
$$

This is the same expression found in part $(b)$, and so gives the same numeric result.
23. At the release point the mass has both kinetic energy and elastic potential energy. The total energy is $\frac{1}{2} m v_{0}^{2}+\frac{1}{2} k x_{0}^{2}$. If friction is to be ignored, then that total energy is constant.
(a) The mass has its maximum speed at a displacement of 0 , and so only has kinetic energy at that point.

$$
\frac{1}{2} m v_{0}^{2}+\frac{1}{2} k x_{0}^{2}=\frac{1}{2} m v_{\max }^{2} \rightarrow v_{\max }=\sqrt{\sqrt{v_{0}^{2}+\frac{k}{m} x_{0}^{2}}}
$$

(b) The mass has a speed of 0 at its maximum stretch from equilibrium, and so only has potential energy at that point.

$$
\frac{1}{2} m v_{0}^{2}+\frac{1}{2} k x_{0}^{2}=\frac{1}{2} k x_{\max }^{2} \quad \rightarrow \quad x_{\max }=\sqrt{\sqrt{x_{0}^{2}+\frac{m}{k} v_{0}^{2}}}
$$

24. (a) The work done against gravity is the change in potential energy.

$$
W_{\substack{\text { against } \\ \text { gravity }}}=\Delta U=m g\left(y_{2}-y_{1}\right)=(75.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(125 \mathrm{~m})=9.19 \times 10^{4} \mathrm{~J}
$$

(b) The work done by the force on the pedals in one revolution is equal to the average tangential force times the circumference of the circular path of the pedals. That work is also equal to the potential energy change of the bicycle during that revolution, assuming that the speed of the bicycle is constant. Note that a vertical rise on the incline is related to the distance along the incline by rise $=$ distance $\times(\sin \theta)$.

$$
\begin{aligned}
& W_{\substack{\text { pedal } \\
\text { force }}}=F_{\text {tan }} 2 \pi r=\Delta U_{\substack{\text { grav } \\
1 \text { rev }}}=m g(\Delta y)_{1 \text { rev }}=m g d_{1 \text { rev }} \sin \theta \rightarrow \\
& F_{\text {tan }}=\frac{m g d_{1 \text { rev }} \sin \theta}{2 \pi r}=\frac{(75.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(5.10 \mathrm{~m}) \sin 9.50^{\circ}}{2 \pi(0.180 \mathrm{~m})}=547 \mathrm{~N}
\end{aligned}
$$

25. Since there are no dissipative forces in the problem, the mechanical energy of the pendulum bob is conserved. Subscript 1 represents the bob at the release point, and subscript 2 represents the ball at some subsequent position. The lowest point in the swing of the pendulum is the zero location for potential energy $(y=0)$. We have $v_{1}=0$ and $y_{1}=\ell(1-\cos \theta)$. The "second" point for the energy conservation will vary from part to part of the problem.

(a) The second point is at the bottom of the swing, so $y_{2}=0$.

$$
\begin{aligned}
& E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2} \rightarrow m g \ell\left(1-\cos 30.0^{\circ}\right)=\frac{1}{2} m v_{2}^{2} \rightarrow \\
& v_{2}=\sqrt{2 g \ell\left(1-\cos 30.0^{\circ}\right)}=\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(2.00 \mathrm{~m})\left(1-\cos 30.0^{\circ}\right)}=2.29 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(b) The second point is displaced from equilibrium by $15.0^{\circ}$, so $y_{2}=\ell\left(1-\cos 15.0^{\circ}\right)$.

$$
\begin{aligned}
& E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2} \rightarrow \\
& m g \ell\left(1-\cos 30.0^{\circ}\right)=\frac{1}{2} m v_{2}^{2}+m g \ell\left(1-\cos 15.0^{\circ}\right) \rightarrow v_{2}=\sqrt{2 g \ell\left(\cos 15.0^{\circ}-\cos 30.0^{\circ}\right)} \\
& \qquad=\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(2.00 \mathrm{~m})\left(\cos 15.0^{\circ}-\cos 30.0^{\circ}\right)}=1.98 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(c) The second point is displaced from equilibrium by $-15.0^{\circ}$. The pendulum bob is at the same height at $-15.0^{\circ}$ as it was at $15.0^{\circ}$, and so the speed is the same. Also, since $\cos (-\theta)=\cos (\theta)$, the mathematics is identical. Thus $v_{2}=1.98 \mathrm{~m} / \mathrm{s}$.
(d) The tension always pulls radially on the pendulum bob, and so is related to the centripetal force on the bob. The net centripetal force is always $m v^{2} / r$. Consider the free body diagram for the pendulum bob at each position.
(a) $F_{\mathrm{T}}-m g=\frac{m v^{2}}{r} \rightarrow F_{\mathrm{T}}=m\left(g+\frac{v^{2}}{\ell}\right)=m\left(g+\frac{2 g \ell\left(1-\cos 30.0^{\circ}\right)}{\ell}\right)$

$$
=m g\left(3-2 \cos 30.0^{\circ}\right)=(0.0700 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(3-2 \cos 30.0^{\circ}\right)=0.870 \mathrm{~N}
$$

(b) $F_{\mathrm{T}}-m g \cos \theta=\frac{m v^{2}}{r} \rightarrow F_{\mathrm{T}}=m\left(g \cos \theta+\frac{v^{2}}{\ell}\right)$


$$
\begin{aligned}
& =m\left(g \cos 15.0^{\circ}+\frac{2 g \ell\left(\cos 15.0^{\circ}-\cos 30.0^{\circ}\right)}{\ell}\right) \\
& =m g\left(3 \cos 15.0^{\circ}-2 \cos 30.0^{\circ}\right) \\
& =(0.0700 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(3 \cos 15.0^{\circ}-2 \cos 30.0^{\circ}\right)=0.800 \mathrm{~N}
\end{aligned}
$$

(c) Again, as earlier, since the cosine and the speed are the same for $-15.0^{\circ}$ as for $15.0^{\circ}$, the tension will be the same, 0.800 N .
(e) Again use conservation of energy, but now we have $v_{1}=v_{0}=1.20 \mathrm{~m} / \mathrm{s}$.
(a) The second point is at the bottom of the swing, so $y_{2}=0$.

$$
\begin{aligned}
& \frac{1}{2} m v_{1}^{2}+m g \ell\left(1-\cos 30.0^{\circ}\right)=\frac{1}{2} m v_{2}^{2} \rightarrow v_{2}=\sqrt{v_{1}^{2}+2 g \ell\left(1-\cos 30.0^{\circ}\right)} \\
& \quad=\sqrt{(1.20 \mathrm{~m} / \mathrm{s})^{2}+2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(2.00 \mathrm{~m})\left(1-\cos 30.0^{\circ}\right)}=2.59 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(b) The second point is displaced from equilibrium by $15.0^{\circ}$, so $y_{2}=\ell\left(1-\cos 15.0^{\circ}\right)$.

$$
\begin{aligned}
& \frac{1}{2} m v_{1}^{2}+m g \ell\left(1-\cos 30.0^{\circ}\right)=\frac{1}{2} m v_{2}^{2}+m g \ell\left(1-\cos 15.0^{\circ}\right) \rightarrow \\
& v_{2}=\sqrt{v_{1}^{2}+2 g \ell\left(\cos 15.0^{\circ}-\cos 30.0^{\circ}\right)} \\
& =\sqrt{(1.20 \mathrm{~m} / \mathrm{s})^{2}+2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(2.00 \mathrm{~m})\left(\cos 15.0^{\circ}-\cos 30.0^{\circ}\right)}=2.31 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(c) As before, the pendulum bob is at the same height at $-15.0^{\circ}$ as it was at $15.0^{\circ}$, and so the speed is the same. Thus $v_{2}=2.31 \mathrm{~m} / \mathrm{s}$.
26. The maximum acceleration of 5.0 g occurs where the force is at a maximum. The maximum force occurs at the maximum displacement from the equilibrium of the spring. The acceleration and the displacement are related by Newton's second law and the spring law, $F_{\text {net }}=F_{\text {spring }} \rightarrow m a=-k x$ $\rightarrow x=-\frac{m}{k} a$. Also, by conservation of energy, the initial kinetic energy of the car will become the final potential energy stored in the spring.

$$
\begin{aligned}
& E_{\text {initial }}=E_{\text {final }} \rightarrow \frac{1}{2} m v_{0}^{2}=\frac{1}{2} k x_{\max }^{2}=\frac{1}{2} k\left(\frac{m}{k} a_{\max }\right)^{2}=\frac{1}{2} \frac{m^{2}}{k}(5.0 g)^{2} \rightarrow \\
& k=\frac{m(5.0 g)^{2}}{v_{0}^{2}}=\frac{(1200 \mathrm{~kg}) 25\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)^{2}}{\left[95 \mathrm{~km} / \mathrm{h}\left(\frac{1.0 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)\right]^{2}}=4100 \mathrm{~N} / \mathrm{m}
\end{aligned}
$$

27. The maximum acceleration of 5.0 g occurs where the force is at a maximum. The maximum force occurs at the bottom of the motion, where the spring is at its maximum compression. Write Newton's second law for the elevator at the bottom of the motion, with up as the positive direction.


$$
F_{\text {net }}=F_{\text {spring }}-M g=M a=5.0 \mathrm{Mg} \quad \rightarrow \quad F_{\text {spring }}=6.0 \mathrm{Mg}
$$

Now consider the diagram for the elevator at various points in its motion. If there are no non-conservative forces, then mechanical energy is conserved. Subscript 1 represents the elevator at the start of its fall, and subscript 2 represents the elevator at the bottom of its fall. The bottom of the fall is the zero location for gravitational potential energy $(y=0)$.
There is also a point at the top of the spring that is the zero location for elastic potential energy $(x=0)$. We have $v_{1}=0, y_{1}=x+h, x_{1}=0, \quad v_{2}=0, y_{2}=0$, and $x_{2}=x$. Apply conservation of energy.


$$
\begin{aligned}
& E_{1}=E_{2} \rightarrow \frac{1}{2} M v_{1}^{2}+M g y_{1}+\frac{1}{2} k x_{1}^{2}=\frac{1}{2} M v_{2}^{2}+M g y_{2}+\frac{1}{2} k x_{2}^{2} \rightarrow \\
& 0+M g(x+h)+0=0+0+\frac{1}{2} k x^{2} \rightarrow M g(x+h)=\frac{1}{2} k x^{2} \\
& F_{\text {spring }}=6.0 M g=k x \rightarrow x=\frac{6.0 M g}{k} \rightarrow M g\left(\frac{6 M g}{k}+h\right)=\frac{1}{2} k\left(\frac{6 M g}{k}\right)^{2} \rightarrow k=\frac{12 M g}{h}
\end{aligned}
$$

28. (a) The skier, while in contact with the sphere, is moving in a circular path, and so must have some component of the net force towards the center of the circle. See the free body diagram.

$$
F_{\text {radial }}=m g \cos \theta-F_{N}=m \frac{v^{2}}{r}
$$

If the skier loses contact with the sphere, the normal force is 0 . Use that relationship to find the critical angle and speed.


$$
m g \cos \theta_{\text {crit }}=m \frac{v_{\text {crit }}^{2}}{r} \rightarrow \cos \theta_{\text {crit }}=\frac{v_{\text {crit }}^{2}}{r g}
$$

Using conservation of mechanical energy, the velocity can be found as a function of angle. Let subscript 1 represent the skier at the top of the sphere, and subscript 2 represent the skier at angle $\theta$. The top of the sphere is the zero location for gravitational potential energy $(y=0)$. There is also a point at the top of the spring that is the zero location for elastic potential energy $(x=0)$. We have $v_{1}=0, y_{1}=0$, and $y_{2}=-(r-r \cos \theta)$.

$$
\begin{aligned}
& E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2} \rightarrow 0=\frac{1}{2} m v_{2}^{2}-m g(r-r \cos \theta) \rightarrow \\
& v_{2}=\sqrt{2 g(r-r \cos \theta)}
\end{aligned}
$$

Combine the two relationships to find the critical angle.

$$
\cos \theta_{\text {crit }}=\frac{v_{\text {crit }}^{2}}{r g}=\frac{2 g\left(r-r \cos \theta_{\text {crit }}\right)}{r g}=2-2 \cos \theta_{\text {crit }} \rightarrow \theta_{\text {crit }}=\cos ^{-1} \frac{2}{3} \approx 48^{\circ}
$$

(b) If friction is present, another force will be present, tangential to the surface of the sphere. The friction force will not affect the centripetal relationship of $\cos \theta_{\text {crit }}=\frac{v_{\text {crit }}^{2}}{r g}$. But the friction will reduce the speed at any given angle, and so the skier will be at a greater angle before the critical speed is reached.
29. Use conservation of energy, where all of the kinetic energy is transformed to thermal energy.

$$
E_{\text {initial }}=E_{\text {final }} \rightarrow \frac{1}{2} m v^{2}=E_{\text {thermal }}=\frac{1}{2}(2)(56,000 \mathrm{~kg})\left[(95 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)\right]^{2}=3.9 \times 10^{7} \mathrm{~J}
$$

30. Apply the conservation of energy to the child, considering work done by gravity and thermal energy. Subscript 1 represents the child at the top of the slide, and subscript 2 represents the child at the bottom of the slide. The ground is the zero location for potential energy $(y=0)$. We have $v_{1}=0$, $y_{1}=2.2 \mathrm{~m}, v_{2}=1.25 \mathrm{~m} / \mathrm{s}$, and $y_{2}=0$. Solve for the work changed into thermal energy.

$$
\begin{aligned}
& E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2}+E_{\text {thermal }} \rightarrow \\
& E_{\text {thermal }}=m g y_{1}-\frac{1}{2} m v_{2}^{2}=(16.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(2.20 \mathrm{~m})-\frac{1}{2}(16.0 \mathrm{~kg})(1.25 \mathrm{~m} / \mathrm{s})^{2}=332 \mathrm{~J}
\end{aligned}
$$

31. (a) See the free-body diagram for the ski. Write Newton's second law for forces perpendicular to the direction of motion, noting that there is no acceleration perpendicular to the plane.

$$
\begin{aligned}
& \sum F_{\perp}=F_{\mathrm{N}}-m g \cos \theta \rightarrow F_{\mathrm{N}}=m g \cos \theta \rightarrow \\
& F_{\mathrm{ff}}=\mu_{k} F_{\mathrm{N}}=\mu_{k} m g \cos \theta
\end{aligned}
$$



Now use conservation of energy, including the non-conservative friction force. Subscript 1 represents the ski at the top of the slope, and subscript 2 represents the ski at the bottom of the slope. The location of the ski at the bottom of the incline is the zero location for gravitational potential energy $(y=0)$. We have $v_{1}=0, y_{1}=\ell \sin \theta$, and $y_{2}=0$. Write the conservation of energy condition, and solve for the final speed. Note that $F_{\mathrm{fr}}=\mu_{k} F_{\mathrm{N}}=\mu_{k} m g \cos \theta$.

$$
\begin{aligned}
& \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2}+F_{\mathrm{fr}} \ell \rightarrow m g \ell \sin \theta=\frac{1}{2} m v_{2}^{2}+\mu_{k} m g \ell \cos \theta \rightarrow \\
& v_{2}=\sqrt{2 g \ell\left(\sin \theta-\mu_{k} \cos \theta\right)}=\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(85 \mathrm{~m})\left(\sin 28^{\circ}-0.090 \cos 28^{\circ}\right)} \\
& \quad=25.49 \mathrm{~m} / \mathrm{s} \approx 25 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(b) Now, on the level ground, $F_{\mathrm{fr}}=\mu_{k} m g$, and there is no change in potential energy. We again use conservation of energy, including the non-conservative friction force, to relate position 2 with position 3. Subscript 3 represents the ski at the end of the travel on the level, having traveled a distance $\ell_{3}$ on the level. We have $v_{2}=25.49 \mathrm{~m} / \mathrm{s}, y_{2}=0, v_{3}=0$, and $y_{3}=0$.

$$
\begin{aligned}
& \frac{1}{2} m v_{2}^{2}+m g y_{2}=\frac{1}{2} m v_{3}^{2}+m g y_{3}+F_{\mathrm{fr}} \ell_{3} \rightarrow \frac{1}{2} m v_{2}^{2}=\mu_{k} m g \ell_{3} \rightarrow \\
& \ell_{3}=\frac{v_{2}^{2}}{2 g \mu_{k}}=\frac{(25.49 \mathrm{~m} / \mathrm{s})^{2}}{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.090)}=368.3 \mathrm{~m} \approx 370 \mathrm{~m}
\end{aligned}
$$

32. (a) Apply energy conservation with no non-conservative work. Subscript 1 represents the ball as it is dropped, and subscript 2 represents the ball as it reaches the ground. The ground is the zero location for gravitational potential energy. We have $v_{1}=0, y_{1}=14.0 \mathrm{~m}$, and $y_{2}=0$. Solve for

$$
\begin{aligned}
& v_{2} . \\
& \qquad \begin{array}{l}
E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2} \rightarrow m g y_{1}=\frac{1}{2} m v_{2}^{2} \rightarrow \\
v_{2}=\sqrt{2 g y_{1}}=\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(14.0 \mathrm{~m})}=16.6 \mathrm{~m} / \mathrm{s}
\end{array}
\end{aligned}
$$

(b) Apply energy conservation, but with non-conservative work due to friction included. The energy dissipated will be given by $F_{\mathrm{fr}} d$. The distance $d$ over which the frictional force acts will be the 14.0 m distance of fall. With the same parameters as above, and $v_{2}=8.00 \mathrm{~m} / \mathrm{s}$, solve for the force of friction.

$$
\begin{aligned}
& \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2}+F_{\mathrm{fr}} d \rightarrow m g y_{1}=\frac{1}{2} m v_{2}^{2}+F_{\mathrm{fr}} d \rightarrow \\
& F_{\mathrm{fr}}=m\left(g \frac{y_{1}}{d}-\frac{v_{2}^{2}}{2 d}\right)=(0.145 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}-\frac{(8.00 \mathrm{~m} / \mathrm{s})^{2}}{2(14.0 \mathrm{~m})}\right)=1.09 \mathrm{~N}, \text { upwards }
\end{aligned}
$$

33. We apply the work-energy theorem. There is no need to use potential energy since the crate moves along the level floor, and there are no springs in the problem. There are two forces doing work in this problem - the pulling force and friction. The starting speed is $v_{0}=0$. Note that the two forces do work over different distances.


$$
\begin{aligned}
& W_{\mathrm{net}}=W_{\mathrm{P}}+W_{\mathrm{fr}}=F_{\mathrm{p}} d_{\mathrm{p}} \cos 0^{\circ}+F_{\mathrm{fr}} d_{\mathrm{fr}} \cos 180^{\circ}=\Delta K=\frac{1}{2} m\left(v_{f}^{2}-v_{i}^{2}\right) \rightarrow \\
& F_{\mathrm{P}} d_{\mathrm{p}}-\mu_{k} m g d_{\mathrm{fr}}=\frac{1}{2} m v_{f}^{2} \rightarrow v_{f}=\sqrt{\frac{2}{m}\left(F_{\mathrm{p}} d_{\mathrm{p}}-\mu_{k} m g d_{\mathrm{fr}}\right)} \\
& \quad=\sqrt{\frac{2}{(96 \mathrm{~kg})}\left[(350 \mathrm{~N})(30 \mathrm{~m})-(0.25)(96 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(15 \mathrm{~m})\right]}=12 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

34. Since there is a non-conservative force, apply energy conservation with the dissipative friction term. Subscript 1 represents the roller coaster at point 1, and subscript 2 represents the roller coaster at point 2. Point 2 is taken as the zero location for gravitational potential energy. We have $v_{1}=1.70 \mathrm{~m} / \mathrm{s}, y_{1}=32 \mathrm{~m}$, and $y_{2}=0$. Solve for $v_{2}$. Note that the dissipated energy is given by $F_{\mathrm{fr}} d=0.23 m g d$.

$$
\begin{aligned}
& \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2}+0.23 m g d \rightarrow v_{2}=\sqrt{-0.46 g d+v_{1}^{2}+2 g y_{1}} \\
& \quad=\sqrt{-0.46\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(45.0 \mathrm{~m})+(1.70 \mathrm{~m} / \mathrm{s})^{2}+2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(32 \mathrm{~m})}=20.67 \mathrm{~m} / \mathrm{s} \approx 21 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

35. Consider the free-body diagram for the skier in the midst of the motion. Write Newton's second law for the direction perpendicular to the plane, with an acceleration of 0 .

$$
\begin{aligned}
& \sum F_{\perp}=F_{\mathrm{N}}-m g \cos \theta=0 \rightarrow F_{\mathrm{N}}=m g \cos \theta \rightarrow \\
& F_{\mathrm{fr}}=\mu_{k} F_{\mathrm{N}}=\mu_{k} m g \cos \theta
\end{aligned}
$$

Apply conservation of energy to the skier, including the dissipative friction force. Subscript 1 represents the skier at the bottom of the slope,
 and subscript 2 represents the skier at the point furthest up the slope. The location of the skier at the bottom of the incline is the zero location for gravitational potential energy $(y=0)$. We have

$$
\begin{aligned}
v_{1}= & 9.0 \mathrm{~m} / \mathrm{s}, \quad y_{1}=0, \quad v_{2}=0, \text { and } y_{2}=d \sin \theta . \\
& \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2}+F_{\mathrm{ff}} d \rightarrow \quad \frac{1}{2} m v_{1}^{2}+0=0+m g d \sin \theta+\mu_{k} m g d \cos \theta \rightarrow \\
& \mu_{k}=\frac{\frac{1}{2} v_{1}^{2}-g d \sin \theta}{g d \cos \theta}=\frac{v_{1}^{2}}{2 g d \cos \theta}-\tan \theta=\frac{(9.0 \mathrm{~m} / \mathrm{s})^{2}}{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(12 \mathrm{~m}) \cos 19^{\circ}}-\tan 19^{\circ}=0.020
\end{aligned}
$$

36. (a) Use conservation of energy to equate the potential energy at the top of the circular track to the kinetic energy at the bottom of the circular track. Take the bottom of the track to the be 0 level for gravitational potential energy.

$$
\begin{aligned}
& E_{\text {top }}=E_{\text {botom }} \rightarrow m g r=\frac{1}{2} m v_{\text {bottom }}^{2} \rightarrow \\
& v_{\text {bottom }}=\sqrt{2 g r}=\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(2.0 \mathrm{~m})}=6.261 \mathrm{~m} / \mathrm{s} \approx 6.3 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(b) The thermal energy produced is the opposite of the work done by the friction force. In this situation, the force of friction is the weight of the object times the coefficient of kinetic friction.

$$
\begin{aligned}
E_{\text {thermal }} & =-W_{\text {friction }}=-\overrightarrow{\mathbf{F}}_{\text {fricition }} \cdot \Delta \overrightarrow{\mathbf{x}}=-F_{\text {friction }} \Delta x \cos \theta=-\mu_{\mathrm{k}} m g \Delta x\left(\cos 180^{\circ}\right)=\mu_{\mathrm{k}} m g \Delta x \\
& =(0.25)(1.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(3.0 \mathrm{~m})=7.35 \mathrm{~J} \approx 7.4 \mathrm{~J}
\end{aligned}
$$

(c) The work done by friction is the change in kinetic energy of the block as it moves from point B to point C .

$$
\begin{aligned}
& W_{\text {fricion }}=\Delta K=K_{\mathrm{C}}-K_{\mathrm{B}}=\frac{1}{2} m\left(v_{\mathrm{C}}^{2}-v_{\mathrm{B}}^{2}\right) \rightarrow \\
& v_{\mathrm{C}}=\sqrt{\frac{2 W_{\text {firicion }}}{m}+v_{\mathrm{B}}^{2}}=\sqrt{\frac{2(-7.35 \mathrm{~J})}{(1.0 \mathrm{~kg})}+(6.261 \mathrm{~m} / \mathrm{s})^{2}}=4.9498 \mathrm{~m} / \mathrm{s} \approx 4.9 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(d) Use conservation of energy to equate the kinetic energy when the block just contacts the spring with the potential energy when the spring is fully compressed and the block has no speed.
There is no friction on the block while compressing the spring.

$$
\begin{aligned}
& E_{\text {initial }}=E_{\text {final }} \rightarrow \frac{1}{2} m v_{\text {contact }}^{2}=\frac{1}{2} k x_{\max }^{2} \rightarrow \\
& k=m \frac{v_{\text {contact }}^{2}}{x_{\text {max }}^{2}}=(1.0 \mathrm{~kg}) \frac{(4.9498 \mathrm{~m} / \mathrm{s})^{2}}{(0.20 \mathrm{~m})^{2}}=612.5 \mathrm{~N} / \mathrm{m} \approx 610 \mathrm{~N} / \mathrm{m}
\end{aligned}
$$

37. Use conservation of energy, including the non-conservative frictional force, as developed in Eq. 815. The block is on a level surface, so there is no gravitational potential energy change to consider. The frictional force is given by $F_{\mathrm{ff}}=\mu_{k} F_{\mathrm{N}}=\mu_{k} m g$, since the normal force is equal to the weight.
Subscript 1 represents the block at the compressed location, and subscript 2 represents the block at the maximum stretched position. The location of the block when the spring is neither stretched nor compressed is the zero location for elastic potential energy $(x=0)$. Take right to be the positive direction. We have $v_{1}=0, x_{1}=-0.050 \mathrm{~m}, v_{2}=0$, and $x_{2}=0.023 \mathrm{~m}$.

$$
\begin{aligned}
& E_{1}=E_{2}+F_{\mathrm{fr}} \ell \rightarrow \frac{1}{2} m v_{1}^{2}+\frac{1}{2} k x_{1}^{2}=\frac{1}{2} m v_{2}^{2}+\frac{1}{2} k x_{2}^{2}+F_{\mathrm{fr}}\left(x_{2}-x_{1}\right) \rightarrow \\
& \frac{1}{2} k x_{1}^{2}=\frac{1}{2} k x_{2}^{2}+\mu_{k} m g\left(x_{2}-x_{1}\right) \rightarrow \\
& \mu_{k}=\frac{k\left(x_{1}^{2}-x_{2}^{2}\right)}{2 m g\left(x_{2}-x_{1}\right)}=\frac{-k\left(x_{2}+x_{1}\right)}{2 m g}=\frac{-(180 \mathrm{~N} / \mathrm{m})[(-0.050 \mathrm{~m})+(0.023 \mathrm{~m})]}{2(0.620 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=0.40
\end{aligned}
$$

38. Use conservation of energy, including the non-conservative frictional force, as developed in Eq. 815. The block is on a level surface, so there is no gravitational potential energy change to consider. Since the normal force is equal to the weight, the frictional force is $F_{\mathrm{fr}}=\mu_{k} F_{\mathrm{N}}=\mu_{k} m g$. Subscript 1 represents the block at the compressed location, and subscript 2 represents the block at the maximum stretched position. The location of the block when the spring is neither stretched nor compressed is the zero location for elastic potential energy $(x=0)$. Take right to be the positive direction. We have $v_{1}=0, x_{1}=-0.18 \mathrm{~m}$, and $v_{2}=0$. The value of the spring constant is found from the fact that
a $25-\mathrm{N}$ force compresses the spring 18 cm , and so $k=F / x=25 \mathrm{~N} / 0.18 \mathrm{~m}=138.9 \mathrm{~N} / \mathrm{m}$. The value of $x_{2}$ must be positive.

$$
\begin{aligned}
& E_{1}=E_{2}+F_{\mathrm{fr}} \ell \rightarrow \frac{1}{2} m v_{1}^{2}+\frac{1}{2} k x_{1}^{2}=\frac{1}{2} m v_{2}^{2}+\frac{1}{2} k x_{2}^{2}+F_{\mathrm{fr}}\left(x_{2}-x_{1}\right) \rightarrow \\
& \frac{1}{2} k x_{1}^{2}=\frac{1}{2} k x_{2}^{2}+\mu_{k} m g\left(x_{2}-x_{1}\right) \rightarrow x_{2}^{2}+\frac{2 \mu_{k} m g}{k} x_{2}-\left(\frac{2 \mu_{k} m g}{k} x_{1}+x_{1}^{2}\right)=0 \rightarrow \\
& x_{2}^{2}+\frac{2(0.30)(0.18)(9.80)}{138.9} x_{2}-\left(\frac{2(0.30)(0.18)(9.80)}{138.9}(-0.18)+(-0.18)^{2}\right)=0 \rightarrow \\
& x_{2}^{2}+0.00762 x_{2}-0.03103=0 \rightarrow x_{2}=0.1724 \mathrm{~m},-0.1800 \mathrm{~m} \rightarrow x_{2}=0.17 \mathrm{~m}
\end{aligned}
$$

39. (a) Calculate the energy of the ball at the two maximum heights, and subtract to find the amount of energy lost. The energy at the two heights is all gravitational potential energy, since the ball has no kinetic energy at those maximum heights.

$$
\begin{aligned}
& E_{\text {lost }}=E_{\text {initial }}-E_{\text {final }}=m g y_{\text {initial }}-m g y_{\text {final }} \\
& \frac{E_{\text {lost }}}{E_{\text {initial }}}=\frac{m g y_{\text {initial }}-m g y_{\text {final }}}{m g y_{\text {initial }}}=\frac{y_{\text {initial }}-y_{\text {final }}}{y_{\text {initial }}}=\frac{2.0 \mathrm{~m}-1.5 \mathrm{~m}}{2.0 \mathrm{~m}}=0.25=25 \%
\end{aligned}
$$

(b) The ball's speed just before the bounce is found from the initial gravitational potential energy, and the ball's speed just after the bounce is found from the ball's final gravitational potential energy.

$$
\begin{aligned}
& U_{\text {initial }}=K_{\text {before }} \rightarrow m g y_{\text {initial }}=\frac{1}{2} m v_{\text {before }}^{2} \rightarrow \\
& v_{\text {before }}=\sqrt{2 g y_{\text {initial }}}=\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(2.0 \mathrm{~m})}=6.3 \mathrm{~m} / \mathrm{s} \\
& U_{\text {final }}=K_{\text {after }} \rightarrow m g y_{\text {final }}=\frac{1}{2} m v_{\text {after }}^{2} \rightarrow \\
& v_{\text {after }}=\sqrt{2 g y_{\text {final }}}=\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(1.5 \mathrm{~m})}=5.4 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(c) The energy "lost" was changed primarily into heat energy - the temperature of the ball and the ground would have increased slightly after the bounce. Some of the energy may have been changed into acoustic energy (sound waves). Some may have been lost due to non-elastic deformation of the ball or ground.
40. Since there is friction in this problem, there will be energy dissipated by friction.

$$
\begin{aligned}
& E_{\text {friction }}+\Delta K+\Delta U=0 \rightarrow E_{\text {friction }}=-\Delta K-\Delta U=\frac{1}{2} m\left(v_{1}^{2}-v_{2}^{2}\right)+m g\left(y_{1}-y_{2}\right) \\
& \quad=\frac{1}{2}(56 \mathrm{~kg})\left[0-(11.0 \mathrm{~m} / \mathrm{s})^{2}\right]+(56 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(230 \mathrm{~m})=1.2 \times 10^{5} \mathrm{~J}
\end{aligned}
$$

41. The change in gravitational potential energy is given by $\Delta U=m g \Delta y$. Assume a mass of 75 kg .

$$
\Delta U=m g \Delta y=(75 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(1.0 \mathrm{~m})=740 \mathrm{~J}
$$

42. (a) Use conservation of energy. Subscript 1 represents the block at the compressed location, and subscript 2 represents the block at its maximum position up the slope. The initial location of the block at the bottom of the plane is taken to be the zero location for gravitational potential energy $(y=0)$. The variable $x$ will represent the amount of spring compression or stretch. We have $v_{1}=0, x_{1}=0.50 \mathrm{~m}, y_{1}=0, v_{2}=0$, and $x_{2}=0$. The distance the block moves up the
plane is given by $d=\frac{y}{\sin \theta}$, so $y_{2}=d \sin \theta$. Solve for $d$.

$$
\begin{aligned}
& E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}+\frac{1}{2} k x_{1}^{2}=\frac{1}{2} m v_{2}^{2}+m g y_{2}+\frac{1}{2} k x_{2}^{2} \rightarrow \\
& \frac{1}{2} k x_{1}^{2}=m g y_{2}=m g d \sin \theta \rightarrow d=\frac{k x_{1}^{2}}{2 m g \sin \theta}=\frac{(75 \mathrm{~N} / \mathrm{m})(0.50 \mathrm{~m})^{2}}{2(2.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \sin 41^{\circ}}=0.73 \mathrm{~m}
\end{aligned}
$$

(b) Now the spring will be stretched at the turning point of the motion. The first half-meter of the block's motion returns the block to the equilibrium position of the spring. After that, the block beings to stretch the spring. Accordingly, we have the same conditions as before except that $x_{2}=d-0.5 \mathrm{~m}$.

$$
\begin{aligned}
& E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}+\frac{1}{2} k x_{1}^{2}=\frac{1}{2} m v_{2}^{2}+m g y_{2}+\frac{1}{2} k x_{2}^{2} \rightarrow \\
& \frac{1}{2} k x_{1}^{2}=m g d \sin \theta+\frac{1}{2} k(d-0.5 \mathrm{~m})
\end{aligned}
$$

This is a quadratic relation in $d$. Solving it gives $d=0.66 \mathrm{~m}$.
(c) The block now moves $d=0.50 \mathrm{~m}$, and stops at the equilibrium point of the spring.

Accordingly, $x_{2}=0$. Apply the method of Section 8-6.

$$
\begin{aligned}
\Delta K & +\Delta U+F_{\mathrm{fr}} \ell=\frac{1}{2} m\left(v_{2}^{2}-v_{1}^{2}\right)+\frac{1}{2} k\left(x_{2}^{2}-x_{1}^{2}\right)+m g\left(y_{2}-y_{1}\right)+\mu_{k} m g d \cos \theta \rightarrow \\
\mu_{k} & =\frac{-\frac{1}{2} k x_{1}^{2}+m g d \sin \theta}{-m g d \cos \theta}=\frac{k x_{1}^{2}}{2 m g d \cos \theta}-\tan \theta \\
& =\frac{(75 \mathrm{~N} / \mathrm{m})(0.50 \mathrm{~m})^{2}}{2(2.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.50 \mathrm{~m}) \cos 41^{\circ}}-\tan 41^{\circ}=0.40
\end{aligned}
$$

43. Because friction does work, Eq. 8-15 applies.
(a) The spring is initially uncompressed, so $x_{0}=0$. The block is stopped at the maximum compression, so $v_{f}=0$.

$$
\begin{aligned}
& \Delta K+\Delta U+F_{\mathrm{fr}} \ell=\frac{1}{2} m\left(v_{f}^{2}-v_{0}^{2}\right)+\frac{1}{2} k\left(x_{f}^{2}-x_{0}^{2}\right)+m g \mu_{\mathrm{k}}\left(x_{f}-x_{0}\right)=0 \rightarrow \\
& \frac{1}{2} k x_{f}^{2}+m g \mu_{\mathrm{k}} x_{f}-\frac{1}{2} m v_{0}^{2}=0 \rightarrow \\
& x_{f}
\end{aligned} \begin{aligned}
& =\frac{-m g \mu_{\mathrm{k}} \pm \sqrt{\left(m g \mu_{\mathrm{k}}\right)^{2}-4\left(\frac{1}{2} k\right)\left(-\frac{1}{2} m v_{0}^{2}\right)}}{2\left(\frac{1}{2} k\right)}=\frac{-m g \mu_{\mathrm{k}} \pm \sqrt{\left(m g \mu_{\mathrm{k}}\right)^{2}+k m v_{0}^{2}}}{k} \\
& \\
& =\frac{m g \mu_{\mathrm{k}}}{k}\left(-1 \pm \sqrt{1+\frac{k m v_{0}^{2}}{\left(m g \mu_{\mathrm{k}}\right)^{2}}}\right) \\
& \\
& =\frac{(2.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.30)}{(120 \mathrm{~N} / \mathrm{m})}\left(-1 \pm \sqrt{1+\frac{(120 \mathrm{~N} / \mathrm{m})(2.0 \mathrm{~kg})(1.3 \mathrm{~m} / \mathrm{s})^{2}}{(2.0 \mathrm{~kg})^{2}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)^{2}(0.30)^{2}}}\right) \\
& \quad=0.1258 \mathrm{~m} \approx 0.13 \mathrm{~m}
\end{aligned}
$$

(b) To remain at the compressed position with the minimum coefficient of static friction, the magnitude of the force exerted by the spring must be the same as the magnitude of the maximum force of static friction.

$$
k x_{f}=\mu_{\mathrm{s}} m g \rightarrow \mu_{\mathrm{s}}=\frac{k x_{f}}{m g}=\frac{(120 \mathrm{~N} / \mathrm{m})(0.1258 \mathrm{~m})}{(2.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=0.7702 \approx 0.77
$$

(c) If static friction is not large enough to hold the block in place, the spring will push the block back towards the equilibrium position. The block will detach from the decompressing spring at the equilibrium position because at that point the spring will begin to slow down while the block continues moving. Use Eq. 8-15 to relate the block at the maximum compression position to the equilibrium position. The block is initially at rest, so $v_{0}=0$. The spring is relaxed at the equilibrium position, so $x_{f}=0$.

$$
\begin{aligned}
& \Delta K+\Delta U+F_{\mathrm{fr}} \ell=\frac{1}{2} m\left(v_{f}^{2}-v_{0}^{2}\right)+\frac{1}{2} k\left(x_{f}^{2}-x_{0}^{2}\right)+m g \mu_{\mathrm{k}}\left(x_{f}-x_{0}\right)=0 \rightarrow \\
& \frac{1}{2} m v_{f}^{2}-\frac{1}{2} k x_{0}^{2}+m g \mu_{\mathrm{k}} x_{0} \\
& \begin{array}{l}
v_{f}
\end{array}=\sqrt{\frac{k}{m} x_{0}^{2}-2 g \mu_{\mathrm{k}} x_{0}}=\sqrt{\frac{(120 \mathrm{~N} / \mathrm{m})}{(2.0 \mathrm{~kg})}(0.1258 \mathrm{~m})^{2}-2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.30)(0.1258 \mathrm{~m})} \\
& \quad=0.458 \mathrm{~m} / \mathrm{s} \approx 0.5 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

44. (a) If there is no air resistance, then conservation of mechanical energy can be used. Subscript 1 represents the glider when at launch, and subscript 2 represents the glider at landing. The landing location is the zero location for elastic potential energy $(y=0)$. We have $y_{1}=3500 \mathrm{~m}$,
$y_{2}=0$, and $v_{1}=480 \mathrm{~km} / \mathrm{h}\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)=133.3 \mathrm{~m} / \mathrm{s}$. Solve for $v_{2}$.

$$
E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2} \rightarrow
$$

$$
v_{2}=\sqrt{v_{1}^{2}+2 g y_{1}}=\sqrt{(133.3 \mathrm{~m} / \mathrm{s})^{2}+2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(3500 \mathrm{~m})}=293.8 \mathrm{~m} / \mathrm{s}\left(\frac{3.6 \mathrm{~km} / \mathrm{h}}{1 \mathrm{~m} / \mathrm{s}}\right)
$$

$$
=1058 \mathrm{~km} / \mathrm{h} \approx 1100 \mathrm{~km} / \mathrm{h}
$$

(b) Now include the work done by the non-conservative frictional force. Consider the diagram of the glider. The distance over which the friction acts is given by
 $\ell=\frac{3500 \mathrm{~m}}{\sin 12^{\circ}}$. Use the same subscript representations as above, with $y_{1}, v_{1}$, and $y_{2}$ as before, and

$$
v_{2}=210 \mathrm{~km} / \mathrm{h}\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)=58.33 \mathrm{~m} / \mathrm{s} \text {. Write the energy conservation equation and solve for }
$$ the frictional force.

$$
\begin{aligned}
E_{1} & =E_{2}+F_{\mathrm{fr}} \ell \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2}+F_{\mathrm{fr}} \ell \rightarrow F_{f}=\frac{m\left(v_{1}^{2}-v_{2}^{2}+2 g y_{1}\right)}{2 \ell} \\
& =\frac{(980 \mathrm{~kg})\left[(133.3 \mathrm{~m} / \mathrm{s})^{2}-(58.33 \mathrm{~m} / \mathrm{s})^{2}+2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(3500 \mathrm{~m})\right]}{2\left(\frac{3500 \mathrm{~m}}{\sin 12^{\circ}}\right)}=2415 \mathrm{~N} \approx 2400 \mathrm{~N}
\end{aligned}
$$

45. (a) Equate the gravitational force to the expression for centripetal force, since the orbit is circular. Let $M_{\mathrm{E}}$ represent the mass of the Earth.

$$
\frac{m_{s} v_{s}^{2}}{r_{s}}=\frac{G M_{\mathrm{E}} m_{s}}{r_{s}^{2}} \rightarrow m_{s} v_{s}^{2}=\frac{G M_{\mathrm{E}} m_{s}}{r_{s}} \rightarrow \frac{1}{2} m_{s} v_{s}^{2}=K=\frac{G M_{\mathrm{E}} m_{s}}{2 r_{s}}
$$

(b) The potential energy is given by Eq. 8-17, $U=-G M_{\mathrm{E}} m_{s} / r_{s}$.
(c) $\frac{K}{U}=\frac{\frac{G M_{\mathrm{E}} m_{s}}{2 r_{s}}}{-\frac{G M_{\mathrm{E}} m_{s}}{r_{s}}}=-\frac{1}{2}$
46. Since air friction is to be ignored, the mechanical energy will be conserved. Subscript 1 represents the rocket at launch, and subscript 2 represents the rocket at its highest altitude. We have $v_{1}=850 \mathrm{~m} / \mathrm{s}, \quad v_{2}=0$, and we take the final altitude to be a distance $h$ above the surface of the Earth.

$$
\begin{aligned}
E_{1} & =E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+\left(-\frac{G M_{\mathrm{E}} m}{r_{\mathrm{E}}}\right)=\frac{1}{2} m v_{2}^{2}+\left(-\frac{G M_{\mathrm{E}} m}{r_{\mathrm{E}}+h}\right) \rightarrow \\
h & =\left(\frac{1}{r_{\mathrm{E}}}-\frac{v_{1}^{2}}{2 G M_{\mathrm{E}}}\right)^{-1}-r_{\mathrm{E}}=r_{\mathrm{E}}\left(\frac{2 G M_{\mathrm{E}}}{r_{\mathrm{E}} v_{0}^{2}}-1\right)^{-1} \\
& =\left(6.38 \times 10^{6} \mathrm{~m}\right)\left(\frac{2\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(5.98 \times 10^{24} \mathrm{~kg}\right)}{\left(6.38 \times 10^{6} \mathrm{~m}\right)(850 \mathrm{~m} / \mathrm{s})^{2}}-1\right)^{-1}=3.708 \times 10^{4} \mathrm{~m} \approx 3.7 \times 10^{4} \mathrm{~m}
\end{aligned}
$$

If we would solve this problem with the approximate gravitation potential energy of $m g h$, we would get an answer of $3.686 \times 10^{4} \mathrm{~m}$, which agrees to 2 significant figures.
47. The escape velocity is given by Eq. 8-19.

$$
\begin{aligned}
& \frac{2 M_{\mathrm{A}} G}{r_{\mathrm{A}}}=2\left(\frac{2 M_{\mathrm{B}} G}{r_{\mathrm{B}}}\right) \rightarrow \frac{r_{\mathrm{A}}}{r_{\mathrm{B}}}=\frac{1}{4}
\end{aligned}
$$

48. Note that the difference in the two distances from the center of the Earth, $r_{2}-r_{1}$, is the same as the height change in the two positions, $y_{2}-y_{1}$. Also, if the two distances are both near the surface of the Earth, then $r_{1} r_{2} \approx r_{\mathrm{E}}^{2}$.

$$
\begin{aligned}
\Delta U & =\left(-\frac{G M_{\mathrm{E}} m}{r_{2}}\right)-\left(-\frac{G M_{\mathrm{E}} m}{r_{1}}\right)=\frac{G M_{\mathrm{E}} m}{r_{1}}-\frac{G M_{\mathrm{E}} m}{r_{2}}=G M_{\mathrm{E}} m\left(\frac{1}{r_{1}}-\frac{1}{r_{2}}\right)=\frac{G M_{\mathrm{E}} m}{r_{1} r_{2}}\left(r_{2}-r_{1}\right) \\
& \approx \frac{G M_{\mathrm{E}} m}{r_{\mathrm{E}}^{2}}\left(y_{2}-y_{1}\right)=m \frac{G M_{\mathrm{E}}}{r_{\mathrm{E}}^{2}}\left(y_{2}-y_{1}\right)=m g\left(y_{2}-y_{1}\right)
\end{aligned}
$$

49. The escape velocity for an object located a distance $r$ from a mass $M$ is given by Eq. 8-19, $v_{\text {esc }}=\sqrt{\frac{2 M G}{r}}$. The orbit speed for an object located a distance $r$ from a mass $M$ is $v_{\text {orb }}=\sqrt{\frac{M G}{r}}$.
(a) $v_{\substack{\text { ses at } \\ \text { surs } \\ \text { surface }}}=\sqrt{\frac{2 M_{\text {Sun }} G}{r_{\text {Sun }}}}=\sqrt{\frac{2\left(2.0 \times 10^{30} \mathrm{~kg}\right)\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)}{7.0 \times 10^{8} \mathrm{~m}}}=6.2 \times 10^{5} \mathrm{~m} / \mathrm{s}$
(b) $v_{\substack{\text { escat at } \\ \text { Eart } \\ \text { orbit }}}=\sqrt{\frac{2 M_{\text {Sum }} G}{r_{\text {Earth orbit }}}}=\sqrt{\frac{2\left(2.0 \times 10^{30} \mathrm{~kg}\right)\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)}{1.50 \times 10^{-11} \mathrm{~m}}}=4.2 \times 10^{4} \mathrm{~m} / \mathrm{s}$

Since $v_{\substack{\text { escat } \\ \text { Eart } \\ \text { orbit }}} \approx 1.4 v_{\substack{\text { Earth } \\ \text { orbit }}}$, the orbiting object will not escape the orbit.
50. (a) The potential energy is given by Eq. 8-17.

$$
\begin{aligned}
U_{\mathrm{A}} & =-\frac{G m M_{\mathrm{E}}}{r_{\mathrm{A}}}=-\frac{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)(950 \mathrm{~kg})\left(5.98 \times 10^{24} \mathrm{~kg}\right)}{\left(6.38 \times 10^{6} \mathrm{~m}+4.20 \times 10^{6} \mathrm{~m}\right)} \\
& =-3.5815 \times 10^{10} \mathrm{~J} \approx-3.6 \times 10^{10} \mathrm{~J} \\
U_{\mathrm{B}} & =-\frac{G m M_{\mathrm{E}}}{r_{\mathrm{B}}}=-\frac{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)(950 \mathrm{~kg})\left(5.98 \times 10^{24} \mathrm{~kg}\right)}{\left(6.38 \times 10^{6} \mathrm{~m}+1.26 \times 10^{7} \mathrm{~m}\right)} \\
& =-1.9964 \times 10^{10} \mathrm{~J} \approx-2.0 \times 10^{10} \mathrm{~J}
\end{aligned}
$$

(b) An expression for the kinetic energy is found by equating the gravitational force to the expression for centripetal force, since the satellites are in circular orbits.

$$
\begin{aligned}
& \frac{m v^{2}}{r}=\frac{G m M_{\mathrm{E}}}{r^{2}} \rightarrow \frac{1}{2} m v^{2}=K=\frac{G m M_{\mathrm{E}}}{2 r}=-\frac{1}{2} U \\
& K_{\mathrm{A}}=\frac{G m M_{\mathrm{E}}}{2 r_{\mathrm{A}}}=-\frac{1}{2}\left(-3.5815 \times 10^{10} \mathrm{~J}\right)=1.7908 \times 10^{10} \mathrm{~J} \approx 1.8 \times 10^{10} \mathrm{~J} \\
& K_{\mathrm{B}}=\frac{G m M_{\mathrm{E}}}{2 r_{\mathrm{B}}}=-\frac{1}{2}\left(-1.9964 \times 10^{10} \mathrm{~J}\right)=0.9982 \times 10^{10} \mathrm{~J} \approx 1.0 \times 10^{10} \mathrm{~J}
\end{aligned}
$$

(c) We use the work-energy theorem to calculate the work done to change the orbit.

$$
\begin{aligned}
W_{\text {Net }} & =\Delta K=W_{\substack{\text { orbit } \\
\text { change }}}+W_{\text {gravity }}=W_{\substack{\text { orbit } \\
\text { change }}}-\Delta U_{\text {gravity }} \rightarrow \underset{\substack{\text { orbit } \\
\text { change }}}{ }=\Delta K+\Delta U_{\text {graviy }} \rightarrow \\
\begin{aligned}
W_{\text {orbit }} \\
\text { change }
\end{aligned} & =\Delta K+\Delta U_{\text {gravity }}=\left(K_{\mathrm{B}}-K_{\mathrm{A}}\right)+\left(U_{\mathrm{B}}-U_{\mathrm{A}}\right)=\left(-\frac{1}{2} U_{\mathrm{B}}+\frac{1}{2} U_{\mathrm{A}}\right)+\left(U_{\mathrm{B}}-U_{\mathrm{A}}\right) \\
& =\frac{1}{2}\left(U_{\mathrm{B}}-U_{\mathrm{A}}\right)=\frac{1}{2}\left(-1.9964 \times 10^{10} \mathrm{~J}--3.5815 \times 10^{10} \mathrm{~J}\right)=7.9 \times 10^{9} \mathrm{~J}
\end{aligned}
$$

51. For a circular orbit, the gravitational force is a centripetal force. The escape velocity is given by Eq. 8-19.

$$
\frac{G M m}{r^{2}}=\frac{m v_{\text {orbit }}^{2}}{r} \rightarrow v_{\text {orbit }}=\sqrt{\frac{M G}{r}} \quad v_{\text {esc }}=\sqrt{\frac{2 M G}{r}}=\sqrt{2} \sqrt{\frac{M G}{r}}=\sqrt{2} v_{\text {orbit }}
$$

52. (a) With the condition that $U=0$ at $r=\infty$, the potential energy is given by $U=-\frac{G M_{\mathrm{E}} m}{r}$. The kinetic energy is found from the fact that for a circular orbit, the gravitational force is a centripetal force.

$$
\begin{aligned}
& \frac{G M_{\mathrm{E}} m}{r^{2}}=\frac{m v_{\text {orbit }}^{2}}{r} \rightarrow m v_{\text {orbit }}^{2}=\frac{G M_{\mathrm{E}} m}{r} \rightarrow K=\frac{1}{2} m v_{\text {orbit }}^{2}=\frac{1}{2} \frac{G M_{\mathrm{E}} m}{r} \\
& E=K+U=\frac{1}{2} \frac{G M_{\mathrm{E}} m}{r}-\frac{G M_{\mathrm{E}} m}{r}=-\frac{1}{2} \frac{G M_{\mathrm{E}} m}{r}
\end{aligned}
$$

(b) As the value of $E$ decreases, since $E$ is negative, the radius $r$ must get smaller. But as the radius gets smaller, the kinetic energy increases, since $K \propto \frac{1}{r}$. If the total energy decreases by 1 Joule, the potential energy decreases by 2 Joules and the kinetic energy increases by 1 Joule.
53. The speed of the surface of the Earth at the equator (relative to the center of the Earth) is given by the following. It is an eastward velocity. Call east the $x$-direction, and up the $y$-direction.

$$
v=\frac{2 \pi r_{\mathrm{E}}}{T}=\frac{2 \pi\left(6.38 \times 10^{6} \mathrm{~m}\right)}{86,400 \mathrm{~s}}=464 \mathrm{~m} / \mathrm{s}
$$

The escape velocity from the Earth (relative to the center of the Earth) is given in Eq. 8-19.
$v_{\text {esc }}=\sqrt{\frac{2 G M_{\mathrm{E}}}{r_{\mathrm{E}}}}=\sqrt{\frac{2\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(5.98 \times 10^{24} \mathrm{~kg}\right)}{6.38 \times 10^{6} \mathrm{~m}}}=11,182 \mathrm{~m} / \mathrm{s}$
(a) With the surface of the Earth traveling east and the rocket velocity to the east, the rocket velocity and surface velocity will add linearly to give the escape velocity.

$$
v_{\substack{\text { rocket relative } \\ \text { to surface of } \\ \text { Earth }}}+464 \mathrm{~m} / \mathrm{s}=11,182 \mathrm{~m} / \mathrm{s} \rightarrow v_{\substack{\text { rocket relative } \\ \text { to surface of } \\ \text { Earth }}}=10,700 \mathrm{~m} / \mathrm{s}
$$

(b) With the surface of the Earth traveling east and the rocket velocity to the west, the rocket velocity will have to be higher than the nominal escape velocity.

$$
v_{\substack{\text { rocket relative } \\ \text { to surface of } \\ \text { Earth }}}+464 \mathrm{~m} / \mathrm{s}=-11,182 \mathrm{~m} / \mathrm{s} \rightarrow \underset{\substack{\text { to surface of } \\ \text { Earth }}}{v_{\text {rocket elative }}=11,646 \mathrm{~m} / \mathrm{s} \approx 11,600 \mathrm{~m} / \mathrm{s}}
$$

(c) When fired vertically upward, the rocket velocity and the Earth's velocity are at right angles to each other, and so add according to the Pythagorean theorem to give the escape velocity.

$$
\begin{aligned}
& v_{\text {rocket relative }}^{2}+(464 \mathrm{~m} / \mathrm{s})^{2}=(11,182 \mathrm{~m} / \mathrm{s})^{2} \rightarrow \underset{\substack{\text { rocket relative } \\
\text { to surface of } \\
\text { Earth }}}{2}=11,172 \mathrm{~m} / \mathrm{s} \approx 11,200 \mathrm{~m} / \mathrm{s} \\
& \text { Earth }
\end{aligned}
$$

54. (a) Since air friction is to be ignored, the mechanical energy will be conserved. Subscript 1 represents the rocket at launch, and subscript 2 represents the rocket at its highest altitude. We have $v_{1}=v_{0}, v_{2}=0, r_{1}=r_{\mathrm{E}}$, and $r_{2}=r_{\mathrm{E}}+h$ where we take the final altitude to be a distance $h$ above the surface of the Earth.

$$
\begin{aligned}
& E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{0}^{2}+\left(-\frac{G M_{\mathrm{E}} m}{r_{\mathrm{E}}}\right)=\frac{1}{2} m v_{2}^{2}+\left(-\frac{G M_{\mathrm{E}} m}{r_{\mathrm{E}}+h}\right)=\left(-\frac{G M_{\mathrm{E}} m}{r_{\mathrm{E}}+h}\right) \rightarrow \\
& h=\left(\frac{1}{r_{\mathrm{E}}}-\frac{v_{0}^{2}}{2 G M_{\mathrm{E}}}\right)^{-1}-r_{\mathrm{E}}=r_{\mathrm{E}}\left(\frac{2 G M_{\mathrm{E}}}{r_{\mathrm{E}} \mathrm{v}_{0}^{2}}-1\right)^{-1}
\end{aligned}
$$

(b) $\quad h=r_{\mathrm{E}}\left(\frac{2 G M_{\mathrm{E}}}{r_{\mathrm{E}} v_{0}^{2}}-1\right)^{-1}$

$$
=\left(6.38 \times 10^{6} \mathrm{~m}\right)\left(\frac{2\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(5.98 \times 10^{24} \mathrm{~kg}\right)}{\left(6.38 \times 10^{6} \mathrm{~m}\right)(8350 \mathrm{~m} / \mathrm{s})^{2}}-1\right)^{-1}=8.0 \times 10^{6} \mathrm{~m}
$$

55. (a) From Eq. 8-19, the escape velocity at a distance $r \geq r_{\mathrm{E}}$ from the center of the Earth is

$$
\begin{aligned}
& v_{\mathrm{esc}}=\sqrt{\frac{2 G M_{\mathrm{E}}}{r}} . \\
& \quad v_{\mathrm{esc}}=\sqrt{\frac{2 G M_{\mathrm{E}}}{r}}=r^{-1 / 2} \sqrt{2 G M_{\mathrm{E}}} \rightarrow \frac{d v_{\mathrm{esc}}}{d r}=-\frac{1}{2} r^{-3 / 2} \sqrt{2 G M_{\mathrm{E}}}=-\sqrt{\frac{G M_{\mathrm{E}}}{2 r^{3}}}
\end{aligned}
$$

(b) $\Delta v_{\text {esc }} \approx \frac{d v_{\text {esc }}}{d r} \Delta r=-\sqrt{\frac{G M_{\mathrm{E}}}{2 r^{3}}} \Delta r=-\sqrt{\frac{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(5.98 \times 10^{24} \mathrm{~kg}\right)}{2\left(6.38 \times 10^{6} \mathrm{~m}\right)^{3}}}\left(3.2 \times 10^{5} \mathrm{~m}\right)$

$$
=-280 \mathrm{~m} / \mathrm{s}
$$

The escape velocity has decreased by $280 \mathrm{~m} / \mathrm{s}$, and so is $v_{\text {esc }}=1.12 \times 10^{4} \mathrm{~m} / \mathrm{s}-280 \mathrm{~m} / \mathrm{s}=$ $1.09 \times 10^{4} \mathrm{~m} / \mathrm{s}$.
56. (a) Since air friction is to be ignored, the mechanical energy will be conserved. Subscript 1 represents the meteorite at the high altitude, and subscript 2 represents the meteorite just before it hits the sand. We have $v_{1}=90.0 \mathrm{~m} / \mathrm{s}, r_{1}=r_{\mathrm{E}}+h=r_{\mathrm{E}}+850 \mathrm{~km}$, and $r_{2}=r_{\mathrm{E}}$.

$$
\begin{aligned}
& E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+\left(-\frac{G M_{\mathrm{E}} m}{r_{\mathrm{E}}+h}\right)=\frac{1}{2} m v_{2}^{2}+\left(-\frac{G M_{\mathrm{E}} m}{r_{\mathrm{E}}}\right) \rightarrow \\
& v_{2}=\sqrt{v_{1}^{2}+2 G M_{\mathrm{E}}\left(\frac{1}{r_{\mathrm{E}}}-\frac{1}{r_{\mathrm{E}}+h}\right)}=3835.1 \mathrm{~m} / \mathrm{s} \approx 3840 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(b) We use the work-energy theorem, where work is done both by gravity (over a short distance) and the sand. The initial speed is $3835.1 \mathrm{~m} / \mathrm{s}$, and the final speed is 0 .

$$
\begin{aligned}
W_{\mathrm{net}} & =W_{\mathrm{G}}+W_{\mathrm{fr}}=m g d+W_{\mathrm{fr}}=\Delta K=\frac{1}{2} m\left(v_{f}^{2}-v_{i}^{2}\right) \rightarrow \\
W_{\mathrm{fr}} & =-\frac{1}{2} m v_{i}^{2}-m g d=-\frac{1}{2}(575 \mathrm{~kg})(3835.1 \mathrm{~m} / \mathrm{s})^{2}-(575 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(3.25 \mathrm{~m}) \\
& =-4.23 \times 10^{9} \mathrm{~J}
\end{aligned}
$$

(c) The average force is the magnitude of the work done, divided by the distance moved in the sand.

$$
F_{\text {sand }}=\frac{\left|W_{\text {sand }}\right|}{d_{\text {sand }}}=\frac{4.23 \times 10^{9} \mathrm{~J}}{3.25 \mathrm{~m}}=1.30 \times 10^{9} \mathrm{~N}
$$

(d) The work done by the sand shows up as thermal energy, so $4.23 \times 10^{9} \mathrm{~J}$ of thermal energy is produced.
57. The external work required $\left(W_{\text {othere }}\right)$ is the change in the mechanical energy of the satellite. Note the following, from the work-energy theorem.

$$
W_{\text {total }}=W_{\text {gravity }}+W_{\text {other }} \rightarrow \Delta K=-\Delta U+W_{\text {other }} \rightarrow W_{\text {other }}=\Delta K+\Delta U=\Delta(K+U)=\Delta E_{\text {mech }}
$$

From problem 52, we know that the mechanical energy is given by $E=-\frac{1}{2} \frac{G M m}{r}$.

$$
\begin{aligned}
E=-\frac{1}{2} \frac{G M m}{r} \rightarrow \Delta E & =\left(-\frac{1}{2} \frac{G M m}{r}\right)_{\text {final }}-\left(-\frac{1}{2} \frac{G M m}{r}\right)_{\text {initial }}=\left(\frac{1}{2} \frac{G M m}{r}\right)_{\text {initial }}-\left(\frac{1}{2} \frac{G M m}{r}\right)_{\text {final }} \\
& =\left(\frac{1}{2} \frac{G M m}{2 r_{\mathrm{E}}}\right)_{\text {initial }}-\left(\frac{1}{2} \frac{G M m}{3 r_{\mathrm{E}}}\right)_{\text {final }}=\frac{G M m}{12 r_{\mathrm{E}}}
\end{aligned}
$$

58. (a) The work to put $m_{1}$ in place is 0 , because it is still infinitely distant from the other two masses. The work to put $m_{2}$ in place is the potential energy of the 2-mass system, $-\frac{G m_{1} m_{2}}{r_{12}}$. The work to put $m_{3}$ in place is the potential energy of the $m_{1}-m_{3}$ combination, $-\frac{G m_{1} m_{3}}{r_{13}}$, and the potential energy of the $m_{2}-m_{3}$ combination, $-\frac{G m_{2} m_{3}}{r_{23}}$. The total work is the sum of all of these potential energies, and so $W=-\frac{G m_{1} m_{2}}{r_{12}}-\frac{G m_{1} m_{3}}{r_{13}}-\frac{G m_{2} m_{3}}{r_{23}} \rightarrow$ $W=-G\left(\frac{m_{1} m_{2}}{r_{12}}+\frac{m_{1} m_{3}}{r_{13}}+\frac{m_{2} m_{3}}{r_{23}}\right)$. Notice that the work is negative, which is a result of the masses being gravitationally attracted towards each other.
(b) This formula gives the potential energy of the entire system. Potential energy does not "belong" to a single object, but rather to the entire system of objects that interact to give the potential energy.
(c) Actually, $|W|=G\left(\frac{m_{1} m_{2}}{r_{12}}+\frac{m_{1} m_{3}}{r_{13}}+\frac{m_{2} m_{3}}{r_{23}}\right)$ is the binding energy of the system. It would take that much work (a positive quantity) to separate the masses infinitely far from each other.
59. Since air friction is to be ignored, the mechanical energy will be conserved. Subscript 1 represents the asteroid at high altitude, and subscript 2 represents the asteroid at the Earth's surface. We have $v_{1}=660 \mathrm{~m} / \mathrm{s}, r_{1}=r_{\mathrm{E}}+5.0 \times 10^{9} \mathrm{~m}$, and $r_{2}=r_{\mathrm{E}}$.

$$
\begin{aligned}
& E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+\left(-\frac{G M_{\mathrm{E}} m}{r_{1}}\right)=\frac{1}{2} m v_{2}^{2}+\left(-\frac{G M_{\mathrm{E}} m}{r_{2}}\right) \rightarrow v_{2}=\sqrt{v_{1}^{2}+2 G M_{\mathrm{E}}\left(\frac{1}{r_{2}}-\frac{1}{r_{1}}\right)} \\
& =\sqrt{(660 \mathrm{~m} / \mathrm{s})^{2}+\left\{\left(\frac{2\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(5.98 \times 10^{24} \mathrm{~kg}\right) \cdot}{6.38 \times 10^{6} \mathrm{~m}+5.0 \times 10^{9} \mathrm{~m}}-\frac{1}{6.38 \times 10^{6} \mathrm{~m}}\right)\right\}}=1.12 \times 10^{4} \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

60. Calculate the density of the shell. Use that density to calculate the potential due to a full sphere of radius $r_{1}$, and then subtract the potential due to a mass of radius $r_{2}$.

$$
\begin{aligned}
& \rho=\frac{M}{\frac{4}{3} \pi\left(r_{1}^{3}-r_{2}^{3}\right)} \quad M_{\text {full }}^{\text {sphere }}
\end{aligned}=\frac{M}{\frac{4}{3} \pi\left(r_{1}^{3}-r_{2}^{3}\right)^{3} \frac{4}{3} \pi r_{1}^{3} \quad M_{\substack{\text { inner } \\
\text { sphere }}}=\frac{M}{\frac{4}{3} \pi\left(r_{1}^{3}-r_{2}^{3}\right)^{3}} \frac{4}{3} \pi r_{2}^{3}} \begin{aligned}
U_{\text {shell }} & =U_{\substack{\text { full } \\
\text { sphere }}}-U_{\substack{\text { inner } \\
\text { sphere }}}=-\frac{G M_{\text {full }}^{\text {sphere }}}{m}\left(-\frac{G M_{\text {inner }}^{\text {sphere }}}{m}\right)=-\frac{G m}{r}\left(M_{\substack{\text { full } \\
\text { sphere }}}-M_{\substack{\text { inner } \\
\text { sphere }}}\right) \\
& =-\frac{G m}{r}\left(\frac{M}{\frac{4}{3} \pi\left(r_{1}^{3}-r_{2}^{3}\right)^{3}} \frac{4}{3} \pi r_{1}^{3}-\frac{M}{\frac{4}{3} \pi\left(r_{1}^{3}-r_{2}^{3}\right)^{3}} \frac{4}{3} \pi r_{2}^{3}\right)=-\frac{G m M}{r}\left(\frac{r_{1}^{3}}{\left(r_{1}^{3}-r_{2}^{3}\right)}-\frac{r_{2}^{3}}{\left(r_{1}^{3}-r_{2}^{3}\right)}\right) \\
& =-G m M / r
\end{aligned}
$$

61. (a) The escape speed from the surface of the Earth is $v_{\mathrm{E}}=\sqrt{2 G M_{\mathrm{E}} / r_{\mathrm{E}}}$. The escape velocity from the gravitational field of the sun, is $v_{\mathrm{S}}=\sqrt{2 G M_{\mathrm{S}} / r_{\mathrm{SE}}}$. In the reference frame of the Earth, if the spacecraft leaves the surface of the Earth with speed $v$ (assumed to be greater than the escape velocity of Earth), then the speed $v^{\prime}$ at a distance far from Earth, relative to the Earth, is found from energy conservation.

$$
\frac{1}{2} m v^{2}-\frac{G M_{\mathrm{E}} m}{r_{\mathrm{E}}^{2}}=\frac{1}{2} m v^{\prime 2} \rightarrow v^{\prime 2}=v^{2}-\frac{2 G M_{\mathrm{E}}}{r_{\mathrm{E}}^{2}}=v^{2}-v_{\mathrm{E}}^{2} \rightarrow v^{2}=v^{\prime 2}+v_{\mathrm{E}}^{2}
$$

The reference frame of the Earth is orbiting the sun with speed $v_{0}$. If the rocket is moving with speed $v^{\prime}$ relative to the Earth, and the Earth is moving with speed $v_{0}$ relative to the Sun, then the speed of the rocket relative to the Sun is $v^{\prime}+v_{0}$ (assuming that both speeds are in the same direction). This is to be the escape velocity from the Sun, and so $v_{\mathrm{S}}=v^{\prime}+v_{0}$, or $v^{\prime}=v_{\mathrm{S}}-v_{0}$. Combine this with the relationship from above.

$$
\begin{aligned}
& v^{2}=v^{\prime 2}+v_{\mathrm{E}}^{2}=\left(v_{\mathrm{S}}-v_{0}\right)^{2}+v_{\mathrm{E}}^{2} \rightarrow \sqrt{v=\sqrt{\left(v_{\mathrm{S}}-v_{0}\right)^{2}+v_{\mathrm{E}}^{2}}} \\
& v_{\mathrm{E}}=\sqrt{\frac{2 G M_{\mathrm{E}}}{r_{\mathrm{E}}}}=\sqrt{\frac{2\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(5.98 \times 10^{24} \mathrm{~kg}\right)}{6.38 \times 10^{6} \mathrm{~m}}}=1.118 \times 10^{4} \mathrm{~m} / \mathrm{s} \\
& v_{\mathrm{S}}=\sqrt{\frac{2 G M_{\mathrm{S}}}{r_{\mathrm{SE}}}}=\sqrt{\frac{2\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(1.99 \times 10^{30} \mathrm{~kg}\right)}{1.496 \times 10^{11} \mathrm{~m}}}=4.212 \times 10^{4} \mathrm{~m} / \mathrm{s} \\
& v_{0}=\frac{2 \pi r_{\mathrm{SE}}}{T_{\mathrm{SE}}}=\frac{2 \pi\left(1.496 \times 10^{11} \mathrm{~m}\right)}{\left(3.156 \times 10^{7} \mathrm{~s}\right)}=2.978 \times 10^{4} \mathrm{~m} / \mathrm{s} \\
& v=\sqrt{\left(v_{\mathrm{S}}-v_{0}\right)^{2}+v_{\mathrm{E}}^{2}}=\sqrt{\left(4.212 \times 10^{4} \mathrm{~m} / \mathrm{s}-2.978 \times 10^{4} \mathrm{~m} / \mathrm{s}\right)^{2}+\left(1.118 \times 10^{4} \mathrm{~m} / \mathrm{s}\right)^{2}} \\
& =1.665 \times 10^{4} \mathrm{~m} / \mathrm{s} \approx 16.7 \mathrm{~km} / \mathrm{s}
\end{aligned}
$$

(b) Calculate the kinetic energy for a 1.00 kg mass moving with a speed of $1.665 \times 10^{4} \mathrm{~m} / \mathrm{s}$. This is the energy required per kilogram of spacecraft mass.

$$
K=\frac{1}{2} m v^{2}=\frac{1}{2}(1.00 \mathrm{~kg})\left(1.665 \times 10^{4} \mathrm{~m} / \mathrm{s}\right)^{2}=1.39 \times 10^{8} \mathrm{~J}
$$

62. The work necessary to lift the piano is the work done by an upward force, equal in magnitude to the weight of the piano. Thus $W=F d \cos 0^{\circ}=m g h$. The average power output required to lift the piano is the work done divided by the time to lift the piano.

$$
P=\frac{W}{t}=\frac{m g h}{t} \rightarrow t=\frac{m g h}{P}=\frac{(335 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(16.0 \mathrm{~m})}{1750 \mathrm{~W}}=30.0 \mathrm{~s}
$$

63. The 18 hp is the power generated by the engine in creating a force on the ground to propel the car forward. The relationship between the power and the force is Eq. 8-21 with the force and velocity in the same direction, $P=F v$. Thus the force to propel the car forward is found by $F=P / v$. If the car has a constant velocity, then the total resistive force must be of the same magnitude as the engine force, so that the net force is zero. Thus the total resistive force is also found by $F=P / v$.

$$
F=\frac{P}{v}=\frac{(18 \mathrm{hp})(746 \mathrm{~W} / 1 \mathrm{hp})}{(95 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)}=510 \mathrm{~N}
$$

64. (a) $K=\frac{1}{2} m v^{2}=\frac{1}{2}(85 \mathrm{~kg})(5.0 \mathrm{~m} / \mathrm{s})^{2}=1062.5 \mathrm{~J} \approx 1100 \mathrm{~J}$
(b) The power required to stop him is the change in energy of the player, divided by the time to carry out the energy change.

$$
P=\frac{1062.5 \mathrm{~J}}{1.0 \mathrm{~s}}=1062.5 \mathrm{~W} \approx 1100 \mathrm{~W}
$$

65. The energy transfer from the engine must replace the lost kinetic energy. From the two speeds, calculate the average rate of loss in kinetic energy while in neutral.

$$
\begin{aligned}
& v_{1}=95 \mathrm{~km} / \mathrm{h}\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)=26.39 \mathrm{~m} / \mathrm{s} \quad v_{2}=65 \mathrm{~km} / \mathrm{h}\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)=18.06 \mathrm{~m} / \mathrm{s} \\
& \Delta K E=\frac{1}{2} m v_{2}^{2}-\frac{1}{2} m v_{1}^{2}=\frac{1}{2}(1080 \mathrm{~kg})\left[(18.06 \mathrm{~m} / \mathrm{s})^{2}-(26.39 \mathrm{~m} / \mathrm{s})^{2}\right]=-1.999 \times 10^{5} \mathrm{~J} \\
& P=\frac{W}{t}=\frac{1.999 \times 10^{5} \mathrm{~J}}{7.0 \mathrm{~s}}=2.856 \times 10^{4} \mathrm{~W}, \text { or }\left(2.856 \times 10^{4} \mathrm{~W}\right) \frac{1 \mathrm{hp}}{746 \mathrm{~W}}=38.29 \mathrm{hp}
\end{aligned}
$$

So $2.9 \times 10^{4} \mathrm{~W}$ or 38 hp is needed from the engine.
66. Since $P=\frac{W}{t}$, we have $W=P t=3.0 \mathrm{hp}\left(\frac{746 \mathrm{~W}}{1 \mathrm{hp}}\right)(1 \mathrm{hr})\left(\frac{3600 \mathrm{~s}}{1 \mathrm{~h}}\right)=8.1 \times 10^{6} \mathrm{~J}$.
67. The power is the force that the motor can provide times the velocity, as given in Eq. 8-21. The force provided by the motor is parallel to the velocity of the boat. The force resisting the boat will be the same magnitude as the force provided by the motor, since the boat is not accelerating, but in the opposite direction to the velocity.

$$
P=\overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{v}}=F v \rightarrow F=\frac{P}{v}=\frac{(55 \mathrm{hp})(746 \mathrm{~W} / 1 \mathrm{hp})}{(35 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)}=4220 \mathrm{~N} \approx 4200 \mathrm{~N}
$$

So the force resisting the boat is 4200 N , opposing the velocity.
68. The average power is the energy transformed per unit time. The energy transformed is the change in kinetic energy of the car.

$$
\begin{aligned}
P & =\frac{\text { energy transformed }}{\text { time }}=\frac{\Delta K}{t}=\frac{\frac{1}{2} m\left(v_{2}^{2}-v_{1}^{2}\right)}{t}=\frac{(1400 \mathrm{~kg})\left[(95 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)\right]^{2}}{2(7.4 \mathrm{~s})} \\
& =6.6 \times 10^{4} \mathrm{~W} \approx 88 \mathrm{hp}
\end{aligned}
$$

69. The minimum force needed to lift the football player vertically is equal to his weight, $m g$. The distance over which that force would do work would be the change in height, $\Delta y=(78 \mathrm{~m}) \sin 33^{\circ}$. So the work done in raising the player is $W=m g \Delta y$ and the power output required is the work done per unit time.

$$
P=\frac{W}{t}=\frac{m g \Delta y}{t}=\frac{(92 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(78 \mathrm{~m}) \sin 33^{\circ}}{75 \mathrm{sec}}=510 \mathrm{~W}
$$

70. The force to lift the water is equal to its weight, and so the work to lift the water is equal to the weight times the vertical displacement. The power is the work done per unit time.

$$
P=\frac{W}{t}=\frac{m g h}{t}=\frac{(21.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(3.50 \mathrm{~m})}{60 \mathrm{sec}}=12.0 \mathrm{~W}
$$

71. The force to lift a person is equal to the person's weight, so the work to lift a person up a vertical distance $h$ is equal to $m g h$. The work needed to lift $N$ people is Nmgh , and so the power needed is the total work divided by the total time. We assume the mass of the average person to be 70 kg .

$$
P=\frac{W}{t}=\frac{N m g h}{t}=\frac{47000(70 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(200 \mathrm{~m})}{3600 \mathrm{~s}}=1.79 \times 10^{6} \mathrm{~W} \approx 2 \times 10^{6} \mathrm{~W} .
$$

72. We represent all 30 skiers as one person on the free-body diagram. The engine must supply the pulling force. The skiers are moving with constant velocity, and so their net force must be 0 .

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{N}}-m g \cos \theta=0 \rightarrow F_{\mathrm{N}}=m g \cos \theta \\
& \sum F_{x}=F_{\mathrm{P}}-m g \sin \theta-F_{\mathrm{fr}}=0 \rightarrow \\
& F_{\mathrm{P}}=m g \sin \theta+F_{\mathrm{fr}}=m g \sin \theta+\mu_{\mathrm{k}} m g \cos \theta
\end{aligned}
$$



The work done by $F_{\mathrm{P}}$ in pulling the skiers a distance $d$ is $F_{\mathrm{P}} d$ since the force is parallel to the displacement. Finally, the power needed is the work done divided by the time to move the skiers up the incline.

$$
\begin{aligned}
P & =\frac{W}{t}=\frac{F_{\mathrm{p}} d}{t}=\frac{m g\left(\sin \theta+\mu_{\mathrm{k}} \cos \theta\right) d}{t} \\
& =\frac{30(75 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(\sin 23^{\circ}+0.10 \cos 23^{\circ}\right)(220 \mathrm{~m})}{120 \mathrm{~s}}=19516 \mathrm{~W}\left(\frac{1 \mathrm{hp}}{746 \mathrm{~W}}\right)=26 \mathrm{hp}
\end{aligned}
$$

73. The net rate of work done is the power, which can be found by $P=F v=m a v$. The velocity is given by $v=\frac{d x}{d t}=15.0 t^{2}-16.0 t-44$ and $a=\frac{d v}{d t}=30.0 t-16.0$.
(a)

$$
\begin{align*}
P & =m a v=(0.28 \mathrm{~kg})\left([30.0(2.0)-16.0] \mathrm{m} / \mathrm{s}^{2}\right)\left[15.0(2.0)^{2}-16.0(2.0)-44\right] \mathrm{m} / \mathrm{s} \\
& =-197.1 \mathrm{~W} \approx-2.0 \times 10^{2} \mathrm{~W} \\
P & =m a v=(0.28 \mathrm{~kg})\left([30.0(4.0)-16.0] \mathrm{m} / \mathrm{s}^{2}\right)\left[15.0(4.0)^{2}-16.0(4.0)-44\right] \mathrm{m} / \mathrm{s}  \tag{b}\\
& =3844 \mathrm{~W} \approx 3800 \mathrm{~W}
\end{align*}
$$

The average net power input is the work done divided by the elapsed time. The work done is the change in kinetic energy. Note $v(0)=-44 \mathrm{~m} / \mathrm{s}, v(2.0)=15.0(2.0)^{2}-16.0(2.0)-44=-16 \mathrm{~m} / \mathrm{s}$, and $v(4.0)=15.0(4.0)^{2}-16.0(4.0)-44=132 \mathrm{~m} / \mathrm{s}$.
(c) $P_{\substack{\text { avg } \\ 0.020}}=\frac{\Delta K}{\Delta t}=\frac{\frac{1}{2} m\left(v_{f}^{2}-v_{i}^{2}\right)}{\Delta t}=\frac{\frac{1}{2}(0.28 \mathrm{~kg})\left[(-16 \mathrm{~m} / \mathrm{s})^{2}-(-44 \mathrm{~m} / \mathrm{s})^{2}\right]}{2.0 \mathrm{~s}}=-120 \mathrm{~W}$
(d) $P_{\substack{\text { avg } \\ 2.0 \text { to } 4.0}}=\frac{\Delta K}{\Delta t}=\frac{\frac{1}{2} m\left(v_{f}^{2}-v_{i}^{2}\right)}{\Delta t}=\frac{\frac{1}{2}(0.28 \mathrm{~kg})\left[(132 \mathrm{~m} / \mathrm{s})^{2}-(16 \mathrm{~m} / \mathrm{s})^{2}\right]}{2.0 \mathrm{~s}}=1200 \mathrm{~W}$
74. First, consider a free-body diagram for the cyclist going down hill. Write Newton's second law for the $x$ direction, with an acceleration of 0 since the cyclist has a constant speed.

$$
\sum F_{x}=m g \sin \theta-F_{\mathrm{fr}}=0 \rightarrow F_{\mathrm{fr}}=m g \sin \theta
$$



Now consider the diagram for the cyclist going up the hill. Again, write Newton's second law for the $x$ direction, with an acceleration of 0 .

$$
\sum F_{x}=F_{\mathrm{fr}}-F_{\mathrm{P}}+m g \sin \theta=0 \rightarrow F_{\mathrm{p}}=F_{\mathrm{fr}}+m g \sin \theta
$$

Assume that the friction force is the same when the speed is the same, so the friction force when going uphill is the same magnitude as when going downhill.

$$
F_{\mathrm{P}}=F_{\mathrm{fr}}+m g \sin \theta=2 m g \sin \theta
$$



The power output due to this force is given by Eq. 8-21, with the force and velocity parallel.

$$
P=F_{\mathrm{P}} v=2 m g v \sin \theta=2(75 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(4.0 \mathrm{~m} / \mathrm{s}) \sin 6.0^{\circ}=610 \mathrm{~W}
$$

75. The potential energy is given by $U(x)=\frac{1}{2} k x^{2}$ and so has a parabolic shape. The total energy of the object is $E=\frac{1}{2} k x_{0}^{2}$. The object, when released, will gain kinetic energy and lose potential energy until it reaches the equilibrium at $x=0$, where it will have its maximum kinetic energy and maximum speed. Then it continues to move to the left, losing kinetic energy and gaining potential energy, until it reaches its extreme point of $x=x_{0}$. Then the motion reverses, until the object
 reaches its original position. Then it will continue this oscillatory motion between $x=0$ and $x=x_{0}$.
76. (a) The total energy is $E=\frac{1}{2} k x_{0}^{2}=\frac{1}{2}(160 \mathrm{~N} / \mathrm{m})(1.0 \mathrm{~m})^{2}=80 \mathrm{~J}$. The answer has 2 significant figures.
(b) The kinetic energy is the total energy minus the potential energy.

$$
K=E-U=E-\frac{1}{2} k x^{2}=80 \mathrm{~J}-\frac{1}{2}(160 \mathrm{~N} / \mathrm{m})(0.50 \mathrm{~m})^{2}=60 \mathrm{~J}
$$

The answer has 2 significant figures.
(c) The maximum kinetic energy is the total energy, 80 J .
(d) The maximum speed occurs at $x=0$, the equilibrium position at the center of the motion. Use the maximum kinetic energy (which is equal to the total energy) to find the maximum speed.

$$
K_{\max }=\frac{1}{2} m v_{\max }^{2} \rightarrow v_{\max }=\sqrt{\frac{2 K_{\max }}{m}}=\sqrt{\frac{2(80 \mathrm{~J})}{5.0 \mathrm{~kg}}}=5.7 \mathrm{~m} / \mathrm{s}
$$

(e) The maximum acceleration occurs at the maximum displacement, $x=1.0 \mathrm{~m}$, since

$$
\begin{aligned}
F= & m a=-k x \rightarrow|a|=\frac{k|x|}{m} . \\
& \left|a_{\max }\right|=\frac{k\left|x_{\max }\right|}{m}=\frac{(160 \mathrm{~N} / \mathrm{m})(1.0 \mathrm{~m})}{5.0 \mathrm{~kg}}=32 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

77. (a) To find possible minima and maxima, set the first derivative of the function equal to 0 and solve for the values of $r$.

$$
\begin{aligned}
& U(r)=-\frac{a}{r^{6}}+\frac{b}{r^{12}}=\frac{1}{r^{12}}\left(b-a r^{6}\right) \rightarrow \frac{d U}{d r}=6 \frac{a}{r^{7}}-12 \frac{b}{r^{13}} \\
& \frac{d U}{d r}=0 \rightarrow \frac{a}{r^{7}}=2 \frac{b}{r^{13}} \rightarrow r_{\text {crit }}=\left(\frac{2 b}{a}\right)^{1 / 6}, \infty
\end{aligned}
$$

The second derivative test is used to determine the actual type of critical points found.

$$
\begin{aligned}
& \frac{d^{2} U}{d r^{2}}=-42 \frac{a}{r^{8}}+156 \frac{b}{r^{14}}=\frac{1}{r^{14}}\left(156 b-42 a r^{6}\right) \\
& \left.\frac{d^{2} U}{d r^{2}}\right|_{\left(\frac{2 b}{a}\right)^{1 / 6}}=\frac{1}{\left(\frac{2 b}{a}\right)^{14 / 6}}\left(156 b-42 a \frac{2 b}{a}\right)=\frac{1}{\left(\frac{2 b}{a}\right)^{14 / 6}}(156 b-84 b)>0 \rightarrow r_{\mathrm{crit}}=\left(\frac{2 b}{a}\right)^{1 / 6}
\end{aligned}
$$

Thus there is a minimum at $r=\left(\frac{2 b}{a}\right)^{1 / 6}$. We also must check the endpoints of the function.
We see from the form $U(r)=\frac{1}{r^{12}}\left(b-a r^{6}\right)$ that as $r \rightarrow 0, U(r) \rightarrow \infty$, and so there is a maximum at $r=0$.
(b) Solve $U(r)=0$ for the distance.

$$
\begin{aligned}
& U(r)=-\frac{a}{r^{6}}+\frac{b}{r^{12}}=\frac{1}{r^{12}}\left(b-a r^{6}\right)=0 \rightarrow \frac{1}{r^{12}}=0 \text { or }\left(b-a r^{6}\right)=0 \rightarrow \\
& r=\infty ; r=\left(\frac{b}{a}\right)^{1 / 6}
\end{aligned}
$$

(c) See the adjacent graph. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH08.XLS," on tab "Problem 8.77c."
(d) For $E<0$, there will be bound oscillatory motion between two turning points. This could represent a chemical bond type of situation. For $E>0$, the motion will be unbounded, and so the atoms will not stay
 together.
(e) The force is the opposite of the slope of the potential energy graph.

$$
F>0 \text { for } r<\left(\frac{2 b}{a}\right)^{1 / 6} ; F<0 \text { for }\left(\frac{2 b}{a}\right)^{1 / 6}<r<\infty \quad ; F=0 \text { for } r=\left(\frac{2 b}{a}\right)^{1 / 6}, r=\infty
$$

(f) $F(r)=-\frac{d U}{d r}=\frac{12 b}{r^{13}}-\frac{6 a}{r^{7}}$
78. The binding energy will be $U(\infty)-U\left(r_{\mathrm{U} \text { min }}\right)$. The value of $r$ for which $U(r)$ has a minimum is found in problem 77 to be $r=\left(\frac{2 b}{a}\right)^{1 / 6}$.

$$
U(\infty)-U\left(r_{\mathrm{U} \text { min }}\right)=0-U\left(r=\left(\frac{2 b}{a}\right)^{1 / 6}\right)=0-\left[-\frac{a}{\left(\frac{2 b}{a}\right)}+\frac{b}{\left(\frac{2 b}{a}\right)^{2}}\right]=0-\left[-\frac{a^{2}}{2 b}+\frac{a^{2} b}{4 b^{2}}\right]=\frac{a^{2}}{4 b}
$$

Notice that this is just the depth of the potential well.
79. The power must exert a force equal to the weight of the elevator, through the vertical height, in the given time.

$$
P=\frac{m g h}{t}=\frac{(885 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(32.0 \mathrm{~m})}{(11.0 \mathrm{~s})}=2.52 \times 10^{4} \mathrm{~W}
$$

80. Since there are no non-conservative forces, the mechanical energy of the projectile will be conserved. Subscript 1 represents the projectile at launch and subscript 2 represents the projectile as it strikes the ground. The ground is the zero location for potential energy $(y=0)$. We have $v_{1}=165 \mathrm{~m} / \mathrm{s}, y_{1}=135 \mathrm{~m}$, and $y_{2}=0$. Solve for $v_{2}$.

$$
\begin{aligned}
& E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2} \rightarrow \\
& v_{2}=\sqrt{v_{1}^{2}+2 g y_{1}}=\sqrt{(165 \mathrm{~m} / \mathrm{s})^{2}+2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(135 \mathrm{~m})}=173 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Notice that the launch angle does not enter the problem, and so does not influence the final speed.
81. (a) Use conservation of mechanical energy, assuming there are no non-conservative forces.

Subscript 1 represents the water at the top of the dam, and subscript 2 represents the water as it strikes the turbine blades. The level of the turbine blades is the zero location for potential energy $(y=0)$. Assume that the water goes over the dam with an approximate speed of 0 . We have $v_{1}=0, y_{1}=80 \mathrm{~m}$, and $y_{2}=0$. Solve for $v_{2}$.

$$
\begin{aligned}
& E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2} \rightarrow m g y_{1}=\frac{1}{2} m v_{2}^{2} \rightarrow \\
& v_{2}=\sqrt{2 g y_{1}}=\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(88 \mathrm{~m})}=41.53 \mathrm{~m} / \mathrm{s} \approx 42 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(b) The energy of the water at the level of the turbine blades is all kinetic energy, and so is given by $\frac{1}{2} m v_{2}^{2} .55 \%$ of that energy gets transferred to the turbine blades. The rate of energy transfer to the turbine blades is the power developed by the water.

$$
P=0.55\left(\frac{1}{2} \frac{m}{t} v_{2}^{2}\right)=\frac{(0.55)(550 \mathrm{~kg} / \mathrm{s})(41.53 \mathrm{~m} / \mathrm{s})^{2}}{2}=2.6 \times 10^{5} \mathrm{~W}
$$

82. First, define three speeds:
$v_{0}=12 \mathrm{~km} / \mathrm{h}=$ speed when coasting downhill.
$v_{1}=32 \mathrm{~km} / \mathrm{h}=$ speed when pedaling downhill.
$v_{2}=$ Speed when climbing the hill.
For coasting downhill at a constant speed, consider the first free-body diagram shown. The net force on the bicyclist must be 0 . Write Newton's second law for the $x$ direction.


$$
\sum F_{x}=F_{\mathrm{fr} 0}-m g \sin \theta=0 \rightarrow F_{\mathrm{ff} 0}=m g \sin \theta
$$

Note that this occurs at $v=v_{0}$.
When pumping hard downhill, the speed is $v_{1}=\frac{32}{12} v_{o}=\frac{8}{3} v_{o}$. Since the frictional force is proportional to $v^{2}$, the frictional force increases by a factor of $\left(\frac{8}{3}\right)^{2}: F_{\mathrm{ff} 1}=\left(\frac{8}{3}\right)^{2} F_{\mathrm{ff} 0}=\frac{64}{9} m g \sin \theta$. See the second freebody diagram. There is a new force, $\overrightarrow{\mathbf{F}}_{\mathrm{Pl}}$, created by the bicyclist.
 Since the cyclist is moving at a constant speed, the net force in the $x$ direction must still be 0 . Solve for $F_{\mathrm{P} 1}$, and calculate the power associated with the force.

$$
\begin{aligned}
& \sum F_{x}=F_{\mathrm{fr} 1}-m g \sin \theta-F_{\mathrm{P} 1}=0 \rightarrow F_{\mathrm{P} 1}=F_{\mathrm{fr} 1}-m g \sin \theta=\left(\frac{64}{9}-1\right) m g \sin \theta=\frac{55}{9} m g \sin \theta \\
& P_{1}=F_{\mathrm{ft} 1} v_{1}=\frac{55}{9} m g v_{1} \sin \theta
\end{aligned}
$$

Now consider the cyclist going uphill. The speed of the cyclist going up the hill is $v_{2}$. Since the frictional force is proportional to $v^{2}$, the frictional force is given by $F_{\mathrm{f} 2}=\left(v_{2} / v_{0}\right)^{2} m g \sin \theta$. See the third freebody diagram. There is a new force, $\overrightarrow{\mathbf{F}}_{\mathrm{P} 2}$, created by the bicyclist. Since the cyclist is moving at a constant speed, the net force in the $x$ direction must still be 0 .

$$
\sum F_{x}=F_{\mathrm{P} 2}-m g \sin \theta-F_{\mathrm{f} 2}=0
$$

The power output of the cyclist while pedaling uphill is the same as when pedaling going downhill.

$$
P_{2}=P_{1}=\frac{55}{9} m g v_{1} \sin \theta \rightarrow F_{P 2} v_{2}=\frac{55}{9} m g v_{1} \sin \theta \rightarrow F_{P 2}=\frac{55}{9} m g\left(v_{1} / v_{2}\right) \sin \theta
$$

Combine this information with Newton's second law equation for the bicyclist going uphill.

$$
F_{\mathrm{P} 2}-m g \sin \theta-F_{\mathrm{fr} 2}=\frac{55}{9} m g\left(v_{1} / v_{2}\right) \sin \theta-m g \sin \theta-\left(v_{2} / v_{0}\right)^{2} m g \sin \theta=0
$$

This simplifies to the following cubic equation: $v_{2}^{3}+v_{2} v_{0}^{2}-\frac{55}{9} v_{1} v_{0}^{2}=0$. Note that since every term has speed to the third power, there is no need to do unit conversions. Numerically, this equation is $v_{2}^{3}+144 v_{2}-28160=0$, when the speed is in $\mathrm{km} / \mathrm{h}$. Solving this cubic equation (with a spreadsheet, for example) gives $v_{2}=28.847 \mathrm{~km} / \mathrm{h} \approx 29 \mathrm{~km} / \mathrm{h}$.
83. (a) The speed $v_{\mathrm{B}}$ can be found from conservation of mechanical energy. Subscript A represents the skier at the top of the jump, and subscript B represents the skier at the end of the ramp. Point B is taken as the zero location for potential energy $(y=0)$. We have $v_{1}=0, y_{1}=40.6 \mathrm{~m}$, and $y_{2}=0$. Solve for $v_{2}$.

$$
\begin{aligned}
& E_{\mathrm{A}}=E_{\mathrm{B}} \rightarrow \frac{1}{2} m v_{\mathrm{A}}^{2}+m g y_{\mathrm{A}}=\frac{1}{2} m v_{\mathrm{B}}^{2}+m g y_{\mathrm{B}} \rightarrow m g y_{\mathrm{A}}=\frac{1}{2} m v_{\mathrm{B}}^{2} \rightarrow \\
& v_{\mathrm{B}}=\sqrt{2 g y_{\mathrm{A}}}=\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(40.6 \mathrm{~m})}=28.209 \mathrm{~m} / \mathrm{s} \approx 28.2 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(b) Now we use projectile motion. We take the origin of coordinates to be the point on the ground directly under the end of the ramp. Then an equation to describe the slope is $y_{\text {slope }}=-x \tan 30^{\circ}$. The equations of projectile motion can be used to find an expression for the parabolic path that the skier follows after leaving the ramp. We take up to be the positive vertical direction. The initial $y$-velocity is 0 , and the $x$-velocity is $v_{\mathrm{B}}$ as found above.

$$
x=v_{\mathrm{B}} t ; y_{\text {proj }}=y_{0}-\frac{1}{2} g t^{2}=y_{0}-\frac{1}{2} g\left(x / v_{\mathrm{B}}\right)^{2}
$$

The skier lands at the intersection of the two paths, so $y_{\text {slope }}=y_{\text {proj }}$.

$$
\begin{aligned}
& y_{\text {slope }}=y_{\text {proj }} \rightarrow-x \tan 30^{\circ}=y_{0}-\frac{1}{2} g\left(\frac{x}{v_{\mathrm{B}}}\right)^{2} \rightarrow g x^{2}-x\left(2 v_{\mathrm{B}}^{2} \tan 30^{\circ}\right)-2 y_{0} v_{\mathrm{B}}^{2}=0 \rightarrow \\
& x=\frac{\left(2 v_{\mathrm{B}}^{2} \tan 30^{\circ}\right) \pm \sqrt{\left(2 v_{\mathrm{B}}^{2} \tan 30^{\circ}\right)^{2}+8 g y_{0} v_{\mathrm{B}}^{2}}}{2 g}=\frac{\left(v_{\mathrm{B}}^{2} \tan 30^{\circ}\right) \pm \sqrt{\left(v_{\mathrm{B}}^{2} \tan 30^{\circ}\right)^{2}+2 g y_{0} v_{\mathrm{B}}^{2}}}{g}
\end{aligned}
$$

Solving this with the given values gives $x=-7.09 \mathrm{~m}, 100.8 \mathrm{~m}$. The positive root is taken.
Finally, $s \cos 30.0^{\circ}=x \rightarrow s=\frac{x}{\cos 30.0^{\circ}}=\frac{100.8 \mathrm{~m}}{\cos 30.0^{\circ}}=116 \mathrm{~m}$.
84. (a) The slant of the jump at point B does not affect the energy conservation calculations from problem 83 , and so this part of the problem is solved exactly as in problem 83 , and the answer is exactly the same as in problem $83: v_{\mathrm{B}}=28.209 \mathrm{~m} / \mathrm{s} \approx 28.2 \mathrm{~m} / \mathrm{s}$.
(b) The projectile motion is now different because the velocity at point B is not purely horizontal. We have that $v_{\mathrm{B}}=28.209 \mathrm{~m} / \mathrm{s}$ and $v_{\mathrm{B} y}=3.0 \mathrm{~m} / \mathrm{s}$. Use the Pythagorean theorem to find $v_{\mathrm{B} x}$.

$$
v_{\mathrm{B} x}=\sqrt{v_{\mathrm{B}}^{2}-v_{\mathrm{B} y}^{2}}=\sqrt{(28.209 \mathrm{~m} / \mathrm{s})^{2}-(3.0 \mathrm{~m} / \mathrm{s})^{2}}=28.049 \mathrm{~m} / \mathrm{s}
$$

We take the origin of coordinates to be the point on the ground directly under the end of the ramp. Then an equation to describe the slope is $y_{\text {slope }}=-x \tan 30^{\circ}$. The equations of projectile motion can be used to find an expression for the parabolic path that the skier follows after leaving the ramp. We take up to be the positive vertical direction.

$$
x=v_{\mathrm{B} x} t ; y_{\mathrm{proj}}=y_{0}+v_{\mathrm{B} y} t-\frac{1}{2} g t^{2}=y_{0}+v_{\mathrm{B} y}\left(\frac{x}{v_{\mathrm{B} x}}\right)-\frac{1}{2} g\left(\frac{x}{v_{\mathrm{B} x}}\right)^{2}
$$

The skier lands at the intersection of the two paths, so $y_{\text {slope }}=y_{\text {proj }}$.

$$
\begin{aligned}
& y_{\text {slope }}=y_{\text {proj }} \rightarrow-x \tan 30^{\circ}=y_{0}+v_{\mathrm{B} y}\left(\frac{x}{v_{\mathrm{B} x}}\right)-\frac{1}{2} g\left(\frac{x}{v_{\mathrm{B} x}}\right)^{2} \rightarrow \\
& g x^{2}-x\left[2 v_{\mathrm{B} x}\left(v_{\mathrm{B} x} \tan 30^{\circ}+v_{\mathrm{B} y}\right)\right]-2 y_{0} v_{\mathrm{B} x}^{2}=0 \rightarrow \\
& x=\frac{\left[2 v_{\mathrm{B} x}\left(v_{\mathrm{B} x} \tan 30^{\circ}+v_{\mathrm{B} y}\right)\right] \pm \sqrt{\left[2 v_{\mathrm{B} x}\left(v_{\mathrm{B} x} \tan 30^{\circ}+v_{\mathrm{B} y}\right)\right]^{2}-8 g y_{0} v_{\mathrm{B} x}^{2}}}{2 g}
\end{aligned}
$$

Solving this with the given values gives $x=-6.09 \mathrm{~m}, 116.0 \mathrm{~m}$. The positive root is taken.
Finally, $s \cos 30.0^{\circ}=x \rightarrow s=\frac{x}{\cos 30.0^{\circ}}=\frac{116.0 \mathrm{~m}}{\cos 30.0^{\circ}}=134 \mathrm{~m}$.
85. (a) The tension in the cord is perpendicular to the path at all times, and so the tension in the cord does not do any work on the ball. Thus only gravity does work on the ball, and so the mechanical energy of the ball is conserved. Subscript 1 represents the ball when it is horizontal, and subscript 2 represents the ball at the lowest point on its path. The lowest point on the path is the zero location for potential energy $(y=0)$. We have $v_{1}=0, y_{1}=\ell$, and $y_{2}=0$. Solve for $v_{2}$.

$$
E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2} \rightarrow m g \ell=\frac{1}{2} m v_{2}^{2} \rightarrow v_{2}=\sqrt{2 g \ell}
$$

(b) Use conservation of energy, to relate points 2 and 3. Point 2 is as described above. Subscript 3 represents the ball at the top of its circular path around the peg. The lowest point on the path is the zero location for potential energy $(y=0)$. We have $v_{2}=\sqrt{2 g \ell}, y_{2}=0$, and

$$
\begin{aligned}
y_{3}= & 2(\ell-h)=2(\ell-0.80 \ell)=0.40 \ell . \text { Solve for } v_{3} . \\
& E_{2}=E_{3} \rightarrow \frac{1}{2} m v_{2}^{2}+m g y_{2}=\frac{1}{2} m v_{3}^{2}+m g y_{3} \rightarrow \frac{1}{2} m(2 g \ell)=\frac{1}{2} m v_{3}^{2}+m g(0.40 \ell) \rightarrow \\
& v_{3}=\sqrt{1.2 g \ell}
\end{aligned}
$$

86. The ball is moving in a circle of radius $(\ell-h)$. If the ball is to complete the circle with the string just going slack at the top of the circle, the force of gravity must supply the centripetal force at the top of the circle. This tells the critical (slowest) speed for the ball to have at the top of the circle.

$$
m g=\frac{m v_{\text {crit }}^{2}}{r} \rightarrow v_{\text {crit }}^{2}=g r=g(\ell-h)
$$

To find another expression for the speed, we use energy conservation. Subscript 1 refers to the ball at the launch point, and subscript 2 refers to the ball at the top of the circular path about the peg. The zero for gravitational potential energy is taken to be the lowest point of the ball's path. Let the speed at point 2 be the critical speed found above.

$$
\begin{aligned}
& E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2} \rightarrow m g \ell=\frac{1}{2} m g(\ell-h)+2 m g(\ell-h) \rightarrow \\
& h=0.6 \ell
\end{aligned}
$$

If $h$ is any smaller than this, then the ball would be moving slower than the critical speed when it reaches the top of the circular path, and would not stay in centripetal motion.
87. Consider the free-body diagram for the coaster at the bottom of the loop. The net force must be an upward centripetal force.

$$
\sum F_{\text {bottom }}=\underset{\substack{\mathrm{N} \\ \text { botom }}}{F_{\mathrm{N}}^{2}}-m g=m v_{\text {botom }}^{2} / R \rightarrow \underset{\text { botom }}{F_{\mathrm{N}}}=m g+m v_{\text {botom }}^{2} / R
$$

Now consider the force diagram at the top of the loop. Again, the net force must be centripetal, and so must be downward.

$$
\sum F_{\text {top }}=\underset{\substack{\mathrm{N} \\ \text { top }}}{F_{\mathrm{c}}}+m g=m v_{\text {top }}^{2} / R \quad \rightarrow \underset{\substack{N \\ \text { top }}}{F_{N}}=m v_{\text {top }}^{2} / R-m g
$$

Assume that the speed at the top is large enough that $F_{N}>0$, and so $v_{\text {top }}>\sqrt{R g}$.


Now apply the conservation of mechanical energy. Subscript 1 represents the coaster at the bottom of the loop, and subscript 2 represents the coaster at the top of the loop. The level of the bottom of the loop is the zero location for potential energy $(y=0)$. We have $y_{1}=0$ and $y_{2}=2 R$.

$$
E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2} \rightarrow v_{\text {botom }}^{2}=v_{\text {top }}^{2}+4 g R
$$

The difference in apparent weights is the difference in the normal forces.

$$
\begin{aligned}
F_{\mathrm{N}}^{\text {botom }}-F_{\text {top }} & =\left(m g+m v_{\text {botom }}^{2} / R\right)-\left(m v_{\text {top }}^{2} / R-m g\right)=2 m g+m\left(v_{\text {botom }}^{2}-v_{\text {top }}^{2}\right) / R \\
& =2 m g+m(4 g R) / R=6 m g
\end{aligned}
$$

Notice that the result does not depend on either $v$ or $R$.
88. The spring constant for the scale can be found from the 0.5 mm compression due to the 760 N force. $k=\frac{F}{x}=\frac{760 \mathrm{~N}}{5.0 \times 10^{-4} \mathrm{~m}}=1.52 \times 10^{6} \mathrm{~N} / \mathrm{m}$. Use conservation of energy for the jump. Subscript 1 represents the initial location, and subscript 2 represents the location at maximum compression of the scale spring. Assume that the location of the uncompressed scale spring is the 0 location for gravitational potential energy. We have $v_{1}=v_{2}=0$ and $y_{1}=1.0 \mathrm{~m}$. Solve for $y_{2}$, which must be negative.

$$
\begin{aligned}
& E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2}+\frac{1}{2} k y_{2}^{2} \rightarrow \\
& m g y_{1}=m g y_{2}+\frac{1}{2} k y_{2}^{2} \rightarrow y_{2}^{2}+2 \frac{m g}{k} y_{2}-2 \frac{m g}{k} y_{1}=y_{2}^{2}+1.00 \times 10^{-3} y_{2}-1.00 \times 10^{-3}=0 \\
& y_{2}=-3.21 \times 10^{-2} \mathrm{~m}, 3.11 \times 10^{-2} \mathrm{~m} \\
& F_{\text {scale }}=k|x|=\left(1.52 \times 10^{6} \mathrm{~N} / \mathrm{m}\right)\left(3.21 \times 10^{-2} \mathrm{~m}\right)=4.9 \times 10^{4} \mathrm{~N}
\end{aligned}
$$

89. (a) The work done by the hiker against gravity is the change in gravitational potential energy.

$$
W_{\mathrm{G}}=m g \Delta y=(65 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(4200 \mathrm{~m}-2800 \mathrm{~m})=8.918 \times 10^{5} \mathrm{~J} \approx 8.9 \times 10^{5} \mathrm{~J}
$$

(b) The average power output is found by dividing the work by the time taken.

$$
\begin{aligned}
& P_{\text {oupput }}=\frac{W_{\text {grav }}}{t}=\frac{8.918 \times 10^{5} \mathrm{~J}}{(5.0 \mathrm{~h})(3600 \mathrm{~s} / 1 \mathrm{~h})}=49.54 \mathrm{~W} \approx 5.0 \times 10^{1} \mathrm{~W} \\
& 49.54 \mathrm{~W}\left(\frac{1 \mathrm{hp}}{746 \mathrm{~W}}\right)=6.6 \times 10^{-2} \mathrm{hp}
\end{aligned}
$$

(c) The output power is the efficiency times the input power.

$$
P_{\text {output }}=0.15 P_{\text {input }} \rightarrow P_{\text {input }}=\frac{P_{\text {output }}}{0.15}=\frac{49.54 \mathrm{~W}}{0.15}=330 \mathrm{~W}=0.44 \mathrm{hp}
$$

90. (a) Draw a free-body diagram for the block at the top of the curve. Since the block is moving in a circle, the net force is centripetal. Write Newton's second law for the block, with down as positive. If the block is to be on the verge of falling off the track, then $F_{\mathrm{N}}=0$.


$$
\sum F_{\mathrm{R}}=F_{\mathrm{N}}+m g=m v^{2} / r \quad \rightarrow \quad m g=m v_{\mathrm{top}}^{2} / r \quad \rightarrow \quad v_{\mathrm{top}}=\sqrt{g r}
$$

Now use conservation of energy for the block. Since the track is frictionless, there are no nonconservative forces, and mechanical energy will be conserved. Subscript 1 represents the block at the release point, and subscript 2 represents the block at the top of the loop. The ground is the zero location for potential energy $(y=0)$. We have $v_{1}=0, y_{1}=h, v_{2}=\sqrt{g r}$, and $y_{2}=2 r$. Solve for $h$.

$$
\begin{aligned}
& E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2} \rightarrow 0+m g h=\frac{1}{2} m g r+2 m g r \rightarrow \\
& h=2.5 r
\end{aligned}
$$

(b) See the free-body diagram for the block at the bottom of the loop. The net force is again centripetal, and must be upwards.

$$
\sum F_{\mathrm{R}}=F_{\mathrm{N}}-m g=m v^{2} / r \quad \rightarrow \quad F_{\mathrm{N}}=m g+m v_{\mathrm{bottom}}^{2} / r
$$

The speed at the bottom of the loop can be found from energy conservation, similar to what was done in part $(a)$ above, by equating the energy at the
 release point (subscript 1) and the bottom of the loop (subscript 2). We now have $v_{1}=0$, $y_{1}=2 h=5 r$, and $y_{2}=0$. Solve for $v_{2}$.

$$
\begin{aligned}
& E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2} \rightarrow 0+5 m g r=\frac{1}{2} m v_{\mathrm{bottom}}^{2}+0 \rightarrow \\
& v_{\mathrm{bottom}}^{2}=10 g r \rightarrow F_{\mathrm{N}}=m g+m v_{\mathrm{bottom}}^{2} / r=m g+10 m g=11 m g
\end{aligned}
$$

(c) Again we use the free body diagram for the top of the loop, but now the normal force does not vanish. We again use energy conservation, with $v_{1}=0, y_{1}=3 r$, and $y_{2}=0$. Solve for $v_{2}$.

$$
\begin{aligned}
& \sum F_{\mathrm{R}}=F_{\mathrm{N}}+m g=m v^{2} / r \rightarrow F_{\mathrm{N}}=m v_{\mathrm{top}}^{2} / r-m g \\
& E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2} \rightarrow 0+3 m g r=\frac{1}{2} m v_{\text {top }}^{2}+0 \rightarrow \\
& v_{\mathrm{top}}^{2}=6 g r \rightarrow F_{\mathrm{N}}=m v_{\mathrm{top}}^{2} / r-m g=6 m g-m g=5 m g
\end{aligned}
$$

(d) On the flat section, there is no centripetal force, and $F_{\mathrm{N}}=m g$.
91. (a) Use conservation of energy for the swinging motion. Subscript 1 represents the student initially grabbing the rope, and subscript 2 represents the student at the top of the swing. The location where the student initially grabs the rope is the zero location for potential energy $(y=0)$. We have $v_{1}=5.0 \mathrm{~m} / \mathrm{s}, y_{1}=0$, and $v_{2}=0$. Solve for $y_{2}$.

$$
\begin{aligned}
& E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2} \rightarrow \\
& \frac{1}{2} m v_{1}^{2}=m g y_{2} \rightarrow y_{2}=\frac{v_{1}^{2}}{2 g}=h
\end{aligned}
$$

Calculate the angle from the relationship in the diagram.

$$
\cos \theta=\frac{\ell-h}{\ell}=1-\frac{h}{\ell}=1-\frac{v_{1}^{2}}{2 g \ell} \rightarrow
$$



$$
\theta=\cos ^{-1}\left(1-\frac{v_{1}^{2}}{2 g \ell}\right)=\cos ^{-1}\left(1-\frac{(5.0 \mathrm{~m} / \mathrm{s})^{2}}{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(10.0 \mathrm{~m})}\right)=29^{\circ}
$$

(b) At the release point, the speed is 0 , and so there is no radial acceleration, since $a_{\mathrm{R}}=v^{2} / r$. Thus the centripetal force must be 0 . Use the free-body diagram to write Newton's second law for the radial direction.

$$
\begin{aligned}
& \sum F_{\mathrm{R}}=F_{\mathrm{T}}-m g \cos \theta=0 \rightarrow \\
& F_{\mathrm{T}}=m g \cos \theta=(56 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \cos 29^{\circ}=480 \mathrm{~N}
\end{aligned}
$$

(c) Write Newton's second law for the radial direction for any angle, and solve for the tension.

$$
\sum F_{\mathrm{R}}=F_{\mathrm{T}}-m g \cos \theta=m v^{2} / r \rightarrow F_{\mathrm{T}}=m g \cos \theta+m v^{2} / r
$$

As the angle decreases, the tension increases, and as the speed increases, the tension increases. Both effects are greatest at the bottom of the swing, and so that is where the tension will be at its maximum.

$$
\underset{\max }{F_{\mathrm{T}}}=m g \cos 0+m v_{1}^{2} / r=(56 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)+\frac{(56 \mathrm{~kg})(5.0 \mathrm{~m} / \mathrm{s})^{2}}{10.0 \mathrm{~m}}=690 \mathrm{~N}
$$

92. (a)

$$
\begin{aligned}
& \text { (a) } \quad F(r)=-\frac{d U(r)}{d r}=-\left[\left(-U_{0}\right)\left(-\frac{r_{0}}{r^{2}}\right) e^{-r / r_{0}}+\left(-U_{0} \frac{r_{0}}{r}\right)\left(-\frac{1}{r_{0}}\right) e^{-r / r_{0}}\right]=-U_{0} \frac{r_{0}}{r} e^{-r / r_{0}}\left(\frac{1}{r}+\frac{1}{r_{0}}\right) \\
& \text { (b) } F\left(3 r_{0}\right) / F\left(r_{0}\right)=\frac{-U_{0} \frac{r_{0}}{3 r_{0}} e^{-3 r_{0} / r_{0}}\left(\frac{1}{3 r_{0}}+\frac{1}{r_{0}}\right)}{-U_{0} \frac{r_{0}}{r_{0}} e^{-r_{0} / r_{0}}\left(\frac{1}{r_{0}}+\frac{1}{r_{0}}\right)}=\frac{2}{9} e^{-2} \approx 0.03 \\
& \text { (c) } F(r)=-\frac{d U(r)}{d r}=-\left[(-C)\left(-\frac{1}{r^{2}}\right)\right]=-\frac{C}{r^{2}} ; F\left(3 r_{0}\right) / F\left(r_{0}\right)=\frac{-C \frac{1}{\left(3 r_{0}\right)^{2}}}{-C \frac{1}{\left(r_{0}\right)^{2}}}=\frac{1}{9} \approx 0.1
\end{aligned}
$$

The Yukawa potential is said to be "short range" because as the above examples illustrate, the Yukawa force "drops off" more quickly then the electrostatic force. The Yukawa force drops by about $97 \%$ when the distance is tripled, while the electrostatic force only drops by about $89 \%$.
93. Energy conservation can be used to find the speed that the water must leave the ground. We take the ground to be the 0 level for gravitational potential energy. The speed at the top will be 0 .

$$
E_{\text {ground }}=E_{\text {top }} \rightarrow \frac{1}{2} m v_{\text {ground }}^{2}=m g y_{\text {top }} \rightarrow v_{\text {ground }}=\sqrt{2 g y_{\text {top }}}=\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(33 \mathrm{~m})}=25.43 \mathrm{~m} / \mathrm{s}
$$

The area of the water stream times the velocity gives a volume flow rate of water. If that is multiplied by the density, then we have a mass flow rate. That is verified by dimensional analysis.

$$
A v \rho \rightarrow\left[\mathrm{~m}^{2}\right][\mathrm{m} / \mathrm{s}]\left[\mathrm{kg} / \mathrm{m}^{3}\right]=[\mathrm{kg} / \mathrm{s}]
$$

Another way to think about it is that $A v \rho$ is the mass that flows out of the hose per second. It takes a minimum force of $A v \rho g$ to lift that mass, and so the work done per second to lift that mass to a height of $y_{\text {top }}$ is $A v \rho g y_{\text {top }}$. That is the power required.

$$
\begin{aligned}
P & =A v \rho g y_{\text {top }}=\pi\left(1.5 \times 10^{-2} \mathrm{~m}\right)^{2}(25.43 \mathrm{~m} / \mathrm{s})\left(1.00 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(33 \mathrm{~m})=5813 \mathrm{~W} \\
& \approx 5800 \mathrm{~W} \text { or } 7.8 \mathrm{hp}
\end{aligned}
$$

94. A free-body diagram for the sled is shown as it moves up the hill. From this we get an expression for the friction force.

$$
\sum F_{y}=F_{\mathrm{N}}-m g \cos \theta=0 \rightarrow F_{\mathrm{N}}=m g \cos \theta \rightarrow F_{\mathrm{fr}}=\mu_{\mathrm{k}} m g \cos \theta
$$

(a) We apply conservation of energy with a frictional force as given in

Eq. 8-15. Subscript 1 refers to the sled at the start of its motion, and
subscript 2 refers to the sled at the top of its motion. Take the starting position of the sled to be the 0 for gravitational potential
 energy. We have $v_{1}=2.4 \mathrm{~m} / \mathrm{s}, y_{1}=0$, and $v_{2}=0$. The relationship between the distance traveled along the incline and the height the sled rises is $y_{2}=d \sin \theta$. Solve for $d$.

$$
\begin{aligned}
& E_{1}=E_{2}+F_{\mathrm{fr}} \ell \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2}+F_{\mathrm{fr}} d \rightarrow \\
& \frac{1}{2} m v_{1}^{2}=m g d \sin \theta+\mu_{\mathrm{k}} m g d \cos \theta \rightarrow \\
& d=\frac{v_{1}^{2}}{2 g\left(\sin \theta+\mu_{\mathrm{k}} \cos \theta\right)}=\frac{(2.4 \mathrm{~m} / \mathrm{s})^{2}}{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(\sin 28^{\circ}+0.25 \cos 28^{\circ}\right)}=0.4258 \mathrm{~m} \approx 0.43 \mathrm{~m}
\end{aligned}
$$

(b) For the sled to slide back down, the friction force will now point UP the hill in the free-body diagram. In order for the sled to slide down, the component of gravity along the hill must be large than the maximum force of static friction.

$$
m g \sin \theta>F_{\mathrm{fr}} \rightarrow m g \sin \theta>\mu_{\mathrm{s}} m g \cos \theta \rightarrow \mu_{\mathrm{s}}<\tan 28^{\circ} \rightarrow \mu_{\mathrm{s}}<0.53
$$

(c) We again apply conservation of energy including work done by friction. Subscript 1 refers to the sled at the top of the incline, and subscript 2 refers to the sled at the bottom of the incline. We have $v_{1}=0, y_{1}=d \sin \theta$, and $y_{2}=0$.

$$
\begin{aligned}
& E_{1}=E_{2}+F_{\mathrm{fr}} \ell \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2}+F_{\mathrm{fr}} d \rightarrow \\
& m g d \sin \theta=\frac{1}{2} m v_{2}^{2}+\mu_{\mathrm{k}} m g d \cos \theta \rightarrow \\
& v_{2}=\sqrt{2 g d\left(\sin \theta-\mu_{\mathrm{k}} \cos \theta\right)}=\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.4258 \mathrm{~m})\left(\sin 28^{\circ}-0.25 \cos 28^{\circ}\right)} \\
& \quad=1.4 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

95. We apply conservation of mechanical energy. We take the surface of the Moon to be the 0 level for gravitational potential energy. Subscript 1 refers to the location where the engine is shut off, and subscript 2 refers to the surface of the Moon. Up is the positive $y$-direction.
(a) We have $v_{1}=0, y_{1}=h, v_{2}=3.0 \mathrm{~m} / \mathrm{s}$, and $y_{2}=0$.

$$
\begin{aligned}
& E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2} \rightarrow m g h=\frac{1}{2} m v_{2}^{2} \rightarrow \\
& h=\frac{v_{2}^{2}}{2 g}=\frac{(3.0 \mathrm{~m} / \mathrm{s})^{2}}{2\left(1.62 \mathrm{~m} / \mathrm{s}^{2}\right)}=2.8 \mathrm{~m}
\end{aligned}
$$

(b) We have the same conditions except $v_{1}=-2.0 \mathrm{~m} / \mathrm{s}$.

$$
\begin{aligned}
& E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g h=\frac{1}{2} m v_{2}^{2} \rightarrow \\
& h=\frac{v_{2}^{2}-v_{1}^{2}}{2 g}=\frac{(3.0 \mathrm{~m} / \mathrm{s})^{2}-(-2.0 \mathrm{~m} / \mathrm{s})^{2}}{2\left(1.62 \mathrm{~m} / \mathrm{s}^{2}\right)}=1.5 \mathrm{~m}
\end{aligned}
$$

(c) We have the same conditions except $v_{1}=2.0 \mathrm{~m} / \mathrm{s}$. And since the speeds, not the velocities, are used in the energy conservation calculation, this is the same as part $(b)$, and so $h=1.5 \mathrm{~m}$.

$$
\begin{aligned}
& E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g h=\frac{1}{2} m v_{2}^{2} \rightarrow \\
& h=\frac{v_{2}^{2}-v_{1}^{2}}{2 g}=\frac{(3.0 \mathrm{~m} / \mathrm{s})^{2}-(-2.0 \mathrm{~m} / \mathrm{s})^{2}}{2\left(1.62 \mathrm{~m} / \mathrm{s}^{2}\right)}=1.5 \mathrm{~m}
\end{aligned}
$$

96. A free-body diagram for the car is shown. We apply conservation of energy with a frictional force as given in Eq. 8-15. Subscript 1 refers to the car at the start of its motion, and subscript 2 refers to the sled at the end of the motion. Take the ending position of the car to be the 0 for gravitational potential energy. We have $v_{1}=95 \mathrm{~km} / \mathrm{h}, y_{2}=0$, and $v_{2}=35 \mathrm{~km} / \mathrm{h}$. The relationship between the distance traveled along the incline and the initial
 height of the car is $y_{1}=d \sin \theta$.

$$
\begin{aligned}
E_{1} & =E_{2}+E_{\mathrm{fr}} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2}+E_{\mathrm{fr}} \rightarrow \\
E_{\mathrm{fr}} & =\frac{1}{2} m\left(v_{1}^{2}-v_{2}^{2}\right)+m g y_{1}=\frac{1}{2} m\left[\left(v_{1}^{2}-v_{2}^{2}\right)+2 g d \sin \theta\right] \\
& =\frac{1}{2}(1500 \mathrm{~kg})\left[\left((95 \mathrm{~km} / \mathrm{h})^{2}-(35 \mathrm{~km} / \mathrm{h})^{2}\right)\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)^{2}+2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(3.0 \times 10^{2} \mathrm{~m}\right) \sin 17^{\circ}\right] \\
& =1.7 \times 10^{6} \mathrm{~J}
\end{aligned}
$$

97. The energy to be stored is the power multiplied by the time: $E=$ Pt. The energy will be stored as the gravitational potential energy increase in the water: $E=\Delta U=m g \Delta y=\rho V g \Delta y$, where $\rho$ is the density of the water, and $V$ is the volume of the water.

$$
P t=\rho V g \Delta y \rightarrow V=\frac{P t}{\rho g \Delta y}=\frac{\left(180 \times 10^{6} \mathrm{~W}\right)(3600 \mathrm{~s})}{\left(1.00 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(380 \mathrm{~m})}=1.7 \times 10^{5} \mathrm{~m}^{3}
$$

98. It is shown in problem 52 that the total mechanical energy for a satellite orbiting in a circular orbit of radius $r$ is $E=-\frac{1}{2} \frac{G m M_{\mathrm{E}}}{r}$. That energy must be equal to the energy of the satellite at the surface of the Earth plus the energy required by fuel.
(a) If launched from the equator, the satellite has both kinetic and potential energy initially. The kinetic energy is from the speed of the equator of the Earth relative to the center of the Earth. In problem 53 that speed is calculated to be $464 \mathrm{~m} / \mathrm{s}$.

$$
\begin{aligned}
& E_{\text {surface }}+E_{\text {fuel }}=E_{\text {orbit }} \rightarrow \frac{1}{2} m v_{0}^{2}-\frac{G m M_{\mathrm{E}}}{R_{\mathrm{E}}}+E_{\text {fuel }}=-\frac{1}{2} \frac{G m M_{\mathrm{E}}}{r} \rightarrow \\
& E_{\text {fuel }}=G m M_{\mathrm{E}}\left(\frac{1}{R_{\mathrm{E}}}-\frac{1}{2 r}\right)-\frac{1}{2} m v_{0}^{2}=\left\{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)(1465 \mathrm{~kg})\left(5.98 \times 10^{24} \mathrm{~kg}\right)\right. \\
& \left.\quad \cdot\left(\frac{1}{6.38 \times 10^{6} \mathrm{~m}}-\frac{1}{2\left(6.38 \times 10^{6} \mathrm{~m}+1.375 \times 10^{6} \mathrm{~m}\right)}\right)\right\}-\frac{1}{2}(1465 \mathrm{~kg})(464 \mathrm{~m} / \mathrm{s})^{2} \\
& =5.38 \times 10^{10} \mathrm{~J}
\end{aligned}
$$

(b) If launched from the North Pole, the satellite has only potential energy initially. There is no initial velocity from the rotation of the Earth.

$$
\begin{aligned}
& E_{\text {surface }}+E_{\text {fuel }}=E_{\text {orbit }} \rightarrow-\frac{G m M_{\mathrm{E}}}{R_{\mathrm{E}}}+E_{\text {fuel }}=-\frac{1}{2} \frac{G m M_{\mathrm{E}}}{r} \rightarrow \\
& E_{\text {fuel }}=G m M_{\mathrm{E}}\left(\frac{1}{R_{\mathrm{E}}}-\frac{1}{2 r}\right)=\left\{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)(1465 \mathrm{~kg})\left(5.98 \times 10^{24} \mathrm{~kg}\right)\right. \\
&\left.\cdot\left(\frac{1}{6.38 \times 10^{6} \mathrm{~m}}-\frac{1}{2\left(6.38 \times 10^{6} \mathrm{~m}+1.375 \times 10^{6} \mathrm{~m}\right)}\right)\right\} \\
&=5.39 \times 10^{10} \mathrm{~J}
\end{aligned}
$$

99. (a) Use energy conservation and equate the energies at A and B . The distance from the center of the Earth to location B is found by the Pythagorean theorem.

$$
\begin{aligned}
r_{\mathrm{B}} & =\sqrt{(13,900 \mathrm{~km})^{2}+(8230 \mathrm{~km})^{2}}=16,150 \mathrm{~km} \\
E_{A} & =E_{B} \rightarrow \frac{1}{2} m v_{A}^{2}+\left(-\frac{G M_{\mathrm{E}} m}{r_{\mathrm{A}}}\right)=\frac{1}{2} m v_{B}^{2}+\left(-\frac{G M_{\mathrm{E}} m}{r_{\mathrm{B}}}\right) \rightarrow \\
v_{B} & =\sqrt{v_{A}^{2}+2 G M_{\mathrm{E}}\left(\frac{1}{r_{\mathrm{B}}}-\frac{1}{r_{\mathrm{A}}}\right)}=\sqrt{(8650 \mathrm{~m} / \mathrm{s})^{2}+\left\{\left(\frac{2\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(5.98 \times 10^{24} \mathrm{~kg}\right) \cdot}{\left(\frac{1}{1.615 \times 10^{7} \mathrm{~m}}-\frac{1}{8.23 \times 10^{6} \mathrm{~m}}\right)}\right.\right.} \\
& =5220 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(b) Use energy conservation and equate the energies at A and C .

$$
\begin{aligned}
r_{\mathrm{C}} & =16,460 \mathrm{~km}+8230 \mathrm{~km}=24,690 \mathrm{~km} \\
E_{A} & =E_{B} \rightarrow \frac{1}{2} m v_{A}^{2}+\left(-\frac{G M_{\mathrm{E}} m}{r_{\mathrm{A}}}\right)=\frac{1}{2} m v_{B}^{2}+\left(-\frac{G M_{\mathrm{E}} m}{r_{\mathrm{B}}}\right) \rightarrow \\
v_{B} & =\sqrt{v_{A}^{2}+2 G M_{\mathrm{E}}\left(\frac{1}{r_{\mathrm{B}}}-\frac{1}{r_{\mathrm{A}}}\right)}=\sqrt{(8650 \mathrm{~m} / \mathrm{s})^{2}+\left\{\begin{array}{l}
2\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(5.98 \times 10^{24} \mathrm{~kg}\right) \cdot \\
\left(\frac{1}{2.469 \times 10^{7} \mathrm{~m}}-\frac{1}{8.23 \times 10^{6} \mathrm{~m}}\right)
\end{array}\right\}} \\
& =3190 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

100. (a) The force is found from the potential function by Eq. 8-7.

$$
\begin{aligned}
F & =-\frac{d U}{d r}=-\frac{d}{d r}\left(-\frac{G M m}{r} e^{-\alpha r}\right)=G M m \frac{d}{d r}\left(\frac{e^{-\alpha r}}{r}\right)=G M m\left(\frac{r\left(-\alpha e^{-\alpha r}\right)-e^{-\alpha r}}{r^{2}}\right) \\
& =-\frac{G M m}{r^{2}} e^{-\alpha r}(1+\alpha r)
\end{aligned}
$$

(b) Find the escape velocity by using conservation of energy to equate the energy at the surface of the Earth to the energy at infinity with a speed of 0 .

$$
E_{R_{\mathrm{E}}}=E_{\infty} \rightarrow \frac{1}{2} m v_{\mathrm{esc}}^{2}-\frac{G M m}{R_{\mathrm{E}}} e^{-\alpha R_{\mathrm{E}}}=0+0 \rightarrow v_{\mathrm{esc}}=\sqrt{\frac{2 G M}{R_{\mathrm{E}}} e^{-\frac{1}{2} \alpha R_{\mathrm{E}}}}
$$

Notice that this escape velocity is smaller than the Newtonian escape velocity by a factor of $e^{-\frac{1}{2} \alpha R_{\mathrm{E}}}$.
101. (a) Assume that the energy of the candy bar is completely converted into a change of potential energy.

$$
E_{\substack{\text { candy } \\ \text { bar }}}=\Delta U=m g \Delta y \rightarrow \Delta y=\frac{E_{\text {candy }}}{\text { bar }}=\frac{1.1 \times 10^{6} \mathrm{~J}}{m g}=\frac{1500 \mathrm{~m}}{(76 \mathrm{~kg})\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)}=
$$

(b) If the person jumped to the ground, the same energy is all converted into kinetic energy.

$$
E_{\substack{\text { candy } \\ \text { bar }}}=\frac{1}{2} m v^{2} \rightarrow v=\sqrt{\frac{2 E_{\text {candy }}}{\text { bar }}} \boldsymbol{m}=\sqrt{\frac{2\left(1.1 \times 10^{6} \mathrm{~J}\right)}{(76 \mathrm{~kg})}}=170 \mathrm{~m} / \mathrm{s}
$$

102. (a) $1 \mathrm{~kW} \cdot \mathrm{~h}=1 \mathrm{~kW} \cdot \mathrm{~h}\left(\frac{1000 \mathrm{~W}}{1 \mathrm{~kW}}\right)\left(\frac{3600 \mathrm{~s}}{1 \mathrm{~h}}\right)\left(\frac{1 \mathrm{~J} / \mathrm{s}}{1 \mathrm{~W}}\right)=3.6 \times 10^{6} \mathrm{~J}$
(b) $\quad(580 \mathrm{~W})(1$ month $)=(580 \mathrm{~W})(1$ month $)\left(\frac{1 \mathrm{~kW}}{1000 \mathrm{~W}}\right)\left(\frac{30 \mathrm{~d}}{1 \text { month }}\right)\left(\frac{24 \mathrm{~h}}{1 \mathrm{~d}}\right)=417.6 \mathrm{~kW} \cdot \mathrm{~h}$

$$
\approx 420 \mathrm{~kW} \cdot \mathrm{~h}
$$

(c) $417.6 \mathrm{~kW} \cdot \mathrm{~h}=417.6 \mathrm{~kW} \cdot \mathrm{~h}\left(\frac{3.6 \times 10^{6} \mathrm{~J}}{1 \mathrm{~kW} \cdot \mathrm{~h}}\right)=1.503 \times 10^{9} \mathrm{~J} \approx 1.5 \times 10^{9} \mathrm{~J}$
(d) $\quad(417.6 \mathrm{~kW} \cdot \mathrm{~h})\left(\frac{\$ 0.12}{1 \mathrm{~kW} \cdot \mathrm{~h}}\right)=\$ 50.11 \approx \$ 50$

Kilowatt-hours is a measure of energy, not power, and so no, the actual rate at which the energy is used does not figure into the bill. They could use the energy at a constant rate, or at a widely varying rate, and as long as the total used is about 420 kilowatt-hours, the price would be about $\$ 50$.
103. The only forces acting on the bungee jumper are gravity and the elastic force from the bungee cord, so the jumper's mechanical energy is conserved. Subscript 1 represents the jumper at the bridge, and subscript 2 represents the jumper at the bottom of the jump. Let the lowest point of the jumper's motion be the zero location for gravitational potential energy $(y=0)$. The zero location for elastic potential energy is the point at which the bungee cord begins to stretch. See the diagram in the textbook. We have $v_{1}=v_{2}=0, y_{1}=h, y_{2}=0$, and the amount of stretch of the cord $x_{2}=h-15$. Solve for $h$.

$$
\begin{aligned}
& E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}+\frac{1}{2} k x_{1}^{2}=\frac{1}{2} m v_{2}^{2}+m g y_{2}+\frac{1}{2} k x_{2}^{2} \rightarrow m g h=\frac{1}{2} k(h-15)^{2} \rightarrow \\
& h^{2}-\left(30+2 \frac{m g}{k}\right) h+225=0 \rightarrow h^{2}-59.4 h+225=0 \rightarrow \\
& h=\frac{59.4 \pm \sqrt{59.4^{2}-4(225)}}{2}=55 \mathrm{~m}, 4 \mathrm{~m} \rightarrow h=60 \mathrm{~m}
\end{aligned}
$$

The larger answer must be taken because $h>15 \mathrm{~m}$. And only 1 significant figure is justified.
104. See the free-body diagram for the patient on the treadmill. We assume that there are no dissipative forces. Since the patient has a constant velocity, the net force parallel to the plane must be 0 . Write Newton's second law for forces parallel to the plane, and then calculate the power output of force $\overrightarrow{\mathbf{F}}_{\mathrm{p}}$.

$$
\begin{aligned}
& \sum F_{\text {parallel }}=F_{\mathrm{P}}-m g \sin \theta=0 \rightarrow F_{\mathrm{P}}=m g \sin \theta \\
& \begin{aligned}
P & =F_{P} v=m g v \sin \theta=(75 \mathrm{~kg})\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)(3.3 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right) \sin 12^{\circ} \\
& =140.1 \mathrm{~W} \approx 140 \mathrm{~W}
\end{aligned}
\end{aligned}
$$



This is 1.5 to 2 times the wattage of typical household light bulbs ( $60-100 \mathrm{~W}$ ).
105. (a) Assume that there are no non-conservative forces on the rock, and so its mechanical energy is conserved. Subscript 1 represents the rock as it leaves the volcano, and subscript 2 represents the rock at its highest point. The location as the rock leaves the volcano is the zero location for PE $(y=0)$. We have $y_{1}=0, y_{2}=500 \mathrm{~m}$, and $v_{2}=0$. Solve for $v_{1}$.

$$
\begin{aligned}
& E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2} \rightarrow \frac{1}{2} m v_{1}^{2}=m g y_{2} \rightarrow \\
& v_{1}=\sqrt{2 g y_{2}}=\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(320 \mathrm{~m})}=79.20 \mathrm{~m} / \mathrm{s} \approx 79 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(b) The power output is the energy transferred to the launched rocks per unit time. The launching energy of a single rock is $\frac{1}{2} m v_{1}^{2}$, and so the energy of 1000 rocks is $1000\left(\frac{1}{2} m v_{1}^{2}\right)$. Divide this energy by the time it takes to launch 1000 rocks ( 1 minute) to find the power output needed to launch the rocks.

$$
P=\frac{1000\left(\frac{1}{2} m v_{1}^{2}\right)}{t}=\frac{500(450 \mathrm{~kg})(79.20 \mathrm{~m} / \mathrm{s})^{2}}{60 \mathrm{sec}}=2.4 \times 10^{7} \mathrm{~W}
$$

106. Assume that there are no non-conservative forces doing work, so the mechanical energy of the jumper will be conserved. Subscript 1 represents the jumper at the launch point of the jump, and subscript 2 represents the jumper at the highest point. The starting height of the jump is the zero location for potential energy $(y=0)$. We have $y_{1}=0, y_{2}=1.1 \mathrm{~m}$, and $v_{2}=6.5 \mathrm{~m} / \mathrm{s}$. Solve for $v_{1}$.

$$
\begin{aligned}
& E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}=\frac{1}{2} m v_{2}^{2}+m g y_{2} \rightarrow \\
& v_{1}=\sqrt{v_{2}^{2}+2 g y_{2}}=\sqrt{(6.5 \mathrm{~m} / \mathrm{s})^{2}+2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(1.1 \mathrm{~m})}=8.0 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

107. (a) The work done by gravity as the elevator falls is the opposite of the change in gravitational potential energy.

$$
\begin{aligned}
W_{\text {grav }} & =-\Delta U_{\text {grav }}=U_{1}-U_{2}=m g\left(y_{1}-y_{2}\right)=(920 \mathrm{~kg})\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)(24 \mathrm{~m}) \\
& =2.164 \times 10^{5} \mathrm{~J} \approx 2.2 \times 10^{5} \mathrm{~J}
\end{aligned}
$$

Gravity is the only force doing work on the elevator as it falls (ignoring friction), so this result is also the net work done on the elevator as it falls.
(b) The net work done on the elevator is equal to its change in kinetic energy. The net work done just before striking the spring is the work done by gravity found above.

$$
\begin{aligned}
& W_{\mathrm{G}}=K_{2}-K_{1} \rightarrow m g\left(y_{1}-y_{2}\right)=\frac{1}{2} m v_{2}-0 \rightarrow \\
& v_{2}=\sqrt{2 g\left(y_{1}-y_{2}\right)}=\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(24 \mathrm{~m})}=21.69 \mathrm{~m} / \mathrm{s} \approx 22 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(c) Use conservation of energy. Subscript 1 represents the elevator just before striking the spring, and subscript 2 represents the elevator at the bottom of its motion. The level of the elevator just before striking the spring is the zero location for both gravitational potential energy and elastic potential energy. We have $v_{1}=21.69 \mathrm{~m} / \mathrm{s}, y_{1}=0$, and $v_{2}=0$. We assume that $y_{2}<0$.

$$
\begin{aligned}
& E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+m g y_{1}+\frac{1}{2} k y_{1}^{2}=\frac{1}{2} m v_{2}^{2}+m g y_{2}+\frac{1}{2} k y_{2}^{2} \rightarrow \\
& y_{2}^{2}+2 \frac{m g}{k} y_{2}-\frac{m}{k} v_{1}^{2}=0 \rightarrow y_{2}=\frac{-\frac{2 m g}{k} \pm \sqrt{\frac{4 m^{2} g^{2}}{k^{2}}+4 \frac{m v_{1}^{2}}{k}}}{2}=\frac{-m g \pm \sqrt{m^{2} g^{2}+m k v_{1}^{2}}}{k}
\end{aligned}
$$

We must choose the negative root so that $y_{2}$ is negative. Thus

$$
\begin{aligned}
y_{2} & =\frac{-(920 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)-\sqrt{(920 \mathrm{~kg})^{2}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)^{2}+(920 \mathrm{~kg})\left(2.2 \times 10^{5} \mathrm{~N} / \mathrm{m}\right)(21.69 \mathrm{~m} / \mathrm{s})^{2}}}{2.2 \times 10^{5} \mathrm{~N} / \mathrm{m}} \\
& =-1.4 \mathrm{~m}
\end{aligned}
$$

108. (a) The plot is included here. To find the crossing point, solve $U(r)=0$ for $r$.

$$
\begin{aligned}
& U(r)=U_{0}\left[\frac{2}{r^{2}}-\frac{1}{r}\right]=0 \rightarrow \\
& \frac{2}{r^{2}}-\frac{1}{r}=0 \rightarrow r=2
\end{aligned}
$$

To find the minimum value, set $\frac{d U}{d r}=0$ and solve for $r$.


$$
\frac{d U}{d r}=U_{0}\left[-\frac{4}{r^{3}}+\frac{1}{r^{2}}\right]=0 \rightarrow-\frac{4}{r^{3}}+\frac{1}{r^{2}}=0 \rightarrow r=4
$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH08.XLS," on tab "Problem 8.108a."
(b) The graph is redrawn with the energy value included. The approximate turning points are indicated by the small dots. An analytic solution to the relationship $U(r)=-0.050 U_{0}$ gives $r \approx 2.3,17.7$. The maximum kinetic energy of the particle occurs at the minimum of the potential energy, and is found from
$E=K+U$.
$E=K+U \rightarrow$

$$
-0.050 U_{0}=K+U(r=4)=K+U_{0}\left(\frac{2}{16}-\frac{1}{4}\right) \rightarrow K=\frac{1}{8} U_{0}-0.050 U_{0}=0.075 U_{0}
$$

109. A point of stable equilibrium will have $\frac{d U}{d x}=0$ and $\frac{d^{2} U}{d x^{2}}>0$, indicating a minimum in the potential equilibrium function.

$$
U(x)=\frac{a}{x}+b x \quad \frac{d U}{d x}=-\frac{a}{x^{2}}+b=0 \quad \rightarrow \quad x^{2}=\frac{a}{b} \rightarrow x= \pm \sqrt{a / b}
$$

But since the problem restricts us to $x>0$, the point of must be $x=\sqrt{a / b}$.

$$
\left.\frac{d^{2} U}{d x^{2}}\right|_{x=\sqrt{a / b}}=\left.\frac{2 a}{x^{3}}\right|_{x=\sqrt{a / b}}=\frac{2 a}{(a / b)^{3 / 2}}=\frac{2}{a b^{3 / 2}}>0 \text {, and so the point } x=\sqrt{a / b} \text { gives a minimum in the }
$$

potential energy function.
110. (a) $U=-\int F d r+C=-\int F_{0}\left[2\left(\frac{\sigma}{r}\right)^{13}-\left(\frac{\sigma}{r}\right)^{7}\right] d r+C=\frac{F_{0} \sigma}{6}\left[\left(\frac{\sigma}{r}\right)^{12}-\left(\frac{\sigma}{r}\right)^{6}\right]+C$
(b) The equilibrium distance occurs at the location where the force is 0 .

$$
F=F_{0}\left[2\left(\frac{\sigma}{r_{0}}\right)^{13}-\left(\frac{\sigma}{r_{0}}\right)^{7}\right]=0 \rightarrow r_{0}=2^{1 / 6} \sigma=2^{1 / 6}\left(3.50 \times 10^{-11} \mathrm{~m}\right)=3.93 \times 10^{-11} \mathrm{~m}
$$

(c) In order to draw the graphs in terms of $r_{0}$, and to scale them to the given constants, the functions have been parameterized as follows.

$$
\begin{aligned}
& F(r)=F_{0}\left[2\left(\frac{\sigma}{r}\right)^{13}-\left(\frac{\sigma}{r}\right)^{7}\right]=F_{0}\left[2\left(\frac{\sigma}{r_{0}}\right)^{13}\left(\frac{r_{0}}{r}\right)^{13}-\left(\frac{\sigma}{r_{0}}\right)^{7}\left(\frac{r_{0}}{r}\right)^{7}\right] \rightarrow \\
& \frac{F(r)}{F_{0}}=\left[2\left(\frac{\sigma}{r_{0}}\right)^{13}\left(\frac{r}{r_{0}}\right)^{-13}-\left(\frac{\sigma}{r_{0}}\right)^{7}\left(\frac{r}{r_{0}}\right)^{-7}\right] \\
& U(r)=\frac{F_{0} \sigma}{6}\left[\left(\frac{\sigma}{r}\right)^{12}-\left(\frac{\sigma}{r}\right)^{6}\right]=\frac{F_{0} \sigma}{6}\left[\left(\frac{\sigma}{r_{0}}\right)^{12}\left(\frac{r}{r_{0}}\right)^{-12}-\left(\frac{\sigma}{r_{0}}\right)^{6}\left(\frac{r}{r_{0}}\right)^{-6}\right] \rightarrow \\
& \frac{U(r)}{F_{0} \sigma}=\frac{1}{6}\left[\left(\frac{\sigma}{r}\right)^{12}-\left(\frac{\sigma}{r}\right)^{6}\right]=\frac{1}{6}\left[\left(\frac{\sigma}{r_{0}}\right)^{12}\left(\frac{r}{r_{0}}\right)^{-12}-\left(\frac{\sigma}{r_{0}}\right)^{6}\left(\frac{r}{r_{0}}\right)^{-6}\right]
\end{aligned}
$$




The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4 ISM CH08.XLS," on tab "Problem 8.110c."

## CHAPTER 9: Linear Momentum

## Responses to Questions

Momentum is conserved if the sum of the external forces acting on an object is zero. In the case of moving objects sliding to a stop, the sum of the external forces is not zero; friction is an unbalanced force. Momentum will not be conserved in that case.
2. With the spring stretched, the system of two blocks and spring has elastic potential energy. When the blocks are released, the spring pulls them back together, converting the potential energy into kinetic energy. The blocks will continue past the equilibrium position and compress the spring, eventually coming to rest as the kinetic energy changes back into potential energy. If no thermal energy is lost, the blocks will continue to oscillate. The center of mass of the system will stay stationary. Since momentum is conserved, and the blocks started at rest, $m_{1} v_{1}=-m_{2} v_{2}$ at all times, if we assume a massless spring.
3. The heavy object will have a greater momentum. If a light object $m_{1}$ and a heavy object $m_{2}$ have the same kinetic energy, then the light object must have a larger velocity than the heavy object. If
$\frac{1}{2} m_{1} v_{1}^{2}=\frac{1}{2} m_{2} v_{2}^{2}$, where $m_{1}<m_{2}$, then $v_{1}=v_{2} \sqrt{\frac{m_{2}}{m_{1}}}$. The momentum of the light object is $m_{1} v_{1}=m_{1} v_{2} \sqrt{\frac{m_{2}}{m_{1}}}=m_{2} v_{2} \sqrt{\frac{m_{1}}{m_{2}}}$. Since the ratio $\frac{m_{1}}{m_{2}}$ is less than 1 , the momentum of the light object will be a fraction of the momentum of the heavy object.
4. The momentum of the person is changed (to zero) by the force of the ground acting on the person. This change in momentum is equal to the impulse on the person, or the average force times the time over which it acts.
5. As the fish swishes its tail back and forth, it moves water backward, away from the fish. If we consider the system to be the fish and the water, then, from conservation of momentum, the fish must move forward.
6. (d) The girl moves in the opposite direction at $2.0 \mathrm{~m} / \mathrm{s}$. Since there are no external forces on the pair, momentum is conserved. The initial momentum of the system (boy and girl) is zero. The final momentum of the girl must be the same in magnitude and opposite in direction to the final momentum of the boy, so that the net final momentum is also zero.
7. (d) The truck and the car will have the same change in the magnitude of momentum because momentum is conserved. (The sum of the changes in momentum must be zero.)
8. Yes. In a perfectly elastic collision, kinetic energy is conserved. In the Earth/ball system, the kinetic energy of the Earth after the collision is negligible, so the ball has the same kinetic energy leaving the floor as it had hitting the floor. The height from which the ball is released determines its potential energy, which is converted to kinetic energy as the ball falls. If it leaves the floor with this same amount of kinetic energy and a velocity upward, it will rise to the same height as it originally had as the kinetic energy is converted back into potential energy.
9. In order to conserve momentum, when the boy dives off the back of the rowboat the boat will move forward.
10. He could have thrown the coins in the direction opposite the shore he was trying to reach. Since the lake is frictionless, momentum would be conserved and he would "recoil" from the throw with a momentum equal in magnitude and opposite in direction to the coins. Since his mass is greater than the mass of the coins, his speed would be less than the speed of the coins, but, since there is no friction, he would maintain this small speed until he hit the shore.
11. When the tennis ball rebounds from a stationary racket, it reverses its component of velocity perpendicular to the racket with very little energy loss. If the ball is hit straight on, and the racket is actually moving forward, the ball can be returned with an energy (and a speed) equal to the energy it had when it was served.
12. Yes. Impulse is the product of the force and the time over which it acts. A small force acting over a longer time could impart a greater impulse than a large force acting over a shorter time.
13. If the force is non-constant, and reverses itself over time, it can give a zero impulse. For example, the spring force would give a zero impulse over one period of oscillation.
14. The collision in which the two cars rebound would probably be more damaging. In the case of the cars rebounding, the change in momentum of each car is greater than in the case in which they stick together, because each car is not only brought to rest but also sent back in the direction from which it came. A greater impulse results from a greater force, and so most likely more damage would occur.
15. (a) No. The ball has external forces acting on it at all points of its path.
(b) If the system is the ball and the Earth, momentum is conserved for the entire path. The forces acting on the ball-Earth system are all internal to the system.
(c) For a piece of putty falling and sticking to a steel plate, if the system is the putty and the Earth, momentum is conserved for the entire path.
16. The impulse imparted to a car during a collision is equal to the change in momentum from its initial speed times mass to zero, assuming the car is brought to rest. The impulse is also equal to the force exerted on the car times the time over which the force acts. For a given change in momentum, therefore, a longer time results in a smaller average force required to stop the car. The "crumple zone" extends the time it takes to bring the car to rest, thereby reducing the force.
17. For maximum power, the turbine blades should be designed so that the water rebounds. The water has a greater change in momentum if it rebounds than if it just stops at the turbine blade. If the water has a greater change in momentum, then, by conservation of momentum, the turbine blades also have a greater change in momentum, and will therefore spin faster.
18. (a) The direction of the change in momentum of the ball is perpendicular to the wall and away from it, or outward.
(b) The direction of the force on the ball is the same as the direction of its change in momentum. Therefore, by Newton's third law, the direction of the force on the wall will be perpendicular to the wall and towards it, or inward.
19. When a ball is thrown into the air, it has only a vertical component of velocity. When the batter hits the ball, usually in or close to the horizontal direction, the ball acquires a component of velocity in the horizontal direction from the bat. If the ball is pitched, then when it is hit by the bat it reverses its
horizontal component of velocity (as it would if it bounced off of a stationary wall) and acquires an additional contribution to its horizontal component of velocity from the bat. Therefore, a pitched ball can be hit farther than one tossed into the air.
20. A perfectly inelastic collision between two objects that initially had momenta equal in magnitude but opposite in direction would result in all the kinetic energy being lost. For instance, imagine sliding two clay balls with equal masses and speeds toward each other across a frictionless surface. Since the initial momentum of the system is zero, the final momentum must be zero as well. The balls stick together, so the only way the final momentum can be zero is if they are brought to rest. In this case, all the kinetic energy would be lost.
21. (b) Elastic collisions conserve both momentum and kinetic energy; inelastic collisions only conserve momentum.
22. Passengers may be told to sit in certain seats in order to balance the plane. If they move during the flight, they could change the position of the center of mass of the plane and affect its flight.
23. You lean backward in order to keep your center of mass over your feet. If, due to the heavy load, your center of mass is in front of your feet, you will fall forward.
24. A piece of pipe is typically uniform, so that its center of mass is at its geometric center. Your arm and leg are not uniform. For instance, the thigh is bigger than the calf, so the center of mass of a leg will be higher than the midpoint.
25.

26. Draw a line from each vertex to the midpoint of the opposite side. The center of mass will be the point at which these lines intersect.
27. When you stand next to a door in the position described, your center of mass is over your heels. If you try to stand on your toes, your center of mass will not be over your area of support, and you will fall over backward.
28. If the car were on a frictionless surface, then the internal force of the engine could not accelerate the car. However, there is friction, which is an external force, between the car tires and the road, so the car can be accelerated.
29. The center of mass of the system of pieces will continue to follow the original parabolic path.
30. Far out in space there are no external forces acting on the rocket, so momentum is conserved. Therefore, to change directions, the rocket needs to expel something (like gas exhaust) in one direction so that the rest of it will move in the opposite direction and conserve momentum.
31. If there were only two particles involved in the decay, then by conservation of momentum, the momenta of the particles would have to be equal in magnitude and opposite in direction, so that the momenta would be required to lie along a line. If the momenta of the recoil nucleus and the electron do not lie along a line, then some other particle must be carrying off some of the momentum.
32. Consider Bob, Jim, and the rope as a system. The center of mass of the system is closer to Bob, because he has more mass. Because there is no net external force on the system, the center of mass will stay stationary. As the two men pull hand-over-hand on the rope they will move toward each other, eventually colliding at the center of mass. Since the CM is on Bob's side of the midline, Jim will cross the midline and lose.
33. The ball that rebounds off the cylinder will give the cylinder a larger impulse and will be more likely to knock it over.

## Solutions to Problems

1. The force on the gas can be found from its change in momentum. The speed of 1300 kg of the gas changes from rest to $4.5 \times 10^{4} \mathrm{~m} / \mathrm{s}$, over the course of one second.

$$
F=\frac{\Delta p}{\Delta t}=\frac{m \Delta v}{\Delta t}=\Delta v \frac{m}{\Delta t}=\left(4.5 \times 10^{4} \mathrm{~m} / \mathrm{s}\right)(1300 \mathrm{~kg} / \mathrm{s})=5.9 \times 10^{7} \mathrm{~N}, \text { opposite to the velocity }
$$

The force on the rocket is the Newton's third law pair (equal and opposite) to the force on the gas, and so is $5.9 \times 10^{7} \mathrm{~N}$ in the direction of the velocity.
2. For a constant force, Eq. 9-2 can be written as $\Delta \overrightarrow{\mathbf{p}}=\overrightarrow{\mathbf{F}} \Delta t$. For a constant mass object, $\Delta \overrightarrow{\mathbf{p}}=m \Delta \overrightarrow{\mathbf{v}}$. Equate the two expressions for $\Delta \overrightarrow{\mathbf{p}}$.

$$
\overrightarrow{\mathbf{F}} \Delta t=m \Delta \overrightarrow{\mathbf{v}} \rightarrow \Delta \overrightarrow{\mathbf{v}}=\frac{\overrightarrow{\mathbf{F}} \Delta t}{m}
$$

If the skier moves to the right, then the speed will decrease, because the friction force is to the left.

$$
\Delta v=-\frac{F \Delta t}{m}=-\frac{(25 \mathrm{~N})(15 \mathrm{~s})}{65 \mathrm{~kg}}=-5.8 \mathrm{~m} / \mathrm{s}
$$

The skier loses $5.8 \mathrm{~m} / \mathrm{s}$ of speed.
3. The force is the derivative of the momentum with respect to time.

$$
\overrightarrow{\mathbf{F}}=\frac{d \overrightarrow{\mathbf{p}}}{d t}=\frac{d\left(4.8 t^{2} \hat{\mathbf{i}}-8.0 \overrightarrow{\mathbf{j}}-8.9 t \overrightarrow{\mathbf{k}}\right)}{d t}=(9.6 t \hat{\mathbf{i}}-8.9 \overrightarrow{\mathbf{k}}) \mathrm{N}
$$

4. The change in momentum is the integral of the force, since the force is the derivative of the momentum.

$$
\overrightarrow{\mathbf{F}}=\frac{d \overrightarrow{\mathbf{p}}}{d t} \rightarrow \overrightarrow{\mathbf{p}}=\int_{t_{1}}^{t_{2}} \overrightarrow{\mathbf{F}} d t=\int_{t=1.0 \mathrm{~s}}^{t=2.0 \mathrm{~s}}\left(26 \hat{\mathbf{i}}-12 t^{2} \overrightarrow{\mathbf{j}}\right) d t=\left(26 t \hat{\mathbf{i}}-4 t^{3} 3 \mathbf{\mathbf { j }}\right)_{t=1.0 \mathrm{~s}}^{t=2.0 \mathrm{~s}},(26 \hat{\mathbf{i}}-28 \overrightarrow{\mathbf{j}}) \mathrm{kg} \cdot \mathrm{~m} / \mathrm{s}
$$

5. The change is momentum is due to the change in direction.

$$
\Delta \overrightarrow{\mathbf{p}}=m\left(\overrightarrow{\mathbf{v}}_{f}-\overrightarrow{\mathbf{v}}_{0}\right)=(0.145 \mathrm{~kg})(30.0 \mathrm{~m} / \mathrm{s} \hat{\mathbf{j}}-30.0 \mathrm{~m} / \mathrm{s} \hat{\mathbf{i}})=4.35 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}(\hat{\mathbf{j}}-\hat{\mathbf{i}})
$$


6. The average force is the change in momentum divided by the elapsed time. Call the direction from the batter to the pitcher the positive x direction, and call upwards the positive y direction. The initial momentum is in the negative x direction, and the final momentum is in the positive y direction. The final $y$-velocity can be found using the height to which the ball rises, with conservation of mechanical energy during the rising motion.

$$
\begin{aligned}
& E_{\text {initial }}=E_{\text {final }} \rightarrow \frac{1}{2} m v_{y}^{2}=m g h \rightarrow v_{y}=\sqrt{2 g h}=\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(36.5 \mathrm{~m})}=26.75 \mathrm{~m} / \mathrm{s} \\
& \overrightarrow{\mathbf{F}}_{\text {avg }}=\frac{\Delta \overrightarrow{\mathbf{p}}}{\Delta t}=\frac{m}{\Delta t}\left(\overrightarrow{\mathbf{v}}_{f}-\overrightarrow{\mathbf{v}}_{0}\right)=(0.145 \mathrm{~kg})\left(\frac{26.75 \hat{\mathbf{j}} \mathrm{~m} / \mathrm{s}-(-32.0 \hat{\mathbf{i}} \mathrm{~m} / \mathrm{s})}{2.5 \times 10^{-3} \mathrm{~s}}\right)=(1856 \hat{\mathbf{i}}+1552 \hat{\mathbf{j}}) \mathrm{N} \\
& F_{\text {avg }}=\sqrt{(1856 \mathrm{~N})^{2}+(1552 \mathrm{~N})^{2}}=2400 \mathrm{~N} ; \theta=\tan ^{-1} \frac{1552 \mathrm{~N}}{1856 \mathrm{~N}}=39.9^{\circ}
\end{aligned}
$$

To alter the course by $35.0^{\circ}$, a velocity perpendicular to the original velocity must be added. Call the direction of the added velocity, $\overrightarrow{\mathbf{v}}_{\text {add }}$, the positive direction. From the diagram, we see that $v_{\text {add }}=v_{\text {orig }} \tan \theta$. The momentum in the perpendicular direction will be conserved, considering that the gases are given perpendicular momentum in the opposite direction of $\overrightarrow{\mathbf{v}}_{\text {add }}$. The gas is expelled oppositely to $\overrightarrow{\mathbf{v}}_{\text {add }}$, and so a negative value is used for $v_{\perp \text { gas }}$.

$$
\begin{aligned}
& p_{\text {before }}=p_{\text {after }} \rightarrow 0=m_{\text {gas }} v_{\perp \text { gas }}+\left(m_{\text {rocket }}-m_{\text {gas }}\right) v_{\text {add }} \rightarrow \\
& m_{\text {gas }}=\frac{m_{\text {rocket }} v_{\text {add }}}{\left(v_{\text {add }}-v_{\perp \text { gas }}\right)}=\frac{(3180 \mathrm{~kg})(115 \mathrm{~m} / \mathrm{s}) \tan 35.0^{\circ}}{\left[(115 \mathrm{~m} / \mathrm{s}) \tan 35.0^{\circ}-(-1750 \mathrm{~m} / \mathrm{s})\right]}=1.40 \times 10^{2} \mathrm{~kg}
\end{aligned}
$$


8. The air is moving with an initial speed of $120 \mathrm{~km} / \mathrm{h}\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)=33.33 \mathrm{~m} / \mathrm{s}$. Thus, in one second, a volume of air measuring $45 \mathrm{mx} 65 \mathrm{~m} \times 33.33 \mathrm{~m}$ will have been brought to rest. By Newton's third law, the average force on the building will be equal in magnitude to the force causing the change in momentum of the air. The mass of the stopped air is its volume times its density.

$$
F=\frac{\Delta p}{\Delta t}=\frac{m \Delta v}{\Delta t}=\frac{V \rho \Delta v}{\Delta t}=\frac{(45 \mathrm{~m})(65 \mathrm{~m})(33.33 \mathrm{~m})\left(1.3 \mathrm{~kg} / \mathrm{m}^{3}\right)(33.33 \mathrm{~m} / \mathrm{s}-0)}{1 \mathrm{~s}}=4.2 \times 10^{6} \mathrm{~N}
$$

9. Consider the motion in one dimension, with the positive direction being the direction of motion of the first car. Let A represent the first car and B represent the second car. Momentum will be conserved in the collision. Note that $v_{\mathrm{B}}=0$.

$$
\begin{aligned}
& p_{\text {initial }}=p_{\text {final }} \rightarrow m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} v_{\mathrm{B}}=\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) v^{\prime} \rightarrow \\
& m_{\mathrm{B}}=\frac{m_{\mathrm{A}}\left(v_{\mathrm{A}}-v^{\prime}\right)}{v^{\prime}}=\frac{(7700 \mathrm{~kg})(18 \mathrm{~m} / \mathrm{s}-5.0 \mathrm{~m} / \mathrm{s})}{5.0 \mathrm{~m} / \mathrm{s}}=2.0 \times 10^{4} \mathrm{~kg}
\end{aligned}
$$

10. Consider the horizontal motion of the objects. The momentum in the horizontal direction will be conserved. Let A represent the car and B represent the load. The positive direction is the direction of the original motion of the car.

$$
p_{\text {initial }}=p_{\text {final }} \rightarrow m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} v_{\mathrm{B}}=\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) v^{\prime} \rightarrow
$$

$$
v^{\prime}=\frac{m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} v_{\mathrm{B}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}=\frac{(9150 \mathrm{~kg})(15.0 \mathrm{~m} / \mathrm{s})+0}{(9150 \mathrm{~kg})+(4350 \mathrm{~kg})}=10.2 \mathrm{~m} / \mathrm{s}
$$

11. Consider the motion in one dimension, with the positive direction being the direction of motion of the alpha particle. Let A represent the alpha particle, with a mass of $m_{\mathrm{A}}$, and let B represent the daughter nucleus, with a mass of $57 \mathrm{~m}_{\mathrm{A}}$. The total momentum must be 0 since the nucleus decayed at rest.

$$
\begin{aligned}
& p_{\text {initial }}=p_{\text {final }} \rightarrow 0=m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}+m_{\mathrm{B}} v_{\mathrm{B}}^{\prime} \rightarrow \\
& v_{\mathrm{B}}^{\prime}=-\frac{m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}}{m_{\mathrm{B}}}=-\frac{m_{\mathrm{A}}\left(2.8 \times 10^{5} \mathrm{~m} / \mathrm{s}\right)}{57 m_{\mathrm{A}}} \rightarrow\left|v_{\mathrm{B}}^{\prime}\right|=4900 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Note that the masses do not have to be converted to kg, since all masses are in the same units, and a ratio of masses is what is significant.
12. The tackle will be analyzed as a one-dimensional momentum conserving situation. Let A represent the halfback and B represent the tackler. We take the direction of the halfback to be the positive direction, so $v_{\mathrm{A}}>0$ and $v_{\mathrm{B}}<0$.

$$
\begin{aligned}
& p_{\text {initial }}=p_{\text {final }} \rightarrow m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} v_{\mathrm{B}}=\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) v^{\prime} \rightarrow \\
& v^{\prime}=\frac{m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} v_{\mathrm{B}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}=\frac{(82 \mathrm{~kg})(5.0 \mathrm{~m} / \mathrm{s})+(130 \mathrm{~kg})(-2.5 \mathrm{~m} / \mathrm{s})}{82 \mathrm{~kg}+130 \mathrm{~kg}}=0.401 \mathrm{~m} / \mathrm{s} \approx 0.4 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

They will be moving it the direction that the halfback was running before the tackle.
13. The throwing of the package is a momentum-conserving action, if the water resistance is ignored. Let A represent the boat and child together, and let B represent the package. Choose the direction that the package is thrown as the positive direction. Apply conservation of momentum, with the initial velocity of both objects being 0 .

$$
\begin{aligned}
& p_{\text {initial }}=p_{\text {final }} \rightarrow\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) v=m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}+m_{\mathrm{B}} v_{\mathrm{B}}^{\prime} \rightarrow \\
& v_{\mathrm{A}}^{\prime}=-\frac{m_{\mathrm{B}} v_{\mathrm{B}}^{\prime}}{m_{\mathrm{A}}}=-\frac{(5.70 \mathrm{~kg})(10.0 \mathrm{~m} / \mathrm{s})}{(24.0 \mathrm{~kg}+35.0 \mathrm{~kg})}=-0.966 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

The boat and child move in the opposite direction as the thrown package, as indicated by the negative velocity.
14. Consider the motion in one dimension, with the positive direction being the direction of motion of the original nucleus. Let A represent the alpha particle, with a mass of 4 u , and let B represent the new nucleus, with a mass of 218 u . Momentum conservation gives the following.

$$
\begin{aligned}
& p_{\text {initial }}=p_{\text {final }} \rightarrow\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) v=m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}+m_{\mathrm{B}} v_{\mathrm{B}}^{\prime} \rightarrow \\
& v_{\mathrm{A}}^{\prime}=\frac{\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) v-m_{\mathrm{B}} v_{\mathrm{B}}^{\prime}}{m_{\mathrm{A}}}=\frac{(222 \mathrm{u})(420 \mathrm{~m} / \mathrm{s})-(218 \mathrm{u})(350 \mathrm{~m} / \mathrm{s})}{4.0 \mathrm{u}}=4200 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Note that the masses do not have to be converted to kg, since all masses are in the same units, and a ratio of masses is what is significant.
15. Momentum will be conserved in one dimension in the explosion. Let A represent the fragment with the larger kinetic energy.

$$
p_{\text {initial }}=p_{\text {final }} \rightarrow 0=m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}+m_{\mathrm{B}} v_{\mathrm{B}}^{\prime} \rightarrow v_{\mathrm{B}}^{\prime}=-\frac{m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}}{m_{\mathrm{B}}}
$$

$$
K_{\mathrm{A}}=2 K_{\mathrm{B}} \rightarrow \frac{1}{2} m_{\mathrm{A}} v_{\mathrm{A}}^{\prime 2}=2\left(\frac{1}{2} m_{\mathrm{B}} v_{\mathrm{B}}^{\prime 2}\right)=m_{\mathrm{B}}\left(-\frac{m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}}{m_{\mathrm{B}}}\right)^{2} \quad \rightarrow \frac{m_{\mathrm{A}}}{m_{\mathrm{B}}}=\frac{1}{2}
$$

The fragment with the larger kinetic energy has half the mass of the other fragment.
16. Consider the motion in one dimension with the positive direction being the direction of motion of the bullet. Let A represent the bullet and B represent the block. Since there is no net force outside of the block-bullet system (like friction with the table), the momentum of the block and bullet combination is conserved. Note that $v_{B}=0$.

$$
\begin{aligned}
& p_{\text {initial }}=p_{\text {final }} \rightarrow m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} v_{\mathrm{B}}=m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}+m_{\mathrm{B}} v_{\mathrm{B}}^{\prime} \rightarrow \\
& v_{\mathrm{B}}^{\prime}=\frac{m_{\mathrm{A}}\left(v_{\mathrm{A}}-v_{\mathrm{A}}^{\prime}\right)}{m_{\mathrm{B}}}=\frac{(0.022 \mathrm{~kg})(210 \mathrm{~m} / \mathrm{s}-150 \mathrm{~m} / \mathrm{s})}{2.0 \mathrm{~kg}}=0.66 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

17. Momentum will be conserved in two dimensions. The fuel was ejected in the $y$ direction as seen by an observer at rest, and so the fuel had no $x$-component of velocity in that reference frame.

$$
\begin{array}{ll}
p_{x}: & m_{\text {rocket }} v_{0}=\left(m_{\text {rocket }}-m_{\text {fuel }}\right) v_{x}^{\prime}+m_{\text {fuel }} 0=\frac{2}{3} m_{\text {rocket }} v_{x}^{\prime} \rightarrow v_{x}^{\prime}=\frac{3}{2} v_{0} \\
p_{y}: & 0=m_{\text {fuel }} v_{\text {fuel }}+\left(m_{\text {rocket }}-m_{\text {fuel }}\right) v_{y}^{\prime}=\frac{1}{3} m_{\text {rocket }}\left(2 v_{0}\right)+\frac{2}{3} m_{\text {rocket }} v_{y}^{\prime} \rightarrow v_{y}^{\prime}=-v_{0}
\end{array}
$$

Thus $\overrightarrow{\mathbf{v}}^{\prime}=\frac{3}{2} v_{0} \hat{\mathbf{i}}-v_{0} \mathbf{\mathbf { j }}$.
18. Since the neutron is initially at rest, the total momentum of the three particles after the decay must also be zero. Thus $0=\overrightarrow{\mathbf{p}}_{\text {proton }}+\overrightarrow{\mathbf{p}}_{\text {electron }}+\overrightarrow{\mathbf{p}}_{\text {neutrino }}$. Solve for any one of the three in terms of the other two: $\overrightarrow{\mathbf{p}}_{\text {proton }}=-\left(\overrightarrow{\mathbf{p}}_{\text {electron }}+\overrightarrow{\mathbf{p}}_{\text {neutrino }}\right)$. Any two vectors are always coplanar, since they can be translated so that they share initial points. So in this case the common initial point and their two terminal points of the electron and neutrino momenta define a plane, which contains their sum. Then, since the proton momentum is just the opposite of the sum of the other two momenta, it is in the same plane.
19. Since no outside force acts on the two masses, their total momentum is conserved.

$$
\begin{aligned}
m_{1} \overrightarrow{\mathbf{v}}_{1} & =m_{1} \overrightarrow{\mathbf{v}}_{1}^{\prime}+m_{2} \overrightarrow{\mathbf{v}}_{2}^{\prime} \rightarrow \\
\overrightarrow{\mathbf{v}}_{2}^{\prime} & =\frac{m_{1}}{m_{2}}\left(\overrightarrow{\mathbf{v}}_{1}-\overrightarrow{\mathbf{v}}_{1}^{\prime}\right)=\frac{2.0 \mathrm{~kg}}{3.0 \mathrm{~kg}}[(4.0 \hat{\mathbf{i}}+5.0 \hat{\mathbf{j}}-2.0 \hat{\mathbf{k}}) \mathrm{m} / \mathrm{s}-(-2.0 \hat{\mathbf{i}}+3.0 \hat{\mathbf{k}}) \mathrm{m} / \mathrm{s}] \\
& =\frac{2.0 \mathrm{~kg}}{3.0 \mathrm{~kg}}[(6.0 \hat{\mathbf{i}}+5.0 \hat{\mathbf{j}}-5.0 \hat{\mathbf{k}}) \mathrm{m} / \mathrm{s}] \\
& =(4.0 \hat{\mathbf{i}}+3.3 \hat{\mathbf{j}}-3.3 \hat{\mathbf{k}}) \mathrm{m} / \mathrm{s}
\end{aligned}
$$

20. (a) Consider the motion in one dimension with the positive direction being the direction of motion before the separation. Let A represent the upper stage (that moves away faster) and B represent the lower stage. It is given that $m_{\mathrm{A}}=m_{\mathrm{B}}, v_{\mathrm{A}}=v_{\mathrm{B}}=v$, and $v_{\mathrm{B}}^{\prime}=v_{\mathrm{A}}^{\prime}-v_{\text {rel }}$. Momentum conservation gives the following.

$$
\begin{aligned}
& p_{\text {initial }}=p_{\text {firal }} \rightarrow\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) v=m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}+m_{\mathrm{B}} v_{\mathrm{B}}^{\prime}=m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}+m_{\mathrm{B}}\left(v_{\mathrm{A}}^{\prime}-v_{\text {rel }}\right) \rightarrow \\
& v_{\mathrm{A}}^{\prime}=\frac{\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) v+m_{\mathrm{B}} v_{\text {rel }}}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)}=\frac{(925 \mathrm{~kg})\left(6.60 \times 10^{3} \mathrm{~m} / \mathrm{s}\right)+\frac{1}{2}(925 \mathrm{~kg})\left(2.80 \times 10^{3} \mathrm{~m} / \mathrm{s}\right)}{(925 \mathrm{~kg})}
\end{aligned}
$$

$$
\begin{aligned}
& =8.00 \times 10^{3} \mathrm{~m} / \mathrm{s}, \text { away from Earth } \\
v_{\mathrm{B}}^{\prime} & =v_{\mathrm{A}}^{\prime}-v_{\mathrm{rel}}=8.00 \times 10^{3} \mathrm{~m} / \mathrm{s}-2.80 \times 10^{3} \mathrm{~m} / \mathrm{s}=5.20 \times 10^{3} \mathrm{~m} / \mathrm{s}, \text { away from Earth }
\end{aligned}
$$

(b) The change in kinetic energy was supplied by the explosion.

$$
\begin{aligned}
\Delta K & =K_{\text {final }}-K_{\text {initial }}=\left(\frac{1}{2} m_{\mathrm{A}} v_{\mathrm{A}}^{\prime 2}+\frac{1}{2} m_{\mathrm{B}} v_{\mathrm{B}}^{\prime 2}\right)-\frac{1}{2}\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) v^{2} \\
& =\frac{1}{2}(462.5 \mathrm{~kg})\left[\left(8.00 \times 10^{3} \mathrm{~m} / \mathrm{s}\right)^{2}+\left(5.20 \times 10^{3} \mathrm{~m} / \mathrm{s}\right)^{2}\right]-\frac{1}{2}(925 \mathrm{~kg})\left(6.60 \times 10^{3} \mathrm{~m} / \mathrm{s}\right)^{2} \\
& =9.065 \times 10^{8} \mathrm{~J} \approx 9 \times 10^{8} \mathrm{~J}
\end{aligned}
$$

21. (a) For the initial projectile motion, the horizontal velocity is constant. The velocity at the highest point, immediately before the explosion, is exactly that horizontal velocity, $v_{x}=v_{0} \cos \theta$. The explosion is an internal force, and so the momentum is conserved during the explosion. Let $\overrightarrow{\mathbf{v}}_{3}$ represent the velocity of the third fragment.

$$
\begin{aligned}
& \overrightarrow{\mathbf{p}}_{\text {before }}=\overrightarrow{\mathbf{p}}_{\text {after }} \rightarrow m v_{0} \cos \theta \hat{\mathbf{i}}=\frac{1}{3} m v_{0} \cos \theta \hat{\mathbf{i}}+\frac{1}{3} m v_{0} \cos \theta(-\hat{\mathbf{j}})+\frac{1}{3} m \overrightarrow{\mathbf{v}}_{3} \rightarrow \\
& \overrightarrow{\mathbf{v}}_{3}=2 v_{0} \cos \theta \hat{\mathbf{i}}+v_{0} \cos \theta \hat{\mathbf{j}}=2(116 \mathrm{~m} / \mathrm{s}) \cos 60.0^{\circ} \hat{\mathbf{i}}+(116 \mathrm{~m} / \mathrm{s}) \cos 60.0^{\circ} \hat{\mathbf{j}} \\
& \quad=(116 \mathrm{~m} / \mathrm{s}) \hat{\mathbf{i}}+(58.0 \mathrm{~m} / \mathrm{s}) \hat{\mathbf{j}}
\end{aligned}
$$

This is 130 ms at an angle of $26.6^{\circ}$ above the horizontal.
(b) The energy released in the explosion is $K_{\text {affer }}-K_{\text {beffre }}$. Note that $v_{3}^{2}=\left(2 v_{0} \cos \theta\right)^{2}+\left(v_{0} \cos \theta\right)^{2}$ $=5 v_{0}^{2} \cos ^{2} \theta$.

$$
\begin{aligned}
K_{\text {affer }}-K_{\text {beforer }} & =\left[\frac{1}{2}\left(\frac{1}{3} m\right)\left(v_{0} \cos \theta\right)^{2}+\frac{1}{2}\left(\frac{1}{3} m\right)\left(v_{0} \cos \theta\right)^{2}+\frac{1}{2}\left(\frac{1}{3} m\right) v_{3}^{2}\right]-\frac{1}{2} m\left(v_{0} \cos \theta\right)^{2} \\
& =\frac{1}{2} m\left\{\left[\frac{1}{3} v_{0}^{2} \cos ^{2} \theta+\frac{1}{3} v_{0}^{2} \cos ^{2} \theta+\frac{1}{3}\left(5 v_{0}^{2} \cos ^{2} \theta\right)\right]-v_{0}^{2} \cos ^{2} \theta\right\} \\
& =\frac{1}{2} \frac{4}{3} m v_{0}^{2} \cos ^{2} \theta=\frac{2}{3}(224 \mathrm{~kg})(116 \mathrm{~m} / \mathrm{s})^{2} \cos ^{2} 60.0^{\circ}=5.02 \times 10^{5} \mathrm{~J}
\end{aligned}
$$

22. Choose the direction from the batter to the pitcher to be the positive direction. Calculate the average force from the change in momentum of the ball.

$$
\begin{aligned}
& \Delta p=F_{\text {avg }} \Delta t=m \Delta v \rightarrow \\
& F_{\text {avg }}=m \frac{\Delta v}{\Delta t}=(0.145 \mathrm{~kg})\left(\frac{56.0 \mathrm{~m} / \mathrm{s}-(-35.0 \mathrm{~m} / \mathrm{s})}{5.00 \times 10^{-3} \mathrm{~s}}\right)=2640 \mathrm{~N}, \text { towards the pitcher }
\end{aligned}
$$

23. (a) The impulse is the change in momentum. The direction of travel of the struck ball is the positive direction.

$$
\Delta p=m \Delta v=\left(4.5 \times 10^{-2} \mathrm{~kg}\right)(45 \mathrm{~m} / \mathrm{s}-0)=2.0 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s} \text {, in the forward direction }
$$

(b) The average force is the impulse divided by the interaction time.

$$
\bar{F}=\frac{\Delta p}{\Delta t}=\frac{2.0 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}}{3.5 \times 10^{-3} \mathrm{~s}}=580 \mathrm{~N} \text {, in the forward direction }
$$

24. (a) The impulse given to the nail is the opposite of the impulse given to the hammer. This is the change in momentum. Call the direction of the initial velocity of the hammer the positive direction.

$$
\Delta p_{\text {nail }}=-\Delta p_{\text {hammer }}=m v_{i}-m v_{f}=(12 \mathrm{~kg})(8.5 \mathrm{~m} / \mathrm{s})-0=1.0 \times 10^{2} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}
$$

(b) The average force is the impulse divided by the time of contact.

$$
F_{\mathrm{avg}}=\frac{\Delta p}{\Delta t}=\frac{1.0 \times 10^{2} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}}{8.0 \times 10^{-3} \mathrm{~s}}=1.3 \times 10^{4} \mathrm{~N}
$$

The impulse given the ball is the change in the ball's momentum. From the symmetry of the problem, the vertical momentum of the ball does not change, and so there is no vertical impulse. Call the direction AWAY from the wall the positive direction for momentum perpendicular to the wall.

$$
\begin{aligned}
\Delta p_{\perp} & =m v_{\perp}-m v_{\perp}=m\left(v \sin 45^{\circ}--v \sin 45^{\circ}\right)=2 m v \sin 45^{\circ} \\
& =2\left(6.0 \times 10^{-2} \mathrm{~km}\right)(25 \mathrm{~m} / \mathrm{s}) \sin 45^{\circ}=2.1 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}, \text { to the left }
\end{aligned}
$$

26. (a) The momentum of the astronaut-space capsule combination will be conserved since the only forces are "internal" to that system. Let A represent the astronaut and B represent the space capsule, and let the direction the astronaut moves be the positive direction. Due to the choice of reference frame, $v_{\mathrm{A}}=v_{\mathrm{B}}=0$. We also have $v_{\mathrm{A}}^{\prime}=2.50 \mathrm{~m} / \mathrm{s}$.

$$
\begin{aligned}
& p_{\text {initial }}=p_{\text {final }} \rightarrow m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} v_{B}=0=m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}+m_{\mathrm{B}} v_{\mathrm{B}}^{\prime} \rightarrow \\
& v_{\mathrm{B}}^{\prime}=-v_{\mathrm{A}}^{\prime} \frac{m_{\mathrm{A}}}{m_{\mathrm{B}}}=-(2.50 \mathrm{~m} / \mathrm{s}) \frac{130 \mathrm{~kg}}{1700 \mathrm{~kg}}=-0.1912 \mathrm{~m} / \mathrm{s} \approx-0.19 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

The negative sign indicates that the space capsule is moving in the opposite direction to the astronaut.
(b) The average force on the astronaut is the astronaut's change in momentum, divided by the time of interaction.

$$
F_{\mathrm{avg}}=\frac{\Delta p}{\Delta t}=\frac{m\left(v_{\mathrm{A}}^{\prime}-v_{\mathrm{A}}\right)}{\Delta t}=\frac{(130 \mathrm{~kg})(2.50 \mathrm{~m} / \mathrm{s}-0)}{0.500 \mathrm{~s}}=6.5 \times 10^{2} \mathrm{~N}
$$

(c) $K_{\text {astronaut }}=\frac{1}{2}(130 \mathrm{~kg})(2.50 \mathrm{~m} / \mathrm{s})^{2}=4.0 \times 10^{2} \mathrm{~J}$
$K_{\text {capsule }}=\frac{1}{2}(1700 \mathrm{~kg})(-0.1912 \mathrm{~m} / \mathrm{s})^{2}=31 \mathrm{~J}$
27. If the rain does not rebound, then the final speed of the rain is 0 . By Newton's third law, the force on the pan due to the rain is equal in magnitude to the force on the rain due to the pan. The force on the rain can be found from the change in momentum of the rain. The mass striking the pan is calculated as volume times density.

$$
\begin{aligned}
F_{\mathrm{avg}} & =\frac{\Delta p}{\Delta t}=\frac{\left(m v_{f}-m v_{0}\right)}{\Delta t}=-\frac{m}{\Delta t}\left(v_{f}-v_{0}\right)=\frac{\rho V}{\Delta t} v_{0}=\frac{\rho A h}{\Delta t} v_{0}=\frac{h}{\Delta t} \rho A v_{0} \\
& =\frac{\left(5.0 \times 10^{-2} \mathrm{~m}\right)}{1 \mathrm{~h}\left(\frac{3600 \mathrm{~s}}{1 \mathrm{~h}}\right)}\left(1.00 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)\left(1.0 \mathrm{~m}^{2}\right)(8.0 \mathrm{~m} / \mathrm{s})=0.11 \mathrm{~N}
\end{aligned}
$$

28. (a) The impulse given the ball is the area under the $F$ vs. $t$ graph. Approximate the area as a triangle of "height" 250 N , and "width" 0.04 sec . A width slightly smaller than the base was chosen to compensate for the "inward" concavity of the force graph.

$$
\Delta p=\frac{1}{2}(250 \mathrm{~N})(0.04 \mathrm{~s})=5 \mathrm{~N} \cdot \mathrm{~s}
$$

(b) The velocity can be found from the change in momentum. Call the positive direction the direction of the ball's travel after being served.

$$
\Delta p=m \Delta v=m\left(v_{f}-v_{\mathrm{i}}\right) \rightarrow v_{f}=v_{i}+\frac{\Delta p}{m}=0+\frac{5 \mathrm{~N} \cdot \mathrm{~s}}{6.0 \times 10^{-2} \mathrm{~kg}}=80 \mathrm{~m} / \mathrm{s}
$$

29. Impuse is the change of momentum, Eq. 9-6. This is a one-dimensional configuration.

$$
J=\Delta p=m\left(v_{\text {final }}-v_{0}\right)=(0.50 \mathrm{~kg})(3.0 \mathrm{~m} / \mathrm{s})=1.5 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}
$$

30. (a) See the adjacent graph. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH09.XLS," on tab "Problem 9.30a."
(b) The area is trapezoidal. We estimate values rather than calculate them.

$$
\begin{aligned}
J & \approx \frac{1}{2}(750 \mathrm{~N}+50 \mathrm{~N})(0.0030 \mathrm{~s}) \\
& =1.2 \mathrm{~N} \cdot \mathrm{~s}
\end{aligned}
$$


(c) $J=\int F d t=\int_{0}^{0.0030}\left[740-\left(2.3 \times 10^{5}\right) t\right] d t=\left[740 t-\left(1.15 \times 10^{5}\right) t^{2}\right]_{0}^{0.0030 \mathrm{~s}}$

$$
=(740 \mathrm{~N})(0.0030 \mathrm{~s})-\left(1.15 \times 10^{5} \mathrm{~N} / \mathrm{s}\right)(0.0030 \mathrm{~s})^{2}=1.185 \mathrm{~N} \cdot \mathrm{~s} \approx 1.2 \mathrm{~N} \cdot \mathrm{~s}
$$

(d) The impulse found above is the change in the bullet's momentum

$$
J=\Delta p=m \Delta v \rightarrow m=\frac{J}{\Delta v}=\frac{1.185 \mathrm{~N} \cdot \mathrm{~s}}{260 \mathrm{~m} / \mathrm{s}}=4.558 \times 10^{-3} \mathrm{~kg} \approx 4.6 \mathrm{~g}
$$

(e) The momentum of the bullet-gun combination is conserved during the firing of the bullet. Use this to find the recoil speed of the gun, calling the direction of the bullet's motion the positive direction. The momentum before firing is 0 .

$$
\begin{aligned}
& p_{\text {initial }}=p_{\text {final }} \rightarrow 0=m_{\text {bullet }} v_{\text {bullet }}-m_{\text {gun }} v_{\text {gun }} \rightarrow \\
& v_{\text {gun }}=\frac{m_{\text {bullet }} v_{\text {bullet }}}{m_{\text {gun }}}=\frac{\left(4.558 \times 10^{-3} \mathrm{~kg}\right)(260 \mathrm{~m} / \mathrm{s})}{4.5 \mathrm{~kg}}=0.26 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

31. (a) Since the velocity changes direction, the momentum changes. Take the final velocity to be in the positive direction. Then the initial velocity is in the negative direction. The average force is the change in momentum divided by the time.

$$
F_{\mathrm{avg}}=\frac{\Delta p}{\Delta t}=\frac{(m v--m v)}{\Delta t}=2 \frac{m v}{\Delta t}
$$

(b) Now, instead of the actual time of interaction, use the time between collisions in order to get the average force over a long time.

$$
F_{\text {avg }}=\frac{\Delta p}{t}=\frac{(m v--m v)}{t}=2 \frac{m v}{t}
$$

32. (a) The impulse is the change in momentum. Take upwards to be the positive direction. The velocity just before reaching the ground is found from conservation of mechanical energy.

$$
\begin{aligned}
& E_{\text {initial }}=E_{\text {final }} \rightarrow m g h=\frac{1}{2} m v_{y}^{2} \rightarrow v_{y}=-\sqrt{2 g h}=-\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(3.0 \mathrm{~m})}=7.668 \mathrm{~m} / \mathrm{s} \\
& \overrightarrow{\mathbf{J}}=\Delta \overrightarrow{\mathbf{p}}=m\left(\overrightarrow{\mathbf{v}}_{f}-\overrightarrow{\mathbf{v}}_{0}\right)=(65 \mathrm{~kg})(-7.668 \mathrm{~m} / \mathrm{s})=498 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s} \approx 5.0 \times 10^{2} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}, \text { upwards }
\end{aligned}
$$

(b) The net force on the person is the sum of the upward force from the ground, plus the downward force of gravity.

$$
\begin{aligned}
F_{\text {net }} & =F_{\text {ground }}-m g=m a \rightarrow \\
F_{\text {ground }} & =m(g+a)=m\left(g+\frac{\left(v_{f}^{2}-v_{0}^{2}\right)}{2 \Delta x}\right)=(65 \mathrm{~kg})\left(\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)+\frac{0-(-7.668 \mathrm{~m} / \mathrm{s})^{2}}{2(-0.010 \mathrm{~m})}\right) \\
& =1.9 \times 10^{5} \mathrm{~m} / \mathrm{s}, \text { upwards }
\end{aligned}
$$

This is about 300 times the jumper's weight.
(c) We do this the same as part (b).

$$
\begin{aligned}
F_{\text {ground }} & =m\left(g+\frac{\left(v_{f}^{2}-v_{0}^{2}\right)}{2 \Delta x}\right)=(65 \mathrm{~kg})\left(\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)+\frac{0-(-7.668 \mathrm{~m} / \mathrm{s})^{2}}{2(-0.50 \mathrm{~m})}\right) \\
& =4.5 \times 10^{3} \mathrm{~m} / \mathrm{s}, \text { upwards }
\end{aligned}
$$

This is about 7 times the jumper's weight.
33. Take the upwards direction as positive.
(a) The scale reading as a function of time will be due to two components - the weight of the (stationary) water already in the pan, and the force needed to stop the falling water. The weight of the water in the pan is just the rate of mass being added to the pan, times the acceleration due to gravity, times the elapsed time.

$$
W_{\substack{\text { water } \\ \text { in pan }}}=\frac{\Delta m}{\Delta t}(g)(t)=(0.14 \mathrm{~kg} / \mathrm{s})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) t=(1.372 t) \mathrm{N} \approx(1.4 t) \mathrm{N}
$$

The force needed to stop the falling water is the momentum change per unit time of the water striking the pan, $F_{\substack{\text { to stop } \\ \text { moving } \\ \text { water }}}=\frac{\Delta p}{\Delta t}$. The speed of the falling water when it reaches the pan can be found from energy conservation. We assume the water leaves the faucet with a speed of 0 , and that there is no appreciable friction during the fall.

$$
E_{\substack{\text { water } \\ \text { at faucet }}}=\underset{\substack{\text { water } \\ \text { at pan }}}{E_{2}} \rightarrow m g h=\frac{1}{2} m v^{2} \rightarrow \underset{\substack{\text { at } \\ \text { pan }}}{v_{2}}=-\sqrt{2 g h}
$$

The negative sign is because the water is moving downwards.

$$
F_{\substack{\text { tostop } \\ \text { noving } \\ \text { water }}}=\frac{\Delta p}{\Delta t}=\frac{m_{\text {falling }}}{\Delta t} \Delta v=(0.14 \mathrm{~kg} / \mathrm{s})\left(0--\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(2.5 \mathrm{~m})}\right)=0.98 \mathrm{~N}
$$

This force is constant, as the water constantly is hitting the pan. And we assume the water level is not riding. So the scale reading is the sum of these two terms.

$$
F_{\text {scale }}=F_{\substack{\text { to stop } \\ \text { moving } \\ \text { water }}}+\underset{\substack{\text { water } \\ \text { in pan }}}{W_{2}}=(0.98+1.4 t) \mathrm{N}
$$

(b) After 9.0 s , the reading is $F_{\text {scale }}=(0.98+1.372(0.9 \mathrm{~s})) \mathrm{N}=13.3 \mathrm{~N}$.
(c) In this case, the level of the water rises over time. The height of the water in the cylinder is the volume of water divided by the area of the cylinder.

$$
\left.h_{\text {in }}=\frac{V_{\text {water }}}{\text { tube }} \text { in tube }\right) ~[(0.14 t) \mathrm{kg}]\left(\frac{1 \mathrm{~m}^{3}}{1.0 \times 10^{3} \mathrm{~kg}}\right) \frac{\left[\left(20 \times 10^{-4} \mathrm{~m}^{2}\right)\right.}{A_{\text {tube }}}=0.070 t \mathrm{~m}
$$

The height that the water falls is now $h^{\prime}=(2.5-0.070 t) \mathrm{m}$. Following the same analysis as above, the speed of the water when it strikes the surface of the already-fallen water is now $v^{\prime}=-\sqrt{2 g h^{\prime}}$, and so the force to stop the falling water is given by the following.

$$
F_{\substack{\text { to stop } \\ \text { moving } \\ \text { water }}}=(0.14 \mathrm{~kg} / \mathrm{s})\left(0--\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(2.5-.070 t) \mathrm{m}}\right)=0.6198 \sqrt{(2.5-.070 t)} \mathrm{N}
$$

The scale reading is again the sum of two terms.

$$
\begin{aligned}
F_{\text {scale }} & =F_{\substack{\text { to stop } \\
\text { moving } \\
\text { water }}}+W_{\substack{\text { water } \\
\text { in cylinder }}}=(0.6198 \sqrt{(2.5-.070 t)}+1.372 t) \mathrm{N} \\
& \approx(0.62 \sqrt{(2.5-.070 t)}+1.4 t) \mathrm{N}
\end{aligned}
$$

At $t=9.0 \mathrm{~s}$, the scale reading is as follows.

$$
F_{\text {scale }}==(0.6198 \sqrt{(2.5-.070(9.0))}+1.372(9.0)) \mathrm{N}=13.196 \mathrm{~N} \approx 13.2 \mathrm{~N}
$$

34. Let A represent the $0.060-\mathrm{kg}$ tennis ball, and let B represent the $0.090-\mathrm{kg}$ ball. The initial direction of the balls is the positive direction. We have $v_{\mathrm{A}}=4.50 \mathrm{~m} / \mathrm{s}$ and $v_{\mathrm{B}}=3.00 \mathrm{~m} / \mathrm{s}$. Use Eq. $9-8$ to obtain a relationship between the velocities.

$$
v_{\mathrm{A}}-v_{\mathrm{B}}=-\left(v_{\mathrm{A}}^{\prime}-v_{\mathrm{B}}^{\prime}\right) \rightarrow v_{\mathrm{B}}^{\prime}=1.50 \mathrm{~m} / \mathrm{s}+v_{\mathrm{A}}^{\prime}
$$

Substitute this relationship into the momentum conservation equation for the collision.

$$
\begin{aligned}
& m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} v_{\mathrm{B}}=m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}+m_{\mathrm{B}} v_{\mathrm{B}}^{\prime} \rightarrow m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} v_{\mathrm{B}}=m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}+m_{\mathrm{B}}\left(1.50 \mathrm{~m} / \mathrm{s}+v_{\mathrm{A}}^{\prime}\right) \rightarrow \\
& v_{\mathrm{A}}^{\prime}=\frac{m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}}\left(v_{\mathrm{B}}-1.50 \mathrm{~m} / \mathrm{s}\right)}{m_{\mathrm{A}}+m_{\mathrm{B}}}=\frac{(0.060 \mathrm{~kg})(4.50 \mathrm{~m} / \mathrm{s})+(0.090 \mathrm{~kg})(3.00 \mathrm{~m} / \mathrm{s}-1.50 \mathrm{~m} / \mathrm{s})}{0.150 \mathrm{~kg}} \\
&=2.7 \mathrm{~m} / \mathrm{s} \\
& v_{\mathrm{B}}^{\prime}=1.50 \mathrm{~m} / \mathrm{s}+v_{\mathrm{A}}^{\prime}=4.2 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Both balls move in the direction of the tennis ball's initial motion.
35. Let A represent the $0.450-\mathrm{kg}$ puck, and let B represent the $0.900-\mathrm{kg}$ puck. The initial direction of puck A is the positive direction. We have $v_{\mathrm{A}}=4.80 \mathrm{~m} / \mathrm{s}$ and $v_{\mathrm{B}}=0$. Use Eq. $9-8$ to obtain a relationship between the velocities.

$$
v_{\mathrm{A}}-v_{\mathrm{B}}=-\left(v_{\mathrm{A}}^{\prime}-v_{\mathrm{B}}^{\prime}\right) \rightarrow v_{\mathrm{B}}^{\prime}=v_{\mathrm{A}}+v_{\mathrm{A}}^{\prime}
$$

Substitute this relationship into the momentum conservation equation for the collision.

$$
\begin{aligned}
& m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} v_{\mathrm{B}}=m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}+m_{\mathrm{B}} v_{\mathrm{B}}^{\prime} \rightarrow m_{\mathrm{A}} v_{\mathrm{A}}=m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}+m_{\mathrm{B}}\left(v_{\mathrm{A}}+v_{\mathrm{A}}^{\prime}\right) \rightarrow \\
& v_{\mathrm{A}}^{\prime}=\frac{\left(m_{\mathrm{A}}-m_{\mathrm{B}}\right)}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)} v_{\mathrm{A}}=\frac{-0.450 \mathrm{~kg}}{1.350 \mathrm{~kg}}(4.80 \mathrm{~m} / \mathrm{s})=-1.60 \mathrm{~m} / \mathrm{s}=1.60 \mathrm{~m} / \mathrm{s}(\text { west }) \\
& v_{\mathrm{B}}^{\prime}=v_{\mathrm{A}}+v_{\mathrm{A}}^{\prime}=4.80 \mathrm{~m} / \mathrm{s}-1.60 \mathrm{~m} / \mathrm{s}=3.20 \mathrm{~m} / \mathrm{s}(\text { east })
\end{aligned}
$$

36. (a) Momentum will be conserved in one dimension. Call the direction of the first ball the positive direction. Let A represent the first ball, and B represent the second ball. We have $v_{\mathrm{B}}=0$ and $v_{\mathrm{B}}^{\prime}=\frac{1}{2} v_{\mathrm{A}}$. Use Eq. $9-8$ to obtain a relationship between the velocities.

$$
v_{\mathrm{A}}-v_{\mathrm{B}}=-\left(v_{\mathrm{A}}^{\prime}-v_{\mathrm{B}}^{\prime}\right) \rightarrow v_{\mathrm{A}}^{\prime}=-\frac{1}{2} v_{\mathrm{A}}
$$

Substitute this relationship into the momentum conservation equation for the collision.

$$
\begin{aligned}
& p_{\text {initial }}=p_{\text {final }} \rightarrow m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} v_{\mathrm{B}}=m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}+m_{\mathrm{B}} v_{\mathrm{B}}^{\prime} \rightarrow m_{\mathrm{A}} v_{\mathrm{A}}=-\frac{1}{2} m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} \frac{1}{2} v_{\mathrm{A}} \rightarrow \\
& m_{\mathrm{B}}=3 m_{\mathrm{A}}=3(0.280 \mathrm{~kg})=0.840 \mathrm{~kg}
\end{aligned}
$$

(b) The fraction of the kinetic energy given to the second ball is as follows.

$$
\frac{K_{\mathrm{B}}^{\prime}}{K_{\mathrm{A}}}=\frac{\frac{1}{2} m_{\mathrm{B}} v_{\mathrm{B}}^{\prime 2}}{\frac{1}{2} m_{\mathrm{A}} v_{\mathrm{A}}^{2}}=\frac{3 m_{\mathrm{A}}\left(\frac{1}{2} v_{\mathrm{A}}\right)^{2}}{m_{\mathrm{A}} v_{\mathrm{A}}^{2}}=0.75
$$

37. Let A represent the moving ball, and let B represent the ball initially at rest. The initial direction of the ball is the positive direction. We have $v_{\mathrm{A}}=7.5 \mathrm{~m} / \mathrm{s}, v_{\mathrm{B}}=0$, and $v_{\mathrm{A}}^{\prime}=-3.8 \mathrm{~m} / \mathrm{s}$.
(a) Use Eq. 9-8 to obtain a relationship between the velocities.

$$
v_{\mathrm{A}}-v_{\mathrm{B}}=-\left(v_{\mathrm{A}}^{\prime}-v_{\mathrm{B}}^{\prime}\right) \rightarrow v_{\mathrm{B}}^{\prime}=v_{\mathrm{A}}-v_{\mathrm{B}}+v_{\mathrm{A}}^{\prime}=7.5 \mathrm{~m} / \mathrm{s}-0-3.8 \mathrm{~m} / \mathrm{s}=3.7 \mathrm{~m} / \mathrm{s}
$$

(b) Use momentum conservation to solve for the mass of the target ball.

$$
\begin{aligned}
& m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} v_{\mathrm{B}}=m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}+m_{\mathrm{B}} v_{\mathrm{B}}^{\prime} \rightarrow \\
& m_{\mathrm{B}}=m_{\mathrm{A}} \frac{\left(v_{\mathrm{A}}-v_{\mathrm{A}}^{\prime}\right)}{\left(v_{\mathrm{B}}^{\prime}-v_{\mathrm{B}}\right)}=(0.220 \mathrm{~kg}) \frac{(7.5 \mathrm{~m} / \mathrm{s}--3.8 \mathrm{~m} / \mathrm{s})}{3.7 \mathrm{~m} / \mathrm{s}}=0.67 \mathrm{~kg}
\end{aligned}
$$

38. Use the relationships developed in Example 9-8 for this scenario.

$$
\begin{aligned}
& v_{\mathrm{A}}^{\prime}=v_{\mathrm{A}}\left(\frac{m_{\mathrm{A}}-m_{\mathrm{B}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}\right) \rightarrow \\
& m_{\mathrm{B}}=\left(\frac{v_{\mathrm{A}}-v_{\mathrm{A}}^{\prime}}{v_{\mathrm{A}}^{\prime}+v_{\mathrm{A}}}\right) m_{\mathrm{A}}=\left(\frac{v_{\mathrm{A}}-(-0.350) v_{\mathrm{A}}}{(-0.350) v_{\mathrm{A}}+v_{\mathrm{A}}}\right) m_{\mathrm{A}}=\left(\frac{1.350}{0.650}\right) m_{\mathrm{A}}=2.08 m
\end{aligned}
$$

39. The one-dimensional stationary target elastic collision is analyzed in Example 9-8. The fraction of kinetic energy lost is found as follows.

$$
\begin{aligned}
& \quad \frac{K_{\mathrm{A}}-K_{\mathrm{A}}}{\text { inital }} \begin{array}{l}
K_{\text {final }} \\
K_{\mathrm{A}} \\
\text { inital }
\end{array}=\frac{K_{\mathrm{B}}}{K_{\text {final }}}=\frac{\frac{1}{2} m_{\mathrm{B}} v_{\mathrm{B}}^{\prime 2}}{K_{\mathrm{A}}} \frac{\mathrm{inital}^{2} m_{\mathrm{A}} v_{\mathrm{A}}^{2}}{K_{\mathrm{B}}}=\frac{m_{\mathrm{B}}\left[v_{\mathrm{A}}\left(\frac{2 m_{\mathrm{A}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}\right)^{2}\right]}{m_{\mathrm{A}} v_{\mathrm{A}}^{2}}=\frac{4 m_{\mathrm{A}} m_{\mathrm{B}}}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)^{2}} \\
& \text { (a) } \frac{4 m_{\mathrm{A}} m_{\mathrm{B}}}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)^{2}}=\frac{4(1.01)(1.01)}{(1.01+1.01)^{2}}=1.00
\end{aligned}
$$

All the initial kinetic energy is lost by the neutron, as expected for the target mass equal to the incoming mass.
(b) $\frac{4 m_{\mathrm{A}} m_{\mathrm{B}}}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)^{2}}=\frac{4(1.01)(2.01)}{(1.01+2.01)^{2}}=0.890$
(c) $\frac{4 m_{\mathrm{A}} m_{\mathrm{B}}}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)^{2}}=\frac{4(1.01)(12.00)}{(1.01+12.00)^{2}}=0.286$
(d) $\frac{4 m_{\mathrm{A}} m_{\mathrm{B}}}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)^{2}}=\frac{4(1.01)(208)}{(1.01+208)^{2}}=0.0192$

Since the target is quite heavy, almost no kinetic energy is lost. The incoming particle "bounces off" the heavy target, much as a rubber ball bounces off a wall with approximately no loss in speed.
40. Both momentum and kinetic energy are conserved in this one-dimensional collision. We start with Eq. 9-3 (for a one-dimensional setting) and Eq. 9-8.

$$
m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} v_{\mathrm{B}}=m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}+m_{\mathrm{B}} v_{\mathrm{B}}^{\prime} ; v_{\mathrm{A}}-v_{\mathrm{B}}=-\left(v_{\mathrm{A}}^{\prime}-v_{\mathrm{B}}^{\prime}\right) \rightarrow v_{\mathrm{B}}^{\prime}=v_{\mathrm{A}}-v_{\mathrm{B}}+v_{\mathrm{A}}^{\prime}
$$

Insert the last result above back into the momentum conservation equation.

$$
\begin{aligned}
& m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} v_{\mathrm{B}}=m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}+m_{\mathrm{B}}\left(v_{\mathrm{A}}-v_{\mathrm{B}}+v_{\mathrm{A}}^{\prime}\right)=\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) v_{\mathrm{A}}^{\prime}+m_{\mathrm{B}}\left(v_{\mathrm{A}}-v_{\mathrm{B}}\right) \rightarrow \\
& m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} v_{\mathrm{B}}-m_{\mathrm{B}}\left(v_{\mathrm{A}}-v_{\mathrm{B}}\right)=\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) v_{\mathrm{A}}^{\prime} \rightarrow\left(m_{\mathrm{A}}-m_{\mathrm{B}}\right) v_{\mathrm{A}}+2 m_{\mathrm{B}} v_{\mathrm{B}}=\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) v_{\mathrm{A}}^{\prime} \rightarrow \\
& v_{\mathrm{A}}^{\prime}=v_{\mathrm{A}}\left(\frac{m_{\mathrm{A}}-m_{\mathrm{B}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}\right)+v_{\mathrm{B}}\left(\frac{2 m_{\mathrm{B}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}\right)
\end{aligned}
$$

Do a similar derivation by solving Eq. 9-8 for $v_{\mathrm{A}}^{\prime}$, which gives $v_{\mathrm{A}}^{\prime}=v_{\mathrm{B}}^{\prime}-v_{\mathrm{A}}+v_{\mathrm{B}}$.

$$
\begin{aligned}
& m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} v_{\mathrm{B}}=m_{\mathrm{A}}\left(v_{\mathrm{B}}^{\prime}-v_{\mathrm{A}}+v_{\mathrm{B}}\right)+m_{\mathrm{B}} v_{\mathrm{B}}^{\prime}=m_{\mathrm{A}}\left(-v_{\mathrm{A}}+v_{\mathrm{B}}\right)+\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) v_{\mathrm{B}}^{\prime} \rightarrow \\
& m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} v_{\mathrm{B}}-m_{\mathrm{A}}\left(-v_{\mathrm{A}}+v_{\mathrm{B}}\right)=\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) v_{\mathrm{B}}^{\prime} \rightarrow 2 m_{\mathrm{A}} v_{\mathrm{A}}+\left(m_{\mathrm{B}}-m_{\mathrm{A}}\right) v_{\mathrm{B}}=\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) v_{\mathrm{B}}^{\prime} \rightarrow \\
& v_{\mathrm{B}}^{\prime}=v_{\mathrm{A}}\left(\frac{2 m_{\mathrm{A}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}\right)+v_{\mathrm{B}}\left(\frac{m_{\mathrm{B}}-m_{\mathrm{A}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}\right)
\end{aligned}
$$

41. (a) At the maximum compression of the spring, the blocks will not be moving relative to each other, and so they both have the same forward speed. All of the interaction between the blocks is internal to the mass-spring system, and so momentum conservation can be used to find that common speed. Mechanical energy is also conserved, and so with that common speed, we can find the energy stored in the spring and then the compression of the spring. Let A represent the 3.0 kg block, let B represent the 4.5 kg block, and let $x$ represent the amount of compression of the spring.

$$
\begin{aligned}
& p_{\text {initial }}=p_{\text {final }} \rightarrow m_{\mathrm{A}} v_{\mathrm{A}}=\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) v^{\prime} \rightarrow v^{\prime}=\frac{m_{\mathrm{A}}}{m_{\mathrm{A}}+m_{\mathrm{B}}} v_{\mathrm{A}} \\
& E_{\text {initial }}=E_{\text {final }} \rightarrow \frac{1}{2} m_{\mathrm{A}} v_{\mathrm{A}}^{2}=\frac{1}{2}\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) v^{\prime 2}+\frac{1}{2} k x^{2} \rightarrow \\
& x=\sqrt{\frac{1}{k}\left[m_{\mathrm{A}} v_{\mathrm{A}}^{2}-\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) v^{\prime 2}\right]}=\sqrt{\frac{1}{k} \frac{m_{\mathrm{A}} m_{\mathrm{B}}}{m_{\mathrm{A}}+m_{\mathrm{B}}} v_{\mathrm{A}}^{2}} \\
& =\sqrt{\left(\frac{1}{850 \mathrm{~N} / \mathrm{m}}\right) \frac{(3.0 \mathrm{~kg})(4.5 \mathrm{~kg})}{(7.5 \mathrm{~kg})}(8.0 \mathrm{~m} / \mathrm{s})^{2}}=0.37 \mathrm{~m}
\end{aligned}
$$

(b) This is a stationary target elastic collision in one dimension, and so the results of Example 9-8 may be used.

$$
\begin{aligned}
& v_{\mathrm{A}}^{\prime}=v_{\mathrm{A}}\left(\frac{m_{\mathrm{A}}-m_{\mathrm{B}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}\right)=(8.0 \mathrm{~m} / \mathrm{s})\left(\frac{-1.5 \mathrm{~kg}}{7.5 \mathrm{~kg}}\right)=-1.6 \mathrm{~m} / \mathrm{s} \\
& v_{\mathrm{B}}^{\prime}=v_{\mathrm{A}}\left(\frac{2 m_{\mathrm{A}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}\right)=(8.0 \mathrm{~m} / \mathrm{s})\left(\frac{6.0 \mathrm{~kg}}{7.5 \mathrm{~kg}}\right)=6.4 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(c) Yes, the collision is elastic. All forces involved in the collision are conservative forces.
42. From the analysis in Example 9-11, the initial projectile speed is given by $v=\frac{m+M}{m} \sqrt{2 g h}$. Compare the two speeds with the same masses.

$$
\frac{v_{2}}{v_{1}}=\frac{\frac{m+M}{m} \sqrt{2 g h_{2}}}{\frac{m+M}{m} \sqrt{2 g h_{1}}}=\frac{\sqrt{h_{2}}}{\sqrt{h_{1}}}=\sqrt{\frac{h_{2}}{h_{1}}}=\sqrt{\frac{5.2}{2.6}}=\sqrt{2} \rightarrow v_{2}=\sqrt{2} v_{1}
$$

43. (a) In Example 9-11, $K_{i}=\frac{1}{2} m v^{2}$ and $K_{f}=\frac{1}{2}(m+M) v^{\prime 2}$. The speeds are related by

$$
\begin{aligned}
& v^{\prime}=\frac{m}{m+M} v . \\
& \begin{aligned}
\frac{\Delta K}{K_{i}} & =\frac{K_{f}-K_{i}}{K_{i}}=\frac{\frac{1}{2}(m+M) v^{\prime 2}-\frac{1}{2} m v^{2}}{\frac{1}{2} m v^{2}}=\frac{(m+M)\left(\frac{m}{m+M} v\right)^{2}-m v^{2}}{m v^{2}} \\
& =\frac{\frac{m^{2} v^{2}}{m+M}-m v^{2}}{m v^{2}}=\frac{m}{m+M}-1=\frac{-M}{m+M}
\end{aligned}
\end{aligned}
$$

(b) For the given values, $\frac{-M}{m+M}=\frac{-380 \mathrm{~g}}{396 \mathrm{~g}}=--0.96$. Thus $96 \%$ of the energy is lost.
44. From the analysis in the Example 9-11, we know that

$$
\begin{aligned}
v & =\frac{m+M}{m} \sqrt{2 g h} \rightarrow \\
h & =\frac{1}{2 g}\left(\frac{m v}{m+M}\right)^{2}=\frac{1}{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}\left(\frac{(0.028 \mathrm{~kg})(210 \mathrm{~m} / \mathrm{s})}{0.028 \mathrm{~kg}+3.6 \mathrm{~kg}}\right)^{2} \\
& =0.134 \mathrm{~m} \approx 1.3 \times 10^{-1} \mathrm{~m}
\end{aligned}
$$

From the diagram we see the following.

$$
\begin{aligned}
& \boldsymbol{\ell}^{2}=(\boldsymbol{\ell}-h)^{2}+x^{2} \\
& x=\sqrt{\ell^{2}-(\ell-h)^{2}}=\sqrt{(2.8 \mathrm{~m})^{2}-(2.8 \mathrm{~m}-0.134 \mathrm{~m})^{2}}=0.86 \mathrm{~m}
\end{aligned}
$$


45. Use conservation of momentum in one dimension, since the particles will separate and travel in opposite directions. Call the direction of the heavier particle's motion the positive direction. Let A represent the heavier particle, and B represent the lighter particle. We have $m_{\mathrm{A}}=1.5 m_{\mathrm{B}}$, and

$$
\begin{aligned}
& v_{\mathrm{A}}=v_{\mathrm{B}}=0 . \\
& \qquad p_{\text {initial }}=p_{\text {final }} \rightarrow 0=m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}+m_{\mathrm{B}} v_{\mathrm{B}}^{\prime} \rightarrow v_{\mathrm{A}}^{\prime}=-\frac{m_{\mathrm{B}} v_{\mathrm{B}}^{\prime}}{m_{\mathrm{A}}}=-\frac{2}{3} v_{\mathrm{B}}^{\prime}
\end{aligned}
$$

The negative sign indicates direction. Since there was no mechanical energy before the explosion, the kinetic energy of the particles after the explosion must equal the energy added.

$$
\begin{aligned}
& E_{\text {added }}=K_{A}^{\prime}+K_{B}^{\prime}=\frac{1}{2} m_{A} v_{A}^{\prime 2}+\frac{1}{2} m_{B} v_{B}^{\prime 2}=\frac{1}{2}\left(1.5 m_{B}\right)\left(\frac{2}{3} v_{B}^{\prime}\right)^{2}+\frac{1}{2} m_{B} v_{B}^{\prime 2}=\frac{5}{3}\left(\frac{1}{2} m_{B} v_{B}^{\prime 2}\right)=\frac{5}{3} K_{B}^{\prime} \\
& K_{B}^{\prime}=\frac{3}{5} E_{\text {added }}=\frac{3}{5}(7500 \mathrm{~J})=4500 \mathrm{~J} \quad K_{A}^{\prime}=E_{\text {added }}-K_{B}^{\prime}=7500 \mathrm{~J}-4500 \mathrm{~J}=3000 \mathrm{~J}
\end{aligned}
$$

Thus

$$
K_{A}^{\prime}=3.0 \times 10^{3} \mathrm{~J} \quad K_{B}^{\prime}=4.5 \times 10^{3} \mathrm{~J} .
$$

46. Use conservation of momentum in one dimension. Call the direction of the sports car's velocity the positive $x$ direction. Let A represent the sports car, and B represent the SUV. We have $v_{B}=0$ and $v_{\mathrm{A}}^{\prime}=v_{\mathrm{B}}^{\prime}$. Solve for $v_{\mathrm{A}}$.

$$
p_{\text {initial }}=p_{\text {final }} \rightarrow m_{\mathrm{A}} v_{\mathrm{A}}+0=\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) v_{\mathrm{A}}^{\prime} \rightarrow v_{\mathrm{A}}=\frac{m_{\mathrm{A}}+m_{\mathrm{B}}}{m_{\mathrm{A}}} v_{\mathrm{A}}^{\prime}
$$

The kinetic energy that the cars have immediately after the collision is lost due to negative work done by friction. The work done by friction can also be calculated using the definition of work. We assume the cars are on a level surface, so that the normal force is equal to the weight. The distance the cars slide forward is $\Delta x$. Equate the two expressions for the work done by friction, solve for $v_{\mathrm{A}}^{\prime}$, and use that to find $v_{\mathrm{A}}$.

$$
\begin{aligned}
& W_{\mathrm{fr}}=\left(K_{\text {final }}-K_{\text {initial }}\right)_{\text {after }}^{\text {collision }}=0-\frac{1}{2}\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) v_{\mathrm{A}}^{\prime 2} \\
& W_{\mathrm{fr}}=F_{\mathrm{fr}} \Delta x \cos 180^{\circ}=-\mu_{k}\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) g \Delta x \\
& -\frac{1}{2}\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) v_{\mathrm{A}}^{\prime 2}=-\mu_{k}\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) g \Delta x \rightarrow v_{\mathrm{A}}^{\prime}=\sqrt{2 \mu_{k} g \Delta x} \\
& v_{\mathrm{A}}=\frac{m_{\mathrm{A}}+m_{\mathrm{B}}}{m_{\mathrm{A}}} v_{\mathrm{A}}^{\prime}=\frac{m_{\mathrm{A}}+m_{\mathrm{B}}}{m_{\mathrm{A}}} \sqrt{2 \mu_{k} g \Delta x}=\frac{920 \mathrm{~kg}+2300 \mathrm{~kg}}{920 \mathrm{~kg}} \sqrt{2(0.80)\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)(2.8 \mathrm{~m})} \\
& \quad=23.191 \mathrm{~m} / \mathrm{s} \approx 23 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

47. The impulse on the ball is its change in momentum. Call upwards the positive direction, so that the final velocity is positive, and the initial velocity is negative. The speeds immediately before and immediately after the collision can be found from conservation of energy. Take the floor to be the zero level for gravitational potential energy.

$$
\begin{aligned}
& \text { Falling: } K_{\text {botom }}=U_{\text {top }} \rightarrow \frac{1}{2} m v_{\mathrm{down}}^{2}=m g h_{\text {down }} \rightarrow v_{\mathrm{down}}=-\sqrt{2 g h_{\mathrm{down}}} \\
& \text { Rising: } K_{\text {botom }}=U_{\text {top }} \rightarrow \frac{1}{2} m v_{\mathrm{up}}^{2}=m g h_{\mathrm{up}} \rightarrow v_{\mathrm{up}}=\sqrt{2 g h_{\mathrm{up}}} \\
& J=\Delta p=m \Delta v=m\left(v_{\mathrm{up}}-v_{\mathrm{down}}\right)=m\left(\sqrt{2 g h_{\mathrm{up}}}--\sqrt{2 g h_{\mathrm{down}}}\right)=m \sqrt{2 g}\left(\sqrt{h_{\mathrm{up}}}+\sqrt{h_{\text {down }}}\right) \\
& \left.=(0.012 \mathrm{~kg}) \sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right.}\right)(\sqrt{0.75 \mathrm{~m}}+\sqrt{1.5 \mathrm{~m}})=0.11 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

The direction of the impulse is upwards, so the complete specification of the impulse is $0.11 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}$, upwards.
48. Fraction $K$ lost $=\frac{K_{\text {initial }}-K_{\text {final }}}{K_{\text {initial }}}=\frac{\frac{1}{2} m_{\mathrm{A}} v_{\mathrm{A}}^{2}-\frac{1}{2} m_{\mathrm{B}} v_{\mathrm{B}}^{\prime 2}}{\frac{1}{2} m_{\mathrm{A}} v_{\mathrm{A}}^{2}}=\frac{v_{\mathrm{A}}^{2}-v_{\mathrm{B}}^{\prime 2}}{v_{\mathrm{A}}^{2}}=\frac{(35 \mathrm{~m} / \mathrm{s})^{2}-(25 \mathrm{~m} / \mathrm{s})^{2}}{(35 \mathrm{~m} / \mathrm{s})^{2}}=0.49$
49. (a) For a perfectly elastic collision, Eq. 9-8 says $v_{A}-v_{B}=-\left(v_{A}^{\prime}-v_{B}^{\prime}\right)$. Substitute that into the coefficient of restitution definition.

$$
e=\frac{v_{A}^{\prime}-v_{B}^{\prime}}{v_{B}-v_{A}}=-\frac{\left(v_{A}-v_{B}\right)}{v_{B}-v_{A}}=1 .
$$

For a completely inelastic collision, $v_{A}^{\prime}=v_{B}^{\prime}$. Substitute that into the coefficient of restitution definition.

$$
e=\frac{v_{A}^{\prime}-v_{B}^{\prime}}{v_{B}-v_{A}}=0
$$

(b) Let A represent the falling object and B represent the heavy steel plate. The speeds of the steel plate are $v_{B}=0$ and $v_{B}^{\prime}=0$. Thus $e=-v_{A}^{\prime} / v_{A}$. Consider energy conservation during the falling or rising path. The potential energy of body A at height $h$ is transformed into kinetic energy just before it collides with the plate. Choose down to be the positive direction.

$$
m g h=\frac{1}{2} m v_{\mathrm{A}}^{2} \rightarrow v_{\mathrm{A}}=\sqrt{2 g h}
$$

The kinetic energy of body A immediately after the collision is transformed into potential energy as it rises. Also, since it is moving upwards, it has a negative velocity.

$$
m g h^{\prime}=\frac{1}{2} m v_{\mathrm{A}}^{\prime 2} \rightarrow v_{\mathrm{A}}^{\prime}=-\sqrt{2 g h^{\prime}}
$$

Substitute the expressions for the velocities into the definition of the coefficient of restitution.

$$
e=-v_{A}^{\prime} / v_{A}=-\frac{-\sqrt{2 g h^{\prime}}}{\sqrt{2 g h}} \rightarrow e=\sqrt{h^{\prime} / h}
$$

50. The swinging motion will conserve mechanical energy. Take the zero level for gravitational potential energy to be at the bottom of the arc. For the pendulum to swing exactly to the top of the arc, the potential energy at the top of the arc must be equal to the kinetic energy at the bottom.

$$
K_{\text {bottom }}=U_{\text {top }} \rightarrow \frac{1}{2}(m+M) V_{\text {bottom }}^{2}=(m+M) g(2 L) \rightarrow V_{\text {bottom }}=2 \sqrt{g L}
$$

Momentum will be conserved in the totally inelastic collision at the bottom of the arc. We assume that the pendulum does not move during the collision process.

$$
p_{\text {initial }}=p_{\text {final }} \rightarrow m v=(m+M) V_{\text {botom }} \rightarrow v=\frac{m+M}{m}=2 \frac{m+M}{m} \sqrt{g L}
$$

51. (a) The collision is assumed to happen fast enough that the bullet-block system does not move during the collision. So the totally inelastic collision is described by momentum conservation. The conservation of energy (including the non-conservative work done by friction) can be used to relate the initial kinetic energy of the bullet-block system to the spring compression and the dissipated energy. Let $m$ represent the mass of the bullet, $M$ represent the mass of the block, and $x$ represent the distance the combination moves after the collision

$$
\text { collision: } m v=(m+M) v^{\prime} \rightarrow v=\frac{m+M}{m} v^{\prime}
$$

$$
\text { after collision: } \frac{1}{2}(m+M) v^{\prime 2}=\frac{1}{2} k x^{2}+\mu(m+M) g x \rightarrow v^{\prime}=\sqrt{\frac{k x^{2}}{m+M}+2 \mu g x}
$$

$$
v=\frac{m+M}{m} \sqrt{\frac{k x^{2}}{m+M}+2 \mu g x}
$$

$$
\begin{aligned}
& =\frac{1.000 \mathrm{~kg}}{1.0 \times 10^{-3} \mathrm{~kg}} \sqrt{\frac{(120 \mathrm{~N} / \mathrm{m})(0.050 \mathrm{~m})^{2}}{1.000 \mathrm{~kg}}+2(0.50)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.050 \mathrm{~m})}=888.8 \mathrm{~m} / \mathrm{s} \\
& \approx 890 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(b) The fraction of kinetic energy dissipated in the collision is $\frac{K_{\text {initial }}-K_{\text {final }}}{K_{\text {initial }}}$, where the kinetic energies are calculated immediately before and after the collision.

$$
\begin{aligned}
\frac{K_{\text {initial }}-K_{\text {final }}}{K_{\text {initial }}} & =\frac{\frac{1}{2} m v^{2}-\frac{1}{2}(m+M) v^{\prime 2}}{\frac{1}{2} m v^{2}}=1-\frac{(m+M) v^{\prime 2}}{m v^{2}}=1-\frac{(m+M) v^{\prime 2}}{m\left(\frac{m+M}{m} v^{\prime}\right)^{2}} \\
& =1-\frac{m}{m+M}=1-\frac{0.0010 \mathrm{~kg}}{1.00 \mathrm{~kg}}=0.999
\end{aligned}
$$

52. (a) Momentum is conserved in the one-dimensional collision. Let A represent the baseball and let $B$ represent the brick.

$$
\begin{aligned}
& m_{\mathrm{A}} v_{\mathrm{A}}=m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}+m_{\mathrm{B}} v_{\mathrm{B}}^{\prime} \rightarrow \\
& v_{\mathrm{A}}^{\prime}=\frac{m_{\mathrm{A}} v_{\mathrm{A}}-m_{\mathrm{B}} v_{\mathrm{B}}^{\prime}}{m_{\mathrm{A}}}=\frac{(0.144 \mathrm{~kg})(28.0 \mathrm{~m} / \mathrm{s})-(5.25 \mathrm{~kg})(1.10 \mathrm{~m} / \mathrm{s})}{0.144 \mathrm{~kg}}=-12.10 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

So the baseball's speed in the reverse direction is $12.1 \mathrm{~m} / \mathrm{s}$.
(b) $K_{\text {before }}=\frac{1}{2} m_{\mathrm{A}} v_{\mathrm{A}}^{2}=\frac{1}{2}(0.144 \mathrm{~kg})(28.0 \mathrm{~m} / \mathrm{s})^{2}=56.4 \mathrm{~J}$ $K_{\text {after }}=\frac{1}{2} m_{\mathrm{A}} v_{\mathrm{A}}^{\prime 2}+\frac{1}{2} m_{\mathrm{A}} v_{\mathrm{B}}^{\prime 2}=\frac{1}{2}(0.144 \mathrm{~kg})(1.21 \mathrm{~m} / \mathrm{s})^{2}+\frac{1}{2}(5.25 \mathrm{~kg})(1.10 \mathrm{~m} / \mathrm{s})^{2}=13.7 \mathrm{~J}$
53. In each case, use momentum conservation. Let A represent the $6.0-\mathrm{kg}$ object and let B represent the $10.0-\mathrm{kg}$ object. We have $v_{\mathrm{A}}=5.5 \mathrm{~m} / \mathrm{s}$ and $v_{\mathrm{B}}=-4.0 \mathrm{~m} / \mathrm{s}$. .
(a) In this totally inelastic case, $v_{\mathrm{A}}^{\prime}=v_{\mathrm{B}}^{\prime}$.

$$
\begin{aligned}
& m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} v_{\mathrm{B}}=\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) v_{\mathrm{A}}^{\prime} \rightarrow \\
& v_{\mathrm{B}}^{\prime}=v_{\mathrm{A}}^{\prime}=\frac{m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} v_{\mathrm{B}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}=\frac{(6.0 \mathrm{~kg})(5.5 \mathrm{~m} / \mathrm{s})+(8.0 \mathrm{~kg})(-4.0 \mathrm{~m} / \mathrm{s})}{14.0 \mathrm{~kg}}=7.1 \times 10^{-2} \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(b) In this case, use Eq. 9-8 to find a relationship between the velocities.

$$
\begin{aligned}
& v_{\mathrm{A}}-v_{\mathrm{B}}=-\left(v_{\mathrm{A}}^{\prime}-v_{\mathrm{B}}^{\prime}\right) \rightarrow v_{\mathrm{B}}^{\prime}=v_{\mathrm{A}}-v_{\mathrm{B}}+v_{\mathrm{A}}^{\prime} \\
& m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} v_{\mathrm{B}}=m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}+m_{\mathrm{B}} v_{\mathrm{B}}^{\prime}=m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}+m_{\mathrm{B}}\left(v_{\mathrm{A}}+v_{\mathrm{B}}+v_{\mathrm{A}}^{\prime}\right) \rightarrow \\
& v_{\mathrm{A}}^{\prime}=\frac{\left(m_{\mathrm{A}}-m_{\mathrm{B}}\right) v_{\mathrm{A}}+2 m_{\mathrm{B}} v_{\mathrm{B}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}=\frac{(-2.0 \mathrm{~kg})(5.5 \mathrm{~m} / \mathrm{s})+2(8.0 \mathrm{~kg})(-4.0 \mathrm{~m} / \mathrm{s})}{14.0 \mathrm{~kg}}=-5.4 \mathrm{~m} / \mathrm{s} \\
& v_{\mathrm{B}}^{\prime}=v_{\mathrm{A}}-v_{\mathrm{B}}+v_{\mathrm{A}}^{\prime}=5.5 \mathrm{~m} / \mathrm{s}-(-4.0 \mathrm{~m} / \mathrm{s})-5.4 \mathrm{~m} / \mathrm{s}=4.1 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(c) In this case, $v_{\mathrm{A}}^{\prime}=0$.

$$
\begin{aligned}
& m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} v_{\mathrm{B}}=m_{\mathrm{B}} v_{\mathrm{B}}^{\prime} \rightarrow \\
& v_{\mathrm{B}}^{\prime}=\frac{m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} v_{\mathrm{B}}}{m_{\mathrm{B}}}=\frac{(6.0 \mathrm{~kg})(5.5 \mathrm{~m} / \mathrm{s})+(8.0 \mathrm{~kg})(-4.0 \mathrm{~m} / \mathrm{s})}{8.0 \mathrm{~kg}}=0.13 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

To check for "reasonableness," first note the final directions of motion. A has stopped, and B has gone in the opposite direction. This is reasonable. Secondly, since both objects are moving slower than their original speeds, there has been a loss of kinetic energy. Since the system has lost kinetic energy and the directions are possible, this interaction is "reasonable."
(d) In this case, $v_{\mathrm{B}}^{\prime}=0$.

$$
\begin{aligned}
& m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} v_{\mathrm{B}}=m_{\mathrm{A}} v_{\mathrm{A}}^{\prime} \rightarrow \\
& v_{\mathrm{A}}^{\prime}=\frac{m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} v_{\mathrm{B}}}{m_{\mathrm{A}}}=\frac{(6.0 \mathrm{~kg})(5.5 \mathrm{~m} / \mathrm{s})+(8.0 \mathrm{~kg})(-4.0 \mathrm{~m} / \mathrm{s})}{6.0 \mathrm{~kg}}=0.17 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

This answer is not reasonable because A is still moving in its original direction while B has stopped. Thus A has somehow passed through B. If B has stopped, A should have rebounded in the negative direction.
(e) In this case, $v_{\mathrm{A}}^{\prime}=-4.0 \mathrm{~m} / \mathrm{s}$.

$$
\begin{aligned}
& m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} v_{\mathrm{B}}=m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}+m_{\mathrm{B}} v_{\mathrm{B}}^{\prime} \rightarrow \\
& v_{\mathrm{B}}^{\prime}=\frac{(6.0 \mathrm{~kg})(5.5 \mathrm{~m} / \mathrm{s}--4.0 \mathrm{~m} / \mathrm{s})+(8.0 \mathrm{~kg})(-4.0 \mathrm{~m} / \mathrm{s})}{8.0 \mathrm{~kg}}=3.1 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

The directions are reasonable, in that each object rebounds. Secondly, since both objects are moving slower than their original speeds, there has been a loss of kinetic energy. Since the system has lost kinetic energy and the directions are possible, this interaction is "reasonable."
54. (a)

$$
\begin{array}{ll}
p_{x}: & m_{\mathrm{A}} v_{\mathrm{A}}=m_{\mathrm{A}} v_{\mathrm{A}}^{\prime} \cos \theta_{\mathrm{A}}^{\prime}+m_{\mathrm{B}} v_{\mathrm{B}}^{\prime} \cos \theta_{\mathrm{B}}^{\prime} \\
p_{y}: & 0=m_{\mathrm{A}} v_{\mathrm{A}}^{\prime} \sin \theta_{\mathrm{A}}^{\prime}-m_{\mathrm{B}} v_{\mathrm{B}}^{\prime} \sin \theta_{\mathrm{B}}^{\prime}
\end{array}
$$

(b) Solve the $x$ equation for $\cos \theta_{\mathrm{B}}^{\prime}$ and the $y$ equation for $\sin \theta_{\mathrm{B}}^{\prime}$, and then find the angle from the tangent function.


$$
\begin{aligned}
& \tan \theta_{\mathrm{B}}^{\prime}=\frac{\sin \theta_{\mathrm{B}}^{\prime}}{\cos \theta_{\mathrm{B}}^{\prime}}=\frac{\frac{m_{\mathrm{A}} v_{\mathrm{A}}^{\prime} \sin \theta_{\mathrm{A}}^{\prime}}{m_{\mathrm{B}} v_{\mathrm{B}}^{\prime}}}{\frac{m_{\mathrm{A}}\left(v_{\mathrm{A}}-v_{\mathrm{A}}^{\prime} \cos \theta_{\mathrm{A}}^{\prime}\right)}{m_{\mathrm{B}} v_{\mathrm{B}}^{\prime}}}=\frac{v_{\mathrm{A}}^{\prime} \sin \theta_{\mathrm{A}}^{\prime}}{\left(v_{\mathrm{A}}-v_{\mathrm{A}}^{\prime} \cos \theta_{\mathrm{A}}^{\prime}\right)} \\
& \theta_{B}^{\prime}=\tan ^{-1} \frac{v_{A}^{\prime} \sin \theta_{A}^{\prime}}{v_{A}-v_{A}^{\prime} \cos \theta_{A}^{\prime}}=\tan ^{-1} \frac{(2.10 \mathrm{~m} / \mathrm{s}) \sin 30.0^{\circ}}{2.80 \mathrm{~m} / \mathrm{s}-(2.10 \mathrm{~m} / \mathrm{s}) \cos 30.0^{\circ}}=46.9^{\circ}
\end{aligned}
$$

With the value of the angle, solve the $y$ equation for the velocity.

$$
v_{\mathrm{B}}^{\prime}=\frac{m_{\mathrm{A}} v_{\mathrm{A}}^{\prime} \sin \theta_{\mathrm{A}}^{\prime}}{m_{\mathrm{B}} \sin \theta_{\mathrm{B}}^{\prime}}=\frac{(0.120 \mathrm{~kg})(2.10 \mathrm{~m} / \mathrm{s}) \sin 30.0^{\circ}}{(0.140 \mathrm{~kg}) \sin 46.9^{\circ}}=1.23 \mathrm{~m} / \mathrm{s}
$$

55. Use this diagram for the momenta after the decay. Since there was no momentum before the decay, the three momenta shown must add to 0 in both the $x$ and $y$ directions.

$$
\begin{aligned}
\left(p_{\text {nucleus }}\right)_{x} & =p_{\text {neutrino }} \quad\left(p_{\text {nucleus }}\right)_{y}=p_{\text {electron }} \\
p_{\text {nucleus }} & =\sqrt{\left(p_{\text {nucleus }}\right)_{x}^{2}+\left(p_{\text {nucleus }}\right)_{y}^{2}}=\sqrt{\left(p_{\text {neutrino }}\right)^{2}+\left(p_{\text {electron }}\right)^{2}} \\
& =\sqrt{\left(6.2 \times 10^{-23} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}\right)^{2}+\left(9.6 \times 10^{-23} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}\right)^{2}}=1.14 \times 10^{-22} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}
\end{aligned}
$$



$$
\theta=\tan ^{-1} \frac{\left(p_{\text {nucleus }}\right)_{y}}{\left(p_{\text {nucleus }}\right)_{x}}=\tan ^{-1} \frac{\left(p_{\text {electron }}\right)}{\left(p_{\text {neutrino }}\right)}=\tan ^{-1} \frac{\left(9.6 \times 10^{-23} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}\right)}{\left(6.2 \times 10^{-23} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}\right)}=57^{\circ}
$$

The second nucleus' momentum is $147^{\circ}$ from the electron's momentum, and is $123^{\circ}$ from the neutrino's momentum.
56. Write momentum conservation in the $x$ and $y$ directions, and kinetic energy conservation. Note that both masses are the same. We allow $\overrightarrow{\mathbf{v}}_{\mathrm{A}}^{\prime}$ to have both $x$ and $y$ components.

$$
\begin{aligned}
p_{x}: & m v_{\mathrm{B}}=m v_{\mathrm{A} x}^{\prime} \rightarrow v_{\mathrm{B}}=v_{\mathrm{A} x}^{\prime} \\
p_{y}: & m v_{\mathrm{A}}=m v_{\mathrm{A} y}^{\prime}+m v_{\mathrm{B}}^{\prime} \rightarrow v_{\mathrm{A}}=v_{\mathrm{A} y}^{\prime}+v_{\mathrm{B}}^{\prime} \\
K: & \frac{1}{2} m v_{\mathrm{A}}^{2}+\frac{1}{2} m v_{\mathrm{B}}^{2}=\frac{1}{2} m v_{\mathrm{A}}^{\prime 2}+\frac{1}{2} m v_{\mathrm{B}}^{\prime 2} \rightarrow v_{\mathrm{A}}^{2}+v_{\mathrm{B}}^{2}=v_{\mathrm{A}}^{\prime 2}+v_{\mathrm{B}}^{\prime 2}
\end{aligned}
$$

Substitute the results from the momentum equations into the kinetic energy equation.

$$
\begin{aligned}
& \left(v_{\mathrm{A} y}^{\prime}+v_{\mathrm{B}}^{\prime}\right)^{2}+\left(v_{\mathrm{A} x}^{\prime}\right)^{2}=v_{\mathrm{A}}^{\prime 2}+v_{\mathrm{B}}^{\prime 2} \rightarrow v_{\mathrm{A} y}^{\prime 2}+2 v_{\mathrm{A} y}^{\prime 2} v_{\mathrm{B}}^{\prime}+v_{\mathrm{B}}^{\prime 2}+v_{\mathrm{A} y}^{\prime 2}=v_{\mathrm{A}}^{\prime 2}+v_{\mathrm{B}}^{\prime 2} \rightarrow \\
& v_{\mathrm{A}}^{\prime 2}+2 v_{\mathrm{A} y}^{\prime 2} v_{\mathrm{B}}^{\prime}+v_{\mathrm{B}}^{\prime 2}=v_{\mathrm{A}}^{\prime 2}+v_{\mathrm{B}}^{\prime 2} \rightarrow 2 v_{\mathrm{A} y}^{2} v_{\mathrm{B}}^{\prime}=0 \rightarrow v_{\mathrm{A} y}^{\prime}=0 \text { or } v_{\mathrm{B}}^{\prime}=0
\end{aligned}
$$

Since we are given that $v_{\mathrm{B}}^{\prime} \neq 0$, we must have $v_{\mathrm{A} y}^{\prime}=0$. This means that the final direction of A is the $x$ direction. Put this result into the momentum equations to find the final speeds.
$v_{\mathrm{A}}^{\prime}=v_{\mathrm{A} x}^{\prime}=v_{\mathrm{B}}=3.7 \mathrm{~m} / \mathrm{s} \quad v_{\mathrm{B}}^{\prime}=v_{\mathrm{A}}=2.0 \mathrm{~m} / \mathrm{s}$
57. (a) Let A represent the incoming nucleus, and B represent the target particle. Take the $x$ direction to be in the direction of the initial velocity of mass A (to the right in the diagram), and the $y$ direction to be up in the diagram. Momentum is conserved in two dimensions, and gives the following relationships.

$$
\begin{array}{ll}
p_{x}: & m_{\mathrm{A}} v_{\mathrm{A}}=m_{\mathrm{B}} v_{\mathrm{B}}^{\prime} \cos \theta \rightarrow \quad v=2 v_{\mathrm{B}}^{\prime} \cos \theta \\
p_{y}: & 0=m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}-m_{\mathrm{B}} v_{\mathrm{B}}^{\prime} \sin \theta \rightarrow \quad v_{\mathrm{A}}^{\prime}=2 v_{\mathrm{B}}^{\prime} \sin \theta
\end{array}
$$



The collision is elastic, and so kinetic energy is also conserved.

$$
K: \quad \frac{1}{2} m_{\mathrm{A}} v_{\mathrm{A}}^{2}=\frac{1}{2} m_{\mathrm{A}} v_{\mathrm{A}}^{\prime 2}+\frac{1}{2} m_{\mathrm{B}} v_{\mathrm{B}}^{\prime 2} \rightarrow v^{2}=v_{\mathrm{A}}^{\prime 2}+2 v_{\mathrm{B}}^{\prime 2} \rightarrow v^{2}-v_{\mathrm{A}}^{\prime 2}=2 v_{\mathrm{B}}^{\prime 2}
$$

Square the two momentum equations and add them together.

$$
v=2 v_{\mathrm{B}}^{\prime} \cos \theta ; v_{\mathrm{A}}^{\prime}=2 v_{\mathrm{B}}^{\prime} \sin \theta \rightarrow v^{2}=4 v_{\mathrm{B}}^{\prime 2} \cos ^{2} \theta ; v_{\mathrm{A}}^{\prime 2}=4 v_{\mathrm{B}}^{\prime 2} \sin ^{2} \theta \rightarrow v^{2}+v_{\mathrm{A}}^{\prime 2}=4 v_{\mathrm{B}}^{\prime 2}
$$

Add these two results together and use them in the $x$ momentum expression to find the angle.

$$
\begin{aligned}
& v^{2}-v_{\mathrm{A}}^{\prime 2}=2 v_{\mathrm{B}}^{\prime 2} ; v^{2}+v_{\mathrm{A}}^{\prime 2}=4 v_{\mathrm{B}}^{\prime 2} \rightarrow 2 v^{2}=6 v_{\mathrm{B}}^{\prime 2} \rightarrow v_{\mathrm{B}}^{\prime}=\frac{v}{\sqrt{3}} \\
& \cos \theta=\frac{v}{2 v_{\mathrm{B}}^{\prime}}=\frac{v}{2 \frac{v}{\sqrt{3}}}=\frac{\sqrt{3}}{2} \rightarrow \theta=30^{\circ}
\end{aligned}
$$

(b) From above, we already have $v_{\mathrm{B}}^{\prime}=\frac{v}{\sqrt{3}}$. Use that in the $y$ momentum equation.

$$
v_{\mathrm{A}}^{\prime}=2 v_{\mathrm{B}}^{\prime} \sin \theta=2 \frac{v}{\sqrt{3}} \sin 30^{\circ}=v_{\mathrm{A}}^{\prime}=\frac{v}{\sqrt{3}}
$$

(c) The fraction transferred is the final energy of the target particle divided by the original kinetic energy.

$$
\frac{K_{\text {target }}}{K_{\text {original }}}=\frac{\frac{1}{2} m_{\mathrm{B}} v_{B}^{\prime 2}}{\frac{1}{2} m_{\mathrm{A}} v_{\mathrm{A}}^{2}}=\frac{\frac{1}{2}\left(2 m_{\mathrm{A}}\right)\left(v^{2} / 3\right)}{\frac{1}{2} m_{\mathrm{A}} v^{2}}=\frac{2}{3}
$$

58. Let n represent the incoming neutron, and let He represent the helium nucleus. We take $m_{\text {не }}=4 m_{\mathrm{n}}$. Take the $x$ direction to be the direction of the initial velocity of the neutron (to the right in the diagram), and the $y$ direction to be up in the diagram. Momentum is conserved in two dimensions, and gives the following relationships.

$$
\begin{aligned}
p_{x}: & m_{\mathrm{n}} v_{\mathrm{n}}=m_{\mathrm{n}} v_{\mathrm{n}}^{\prime} \cos \theta_{\mathrm{n}}^{\prime}+m_{\mathrm{He}} v_{\mathrm{He}}^{\prime} \cos \theta_{\mathrm{He}}^{\prime} \rightarrow \\
& v_{\mathrm{n}}-4 v_{\mathrm{He}}^{\prime} \cos \theta_{\mathrm{He}}^{\prime}=v_{\mathrm{n}}^{\prime} \cos \theta_{\mathrm{n}}^{\prime} \\
p_{y}: & 0=m_{\mathrm{n}} v_{\mathrm{n}}^{\prime} \sin \theta_{\mathrm{n}}^{\prime}-m_{\mathrm{He}}^{\prime} v_{\mathrm{He}}^{\prime} \sin \theta_{\mathrm{He}}^{\prime} \rightarrow 4 v_{\mathrm{He}}^{\prime} \sin \theta_{\mathrm{He}}^{\prime}=v_{\mathrm{n}}^{\prime} \sin \theta_{\mathrm{n}}^{\prime}
\end{aligned}
$$



> The collision is elastic, and so kinetic energy is also conserved.

$$
K: \quad \frac{1}{2} m_{\mathrm{n}} v_{\mathrm{n}}^{2}=\frac{1}{2} m_{\mathrm{n}} v_{\mathrm{n}}^{\prime 2}+\frac{1}{2} m_{\mathrm{He}} v_{\mathrm{He}}^{\prime 2} \rightarrow v_{\mathrm{n}}^{2}=v_{\mathrm{n}}^{\prime 2}+4 v_{\mathrm{He}}^{\prime 2} \quad \rightarrow v_{\mathrm{n}}^{\prime 2}=v_{\mathrm{n}}^{2}-4 v_{\mathrm{He}}^{\prime 2}
$$

This is a set of three equations in the three unknowns $v_{\mathrm{n}}^{\prime}$, $v_{\mathrm{He}}^{\prime}$, and $\theta_{\mathrm{n}}^{\prime}$. We can eliminate $\theta_{\mathrm{n}}^{\prime}$ by squaring and adding the momentum equations. That can be combined with the kinetic energy equation to solve for one of the unknown speeds.

$$
\begin{aligned}
& \left(v_{\mathrm{n}}-4 v_{\mathrm{He}}^{\prime} \cos \theta_{\mathrm{He}}^{\prime}\right)^{2}=\left(v_{\mathrm{n}}^{\prime} \cos \theta_{\mathrm{n}}^{\prime}\right)^{2} ; \quad\left(4 v_{\mathrm{He}}^{\prime} \sin \theta_{\mathrm{He}}^{\prime}\right)^{2}=\left(v_{\mathrm{n}}^{\prime} \sin \theta_{\mathrm{n}}^{\prime}\right)^{2} \rightarrow \\
& v_{\mathrm{n}}^{2}-8 v_{\mathrm{n}}^{\prime} v_{\mathrm{He}}^{\prime} \cos \theta_{\mathrm{He}}^{\prime}+16 v_{\mathrm{He}}^{\prime 2} \cos ^{2} \theta_{\mathrm{He}}^{\prime}+16 v_{\mathrm{He}}^{\prime 2} \sin ^{2} \theta_{\mathrm{He}}^{\prime}=v_{\mathrm{n}}^{\prime 2} \cos ^{2} \theta_{\mathrm{n}}^{\prime}+{v_{\mathrm{n}}^{\prime 2} \sin ^{2} \theta_{\mathrm{n}}^{\prime} \rightarrow}_{v_{\mathrm{n}}^{2}-8 v_{\mathrm{n}} v_{\mathrm{He}}^{\prime} \cos \theta_{\mathrm{He}}^{\prime}+16 v_{\mathrm{He}}^{\prime 2}=v_{\mathrm{n}}^{\prime 2}=v_{\mathrm{n}}^{2}-4 v_{\mathrm{He}}^{\prime 2} \rightarrow}^{v_{\mathrm{He}}^{\prime}=0.4 v_{\mathrm{n}} \cos \theta_{\mathrm{He}}^{\prime}=0.4\left(6.2 \times 10^{5} \mathrm{~m} / \mathrm{s}\right) \cos 45^{\circ}=1.754 \times 10^{5} \mathrm{~m} / \mathrm{s}} \\
& v_{\mathrm{n}}^{\prime 2}=v_{\mathrm{n}}^{2}-4 v_{\mathrm{He}}^{\prime 2} \rightarrow v_{\mathrm{n}}^{\prime}=\sqrt{v_{\mathrm{n}}^{2}-4 v_{\mathrm{He}}^{\prime 2}}=\sqrt{\left(6.2 \times 10^{5} \mathrm{~m} / \mathrm{s}\right)^{2}-4\left(1.754 \times 10^{5} \mathrm{~m} / \mathrm{s}\right)}=5.112 \times 10^{5} \mathrm{~m} / \mathrm{s} \\
& 4 v_{\mathrm{He}}^{\prime} \sin \theta_{\mathrm{He}}^{\prime}=v_{\mathrm{n}}^{\prime} \sin \theta_{\mathrm{n}}^{\prime} \rightarrow \theta_{\mathrm{n}}^{\prime}=\sin ^{-1}\left(4 \frac{v_{\mathrm{He}}^{\prime}}{v_{\mathrm{n}}^{\prime}} \sin \theta_{\mathrm{He}}^{\prime}\right)=\sin ^{-1}\left(4 \frac{\left(1.754 \times 10^{5} \mathrm{~m} / \mathrm{s}\right)}{\left(5.112 \times 10^{5} \mathrm{~m} / \mathrm{s}\right)} \sin 45^{\circ}\right)=76^{\circ}
\end{aligned}
$$

To summarize: $v_{\mathrm{n}}^{\prime}=5.1 \times 10^{5} \mathrm{~m} / \mathrm{s}, \quad v_{\mathrm{He}}^{\prime}=1.8 \times 10^{5} \mathrm{~m} / \mathrm{s}, \theta_{\mathrm{n}}^{\prime}=76^{\circ}$.
59. Let A represent the incoming neon atom, and B represent the target atom. A momentum diagram of the collision looks like the first figure. The figure can be re-drawn as a triangle, the second figure, since $m_{\mathrm{A}} \overrightarrow{\mathbf{v}}_{\mathrm{A}}=m_{\mathrm{A}} \overrightarrow{\mathbf{v}}_{\mathrm{A}}^{\prime}+m_{\mathrm{B}} \overrightarrow{\mathbf{v}}_{\mathrm{B}}^{\prime}$. Write the law of sines for this triangle, relating each final momentum magnitude to the initial momentum magnitude.

$$
\begin{aligned}
& \frac{m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}}{m_{\mathrm{A}} v_{\mathrm{A}}}=\frac{\sin \phi}{\sin \alpha} \rightarrow v_{\mathrm{A}}^{\prime}=v_{\mathrm{A}} \frac{\sin \phi}{\sin \alpha} \\
& \frac{m_{\mathrm{B}} v_{\mathrm{B}}^{\prime}}{m_{\mathrm{A}} v_{\mathrm{A}}}=\frac{\sin \theta}{\sin \alpha} \rightarrow v_{\mathrm{B}}^{\prime}=v_{\mathrm{A}} \frac{m_{\mathrm{A}}}{m_{\mathrm{B}}} \frac{\sin \theta}{\sin \alpha}
\end{aligned}
$$

The collision is elastic, so write the kinetic energy conservation equation,
 and substitute the results from above. Also note that $\alpha=180.0-55.6^{\circ}-50.0^{\circ}=74.4^{\circ}$.

$$
\frac{1}{2} m_{\mathrm{A}} v_{\mathrm{A}}^{2}=\frac{1}{2} m_{\mathrm{A}} v_{\mathrm{A}}^{\prime 2}+\frac{1}{2} m_{\mathrm{B}} v_{\mathrm{B}}^{\prime 2} \rightarrow m_{\mathrm{A}} v_{\mathrm{A}}^{2}=m_{\mathrm{A}}\left(v_{\mathrm{A}} \frac{\sin \phi}{\sin \alpha}\right)^{2}+m_{\mathrm{B}}\left(v_{\mathrm{A}} \frac{m_{\mathrm{A}}}{m_{\mathrm{B}}} \frac{\sin \theta}{\sin \alpha}\right)^{2} \rightarrow
$$

$$
m_{\mathrm{B}}=\frac{m_{\mathrm{A}} \sin ^{2} \theta}{\sin ^{2} \alpha-\sin ^{2} \phi}=\frac{(20.0 \mathrm{u}) \sin ^{2} 55.6^{\circ}}{\sin ^{2} 74.4-\sin ^{2} 50.0^{\circ}}=39.9 \mathrm{u}
$$

60. Use the coordinate system indicated in the diagram. We start with the conditions for momentum and kinetic energy conservation.

$$
\begin{aligned}
p_{x}: & m_{\mathrm{A}} v_{\mathrm{A}}=m_{\mathrm{A}} v_{\mathrm{A}}^{\prime} \cos \theta_{\mathrm{A}}^{\prime}+m_{\mathrm{B}} v_{\mathrm{B}}^{\prime} \cos \theta_{\mathrm{B}}^{\prime} \rightarrow \\
& m_{\mathrm{A}} v_{\mathrm{A}}-m_{\mathrm{A}} v_{\mathrm{A}}^{\prime} \cos \theta_{\mathrm{A}}^{\prime}=m_{\mathrm{B}}^{\prime} v_{\mathrm{B}}^{\prime} \cos \theta_{\mathrm{B}}^{\prime} \\
p_{y}: & 0=m_{\mathrm{A}}^{\prime} v_{\mathrm{A}}^{\prime} \sin \theta_{\mathrm{A}}^{\prime}-m_{\mathrm{B}}^{\prime} v_{\mathrm{B}}^{\prime} \sin \theta_{\mathrm{B}}^{\prime} \rightarrow \\
& m_{\mathrm{A}} v_{\mathrm{A}}^{\prime} \sin \theta_{\mathrm{A}}^{\prime}=m_{\mathrm{B}} v_{\mathrm{B}}^{\prime} \sin \theta_{\mathrm{B}}^{\prime} \\
K: & \frac{1}{2} m_{\mathrm{A}} v_{\mathrm{A}}^{2}=\frac{1}{2} m_{\mathrm{A}} v_{\mathrm{A}}^{\prime 2}+\frac{1}{2} m_{\mathrm{B}} v_{\mathrm{B}}^{\prime 2} \rightarrow m_{\mathrm{A}}\left(v_{\mathrm{A}}^{2}-v_{\mathrm{A}}^{\prime 2}\right)=m_{\mathrm{B}} v_{\mathrm{B}}^{\prime 2} \rightarrow m_{\mathrm{A}} m_{\mathrm{B}}\left(v_{\mathrm{A}}^{2}-v_{\mathrm{A}}^{\prime 2}\right)=m_{\mathrm{B}}^{2} v_{\mathrm{B}}^{\prime 2}
\end{aligned}
$$



Note that from the kinetic energy relationship, since the right side of the equation is positive, we must have $v_{\mathrm{A}} \geq v_{\mathrm{A}}^{\prime} \geq 0$.
Now we may eliminate $\theta_{\mathrm{B}}^{\prime}$ by squaring the two momentum relationships and adding them.

$$
\begin{aligned}
& \left(m_{\mathrm{A}} v_{\mathrm{A}}-m_{\mathrm{A}} v_{\mathrm{A}}^{\prime} \cos \theta_{\mathrm{A}}^{\prime}\right)^{2}+\left(m_{\mathrm{A}} v_{\mathrm{A}}^{\prime} \sin \theta_{\mathrm{A}}^{\prime}\right)^{2}=\left(m_{\mathrm{B}} v_{\mathrm{B}}^{\prime} \cos \theta_{\mathrm{B}}^{\prime}\right)^{2}+\left(m_{\mathrm{B}} v_{\mathrm{B}}^{\prime} \sin \theta_{\mathrm{B}}^{\prime}\right)^{2} \rightarrow \\
& \left(m_{\mathrm{A}} v_{\mathrm{A}}\right)^{2}-\left(2 m_{\mathrm{A}}^{2} v_{\mathrm{A}} v_{\mathrm{A}}^{\prime} \cos \theta_{\mathrm{A}}^{\prime}\right)+\left(m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}\right)^{2}=\left(m_{\mathrm{B}} v_{\mathrm{B}}^{\prime}\right)^{2}
\end{aligned}
$$

Combining the previous result with the conservation of energy result gives the following.
$\left(m_{A} v_{\mathrm{A}}\right)^{2}-\left(2 m_{\mathrm{A}}^{2} v_{\mathrm{A}} v_{\mathrm{A}}^{\prime} \cos \theta_{\mathrm{A}}^{\prime}\right)+\left(m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}\right)^{2}=\left(m_{\mathrm{B}} v_{\mathrm{B}}^{\prime}\right)^{2}=m_{\mathrm{A}} m_{\mathrm{B}}\left(v_{\mathrm{A}}^{2}-v_{\mathrm{A}}^{\prime 2}\right) \rightarrow$ $\cos \theta_{\mathrm{A}}^{\prime}=\frac{1}{2}\left[\left(1-\frac{m_{\mathrm{B}}}{m_{\mathrm{A}}}\right) \frac{v_{\mathrm{A}}}{v_{\mathrm{A}}^{\prime}}+\left(1+\frac{m_{\mathrm{B}}}{m_{\mathrm{A}}}\right) \frac{v_{\mathrm{A}}^{\prime}}{v_{\mathrm{A}}}\right] ;$ still with $v_{\mathrm{A}} \geq v_{\mathrm{A}}^{\prime} \geq 0$
(a) Consider $m_{\mathrm{A}}<m_{\mathrm{B}}$. If $v_{\mathrm{A}}^{\prime}=v_{\mathrm{A}}$, its maximum value, then
$\cos \theta_{\mathrm{A}}^{\prime}=\frac{1}{2}\left[\left(1-\frac{m_{\mathrm{B}}}{m_{\mathrm{A}}}\right) \frac{v_{\mathrm{A}}}{v_{\mathrm{A}}^{\prime}}+\left(1+\frac{m_{\mathrm{B}}}{m_{\mathrm{A}}}\right) \frac{v_{\mathrm{A}}^{\prime}}{v_{\mathrm{A}}}\right]=1 \rightarrow \theta_{\mathrm{A}}^{\prime}=0$. As $v_{\mathrm{A}}^{\prime}$ decreases towards 0 , eventually the first term in the expression for $\cos \theta_{\mathrm{A}}^{\prime}$ will dominate, since it has $\frac{v_{\mathrm{A}}}{v_{\mathrm{A}}^{\prime}}$ as a factor. That term will also be negative because $m_{\mathrm{A}}<m_{\mathrm{B}}$. The expression for $\cos \theta_{\mathrm{A}}^{\prime}$ will eventually become negative and approach $-\infty$ in a continuous fashion. Thus $\cos \theta_{\mathrm{A}}^{\prime}$ will for some value of $\frac{v_{\mathrm{A}}}{v_{\mathrm{A}}^{\prime}}$ have the value of -1 , indicating that there is some allowable value of $v_{\mathrm{A}}^{\prime}$ that causes $\theta_{\mathrm{A}}^{\prime}=180^{\circ}$, and so all scattering angles are possible.
A plot of $\frac{v_{\mathrm{A}}^{\prime}}{v_{\mathrm{A}}}$ vs. $\theta_{\mathrm{A}}^{\prime}$ is helpful in seeing
this. Here is such a plot for $m_{\mathrm{A}}=0.5 m_{\mathrm{B}}$.
Note that it indicates that the speed of the incident particle will range from a minimum of about $0.35 v_{\mathrm{A}}$ for a complete backscatter (a one-dimensional collision) to $1.00 v_{\mathrm{A}}$, which essentially means a

"miss" - no collision. We also see that the graph is monotonically decreasing, which means that there are no analytical extrema to consider in the analysis.
(b) Now consider $m_{\mathrm{A}}>m_{\mathrm{B}}$. If $v_{\mathrm{A}}^{\prime}=v_{\mathrm{A}}$, its maximum value, then again we will have $\cos \theta_{\mathrm{A}}^{\prime}=\frac{1}{2}\left[\left(1-\frac{m_{\mathrm{B}}}{m_{\mathrm{A}}}\right) \frac{v_{\mathrm{A}}}{v_{\mathrm{A}}^{\prime}}+\left(1+\frac{m_{\mathrm{B}}}{m_{\mathrm{A}}}\right) \frac{v_{\mathrm{A}}^{\prime}}{v_{\mathrm{A}}}\right]=1 \rightarrow \theta_{\mathrm{A}}^{\prime}=0$. As $v_{\mathrm{A}}^{\prime}$ decreases towards 0 , eventually the first term in the expression for $\cos \theta_{\mathrm{A}}^{\prime}$ will dominate, since it has $\frac{v_{\mathrm{A}}}{v_{\mathrm{A}}^{\prime}}$ as a factor. But both terms in the expression are positive, since $m_{\mathrm{A}}>m_{\mathrm{B}}$. So the expression for $\cos \theta_{\mathrm{A}}^{\prime}$ will eventually approach $+\infty$ in a continuous fashion, and will never be negative. Thus there will not be any scattering angles bigger than $90^{\circ}$ in any case. But is there a maximum angle, corresponding to a minimum value of $\cos \theta_{\mathrm{A}}^{\prime}$ ? We look for such a point by calculating the derivative $\frac{d}{d v_{\mathrm{A}}^{\prime}} \cos \theta_{\mathrm{A}}^{\prime}$.

$$
\frac{d}{d v_{\mathrm{A}}^{\prime}} \cos \theta_{\mathrm{A}}^{\prime}=\frac{1}{2}\left[-\left(1-\frac{m_{\mathrm{B}}}{m_{\mathrm{A}}}\right) \frac{v_{\mathrm{A}}}{v_{\mathrm{A}}^{\prime 2}}+\left(1+\frac{m_{\mathrm{B}}}{m_{\mathrm{A}}}\right) \frac{1}{v_{\mathrm{A}}}\right]=0 \rightarrow v_{\mathrm{A}}^{\prime}=v_{\mathrm{A}}\left[\frac{\left(1-\frac{m_{\mathrm{B}}}{m_{\mathrm{A}}}\right)}{\left(1+\frac{m_{\mathrm{B}}}{m_{\mathrm{A}}}\right)}\right]^{1 / 2}
$$

Using this critical value gives the following value for $\cos \theta_{1}^{\prime}$, which we label as $\cos \phi$.

$$
\begin{aligned}
& \cos \phi=\frac{1}{2}\left[( 1 - \frac { m _ { \mathrm { B } } } { m _ { \mathrm { A } } } ) \left[\left(\frac{\left.1+\frac{m_{\mathrm{B}}}{m_{\mathrm{A}}}\right)}{\left(1-\frac{m_{\mathrm{B}}}{m_{\mathrm{A}}}\right)}\right]^{1 / 2}+\left(1+\frac{m_{\mathrm{B}}}{m_{\mathrm{A}}}\right)\left[\left(\frac{\left(1-\frac{m_{\mathrm{B}}}{m_{\mathrm{A}}}\right)}{\left(1+\frac{m_{\mathrm{B}}}{m_{\mathrm{A}}}\right)}\right]^{1 / 2}\right]=\left(1-\left(\frac{m_{\mathrm{B}}}{m_{\mathrm{A}}}\right)^{2}\right)^{1 / 2} \rightarrow\right.\right. \\
& \cos ^{2} \phi=1-\left(\frac{m_{\mathrm{B}}}{m_{\mathrm{A}}}\right)^{2}
\end{aligned}
$$

This gives the largest possible scattering angle for the given mass ratio. Again, a plot is instructive. Here is such a plot for $m_{\mathrm{A}}=2 m_{\mathrm{B}}$. We find the maximum scattering angle according to the equation above.

$$
\begin{aligned}
& \cos ^{2} \phi=1-\left(\frac{m_{\mathrm{B}}}{m_{\mathrm{A}}}\right)^{2} \rightarrow \\
& \phi=\cos ^{-1} \sqrt{\left[1-\left(\frac{m_{\mathrm{B}}}{m_{\mathrm{A}}}\right)^{2}\right]} \\
& =\cos ^{-1} \sqrt{\left[1-(0.5)^{2}\right]}=30^{\circ}
\end{aligned}
$$

The equation and the graph agree. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH09.XLS," on tab "Problem 9.60b."
61. To do this problem with only algebraic manipulations is complicated. We use a geometric approach instead. See the diagram of the geometry.

Momentum conservation: $m \overrightarrow{\mathbf{v}}_{\mathrm{A}}=m \overrightarrow{\mathbf{v}}_{\mathrm{A}}^{\prime}+m \overrightarrow{\mathbf{v}}_{\mathrm{B}}^{\prime} \rightarrow \overrightarrow{\mathbf{v}}_{\mathrm{A}}=\overrightarrow{\mathbf{v}}_{\mathrm{A}}^{\prime}+\overrightarrow{\mathbf{v}}_{\mathrm{B}}^{\prime}$
Kinetic energy conservation: $\frac{1}{2} m v_{\mathrm{A}}^{2}=\frac{1}{2} m v_{\mathrm{A}}^{\prime 2}+\frac{1}{2} m v_{\mathrm{B}}^{\prime 2} \rightarrow v_{\mathrm{A}}^{2}=v_{\mathrm{A}}^{\prime 2}+v_{\mathrm{B}}^{\prime 2}$ The momentum equation can be illustrated as a vector summation diagram, and the kinetic energy equation relates the magnitudes of the vectors in that summation diagram. Examination of the energy equation shows that it is identical to the Pythagorean theorem. The only way that the Pythagorean theorem can hold true is if the angle $\alpha$ in the diagram is a right angle. If $\alpha$ is a right angle, then $\theta+\phi=90^{\circ}$, and so the angle between the final velocity
 vectors must be $90^{\circ}$.
62. Find the CM relative to the front of the car.

$$
\begin{aligned}
x_{\mathrm{CM}} & =\frac{m_{\text {car }} x_{\text {car }}+m_{\text {front }} x_{\text {front }}+m_{\text {back }} x_{\text {back }}}{m_{\text {car }}+m_{\text {froont }}+m_{\text {back }}} \\
& =\frac{(1250 \mathrm{~kg})(2.50 \mathrm{~m})+2(70.0 \mathrm{~kg})(2.80 \mathrm{~m})+3(70.0 \mathrm{~kg})(3.90 \mathrm{~m})}{1250 \mathrm{~kg}+2(70.0 \mathrm{~kg})+3(70.0 \mathrm{~kg})}=2.71 \mathrm{~m}
\end{aligned}
$$

63. Choose the carbon atom as the origin of coordinates.

$$
x_{\mathrm{CM}}=\frac{m_{\mathrm{C}} x_{\mathrm{C}}+m_{\mathrm{o}} x_{\mathrm{O}}}{m_{\mathrm{C}}+m_{\mathrm{O}}}=\frac{(12 \mathrm{u})(0)+(16 \mathrm{u})\left(1.13 \times 10^{-10} \mathrm{~m}\right)}{12 \mathrm{u}+16 \mathrm{u}}=6.5 \times 10^{-11} \mathrm{~m} \text { from the } \mathrm{C} \text { atom. }
$$

64. By the symmetry of the problem, since the centers of the cubes are along a straight line, the vertical CM coordinate will be 0 , and the depth CM coordinate will be 0 . The only CM coordinate to calculate is the one along the straight line joining the centers. The mass of each cube will be the volume times the density, and so $m_{1}=\rho\left(\ell_{0}\right)^{3}, m_{2}=\rho\left(2 \ell_{0}\right)^{3}, m_{3}=\rho\left(3 \ell_{0}\right)^{3}$. Measuring from the left edge of the smallest block, the locations of the CMs of the individual cubes are $x_{1}=\frac{1}{2} \ell_{0}$, $x_{2}=2 \ell_{0}, x_{3}=4.5 \ell_{0}$. Use Eq. 9-10 to calculate the CM of the system.

$$
\begin{aligned}
x_{\mathrm{CM}} & =\frac{m_{1} x_{1}+m_{2} x_{2}+m_{3} x_{3}}{m_{1}+m_{2}+m_{3}}=\frac{\rho \ell_{0}^{3}\left(\frac{1}{2} \ell_{0}\right)+8 \rho \ell_{0}^{3}\left(2 \ell_{0}\right)+27 \rho \ell_{0}^{3}\left(4.5 \ell_{0}\right)}{\rho \ell_{0}^{3}+8 \rho \ell_{0}^{3}+27 \rho \ell_{0}^{3}} \\
& =3.8 \ell_{0} \text { from the left edge of the smallest cube }
\end{aligned}
$$

65. Consider this diagram of the cars on the raft. Notice that the origin of coordinates is located at the CM of the raft. Reference all distances to that location.

$$
\begin{aligned}
& x_{C M}=\frac{(1350 \mathrm{~kg})(9 \mathrm{~m})+(1350 \mathrm{~kg})(9 \mathrm{~m})+(1350 \mathrm{~kg})(-9 \mathrm{~m})}{3(1350 \mathrm{~kg})+6200 \mathrm{~kg}}=1.2 \mathrm{~m} \\
& y_{C M}=\frac{(1350 \mathrm{~kg})(9 \mathrm{~m})+(1350 \mathrm{~kg})(-9 \mathrm{~m})+(1350 \mathrm{~kg})(-9 \mathrm{~m})}{3(1350 \mathrm{~kg})+6200 \mathrm{~kg}}=-1.2 \mathrm{~m}
\end{aligned}
$$


66. Consider the following. We start with a full circle of radius $2 R$, with its CM at the origin. Then we draw a circle of radius $R$, with its CM at the coordinates $(0.80 R, 0)$. The full circle can now be labeled as a "gray" part and a "white" part. The $y$ coordinate of the CM of the entire circle, the CM of the gray part, and the CM of the
 white part are all at $y=0$ by the symmetry of the system. The $x$ coordinate of the entire circle is at $x_{\mathrm{CM}}=0$, and can be calculated by $x_{\mathrm{CM}}=\frac{m_{\text {gray }} x_{\text {gray }}+m_{\text {white }} x_{\text {white }}}{m_{\text {total }}}$. Rearrange this equation to solve for the CM of the "gray" part.

$$
\begin{aligned}
x_{\mathrm{CM}} & =\frac{m_{\text {gray }} x_{\text {gray }}+m_{\text {white }} x_{\text {white }}}{m_{\text {total }}} \rightarrow \\
x_{\text {gray }} & =\frac{m_{\text {total }} x_{\mathrm{CM}}-m_{\text {white }} x_{\text {white }}}{m_{\text {gray }}}=\frac{m_{\text {total }} x_{\mathrm{CM}}-m_{\text {white }} x_{\text {white }}}{m_{\text {total }}-m_{\text {white }}}=\frac{-m_{\text {white }} x_{\text {white }}}{m_{\text {total }}-m_{\text {white }}}
\end{aligned}
$$

This is functionally the same as treating the white part of the figure as a hole of negative mass. The mass of each part can be found by multiplying the area of the part times the uniform density of the plate.

$$
x_{\text {gray }}=\frac{-m_{\text {white }} x_{\text {white }}}{m_{\text {total }}-m_{\text {white }}}=\frac{-\rho \pi R^{2}(0.80 R)}{\rho \pi(2 R)^{2}-\rho \pi R^{2}}=\frac{-0.80 R}{3}=-0.27 R
$$

The negative sign indicates that the CM of the "gray" part is to the left of the center of the circle of radius $2 R$.
67. From the symmetry of the wire, we know that $x_{\mathrm{CM}}=0$. Consider an infinitesimal piece of the wire, with mass $d m$, and coordinates $(x, y)=(r \cos \theta, r \sin \theta)$. If the length of that piece of wire is $d \ell$, then since the wire is uniform, we have $d m=\frac{M}{\pi r} d \boldsymbol{\ell}$. And from the diagram and the definition of radian angle measure, we have $d \boldsymbol{\ell}=r d \theta$.


Thus $d m=\frac{M}{\pi r} r d \theta=\frac{M}{\pi} d \theta$. Now apply Eq. 9-13.

$$
y_{\mathrm{CM}}=\frac{1}{M} \int y d m=\frac{1}{M} \int_{0}^{\pi} r \sin \theta \frac{M}{\pi} d \theta=\frac{r}{\pi} \int_{0}^{\pi} \sin \theta d \theta=\frac{2 r}{\pi}
$$

Thus the coordinates of the center of mass are $\left(x_{\mathrm{CM}}, y_{\mathrm{CM}}\right)=\left(0, \frac{2 r}{\pi}\right)$.
68. From the symmetry of the hydrogen equilateral triangle, and the fact that the nitrogen atom is above the center of that triangle, the center of mass will be perpendicular to the plane of the hydrogen atoms, on a line from the center of the hydrogen triangle to the nitrogen atom. We find the height of the center of mass above the triangle from the heights of the individual atoms. The masses can be expressed in any consistent units, and so atomic mass units from the periodic table will be used.

$$
z_{\mathrm{CM}}=\frac{3 m_{\mathrm{H}} z_{\mathrm{H}}+m_{\mathrm{N}} z_{\mathrm{N}}}{m_{\text {total }}}=\frac{3(1.008 \mathrm{u})(0)+(14.007 \mathrm{u})(0.037 \mathrm{~nm})}{3(1.008 \mathrm{u})+(14.007 \mathrm{u})}=0.030 \mathrm{~nm}
$$

And so the center of mass is 0.030 nm above the center of the hydrogen triangle.
69. Let the tip of the cone be at the origin, and the symmetry axis of the cone be vertical. From the symmetry of the cone, we know that $x_{\mathrm{CM}}=y_{\mathrm{CM}}=0$, and so the center of mass lies on the $z$ axis. We have from Eq. 9-13 that $z_{\mathrm{CM}}=\frac{1}{M} \int z d m$. The mass can be expressed as $M=\int d m$, and so $z_{\mathrm{CM}}=\frac{\int z d m}{\int d m}$. Since the object is uniform, we can express the mass as the uniform density $\rho$ times the volume, for any part of the cone. That results in the
 following.

$$
z_{\mathrm{CM}}=\frac{\int z d m}{\int d m}=\frac{\int z \rho d V}{\int \rho d V}
$$

From the diagram, a disk of radius $r$ and thickness $d z$ has a volume of $d V=\pi r^{2} d z$. Finally, the geometry of the cone is such that $r / z=R / h$, and so $r=z R / h$. Combine these relationships and integrate over the $z$ dimension to find the center of mass.

$$
z_{\mathrm{CM}}=\frac{\int z \rho d V}{\int \rho d V}=\frac{\rho \int z \pi r^{2} d z}{\rho \int \pi r^{2} d z}=\frac{\rho \pi \int z(z R / h)^{2} d z}{\rho \pi \int(z R / h)^{2} d z}=\frac{\rho \pi(R / h)^{2} \int z^{3} d z}{\rho \pi(R / h)^{2} \int z^{2} d z}=\frac{\int_{0}^{h} z^{3} d z}{\int_{0}^{h} z^{2} d z}=\frac{h^{4} / 4}{h^{3} / 3}=\frac{3}{4} h
$$

Thus the center of mass is at $\left(0 \hat{\mathbf{i}}+0 \hat{\mathbf{j}}+\frac{3}{4} h \hat{\mathbf{k}}\right)$.
70. Let the peak of the pyramid be directly above the origin, and the base edges of the pyramid be parallel to the $x$ and $y$ axes. From the symmetry of the pyramid, we know that $x_{\mathrm{CM}}=y_{\mathrm{CM}}=0$, and so the center of mass lies on the $z$ axis. We have from Eq. 9-13 that $z_{\mathrm{CM}}=\frac{1}{M} \int z d m$.
The mass can be expressed as $M=\int d m$, and so
$z_{\mathrm{CM}}=\frac{\int z d m}{\int d m}$. Since the object is uniform, we can
 express the mass as the uniform density $\rho$ times the volume, for any part of the pyramid. That results in the following.

$$
z_{\mathrm{CM}}=\frac{\int z d m}{\int d m}=\frac{\int z \rho d V}{\int \rho d V}
$$

From the diagram, for the differential volume we use a square disk of side $\ell$ and thickness $d z$, which has a volume of $d V=\ell^{2} d z$. The geometry of the pyramid is such that $\ell=\frac{s}{h}(h-z)$. That can be checked from the fact that $\ell$ is a linear function of $z, \ell=s$ for $z=0$, and $\ell=0$ for $z=h$. We can relate $s$ to $h$ by expressing the length of an edge in terms of the coordinates of the endpoints
of an edge. One endpoint of each edge is at ( $x= \pm s / 2, y= \pm s / 2, z=0$ ), and the other endpoint of each edge is at $(x=0, y=0, z=h)$. Using the Pythagorean theorem and knowing the edge length is $s$ gives the following relationship.

$$
s^{2}=(s / 2)^{2}+(s / 2)^{2}+h^{2} \rightarrow h=s / \sqrt{2}
$$

We combine these relationships and integrate over the $z$ dimension to find the center of mass.

$$
\begin{aligned}
z_{\mathrm{CM}} & =\frac{\int z d m}{\int d m}=\frac{\int z \rho d V}{\int \rho d V}=\frac{\rho \int z \ell^{2} d z}{\rho \int \ell^{2} d z}=\frac{\rho \int_{0}^{h} z\left[\frac{s}{h}(h-z)\right]^{2} d z}{\rho \int_{0}^{h}\left[\frac{s}{h}(h-z)\right]^{2} d z}=\frac{\int_{0}^{h} z[(h-z)]^{2} d z}{\int_{0}^{h}[(h-z)]^{2} d z} \\
& =\frac{\int_{0}^{h}\left(h^{2} z-2 h z^{2}+z^{3}\right) d z}{\int_{0}^{h}\left(h^{2}-2 h z+z^{2}\right) d z}=\frac{\left(\frac{1}{2} h^{2} z^{2}-\frac{2}{3} h z^{3}+\frac{1}{4} z^{4}\right)_{0}^{h}}{\left(h^{2} z-h z^{2}+\frac{1}{3} z^{3}\right)_{0}^{h}}=\frac{1}{4} h=\frac{1}{4} \frac{s}{\sqrt{2}}=\frac{s}{4 \sqrt{2}}
\end{aligned}
$$

Thus the center of mass is at $\left(0 \hat{\mathbf{i}}+0 \hat{\mathbf{j}}+\frac{s}{4 \sqrt{2}} \hat{\mathbf{k}}\right)$.
71. Let the radius of the semicircular plate be $R$, with the center at the origin. From the symmetry of the semicircle, we know that $x_{\mathrm{CM}}=0$, and so the center of mass lies on the $y$ axis. We have from Eq. 9-13 that $y_{\mathrm{CM}}=\frac{1}{M} \int y d m$. The mass can be expressed as $M=\int d m$, and
 so $y_{\mathrm{CM}}=\frac{\int y d m}{\int d m}$. Since the object is uniform, we can express the mass as a uniform density $\sigma$ times the area, for any part of the semicircle. That results in the following.

$$
y_{\mathrm{CM}}=\frac{\int y d m}{\int d m}=\frac{\int y \sigma d A}{\int \sigma d A}
$$

From the diagram, for the differential area we use a semicircular strip of width $d r$ and length $\pi r$, which has a differential area of $d A=\pi r d r$. And from problem 67, the y coordinate of the center of mass of that strip is $\frac{2 r}{\pi}$. (Note the discussion immediately before Example $9-17$ which mentions using the center of mass of individual objects to find the center of mass of an extended object.) We combine these relationships and integrate over the $z$ dimension to find the center of mass.

$$
y_{\mathrm{CM}}=\frac{\int y d m}{\int d m}=\frac{\int y \sigma d A}{\int \sigma d A}=\frac{\sigma \int_{0}^{R} \frac{2 r}{\pi} \pi r d r}{\sigma \int_{0}^{R} \pi r d r}=\frac{2 \int_{0}^{R} r^{2} d r}{\pi \int_{0}^{R} r d r}=\frac{\frac{2}{3} R^{3}}{\frac{1}{2} \pi R^{2}}=\frac{4 R}{3 \pi}
$$

Thus the center of mass is at $\left(0 \hat{\mathbf{i}}+\frac{4 R}{3 \pi} \hat{\mathbf{j}}\right)$.
72. From Eq. 9-15, we see that $\overrightarrow{\mathbf{v}}_{\mathrm{CM}}=\frac{1}{M} \sum m_{i} \overrightarrow{\mathbf{v}}_{i}$.

$$
\begin{aligned}
\overrightarrow{\mathbf{v}}_{\mathrm{CM}} & =\frac{(35 \mathrm{~kg})(12 \hat{\mathbf{i}}-16 \hat{\mathbf{j}}) \mathrm{m} / \mathrm{s}+(25 \mathrm{~kg})(-20 \hat{\mathbf{i}}+14 \hat{\mathbf{j}}) \mathrm{m} / \mathrm{s}}{(35 \mathrm{~kg}+25 \mathrm{~kg})} \\
& =\frac{[(35)(12)-(25)(20)] \hat{\mathbf{i}} \mathrm{kg} \cdot \mathrm{~m} / \mathrm{s}+[(35)(-12)+(25)(24)] \hat{\mathbf{j}} \mathrm{kg} \cdot \mathrm{~m} / \mathrm{s}}{(60 \mathrm{~kg})} \\
= & \frac{-80 \hat{\mathbf{i}} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}-210 \hat{\mathbf{j}} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}}{(60 \mathrm{~kg})}=-1.3 \hat{\mathbf{i}} \mathrm{~m} / \mathrm{s}-3.5 \hat{\mathbf{j}} \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

73. (a) Find the CM relative to the center of the Earth.

$$
\begin{aligned}
x_{C M} & =\frac{m_{E} x_{E}+m_{M} x_{M}}{m_{E}+m_{M}}=\frac{\left(5.98 \times 10^{24} \mathrm{~kg}\right)(0)+\left(7.35 \times 10^{22} \mathrm{~kg}\right)\left(3.84 \times 10^{8} \mathrm{~m}\right)}{5.98 \times 10^{24} \mathrm{~kg}+7.35 \times 10^{22} \mathrm{~kg}} \\
& =4.66 \times 10^{6} \mathrm{~m} \text { from the center of the Earth }
\end{aligned}
$$

This is actually inside the volume of the Earth, since $R_{E}=6.38 \times 10^{6} \mathrm{~m}$.
(b) It is this Earth-Moon CM location that actually traces out the orbit as discussed in an earlier chapter. The Earth and Moon will orbit about this orbit path in (approximately) circular orbits. The motion of the Moon, for example, around the Sun would then be a sum of two motions: i) the motion of the Moon about the Earth-Moon CM; and ii) the motion of the Earth-Moon CM about the Sun. To an external observer, the Moon's motion would appear to be a small radius, higher frequency circular motion (motion about the Earth-Moon CM) combined with a large radius, lower frequency circular motion (motion about the Sun). The Earth's motion would be similar, but since the center of mass of that Earth-Moon motion is inside the Earth, the Earth would be observed to "wobble" about that CM.
74. The point that will follow a parabolic trajectory is the center of mass of the mallet. Find the CM relative to the bottom of the mallet. Each part of the hammer (handle and head) can be treated as a point mass located at the CM of the respective piece. So the CM of the handle is 12.0 cm from the bottom of the handle, and the CM of the head is 28.0 cm from the bottom of the handle.

$$
x_{\mathrm{CM}}=\frac{m_{\text {handle }} x_{\text {handle }}+m_{\text {head }} x_{\text {head }}}{m_{\text {handle }}+m_{\text {head }}}=\frac{(0.500 \mathrm{~kg})(12.0 \mathrm{~cm})+(2.80 \mathrm{~kg})(28.0 \mathrm{~cm})}{3.30 \mathrm{~kg}}=25.6 \mathrm{~cm}
$$

Note that this is inside the head of the mallet. The mallet will rotate about this point as it flies through the air, giving it a wobbling kind of motion.
75. (a) Measure all distances from the original position of the woman.

$$
\begin{aligned}
x_{\mathrm{CM}} & =\frac{m_{\mathrm{w}} x_{\mathrm{w}}+m_{\mathrm{M}} x_{\mathrm{M}}}{m_{\mathrm{w}}+m_{\mathrm{M}}}=\frac{(55 \mathrm{~kg})(0)+(72 \mathrm{~kg})(10.0 \mathrm{~m})}{127 \mathrm{~kg}}=5.669 \mathrm{~m} \\
& \approx 5.7 \mathrm{~m} \text { from the woman }
\end{aligned}
$$

(b) Since there is no force external to the man-woman system, the CM will not move, relative to the original position of the woman. The woman's distance will no longer be 0 , and the man's distance has changed to 7.5 m .

$$
x_{\mathrm{CM}}=\frac{m_{\mathrm{w}} x_{\mathrm{w}}+m_{\mathrm{M}} x_{\mathrm{M}}}{m_{\mathrm{w}}+m_{\mathrm{M}}}=\frac{(55 \mathrm{~kg}) x_{\mathrm{w}}+(72 \mathrm{~kg})(7.5 \mathrm{~m})}{127 \mathrm{~kg}}=5.669 \mathrm{~m} \rightarrow
$$

$$
\begin{aligned}
& x_{\mathrm{w}}=\frac{(5.669 \mathrm{~m})(127 \mathrm{~kg})-(72 \mathrm{~kg})(7.5 \mathrm{~m})}{55 \mathrm{~kg}}=3.272 \mathrm{~m} \\
& x_{\mathrm{M}}-x_{\mathrm{w}}=7.5 \mathrm{~m}-3.272 \mathrm{~m}=4.228 \mathrm{~m} \approx 4.2 \mathrm{~m}
\end{aligned}
$$

(c) When the man collides with the woman, he will be at the original location of the center of mass.

$$
\underset{\substack{\mathrm{M} \\ \text { final }}}{ }-x_{\substack{\mathrm{M} \\ \text { initial }}}=5.669 \mathrm{~m}-10.0 \mathrm{~m}=-4.331 \mathrm{~m}
$$

He has moved 4.3 m from his original position.
76. (a) As in Example 9-18, the CM of the system follows the parabolic trajectory. Part I will again fall vertically, the CM will "land" a distance $d$ from part I (as in Fig. 9-32), and part II will land a distance $x$ to the right of the CM. We measure horizontal distances from the point underneath the explosion.

$$
x_{\mathrm{CM}}=\frac{m_{\mathrm{I}} x_{\mathrm{I}}+m_{\mathrm{II}} x_{\mathrm{II}}}{m_{\mathrm{I}}+m_{\mathrm{II}}} \rightarrow x_{\mathrm{II}}=\frac{x_{\mathrm{CM}}\left(m_{\mathrm{I}}+m_{\mathrm{II}}\right)-m_{\mathrm{I}} x_{\mathrm{I}}}{m_{\mathrm{II}}}=\frac{d\left(m_{\mathrm{I}}+3 m_{\mathrm{I}}\right)-m_{\mathrm{I}}(0)}{3 m_{\mathrm{I}}}=\frac{4}{3} d
$$

Therefore part II lands a total distance $\frac{7}{3} d$ from the starting point.
(b) Use a similar analysis for this case, but with $m_{\mathrm{I}}=3 m_{\text {II }}$.

$$
x_{\mathrm{CM}}=\frac{m_{\mathrm{I}} x_{\mathrm{I}}+m_{\mathrm{II}} x_{\mathrm{II}}}{m_{\mathrm{I}}+m_{\mathrm{II}}} \rightarrow x_{\mathrm{II}}=\frac{x_{\mathrm{CM}}\left(m_{\mathrm{I}}+m_{\mathrm{II}}\right)-m_{\mathrm{I}} x_{\mathrm{I}}}{m_{\mathrm{II}}}=\frac{d\left(3 m_{\mathrm{II}}+m_{\mathrm{II}}\right)-3 m_{\mathrm{II}}(0)}{m_{\mathrm{II}}}=4 d
$$

Therefore part II lands a total distance 5 5d from the starting point.
77. Calculate the CM relative to the $55-\mathrm{kg}$ person's seat, at one end of the boat. See the first diagram. Be sure to include the boat's mass.

$$
\begin{aligned}
x_{\mathrm{CM}} & =\frac{m_{\mathrm{A}} x_{\mathrm{A}}+m_{\mathrm{B}} x_{\mathrm{B}}+m_{\mathrm{C}} x_{\mathrm{C}}}{m_{\mathrm{A}}+m_{\mathrm{B}}+m_{\mathrm{C}}} \\
& =\frac{(55 \mathrm{~kg})(0)+(78 \mathrm{~kg})(1.5 \mathrm{~m})+(85 \mathrm{~kg})(3.0 \mathrm{~m})}{218 \mathrm{~kg}}=1.706 \mathrm{~m}
\end{aligned}
$$



Now, when the passengers exchange positions, the boat will move some distance " $d$ " as shown, but the CM will not move. We measure the location of the CM from the same place as before, but now the boat has moved relative to that origin.

$$
\begin{aligned}
& x_{\mathrm{CM}}=\frac{m_{\mathrm{A}} x_{\mathrm{A}}+m_{\mathrm{B}} x_{\mathrm{B}}+m_{\mathrm{C}} x_{\mathrm{C}}}{m_{\mathrm{A}}+m_{\mathrm{B}}+m_{\mathrm{C}}} \\
& 1.706 \mathrm{~m}=\frac{(85 \mathrm{~kg})(d)+(78 \mathrm{~kg})(1.5 \mathrm{~m}+d)+(55 \mathrm{~kg})(3.0 \mathrm{~m}+d)}{218 \mathrm{~kg}}=\frac{218 d \mathrm{~kg} \cdot \mathrm{~m}+282 \mathrm{~kg} \cdot \mathrm{~m}}{218 \mathrm{~kg}} \\
& d=0.412 \mathrm{~m}
\end{aligned}
$$

Thus the boat will move 0.41 m towards the initial position of the 85 kg person.
78. Because the interaction between the worker and the flatcar is internal to the worker-flatcar system, their total momentum will be conserved, and the center of mass of the system will move with a constant velocity relative to the ground. The velocity of the center of mass is $6.0 \mathrm{~m} / \mathrm{s}$. Once the worker starts to move, the velocity of the flatcar relative to the ground will be taken as $v_{\text {car }}$ and the velocity of the worker relative to the ground will then be $v_{\text {car }}+2.0 \mathrm{~m} / \mathrm{s}$. Apply Eq. $9-15$, in one dimension. Letter A represents the worker, and letter B represents the flatcar.

$$
\begin{aligned}
& v_{\mathrm{CM}}=\frac{m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} v_{\mathrm{B}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}=\frac{m_{\mathrm{A}}\left(v_{\mathrm{car}}+2.0 \mathrm{~m} / \mathrm{s}\right)+m_{\mathrm{B}} v_{\mathrm{car}}}{m_{\mathrm{A}}+m_{\mathrm{B}}} \rightarrow \\
& v_{\text {car }}=v_{\mathrm{CM}}-\frac{m_{\mathrm{A}}}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)}(2.0 \mathrm{~m} / \mathrm{s})=6.0 \mathrm{~m} / \mathrm{s}-\frac{95 \mathrm{~kg}}{375 \mathrm{~kg}}(2.0 \mathrm{~m} / \mathrm{s})=5.493 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

The flatcar moves this speed while the worker is walking. The worker walks 25 m along the flatcar at a relative speed of $2.0 \mathrm{~m} / \mathrm{s}$, and so he walks for 12.5 s .

$$
\Delta x_{\mathrm{car}}=v_{\mathrm{car}} \Delta t=(5.493 \mathrm{~m} / \mathrm{s})(12.5 \mathrm{~s})=68.66 \mathrm{~m} \approx 69 \mathrm{~m}
$$

79. Call the origin of coordinates the CM of the balloon, gondola, and person at rest. Since the CM is at rest, the total momentum of the system relative to the ground is 0 . The man climbing the rope cannot change the total momentum of the system, and so the CM must stay at rest. Call the upward direction positive. Then the velocity of the man with respect to the balloon is $-v$. Call the velocity of the balloon with respect to the ground $\nu_{\mathrm{BG}}$. Then the velocity of the man with respect to the ground is $v_{\mathrm{MG}}=-v+v_{\mathrm{BG}}$. Apply conservation of linear momentum in one dimension.

$$
0=m v_{\mathrm{MG}}+M v_{\mathrm{BG}}=m\left(-v+v_{\mathrm{BG}}\right)+M v_{\mathrm{BG}} \rightarrow v_{\mathrm{BG}}=v \frac{m}{m+M}, \text { upward }
$$

If the passenger stops, the balloon also stops, and the CM of the system remains at rest.
80. Use Eq. 9-19a. Call upwards the positive direction. The external force is gravity, acting downwards. The exhaust is in the negative direction, and the rate of change of mass is negative.

$$
\begin{aligned}
& \sum \overrightarrow{\mathbf{F}}_{\text {ext }}=M \frac{d \overrightarrow{\mathbf{v}}}{d t}-\overrightarrow{\mathbf{v}}_{\text {rel }} \frac{d M}{d t} \rightarrow-M g=M a+v_{\text {exhaust }} \frac{d M}{d t} \rightarrow \\
& v_{\text {exhaust }}=\frac{-4.0 M g}{d M / d t}=\frac{-4.0(3500 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{-27 \mathrm{~kg} / \mathrm{s}}=5100 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

81. The external force on the belt is the force supplied by the motor and the oppositely-directed force of friction. Use Eq. 9-19 in one dimension. The belt is to move at a constant speed, so the acceleration of the loaded belt is 0 .

$$
\begin{aligned}
& M \frac{d v}{d t}=F_{\text {ext }}+v_{\text {rel }} \frac{d M}{d t} \rightarrow M(0)=F_{\text {motor }}+F_{\text {friction }}+(-v) \frac{d M}{d t} \rightarrow \\
& F_{\text {motor }}=(v) \frac{d M}{d t}-F_{\text {friction }}=(2.20 \mathrm{~m} / \mathrm{s})(75.0 \mathrm{~kg} / \mathrm{s})-(-150 \mathrm{~N})=315 \mathrm{~N}
\end{aligned}
$$

The required power output from the motor is then found as the product of the force and the velocity.

$$
P_{\text {motor }}=F_{\text {motor }} v=(315 \mathrm{~N})(2.20 \mathrm{~m} / \mathrm{s})=693 \mathrm{~W}\left(\frac{1 \mathrm{hp}}{746 \mathrm{~W}}\right)=0.93 \mathrm{hp}
$$

When the gravel drops from the conveyor belt, it is not accelerated in the horizontal direction by the belt and so has no further force interaction with the belt. The "new" gravel dropping on the belt must still be accelerated, so the power required is constant.
82. The thrust is, in general, given as $v_{\text {rel }} \frac{d M}{d t}$.
(a) The mass is ejected at a rate of $4.2 \mathrm{~kg} / \mathrm{s}$, with a relative speed of $550 \mathrm{~m} / \mathrm{s}$ opposite to the direction of travel.

$$
F_{\substack{\text { thrust } \\ \text { fuel }}}=v_{\text {rel }} \frac{d M_{\text {fuel }}}{d t}=(-550 \mathrm{~m} / \mathrm{s})(-4.2 \mathrm{~kg} / \mathrm{s})=2310 \mathrm{~N} \approx 2300 \mathrm{~N}
$$

(b) The mass is first added at a rate of $120 \mathrm{~kg} / \mathrm{s}$, with a relative speed of $270 \mathrm{~m} / \mathrm{s}$ opposite to the direction of travel, and then ejected at a rate of $120 \mathrm{~kg} / \mathrm{s}$, with a relative speed of $550 \mathrm{~m} / \mathrm{s}$ opposite to the direction of travel.

$$
\begin{aligned}
F_{\text {thrust }}^{\text {air }} & =v_{\text {rel }} \frac{d M_{\text {air }}}{d t}=(-270 \mathrm{~m} / \mathrm{s})(120 \mathrm{~kg} / \mathrm{s})+(-550 \mathrm{~m} / \mathrm{s})(-120 \mathrm{~kg} / \mathrm{s})=33600 \mathrm{~N} \\
& \approx 3.4 \times 10^{4} \mathrm{~N}
\end{aligned}
$$

(c) The power developed is the force of thrust times the velocity of the airplane.

$$
\begin{aligned}
P & =\left(\begin{array}{c}
F_{\text {thrust }}^{\text {fuel }}
\end{array}+\underset{\substack{\text { thrust } \\
\text { air }}}{ }\right) v=(2310 \mathrm{~N}+33600 \mathrm{~N})(270 \mathrm{~m} / \mathrm{s})=9.696 \times 10^{6} \mathrm{~W}\left(\frac{1 \mathrm{hp}}{746 \mathrm{~W}}\right) \\
& =1.3 \times 10^{4} \mathrm{hp}
\end{aligned}
$$

83. We apply Eq. $9-19$ b in one dimension, with "away" from the Earth as the positive direction, and "towards" the Earth as the negative direction. The external force is the force of gravity at that particular altitude, found from Eq. 6-1.

$$
\begin{aligned}
& M \frac{d v}{d t}=F_{\mathrm{ext}}+v_{\mathrm{rel}} \frac{d M}{d t} \rightarrow \\
& \begin{aligned}
& \frac{d M}{d t}=\frac{1}{v_{\mathrm{rel}}}\left(M \frac{d v}{d t}-F_{\mathrm{ext}}\right)=\frac{1}{v_{\mathrm{rel}}}\left(M \frac{d v}{d t}--\frac{G M_{\mathrm{Earth}} M}{r^{2}}\right) \\
&=\frac{(25000 \mathrm{~kg})}{(-1300 \mathrm{~m} / \mathrm{s})}\left[1.5 \mathrm{~m} / \mathrm{s}^{2}+\frac{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(5.98 \times 10^{24} \mathrm{~kg}\right)}{\left(6.38 \times 10^{6} \mathrm{~m}+6.4 \times 10^{6} \mathrm{~m}\right)^{2}}\right]=-76 \mathrm{~kg} / \mathrm{s} \\
&
\end{aligned}
\end{aligned}
$$

The negative sign means that the mass is being ejected rather than absorbed.
84. Because the sand is leaking out of the hole, rather than being pushed out the hole, there is no relative velocity of the leaking sand with respect to the sled (during the leaking process). Thus there is no "thrust" in this situation, and so the problem is the same as if there were no hole in the sled. From the free body diagram, we see that the acceleration down the plane will be $a=g \sin \theta$, as analyzed several times in Chapter 4. Use the constant acceleration relationships to find the time.


$$
x=x_{0}+v_{y 0} t+\frac{1}{2} a_{x} t^{2} \rightarrow t=\sqrt{\frac{2 x}{a_{x}}}=\sqrt{\frac{2(120 \mathrm{~m})}{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(\sin 32^{\circ}\right)}}=6.8 \mathrm{~s}
$$

85. It is proven in the solution to problem 61 that in an elastic collision between two objects of equal mass, with the target object initially stationary, the angle between the final velocities of the objects is $90^{\circ}$. For this specific circumstance, see the diagram. We assume that the target ball is hit "correctly" so that it goes in the pocket. Find $\theta_{1}$ from the geometry of the "left' triangle: $\theta_{1}=\tan ^{-1} \frac{1.0}{\sqrt{3.0}}=30^{\circ}$. Find $\theta_{2}$ from

the geometry of the "right" triangle: $\theta_{2}=\tan ^{-1} \frac{3.0}{\sqrt{3.0}}=60^{\circ}$. Since the balls will separate at a $90^{\circ}$ angle, if the target ball goes in the pocket, this does appear to be a good possibility of a scratch shot.
86. The force stopping the wind is exerted by the person, so the force on the person would be equal in magnitude and opposite in direction to the force stopping the wind. Calculate the force from Eq. 9-2, in magnitude only.

$$
\begin{aligned}
& \frac{m_{\text {wind }}}{\Delta t}=\frac{45 \mathrm{~kg} / \mathrm{s}}{\mathrm{~m}^{2}}(1.60 \mathrm{~m})(0.50 \mathrm{~m})=36 \mathrm{~kg} / \mathrm{s} \quad \Delta v_{\text {wind }}=120 \mathrm{~km} / \mathrm{h}\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)=33.33 \mathrm{~m} / \mathrm{s} \\
& \begin{aligned}
F_{\text {on person }} & =F_{\text {on wind }}=\frac{\Delta p_{\text {wind }}}{\Delta t}=\frac{m_{\text {wind }} \Delta v_{\text {wind }}}{\Delta t}=\frac{m_{\text {wind }}}{\Delta t} \Delta v_{\text {wind }}=(36 \mathrm{~kg} / \mathrm{s})(33.33 \mathrm{~m} / \mathrm{s}) \\
& =1200 \mathrm{~N}
\end{aligned}
\end{aligned}
$$

The typical maximum frictional force is $F_{\mathrm{fr}}=\mu_{s} m g=(1.0)(75 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=740 \mathrm{~N}$, and so we see that $F_{\text {on person }}>F_{\mathrm{fr}}$. The wind is literally strong enough to blow a person off his feet.
87. Consider conservation of energy during the rising and falling of the ball, between contacts with the floor. The gravitational potential energy at the top of a path will be equal to the kinetic energy at the start and the end of each rising-falling cycle. Thus $m g h=\frac{1}{2} m v^{2}$ for any particular bounce cycle, and so for an interaction with the floor, the ratio of the energies before and after the bounce is
$\frac{K_{\text {after }}}{K_{\text {before }}}=\frac{m g h^{\prime}}{m g h}=\frac{1.20 \mathrm{~m}}{1.50 \mathrm{~m}}=0.80$. We assume that each bounce will further reduce the energy to $80 \%$ of its pre-bounce amount. The number of bounces to lose $90 \%$ of the energy can be expressed as follows.

$$
(0.8)^{n}=0.1 \rightarrow n=\frac{\log 0.1}{\log 0.8}=10.3
$$

Thus after 11 bounces, more than $90 \%$ of the energy is lost.
As an alternate method, after each bounce, $80 \%$ of the available energy is left. So after 1 bounce, $80 \%$ of the original energy is left. After the second bounce, only $80 \%$ of $80 \%$, or $64 \%$ of the available energy is left. After the third bounce, $51 \%$. After the fourth bounce, $41 \%$. After the fifth bounce, $33 \%$. After the sixth bounce, $26 \%$. After the seventh bounce, $21 \%$. After the eight bounce, $17 \%$. After the ninth bounce, $13 \%$. After the tenth bounce, $11 \%$. After the eleventh bounce, $9 \%$ is left. So again, it takes 11 bounces.
88. Since the collision is elastic, both momentum (in two dimensions) and kinetic energy are conserved. Write the three conservation equations and use them to solve for the desired quantities. The positive $x$ direction in the diagram is taken to the right, and the positive $y$ direction is taken towards the top of the picture.

$$
\begin{aligned}
& \underset{\substack{x \\
\text { intital }}}{p_{\text {final }}} \underset{p_{x}}{ } \rightarrow 0=m v_{\text {pin }} \sin 75^{\circ}-M v_{\text {ball }} \sin \theta \rightarrow v_{\text {pin }} \sin 75^{\circ}=5 v_{\text {ball }} \sin \theta \\
& p_{\text {initial }}=p_{\text {final }} \rightarrow M(13.0 \mathrm{~m} / \mathrm{s})=m v_{\text {pin }} \cos 75^{\circ}+M v_{\text {ball }} \cos \theta \rightarrow \\
& 65.0 \mathrm{~m} / \mathrm{s}-v_{\text {pin }} \cos 75^{\circ}=5 v_{\text {ball }} \cos \theta
\end{aligned}
$$

$$
\begin{aligned}
K_{\text {initial }}=K_{\text {final }} \rightarrow & \frac{1}{2} M(13.0 \mathrm{~m} / \mathrm{s})^{2}=\frac{1}{2} m v_{\text {pin }}^{2}+\frac{1}{2} M v_{\text {ball }}^{2} \rightarrow 845 \mathrm{~m}^{2} / \mathrm{s}^{2}=v_{\text {pin }}^{2}+5 v_{\text {ball }}^{2} \rightarrow \\
& 845 \mathrm{~m}^{2} / \mathrm{s}^{2}-v_{\text {pin }}^{2}=5 v_{\text {ball }}^{2}
\end{aligned}
$$

Square the two momentum equations and add them to eliminate the dependence on $\theta$.

$$
\begin{aligned}
& v_{\text {pin }}^{2} \sin ^{2} 75^{\circ}=25 v_{\text {ball }}^{2} \sin ^{2} \theta ;(65.0)^{2}-2(65.0) v_{\text {pin }} \cos 75^{\circ}+v_{\text {pin }}^{2} \cos ^{2} 75^{\circ}=25 v_{\text {ball }}^{2} \cos ^{2} \theta \rightarrow \\
& v_{\text {pin }}^{2} \sin ^{2} 75^{\circ}+(65.0)^{2}-2(65.0) v_{\text {pin }} \cos 75^{\circ}+v_{\text {pin }}^{2} \cos ^{2} 75^{\circ}=25 v_{\text {ball }}^{2} \sin ^{2} \theta+25 v_{\text {ball }}^{2} \cos ^{2} \theta+\rightarrow \\
& (65.0)^{2}-130 v_{\text {pin }} \cos 75^{\circ}+v_{\text {pin }}^{2}=25 v_{\text {ball }}^{2}=5\left(5 v_{\text {ball }}^{2}\right)
\end{aligned}
$$

Substitute from the kinetic energy equation.

$$
\begin{aligned}
& (65.0)^{2}-130 v_{\text {pin }} \cos 75^{\circ}+v_{\text {pin }}^{2}=5\left(845-v_{\text {pin }}^{2}\right) \rightarrow 4225-130 v_{\text {pin }} \cos 75^{\circ}+v_{\text {pin }}^{2}=4225-5 v_{\text {pin }}^{2} \\
& 6 v_{\text {pin }}^{2}=130 v_{\text {pin }} \cos 75^{\circ} \rightarrow v_{\text {pin }}=5.608 \mathrm{~m} / \mathrm{s} \\
& 845-v_{\text {pin }}^{2}=5 v_{\text {ball }}^{2} \rightarrow v_{\text {ball }}=\sqrt{\frac{1}{5}\left(845-v_{\text {pin }}^{2}\right)}=\sqrt{\frac{1}{5}\left(845-(5.608)^{2}\right)}=12.756 \mathrm{~m} / \mathrm{s} \\
& v_{\text {pin }} \sin 75^{\circ}=5 v_{\text {ball }} \sin \theta \rightarrow \theta=\sin ^{-1}\left(\frac{v_{\text {pin }} \sin 75^{\circ}}{5 v_{\text {ball }}}\right)=\sin ^{-1}\left(\frac{(5.608) \sin 75^{\circ}}{5(12.756)}\right)=4.87^{\circ}
\end{aligned}
$$

So the final answers are as follows.
(a) $v_{\text {pin }}=5.608 \mathrm{~m} / \mathrm{s} \approx 5.6 \mathrm{~m} / \mathrm{s}$
(b) $v_{\text {ball }}=12.756 \mathrm{~m} / \mathrm{s} \approx 13 \mathrm{~m} / \mathrm{s}$
(c) $\theta=4.87^{\circ} \approx 4.9^{\circ}$
89. This is a ballistic "pendulum" of sorts, similar to Example $9-11$ in the textbook. There is no difference in the fact that the block and bullet are moving vertically instead of horizontally. The collision is still totally inelastic and conserves momentum, and the energy is still conserved in the rising of the block and embedded bullet after the collision. So we simply quote the equation from that example.

$$
\begin{aligned}
& v=\frac{m+M}{m} \sqrt{2 g h} \rightarrow \\
& h=\frac{1}{2 g}\left(\frac{m v}{m+M}\right)^{2}=\frac{1}{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}\left(\frac{(0.0240 \mathrm{~kg})(310 \mathrm{~m} / \mathrm{s})}{0.0240 \mathrm{~kg}+1.40 \mathrm{~kg}}\right)^{2}=1.4 \mathrm{~m}
\end{aligned}
$$

90. The initial momentum is 0 , and the net external force on the puck is 0 . Thus momentum will be conserved in two dimensions.

$$
\begin{aligned}
& \overrightarrow{\mathbf{p}}_{\text {initial }}=\overrightarrow{\mathbf{p}}_{\text {initial }} \rightarrow 0=m v \hat{i}+2 m(2 v) \hat{\mathbf{j}}+m \overrightarrow{\mathbf{v}}_{3} \rightarrow \overrightarrow{\mathbf{v}}_{3}=-v \hat{i}-4 \hat{\mathbf{j}} \\
& v_{3}=\sqrt{(-v)^{2}+(-4 v)^{2}}=\sqrt{17} v \quad \theta_{3}=\tan ^{-1} \frac{-4 v}{-v}=256^{\circ}
\end{aligned}
$$

91. The fraction of energy transformed is $\frac{K_{\text {initial }}-K_{\text {final }}}{K_{\text {initial }}}$.

$$
\begin{aligned}
\frac{K_{\text {intial }}-K_{\text {final }}}{K_{\text {initial }}} & =\frac{\frac{1}{2} m_{\mathrm{A}} v_{\mathrm{A}}^{2}-\frac{1}{2}\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) v^{2}}{\frac{1}{2} m_{\mathrm{A}} v_{\mathrm{A}}^{2}}=\frac{m_{\mathrm{A}} v_{\mathrm{A}}^{2}-\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)\left(\frac{m_{\mathrm{A}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}\right)^{2} v_{\mathrm{A}}^{2}}{m_{\mathrm{A}} v_{\mathrm{A}}^{2}} \\
& =1-\frac{m_{\mathrm{A}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}=\frac{m_{\mathrm{B}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}=\frac{1}{2}
\end{aligned}
$$

92. Momentum will be conserved in the horizontal direction. Let A represent the railroad car, and B represent the snow. For the horizontal motion, $v_{B}=0$ and $v_{B}^{\prime}=v_{A}^{\prime}$. Momentum conservation in the horizontal direction gives the following.

$$
\begin{aligned}
& p_{\text {intital }}=p_{\text {final }} \rightarrow m_{\mathrm{A}} v_{\mathrm{A}}=(m t+m t) v_{\mathrm{A}}^{\prime} \\
& v_{\mathrm{A}}^{\prime}=\frac{m_{\mathrm{A}} v_{\mathrm{A}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}=\frac{(4800 \mathrm{~kg})(8.60 \mathrm{~m} / \mathrm{s})}{4800 \mathrm{~kg}+\left(\frac{3.80 \mathrm{~kg}}{\min }\right)(60.0 \mathrm{~min})}=8.210 \mathrm{~m} / \mathrm{s} \approx 8.2 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

93. (a) We consider only the horizontal direction (the direction of motion of the railroad car). There is no external force in the horizontal direction. In Eq. 9-19b, the relative velocity (in the horizontal direction) of the added mass is the opposite of the horizontal velocity of the moving mass, since the added mass is moving straight down.

$$
\begin{aligned}
& M \frac{d v}{d t}=F_{\text {ext }}+v_{\text {rel }} \frac{d M}{d t} \rightarrow M \frac{d v}{d t}=-v \frac{d M}{d t} \rightarrow \frac{d v}{v}=-\frac{d M}{M} \rightarrow \int_{v_{0}}^{v_{f}} \frac{d v}{v}=-\int_{M_{0}}^{M_{f}} \frac{d M}{M} \rightarrow \\
& \ln \frac{v_{f}}{v_{0}}=-\ln \frac{M_{f}}{M_{0}}=\ln \frac{M_{0}}{M_{\mathrm{f}}} \rightarrow \\
& v_{\mathrm{f}}=v_{0} \frac{M_{0}}{M_{f}}=v_{0} \frac{M_{0}}{M_{0}+\frac{d M}{d t} t}
\end{aligned}
$$

(b) Evaluate the speed at $t=60.0 \mathrm{~min}$.

$$
v(t=60.0)=v_{0} \frac{M_{0}}{M_{0}+\frac{d M}{d t} t}=\frac{4800 \mathrm{~kg}}{4800 \mathrm{~kg}+(3.80 \mathrm{~kg} / \mathrm{min})(60.0 \mathrm{~min})}=8.2 \mathrm{~m} / \mathrm{s}
$$

This agrees with the previous problem.
94. (a) No, there is no net external force on the system. In particular, the spring force is internal to the system.
(b) Use conservation of momentum to determine the ratio of speeds. Note that the two masses will be moving in opposite directions. The initial momentum, when the masses are released, is 0 .
$\begin{aligned} & p_{\text {initial }}=p_{\text {later }} \rightarrow 0=m_{\mathrm{A}} v_{\mathrm{A}}-m_{\mathrm{B}} v_{\mathrm{B}} \rightarrow v_{\mathrm{A}} / v_{\mathrm{B}}=m_{\mathrm{B}} / m_{\mathrm{A}} \\ & \text { (c) } \frac{K_{\mathrm{A}}}{K_{\mathrm{B}}}=\frac{\frac{1}{2} m_{\mathrm{A}} v_{\mathrm{A}}^{2}}{\frac{1}{2} m_{\mathrm{B}} v_{\mathrm{B}}^{2}}=\frac{m_{\mathrm{A}}}{m_{\mathrm{B}}}\left(\frac{v_{\mathrm{A}}}{v_{\mathrm{B}}}\right)^{2}=\frac{m_{\mathrm{A}}}{m_{\mathrm{B}}}\left(\frac{m_{\mathrm{B}}}{m_{\mathrm{A}}}\right)^{2}=m_{\mathrm{B}} / m_{\mathrm{A}}\end{aligned}$
(d) The center of mass was initially at rest. Since there is no net external force on the system, the center of mass does not move, and so stays at rest.
(e) With friction present, there could be a net external force on the system, because the forces of friction on the two masses would not necessarily be equal in magnitude. If the two friction forces are not equal in magnitude, the ratios found above would not be valid. Likewise, the center of mass would not necessarily be at rest with friction present.
95. We assume that all motion is along a single direction. The distance of sliding can be related to the change in the kinetic energy of a car, as follows.

$$
\begin{aligned}
& W_{\mathrm{fr}}=\Delta K=\frac{1}{2} m\left(v_{f}^{2}-v_{i}^{2}\right) \quad W_{\mathrm{fr}}=F_{\mathrm{fr}} \Delta x \cos 180^{\circ} \theta=-\mu_{k} F_{\mathrm{N}} \Delta x=-\mu_{k} m g \Delta x \rightarrow \\
& -\mu_{k} g \Delta x=\frac{1}{2}\left(v_{f}^{2}-v_{i}^{2}\right)
\end{aligned}
$$

For post-collision sliding, $v_{f}=0$ and $v_{i}$ is the speed immediately after the collision, $v^{\prime}$. Use this relationship to find the speed of each car immediately after the collision.

$$
\begin{array}{ll}
\operatorname{Car} \mathrm{A}: & -\mu_{k} g \Delta x_{\mathrm{A}}^{\prime}=-\frac{1}{2} v_{\mathrm{A}}^{\prime 2} \rightarrow v_{\mathrm{A}}^{\prime}=\sqrt{2 \mu_{k} g \Delta x_{\mathrm{A}}^{\prime}}=\sqrt{2(0.60)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(18 \mathrm{~m})}=14.55 \mathrm{~m} / \mathrm{s} \\
\operatorname{Car} \mathrm{~B}: & -\mu_{k} g \Delta x_{\mathrm{B}}^{\prime}=-\frac{1}{2} v_{\mathrm{B}}^{\prime 2} \rightarrow v_{\mathrm{B}}^{\prime}=\sqrt{2 \mu_{k} g \Delta x_{\mathrm{B}}^{\prime}}=\sqrt{2(0.60)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(30 \mathrm{~m})}=18.78 \mathrm{~m} / \mathrm{s}
\end{array}
$$

During the collision, momentum is conserved in one dimension. Note that $v_{\mathrm{B}}=0$.

$$
\begin{aligned}
& p_{\text {initial }}=p_{\text {final }} \rightarrow m_{\mathrm{A}} v_{\mathrm{A}}=m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}+m_{\mathrm{B}} v_{\mathrm{B}}^{\prime} \\
& v_{A}=\frac{m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}+m_{\mathrm{B}} v_{\mathrm{B}}^{\prime}}{m_{\mathrm{A}}}=\frac{(1500 \mathrm{~kg})(14.55 \mathrm{~m} / \mathrm{s})+(1100 \mathrm{~kg})(18.78 \mathrm{~m} / \mathrm{s})}{1500 \mathrm{~kg}}=28.32 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

For pre-collision sliding, again apply the friction-energy relationship, with $v_{f}=v_{\mathrm{A}}$ and $v_{i}$ is the speed when the brakes were first applied.

$$
\begin{aligned}
& -\mu_{k} g \Delta x_{\mathrm{A}}=\frac{1}{2}\left(v_{\mathrm{A}}^{2}-v_{i}^{2}\right) \rightarrow v_{i}=\sqrt{v_{A}^{2}+2 \mu_{k} g \Delta x_{A}}=\sqrt{(28.32 \mathrm{~m} / \mathrm{s})^{2}+2(0.60)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(15 \mathrm{~m})} \\
& =31.23 \mathrm{~m} / \mathrm{s}\left(\frac{1 \mathrm{mi} / \mathrm{h}}{0.447 \mathrm{~m} / \mathrm{s}}\right)=70 \mathrm{mi} / \mathrm{h}
\end{aligned}
$$

This is definitely over the speed limit.
96. (a) The meteor striking and coming to rest in the Earth is a totally inelastic collision. Let A represent the Earth and B represent the meteor. Use the frame of reference in which the Earth is at rest before the collision, and so $v_{\mathrm{A}}=0$. Write momentum conservation for the collision.

$$
\begin{aligned}
& m_{\mathrm{B}} v_{\mathrm{B}}=\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) v^{\prime} \rightarrow \\
& v^{\prime}=v_{\mathrm{B}} \frac{m_{\mathrm{B}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}=\left(2.5 \times 10^{4} \mathrm{~m} / \mathrm{s}\right) \frac{2.0 \times 10^{8} \mathrm{~kg}}{6.0 \times 10^{24} \mathrm{~kg}+2.0 \times 10^{8} \mathrm{~kg}}=8.3 \times 10^{-13} \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

This is so small as to be considered 0 .
(b) The fraction of the meteor's kinetic energy transferred to the Earth is the final kinetic energy of the Earth divided by the initial kinetic energy of the meteor.

$$
\frac{K_{\text {final }}}{K_{\text {Earth }}}=\frac{\frac{1}{2} m_{\mathrm{A}} v^{\prime 2}}{K_{\text {initial }}} \text { meteor }-2 m_{\mathrm{B}} v_{\mathrm{B}}^{2}=\frac{\frac{1}{2}\left(6.0 \times 10^{24} \mathrm{~kg}\right)\left(8.3 \times 10^{-13} \mathrm{~m} / \mathrm{s}\right)^{2}}{\frac{1}{2}\left(2.0 \times 10^{8} \mathrm{~kg}\right)\left(2.5 \times 10^{4} \mathrm{~m} / \mathrm{s}\right)^{2}}=3.3 \times 10^{-17}
$$

(c) The Earth's change in kinetic energy can be calculated directly.

$$
\Delta K_{\text {Earth }}=\underset{\substack{\text { final } \\ \text { Earth }}}{ }-K_{\substack{\text { initial } \\ \text { Earth }}}=\frac{1}{2} m_{\mathrm{A}} v^{\prime 2}-0=\frac{1}{2}\left(6.0 \times 10^{24} \mathrm{~kg}\right)\left(8.3 \times 10^{-13} \mathrm{~m} / \mathrm{s}\right)^{2}=2.1 \mathrm{~J}
$$

97. Since the only forces on the astronauts are internal to the 2 -astronaut system, their CM will not change. Call the CM location the origin of coordinates. That is also the original location of the two astronauts.

$$
x_{\mathrm{CM}}=\frac{m_{\mathrm{A}} x_{\mathrm{A}}+m_{\mathrm{B}} x_{\mathrm{B}}}{m_{\mathrm{A}}+m_{\mathrm{B}}} \rightarrow 0=\frac{(60 \mathrm{~kg})(12 \mathrm{~m})+(80 \mathrm{~kg}) x_{\mathrm{B}}}{140 \mathrm{~kg}} \rightarrow x=-9 \mathrm{~m}
$$

Their distance apart is $x_{\mathrm{A}}-x_{\mathrm{B}}=12 \mathrm{~m}-(-9 \mathrm{~m})=21 \mathrm{~m}$.
98. This is a ballistic "pendulum" of sorts, similar to Example 9-11 in the textbook. The mass of the bullet is $m$, and the mass of the block of wood is $M$. The speed of the bullet before the collision is $v$, and the speed of the combination after the collision is $v^{\prime}$. Momentum is conserved in the totally inelastic collision, and so $m v=(m+M) v^{\prime}$. The kinetic energy present immediately after the collision is lost due to negative work being done by friction.

$$
\begin{aligned}
& W_{\mathrm{fr}}=\Delta K=\frac{1}{2} m\left(v_{f}^{2}-v_{i}^{2}\right)_{\substack{\text { after } \\
\text { collision }}} \quad W_{\mathrm{fr}}=F_{\mathrm{fr}} \Delta x \cos 180^{\circ} \theta=-\mu_{k} F_{\mathrm{N}} \Delta x=-\mu_{k} m g \Delta x \rightarrow \\
& -\mu_{k} g \Delta x=\frac{1}{2}\left(v_{f}^{2}-v_{i}^{2}\right)=-\frac{1}{2} v^{\prime 2} \rightarrow v^{\prime}=\sqrt{2 \mu_{k} g \Delta x}
\end{aligned}
$$

Use this expression for $v^{\prime}$ in the momentum conservation equation in one dimension in order to solve for $v$.

$$
\begin{aligned}
m v & =(m+M) v^{\prime}=(m+M) \sqrt{2 \mu_{k} g \Delta x} \rightarrow \\
v & =\left(\frac{m+M}{m}\right) \sqrt{2 \mu_{k} g \Delta x}=\left(\frac{0.022 \mathrm{~kg}+1.35 \mathrm{~kg}}{0.022 \mathrm{~kg}}\right) \sqrt{2(0.28)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(8.5 \mathrm{~m})} \\
& =4.3 \times 10^{2} \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

99. (a) Conservation of mechanical energy can be used to find the velocity of the lighter ball before impact. The potential energy of the ball at the highest point is equal to the kinetic energy of the ball just before impact. Take the lowest point in the swing as the zero location for gravitational potential energy.

$$
\begin{aligned}
& E_{\text {initial }}=E_{\text {final }} \rightarrow m_{\mathrm{A}} g \ell(1-\cos \theta)=\frac{1}{2} m_{\mathrm{A}} v_{\mathrm{A}}^{2} \rightarrow \\
& v_{\mathrm{A}}=\sqrt{2 g \ell(1-\cos \theta)}=\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.30 \mathrm{~m})\left(1-\cos 66^{\circ}\right)} \\
& \quad=1.868 \mathrm{~m} / \mathrm{s} \approx 1.9 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(b) This is an elastic collision with a stationary target. Accordingly, the relationships developed in Example 9-8 are applicable.

$$
\begin{aligned}
& v_{\mathrm{A}}^{\prime}=v_{\mathrm{A}}\left(\frac{m_{\mathrm{A}}-m_{\mathrm{B}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}\right)=(1.868 \mathrm{~m} / \mathrm{s})\left(\frac{0.045 \mathrm{~kg}-0.065 \mathrm{~kg}}{0.045 \mathrm{~kg}+0.065 \mathrm{~kg}}\right)=-0.3396 \mathrm{~m} / \mathrm{s}=-0.34 \mathrm{~m} / \mathrm{s} \\
& v_{\mathrm{B}}^{\prime}=v_{\mathrm{A}}\left(\frac{2 m_{\mathrm{A}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}\right)=(1.868 \mathrm{~m} / \mathrm{s})\left(\frac{2(0.045 \mathrm{~kg})}{0.045 \mathrm{~kg}+0.065 \mathrm{~kg}}\right)=1.528 \mathrm{~m} / \mathrm{s}=1.5 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(c) We can again use conservation of energy for each ball after the collision. The kinetic energy of each ball immediately after the collision will become gravitational potential energy as each ball rises.

$$
\begin{aligned}
& E_{\text {initial }}=E_{\text {final }} \rightarrow \frac{1}{2} m v^{2}=m g h \rightarrow h=\frac{v^{2}}{2 g} \\
& h_{\mathrm{A}}=\frac{v_{\mathrm{A}}^{2}}{2 g}=\frac{(-0.3396 \mathrm{~m} / \mathrm{s})^{2}}{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=5.9 \times 10^{-3} \mathrm{~m} ; h_{\mathrm{B}}=\frac{v_{\mathrm{B}}^{2}}{2 g}=\frac{(1.528 \mathrm{~m} / \mathrm{s})^{2}}{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=0.12 \mathrm{~m}
\end{aligned}
$$

100. (a) Use conservation of energy to find the speed of mass $m$ before the collision. The potential energy at the starting point is all transformed into kinetic energy just before the collision.

$$
m g h_{\mathrm{A}}=\frac{1}{2} m v_{\mathrm{A}}^{2} \rightarrow v_{\mathrm{A}}=\sqrt{2 g h_{\mathrm{A}}}=\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(3.60 \mathrm{~m})}=8.40 \mathrm{~m} / \mathrm{s}
$$

Use Eq. 9-8 to obtain a relationship between the velocities, noting that $v_{B}=0$.

$$
v_{\mathrm{A}}-v_{\mathrm{B}}=v_{\mathrm{B}}^{\prime}-v_{\mathrm{A}}^{\prime} \rightarrow v_{\mathrm{B}}^{\prime}=v_{\mathrm{A}}^{\prime}+v_{\mathrm{A}}
$$

Apply momentum conservation for the collision, and substitute the result from Eq. 9-8.

$$
\begin{aligned}
& m v_{\mathrm{A}}=m v_{\mathrm{A}}^{\prime}+M v_{\mathrm{B}}^{\prime}=m v_{\mathrm{A}}^{\prime}+M\left(v_{\mathrm{A}}+v_{\mathrm{A}}^{\prime}\right) \rightarrow \\
& v_{\mathrm{A}}^{\prime}=\frac{m-M}{m+M} v_{\mathrm{A}}=\left(\frac{2.20 \mathrm{~kg}-7.00 \mathrm{~kg}}{9.20 \mathrm{~kg}}\right)(8.4 \mathrm{~m} / \mathrm{s})=-4.38 \mathrm{~m} / \mathrm{s} \approx-4.4 \mathrm{~m} / \mathrm{s} \\
& v_{\mathrm{B}}^{\prime}=v_{\mathrm{A}}^{\prime}+v_{\mathrm{A}}=-4.4 \mathrm{~m} / \mathrm{s}+8.4 \mathrm{~m} / \mathrm{s}=4.0 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(b) Again use energy conservation to find the height to which mass $m$ rises after the collision. The kinetic energy of $m$ immediately after the collision is all transformed into potential energy. Use the angle of the plane to change the final height into a distance along the incline.

$$
\begin{aligned}
& \frac{1}{2} m v_{\mathrm{A}}^{\prime 2}=m g h_{\mathrm{A}}^{\prime} \rightarrow h_{\mathrm{A}}^{\prime}=\frac{v_{\mathrm{A}}^{\prime 2}}{2 g} \\
& d_{\mathrm{A}}^{\prime}=\frac{h_{\mathrm{A}}^{\prime}}{\sin 30^{\circ}}=\frac{v_{\mathrm{A}}^{\prime 2}}{2 g \sin 30^{\circ}}=\frac{(-4.38 \mathrm{~m} / \mathrm{s})^{2}}{2\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right) g \sin 30^{\circ}}=1.96 \mathrm{~m} \approx 2.0 \mathrm{~m}
\end{aligned}
$$

101. Let A represent mass $m$ and B represent mass $M$. Use Eq. $9-8$ to obtain a relationship between the velocities, noting that $v_{\mathrm{B}}=0$.

$$
v_{\mathrm{A}}-v_{\mathrm{B}}=v_{\mathrm{B}}^{\prime}-v_{\mathrm{A}}^{\prime} \rightarrow v_{\mathrm{A}}^{\prime}=v_{\mathrm{B}}^{\prime}-v_{\mathrm{A}}
$$

After the collision, $v_{\mathrm{A}}^{\prime}<0$ since $m$ is moving in the negative direction. For there to be a second collision, then after $m$ moves up the ramp and comes back down, with a positive velocity at the bottom of the incline of $-v_{\mathrm{A}}^{\prime}$, the speed of $m$ must be greater than the speed of $M$ so that $m$ can catch
M. Thus $-v_{\mathrm{A}}^{\prime}>v_{\mathrm{B}}^{\prime}$, or $v_{\mathrm{A}}^{\prime}<-v_{\mathrm{B}}^{\prime}$. Substitute the result from Eq. $9-8$ into the inequality.

$$
v_{\mathrm{B}}^{\prime}-v_{\mathrm{A}}<-v_{\mathrm{B}}^{\prime} \rightarrow v_{\mathrm{B}}^{\prime}<\frac{1}{2} v_{\mathrm{A}}
$$

Now write momentum conservation for the original collision, and substitute the result from Eq. 9-8.

$$
m v_{\mathrm{A}}=m v_{\mathrm{A}}^{\prime}+M v_{\mathrm{B}}^{\prime}=m\left(v_{\mathrm{B}}^{\prime}-v_{\mathrm{A}}\right)+M v_{\mathrm{B}}^{\prime} \quad \rightarrow \quad v_{\mathrm{B}}^{\prime}=\frac{2 m}{m+M} v_{\mathrm{A}}
$$

Finally, combine the above result with the inequality from above.

$$
\frac{2 m}{m+M} v_{\mathrm{A}}<\frac{1}{2} v_{\mathrm{A}} \rightarrow 4 m<m+M \rightarrow m<\frac{1}{3} M=2.33 \mathrm{~kg}
$$

102. Call the final direction of the joined objects the positive $x$ axis. A diagram of the collision is shown. Momentum will be conserved in both the $x$ and $y$ directions. Note that $v_{\mathrm{A}}=v_{\mathrm{B}}=v$ and $v^{\prime}=v / 3$.

$$
\begin{aligned}
& p_{y}: \quad-m v \sin \theta_{1}+m v \sin \theta_{2}=0 \rightarrow \sin \theta_{1}=\sin \theta_{2} \rightarrow \theta_{1}=\theta_{2} \\
& p_{x}: m v \cos \theta_{1}+m v \cos \theta_{2}=(2 m)(v / 3) \rightarrow \cos \theta_{1}+\cos \theta_{2}=\frac{2}{3} \\
& \cos \theta_{1}+\cos \theta_{2}=2 \cos \theta_{1}=\frac{2}{3} \rightarrow \theta_{1}=\cos ^{-1} \frac{1}{3}=70.5^{\circ}=\theta_{2} \\
& \theta_{1}+\theta_{2}=141^{\circ}
\end{aligned}
$$

103. The original horizontal distance can be found from the range formula from Example 3-10.

$$
R=v_{0}^{2} \sin 2 \theta_{0} / g=(25 \mathrm{~m} / \mathrm{s})^{2}\left(\sin 56^{\circ}\right) /\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)=52.87 \mathrm{~m}
$$

The height at which the objects collide can be found from Eq. 2-12c for the vertical motion, with $v_{y}=0$ at the top of the path. Take up to be positive.

$$
v_{y}^{2}=v_{y 0}^{2}+2 a\left(y-y_{0}\right) \rightarrow\left(y-y_{0}\right)=\frac{v_{y}^{2}-v_{y 0}^{2}}{2 a}=\frac{0-\left[(25 \mathrm{~m} / \mathrm{s}) \sin 28^{\circ}\right]^{2}}{2\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=7.028 \mathrm{~m}
$$

Let $m$ represent the bullet and $M$ the skeet. When the objects collide, the skeet is moving horizontally at $v_{0} \cos \theta=(25 \mathrm{~m} / \mathrm{s}) \cos 28^{\circ}=22.07 \mathrm{~m} / \mathrm{s}=v_{x}$, and the bullet is moving vertically at $v_{y}=230 \mathrm{~m} / \mathrm{s}$. Write momentum conservation in both directions to find the velocities after the totally inelastic collision.

$$
\begin{aligned}
& p_{x}: M v_{x}=(M+m) v_{x}^{\prime} \rightarrow v_{x}^{\prime}=\frac{M v_{x}}{M+m}=\frac{(0.25 \mathrm{~kg})(22.07 \mathrm{~m} / \mathrm{s})}{(0.25+0.015) \mathrm{kg}}=20.82 \mathrm{~m} / \mathrm{s} \\
& p_{y}: m v_{y}=(M+m) v_{y}^{\prime} \rightarrow v_{y}^{\prime}=\frac{m v_{y}}{M+m}=\frac{(0.015 \mathrm{~kg})(230 \mathrm{~m} / \mathrm{s})}{(0.25+0.015) \mathrm{kg}}=13.02 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(a) The speed $v_{y}^{\prime}$ can be used as the starting vertical speed in Eq. 2-12c to find the height that the skeet-bullet combination rises above the point of collision.

$$
\begin{aligned}
& v_{y}^{2}=v_{y 0}^{2}+2 a\left(y-y_{0}\right)_{\mathrm{extra}} \rightarrow \\
& \left(y-y_{0}\right)_{\mathrm{extra}}=\frac{v_{y}^{2}-v_{y 0}^{2}}{2 a}=\frac{0-(13.02 \mathrm{~m} / \mathrm{s})^{2}}{2\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=8.649 \mathrm{~m} \approx 8.6 \mathrm{~m}
\end{aligned}
$$

(b) From Eq. 2-12b applied to the vertical motion after the collision, we can find the time for the skeet-bullet combination to reach the ground.

$$
\begin{aligned}
& y=y_{0}+v_{y}^{\prime} t+\frac{1}{2} a t^{2} \rightarrow 0=8.649 \mathrm{~m}+(13.02 \mathrm{~m} / \mathrm{s}) t+\frac{1}{2}\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right) t^{2} \rightarrow \\
& 4.9 t^{2}-13.02 t-8.649=0 \rightarrow t=3.207 \mathrm{~s},-0.550 \mathrm{~s}
\end{aligned}
$$

The positive time root is used to find the horizontal distance traveled by the combination after the collision.

$$
x_{\text {after }}=v_{x}^{\prime} t=(20.82 \mathrm{~m} / \mathrm{s})(3.207 \mathrm{~s})=66.77 \mathrm{~m}
$$

If the collision would not have happened, the skeet would have gone $\frac{1}{2} R$ horizontally from this point.

$$
\Delta x=x_{\text {after }}-\frac{1}{2} R=66.77 \mathrm{~m}-\frac{1}{2}(52.87 \mathrm{~m})=40.33 \mathrm{~m} \approx 40 \mathrm{~m}
$$

Note that the answer is correct to 2 significant figures.
104. In this interaction, energy is conserved (initial potential energy of mass - compressed spring system $=$ final kinetic energy of moving blocks) and momentum is conserved, since the net external force is 0 . Use these two relationships to find the final speeds.

$$
\begin{aligned}
& p_{\text {initial }}=p_{\text {final }} \rightarrow 0=m v_{m}-3 m v_{3 m} \rightarrow v_{m}=3 v_{3 m} \\
& E_{\text {initial }}=E_{\text {final }} \rightarrow U_{\substack{\text { spring } \\
\text { initial }}}=K_{\text {final }} \rightarrow \frac{1}{2} k D^{2}=\frac{1}{2} m v_{m}^{2}+\frac{1}{2} 3 m v_{3 m}^{2}=\frac{1}{2} m\left(3 v_{3 m}\right)^{2}+\frac{1}{2} 3 m v_{3 m}^{2}=6 m v_{3 m}^{2} \\
& \frac{1}{2} k D^{2}=6 m v_{3 m}^{2} \rightarrow v_{3 m}=D \sqrt{\frac{k}{12 m}} ; v_{m}=3 D \sqrt{\frac{k}{12 m}}
\end{aligned}
$$

105. The interaction between the planet and the spacecraft is elastic, because the force of gravity is conservative. Thus kinetic energy is conserved in the interaction. Consider the problem a 1 dimensional collision, with A representing the spacecraft and B representing Saturn. Because the mass of Saturn is so much bigger than the mass of the spacecraft, Saturn's speed is not changed appreciably during the interaction. Use Eq. 9-8, with $v_{\mathrm{A}}=10.4 \mathrm{~km} / \mathrm{s}$ and $v_{\mathrm{B}}=v_{\mathrm{B}}^{\prime}=-9.6 \mathrm{~km} / \mathrm{s}$.

$$
v_{\mathrm{A}}-v_{\mathrm{B}}=-v_{\mathrm{A}}^{\prime}+v_{\mathrm{B}}^{\prime} \rightarrow v_{\mathrm{A}}^{\prime}=2 v_{\mathrm{B}}-v_{\mathrm{A}}=2(-9.6 \mathrm{~km} / \mathrm{s})-10.4 \mathrm{~km} / \mathrm{s}=-29.6 \mathrm{~km} / \mathrm{s}
$$

Thus there is almost a threefold increase in the spacecraft's speed, and it reverses direction.
106. Let the original direction of the cars be the positive direction. We have $v_{\mathrm{A}}=4.50 \mathrm{~m} / \mathrm{s}$ and $v_{\mathrm{B}}=3.70 \mathrm{~m} / \mathrm{s}$.
(a) Use Eq. 9-8 to obtain a relationship between the velocities.

$$
v_{\mathrm{A}}-v_{\mathrm{B}}=-\left(v_{\mathrm{A}}^{\prime}-v_{\mathrm{B}}^{\prime}\right) \rightarrow v_{\mathrm{B}}^{\prime}=v_{\mathrm{A}}-v_{\mathrm{B}}+v_{\mathrm{A}}^{\prime}=0.80 \mathrm{~m} / \mathrm{s}+v_{\mathrm{A}}^{\prime}
$$

Substitute this relationship into the momentum conservation equation for the collision.

$$
\begin{aligned}
& m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} v_{\mathrm{B}}=m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}+m_{\mathrm{B}} v_{\mathrm{B}}^{\prime} \rightarrow m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} v_{\mathrm{B}}=m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}+m_{\mathrm{B}}\left(0.80 \mathrm{~m} / \mathrm{s}+v_{\mathrm{A}}^{\prime}\right) \rightarrow \\
& v_{\mathrm{A}}^{\prime}=\frac{m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}}\left(v_{\mathrm{B}}-0.80 \mathrm{~m} / \mathrm{s}\right)}{m_{\mathrm{A}}+m_{\mathrm{B}}}=\frac{(450 \mathrm{~kg})(4.50 \mathrm{~m} / \mathrm{s})+(490 \mathrm{~kg})(2.90 \mathrm{~m} / \mathrm{s})}{940 \mathrm{~kg}}=3.666 \mathrm{~m} / \mathrm{s} \\
& \approx 3.67 \mathrm{~m} / \mathrm{s} ; v_{\mathrm{B}}^{\prime}=0.80 \mathrm{~m} / \mathrm{s}+v_{\mathrm{A}}^{\prime}=4.466 \mathrm{~m} / \mathrm{s} \approx 4.47 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(b) Calculate $\Delta p=p^{\prime}-p$ for each car.

$$
\begin{aligned}
\Delta p_{\mathrm{A}} & =m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}-m_{\mathrm{A}} v_{\mathrm{A}}=(450 \mathrm{~kg})(3.666 \mathrm{~m} / \mathrm{s}-4.50 \mathrm{~m} / \mathrm{s})=-3.753 \times 10^{2} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s} \\
& \approx-380 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s} \\
\Delta p_{\mathrm{B}} & =m_{\mathrm{B}} v_{\mathrm{B}}^{\prime}-m_{\mathrm{B}} v_{\mathrm{B}}=(490 \mathrm{~kg})(4.466 \mathrm{~m} / \mathrm{s}-3.70 \mathrm{~m} / \mathrm{s})=3.753 \times 10^{2} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s} \\
& \approx 380 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

The two changes are equal and opposite because momentum was conserved.
107. Let A represent the cube of mass $M$ and B represent the cube of mass $m$. Find the speed of A immediately before the collision, $v_{\mathrm{A}}$, by using energy conservation.

$$
M g h=\frac{1}{2} M v_{\mathrm{A}}^{2} \rightarrow v_{\mathrm{A}}=\sqrt{2 g h}=\sqrt{2\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)(0.35 \mathrm{~m})}=2.619 \mathrm{~m} / \mathrm{s}
$$

Use Eq. $9-8$ for elastic collisions to obtain a relationship between the velocities in the collision. We have $v_{\mathrm{B}}=0$ and $M=2 m$.

$$
v_{\mathrm{A}}-v_{\mathrm{B}}=-\left(v_{\mathrm{A}}^{\prime}-v_{\mathrm{B}}^{\prime}\right) \rightarrow v_{\mathrm{B}}^{\prime}=v_{\mathrm{A}}+v_{\mathrm{A}}^{\prime}
$$

Substitute this relationship into the momentum conservation equation for the collision.

$$
\begin{aligned}
& m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} v_{\mathrm{B}}=m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}+m_{\mathrm{B}} v_{\mathrm{B}}^{\prime} \rightarrow m_{\mathrm{A}} v_{\mathrm{A}}=m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}+m_{\mathrm{B}}\left(v_{\mathrm{A}}+v_{\mathrm{A}}^{\prime}\right) \rightarrow \\
& 2 m v_{\mathrm{A}}=2 m v_{\mathrm{A}}^{\prime}+m\left(v_{\mathrm{A}}+v_{\mathrm{A}}^{\prime}\right) \rightarrow v_{\mathrm{A}}^{\prime}=\frac{v_{\mathrm{A}}}{3}=\frac{\sqrt{2 g h}}{3}=\frac{\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.35 \mathrm{~m})}}{3}=0.873 \mathrm{~m} / \mathrm{s} \\
& v_{\mathrm{B}}^{\prime}=v_{\mathrm{A}}+v_{\mathrm{A}}^{\prime}=\frac{4}{3} v_{\mathrm{A}}=3.492 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Each mass is moving horizontally initially after the collision, and so each has a vertical velocity of 0 as they start to fall. Use constant acceleration Eq. $2-12 b$ with down as positive and the table top as the vertical origin to find the time of fall.

$$
y=y_{0}+v_{0} t+\frac{1}{2} a t^{2} \rightarrow H=0+0+\frac{1}{2} g t^{2} \quad \rightarrow \quad t=\sqrt{2 H / g}
$$

Each cube then travels a horizontal distance found by $\Delta x=v_{x} \Delta t$.

$$
\begin{aligned}
& \Delta x_{m}=v_{A}^{\prime} \Delta t=\frac{\sqrt{2 g h}}{3} \sqrt{\frac{2 H}{g}}=\frac{2}{3} \sqrt{h H}=\frac{2}{3} \sqrt{(0.35 \mathrm{~m})(0.95 \mathrm{~m})}=0.3844 \mathrm{~m} \approx 0.38 \mathrm{~m} \\
& \Delta x_{M}=v_{B}^{\prime} \Delta t=\frac{4 \sqrt{2 g h}}{3} \sqrt{\frac{2 H}{g}}=\frac{8}{3} \sqrt{h H}=\frac{8}{3} \sqrt{(0.35 \mathrm{~m})(0.95 \mathrm{~m})}=1.538 \mathrm{~m} \approx 1.5 \mathrm{~m}
\end{aligned}
$$

108. (a) Momentum is conserved in the $z$ direction. The initial $z$-momentum is 0 .

$$
\begin{aligned}
& p_{z_{\text {befrore }}}=p_{z} \rightarrow 0=m_{\text {sater }} \rightarrow 0 \text { sallite } \\
& v_{z \text { satellite }}+m_{\text {shutule }} v_{z \text { shutle }} \rightarrow \\
& v_{z \text { shututle }}=-\frac{m_{\text {satellite }} v_{z \text { satellite }}}{m_{\text {shuttle }}}=-\frac{850 \mathrm{~kg}}{92,000 \mathrm{~kg}}(0.30 \mathrm{~m} / \mathrm{s})=-2.8 \times 10^{-3} \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

And so the component in the minus $z$ direction is $2.8 \times 10^{-3} \mathrm{~m} / \mathrm{s}$.
(b) The average force is the change in momentum per unit time. The force on the satellite is in the positive $z$ direction.

$$
F_{\text {avg }}=\frac{\Delta p}{\Delta t}=\frac{m \Delta v}{\Delta t}=\frac{(850 \mathrm{~kg})(0.30 \mathrm{~m} / \mathrm{s})}{4.0 \mathrm{~s}}=64 \mathrm{~N}
$$

109. (a) The average force is the momentum change divided by the elapsed time.

$$
F_{\text {avg }}=\frac{\Delta p}{\Delta t}=\frac{m \Delta v}{\Delta t}=\frac{(1500 \mathrm{~kg})(0-45 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)}{0.15 \mathrm{~s}}=-1.25 \times 10^{5} \mathrm{~N} \approx-1.3 \times 10^{5} \mathrm{~N}
$$

The negative sign indicates direction - that the force is in the opposite direction to the original direction of motion.
(b) Use Newton's second law.

$$
F_{\text {avg }}=m a_{\text {avg }} \rightarrow a_{\text {avg }}=\frac{F_{\text {avg }}}{m}=\frac{-1.25 \times 10^{5} \mathrm{~N}}{1500 \mathrm{~kg}}=-83.33 \mathrm{~m} / \mathrm{s}^{2} \approx-83 \mathrm{~m} / \mathrm{s}^{2}
$$

110. (a) In the reference frame of the Earth, the final speed of the Earth-asteroid system is essentially 0, because the mass of the Earth is so much greater than the mass of the asteroid. It is like throwing a ball of mud at the wall of a large building - the smaller mass stops, and the larger mass doesn't move appreciably. Thus all of the asteroid's original kinetic energy can be released as destructive energy.

$$
K_{\text {orig }}=\frac{1}{2} m v_{0}^{2}=\frac{1}{2}\left[\left(3200 \mathrm{~kg} / \mathrm{m}^{3}\right) \frac{4}{3} \pi\left(1.0 \times 10^{3} \mathrm{~m}\right)^{3}\right]\left(1.5 \times 10^{4} \mathrm{~m} / \mathrm{s}\right)^{2}=1.507 \times 10^{21} \mathrm{~J}
$$

$$
\approx 1.5 \times 10^{21} \mathrm{~J}
$$

(b) $1.507 \times 10^{21} \mathrm{~J}\left(\frac{1 \text { bomb }}{4.0 \times 10^{16} \mathrm{~J}}\right)=38,000$ bombs
111. We apply Eq. 9-19b, with no external forces. We also assume that the motion is all in one dimension.

$$
\begin{aligned}
& M \frac{d \overrightarrow{\mathbf{v}}}{d t}=\overrightarrow{\mathbf{v}}_{\text {rel }} \frac{d M}{d t} \rightarrow M d v=v_{\text {rel }} d M \rightarrow \frac{1}{v_{\text {rel }}} d v=\frac{1}{M} d M \rightarrow \\
& \frac{1}{v_{\text {rel }}} \int_{0}^{v_{\text {forl }}} d v=\int_{M_{0}}^{M_{\text {farl }}} \frac{1}{M} d M \rightarrow \frac{v_{\text {final }}}{v_{\text {rel }}}=\ln \frac{M_{\text {final }}}{M_{0}} \rightarrow M_{\text {final }}=M_{0} e^{v_{\text {firial }} / v_{\text {rel }}} \rightarrow \\
& M_{\text {ejected }}=M_{0}-M_{\text {final }}=M_{0}\left(1-e^{v_{\text {faim }} / v_{\text {rel }}}\right)=(210 \mathrm{~kg})\left(1-e^{2.0 /(-35)}\right)=11.66 \mathrm{~kg} \approx 12 \mathrm{~kg}
\end{aligned}
$$

112. (a) We take the CM of the system as the origin of coordinates. Then at any time, we consider the $x$ axis to be along the line connecting the star and the planet. Use the definition of center of mass:

$$
x_{\mathrm{CM}}=\frac{m_{\mathrm{A}} r_{\mathrm{A}}+m_{\mathrm{B}}\left(-r_{\mathrm{B}}\right)}{m_{\mathrm{A}}+m_{\mathrm{B}}}=0 \rightarrow r_{\mathrm{A}}=\frac{m_{\mathrm{B}}}{m_{\mathrm{A}}} r_{\mathrm{B}}
$$

(b)

$$
r_{\mathrm{A}}=\frac{m_{\mathrm{B}}}{m_{\mathrm{A}}} r_{\mathrm{B}}=\frac{1.0 \times 10^{-3} m_{\mathrm{A}}}{m_{\mathrm{A}}}\left(8.0 \times 10^{11} \mathrm{~m}\right)=8.0 \times 10^{8} \mathrm{~m}
$$


(c) The geometry of this situation is illustrated in the adjacent diagram. For small angles in
 radian measure, $\theta \approx \tan \theta \approx \sin \theta$.

$$
\theta \approx \tan \theta \approx \frac{2 r_{\mathrm{A}}}{d} \rightarrow d=\frac{2 r_{\mathrm{A}}}{\theta}=\frac{2\left(8.0 \times 10^{8} \mathrm{~m}\right)}{\left(\frac{1}{1000}\right)\left(\frac{1}{3600}\right) \frac{\pi}{180}}=3.30 \times 10^{17} \mathrm{~m}\left(\frac{1 \mathrm{ly}}{9.46 \times 10^{15} \mathrm{~m}}\right)=351 \mathrm{y}
$$

(d) We assume that stars are distributed uniformly, with an average interstellar distance of 4 ly. If we think about each star having a spherical "volume" associated with it, that volume would have a radius of 2 ly (half the distance to an adjacent star). Each star would have a volume of $\frac{4}{3} \pi r_{\text {star }}^{3}=\frac{4}{3} \pi(21 y)^{3}$. If wobble can be detected from a distance of 35 ly , the volume over which to
star
wobble can be detected is $\frac{4}{3} \pi r_{\text {dececable }}^{3}$ wobble $-\frac{4}{3} \pi(351 \mathrm{y})^{3}$.

$$
\left.\left.\# \text { stars }=\frac{\frac{4}{3} \pi r_{\text {detectable }}^{3}}{\substack{\text { wobble }}} \frac{(35 \mathrm{ly})^{3}}{\frac{4}{3} \pi r_{\text {star }}^{3}} \begin{array}{c}
\text { to star }
\end{array}\right) ~(21 \mathrm{y})^{3}\right) ~ 5400 \mathrm{stars}
$$

113. This is a totally inelastic collision in one dimension. Call the direction of the Asteroid A the positive direction.

$$
p_{\text {initial }}=p_{\text {final }} \rightarrow m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} v_{\mathrm{B}}=\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) v^{\prime} \rightarrow
$$

$$
\begin{aligned}
v^{\prime} & =\frac{m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} v_{\mathrm{B}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}=\frac{\left(7.5 \times 10^{12} \mathrm{~kg}\right)(3.3 \mathrm{~km} / \mathrm{s})+\left(1.45 \times 10^{13} \mathrm{~kg}\right)(-1.4 \mathrm{~km} / \mathrm{s})}{7.5 \times 10^{12} \mathrm{~kg}+1.45 \times 10^{13} \mathrm{~kg}} \\
& =0.2 \mathrm{~km} / \mathrm{s}, \text { in the original direction of asteroid A }
\end{aligned}
$$

114. (a) The elastic, stationary-target one-dimensional collision is analyzed in Example 9-8. We can use the relationships derived there to find the final velocity of the target.

$$
v_{\mathrm{B}}^{\prime}=v_{\mathrm{A}}\left(\frac{2 m_{\mathrm{A}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}\right)=\frac{2 m_{\mathrm{A}} v_{\mathrm{A}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}=\frac{2 v_{\mathrm{A}}}{1+m_{\mathrm{B}} / m_{\mathrm{A}}}
$$

Note that since $m_{\mathrm{B}}<m_{\mathrm{A}}, v_{\mathrm{B}}^{\prime}>v_{\mathrm{A}}$.
(b) In this scenario, the first collision would follow the same calculation as above, giving $v_{\mathrm{C}}^{\prime}$. Then particle C is incident on particle B , and using the same calculation as above, would give $v_{\mathrm{B}}^{\prime}$.

$$
\begin{aligned}
& v_{\mathrm{C}}^{\prime}=v_{\mathrm{A}}\left(\frac{2 m_{\mathrm{A}}}{m_{\mathrm{A}}+m_{\mathrm{C}}}\right) \\
& v_{\mathrm{B}}^{\prime}=v_{\mathrm{C}}^{\prime}\left(\frac{2 m_{\mathrm{C}}}{m_{\mathrm{B}}+m_{\mathrm{C}}}\right)=v_{\mathrm{A}}\left(\frac{2 m_{\mathrm{A}}}{m_{\mathrm{A}}+m_{\mathrm{C}}}\right)\left(\frac{2 m_{\mathrm{C}}}{m_{\mathrm{B}}+m_{\mathrm{C}}}\right)=4 v_{\mathrm{A}} \frac{m_{\mathrm{A}} m_{\mathrm{C}}}{\left(m_{\mathrm{A}}+m_{\mathrm{C}}\right)\left(m_{\mathrm{B}}+m_{\mathrm{C}}\right)}
\end{aligned}
$$

(c) To find the value of $m_{\mathrm{C}}$ that gives the maximum $v_{\mathrm{B}}^{\prime}$, set $\frac{d v_{\mathrm{B}}^{\prime}}{d m_{\mathrm{C}}}=0$ and solve for $m_{\mathrm{C}}$.

$$
\begin{aligned}
& \frac{d v_{\mathrm{B}}^{\prime}}{d m_{\mathrm{C}}}=4 v_{\mathrm{A}} m_{\mathrm{A}} \frac{\left[\left(m_{\mathrm{A}}+m_{\mathrm{C}}\right)\left(m_{\mathrm{B}}+m_{\mathrm{C}}\right)-m_{\mathrm{C}}\left(m_{\mathrm{A}}+m_{\mathrm{B}}+2 m_{\mathrm{C}}\right)\right]}{\left(m_{\mathrm{A}}+m_{\mathrm{C}}\right)^{2}\left(m_{\mathrm{B}}+m_{\mathrm{C}}\right)^{2}}=0 \rightarrow \\
& \left(m_{\mathrm{A}}+m_{\mathrm{C}}\right)\left(m_{\mathrm{B}}+m_{\mathrm{C}}\right)-m_{\mathrm{C}}\left(m_{\mathrm{A}}+m_{\mathrm{B}}+2 m_{\mathrm{C}}\right)=0 \rightarrow \\
& m_{\mathrm{A}} m_{\mathrm{B}}-m_{\mathrm{C}}^{2}=0 \rightarrow m_{\mathrm{C}}=\sqrt{m_{\mathrm{A}} m_{\mathrm{B}}}
\end{aligned}
$$

(d) The graph is shown here. The numeric maximum of the graph has $v_{\mathrm{B}}^{\prime}=4.5 \mathrm{~m} / \mathrm{s}$ and occurs at $m_{\mathrm{C}}=6.0 \mathrm{~kg}$. According to the analysis from part (c), the value of $m_{\mathrm{C}}=\sqrt{m_{\mathrm{A}} m_{\mathrm{B}}}=$
$\sqrt{(18.0 \mathrm{~kg})(2.0 \mathrm{~kg})}=6.0 \mathrm{~kg}$, and gives a speed of $v_{\mathrm{B}}^{\prime}=\frac{4 v_{\mathrm{A}} m_{\mathrm{A}} m_{\mathrm{C}}}{\left(m_{\mathrm{A}}+m_{\mathrm{C}}\right)\left(m_{\mathrm{B}}+m_{\mathrm{C}}\right)}$
$=\frac{4(2.0 \mathrm{~m} / \mathrm{s})(18.0 \mathrm{~kg})(6.0 \mathrm{~kg})}{(24.0 \mathrm{~kg})(8.0 \mathrm{~kg})}$.
$=4.5 \mathrm{~m} / \mathrm{s}$.
The numeric results agree with the analytical results. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH09.XLS," on tab "Problem 9. 114 d ."


## CHAPTER 10: Rotational Motion

## Responses to Questions

1. The odometer will register a distance greater than the distance actually traveled. The odometer counts the number of revolutions and the calibration gives the distance traveled per revolution ( $2 \pi r$ ). The smaller tire will have a smaller radius, and a smaller actual distance traveled per revolution.
2. A point on the rim of a disk rotating with constant angular velocity has no tangential acceleration since the tangential speed is constant. It does have radial acceleration. Although the point's speed is not changing, its velocity is, since the velocity vector is changing direction. The point has a centripetal acceleration, which is directed radially inward. If the disk's angular velocity increases uniformly, the point on the rim will have both radial and tangential acceleration, since it is both moving in a circle and speeding up. The magnitude of the radial component of acceleration will increase in the case of the disk with a uniformly increasing angular velocity, although the tangential component will be constant. In the case of the disk rotating with constant angular velocity, neither component of linear acceleration will change.
3. No. The relationship between the parts of a non-rigid object can change. Different parts of the object may have different values of $\omega$.
4. Yes. The magnitude of the torque exerted depends not only on the magnitude of the force but also on the lever arm, which involves both the distance from the force to the axis of rotation and the angle at which the force is applied. A small force applied with a large lever arm could create a greater torque than a larger force with a smaller lever arm.
5. When you do a sit-up, you are rotating your trunk about a horizontal axis through your hips. When your hands are behind your head, your moment of inertia is larger than when your hands are stretched out in front of you. The sit-up with your hands behind your head will require more torque, and therefore will be "harder" to do.
6. Running involves rotating the leg about the point where it is attached to the rest of the body. Therefore, running fast requires the ability to change the leg's rotation easily. The smaller the moment of inertia of an object, the smaller the resistance to a change in its rotational motion. The closer the mass is to the axis of rotation, the smaller the moment of inertia. Concentrating flesh and muscle high and close to the body minimizes the moment of inertia and increases the angular acceleration possible for a given torque, improving the ability to run fast.

No. If two equal and opposite forces act on an object, the net force will be zero. If the forces are not co-linear, the two forces will produce a torque. No. If an unbalanced force acts through the axis of rotation, there will be a net force on the object, but no net torque.
8. The speed of the ball will be the same on both inclines. At the top of the incline, the ball has gravitational potential energy. This energy becomes converted to translational and rotational kinetic energy as the ball rolls down the incline. Since the inclines have the same height, the ball will have the same initial potential energy and therefore the same final kinetic energy and the same speed in both cases.
9. Roll the spheres down an incline. The hollow sphere will have a great moment of inertia and will take longer to reach the bottom of the incline.
10. The two spheres will reach the bottom at the same time with the same speed. The larger, more massive sphere will have the greater total kinetic energy at the bottom, since the total kinetic energy can be stated in terms of mass and speed.
11. A tightrope walker carries a long, narrow beam in order to increase his or her moment of inertia, making rotation (and falling off the wire) more difficult. The greater moment of inertia increases the resistance to change in angular motion, giving the walker more time to compensate for small shifts in position.
12. The moment of inertia of a solid sphere is given by $\frac{2}{5} M R^{2}$ and that of a solid cylinder is given by $\frac{1}{2} M R^{2}$. The solid sphere, with a smaller moment of inertia and therefore a smaller resistance to change in rotational motion, will reach the bottom of the incline first and have the greatest speed. Since both objects begin at the same height and have the same mass, they have the same initial potential energy. Since the potential energy is completely converted to kinetic energy at the bottom of the incline, the two objects will have the same total kinetic energy. However, the cylinder will have a greater rotational kinetic energy because its greater moment of inertia more than compensates for its lower velocity. At the bottom, $v_{\text {sphere }}=\sqrt{\frac{10}{7} g h}$ and $v_{\text {cylinder }}=\sqrt{\frac{4}{3} g h}$. Since rotational kinetic energy is $K_{\text {rot }}=\frac{1}{2} I \omega^{2}$, then $\underset{\substack{\text { rot } \\ \text { sphere }}}{ }=\frac{2}{7} m g h$ and $K_{\substack{\text { rot } \\ \text { cylinder }}}=\frac{1}{3} m g h$.
13. The moment of inertia will be least about an axis parallel to the spine of the book, passing through the center of the book. For this choice, the mass distribution for the book will be closest to the axis.
14. Larger. The moment of inertia depends on the distribution of mass. Imagine the disk as a collection of many little bits of mass. Moving the axis of rotation to the edge of the disk increases the average distance of the bits of mass to the axis, and therefore increases the moment of inertia. (See the Parallel Axis theorem.)
15. If the angular velocity vector of a wheel on an axle points west, the wheel is rotating such that the linear velocity vector of a point at the top of the wheel points north. If the angular acceleration vector points east (opposite the angular velocity vector), then the wheel is slowing down and the linear acceleration vector for the point on the top of the wheel points south. The angular speed of the wheel is decreasing.

## Solutions to Problems

(a) $\left(45.0^{\circ}\right)\left(2 \pi \mathrm{rad} / 360^{\circ}\right)=\pi / 4 \mathrm{rad}=0.785 \mathrm{rad}$
(b) $\left(60.0^{\circ}\right)\left(2 \pi \mathrm{rad} / 360^{\circ}\right)=\pi / 3 \mathrm{rad}=1.05 \mathrm{rad}$
(c) $\left(90.0^{\circ}\right)\left(2 \pi \mathrm{rad} / 360^{\circ}\right)=\pi / 2 \mathrm{rad}=1.57 \mathrm{rad}$
(d) $\left(360.0^{\circ}\right)\left(2 \pi \mathrm{rad} / 360^{\circ}\right)=2 \pi \mathrm{rad}=6.283 \mathrm{rad}$
(e) $\left(445^{\circ}\right)\left(2 \pi \mathrm{rad} / 360^{\circ}\right)=89 \pi / 36 \mathrm{rad}=7.77 \mathrm{rad}$
2. The subtended angle (in radians) is the diameter of the Sun divided by the Earth - Sun distance.

$$
\begin{aligned}
& \theta=\frac{\text { diameter of Sun }}{r_{\text {Earth-Sun }}} \rightarrow \\
& \text { radius of Sun }=\frac{1}{2} \theta r_{\text {Earth-Sun }}=\frac{1}{2}\left(0.5^{\circ}\right)\left(\frac{\pi \mathrm{rad}}{180^{\circ}}\right)\left(1.5 \times 10^{11} \mathrm{~m}\right)=6.545 \times 10^{8} \mathrm{~m} \approx 7 \times 10^{8} \mathrm{~m}
\end{aligned}
$$

3. We find the diameter of the spot from the definition of radian angle measure.

$$
\theta=\frac{\text { diameter }}{r_{\text {Earth-Moon }}} \rightarrow \text { diameter }=\theta r_{\text {Earth-Moon }}=\left(1.4 \times 10^{-5} \mathrm{rad}\right)\left(3.8 \times 10^{8} \mathrm{~m}\right)=5300 \mathrm{~m}
$$

4. The initial angular velocity is $\omega_{o}=\left(6500 \frac{\mathrm{rev}}{\mathrm{min}}\right)\left(\frac{2 \pi \mathrm{rad}}{1 \mathrm{rev}}\right)\left(\frac{1 \mathrm{~min}}{60 \mathrm{sec}}\right)=681 \mathrm{rad} / \mathrm{s}$. Use the definition of angular acceleration.

$$
\alpha=\frac{\Delta \omega}{\Delta t}=\frac{0-681 \mathrm{rad} / \mathrm{s}}{4.0 \mathrm{~s}}=-170 \mathrm{rad} / \mathrm{s}^{2}
$$

5. (a) $\omega=\left(\frac{2500 \mathrm{rev}}{1 \mathrm{~min}}\right)\left(\frac{2 \pi \mathrm{rad}}{1 \mathrm{rev}}\right)\left(\frac{1 \mathrm{~min}}{60 \mathrm{~s}}\right)=261.8 \mathrm{rad} / \mathrm{sec} \approx 260 \mathrm{rad} / \mathrm{sec}$
(b) $v=\omega r=(261.8 \mathrm{rad} / \mathrm{sec})(0.175 \mathrm{~m})=46 \mathrm{~m} / \mathrm{s}$

$$
a_{\mathrm{R}}=\omega^{2} r=(261.8 \mathrm{rad} / \mathrm{sec})^{2}(0.175 \mathrm{~m})=1.2 \times 10^{4} \mathrm{~m} / \mathrm{s}^{2}
$$

6. In each revolution, the wheel moves forward a distance equal to its circumference, $\pi d$.

$$
\Delta x=N_{\mathrm{rev}}(\pi d) \rightarrow N=\frac{\Delta x}{\pi d}=\frac{7200 \mathrm{~m}}{\pi(0.68 \mathrm{~m})}=3400 \mathrm{rev}
$$

7. The angular velocity is expressed in radians per second. The second hand makes 1 revolution every 60 seconds, the minute hand makes 1 revolution every 60 minutes, and the hour hand makes 1 revolution every 12 hours.
(a) Second hand: $\omega=\left(\frac{1 \mathrm{rev}}{60 \mathrm{sec}}\right)\left(\frac{2 \pi \mathrm{rad}}{1 \mathrm{rev}}\right)=\frac{\pi}{30} \mathrm{rad} / \mathrm{sec} \approx 1.05 \times 10^{-1} \frac{\mathrm{rad}}{\mathrm{sec}}$
(b) Minute hand: $\omega=\left(\frac{1 \mathrm{rev}}{60 \mathrm{~min}}\right)\left(\frac{2 \pi \mathrm{rad}}{1 \mathrm{rev}}\right)\left(\frac{1 \mathrm{~min}}{60 \mathrm{~s}}\right)=\frac{\pi}{1800} \frac{\mathrm{rad}}{\mathrm{sec}} \approx 1.75 \times 10^{-3} \frac{\mathrm{rad}}{\mathrm{sec}}$
(c) Hour hand: $\omega=\left(\frac{1 \mathrm{rev}}{12 \mathrm{~h}}\right)\left(\frac{2 \pi \mathrm{rad}}{1 \mathrm{rev}}\right)\left(\frac{1 \mathrm{~h}}{3600 \mathrm{~s}}\right)=\frac{\pi}{21,600} \frac{\mathrm{rad}}{\mathrm{sec}} \approx 1.45 \times 10^{-4} \frac{\mathrm{rad}}{\mathrm{sec}}$
(d) The angular acceleration in each case is 0 , since the angular velocity is constant.
8. The angular speed of the merry-go-round is $2 \pi \mathrm{rad} / 4.0 \mathrm{~s}=1.57 \mathrm{rad} / \mathrm{s}$.
(a) $v=\omega r=(1.57 \mathrm{rad} / \mathrm{sec})(1.2 \mathrm{~m})=1.9 \mathrm{~m} / \mathrm{s}$
(b) The acceleration is radial. There is no tangential acceleration.

$$
a_{\mathrm{R}}=\omega^{2} r=(1.57 \mathrm{rad} / \mathrm{sec})^{2}(1.2 \mathrm{~m})=3.0 \mathrm{~m} / \mathrm{s}^{2} \text { towards the center }
$$

9. Each location will have the same angular velocity (1 revolution per day), but the radius of the circular path varies with the location. From the diagram, we see $r=R \cos \theta$, where $R$ is the radius of the Earth, and $r$ is the radius at latitude $\theta$.
(a) $v=\omega r=\frac{2 \pi}{T} r=\left(\frac{2 \pi \mathrm{rad}}{1 \text { day }}\right)\left(\frac{1 \text { day }}{86400 \mathrm{~s}}\right)\left(6.38 \times 10^{6} \mathrm{~m}\right)=464 \mathrm{~m} / \mathrm{s}$

(b) $v=\omega r=\frac{2 \pi}{T} r=\left(\frac{2 \pi \mathrm{rad}}{1 \text { day }}\right)\left(\frac{1 \text { day }}{86400 \mathrm{~s}}\right)\left(6.38 \times 10^{6} \mathrm{~m}\right) \cos 66.5^{\circ}=185 \mathrm{~m} / \mathrm{s}$
(c) $v=\omega r=\frac{2 \pi}{T} r=\left(\frac{2 \pi \mathrm{rad}}{1 \text { day }}\right)\left(\frac{1 \text { day }}{86400 \mathrm{~s}}\right)\left(6.38 \times 10^{6} \mathrm{~m}\right) \cos 45.0^{\circ}=328 \mathrm{~m} / \mathrm{s}$
10. (a) The Earth makes one orbit around the Sun in one year.

$$
\omega_{\text {orbit }}=\frac{\Delta \theta}{\Delta t}=\left(\frac{2 \pi \mathrm{rad}}{1 \text { year }}\right)\left(\frac{1 \text { year }}{3.16 \times 10^{7} \mathrm{~s}}\right)=1.99 \times 10^{-7} \mathrm{rad} / \mathrm{s}
$$

(b) The Earth makes one revolution about its axis in one day.

$$
\omega_{\text {rotation }}=\frac{\Delta \theta}{\Delta t}=\left(\frac{2 \pi \mathrm{rad}}{1 \text { day }}\right)\left(\frac{1 \text { day }}{86,400 \mathrm{~s}}\right)=7.27 \times 10^{-5} \mathrm{rad} / \mathrm{s}
$$

11. The centripetal acceleration is given by $a=\omega^{2} r$. Solve for the angular velocity.

$$
\omega=\sqrt{\frac{a}{r}}=\sqrt{\frac{(100,000)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{0.070 \mathrm{~m}}}=3741 \frac{\mathrm{rad}}{\mathrm{~s}}\left(\frac{1 \mathrm{rev}}{2 \pi \mathrm{rad}}\right)\left(\frac{60 \mathrm{~s}}{1 \mathrm{~min}}\right)=3.6 \times 10^{4} \mathrm{rpm}
$$

12. Convert the rpm values to angular velocities.

$$
\begin{aligned}
& \omega_{0}=\left(130 \frac{\mathrm{rev}}{\mathrm{~min}}\right)\left(\frac{2 \pi \mathrm{rad}}{1 \mathrm{rev}}\right)\left(\frac{1 \mathrm{~min}}{60 \mathrm{sec}}\right)=13.6 \mathrm{rad} / \mathrm{s} \\
& \omega=\left(280 \frac{\mathrm{rev}}{\mathrm{~min}}\right)\left(\frac{2 \pi \mathrm{rad}}{1 \mathrm{rev}}\right)\left(\frac{1 \mathrm{~min}}{60 \mathrm{sec}}\right)=29.3 \mathrm{rad} / \mathrm{s}
\end{aligned}
$$

(a) The angular acceleration is found from Eq. 10-3a.

$$
\alpha=\frac{\omega-\omega_{0}}{t}=\frac{29.3 \mathrm{rad} / \mathrm{s}-13.6 \mathrm{rad} / \mathrm{s}}{4.0 \mathrm{~s}}=3.93 \mathrm{rad} / \mathrm{s}^{2} \approx 3.9 \mathrm{rad} / \mathrm{s}^{2}
$$

(b) To find the components of the acceleration, the instantaneous angular velocity is needed.

$$
\omega=\omega_{0}+\alpha t=13.6 \mathrm{rad} / \mathrm{s}+\left(3.93 \mathrm{rad} / \mathrm{s}^{2}\right)(2.0 \mathrm{~s})=21.5 \mathrm{rad} / \mathrm{s}
$$

The instantaneous radial acceleration is given by $a_{\mathrm{R}}=\omega^{2} r$.

$$
a_{\mathrm{R}}=\omega^{2} r=(21.5 \mathrm{rad} / \mathrm{s})^{2}(0.35 \mathrm{~m})=160 \mathrm{~m} / \mathrm{s}^{2}
$$

The tangential acceleration is given by $a_{\mathrm{tan}}=\alpha r$.

$$
a_{\mathrm{tan}}=\alpha r=\left(3.93 \mathrm{rad} / \mathrm{s}^{2}\right)(0.35 \mathrm{~m})=1.4 \mathrm{~m} / \mathrm{s}^{2}
$$

13. (a) The angular rotation can be found from Eq. 10-3a. The initial angular frequency is 0 and the final frequency is 1 rpm .

$$
\alpha=\frac{\omega-\omega_{0}}{t}=\frac{\left(1.0 \frac{\mathrm{rev}}{\mathrm{~min}}\right)\left(\frac{2 \pi \mathrm{rad}}{1 \mathrm{rev}}\right)\left(\frac{1.0 \mathrm{~min}}{60 \mathrm{~s}}\right)-0}{720 \mathrm{~s}}=1.454 \times 10^{-4} \mathrm{rad} / \mathrm{s}^{2} \approx 1.5 \times 10^{-4} \mathrm{rad} / \mathrm{s}^{2}
$$

(b) After $7.0 \mathrm{~min}(420 \mathrm{~s})$, the angular speed is as follows.

$$
\omega=\omega_{0}+\alpha t=0+\left(1.454 \times 10^{-4} \mathrm{rad} / \mathrm{s}^{2}\right)(420 \mathrm{~s})=6.107 \times 10^{-2} \mathrm{rad} / \mathrm{s}
$$

Find the components of the acceleration of a point on the outer skin from the angular speed and the radius.

$$
\begin{aligned}
& a_{\mathrm{tan}}=\alpha R=\left(1.454 \times 10^{-4} \mathrm{rad} / \mathrm{s}^{2}\right)(4.25 \mathrm{~m})=6.2 \times 10^{-4} \mathrm{~m} / \mathrm{s}^{2} \\
& a_{\mathrm{rad}}=\omega^{2} R=\left(6.107 \times 10^{-2} \mathrm{rad} / \mathrm{s}\right)^{2}(4.25 \mathrm{~m})=1.6 \times 10^{-2} \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

14. The tangential speed of the turntable must be equal to the tangential speed of the roller, if there is no slippage.

$$
v_{1}=v_{2} \rightarrow \omega_{1} R_{1}=\omega_{2} R_{2} \rightarrow \omega_{1} / \omega_{2}=R_{2} / R_{1}
$$

15. (a) The direction of $\omega_{1}$ is along the axle of the wheel, to the left. That is the $-\hat{\mathbf{i}}$ direction. The direction of $\omega_{2}$ is also along its axis of rotation, so it is straight up. That is the $+\hat{\mathbf{k}}$ direction. That is also the angular velocity of the axis of the wheel.
(b) At the instant shown in the textbook, we have the vector relationship as shown in the diagram.

$$
\begin{aligned}
& \omega=\sqrt{\omega_{1}^{2}+\omega_{2}^{2}}=\sqrt{(44.0 \mathrm{rad} / \mathrm{s})^{2}+(35.0 \mathrm{rad} / \mathrm{s})^{2}}=56.2 \mathrm{rad} / \mathrm{s} \\
& \theta=\tan ^{-1} \frac{\omega_{2}}{\omega_{2}}=\tan ^{-1} \frac{35.0}{44.0}=38.5^{\circ}
\end{aligned}
$$


(c) Angular acceleration is given by $\overrightarrow{\boldsymbol{\alpha}}=\frac{d \overrightarrow{\boldsymbol{\omega}}}{d t}$. Since $\overrightarrow{\boldsymbol{\omega}}=\overrightarrow{\boldsymbol{\omega}}_{1}+\overrightarrow{\boldsymbol{\omega}}_{2}$, and $\overrightarrow{\boldsymbol{\omega}}_{2}$ is a constant $35.0 \hat{\mathbf{k}} \mathrm{rad} / \mathrm{s}, \quad \overrightarrow{\boldsymbol{\alpha}}=\frac{d \overrightarrow{\boldsymbol{\omega}}_{1}}{d t} . \quad \overrightarrow{\boldsymbol{\omega}}_{1}$ is rotating counterclockwise about the $z$ axis with the angular velocity of $\omega_{2}$, and so if the figure is at $t=0$, then $\overrightarrow{\boldsymbol{\omega}}_{1}=\omega_{1}\left(-\cos \omega_{2} t \hat{\mathbf{i}}-\sin \omega_{2} t \hat{\mathbf{j}}\right)$.

$$
\begin{aligned}
& \overrightarrow{\boldsymbol{\alpha}}=\frac{d \overrightarrow{\boldsymbol{\omega}}}{d t}=\frac{d\left(\overrightarrow{\boldsymbol{\omega}}_{1}+\overrightarrow{\boldsymbol{\omega}}_{2}\right)}{d t}=\frac{d \overrightarrow{\boldsymbol{\omega}}_{1}}{d t}=\frac{d\left[\omega_{1}\left(-\cos \omega_{2} t \hat{\mathbf{i}}-\sin \omega_{2} \hat{\mathbf{j}}\right)\right]}{d t}=\omega_{1} \omega_{2}\left(\sin \omega_{2} t \hat{\mathbf{i}}-\cos \omega_{2} t \hat{\mathbf{j}}\right) \\
& \overrightarrow{\boldsymbol{\alpha}}(t=0)=\omega_{1} \omega_{2}(-\hat{\mathbf{j}})=-(44.0 \mathrm{rad} / \mathrm{s})(35.0 \mathrm{rad} / \mathrm{s}) \hat{\mathbf{j}}=-1540 \mathrm{rad} / \mathrm{s}^{2} \hat{\mathbf{j}}
\end{aligned}
$$

16. (a) For constant angular acceleration:

$$
\begin{aligned}
\alpha & =\frac{\omega-\omega_{o}}{t}=\frac{1200 \mathrm{rev} / \mathrm{min}-3500 \mathrm{rev} / \mathrm{min}}{2.5 \mathrm{~s}}=\frac{-2300 \mathrm{rev} / \mathrm{min}}{2.5 \mathrm{~s}}\left(\frac{2 \pi \mathrm{rad}}{1 \mathrm{rev}}\right)\left(\frac{1 \mathrm{~min}}{60 \mathrm{~s}}\right) \\
& =-96.34 \mathrm{rad} / \mathrm{s}^{2} \approx-96 \mathrm{rad} / \mathrm{s}^{2}
\end{aligned}
$$

(b) For the angular displacement, given constant angular acceleration:

$$
\theta=\frac{1}{2}\left(\omega_{o}+\omega\right) t=\frac{1}{2}(3500 \mathrm{rev} / \mathrm{min}+1200 \mathrm{rev} / \mathrm{min})(2.5 \mathrm{~s})\left(\frac{1 \mathrm{~min}}{60 \mathrm{~s}}\right)=98 \mathrm{rev}
$$

17. The angular displacement can be found from Eq. 10-9d.

$$
\theta=\bar{\omega} t=\frac{1}{2}\left(\omega_{o}+\omega\right) t=\frac{1}{2}(0+15000 \mathrm{rev} / \mathrm{min})(220 \mathrm{~s})(1 \mathrm{~min} / 60 \mathrm{~s})=2.8 \times 10^{4} \mathrm{rev}
$$

18. (a) The angular acceleration can be found from Eq. $10-9 \mathrm{~b}$ with $\omega_{o}=0$.

$$
\alpha=\frac{2 \theta}{t^{2}}=\frac{2(20 \mathrm{rev})}{(1.0 \mathrm{~min})^{2}}=4.0 \times 10^{1} \mathrm{rev} / \mathrm{min}^{2}
$$

(b) The final angular speed can be found from $\theta=\frac{1}{2}\left(\omega_{o}+\omega\right) t$, with $\omega_{o}=0$.

$$
\omega=\frac{2 \theta}{t}-\omega_{o}=\frac{2(20 \mathrm{rev})}{1.0 \mathrm{~min}}=4.0 \times 10^{1} \mathrm{rpm}
$$

19. (a) The angular acceleration can be found from Eq. 10-9c.

$$
\alpha=\frac{\omega^{2}-\omega_{o}^{2}}{2 \theta}=\frac{0-(850 \mathrm{rev} / \mathrm{min})^{2}}{2(1350 \mathrm{rev})}=\left(-267.6 \frac{\mathrm{rev}}{\mathrm{~min}^{2}}\right)\left(\frac{2 \pi \mathrm{rad}}{1 \mathrm{rev}}\right)\left(\frac{1 \mathrm{~min}}{60 \mathrm{~s}}\right)^{2}=-0.47 \frac{\mathrm{rad}}{\mathrm{~s}^{2}}
$$

(b) The time to come to a stop can be found from $\theta=\frac{1}{2}\left(\omega_{o}+\omega\right) t$.

$$
t=\frac{2 \theta}{\omega_{o}+\omega}=\frac{2(1350 \mathrm{rev})}{850 \mathrm{rev} / \mathrm{min}}\left(\frac{60 \mathrm{~s}}{1 \mathrm{~min}}\right)=190 \mathrm{~s}
$$

20. We start with $\alpha=\frac{d \omega}{d t}$. We also assume that $\alpha$ is constant, that the angular speed at time $t=0$ is $\omega_{0}$, and that the angular displacement at time $t=0$ is 0 .

$$
\begin{aligned}
& \alpha=\frac{d \omega}{d t} \rightarrow d \omega=\alpha d t \rightarrow \int_{\omega_{0}}^{\omega} d \omega=\int_{0}^{t} \alpha d t \rightarrow \omega-\omega_{0}=\alpha t \rightarrow \omega=\omega_{0}+\alpha t \\
& \omega=\omega_{0}+\alpha t=\frac{d \theta}{d t} \rightarrow d \theta=\left(\omega_{0}+\alpha t\right) d t \rightarrow \int_{0}^{\theta} d \theta=\int_{0}^{t}\left(\omega_{0}+\alpha t\right) d t \rightarrow \theta=\omega_{0} t+\frac{1}{2} \alpha t^{2}
\end{aligned}
$$

21. Since there is no slipping between the wheels, the tangential component of the linear acceleration of each wheel must be the same.
(a) $\underset{\substack{\text { tan } \\ \text { small }}}{a_{\text {lat }}=a_{\text {tan }}^{\text {large }}} \rightarrow \alpha_{\text {small }} r_{\text {small }}=\alpha_{\text {large }} r_{\text {large }} \rightarrow$

$$
\alpha_{\text {large }}=\alpha_{\text {smal }} \frac{r_{\text {small }}}{r_{\text {large }}}=\left(7.2 \mathrm{rad} / \mathrm{s}^{2}\right)\left(\frac{2.0 \mathrm{~cm}}{21.0 \mathrm{~cm}}\right)=0.6857 \mathrm{rad} / \mathrm{s}^{2} \approx 0.69 \mathrm{rad} / \mathrm{s}^{2}
$$

(b) Assume the pottery wheel starts from rest. Convert the speed to an angular speed, and then use Eq. 10-9a.

$$
\begin{aligned}
& \omega=\left(65 \frac{\mathrm{rev}}{\mathrm{~min}}\right)\left(\frac{2 \pi \mathrm{rad}}{1 \mathrm{rev}}\right)\left(\frac{1 \mathrm{~min}}{60 \mathrm{~s}}\right)=6.807 \mathrm{rad} / \mathrm{s} \\
& \omega=\omega_{0}+\alpha t \rightarrow t=\frac{\omega-\omega_{0}}{\alpha}=\frac{6.807 \mathrm{rad} / \mathrm{s}}{0.6857 \mathrm{rad} / \mathrm{s}^{2}}=9.9 \mathrm{~s}
\end{aligned}
$$

22. We are given that $\theta=8.5 t-15.0 t^{2}+1.6 t^{4}$.
(a) $\omega=\frac{d \theta}{d t}=8.5-30.0 t+6.4 t^{3}$, where $\omega$ is in $\mathrm{rad} / \mathrm{sec}$ and $t$ is in sec.
(b) $\alpha=\frac{d \omega}{d t}=-30.0+19.2 t^{2}$, where $\alpha$ is in $\mathrm{rad} / \sec ^{2}$ and $t$ is in sec.
(c) $\omega(3.0)=8.5-30.0(3.0)+6.4(3.0)^{3}=91 \mathrm{rad} / \mathrm{s}$
$\alpha(3.0)=-30.0+19.2(3.0)^{2}=140 \mathrm{rad} / \mathrm{s}^{2}$
(d) The average angular velocity is the angular displacement divided by the elapsed time.

$$
\begin{aligned}
\omega_{\text {avg }} & =\frac{\Delta \theta}{\Delta t}=\frac{\theta(3.0)-\theta(2.0)}{3.0 \mathrm{~s}-2.0 \mathrm{~s}} \\
& =\frac{\left[8.5(3.0)-15.0(3.0)^{2}+1.6(3.0)^{4}\right]-\left[8.5(2.0)-15.0(2.0)^{2}+1.6(2.0)^{4}\right]}{1.0 \mathrm{~s}} \\
& =38 \mathrm{rad} / \mathrm{s}
\end{aligned}
$$

(e) The average angular acceleration is the change in angular velocity divided by the elapsed time.

$$
\begin{aligned}
\alpha_{\text {avg }} & =\frac{\Delta \omega}{\Delta t}=\frac{\omega(3.0)-\omega(2.0)}{3.0 \mathrm{~s}-2.0 \mathrm{~s}} \\
& =\frac{\left[8.5-30.0(3.0)+6.4(3.0)^{3}\right]-\left[8.5-30.0(2.0)+6.4(2.0)^{3}\right]}{1.0 \mathrm{~s}}=92 \mathrm{rad} / \mathrm{s}^{2}
\end{aligned}
$$

23. (a) The angular velocity is found by integrating the angular acceleration function.

$$
\alpha=\frac{d \omega}{d t} \rightarrow d \omega=\alpha d t \rightarrow \int_{0}^{\omega} d \omega=\int_{0}^{t} \alpha d t=\int_{0}^{t}\left(5.0 t^{2}-8.5 t\right) d t \rightarrow \omega=\frac{1}{3} 5.0 t^{3}-\frac{1}{2} 8.5 t^{2}
$$

(b) The angular position is found by integrating the angular velocity function.

$$
\begin{aligned}
& \omega=\frac{d \theta}{d t} \rightarrow d \theta=\omega d t \rightarrow \int_{0}^{\theta} d \theta=\int_{0}^{t} \omega d t=\int_{0}^{t}\left(\frac{1}{3} 5.0 t^{3}-\frac{1}{2} 8.5 t^{2}\right) d t \rightarrow \\
& \theta=\frac{1}{12} 5.0 t^{4}-\frac{1}{6} 8.5 t^{3}
\end{aligned}
$$

(c) $\omega(2.0 \mathrm{~s})=\frac{1}{3} 5.0(2.0)^{3}-\frac{1}{2} 8.5(2.0)^{2}=-3.7 \mathrm{rad} / \mathrm{s} \approx-4 \mathrm{rad} / \mathrm{s}$

$$
\theta(2.0 \mathrm{~s})=\frac{1}{12} 5.0(2.0)^{4}-\frac{1}{6} 8.5(2.0)^{3}=-4.67 \mathrm{rad} \approx-5 \mathrm{rad}
$$

24. (a) The maximum torque will be exerted by the force of her weight, pushing tangential to the circle in which the pedal moves.

$$
\tau=r_{\perp} F=r_{\perp} m g=(0.17 \mathrm{~m})(62 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=1.0 \times 10^{2} \mathrm{~m} \cdot \mathrm{~N}
$$

(b) She could exert more torque by pushing down harder with her legs, raising her center of mass. She could also pull upwards on the handle bars as she pedals, which will increase the downward force of her legs.
25. Each force is oriented so that it is perpendicular to its lever arm. Call counterclockwise torques positive. The torque due to the three applied forces is given by the following.

$$
\tau_{\substack{\text { applied } \\ \text { forces }}}=(28 \mathrm{~N})(0.24 \mathrm{~m})-(18 \mathrm{~N})(0.24 \mathrm{~m})-(35 \mathrm{~N})(0.12 \mathrm{~m})=-1.8 \mathrm{~m} \cdot \mathrm{~N}
$$

Since this torque is clockwise, we assume the wheel is rotating clockwise, and so the frictional torque is counterclockwise. Thus the net torque is as follows.

$$
\begin{aligned}
\tau_{\text {net }} & =(28 \mathrm{~N})(0.24 \mathrm{~m})-(18 \mathrm{~N})(0.24 \mathrm{~m})-(35 \mathrm{~N})(0.12 \mathrm{~m})+0.40 \mathrm{~m} \cdot \mathrm{~N}=-1.4 \mathrm{~m} \cdot \mathrm{~N} \\
& =1.4 \mathrm{~m} \cdot \mathrm{~N}, \text { clockwise }
\end{aligned}
$$

26. The torque is calculated by $\tau=r F \sin \theta$. See the diagram, from the top view.
(a) For the first case, $\theta=90^{\circ}$.

$$
\tau=r F \sin \theta=(0.96 \mathrm{~m})(32 \mathrm{~N}) \sin 90^{\circ}=31 \mathrm{~m} \cdot \mathrm{~N}
$$

(b) For the second case, $\theta=60.0^{\circ}$.

$$
\tau=r F \sin \theta=(0.96 \mathrm{~m})(32 \mathrm{~N}) \sin 60.0^{\circ}=27 \mathrm{~m} \cdot \mathrm{~N}
$$


27. There is a counterclockwise torque due to the force of gravity on the left block, and a clockwise torque due to the force of gravity on the right block. Call clockwise the positive direction.

$$
\sum \tau=m g \ell_{2}-m g \ell_{1}=m g\left(\ell_{2}-\ell_{1}\right), \text { clockwise }
$$

28. The lever arm to the point of application of the force is along the $x$ axis. Thus the perpendicular part of the force is the $y$ component. Use Eq. 10-10b.

$$
\tau=R F_{\perp}=(0.135 \mathrm{~m})(43.4 \mathrm{~N})=5.86 \mathrm{~m} \cdot \mathrm{~N}, \text { counterclockwise }
$$

29. The force required to produce the torque can be found from $\tau=r F \sin \theta$. The force is applied perpendicularly to the wrench, so $\theta=90^{\circ}$.

$$
F=\frac{\tau}{r}=\frac{75 \mathrm{~m} \cdot \mathrm{~N}}{0.28 \mathrm{~m}}=270 \mathrm{~N}
$$

The net torque still must be $75 \mathrm{~m} \cdot \mathrm{~N}$. This is produced by 6 forces, one at each of the 6 points. We assume that those forces are also perpendicular to their lever arms.

$$
\tau_{\text {net }}=\left(6 F_{\text {point }}\right) r_{\text {point }} \rightarrow F_{\text {point }}=\frac{\tau}{6 r}=\frac{75 \mathrm{~m} \cdot \mathrm{~N}}{6(0.0075 \mathrm{~m})}=1700 \mathrm{~N}
$$

30. For each torque, use Eq. 10-10c. Take counterclockwise torques to be positive.
(a) Each force has a lever arm of 1.0 m .

$$
\tau_{\text {about }}=-(1.0 \mathrm{~m})(56 \mathrm{~N}) \sin 30^{\circ}+(1.0 \mathrm{~m})(52 \mathrm{~N}) \sin 60^{\circ}=17 \mathrm{~m} \cdot \mathrm{~N}
$$

(b) The force at C has a lever arm of 1.0 m , and the force at the top has a lever arm of 2.0 m .

$$
\tau_{\text {about }}=-(2.0 \mathrm{~m})(56 \mathrm{~N}) \sin 30^{\circ}+(1.0 \mathrm{~m})(65 \mathrm{~N}) \sin 45^{\circ}=-10 \mathrm{~m} \cdot \mathrm{~N} \quad(2 \text { sig fig })
$$

The negative sign indicates a clockwise torque.
31. For a sphere rotating about an axis through its center, the moment of inertia is as follows.

$$
I=\frac{2}{5} M R^{2}=\frac{2}{5}(10.8 \mathrm{~kg})(0.648 \mathrm{~m})^{2}=1.81 \mathrm{~kg} \cdot \mathrm{~m}^{2}
$$

32. Since all of the significant mass is located at the same distance from the axis of rotation, the moment of inertia is given by $I=M R^{2}$.

$$
I=M R^{2}=(1.1 \mathrm{~kg})\left(\frac{1}{2}(0.67 \mathrm{~m})\right)^{2}=0.12 \mathrm{~kg} \cdot \mathrm{~m}^{2}
$$

The hub mass can be ignored because its distance from the axis of rotation is very small, and so it has a very small rotational inertia.
33. (a) The torque exerted by the frictional force is $\tau=r F_{\mathrm{fr}} \sin \theta$. The force of friction is assumed to be tangential to the clay, and so $\theta=90^{\circ}$.

$$
\tau_{\text {total }}=r F_{\mathrm{fr}} \sin \theta=\left(\frac{1}{2}(0.12 \mathrm{~m})\right)(1.5 \mathrm{~N}) \sin 90^{\circ}=0.090 \mathrm{~m} \cdot \mathrm{~N}
$$

(b) The time to stop is found from $\omega=\omega_{o}+\alpha t$, with a final angular velocity of 0 . The angular acceleration can be found from $\tau_{\text {total }}=I \alpha$.


The net torque (and angular acceleration) is negative since the object is slowing.

$$
t=\frac{\omega-\omega_{o}}{\alpha}=\frac{\omega-\omega_{o}}{\tau / I}=\frac{0-(1.6 \mathrm{rev} / \mathrm{s})(2 \pi \mathrm{rad} / \mathrm{rev})}{(-0.090 \mathrm{~m} \cdot \mathrm{~N}) /\left(0.11 \mathrm{~kg} \cdot \mathrm{~m}^{2}\right)}=12 \mathrm{~s}
$$

34. The oxygen molecule has a "dumbbell" geometry, rotating about the dashed line, as shown in the diagram. If the total mass is $M$, then each atom has a mass of $M / 2$. If the distance between them is $d$, then the distance from the axis of rotation to each
 atom is $d / 2$. Treat each atom as a particle for calculating the moment of inertia.

$$
\begin{aligned}
& I=(M / 2)(d / 2)^{2}+(M / 2)(d / 2)^{2}=2(M / 2)(d / 2)^{2}=\frac{1}{4} M d^{2} \rightarrow \\
& d=\sqrt{4 I / M}=\sqrt{4\left(1.9 \times 10^{-46} \mathrm{~kg} \cdot \mathrm{~m}^{2}\right) /\left(5.3 \times 10^{-26} \mathrm{~kg}\right)}=1.2 \times 10^{-10} \mathrm{~m}
\end{aligned}
$$

35. The torque can be calculated from $\tau=I \alpha$. The rotational inertia of a rod about its end is given by $I=\frac{1}{3} M L^{2}$.

$$
\tau=I \alpha=\frac{1}{3} M L^{2} \frac{\Delta \omega}{\Delta t}=\frac{1}{3}(2.2 \mathrm{~kg})(0.95 \mathrm{~m})^{2} \frac{(2.7 \mathrm{rev} / \mathrm{s})(2 \pi \mathrm{rad} / \mathrm{rev})}{0.20 \mathrm{~s}}=56 \mathrm{~m} \cdot \mathrm{~N}
$$

36. (a) The moment of inertia of a cylinder is $\frac{1}{2} M R^{2}$.

$$
I=\frac{1}{2} M R^{2}=\frac{1}{2}(0.380 \mathrm{~kg})(0.0850 \mathrm{~m})^{2}=1.373 \times 10^{-3} \mathrm{~kg} \cdot \mathrm{~m}^{2} \approx 1.37 \times 10^{-3} \mathrm{~kg} \cdot \mathrm{~m}^{2}
$$

(b) The wheel slows down "on its own" from 1500 rpm to rest in 55.0 s . This is used to calculate the frictional torque.

$$
\begin{aligned}
\tau_{\mathrm{fr}} & =I \alpha_{\mathrm{fr}}=I \frac{\Delta \omega}{\Delta t}=\left(1.373 \times 10^{-3} \mathrm{~kg} \cdot \mathrm{~m}^{2}\right) \frac{(0-1500 \mathrm{rev} / \mathrm{min})(2 \pi \mathrm{rad} / \mathrm{rev})(1 \mathrm{~min} / 60 \mathrm{~s})}{55.0 \mathrm{~s}} \\
& =-3.921 \times 10^{-3} \mathrm{~m} \cdot \mathrm{~N}
\end{aligned}
$$

The net torque causing the angular acceleration is the applied torque plus the (negative) frictional torque.

$$
\begin{aligned}
\sum \tau & =\tau_{\text {applied }}+\tau_{\mathrm{fr}}=I \alpha \rightarrow \tau_{\text {applied }}=I \alpha-\tau_{\mathrm{fr}}=I \frac{\Delta \omega}{\Delta t}-\tau_{\mathrm{fr}} \\
& =\left(1.373 \times 10^{-3} \mathrm{~kg} \cdot \mathrm{~m}^{2}\right) \frac{(1750 \mathrm{rev} / \mathrm{min})(2 \pi \mathrm{rad} / \mathrm{rev})(1 \mathrm{~min} / 60 \mathrm{~s})}{5.00 \mathrm{~s}}-\left(-3.921 \times 10^{-3} \mathrm{~m} \cdot \mathrm{~N}\right) \\
& =5.42 \times 10^{-2} \mathrm{~m} \cdot \mathrm{~N}
\end{aligned}
$$

37. (a) The small ball can be treated as a particle for calculating its moment of inertia.

$$
I=M R^{2}=(0.650 \mathrm{~kg})(1.2 \mathrm{~m})^{2}=0.94 \mathrm{~kg} \cdot \mathrm{~m}^{2}
$$

(b) To keep a constant angular velocity, the net torque must be zero, and so the torque needed is the same magnitude as the torque caused by friction.

$$
\sum \tau=\tau_{\text {applied }}-\tau_{\mathrm{ff}}=0 \rightarrow \tau_{\text {applied }}=\tau_{\mathrm{ff}}=F_{\mathrm{fr}} r=(0.020 \mathrm{~N})(1.2 \mathrm{~m})=2.4 \times 10^{-2} \mathrm{~m} \cdot \mathrm{~N}
$$

38. (a) The torque gives angular acceleration to the ball only, since the arm is considered massless. The angular acceleration of the ball is found from the given tangential acceleration.

$$
\begin{aligned}
\tau & =I \alpha=M R^{2} \alpha=M R^{2} \frac{a_{\mathrm{tan}}}{R}=M R a_{\mathrm{tan}}=(3.6 \mathrm{~kg})(0.31 \mathrm{~m})\left(7.0 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =7.812 \mathrm{~m} \cdot \mathrm{~N} \approx 7.8 \mathrm{~m} \cdot \mathrm{~N}
\end{aligned}
$$

(b) The triceps muscle must produce the torque required, but with a lever arm of only 2.5 cm , perpendicular to the triceps muscle force.

$$
\tau=F r_{\perp} \rightarrow F=\tau / r_{\perp}=7.812 \mathrm{~m} \cdot \mathrm{~N} /\left(2.5 \times 10^{-2} \mathrm{~m}\right)=310 \mathrm{~N}
$$

39. (a) The angular acceleration can be found from the following.

$$
\alpha=\frac{\Delta \omega}{\Delta t}=\frac{\omega}{t}=\frac{v / r}{t}=\frac{(8.5 \mathrm{~m} / \mathrm{s}) /(0.31 \mathrm{~m})}{0.35 \mathrm{~s}}=78.34 \mathrm{rad} / \mathrm{s}^{2} \approx 78 \mathrm{rad} / \mathrm{s}^{2}
$$

(b) The force required can be found from the torque, $\operatorname{since} \tau=F r \sin \theta$. In this situation the force is perpendicular to the lever arm, and so $\theta=90^{\circ}$. The torque is also given by $\tau=I \alpha$, where $I$ is the moment of inertia of the arm-ball combination. Equate the two expressions for the torque, and solve for the force.

$$
F r \sin \theta=I \alpha
$$

$$
\begin{aligned}
F & =\frac{I \alpha}{r \sin \theta}=\frac{m_{\text {ball }} d_{\text {ball }}^{2}+\frac{1}{3} m_{\text {arm }} L_{\text {arm }}^{2}}{r \sin 90^{\circ}} \alpha \\
& =\frac{(1.00 \mathrm{~kg})(0.31 \mathrm{~m})^{2}+\frac{1}{3}(3.7 \mathrm{~kg})(0.31 \mathrm{~m})^{2}}{(0.025 \mathrm{~m})}\left(78.34 \mathrm{rad} / \mathrm{s}^{2}\right)=670 \mathrm{~N}
\end{aligned}
$$

40. (a) To calculate the moment of inertia about the $y$ axis (vertical), use the following.

$$
\begin{aligned}
I & =\sum M_{i} R_{i x}^{2}=m(0.50 \mathrm{~m})^{2}+M(0.50 \mathrm{~m})^{2}+m(1.00 \mathrm{~m})^{2}+M(1.00 \mathrm{~m})^{2} \\
& =(m+M)\left[(0.50 \mathrm{~m})^{2}+(1.00 \mathrm{~m})^{2}\right]=(5.3 \mathrm{~kg})\left[(0.50 \mathrm{~m})^{2}+(1.00 \mathrm{~m})^{2}\right]=6.6 \mathrm{~kg} \cdot \mathrm{~m}^{2}
\end{aligned}
$$

(b) To calculate the moment of inertia about the $x$-axis (horizontal), use the following.

$$
I=\sum M_{i} R_{i y}^{2}=(2 m+2 M)(0.25 \mathrm{~m})^{2}=0.66 \mathrm{~kg} \cdot \mathrm{~m}^{2}
$$

(c) Because of the larger $I$ value, it is ten times harder to accelerate the array about the vertical axis.
41. The torque required is equal to the angular acceleration times the moment of inertia. The angular acceleration is found using Eq. 10-9a. Use the moment of inertia of a solid cylinder.

$$
\begin{aligned}
& \omega=\omega_{0}+\alpha t \rightarrow \alpha=\omega / t \\
& \tau=I \alpha=\left(\frac{1}{2} M R_{0}^{2}\right)\left(\frac{\omega}{t}\right)=\frac{M R_{0}^{2} \omega}{2 t}=\frac{(31000 \mathrm{~kg})(7.0 \mathrm{~m})^{2}(0.68 \mathrm{rad} / \mathrm{s})}{2(24 \mathrm{~s})}=2.2 \times 10^{4} \mathrm{~m} \cdot \mathrm{~N}
\end{aligned}
$$

42. The torque supplied is equal to the angular acceleration times the moment of inertia. The angular acceleration is found using Eq. $10-9 \mathrm{~b}$, with $\omega_{0}=0$. Use the moment of inertia of a sphere.

$$
\begin{aligned}
& \theta=\omega_{0}+\frac{1}{2} \alpha t^{2} \rightarrow \alpha=\frac{2 \theta}{t^{2}} ; \tau=I \alpha=\left(\frac{2}{5} M r_{0}^{2}\right)\left(\frac{2 \theta}{t^{2}}\right) \rightarrow \\
& M=\frac{5 \tau t^{2}}{4 r_{0}^{2} \theta}=\frac{5(10.8 \mathrm{~m} \cdot \mathrm{~N})(15.0 \mathrm{~s})^{2}}{4(0.36 \mathrm{~m})^{2}(360 \pi \mathrm{rad})}=21 \mathrm{~kg}
\end{aligned}
$$

43. The applied force causes torque, which gives the pulley an angular acceleration. Since the applied force varies with time, so will the angular acceleration. The variable acceleration will be integrated to find the angular velocity. Finally, the speed of a point on the rim is the tangential velocity of the rim of the wheel.

$$
\begin{aligned}
& \sum \tau=R_{0} F_{T}=I \alpha \rightarrow \alpha=\frac{R_{0} F_{\mathrm{T}}}{I}=\frac{d \omega}{d t} \rightarrow d \omega=\frac{R_{0} F_{\mathrm{T}}}{I} d t \rightarrow \int_{\omega_{0}}^{\omega} d \omega=\int_{0}^{t} \frac{R_{0} F_{\mathrm{T}}}{I} d t \rightarrow \\
& \omega=\frac{v}{R_{0}}=\omega_{0}+\frac{R_{0}}{I} \int_{0}^{t} F_{\mathrm{T}} d t=\frac{R_{0}}{I} \int_{0}^{t} F_{\mathrm{T}} d t \rightarrow \\
& v_{\mathrm{T}}=\omega R_{0}=\frac{R_{0}^{2}}{I} \int_{0}^{t} F_{\mathrm{T}} d t=\frac{R_{0}^{2}}{I} \int_{0}^{t}\left(3.00 t-0.20 t^{2}\right) d t=\frac{R_{0}^{2}}{I}\left[\left(\frac{3}{2} t^{2}-\frac{0.20}{3} t^{3}\right) \mathrm{N} \cdot \mathrm{~s}\right] \\
& v(t=8.0 \mathrm{~s})=\frac{(0.330 \mathrm{~m})^{2}}{\left(0.385 \mathrm{~kg} \cdot \mathrm{~m}^{2}\right)}\left[\left(\frac{3}{2}(8.0 \mathrm{~s})^{2}-\frac{0.20}{3}(8.0 \mathrm{~s})^{3}\right) \mathrm{N} \cdot \mathrm{~s}\right]=17.499 \mathrm{~m} / \mathrm{s} \approx 17 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

44. The torque needed is the moment of inertia of the system (merry-go-round and children) times the angular acceleration of the system. Let the subscript "mgr" represent the merry-go-round.

$$
\begin{aligned}
\tau & =I \alpha=\left(I_{\mathrm{mgr}}+I_{\text {children }}\right) \frac{\Delta \omega}{\Delta t}=\left(\frac{1}{2} M_{\mathrm{mgr}} R^{2}+2 m_{\text {child }} R^{2}\right) \frac{\omega-\omega_{0}}{t} \\
& =\left[\frac{1}{2}(760 \mathrm{~kg})+2(25 \mathrm{~kg})\right](2.5 \mathrm{~m})^{2} \frac{(15 \mathrm{rev} / \mathrm{min})(2 \pi \mathrm{rad} / \mathrm{rev})(1 \mathrm{~min} / 60 \mathrm{~s})}{10.0 \mathrm{~s}} \\
& =422.15 \mathrm{~m} \cdot \mathrm{~N} \approx 420 \mathrm{~m} \cdot \mathrm{~N}
\end{aligned}
$$

The force needed is calculated from the torque and the radius. We are told that the force is directed perpendicularly to the radius.

$$
\tau=F_{\perp} R \sin \theta \rightarrow F_{\perp}=\tau / R=422.15 \mathrm{~m} \cdot \mathrm{~N} / 2.5 \mathrm{~m}=170 \mathrm{~N}
$$

45. Each mass is treated as a point particle. The first mass is at the axis of rotation; the second mass is a distance $\ell$ from the axis of rotation; the third mass is $2 \ell$ from the axis, and the fourth mass
 is $3 \ell$ from the axis.
(a) $I=M \ell^{2}+M(2 \ell)^{2}+M(3 \ell)^{2}=14 M \ell^{2}$
(b) The torque to rotate the rod is the perpendicular component of force times the lever arm, and is also the moment of inertia times the angular acceleration.

$$
\tau=I \alpha=F_{\perp} r \rightarrow F_{\perp}=\frac{I \alpha}{r}=\frac{14 M \ell^{2} \alpha}{3 \ell}=\frac{14}{3} M \ell \alpha
$$

(c) The force must be perpendicular to the rod connecting the masses, and perpendicular to the axis of rotation. An appropriate direction is shown in the diagram.
46. (a) The free body diagrams are shown. Note that only the forces producing torque are shown on the pulley. There would also be a gravity force on the pulley (since it has mass) and a normal force from the pulley's suspension, but they are not shown.
(b) Write Newton's second law for the two blocks, taking the positive $x$ direction as shown in the free body diagrams.

$$
\begin{aligned}
& m_{\mathrm{A}}: \sum F_{x}=F_{\mathrm{TA}}-m_{\mathrm{A}} g \sin \theta_{\mathrm{A}}=m_{\mathrm{A}} a \rightarrow \\
& F_{\mathrm{TA}}=m_{\mathrm{A}}\left(g \sin \theta_{\mathrm{A}}+a\right) \\
&=(8.0 \mathrm{~kg})\left[\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \sin 32^{\circ}+1.00 \mathrm{~m} / \mathrm{s}^{2}\right]=49.55 \mathrm{~N} \\
& \approx 50 \mathrm{~N}(2 \operatorname{sig} \mathrm{fig}) \\
& m_{\mathrm{B}}: \sum F_{x}=m_{\mathrm{B}} g \sin \theta_{\mathrm{B}}-F_{\mathrm{TB}}=m_{\mathrm{B}} a \rightarrow \\
& F_{\mathrm{TB}}=m_{\mathrm{B}}\left(g \sin \theta_{\mathrm{B}}-a\right) \\
&=(10.0 \mathrm{~kg})\left[\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \sin 61^{\circ}-1.00 \mathrm{~m} / \mathrm{s}^{2}\right]=75.71 \mathrm{~N} \\
& \approx 76 \mathrm{~N}
\end{aligned}
$$




(c) The net torque on the pulley is caused by the two tensions. We take clockwise torques as positive.

$$
\sum \tau=\left(F_{\mathrm{TB}}-F_{\mathrm{TB}}\right) R=(75.71 \mathrm{~N}-49.55 \mathrm{~N})(0.15 \mathrm{~m})=3.924 \mathrm{~m} \cdot \mathrm{~N} \approx 3.9 \mathrm{~m} \cdot \mathrm{~N}
$$

Use Newton's second law to find the rotational inertia of the pulley. The tangential acceleration of the pulley's rim is the same as the linear acceleration of the blocks, assuming that the string doesn't slip.

$$
\begin{aligned}
& \sum \tau=I \alpha=I \frac{a}{R}=\left(F_{\mathrm{TB}}-F_{\mathrm{TB}}\right) R \rightarrow \\
& I=\frac{\left(F_{\mathrm{TB}}-F_{\mathrm{TB}}\right) R^{2}}{a}=\frac{(75.71 \mathrm{~N}-49.55 \mathrm{~N})(0.15 \mathrm{~m})^{2}}{1.00 \mathrm{~m} / \mathrm{s}^{2}}=0.59 \mathrm{~kg} \cdot \mathrm{~m}^{2}
\end{aligned}
$$

47. (a) The moment of inertia of a thin rod, rotating about its end, is $\frac{1}{3} M L^{2}$. There are three blades to add together.

$$
I_{\text {total }}=3\left(\frac{1}{3} M \ell^{2}\right)=M \ell^{2}=(135 \mathrm{~kg})(3.75 \mathrm{~m})^{2}=1898 \mathrm{~kg} \cdot \mathrm{~m}^{2} \approx 1.90 \times 10^{3} \mathrm{~kg} \cdot \mathrm{~m}^{2}
$$

(b) The torque required is the rotational inertia times the angular acceleration, assumed constant.

$$
\tau=I_{\text {total }} \alpha=I_{\text {total }} \frac{\omega-\omega_{0}}{t}=\left(1898 \mathrm{~kg} \cdot \mathrm{~m}^{2}\right) \frac{(5.0 \mathrm{rev} / \mathrm{sec})(2 \pi \mathrm{rad} / \mathrm{rev})}{8.0 \mathrm{~s}}=7500 \mathrm{~m} \cdot \mathrm{~N}
$$

48. The torque on the rotor will cause an angular acceleration given by $\alpha=\tau / I$. The torque and angular acceleration will have the opposite sign of the initial angular velocity because the rotor is being brought to rest. The rotational inertia is that of a solid cylinder. Substitute the expressions for angular acceleration and rotational inertia into the equation $\omega^{2}=\omega_{o}^{2}+2 \alpha \theta$, and solve for the angular displacement.

$$
\begin{aligned}
\omega^{2} & =\omega_{o}^{2}+2 \alpha \theta \rightarrow \theta=\frac{\omega^{2}-\omega_{o}^{2}}{2 \alpha}=\frac{0-\omega_{o}^{2}}{2(\tau / I)}=\frac{-\omega_{o}^{2}}{2\left(\tau / \frac{1}{2} M R^{2}\right)}=\frac{-M R^{2} \omega_{o}^{2}}{4 \tau} \\
& =\frac{-(3.80 \mathrm{~kg})(0.0710 \mathrm{~m})^{2}\left[\left(10,300 \frac{\mathrm{rev}}{\mathrm{~min}}\right)\left(\frac{2 \pi \mathrm{rad}}{1 \mathrm{rev}}\right)\left(\frac{1 \mathrm{~min}}{60 \mathrm{~s}}\right)\right]^{2}}{4(-1.20 \mathrm{~N} \cdot \mathrm{~m})}=4643 \mathrm{rad}\left(\frac{1 \mathrm{rev}}{2 \pi \mathrm{rad}}\right) \\
& =739 \mathrm{rev}
\end{aligned}
$$

The time can be found from $\theta=\frac{1}{2}\left(\omega_{o}+\omega\right) t$.

$$
t=\frac{2 \theta}{\omega_{o}+\omega}=\frac{2(739 \mathrm{rev})}{10,300 \mathrm{rev} / \mathrm{min}}\left(\frac{60 \mathrm{~s}}{1 \mathrm{~min}}\right)=8.61 \mathrm{~s}
$$

49. 

(a) Thin hoop, radius $R_{0}$

$$
\begin{aligned}
& I=M k^{2}=M R_{0}^{2} \rightarrow k=R_{0} \\
& I=M k^{2}=\frac{1}{2} M R_{0}^{2}+\frac{1}{12} M w^{2} \rightarrow k=\sqrt{\sqrt{\frac{1}{2} R_{0}^{2}+\frac{1}{12} w^{2}}} \\
& I=M k^{2}=\frac{1}{2} M R_{0}^{2} \rightarrow k=\sqrt{\frac{1}{2}} R_{0} \\
& I=M k^{2}=\frac{1}{2} M\left(R_{1}^{2}+R_{2}^{2}\right) \rightarrow k=\sqrt{\sqrt{\frac{1}{2}\left(R_{1}^{2}+R_{2}^{2}\right)}} \\
& I=M k^{2}=\frac{2}{5} M r_{0}^{2} \rightarrow k=\sqrt{\frac{2}{5} r_{0}} \\
& I=M k^{2}=\frac{1}{12} M \ell^{2} \rightarrow k=\sqrt{\frac{1}{12}} \ell \\
& I=M k^{2}=\frac{1}{3} M \ell^{2} \rightarrow k=\sqrt{\frac{1}{3}} \ell \\
& I=M k^{2}=\frac{1}{12} M\left(\ell^{2}+w^{2}\right) \rightarrow k=\sqrt{\frac{1}{12}\left(\ell^{2}+w^{2}\right)}
\end{aligned}
$$

(b) Thin hoop, radius $R_{0}$, width $w$
(c) Solid cylinder
(d) Hollow cylinder
(e) Uniform sphere
(f) Long rod, through center
(g) Long rod, through end
(h) Rectangular thin plate
50. The firing force of the rockets will create a net torque, but no net force. Since each rocket fires tangentially, each force has a lever arm equal to the radius of the satellite, and each force is perpendicular to the lever arm. Thus $\tau_{\text {net }}=4 F R$. This torque will cause an angular acceleration according to $\tau=I \alpha$, where $I=\frac{1}{2} M R^{2}+4 m R^{2}$, combining a cylinder of mass $M$ and radius $R$ with 4 point masses of mass $m$ and lever arm $R$ each. The angular acceleration can be found from the kinematics by $\alpha=\frac{\Delta \omega}{\Delta t}$. Equating the two expressions for the torque and substituting enables us to solve for the force.

$$
\begin{aligned}
4 F R & =I \alpha=\left(\frac{1}{2} M+4 m\right) R^{2} \frac{\Delta \omega}{\Delta \tau} \rightarrow F=\frac{\left(\frac{1}{2} M+4 m\right) R \Delta \omega}{4 \Delta t} \\
& =\frac{\left(\frac{1}{2}(3600 \mathrm{~kg})+4(250 \mathrm{~kg})\right)(4.0 \mathrm{~m})(32 \mathrm{rev} / \mathrm{min})(2 \pi \mathrm{rad} / \mathrm{rev})(1 \mathrm{~min} / 60 \mathrm{~s})}{4(5.0 \mathrm{~min})(60 \mathrm{~s} / \mathrm{min})}=31.28 \mathrm{~N} \\
& \approx 31 \mathrm{~N}
\end{aligned}
$$

51. We assume that $m_{\mathrm{B}}>m_{\mathrm{A}}$, and so $m_{\mathrm{B}}$ will accelerate down, $m_{\mathrm{A}}$ will accelerate up, and the pulley will accelerate clockwise. Call the direction of acceleration the positive direction for each object. The masses will have the same acceleration since they are connected by a cord. The rim of the pulley will have that same acceleration since the cord is making it rotate, and so $\alpha_{\text {pulley }}=a / R$. From the free-body diagrams for each object, we have the following.

$$
\begin{aligned}
& \sum F_{y \mathrm{~A}}=F_{\mathrm{TA}}-m_{\mathrm{A}} g=m_{\mathrm{A}} a \rightarrow F_{\mathrm{TA}}=m_{\mathrm{A}} g+m_{\mathrm{A}} a \\
& \sum F_{y \mathrm{~B}}=m_{\mathrm{B}} g-F_{\mathrm{TB}}=m_{\mathrm{B}} a \rightarrow F_{\mathrm{TB}}=m_{\mathrm{B}} g-m_{\mathrm{B}} a \\
& \sum \tau=F_{\mathrm{TB}} r-F_{\mathrm{TA}} r=I \alpha=I \frac{a}{R}
\end{aligned}
$$



Substitute the expressions for the tensions into the torque equation, and solve for the acceleration.

$$
\begin{aligned}
& F_{\mathrm{TB}} R-F_{\mathrm{TA}} R=I \frac{a}{R} \rightarrow\left(m_{\mathrm{B}} g-m_{\mathrm{B}} a\right) R-\left(m_{\mathrm{A}} g+m_{\mathrm{A}} a\right) R=I \frac{a}{R} \rightarrow \\
& a=\frac{\left(m_{\mathrm{B}}-m_{\mathrm{A}}\right)}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}+I / R^{2}\right)} g
\end{aligned}
$$

If the moment of inertia is ignored, then from the torque equation we see that $F_{\text {тв }}=F_{\text {TA }}$, and the acceleration will be $a_{I=0}=\frac{\left(m_{\mathrm{B}}-m_{\mathrm{A}}\right)}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)} g$. We see that the acceleration with the moment of inertia included will be smaller than if the moment of inertia is ignored.
52. (a) The free body diagram and analysis from problem 51 are applicable here, for the no-friction case.

$$
\begin{aligned}
a & =\frac{\left(m_{\mathrm{B}}-m_{\mathrm{A}}\right)}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}+I / r^{2}\right)} g=\frac{\left(m_{\mathrm{B}}-m_{\mathrm{A}}\right)}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}+\frac{1}{2} m_{\mathrm{P}} r^{2} / r^{2}\right)} g=\frac{\left(m_{\mathrm{B}}-m_{\mathrm{A}}\right)}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}+\frac{1}{2} m_{\mathrm{P}}\right)} g \\
& =\frac{(3.80 \mathrm{~kg}-3.15 \mathrm{~kg})}{(3.80 \mathrm{~kg}+3.15 \mathrm{~kg}+0.40 \mathrm{~kg})}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=0.8667 \mathrm{~m} / \mathrm{s}^{2} \approx 0.87 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

(b) With a frictional torque present, the torque equation from problem 51 would be modified, and the analysis proceeds as follows.

$$
\begin{aligned}
& \sum \tau=F_{\mathrm{TB}} r-F_{\mathrm{TA}} r-\tau_{\mathrm{fr}}=I \alpha=I \frac{a}{r} \rightarrow\left(m_{\mathrm{B}} g-m_{\mathrm{B}} a\right) r-\left(m_{\mathrm{A}} g+m_{\mathrm{A}} a\right) r-\tau_{\mathrm{ff}}=I \frac{a}{r} \rightarrow \\
& \tau_{\mathrm{fr}}=r\left[\left(m_{\mathrm{B}}-m_{\mathrm{A}}\right) g-\left(m_{\mathrm{B}}+m_{\mathrm{A}}+\frac{I}{r^{2}}\right) a\right]=r\left[\left(m_{\mathrm{B}}-m_{\mathrm{A}}\right) g-\left(m_{\mathrm{B}}+m_{\mathrm{A}}+\frac{1}{2} m_{\mathrm{p}}\right) a\right]
\end{aligned}
$$

The acceleration can be found from the kinematical data and Eq. 2-12a.

$$
\begin{aligned}
v & =v_{0}+a t \rightarrow a=\frac{v-v_{0}}{t}=\frac{0-0.20 \mathrm{~m} / \mathrm{s}}{6.2 \mathrm{~s}}=-0.03226 \mathrm{~m} / \mathrm{s}^{2} \\
\tau_{\mathrm{fr}} & =r\left[\left(m_{\mathrm{B}}-m_{\mathrm{A}}\right) g-\left(m_{\mathrm{B}}+m_{\mathrm{A}}+\frac{1}{2} m_{\mathrm{p}}\right) a\right] \\
& =(0.040 \mathrm{~m})\left[(0.65 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)-(7.35 \mathrm{~kg})\left(-0.03226 \mathrm{~m} / \mathrm{s}^{2}\right)\right]=0.26 \mathrm{~m} \cdot \mathrm{~N}
\end{aligned}
$$

53. A top view diagram of the hammer is shown, just at the instant of release, along with the acceleration vectors.
(a) The angular acceleration is found from Eq. 10-9c.

$$
\begin{aligned}
\omega^{2}=\omega_{0}^{2} & +2 \alpha \Delta \theta \rightarrow \alpha=\frac{\omega^{2}-\omega_{0}^{2}}{2 \Delta \theta}=\frac{(v / r)^{2}-0}{2 \Delta \theta} \\
& =\frac{[(26.5 \mathrm{~m} / \mathrm{s}) /(1.20 \mathrm{~m})]^{2}}{2(8 \pi \mathrm{rad})}=9.702 \mathrm{rad} / \mathrm{s}^{2} \approx 9.70 \mathrm{rad} / \mathrm{s}^{2}
\end{aligned}
$$


(b) The tangential acceleration is found from the angular acceleration and the radius.

$$
a_{\mathrm{tan}}=\alpha r=\left(9.702 \mathrm{rad} / \mathrm{s}^{2}\right)(1.20 \mathrm{~m})=11.64 \mathrm{~m} / \mathrm{s}^{2} \approx 11.6 \mathrm{~m} / \mathrm{s}^{2}
$$

(c) The centripetal acceleration is found from the speed and the radius.

$$
a_{\mathrm{rad}}=v^{2} / r=(26.5 \mathrm{~m} / \mathrm{s})^{2} /(1.20 \mathrm{~m})=585.2 \mathrm{~m} / \mathrm{s}^{2} \approx 585 \mathrm{~m} / \mathrm{s}^{2}
$$

(d) The net force is the mass times the net acceleration. It is in the same direction as the net acceleration.

$$
F_{\mathrm{net}}=m a_{\mathrm{net}}=m \sqrt{a_{\mathrm{tan}}^{2}+a_{\mathrm{rad}}^{2}}=(7.30 \mathrm{~kg}) \sqrt{\left(11.64 \mathrm{~m} / \mathrm{s}^{2}\right)^{2}+\left(585.2 \mathrm{~m} / \mathrm{s}^{2}\right)^{2}}=4270 \mathrm{~N}
$$

(e) Find the angle from the two acceleration vectors.

$$
\theta=\tan ^{-1} \frac{a_{\mathrm{tan}}}{a_{\mathrm{rad}}}=\tan ^{-1} \frac{11.64 \mathrm{~m} / \mathrm{s}^{2}}{585.2 \mathrm{~m} / \mathrm{s}^{2}}=1.14^{\circ}
$$

54. (a) See the free body diagram for the falling rod. The axis of rotation would be coming out of the paper at the point of contact with the floor. There are contact forces between the rod and the table (the friction force and the normal force), but they act through the axis of rotation and so cause no torque. Thus only gravity causes torque. Write Newton's second law for the rotation of the rod. Take counterclockwise to be the positive direction for rotational quantities. Thus in the diagram, the angle is positive, but the torque is negative.

$$
\begin{aligned}
& \sum \tau=I \alpha=-m g\left(\frac{1}{2} \ell \cos \phi\right)=\frac{1}{3} m \ell^{2} \frac{d \omega}{d t} \rightarrow \\
& -\frac{3 g}{2 \ell} \cos \phi=\frac{d \omega}{d t}=\frac{d \omega}{d \phi} \frac{d \phi}{d t}=\frac{d \omega}{d \phi} \phi \rightarrow \frac{3 g}{2 \ell} \cos \phi d \phi=-\omega d \omega \rightarrow \\
& \frac{3 g}{2 \ell} \int_{\pi / 2}^{\phi} \cos \phi d \phi=-\int_{0}^{\omega} \omega d \omega \rightarrow \frac{3 g}{2 \ell}(\sin \phi-1)=-\frac{1}{2} \omega^{2} \rightarrow \omega=\sqrt{\frac{3 g}{\ell}(1-\sin \phi)}
\end{aligned}
$$



The speed of the tip is the tangential speed of the tip, since the rod is rotating. At the tabletop, $\phi=0$.

$$
v=\omega \ell=\sqrt{3 g \ell(1-\sin \phi)} \rightarrow v(0)=\sqrt{3 g \ell}
$$

55. The parallel axis theorem is given in Eq. 10-17. The distance from the center of mass of the rod to the end of the rod is $h=\frac{1}{2} \ell$.

$$
I=I_{\mathrm{CM}}+M h^{2}=\frac{1}{12} M \ell^{2}+M\left(\frac{1}{2} \ell\right)^{2}=\left(\frac{1}{12}+\frac{1}{4}\right) M \ell^{2}=\frac{1}{3} M \ell^{2}
$$

56. We can consider the door to be made of a large number of thin horizontal rods, each of length $\ell=1.0 \mathrm{~m}$, and rotating about one end. Two such rods are shown in the diagram. The moment of inertia of one of these rods is $\frac{1}{3} m_{i} \ell^{2}$, where $m_{i}$ is the mass of a single rod. For a collection of identical rods, then, the moment of inertia would be $I=\sum_{i} \frac{1}{3} m_{i} \ell^{2}=\frac{1}{3} M \ell^{2}$. The height of the door does not enter into the calculation directly.

$$
I=\frac{1}{3} M \ell^{2}=\frac{1}{3}(19.0 \mathrm{~kg})(1.0 \mathrm{~m})^{2}=6.3 \mathrm{~kg} \cdot \mathrm{~m}^{2}
$$


57. (a) The parallel axis theorem (Eq. 10-17) is to be applied to each sphere. The distance from the center of mass of each sphere to the axis of rotation is $h=1.5 r_{0}$.

$$
I_{\substack{\text { for one } \\ \text { sphere }}}=I_{\mathrm{CM}}+M h^{2}=\frac{2}{5} M r_{0}^{2}+M\left(1.5 r_{0}\right)^{2}=2.65 M r_{0}^{2} \rightarrow I_{\text {total }}=5.3 M r_{0}^{2}
$$

(b) Treating each mass as a point mass, the point mass would be a distance of $1.5 r_{0}$ from the axis of rotation.

$$
\begin{aligned}
& I_{\text {approx }}=2\left[M\left(1.5 r_{0}\right)^{2}\right]=4.5 M r_{0}^{2} \\
& \begin{aligned}
\% \text { error } & =\left(\frac{I_{\text {approx }}-I_{\text {exact }}}{I_{\text {exact }}}\right)(100)=\left[\frac{4.5 M r_{0}^{2}-5.3 M r_{0}^{2}}{5.3 M r_{0}^{2}}\right](100)=\left[\frac{4.5-5.3}{5.3}\right](100) \\
& =-15 \%
\end{aligned}
\end{aligned}
$$

The negative sign means that the approximation is smaller than the exact value, by about $15 \%$.
58. (a) Treating the ball as a point mass, the moment of inertia about AB is $I=M R_{0}^{2}$.
(b) The parallel axis theorem is given in Eq. 10-17. The distance from the center of mass of the ball to the axis of rotation is $h=R_{0}$.
$\begin{aligned} I= & I_{\mathrm{CM}}+M h^{2}=\boxed{\frac{2}{5} M r_{1}^{2}+M R_{0}^{2}} \\ \text { (c) } \% \text { error } & =\left(\frac{I_{\text {approx }}-I_{\text {exact }}}{I_{\text {exact }}}\right)(100)=\left[\frac{M R_{0}^{2}-\left(\frac{2}{5} M r_{1}^{2}+M R_{0}^{2}\right)}{\frac{2}{5} M r_{1}^{2}+M R_{0}^{2}}\right](100)=\frac{-\frac{2}{5} M r_{1}^{2}}{\frac{2}{5} M r_{1}^{2}+M R_{0}^{2}}(100) \\ & =\frac{-1}{1+\frac{5}{2}\left(R_{0} / r_{1}\right)^{2}}(100)=-\frac{-1}{1+\frac{5}{2}(1.0 / 0.090)^{2}}(100)=-0.32295 \approx-0.32\end{aligned}$
The negative sign means that the approximation is smaller than the exact value, by about $0.32 \%$.
59. The $1.50-\mathrm{kg}$ weight is treated as a point mass. The origin is placed at the center of the wheel, with the $x$ direction to the right. Let A represent the wheel and $B$ represent the weight.
(a) $x_{\mathrm{CM}}=\frac{m_{\mathrm{A}} x_{\mathrm{A}}+m_{\mathrm{B}} x_{\mathrm{B}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}=\frac{(7.0 \mathrm{~kg})(0)+(1.50 \mathrm{~kg})(0.22 \mathrm{~m})}{8.50 \mathrm{~kg}}$

$$
=3.88 \times 10^{-2} \mathrm{~m} \approx 0.039 \mathrm{~m}
$$


(b) The moment of inertia of the wheel is found from the parallel axis theorem.

$$
\begin{aligned}
I & =I_{\text {wheel }}+I_{\text {weight }}=I_{\mathrm{CM}} \\
& =\frac{1}{2} M_{\text {wheel }} R^{2}+M_{\text {wheel }} x_{\mathrm{CM}}^{2}+M_{\text {weel }} x_{\mathrm{CM}}^{2}+M_{\text {weight }}\left(x_{\text {weight }}-x_{\mathrm{CM}}\right)^{2} \\
& =(7.0 \mathrm{~kg})\left(\frac{1}{2}(0.32 \mathrm{~m})^{2}+\left(0.0388 \mathrm{~m}^{2}\right)^{2}\right)+(1.50 \mathrm{~kg})(0.22 \mathrm{~m}-0.0388 \mathrm{~m})^{2}=0.42 \mathrm{~kg} \cdot \mathrm{~m}^{2}
\end{aligned}
$$

60. We calculate the moment of inertia about one end, and then use the parallel axis theorem to find the moment of inertia about the center. Let the mass of the rod be $M$, and use Eq. 10-16. A small mass $d M$ can be found as a small length $d x$
 times the mass per unit length of the rod.

$$
\begin{aligned}
& I_{\text {end }}=\int R^{2} d M=\int_{0}^{\ell} x^{2} \frac{M}{\ell} d x=\frac{M}{\ell} \frac{\ell^{3}}{3}=\frac{1}{3} M \ell^{2} \\
& I_{\text {end }}=I_{\mathrm{CM}}+M\left(\frac{1}{2} \ell\right)^{2} \rightarrow I_{\mathrm{CM}}=I_{\text {end }}-M\left(\frac{1}{2} \ell\right)^{2}=\frac{1}{3} M \ell^{2}-\frac{1}{4} M \ell^{2}=\frac{1}{12} M \ell^{2}
\end{aligned}
$$

61. (a) We choose coordinates so that the center of the plate is at the origin. Divide the plate up into differential rectangular elements, each with an area of $d A=d x d y$. The mass of an element is $d m=\left(\frac{M}{\ell w}\right) d x d y$. The distance of that element from the axis of rotation is $R=\sqrt{x^{2}+y^{2}}$. Use Eq. 10-16 to calculate the
 moment of inertia.

$$
\begin{aligned}
I_{\text {center }} & =\int R^{2} d M=\int_{-w / 2}^{w / 2} \int_{-\ell / 2}^{\ell / 2}\left(x^{2}+y^{2}\right) \frac{M}{\ell w} d x d y=\frac{4 M}{\ell w} \int_{0}^{w / 2} \int_{0}^{\ell / 2}\left(x^{2}+y^{2}\right) d x d y \\
& =\frac{4 M}{\ell w} \int_{0}^{w / 2}\left[\frac{1}{3}\left(\frac{1}{2} \ell\right)^{3}+\left(\frac{1}{2} \ell\right) y^{2}\right] d y=\frac{2 M}{w} \int_{0}^{w / 2}\left[\frac{1}{12} \ell^{2}+y^{2}\right] d y \\
& =\frac{2 M}{w}\left[\frac{1}{12} \ell^{2}\left(\frac{1}{2} w\right)+\frac{1}{3}\left(\frac{1}{2} w\right)^{3}\right]=\frac{1}{12} M\left(\ell^{2}+w^{2}\right)
\end{aligned}
$$

(b) For the axis of rotation parallel to the $w$ dimension (so the rotation axis is in the $y$ direction), we can consider the plate to be made of a large number of thin rods, each of length $\ell$, rotating about an axis through their center. The moment of inertia of one of these rods is $\frac{1}{12} m_{i} \ell^{2}$, where $m_{i}$ is the mass of a single rod. For a collection of identical rods,
 then, the moment of inertia would be $I_{y}=\sum_{i} \frac{1}{12} m_{i} \ell^{2}=\frac{1}{12} M \ell^{2}$. A similar argument would give $I_{x}=\frac{1}{12} M w^{2}$. This illustrates the perpendicular axis theorem, Eq. 10-18, $I_{z}=I_{x}+I_{y}$.
62. Work can be expressed in rotational quantities as $W=\tau \Delta \theta$, and so power can be expressed in rotational quantities as $P=\frac{W}{\Delta t}=\tau \frac{\Delta \theta}{\Delta t}=\tau \omega$.

$$
P=\tau \omega=(255 \mathrm{~m} \cdot \mathrm{~N})\left(3750 \frac{\mathrm{rev}}{\mathrm{~min}}\right)\left(\frac{2 \pi \mathrm{rad}}{1 \mathrm{rev}}\right)\left(\frac{1 \mathrm{~min}}{60 \mathrm{~s}}\right)\left(\frac{1 \mathrm{hp}}{746 \mathrm{~W}}\right)=134 \mathrm{hp}
$$

63. The energy required to bring the rotor up to speed from rest is equal to the final rotational kinetic energy of the rotor.

$$
K_{\mathrm{rot}}=\frac{1}{2} I \omega^{2}=\frac{1}{2}\left(4.25 \times 10^{-2} \mathrm{~kg} \cdot \mathrm{~m}^{2}\right)\left[9750 \frac{\mathrm{rev}}{\min }\left(\frac{2 \pi \mathrm{rad}}{1 \mathrm{rev}}\right)\left(\frac{1 \mathrm{~min}}{60 \mathrm{~s}}\right)\right]^{2}=2.22 \times 10^{4} \mathrm{~J}
$$

64. To maintain a constant angular speed $\omega_{\text {steady }}$ will require a torque $\tau_{\text {motor }}$ to oppose the frictional torque. The power required by the motor is $P=\tau_{\text {motor }} \omega_{\text {steady }}=-\tau_{\text {friction }} \omega_{\text {steady }}$.

$$
\begin{aligned}
\tau_{\text {friction }} & =I \alpha_{\text {friction }}=\frac{1}{2} M R^{2}\left(\frac{\omega_{f}-\omega_{0}}{t}\right) \rightarrow \\
P_{\text {motor }} & =\frac{1}{2} M R^{2}\left(\frac{\omega_{0}-\omega_{f}}{t}\right) \omega_{\text {steady }}=\frac{1}{2}(220 \mathrm{~kg})(5.5 \mathrm{~m})^{2} \frac{\left[(3.8 \mathrm{rev} / \mathrm{s})\left(\frac{2 \pi \mathrm{rad}}{\mathrm{rev}}\right)\right]^{2}}{16 \mathrm{~s}}=1.186 \times 10^{5} \mathrm{~W} \\
& =1.186 \times 10^{5} \mathrm{~W}\left(\frac{1 \mathrm{hp}}{746 \mathrm{~W}}\right)=158.9 \mathrm{hp} \approx 160 \mathrm{hp}
\end{aligned}
$$

65. The work required is the change in rotational kinetic energy. The initial angular velocity is 0 .

$$
W=\Delta K_{\mathrm{rot}}=\frac{1}{2} I \omega_{f}^{2}-\frac{1}{2} I \omega_{i}^{2}=\frac{1}{2}\left(\frac{1}{2} M R^{2}\right) \omega_{f}^{2}=\frac{1}{4}(1640 \mathrm{~kg})(7.50 \mathrm{~m})^{2}\left(\frac{2 \pi \mathrm{rad}}{8.00 \mathrm{~s}}\right)^{2}=1.42 \times 10^{4} \mathrm{~J}
$$

66. Mechanical energy will be conserved. The rotation is about a fixed axis, so $K_{\text {tot }}=K_{\text {rot }}=\frac{1}{2} I \omega^{2}$. For gravitational potential energy, we can treat the object as if all of its mass were at its center of mass. Take the lowest point of the center of mass as the zero location for gravitational potential energy.

$$
\begin{aligned}
& E_{\text {initial }}=E_{\text {final }} \rightarrow U_{\text {initial }}=K_{\text {final }} \rightarrow \\
& M g \frac{1}{2} \ell(1-\cos \theta)=\frac{1}{2} I \omega_{\text {bottom }}^{2}=\frac{1}{2}\left(\frac{1}{3} M \ell^{2}\right) \omega_{\text {bottom }}^{2} \rightarrow \\
& \omega_{\text {bottom }}=\sqrt{\frac{3 g}{\ell}(1-\cos \theta)} ; v_{\text {bottom }}=\omega_{\text {bottom }} \ell=\sqrt{3 g \ell(1-\cos \theta)}
\end{aligned}
$$

The only force doing work in this system is gravity, so mechanical energy is conserved. The initial state of the system is the configuration with $m_{\mathrm{A}}$ on the ground and all objects at rest. The final state of the system has $m_{\mathrm{B}}$ just reaching the ground, and all objects in motion. Call the zero level of gravitational potential energy to be the ground level. Both masses will have the same speed since
they are connected by the rope. Assuming that the rope does not slip on the pulley, the angular speed of the pulley is related to the speed of the masses by $\omega=v / R$. All objects have an initial speed of 0 .

$$
\begin{aligned}
& E_{i}=E_{f} \rightarrow \\
& \begin{aligned}
\frac{1}{2} m_{\mathrm{A}} v_{i}^{2}+\frac{1}{2} m_{\mathrm{B}} v_{i}^{2}+\frac{1}{2} I \omega_{i}^{2}+m_{\mathrm{A}} g y_{1 i}+m_{\mathrm{B}} g y_{2 i}= & \frac{1}{2} m_{\mathrm{A}} v_{f}^{2}+\frac{1}{2} m_{\mathrm{B}} v_{f}^{2}+\frac{1}{2} I \omega_{f}^{2} \\
& +m_{\mathrm{A}} g y_{1 f}+m_{\mathrm{B}} g y_{2 f}
\end{aligned} \\
& m_{\mathrm{B}} g h=\frac{1}{2} m_{\mathrm{A}} v_{f}^{2}+\frac{1}{2} m_{\mathrm{B}} v_{f}^{2}+\frac{1}{2}\left(\frac{1}{2} M R^{2}\right)\left(\frac{v_{f}^{2}}{R^{2}}\right)+m_{\mathrm{A}} g h
\end{aligned}
$$

$$
v_{f}=\sqrt{\frac{2\left(m_{\mathrm{B}}-m_{\mathrm{A}}\right) g h}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}+\frac{1}{2} M\right)}}=\sqrt{\frac{2(38.0 \mathrm{~kg}-35.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(2.5 \mathrm{~m})}{\left(38.0 \mathrm{~kg}+35.0 \mathrm{~kg}+\left(\frac{1}{2}\right) 3.1 \mathrm{~kg}\right)}}=1.4 \mathrm{~m} / \mathrm{s}
$$

68. (a) The kinetic energy of the system is the kinetic energy of the two masses, since the rod is treated as massless. Let A represent the heavier mass, and B the lighter mass.

$$
\begin{aligned}
K & =\frac{1}{2} I_{\mathrm{A}} \omega_{\mathrm{A}}^{2}+\frac{1}{2} I_{\mathrm{B}} \omega_{\mathrm{B}}^{2}=\frac{1}{2} m_{\mathrm{A}} r_{\mathrm{A}}^{2} \omega_{\mathrm{A}}^{2}+\frac{1}{2} m_{\mathrm{B}} r_{\mathrm{B}}^{2} \omega_{\mathrm{A}}^{2}=\frac{1}{2} r^{2} \omega^{2}\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) \\
& =\frac{1}{2}(0.210 \mathrm{~m})^{2}(5.60 \mathrm{rad} / \mathrm{s})^{2}(7.00 \mathrm{~kg})=4.84 \mathrm{~J}
\end{aligned}
$$

(b) The net force on each object produces centripetal motion, and so can be expressed as $m r \omega^{2}$.

$$
\begin{aligned}
& F_{\mathrm{A}}=m_{\mathrm{A}} r_{\mathrm{A}} \omega_{\mathrm{A}}^{2}=(4.00 \mathrm{~kg})(0.210 \mathrm{~m})(5.60 \mathrm{rad} / \mathrm{s})^{2}=26.3 \mathrm{~N} \\
& F_{\mathrm{B}}=m_{\mathrm{B}} r_{\mathrm{B}} \omega_{\mathrm{B}}^{2}=(3.00 \mathrm{~kg})(0.210 \mathrm{~m})(5.60 \mathrm{rad} / \mathrm{s})^{2}=19.8 \mathrm{~N}
\end{aligned}
$$

These forces are exerted by the rod. Since they are unequal, there would be a net horizontal force on the rod (and hence the axle) due to the masses. This horizontal force would have to be counteracted by the mounting for the rod and axle in order for the rod not to move horizontally. There is also a gravity force on each mass, balanced by a vertical force from the rod, so that there is no net vertical force on either mass.
(c) Take the 4.00 kg mass to be the origin of coordinates for determining the center of mass.

$$
x_{\mathrm{CM}}=\frac{m_{\mathrm{A}} x_{\mathrm{A}}+m_{\mathrm{B}} x_{\mathrm{B}}}{m_{\mathrm{A}}+m_{\mathrm{B}}}=\frac{(4.00 \mathrm{~kg})(0)+(3.00 \mathrm{~kg})(0.420 \mathrm{~m})}{7.00 \mathrm{~kg}}=0.180 \mathrm{~m} \text { from mass A }
$$

So the distance from mass A to the axis of rotation is now 0.180 m , and the distance from mass B to the axis of rotation is now 0.24 m . Re-do the above calculations with these values.

$$
\begin{aligned}
K & =\frac{1}{2} I_{\mathrm{A}} \omega_{\mathrm{A}}^{2}+\frac{1}{2} I_{\mathrm{B}} \omega_{\mathrm{B}}^{2}=\frac{1}{2} m_{\mathrm{A}} r_{\mathrm{A}}^{2} \omega_{\mathrm{A}}^{2}+\frac{1}{2} m_{\mathrm{B}} r_{\mathrm{B}}^{2} \omega_{\mathrm{A}}^{2}=\frac{1}{2} \omega^{2}\left(m_{\mathrm{A}} r_{\mathrm{A}}^{2}+m_{\mathrm{B}} r_{\mathrm{B}}^{2}\right) \\
& =\frac{1}{2}(5.60 \mathrm{rad} / \mathrm{s})^{2}\left[(4.00 \mathrm{~kg})(0.180 \mathrm{~m})^{2}+(3.00 \mathrm{~kg})(0.240 \mathrm{~m})^{2}\right]=4.74 \mathrm{~J} \\
F_{\mathrm{A}} & =m_{\mathrm{A}} r_{\mathrm{A}} \omega_{\mathrm{A}}^{2}=(4.00 \mathrm{~kg})(0.180 \mathrm{~m})(5.60 \mathrm{rad} / \mathrm{s})^{2}=22.6 \mathrm{~N} \\
F_{\mathrm{B}} & =m_{\mathrm{B}} r_{\mathrm{B}} \omega_{\mathrm{B}}^{2}=(3.00 \mathrm{~kg})(0.240 \mathrm{~m})(5.60 \mathrm{rad} / \mathrm{s})^{2}=22.6 \mathrm{~N}
\end{aligned}
$$

Note that the horizontal forces are now equal, and so there will be no horizontal force on the rod or axle.
69. Since the lower end of the pole does not slip on the ground, the friction does no work, and so mechanical energy is conserved. The initial energy is the potential energy, treating all the mass as if it were at the CM. The final energy is rotational kinetic energy, for rotation about the point of contact with the ground. The linear velocity of the falling tip of the rod is its angular velocity
divided by the length.

$$
\begin{aligned}
& E_{\text {intial }}=E_{\text {final }} \rightarrow U_{\text {initial }}=K_{\text {final }} \rightarrow m g h=\frac{1}{2} I \omega^{2} \rightarrow m g L / 2=\frac{1}{2}\left(\frac{1}{3} m L^{2}\right)\left(v_{\text {end }} / L\right)^{2} \rightarrow \\
& v_{\text {end }}=\sqrt{3 g L}=\sqrt{3\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(2.30 \mathrm{~m})}=8.22 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

70. Apply conservation of mechanical energy. Take the bottom of the incline to be the zero location for gravitational potential energy. The energy at the top of the incline is then all gravitational potential energy, and at the bottom of the incline, there is both rotational and translational kinetic energy. Since the cylinder rolls without slipping, the angular velocity is given by $\omega=v / R$.

$$
\begin{aligned}
& E_{\text {top }}=E_{\text {bottom }} \rightarrow M g h=\frac{1}{2} M v^{2}+\frac{1}{2} I_{\mathrm{CM}} \omega^{2}=\frac{1}{2} M v^{2}+\frac{1}{2} \frac{1}{2} M R^{2} \frac{v^{2}}{R^{2}}=\frac{3}{4} M v^{2} \rightarrow \\
& v=\sqrt{\frac{4}{3} g h}=\sqrt{\frac{4}{3}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(7.20 \mathrm{~m})}=9.70 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

71. The total kinetic energy is the sum of the translational and rotational kinetic energies. Since the ball is rolling without slipping, the angular velocity is given by $\omega=v / R$. The rotational inertia of a sphere about an axis through its center is $I=\frac{2}{5} m R^{2}$.

$$
\begin{aligned}
K_{\text {total }} & =K_{\text {trans }}+K_{\text {rot }}=\frac{1}{2} m v^{2}+\frac{1}{2} I \omega^{2}=\frac{1}{2} m v^{2}+\frac{1}{2} \frac{2}{5} m R^{2} \frac{v^{2}}{R^{2}}=\frac{7}{10} m v^{2} \\
& =0.7(7.3 \mathrm{~kg})(3.7 \mathrm{~m} / \mathrm{s})^{2}=7.0 \times 10^{1} \mathrm{~J}
\end{aligned}
$$

72. (a) For the daily rotation about its axis, treat the Earth as a uniform sphere, with an angular frequency of one revolution per day.

$$
\begin{aligned}
K_{\text {daily }}= & \frac{1}{2} I \omega_{\text {daily }}^{2}=\frac{1}{2}\left(\frac{2}{5} M R_{\text {Earth }}^{2}\right) \omega_{\text {daily }}^{2} \\
& =\frac{1}{5}\left(6.0 \times 10^{24} \mathrm{~kg}\right)\left(6.4 \times 10^{6} \mathrm{~m}\right)^{2}\left[\left(\frac{2 \pi \mathrm{rad}}{1 \text { day }}\right)\left(\frac{1 \text { day }}{86,400 \mathrm{~s}}\right)\right]^{2}=2.6 \times 10^{29} \mathrm{~J}
\end{aligned}
$$

(b) For the yearly revolution about the Sun, treat the Earth as a particle, with an angular frequency of one revolution per year.

$$
\begin{aligned}
K_{\text {yearly }} & =\frac{1}{2} I \omega_{\text {yearly }}^{2}=\frac{1}{2}\left(M R_{\text {Sun- }}^{2}\right) \omega_{\text {Earth }}^{2} \\
& =\frac{1}{2}\left(6.0 \times 10^{24} \mathrm{~kg}\right)\left(1.5 \times 10^{11} \mathrm{~m}\right)^{2}\left[\left(\frac{2 \pi \mathrm{rad}}{365 \text { day }}\right)\left(\frac{1 \text { day }}{86,400 \mathrm{~s}}\right)\right]^{2}=2.7 \times 10^{33} \mathrm{~J}
\end{aligned}
$$

Thus the total kinetic energy is $K_{\text {daily }}+K_{\text {yearly }}=2.6 \times 10^{29} \mathrm{~J}+2.7 \times 10^{33} \mathrm{~J}=2.7 \times 10^{33} \mathrm{~J}$. The kinetic energy due to the daily motion is about 10,000 times smaller than that due to the yearly motion.
73. (a) Mechanical energy is conserved as the sphere rolls without slipping down the plane. Take the zero level of gravitational potential energy to the level of the center of mass of the sphere when it is on the level surface at the bottom of the plane. All of the energy is potential energy at the top, and all is
 kinetic energy (of both translation and rotation) at the bottom.

$$
\begin{aligned}
& E_{\text {intial }}=E_{\text {final }} \rightarrow U_{\text {initial }}=K_{\text {final }}=K_{\text {CM }}+K_{\text {rot }} \rightarrow \\
& m g h=m g \ell \sin \theta=\frac{1}{2} m v_{\text {botom }}^{2}+\frac{1}{2} I \omega_{\text {botom }}^{2}=\frac{1}{2} m v_{\text {botom }}^{2}+\frac{1}{2}\left(\frac{2}{5} m r_{0}^{2}\right)\left(\frac{v_{\text {botom }}}{r_{0}}\right)^{2} \rightarrow \\
& v_{\text {bottom }}=\sqrt{\frac{10}{7} g h}=\sqrt{\frac{10}{7} g \ell \sin \theta}=\sqrt{\frac{10}{7}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(10.0 \mathrm{~m}) \sin 30.0^{\circ}}=8.367 \mathrm{~m} / \mathrm{s} \\
& \approx 8.37 \mathrm{~m} / \mathrm{s} \\
& \omega_{\text {botom }}=\frac{v_{\text {bottom }}}{r_{0}}=\frac{8.367 \mathrm{~m} / \mathrm{s}}{0.254 \mathrm{~m}}=32.9 \mathrm{rad} / \mathrm{s} \\
& \text { (b) } \frac{K_{\mathrm{CM}}}{K_{\text {rot }}}=\frac{\frac{1}{2} m v_{\text {botom }}^{2}}{\frac{1}{2} I \omega_{\text {bottom }}^{2}}=\frac{\frac{1}{2} m v_{\text {botom }}^{2}}{\frac{1}{2}\left(\frac{2}{5} m r_{0}^{2}\right)\left(\frac{v_{\text {botom }}}{r_{0}}\right)^{2}}=\frac{5}{2}
\end{aligned}
$$

(c) The translational speed at the bottom, and the ratio of kinetic energies, are both independent of the radius and the mass. The rotational speed at the bottom depends on the radius.
74. (a) Since the center of mass of the spool is stationary, the net force must be 0 . Thus the force on the thread must be equal to the weight of the spool and so $F_{\text {ttread }}=M g$.
(b) By the work-energy theorem, the work done is the change in kinetic energy of the spool The spool has rotational kinetic energy.

$$
W=K_{\text {final }}-K_{\text {initial }}=\frac{1}{2} I \omega^{2}=\frac{1}{2}\left(\frac{1}{2} M R^{2}\right) \omega^{2}=\frac{1}{4} M R^{2} \omega^{2}
$$

75. Use conservation of mechanical energy to equate the energy at points A and B. Call the zero level for gravitational potential energy to be the lowest point on which the ball rolls. Since the ball rolls without slipping, $\omega=v / r_{0}$.

$$
\begin{aligned}
& E_{\mathrm{A}}=E_{\mathrm{B}} \rightarrow U_{\mathrm{A}}=U_{\mathrm{B}}+K_{\mathrm{B} \text { final }}=U_{\mathrm{B}}+K_{\mathrm{BCM}}+K_{\mathrm{B} \text { rot }} \rightarrow \\
& m g R_{0}=m g r_{0}+\frac{1}{2} m v_{\mathrm{B}}^{2}+\frac{1}{2} I \omega_{\mathrm{B}}^{2} \\
& \quad=m g r_{0}+\frac{1}{2} m v_{\mathrm{B}}^{2}+\frac{1}{2}\left(\frac{2}{5} m r_{0}^{2}\right)\left(\frac{v_{\mathrm{B}}}{r_{0}}\right)^{2} \rightarrow v_{\mathrm{B}}=\sqrt{\frac{10}{7} g\left(R_{0}-r_{0}\right)}
\end{aligned}
$$


76. (a) We work in the accelerating reference frame of the car. In the accelerating frame, we must add a fictitious force of magnitude $M a_{\substack{\text { train rel } \\ \text { ground }}}$, in the opposite direction to the acceleration of the train. This is discussed in detail in section 11-8 of the textbook. Since the ball is rolling without slipping, $\alpha=a_{\substack{\text { bal rel } \\ \text { train }}} / R$. See the free-body diagram for the ball in the accelerating reference frame. Write Newton's second law for the horizontal direction and for torques, with clockwise torques as positive. Combine these relationships to find $a_{\substack{\text { ball rel } \\ \text { train }}}$, the acceleration of
 the ball in the accelerated frame.

$$
\begin{aligned}
& \sum \tau=-F_{\mathrm{fr}} R=I \alpha=I \frac{\begin{array}{c}
a_{\text {ball rel }} \\
\text { train }
\end{array}}{R} \rightarrow F_{\mathrm{fr}}=-\frac{2}{5} M a_{\substack{\text { ball rel } \\
\text { train }}} \\
& \sum F_{x}=F_{\mathrm{fr}}-M a_{\substack{\text { train rel } \\
\text { ground }}}=M a_{\substack{\text { ball rel } \\
\text { train }}} \rightarrow-\frac{2}{5} M a_{\substack{\text { ball rel } \\
\text { train }}}-M a_{\substack{\text { train rel } \\
\text { ground }}}=M a_{\substack{\text { ball rel } \\
\text { train }}} \rightarrow \\
& a_{\substack{\text { ball rel } \\
\text { train }}}=-\frac{5}{7} a_{\substack{\text { train rel } \\
\text { ground }}}
\end{aligned}
$$

And so as seen from inside the train, the ball is accelerating backwards.
(b) Use the relative acceleration relationship.

$$
\underset{\substack{\text { ball rel } \\ \text { ground }}}{a_{\text {ball rel }}}+\underset{\substack{\text { train rel } \\ \text { tround }}}{a_{\substack{\text { train rel } \\ \text { ground }}}+a_{\substack{\text { train rel } \\ \text { ground }}}=\frac{5}{7} a_{\text {train }}}
$$

And so as seen from outside the train, the ball is accelerating forwards, but with a smaller acceleration than the train.
77. (a) Use conservation of mechanical energy. Call the zero level for gravitational potential energy to be the lowest point on which the pipe rolls. Since the pipe rolls without slipping, $\omega=v / R$. See the attached diagram.

$$
\begin{aligned}
& E_{\text {initial }}=E_{\text {final }} \rightarrow U_{\text {initial }}=K_{\text {final }}=K_{\mathrm{CM}}+K_{\mathrm{rot}} \\
& m g D \sin \theta=\frac{1}{2} m v_{\text {bottom }}^{2}+\frac{1}{2} I \omega_{\text {bottom }}^{2} \\
& \quad=\frac{1}{2} m v_{\text {bottom }}^{2}+\frac{1}{2}\left(m R^{2}\right)\left(\frac{v_{\text {bottom }}^{2}}{R^{2}}\right)=m v_{\text {bottom }}^{2} \rightarrow \\
& v_{\text {bottom }}=\sqrt{g D \sin \theta}=\sqrt{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(5.60 \mathrm{~m}) \sin 17.5^{\circ}}=4.06 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$


(b) The total kinetic energy at the base of the incline is the same as the initial potential energy.

$$
K_{\text {final }}=U_{\text {initial }}=m g D \sin \theta=(0.545 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(5.60 \mathrm{~m}) \sin 17.5^{\circ}=8.99 \mathrm{~J}
$$

(c) The frictional force supplies the torque for the object to roll without slipping, and the frictional force has a maximum value. Since the object rolls without slipping, $\alpha=a / R$. Use Newton's second law for the directions parallel and perpendicular to the plane, and for the torque, to solve for the coefficient of friction.

$$
\begin{aligned}
& \sum \tau=F_{\mathrm{fr}} R=I \alpha=m R^{2} \frac{a}{R}=m a R \rightarrow F_{\mathrm{ff}}=m a \\
& \sum_{\perp} F_{\perp}=F_{\mathrm{N}}-m g \cos \theta \rightarrow F_{\mathrm{N}}=m g \cos \theta \\
& \sum F_{\mathrm{If}}=m g \sin \theta-F_{\mathrm{fr}}=m a \rightarrow F_{\mathrm{fr}}=\frac{1}{2} m g \sin \theta \\
& F_{\mathrm{fr}} \leq F_{\substack{\text { static } \\
\text { max }}} \rightarrow \frac{1}{2} m g \sin \theta \leq \mu_{\mathrm{s}} F_{N}=\mu_{\mathrm{s}} m g \cos \theta \rightarrow \mu_{\mathrm{s}} \geq \frac{1}{2} \tan \theta \rightarrow \\
& \mu_{\mathrm{s}}=\frac{1}{2} \tan \theta=\frac{1}{2} \tan 17.5^{\circ}=0.158
\end{aligned}
$$

78. (a) While the ball is slipping, the acceleration of the center of mass is constant, and so constant acceleration relationships may be used. Use Eq. 2-12b with results from Example 10-20.

$$
x-x_{0}=v_{0} t+\frac{1}{2} a t^{2}=v_{0}\left(\frac{2 v_{0}}{7 \mu_{k} g}\right)+\frac{1}{2}\left(-\mu_{\mathrm{k}} g\right)\left(\frac{2 v_{0}}{7 \mu_{k} g}\right)^{2}=\frac{12 v_{0}^{2}}{49 \mu_{k} g}
$$

(b) Again make use of the fact that the acceleration is constant. Once the final speed is reached, the angular velocity is given by $\omega=v / r_{0}$.

$$
v=v_{0}+a t=v_{0}+\left(-\mu_{\mathrm{k}} g\right)\left(\frac{2 v_{0}}{7 \mu_{k} g}\right)=\frac{5}{7} v_{0} ; \omega=\frac{5}{7} v_{0} / r_{0}
$$

79. (a) The total kinetic energy included the translational kinetic energy of the car's total mass, and the rotational kinetic energy of the car's wheels. The wheels can be treated as one cylinder. We assume the wheels are rolling without slipping, so that $v_{\mathrm{CM}}=\omega / R_{\text {wheels }}$.

$$
\begin{aligned}
K_{\text {tot }} & =K_{\mathrm{CM}}+K_{\mathrm{rot}}=\frac{1}{2} M_{\mathrm{tot}} v_{\mathrm{CM}}^{2}+\frac{1}{2} I_{\text {wheels }} \omega^{2}=\frac{1}{2} M_{\text {tot }} v_{\mathrm{CM}}^{2}+\frac{1}{2}\left(\frac{1}{2} M_{\text {wheels }} R_{\text {wheels }}^{2}\right) \frac{v_{\mathrm{CM}}^{2}}{R_{\text {wheels }}^{2}} \\
& =\frac{1}{2}\left(M_{\text {tot }}+\frac{1}{2} M_{\text {wheels }}\right) v_{\mathrm{CM}}^{2}=\frac{1}{2}(1170 \mathrm{~kg})\left[(95 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)\right]^{2}=4.074 \times 10^{5} \mathrm{~J} \\
& \approx 4.1 \times 10^{5} \mathrm{~J}
\end{aligned}
$$

(b) The fraction of kinetic energy in the tires and wheels is $\frac{K_{\text {rot }}+K_{\substack{\text { trans } \\ \text { wheels }}}}{K_{\text {tot }}}$.

$$
\begin{aligned}
\frac{K_{\text {rot }}}{K_{\text {tot }}} & =\frac{\frac{1}{2} I_{\text {wheels }} \omega^{2}+\frac{1}{2} M_{\text {wheels }} v_{\mathrm{CM}}^{2}}{\frac{1}{2} M_{\text {tot }} v_{\mathrm{CM}}^{2}+\frac{1}{2} I_{\text {wheels }} \omega^{2}}=\frac{\frac{1}{2}\left(\frac{1}{2} M_{\text {wheels }}+M_{\text {wheels }}\right) v_{\mathrm{CM}}^{2}}{\frac{1}{2}\left(M_{\text {tot }}+\frac{1}{2} M_{\text {wheels }}\right) v_{\mathrm{CM}}^{2}}=\frac{\left(\frac{3}{2} M_{\text {wheels }}\right)}{\left(M_{\text {tot }}+\frac{1}{2} M_{\text {wheels }}\right)} \\
& =\frac{210 \mathrm{~kg}}{1170 \mathrm{~kg}}=0.18
\end{aligned}
$$

(c) A free body diagram for the car is shown, with the frictional force of $\overrightarrow{\mathbf{F}}_{\mathrm{fr}}$ at each wheel to cause the wheels to roll. A separate diagram of one wheel is also shown. Write Newton's second law for the horizontal motion of the car as a whole, and the rotational motion of one wheel. Take clockwise torques as positive. Since the wheels are rolling without slipping, $a_{\text {СМ }}=\alpha / R_{\text {wheels }}$.


$$
\begin{aligned}
& \sum \tau=4 F_{\mathrm{fr}} R=I_{\text {wheels }} \alpha=\frac{1}{2} M_{\text {wheels }} R_{\text {wheels }}^{2} \frac{a_{\mathrm{CM}}}{R_{\text {wheels }}} \rightarrow \\
& \quad F_{\mathrm{fr}}=\frac{1}{8} M_{\text {wheels }} a_{\mathrm{CM}} \\
& \sum F_{x}=F_{\mathrm{tow}}-4 F_{\mathrm{fr}}=M_{\mathrm{tot}} a_{\mathrm{CM}} \rightarrow \\
& F_{\mathrm{tow}}-4\left(\frac{1}{8} M_{\mathrm{wheels}} a_{\mathrm{CM}}\right)=M_{\mathrm{tot}} a_{\mathrm{CM}} \rightarrow \\
& \quad a_{\mathrm{CM}}=\frac{F_{\mathrm{tow}}}{\left(M_{\mathrm{tot}}+\frac{1}{2} M_{\text {wheels }}\right)}=\frac{1500 \mathrm{~N}}{(1170 \mathrm{~kg})}=1.282 \mathrm{~m} / \mathrm{s}^{2} \approx 1.3 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$


(d) If the rotational inertia were ignored, we would have the following.

$$
\begin{aligned}
& \sum F_{x}=F_{\mathrm{tow}}=M_{\mathrm{tot}} a_{\mathrm{CM}} \rightarrow a_{\mathrm{CM}}=\frac{F_{\mathrm{tow}}}{M_{\mathrm{tot}}}=\frac{1500 \mathrm{~N}}{1100 \mathrm{~kg}}=1.364 \mathrm{~m} / \mathrm{s}^{2} \\
& \% \text { error }=\frac{\Delta a_{\mathrm{CM}}}{a_{\mathrm{CM}}} \times 100=\frac{1.364 \mathrm{~m} / \mathrm{s}^{2}-1.282 \mathrm{~m} / \mathrm{s}^{2}}{1.282 \mathrm{~m} / \mathrm{s}^{2}} \times 100=6 \%
\end{aligned}
$$

80. (a) The friction force accelerates the center of mass of the wheel. If the wheel is spinning (and slipping) clockwise in the diagram, then the surface of the wheel that touches the ground is moving to the left, and the friction force is to the right or forward. It acts in the direction of motion of the velocity of the center of mass of the wheel.
(b) Write Newton's second law for the $x$ direction, the $y$ direction, and the rotation. Take clockwise torques (about the center of mass) as positive.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{N}}-M g=0 \rightarrow F_{\mathrm{N}}=M g \\
& \sum F_{x}=F_{\mathrm{fr}}=M a \rightarrow a=\frac{F_{\mathrm{fr}}}{M}=\frac{\mu_{\mathrm{k}} F_{\mathrm{N}}}{M}=\frac{\mu_{\mathrm{k}} M g}{M}=\mu_{\mathrm{k}} g \\
& \sum \tau=-F_{\mathrm{fr}} R=I \alpha \rightarrow \alpha=-\frac{F_{\mathrm{fr}} R}{\frac{1}{2} M R^{2}}=-\frac{2 M g}{M R}=-\frac{2 \mu_{\mathrm{k}} g}{R}
\end{aligned}
$$

Both the acceleration and angular acceleration are constant, and so constant acceleration kinematics may be used to express the velocity and angular velocity.

$$
v=v_{0}+a t=\mu_{\mathrm{k}} g t ; \omega=\omega_{0}+\alpha t=\omega_{0}-\frac{2 \mu_{\mathrm{k}} g}{R} t
$$

Note that the velocity starts at 0 and increases, while the angular velocity starts at $\omega_{0}$ and decreases. Thus at some specific time $T$, the velocity and angular velocity will be $\omega=v / R$, and the ball will roll without slipping. Solve for the value of $T$ needed to make that true.

$$
\omega=v / R \rightarrow \omega_{0}-\frac{2 \mu_{\mathrm{k}} g}{R} T=\mu_{\mathrm{k}} g T / R \quad \rightarrow \quad T=\frac{\omega_{0} R}{3 \mu_{\mathrm{k}} g}
$$

(c) Once the ball starts rolling without slipping, there is no more frictional sliding force, and so the velocity will remain constant.

$$
v_{\text {final }}=\mu_{\mathrm{k}} g T=\mu_{\mathrm{k}} g \frac{\omega_{0} R}{3 \mu_{\mathrm{k}} g}=\frac{1}{3} R \omega_{0}
$$

81. (a) Use conservation of mechanical energy to equate the energy at point A to the energy at point C. Call the zero level for gravitational potential energy to be the lowest point on which the ball rolls. Since the ball rolls without slipping, $\omega=v / r_{0}$. All locations given for the ball are for its center of mass.

$$
\begin{aligned}
& E_{\mathrm{A}}=E_{\mathrm{C}} \rightarrow \\
& U_{\mathrm{A}}=U_{\mathrm{C}}+K_{\mathrm{C}}=U_{\mathrm{C}}+K_{\mathrm{CM}}+K_{\mathrm{C}} \rightarrow \\
& m g R_{0}=m g\left[R_{0}-\left(R_{0}-r_{0}\right) \cos \theta\right]+\frac{1}{2} m v_{\mathrm{C}}^{2}+\frac{1}{2} I \omega_{\mathrm{C}}^{2}
\end{aligned}
$$

$$
=m g\left[R_{0}-\left(R_{0}-r_{0}\right) \cos \theta\right]+\frac{1}{2} m v_{\mathrm{C}}^{2}+\frac{1}{2}\left(\frac{2}{5} m r_{0}^{2}\right) \frac{v_{\mathrm{C}}^{2}}{r_{0}^{2}} \rightarrow
$$

$$
v_{\mathrm{C}}=\sqrt{\frac{10}{7} g\left(R_{0}-r_{0}\right) \cos \theta}=\sqrt{\frac{10}{7}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.245 \mathrm{~m}) \cos 45^{\circ}}=1.557 \mathrm{~m} / \mathrm{s} \approx 1.6 \mathrm{~m} / \mathrm{s}
$$

(b) Once the ball leaves the ramp, it will move as a projectile under the influence of gravity, and the constant acceleration equations may be used to find the distance. The initial location of the ball is given by $x_{0}=\left(R_{0}-r_{0}\right) \sin 45^{\circ}$ and $y_{0}=R_{0}-\left(R_{0}-r_{0}\right) \cos 45^{\circ}$. The initial velocity of the ball
is given by $v_{0 x}=v_{\mathrm{C}} \cos 45^{\circ}$ and $v_{0 y}=v_{\mathrm{C}} \sin 45^{\circ}$. The ball lands when $y=r_{0}=0.015 \mathrm{~m}$. Find the time of flight from the vertical motion, and then find D from the horizontal motion. Take the upward direction as positive for the vertical motion.

$$
\begin{aligned}
& y=y_{0}+v_{0 y} t+\frac{1}{2} a t^{2}=R_{0}-\left(R_{0}-r_{0}\right) \cos 45^{\circ}+v_{\mathrm{C}} \sin 45^{\circ} t-\frac{1}{2} g t^{2} \rightarrow \\
& 4.90 t^{2}-1.101 t-0.07178=0 \rightarrow t=0.277 \mathrm{~s},-0.0528 \mathrm{~s}
\end{aligned}
$$

We use the positive time.

$$
\begin{aligned}
D & =x=x_{0}+v_{0 x} t=\left(R_{0}-r_{0}\right) \sin 45^{\circ}+v_{\mathrm{C}} \cos 45^{\circ} t \\
& =(0.245 \mathrm{~m}) \sin 45^{\circ}+(1.557 \mathrm{~m} / \mathrm{s}) \cos 45^{\circ}(0.277 \mathrm{~s})=0.4782 \mathrm{~m} \approx 0.48 \mathrm{~m}
\end{aligned}
$$

82. Write the rotational version of Newton's second law, with counterclockwise torques as positive.

$$
\tau_{\text {net }}=\tau_{\mathrm{N}}-\tau_{\mathrm{fr}}=F_{\mathrm{N}} \ell-F R=I_{\mathrm{CN}} \alpha_{\mathrm{CM}}=\frac{2}{5} M R^{2} \alpha_{\mathrm{CM}}
$$

Newton's second law for the translational motion, with left as the positive direction, gives the following.

$$
F_{\mathrm{net}}=F=m a \rightarrow a=\frac{F}{m}
$$

If the sphere is rolling without slipping, we have $\alpha_{\mathrm{CM}}=a / R$. Combine these relationships to analyze the relationship between the torques.

$$
\begin{aligned}
& F_{\mathrm{N}} \ell=F R+\frac{2}{5} M R^{2} \alpha_{\mathrm{CM}}=F R+\frac{2}{5} M R^{2} \frac{a}{R}=F R+\frac{2}{5} M a R=F R+\frac{2}{5} F R=\frac{7}{5} F R \rightarrow \\
& \tau_{\mathrm{N}}=\frac{7}{5} \tau_{\mathrm{fr}}
\end{aligned}
$$

And since the torque due to the normal force is larger than the torque due to friction, the sphere has a counterclockwise angular acceleration, and thus the rotational velocity will decrease.
83. Since the spool rolls without slipping, each point on the edge of the spool moves with a speed of $v=r \omega=v_{\mathrm{CM}}$ relative to the center of the spool, where $v_{\mathrm{CM}}$ is the speed of the center of the spool relative to the ground. Since the spool is moving to the right relative to the ground, and the top of the spool is moving to the right relative to the center of the spool, the top of the spool is moving with a speed of $2 v_{\mathrm{CM}}$ relative to the ground. This is the speed of the rope, assuming it is unrolling without slipping and is at the outer edge of the spool. The speed of the rope is the same as the speed of the person, since the person is holding the rope. So the person is walking with a speed of twice that of the center of the spool. Thus if the person moves forward a distance $\ell$, in the same time the center of the spool, traveling with half the speed, moves forward a distance $\ell / 2$. The rope, to stay connected both to the person and to the spool, must therefore unwind by an amount $\ell / 2$ also.
84. The linear speed is related to the angular velocity by $v=\omega R$, and the angular velocity ( $\mathrm{rad} / \mathrm{sec}$ ) is related to the frequency (rev/sec) by Eq. $10-7, \omega=2 \pi f$. Combine these relationships to find values for the frequency.

$$
\begin{aligned}
& \omega=2 \pi f=\frac{v}{R} \rightarrow f=\frac{v}{2 \pi R} ; f_{1}=\frac{v}{2 \pi R_{1}}=\frac{1.25 \mathrm{~m} / \mathrm{s}}{2 \pi(0.025 \mathrm{~m})}\left(\frac{60 \mathrm{~s}}{1 \mathrm{~min}}\right)=480 \mathrm{rpm} \\
& f_{2}=\frac{v}{2 \pi R_{2}}=\frac{1.25 \mathrm{~m} / \mathrm{s}}{2 \pi(0.058 \mathrm{~m})}\left(\frac{60 \mathrm{~s}}{1 \mathrm{~min}}\right)=210 \mathrm{rpm}
\end{aligned}
$$

85. (a) There are two forces on the yo-yo: gravity and string tension. If the top of the string is held fixed, then the tension does no work, and so mechanical energy is conserved. The initial gravitational potential energy is converted into rotational and translational kinetic energy. Since the yo-yo rolls without slipping at the point of contact of the string, the velocity of the CM is related to the angular velocity of the

yo-yo by $v_{\mathrm{CM}}=r \omega$, where $r$ is the radius of the inner hub. Let $m$ be the mass of the inner hub, and $M$ and $R$ be the mass and radius of each outer disk. Calculate the rotational inertia of the yo-yo about its CM, and then use conservation of energy to find the linear speed of the CM. We take the 0 of gravitational potential energy to be at the bottom of its fall.

$$
\begin{aligned}
I_{\mathrm{CM}} & =\frac{1}{2} m r^{2}+2\left(\frac{1}{2} M R^{2}\right)=\frac{1}{2} m r^{2}+M R^{2} \\
& =\frac{1}{2}\left(5.0 \times 10^{-3} \mathrm{~kg}\right)\left(5.0 \times 10^{-3} \mathrm{~m}\right)^{2}+\left(5.0 \times 10^{-2} \mathrm{~kg}\right)\left(3.75 \times 10^{-2} \mathrm{~m}\right)^{2}=7.038 \times 10^{-5} \mathrm{~kg} \cdot \mathrm{~m}^{2} \\
m_{\text {total }} & =m+2 M=5.0 \times 10^{-3} \mathrm{~kg}+2\left(5.0 \times 10^{-2} \mathrm{~kg}\right)=0.105 \mathrm{~kg} \\
U_{\text {initial }} & =K_{\text {final }} \rightarrow \\
m_{\text {total }} g h & =\frac{1}{2} m_{\text {total }} v_{\mathrm{CM}}^{2}+\frac{1}{2} I_{\mathrm{CM}} \omega^{2}=\frac{1}{2} m_{\text {total }} v_{\mathrm{CM}}^{2}+\frac{1}{2} \frac{I_{\mathrm{CM}}}{r^{2}} v_{\mathrm{CM}}^{2}=\left(\frac{1}{2} m_{\text {total }}+\frac{1}{2} \frac{I_{\mathrm{CM}}}{r^{2}}\right) v_{\mathrm{CM}}^{2} \rightarrow \\
v_{\mathrm{CM}} & =\sqrt{\frac{m_{\text {total }} g h}{\frac{1}{2}\left(m_{\text {total }}+\frac{I_{\mathrm{CM}}}{r^{2}}\right)}}=\sqrt{\frac{(0.105 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(1.0 \mathrm{~m})}{\frac{1}{2}\left[(0.105 \mathrm{~kg})+\frac{\left(7.038 \times 10^{-5} \mathrm{~kg} \cdot \mathrm{~m}^{2}\right)}{\left(5.0 \times 10^{-3} \mathrm{~m}\right)^{2}}\right]}}=0.8395=0.84 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(b) Calculate the ratio $K_{\text {rot }} / K_{\text {tot }}$.

$$
\begin{aligned}
\frac{K_{\text {rot }}}{K_{\text {tot }}} & =\frac{K_{\text {rot }}}{U_{\text {initial }}}=\frac{\frac{1}{2} I_{\mathrm{CM}} \omega^{2}}{m_{\text {total }} g h}=\frac{\frac{1}{2} \frac{I_{\mathrm{CM}}}{r^{2}} v_{\mathrm{CM}}^{2}}{m_{\text {total }} g h}=\frac{I_{\mathrm{CM}} v_{\mathrm{CM}}^{2}}{2 r^{2} m_{\text {total }} g h} \\
& =\frac{\left(7.038 \times 10^{-5} \mathrm{~kg} \cdot \mathrm{~m}^{2}\right)(0.8395 \mathrm{~m} / \mathrm{s})^{2}}{2\left(5.0 \times 10^{-3} \mathrm{~m}\right)^{2}(0.105 \mathrm{~kg})\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)(1.0 \mathrm{~m})}=0.96=96 \%
\end{aligned}
$$

86. As discussed in the text, from the reference frame of the axle of the wheel, the points on the wheel are all moving with the same speed of $v=r \omega$, where $v$ is the speed of the axle of the wheel relative to the ground. The top of the tire has a velocity of $v$ to the right relative to the axle, so it has a velocity of $2 v$ to the right relative to the ground.

$$
\begin{aligned}
& v_{\substack{\text { top rel } \\
\text { ground }}}=2 v=2\left(v_{0}+a t\right)=2 a t=2\left(1.00 \mathrm{~m} / \mathrm{s}^{2}\right)(2.5 \mathrm{~s})=5.0 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

87. Assume that the angular acceleration is uniform. Then the torque required to whirl the rock is the moment of inertia of the rock (treated as a particle) times the angular acceleration.

$$
\tau=I \alpha=\left(m r^{2}\right)\left(\frac{\omega-\omega_{0}}{t}\right)=\frac{(0.50 \mathrm{~kg})(1.5 \mathrm{~m})^{2}}{5.0 \mathrm{~s}}\left[\left(85 \frac{\mathrm{rev}}{\mathrm{~min}}\right)\left(\frac{2 \pi \mathrm{rad}}{\mathrm{rev}}\right)\left(\frac{1 \mathrm{~min}}{60 \mathrm{~s}}\right)\right]=2.0 \mathrm{~m} \cdot \mathrm{~N}
$$

That torque comes from the arm swinging the sling, and so comes from the arm muscles.
88. The torque is found from $\tau=I \alpha$. The angular acceleration can be found from $\omega=\omega_{o}+\alpha t$, with an initial angular velocity of 0 . The rotational inertia is that of a cylinder.

$$
\tau=I \alpha=\frac{1}{2} M R^{2}\left(\frac{\omega-\omega_{o}}{t}\right)=0.5(1.4 \mathrm{~kg})(0.20 \mathrm{~m})^{2} \frac{(1800 \mathrm{rev} / \mathrm{s})(2 \pi \mathrm{rad} / \mathrm{rev})}{6.0 \mathrm{~s}}=53 \mathrm{~m} \cdot \mathrm{~N}
$$

89. (a) The linear speed of the chain must be the same as it passes over both sprockets. The linear speed is related to the angular speed by $v=\omega R$, and so $\omega_{R} R_{R}=\omega_{F} R_{F}$. If the spacing of the teeth on the sprockets is a distance $d$, then the number of teeth on a sprocket times the spacing distance must give the circumference of the sprocket.

$$
N d=2 \pi R \text { and so } R=\frac{N d}{2 \pi} \text {. Thus } \omega_{R} \frac{N_{R} d}{2 \pi}=\omega_{F} \frac{N_{F} d}{2 \pi} \rightarrow \frac{\omega_{R}}{\omega_{F}}=\frac{N_{F}}{N_{R}}
$$

(b)

$$
\omega_{R} / \omega_{F}=52 / 13=4.0
$$

(c) $\omega_{R} / \omega_{F}=42 / 28=1.5$
90. The mass of a hydrogen atom is 1.01 atomic mass units. The atomic mass unit is $1.66 \times 10^{-27} \mathrm{~kg}$. Since the axis passes through the oxygen atom, it will have no rotational inertia.
(a) If the axis is perpendicular to the plane of the molecule, then each hydrogen atom is a distance $\ell$ from the axis of rotation.

$$
\begin{aligned}
I_{\text {perp }} & =2 m_{H} \ell^{2}=2(1.01)\left(1.66 \times 10^{-27} \mathrm{~kg}\right)\left(0.96 \times 10^{-9} \mathrm{~m}\right)^{2} \\
& =3.1 \times 10^{-45} \mathrm{~kg} \cdot \mathrm{~m}^{2}
\end{aligned}
$$

(b) If the axis is in the plane of the molecule, bisecting the H-O-H bonds,
 each hydrogen atom is a distance of $\ell_{y}=\ell \sin \theta=\left(9.6 \times 10^{-10} \mathrm{~m}\right) \sin 52^{\circ}$ $=7.564 \times 10^{-10} \mathrm{~m}$. Thus the moment of inertia is as follows.

$$
I_{\text {plane }}=2 m_{H} \ell_{y}^{2}=2(1.01)\left(1.66 \times 10^{-27} \mathrm{~kg}\right)\left(7.564 \times 10^{-10} \mathrm{~m}\right)^{2}=1.9 \times 10^{-45} \mathrm{~kg} \cdot \mathrm{~m}^{2}
$$

91. (a) The initial energy of the flywheel is used for two purposes - to give the car translational kinetic energy 20 times, and to replace the energy lost due to friction, from air resistance and from braking. The statement of the problem leads us to ignore any gravitational potential energy changes.

$$
\begin{aligned}
& W_{\mathrm{fr}}=K_{\text {final }}-K_{\text {initial }} \rightarrow F_{\mathrm{fr}} \Delta x \cos 180^{\circ}=\frac{1}{2} M_{\mathrm{car}} v_{\mathrm{car}}^{2}-K_{\text {flywheel }} \\
& K_{\text {flywheel }}=F_{\mathrm{fr}} \Delta x+\frac{1}{2} M_{\mathrm{car}} v_{\mathrm{car}}^{2}
\end{aligned}
$$

$$
=(450 \mathrm{~N})\left(3.5 \times 10^{5} \mathrm{~m}\right)+(20) \frac{1}{2}(1100 \mathrm{~kg})\left[(95 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)\right]^{2}
$$

$$
=1.652 \times 10^{8} \mathrm{~J} \approx 1.7 \times 10^{8} \mathrm{~J}
$$

(b) $K_{\text {flywheel }}=\frac{1}{2} I \omega^{2}$

$$
\omega=\sqrt{\frac{2 K E}{I}}=\sqrt{\frac{2 K E}{\frac{1}{2} M_{\text {flywheel }} R_{\text {flywheel }}^{2}}}=\sqrt{\frac{2\left(1.652 \times 10^{8} \mathrm{~J}\right)}{\frac{1}{2}(240 \mathrm{~kg})(0.75 \mathrm{~m})^{2}}}=2200 \mathrm{rad} / \mathrm{s}
$$

(c) To find the time, use the relationship that power $=\frac{\text { work }}{\text { time }}$, where the work done by the motor will be equal to the kinetic energy of the flywheel.

$$
P=\frac{W}{t} \rightarrow t=\frac{W}{P}=\frac{\left(1.652 \times 10^{8} \mathrm{~J}\right)}{(150 \mathrm{hp})(746 \mathrm{~W} / \mathrm{hp})}=1.476 \times 10^{3} \mathrm{~s} \approx 25 \mathrm{~min}
$$

92. (a) Assuming that there are no dissipative forces doing work, conservation of mechanical energy may be used to find the final height $h$ of the hoop. Take the bottom of the incline to be the zero level of gravitational potential energy. We assume that the hoop is rolling without sliding, so that $\omega=v / R$. Relate the
 conditions at the bottom of the incline to the conditions at the top by conservation of energy. The hoop has both translational and rotational kinetic energy at the bottom, and the rotational inertia of the hoop is given by $I=m R^{2}$.

$$
\begin{aligned}
& E_{\text {botom }}=E_{\text {top }} \rightarrow \frac{1}{2} m v^{2}+\frac{1}{2} I \omega^{2}=m g h \rightarrow \frac{1}{2} m v^{2}+\frac{1}{2} m R^{2} \frac{v^{2}}{R^{2}}=m g h \rightarrow \\
& h=\frac{v^{2}}{g}=\frac{(3.3 \mathrm{~m} / \mathrm{s})^{2}}{9.80 \mathrm{~m} / \mathrm{s}^{2}}=1.111 \mathrm{~m}
\end{aligned}
$$

The distance along the plane is given by $d=\frac{h}{\sin \theta}=\frac{1.111 \mathrm{~m}}{\sin 15^{\circ}}=4.293 \mathrm{~m} \approx 4.3 \mathrm{~m}$
(b) The time can be found from the constant acceleration linear motion.

$$
\Delta x=\frac{1}{2}\left(v+v_{o}\right) t \rightarrow t=\frac{2 \Delta x}{v+v_{o}}=\frac{2(4.293 \mathrm{~m})}{0+3.3 \mathrm{~m} / \mathrm{s}}=2.602 \mathrm{~s}
$$

This is the time to go up the plane. The time to come back down the plane is the same, and so the total time is 5.2 s .
93. The wheel is rolling about the point of contact with the step, and so all torques are to be taken about that point. As soon as the wheel is off the floor, there will be only two forces that can exert torques on the wheel - the pulling force and the force of gravity. There will not be a normal force of contact between the wheel and the floor once the wheel is off the floor, and any force on the wheel from the point of the step cannot exert a torque about that very point. Calculate the net torque on the wheel, with clockwise torques positive. The minimum force occurs when the net torque is 0 .

$$
\begin{aligned}
& \sum \tau=F(R-h)-m g \sqrt{R^{2}-(R-h)^{2}}=0 \\
& F=\frac{M g \sqrt{R^{2}-(R-h)^{2}}}{R-h}=\frac{M g \sqrt{2 R h-h^{2}}}{R-h}
\end{aligned}
$$

94. Since frictional losses can be ignored, energy will be conserved for the marble. Define the 0 position of gravitational potential energy to be the bottom of the track, so that the bottom of the ball is initially a height $h$ above the 0 position of gravitational potential energy. We also assume that the
marble is rolling without slipping, so $\omega=v / r$, and that the marble is released from rest. The marble has both translational and rotational kinetic energy.
(a) Since $r \ll R$, the marble's CM is very close to the surface of the track. While the marble is on the loop, we then approximate that its CM will be moving in a circle of radius $R$. When the marble is at the top of the loop, we approximate that its CM is a distance of $2 R$ above the 0 position of gravitational potential energy. For the marble to just be on the verge of leaving the track means the normal force between the marble and the track is zero, and so the centripetal force at the top must be equal to the gravitational force on the marble.

$$
\frac{m v_{\text {top of }}^{2}}{R}=m g \rightarrow \underset{\substack{\text { toop of } \\ \text { loop }}}{2}=g R
$$

Use energy conservation to relate the release point to the point at the top of the loop.

$$
\begin{aligned}
& E_{\text {release }}=E_{\substack{\text { top of } \\
\text { loop }}} \rightarrow K_{\text {release }}+U_{\text {release }}=\underset{\substack{\text { top of } \\
\text { loop }}}{K_{\text {too }}}+U_{\substack{\text { top of } \\
\text { loop }}} \\
& 0+m g h=\frac{1}{2} m v_{\substack{\text { top of } \\
\text { loop }}}^{2}+\frac{1}{2} I \omega_{\substack{\text { top of } \\
\text { loop }}}^{2}+m g 2 R=\frac{1}{2} m v_{\substack{\text { top of } \\
\text { loop }}}^{2}+\frac{1}{2}\left(\frac{2}{5} m r^{2}\right) \frac{v_{\text {top of }}^{2}}{r^{2}}+2 m g R \\
& m g h=\frac{7}{10} m v_{\substack{\text { top of } \\
\text { loop }}}^{2}+2 m g R=\frac{7}{10} m g R+2 m g R=2.7 m g R \rightarrow h=2.7 R
\end{aligned}
$$

(b) Since we are not to assume that $r \ll R$, then while the marble is on the loop portion of the track, it is moving in a circle of radius $R-r$, and when at the top of the loop, the bottom of the marble is a height of $2(R-r)$ above the 0 position of gravitational potential energy (see the diagram). For the marble to just be on the verge of leaving the track means the normal force between the marble and the track is zero, and so the centripetal force at the top must be equal to the gravitational force on the marble.

$$
\frac{m v_{\text {top of }}^{2}}{\substack{\text { loop }}}=m g \rightarrow \underset{\substack{\text { top of } \\ \text { loop }}}{2}=g(R-r)
$$

Use energy conservation to equate the energy at the release point to the energy at the top of the loop.

$$
\begin{aligned}
& E_{\text {release }}=E_{\substack{\text { top of } \\
\text { loop }}} \rightarrow K_{\text {release }}+U_{\text {release }}=K_{\substack{\text { top of } \\
\text { loop }}}+U_{\text {top of }}^{\text {loop }} \\
& 0+m g h=\frac{1}{2} m v_{\substack{\text { top of } \\
\text { loop }}}^{2}+\frac{1}{2} I \omega_{\substack{\text { top of } \\
\text { loop }}}^{2}+m g 2(R-r)=\frac{1}{2} m v_{\substack{\text { top of } \\
\text { loop }}}^{2}+\frac{1}{2}\left(\frac{2}{5} m r^{2}\right) \frac{v_{\text {top of }}^{2}}{\text { loop }^{2}}+2 m g(R-r) \\
& m g h=\frac{7}{10} m v_{\substack{\text { top of } \\
\text { loop }}}^{2}+2 m g(R-r)=\frac{7}{10} m g(R-r)+2 m g(R-r)=2.7 m g(R-r) \\
& h=2.7(R-r)
\end{aligned}
$$

95. We calculate the moment of inertia about an axis through the geometric center of the rod. Select a differential element of the rod of length $d x$, a distance $x$ from the
 center of the rod. Because the mass density changes
uniformly from $\lambda_{0}$ at $x=-\frac{1}{2} \ell$ to $3 \lambda_{0}$ at $x=\frac{1}{2} \ell$, the mass density function is $\lambda=2 \lambda_{0}\left(1+\frac{x}{\ell}\right)$.

The mass of the differential element is then $d M=\lambda d x=2 \lambda_{0}\left(1+\frac{x}{\ell}\right) d x$. Use Eq. $10-16$ to calculate the moment of inertia.

$$
I_{\mathrm{end}}=\int R^{2} d M=\int_{-\ell / 2}^{\ell / 2} x^{2} 2 \lambda_{0}\left(1+\frac{x}{\ell}\right) d x=2 \lambda_{0} \int_{-\ell / 2}^{\ell / 2}\left(x^{2}+\frac{x^{3}}{\ell}\right) d x=2 \lambda_{0}\left(\frac{1}{3} x^{3}+\frac{1}{4} \frac{x^{4}}{\ell}\right)_{-\ell / 2}^{\ell / 2}=\frac{1}{6} \lambda_{0} \ell^{3}
$$

96. A free body diagram for the ball while the stick is in contact is shown. Write Newton's second law for the $x$ direction, the $y$ direction, and the rotation. Take clockwise torques (about the center of mass) as positive.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{N}}-M g=0 \rightarrow F_{\mathrm{N}}=M g \\
& \sum F_{x}=F-F_{\mathrm{fr}}=F-\mu_{k} F_{\mathrm{N}}=F-\mu_{\mathrm{k}} M g=M a \rightarrow a=\frac{F}{M}-\mu_{\mathrm{k}} g \\
& \sum \tau=F(h-r)+F_{\mathrm{fr}} r=F(h-r)+\mu_{\mathrm{k}} M g r=I \alpha \rightarrow \\
& \quad \alpha=\frac{F(h-r)+\mu_{\mathrm{k}} M g r}{I}
\end{aligned}
$$



The acceleration and angular acceleration are constant, and so constant acceleration kinematics may be used to find the velocity and angular velocity as functions of time. The object starts from rest.

$$
v_{\mathrm{CM}}=v_{0}+a t=\left(\frac{F}{M}-\mu_{\mathrm{k}} g\right) t ; \omega=\omega_{0}+\alpha t=\left(\frac{F(h-r)+\mu_{\mathrm{k}} M g r}{I}\right) t
$$

At a specific time $t_{\text {release }}$, when the ball loses contact with the pushing stick, the ball is rolling without slipping, and so at that time $\omega=v_{\mathrm{CM}} / r$. Solve for the value of $h$ needed to make that true. The moment of inertia is $I=\frac{2}{5} \mathrm{Mr}^{2}$.

$$
\begin{aligned}
& \omega=v_{\mathrm{CM}} / r \rightarrow\left(\frac{F(h-r)+\mu_{\mathrm{k}} M g r}{I}\right) t_{\text {release }}=\frac{1}{r}\left(\frac{F}{M}-\mu_{\mathrm{k}} g\right) t_{\text {release }} \rightarrow \\
& h=\frac{1}{F}\left[\frac{I}{r}\left(\frac{F}{M}-\mu_{\mathrm{k}} g\right)-\mu_{\mathrm{k}} M g r+F r\right]=\frac{\frac{7}{5}}{F}\left(F-\mu_{\mathrm{k}} M g\right)
\end{aligned}
$$

97. Each wheel supports $1 / 4$ of the weight of the car. For rolling without slipping, there will be static friction between the wheel and the pavement. So for the wheel to be on the verge of slipping, there must be an applied torque that is equal to the torque supplied by the static frictional force. We take counterclockwise torques to the right in the diagram. The bottom wheel would be moving to the left relative to the pavement if it started to slip, so the frictional force is to the right. See the free-body diagram.

$$
\begin{aligned}
\tau_{\substack{\text { applied } \\
\text { min }}} & =\tau_{\substack{\text { static } \\
\text { friction }}}=R F_{\text {fr }}=R \mu_{\mathrm{s}} F_{\mathrm{N}}=R \mu_{\mathrm{s}} \frac{1}{4} \mathrm{mg} \\
& =\frac{1}{4}(0.33 \mathrm{~m})(0.65)(950 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=5.0 \times 10^{2} \mathrm{~m} \cdot \mathrm{~N}
\end{aligned}
$$

98. (a) If there is no friction, then conservation of mechanical energy can be used to find the speed of the block. We assume the cord unrolls from the cylinder without slipping, and so $v_{\text {block }}=v_{\text {cord }}=\omega_{\text {cord }} R$. We take the zero position of gravitational potential energy to be the
bottom of the motion of the block. Since the cylinder does not move vertically, we do not have to consider its gravitational potential energy.

$$
\begin{aligned}
& E_{\text {initial }}=E_{\text {final }} \rightarrow U_{\text {initial }}=K_{\text {final }}=K_{\text {block }}+K_{\text {cylinder }} \rightarrow \\
& m g h=\frac{1}{2} m v^{2}+\frac{1}{2} I \omega^{2} \rightarrow m g D \sin \theta=\frac{1}{2} m v^{2}+\frac{1}{2}\left(\frac{1}{2} M R^{2}\right)\left(\frac{v^{2}}{R^{2}}\right) \rightarrow \\
& v=\sqrt{\frac{2 m g D \sin \theta}{\left(m+\frac{1}{2} M\right)}}=\sqrt{\frac{2(3.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(1.80 \mathrm{~m}) \sin 27^{\circ}}{(19.5 \mathrm{~kg})}}=1.570 \mathrm{~m} / \mathrm{s} \approx 1.6 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(b) The first printing of the textbook has $\mu=0.055$, while later printings will have $\mu=0.035$. The results are fundamentally different in the two cases. Consider the free body diagrams for both the block and the cylinder. We make the following observations and assumptions. Note that for the block to move down the plane from rest, $F_{\mathrm{T}}<m g$. Also note that $m g<0.1 M g$ due to the difference in masses. Thus
 $F_{\mathrm{T}}<0.1 M g$. Accordingly, we will ignore $F_{\mathrm{T}}$ when finding the net vertical and horizontal forces on the cylinder, knowing that we will make less than a $10 \%$ error. Instead of trying to assign a specific direction for the force of friction between the cylinder and the depression $\left(F_{\mathrm{fr} 2}\right)$, we show a torque in the counterclockwise direction (since the cylinder will rotate clockwise). Finally, we assume that $F_{\mathrm{fr} 2}=\mu F_{\mathrm{N} 2}=\mu M g$.


Write Newton's second law to analyze the linear motion of the block and the rotational motion of the cylinder, and solve for the acceleration of the block. We assume the cord unrolls without slipping.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{N}}-m g \cos \theta=0 \rightarrow F_{\mathrm{N}}=m g \cos \theta \\
& \sum F_{x}=m g \sin \theta-F_{\mathrm{T}}-F_{\mathrm{fr} 1}=m g \sin \theta-F_{\mathrm{T}}-\mu m g \cos \theta=m a \\
& \sum \tau=F_{\mathrm{T}} R-\tau_{\mathrm{fr} 2}=F_{\mathrm{T}} R-\mu F_{\mathrm{N} 2} R=F_{\mathrm{T}} R-\mu M g R=I \alpha=I \frac{a}{R}=\frac{1}{2} M R a \rightarrow \\
& \quad F_{\mathrm{T}}-\mu M g=\frac{1}{2} M a
\end{aligned}
$$

Add the $x$ equation to the torque equation.

$$
\begin{aligned}
& \quad m g \sin \theta-F_{\mathrm{T}}-\mu m g \cos \theta=m a ; F_{\mathrm{T}}-\mu M g=\frac{1}{2} M a \rightarrow \\
& m g \sin \theta-\mu M g-\mu m g \cos \theta=m a+\frac{1}{2} M a \rightarrow \\
& a=g \frac{m(\sin \theta-\mu \cos \theta)-\mu M}{\left(m+\frac{1}{2} M\right)} \\
& \text { If } \mu=0.055, a=g \frac{(3.0 \mathrm{~kg})\left(\sin 27^{\circ}-0.055 \cos 27^{\circ}\right)-(0.055)(33 \mathrm{~kg})}{(19.5 \mathrm{~kg})}=-0.302 \mathrm{~m} / \mathrm{s}^{2} . \text { But the }
\end{aligned}
$$

object cannot accelerate UP the plane from rest. So the conclusion is that object will not move with $\mu=0.055$. The small block is not heavy enough to move itself, rotate the cylinder, and overcome friction.

$$
\text { If } \mu=0.035, a=g \frac{(3.0 \mathrm{~kg})\left(\sin 27^{\circ}-0.035 \cos 27^{\circ}\right)-(0.035)(33 \mathrm{~kg})}{(19.5 \mathrm{~kg})}=0.057 \mathrm{~m} / \mathrm{s}^{2} .
$$

Use Eq. 2-12c to find the speed after moving 1.80 m .

$$
v^{2}=v_{0}^{2}+2 a \Delta x \rightarrow v=\sqrt{2\left(0.057 \mathrm{~m} / \mathrm{s}^{2}\right)(1.80 \mathrm{~m})}=0.45 \mathrm{~m} / \mathrm{s} .
$$

99. (a) See the free body diagram. Take clockwise torques as positive. Write Newton's second law for the rotational motion. The angular acceleration is constant, and so constant acceleration relationships can be used. We also use the definition of radian angles, $\Delta \theta=\frac{\Delta s}{R}$.


$$
\sum \tau=F R-\tau_{\mathrm{fr}}=I \alpha_{1} ; \Delta \theta_{1}=\omega_{0} t_{1}+\frac{1}{2} \alpha_{1} t_{1}^{2}=\frac{1}{2} \alpha_{1} t_{1}^{2} ; \Delta s_{1}=R \Delta \theta_{1}
$$

Combine the relationships to find the length unrolled, $\Delta s_{1}$.

$$
\begin{aligned}
\Delta s_{1} & =R \Delta \theta_{1}=R\left(\frac{1}{2} \alpha_{1} t_{1}^{2}\right)=\frac{R t_{1}^{2}}{2 I}\left(F R-\tau_{\mathrm{fr}}\right) \\
& =\frac{(0.076 \mathrm{~m})(1.3 \mathrm{~s})^{2}}{2\left(3.3 \times 10^{-3} \mathrm{~kg} \cdot \mathrm{~m}^{2}\right)}[(2.5 \mathrm{~N})(0.076 \mathrm{~m})-(0.11 \mathrm{~m} \cdot \mathrm{~N})]=1.557 \mathrm{~m} \approx 1.6 \mathrm{~m}
\end{aligned}
$$

(b) Now the external force is removed, but the frictional torque is still present. The analysis is very similar to that in part (a), except that the initial angular velocity is needed. That angular velocity is the final angular velocity from the motion in part $(a)$.

$$
\begin{aligned}
& \omega_{1}=\omega_{0}+\alpha_{1} t_{1}=\left(\frac{F R-\tau_{\mathrm{ff}}}{I}\right) t_{1}=\frac{[(2.5 \mathrm{~N})(0.076 \mathrm{~m})-(0.11 \mathrm{~m} \cdot \mathrm{~N})]}{\left(3.3 \times 10^{-3} \mathrm{~kg} \cdot \mathrm{~m}^{2}\right)}(1.3 \mathrm{~s})=31.515 \mathrm{rad} / \mathrm{s} \\
& \sum \tau=-\tau_{\mathrm{ff}}=I \alpha_{2} ; \omega_{2}^{2}-\omega_{1}^{2}=2 \alpha_{2} \Delta \theta_{2}=-\omega_{1}^{2} ; \Delta s_{2}=R \Delta \theta_{2}
\end{aligned}
$$

Combine the relationships to find the length unrolled, $\Delta s_{2}$.

$$
\begin{aligned}
\Delta s_{2} & =R \Delta \theta_{2}=R\left(\frac{-\omega_{1}^{2}}{2 \alpha_{2}}\right)=R\left(\frac{-\omega_{1}^{2} I}{-2 \tau_{\mathrm{fr}}}\right)=\frac{(0.076 \mathrm{~m})(31.515 \mathrm{rad} / \mathrm{s})^{2}\left(3.3 \times 10^{-3} \mathrm{~kg} \cdot \mathrm{~m}^{2}\right)}{2(0.11 \mathrm{~m} \cdot \mathrm{~N})} \\
& =1.13 \mathrm{~m} \approx 1.1 \mathrm{~m}
\end{aligned}
$$

100. (a) The disk starts from rest, and so the velocity of the center of mass is in the direction of the net force: $\overrightarrow{\mathbf{v}}=\overrightarrow{\mathbf{v}}_{0}+\overrightarrow{\mathbf{a}} t \rightarrow \overrightarrow{\mathbf{v}}=\overrightarrow{\mathbf{F}}_{\text {net }} \frac{t}{m}$. Thus the center of mass moves to the right.
(b) For the linear motion of the center of mass, we may apply constant acceleration equations, where the acceleration is $\frac{F}{m}$.

$$
v^{2}=v_{0}^{2}+2 a \Delta x \rightarrow v=\sqrt{2 \frac{F}{m} \Delta x}=\sqrt{2 \frac{(35 \mathrm{~N})}{(21.0 \mathrm{~kg})}(5.5 \mathrm{~m})}=4.282 \mathrm{~m} / \mathrm{s} \approx 4.3 \mathrm{~m} / \mathrm{s}
$$

(c) The only torque is a constant torque caused by the constant string tension. That can be used to find the angular velocity.

$$
\tau=I \alpha=I\left(\frac{\omega-\omega_{0}}{t}\right)=\frac{I \omega}{t}=F r \quad \rightarrow \omega=\frac{F r t}{I}=\frac{F r t}{\frac{1}{2} m r^{2}}=\frac{2 F t}{m r}
$$

The time can be found from the center of mass motion under constant acceleration.

$$
\Delta x=v_{0} t+\frac{1}{2} a t^{2}=\frac{1}{2} \frac{F}{m} t^{2} \rightarrow t=\sqrt{\frac{2 m \Delta x}{F}}
$$

$$
\begin{aligned}
\omega & =\frac{2 F t}{m r}=\frac{2 F}{m r} \sqrt{\frac{2 m \Delta x}{F}}=\frac{2}{r} \sqrt{\frac{2 F \Delta x}{m}}=\frac{2}{(0.850 \mathrm{~m})} \sqrt{\frac{2(35.0 \mathrm{~N})(5.5 \mathrm{~m})}{(21.0 \mathrm{~kg})}} \\
& =10.07 \mathrm{rad} / \mathrm{s} \approx 10 \mathrm{rad} / \mathrm{s}(2 \mathrm{sig} \mathrm{fig})
\end{aligned}
$$

Note that $v \neq \omega r$ since the disk is NOT rolling without slipping.
(d) The amount of string that has unwrapped is related to the angle through which the disk has turned, by the definition of radian measure, $s=r \Delta \theta$. The angular displacement is found from constant acceleration relationships.

$$
\begin{aligned}
& \Delta \theta=\frac{1}{2}\left(\omega_{0}+\omega\right) t=\frac{1}{2} \omega t=\frac{1}{2}\left(\frac{2 F t}{m r}\right) t=\frac{F t^{2}}{m r}=\frac{F \frac{2 m \Delta x}{F}}{m r}=\frac{2 \Delta x}{r} \\
& s=r \Delta \theta=r \frac{2 \Delta x}{r}=2 \Delta x=11 \mathrm{~m}
\end{aligned}
$$

101. (a) We assume that the front wheel is barely lifted off the ground, so that the only forces that act on the system are the normal force on the bike's rear wheel, the static frictional force on the bike's wheel, and the total weight of the system. We assume that the upward acceleration is zero and the angular acceleration about the center of mass is also zero. Write Newton's second law for the $x$ direction, the $y$ direction, and rotation. Take positive torques to be clockwise.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{N}}-M g=0 \rightarrow F_{\mathrm{N}}=M g \\
& \sum F_{x}=F_{\mathrm{fr}}=M a \rightarrow a=\frac{F_{\mathrm{fr}}}{M} \\
& \sum \tau_{\mathrm{CM}}=F_{\mathrm{N}} x-F_{\mathrm{fr}} y=0
\end{aligned}
$$



Combine these equations to solve for the acceleration.

$$
F_{\mathrm{N}} x-F_{\mathrm{f} y} y=0 \rightarrow M g x=\text { May } \rightarrow a=\frac{x}{y} g
$$

(b) Based on the form of the solution for the acceleration, $a=\frac{x}{y} g$, to minimize the acceleration $x$ should be as small as possible and $y$ should be as large as possible. The rider should move upwards and towards the rear of the bicycle.
(c)

$$
a=\frac{x}{y} g=\frac{0.35 \mathrm{~m}}{0.95 \mathrm{~m}}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=3.6 \mathrm{~m} / \mathrm{s}^{2}
$$

102. We follow the hint given in the problem. The mass of the cutout piece is proportional by area to the mass of the entire piece.

$$
\begin{aligned}
& I_{\text {total }}=\frac{1}{2} M R_{0}^{2}=I_{\text {remainder }}+I_{\text {cutout }} \rightarrow I_{\text {remainder }}=\frac{1}{2} M R_{0}^{2}-I_{\text {cutout }} \\
& I_{\text {cutout }}=\frac{1}{2} m_{\text {cutout }} R_{1}^{2}+m_{\text {cutout }} h^{2} ; m_{\text {cutout }}=\frac{M}{\pi R_{0}^{2}} \pi R_{1}^{2}=M \frac{R_{1}^{2}}{R_{0}^{2}} \rightarrow \\
& I_{\text {remainder }}=\frac{1}{2} M R_{0}^{2}-\left(\frac{1}{2} m_{\text {cutout }} R_{1}^{2}+m_{\text {cutout }} h^{2}\right)=\frac{1}{2} M R_{0}^{2}-M \frac{R_{1}^{2}}{R_{0}^{2}}\left(\frac{1}{2} R_{1}^{2}+h^{2}\right)
\end{aligned}
$$

$$
=\frac{1}{2} \frac{M}{R_{0}^{2}}\left(R_{0}^{4}-R_{1}^{4}-2 R_{1}^{2} h^{2}\right)
$$

103. Since there is no friction at the table, there are no horizontal forces on the rod, and so the center of mass will fall straight down. The moment of inertia of the rod about its center of mass is $\frac{1}{12} M \ell^{2}$. Since there are no dissipative forces, energy will be conserved during the fall. Take the zero level of gravitational potential energy to be at the tabletop. The angular velocity and the center of mass velocity are related by $\omega_{\mathrm{CM}}=\frac{v_{\mathrm{CM}}}{\left(\frac{1}{2} \ell\right)}$.

$$
\begin{aligned}
& E_{\text {initial }}=E_{\text {final }} \rightarrow U_{\text {release }}=K_{\text {final }} \rightarrow M g\left(\frac{1}{2} \ell\right)=\frac{1}{2} M v_{\mathrm{CM}}^{2}+\frac{1}{2} I \omega_{\mathrm{CM}}^{2} \rightarrow \\
& 0 \\
& M g\left(\frac{1}{2} \ell\right)=\frac{1}{2} M v_{\mathrm{CM}}^{2}+\frac{1}{2}\left(\frac{1}{12} M \ell^{2}\right)\left[\frac{v_{\mathrm{CM}}}{\left(\frac{1}{2} \ell\right)}\right]^{2} \rightarrow g \ell=\frac{4}{3} v_{\mathrm{CM}}^{2} \rightarrow v_{\mathrm{CM}}=\sqrt{\frac{3}{4} g \ell}
\end{aligned}
$$

104. (a) The acceleration is found in Example 10-19 to be a constant value, $a=\frac{2}{3} g$, and so constant acceleration kinematics can be used. Take downward to be the positive direction.

$$
v_{y}^{2}=v_{y 0}^{2}+2 a_{y} \Delta y \rightarrow v_{y}=\sqrt{2 a_{y} \Delta y}=\sqrt{2 \frac{2}{3} g h}=\sqrt{\frac{4}{3} g h}
$$

(b) We take the zero level for gravitational potential energy to be the starting height of the yo-yo. Then the final gravitational potential energy is negative.

$$
\begin{aligned}
& E_{\text {initial }}=E_{\text {final }} \rightarrow 0=U_{\text {final }}+K_{\text {final }}=-M g h+\frac{1}{2} M v_{\mathrm{CM}}^{2}+\frac{1}{2} I \omega_{\mathrm{CM}}^{2} \rightarrow \\
& M g h=\frac{1}{2} M v_{\mathrm{CM}}^{2}+\frac{1}{2}\left(\frac{1}{2} M R^{2}\right)\left(\frac{v_{\mathrm{CM}}}{R}\right)^{2} \rightarrow v_{\mathrm{CM}}=\sqrt{\frac{4}{3} g h}
\end{aligned}
$$

105. From the diagram, we see that the torque about the support A is as follows.

$$
\begin{aligned}
\tau & =R_{\perp} F=\left(\ell_{1} \cos \theta+\ell_{2}\right) F \\
& =[(0.300 \mathrm{~m}) \cos \theta+0.200 \mathrm{~m}](500 \mathrm{~N})
\end{aligned}
$$

The graph of torque as a function of angle is shown.


The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH10.XLS," on tab "Problem 10.105."
106. From problem 51, the acceleration is as follows.

$$
a=\frac{\left(m_{\mathrm{B}}-m_{\mathrm{A}}\right)}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}+I / R^{2}\right)} g=\frac{(0.200 \mathrm{~kg})}{\left(0.500 \mathrm{~kg}+I /(0.040 \mathrm{~m})^{2}\right)}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)
$$

(a) The graph is shown here. The spreadsheet used for this problem can be found on the Media Manager, with filename
"PSE4_ISM_CH10.XLS," on tab "Problem 10.106a."
(b) The value of the acceleration with a zero moment of inertia is found as follows.

$$
\begin{aligned}
a & =\frac{(0.200 \mathrm{~kg})}{(0.500 \mathrm{~kg})}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =3.92 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$


(c) A $2.0 \%$ decrease in the acceleration means the acceleration is as follows.
$a=3.92 \mathrm{~m} / \mathrm{s}^{2}(0.98)=3.84 \mathrm{~m} / \mathrm{s}^{2}$. Looking at the graph, that would occur roughly for a moment of inertia of $1.6 \times 10^{-5} \mathrm{~kg} \cdot \mathrm{~m}^{2}$.
(d) Using the value above gives the following pulley mass.

$$
I=\frac{1}{2} m r^{2}=1.6 \times 10^{-5} \mathrm{~kg} \cdot \mathrm{~m}^{2} \rightarrow m=\frac{2 I}{R^{2}}=2\left(\frac{1.6 \times 10^{-5} \mathrm{~kg} \cdot \mathrm{~m}^{2}}{(0.040 \mathrm{~m})^{2}}\right)=0.020 \mathrm{~kg}=20 \text { grams }
$$

## CHAPTER 11: Angular Momentum; General Rotation

## Responses to Questions

(a) With more people at the equator, more mass would be farther from the axis of rotation, and the moment of inertia of the Earth would increase. Due to conservation of angular momentum, the Earth's angular velocity would decrease. The length of the day would increase.
2. No. Once the diver is in the air, there will be no net torque on her and therefore angular momentum will be conserved. If she leaves the board with no initial rotation, her initial angular momentum will be zero. Conservation of angular momentum requires that her final angular momentum will also be zero, so she will not be able to do a somersault.
3. Your angular velocity will stay the same. The angular momentum of the system of you and the stool and the masses is conserved. The masses carry off their angular momentum (until they hit something); you and the stool continue to rotate as before.
4. Once the motorcycle leaves the ground, there is no net torque on it and angular momentum must be conserved. If the throttle is on, the rear wheel will spin faster as it leaves the ground because there is no torque from the ground acting on it. The front of the motorcycle must rise up, or rotate in the direction opposite the rear wheel, in order to conserve angular momentum.
5. As you walk toward the center, the moment of inertia of the system of you + the turntable will decrease. No external torque is acting on the system, so angular momentum must be conserved, and the angular speed of the turntable will increase.
6. When the player is in the air, there is no net torque on him so his total angular momentum must be conserved. If his upper body rotates one direction, his lower body will rotate the other direction to conserve angular momentum.
7. The cross product remains the same. $\overrightarrow{\mathbf{V}}_{1} \times \overrightarrow{\mathbf{V}}_{2}=\left(-\overrightarrow{\mathbf{V}}_{1}\right) \times\left(-\overrightarrow{\mathbf{V}}_{2}\right)$
8. The cross product of the two vectors will be zero if the magnitude of either vector is zero or if the vectors are parallel or anti-parallel to each other.
9. The torque about the CM, which is the cross product between $\mathbf{r}$ and $\mathbf{F}$, depends on $x$ and $z$, but not on $y$.
10. The angular momentum will remain constant. If the particle is moving in a straight line at constant speed, there is no net torque acting on it and therefore its angular momentum must be conserved.
11. No. If two equal and opposite forces act on an object, the net force will be zero. If the forces are not co-linear, the two forces will produce a torque. No. If an unbalanced force acts through the axis of rotation, there will be a net force on the object, but no net torque.
12. At the forward peak of the swinging motion, the child leans forward, increasing the gravitational torque about the axis of rotation by moving her center of mass forward. This increases the angular momentum of the system. At the back peak of the swinging motion, the child leans backward, increasing the gravitational torque about the axis of rotation by moving her center of mass backward.

This again increases the angular momentum of the system. As the angular momentum of the system increases, the swing goes higher.
13. A force directed to the left will produce a torque that will cause the axis of the rotating wheel to move directly upward.
14. In both cases, angular momentum must be conserved. Assuming that the astronaut starts with zero angular momentum, she must move her limbs so that her total angular momentum remains zero. The angular momentum of her limbs must be opposite the angular momentum of the rest of her body.
(a) In order to turn her body upside down, the astronaut could hold her arms straight out from her sides and rotate them from the shoulder in vertical circles. If she rotates them forward, her body will rotate backwards.
(b) To turn her body about-face, she could hold her arms straight out from her sides and then pull one across the front of her body while she pulls the other behind her back. If she moves her arms counterclockwise, her body will twist clockwise.
15. Once the helicopter has left the ground, no external torques act on it and angular momentum must be conserved. If there were only one propeller, then when the angular velocity of the propeller changed, the body of the helicopter would begin to rotate in a direction so as to conserve angular momentum. The second propeller can be in the same plane as the first, but spinning in the opposite direction, or perpendicular to the plane of the first. Either case will stabilize the helicopter.
16. The rotational speed of the wheel will not change. Angular momentum of the entire system is conserved, since no net torque operates on the wheel. The small parts of the wheel that fly off will carry angular momentum with them. The remaining wheel will have a lower angular momentum and a lower rotational kinetic energy since it will have the same angular velocity but a smaller mass, and therefore a smaller moment of inertia. The kinetic energy of the total system is not conserved.
17. (a) Displacement, velocity, acceleration, and momentum are independent of the choice of origin.
(b) Displacement, acceleration, and torque are independent of the velocity of the coordinate system.
18. Turning the steering wheel changes the axis of rotation of the tires, and makes the car turn. The torque is supplied by the friction between the tires and the pavement. (Notice that if the road is slippery or the tire tread is worn, the car will not be able to make a sharp turn.)
19. The Sun will pull on the bulge closer to it more than it pulls on the opposite bulge, due to the inverse-square law of gravity. These forces, and those from the Moon, create a torque which causes the precession of the axis of rotation of the Earth. The precession is about an axis perpendicular to the plane of the orbit. During the equinox, no torque exists, since the forces on the bulges lie along a line.
20. Because of the rotation of the Earth, the plumb bob will be slightly deflected by the Coriolis force, which is a "pseudoforce."
21. Newton's third law is not valid in a rotating reference frame, since there is no reaction to the pseudoforce.
22. In the Northern Hemisphere, the shots would be deflected to the right, with respect to the surface of the Earth, due to the Coriolis effect. In the Southern Hemisphere, the deflection of the shots would be to the left. The gunners had experience in the Northern Hemisphere and so miscalculated the necessary launch direction.

## Solutions to Problems

1. The angular momentum is given by Eq. 11-1.

$$
L=I \omega=M R^{2} \omega=(0.210 \mathrm{~kg})(1.35 \mathrm{~m})^{2}(10.4 \mathrm{rad} / \mathrm{s})=3.98 \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}
$$

2. (a) The angular momentum is given by Eq. 11-1.

$$
\begin{aligned}
L & =I \omega=\frac{1}{2} M R^{2} \omega=\frac{1}{2}(2.8 \mathrm{~kg})(0.18 \mathrm{~m})^{2}\left[\left(\frac{1300 \mathrm{rev}}{1 \mathrm{~min}}\right)\left(\frac{2 \pi \mathrm{rad}}{1 \mathrm{rev}}\right)\left(\frac{1 \mathrm{~min}}{60 \mathrm{~s}}\right)\right] \\
& =6.175 \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s} \approx 6.2 \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}
\end{aligned}
$$

(b) The torque required is the change in angular momentum per unit time. The final angular momentum is zero.

$$
\tau=\frac{L-L_{0}}{\Delta t}=\frac{0-6.175 \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}}{6.0 \mathrm{~s}}=-1.0 \mathrm{~m} \cdot \mathrm{~N}
$$

The negative sign indicates that the torque is used to oppose the initial angular momentum.
3. (a) Consider the person and platform a system for angular momentum analysis. Since the force and torque to raise and/or lower the arms is internal to the system, the raising or lowering of the arms will cause no change in the total angular momentum of the system. However, the rotational inertia increases when the arms are raised. Since angular momentum is conserved, an increase in rotational inertia must be accompanied by a decrease in angular velocity.
(b) $L_{i}=L_{f} \rightarrow I_{i} \omega_{i}=I_{f} \omega_{f} \rightarrow I_{f}=I_{i} \frac{\omega_{i}}{\omega_{f}}=I_{i} \frac{0.90 \mathrm{rev} / \mathrm{s}}{0.70 \mathrm{rev} / \mathrm{s}}=1.286 I_{i} \approx 1.3 I_{i}$

The rotational inertia has increased by a factor of 1.3 .
4. The skater's angular momentum is constant, since no external torques are applied to her.

$$
L_{i}=L_{f} \rightarrow I_{i} \omega_{i}=I_{f} \omega_{f} \rightarrow I_{f}=I_{i} \frac{\omega_{i}}{\omega_{f}}=\left(4.6 \mathrm{~kg} \cdot \mathrm{~m}^{2}\right) \frac{1.0 \mathrm{rev} / 1.5 \mathrm{~s}}{2.5 \mathrm{rev} / \mathrm{s}}=1.2 \mathrm{~kg} \cdot \mathrm{~m}^{2}
$$

She accomplishes this by starting with her arms extended (initial angular velocity) and then pulling her arms in to the center of her body (final angular velocity).
5. There is no net torque on the diver because the only external force (gravity) passes through the center of mass of the diver. Thus the angular momentum of the diver is conserved. Subscript 1 refers to the tuck position, and subscript 2 refers to the straight position.

$$
L_{1}=L_{2} \rightarrow I_{1} \omega_{1}=I_{2} \omega_{2} \rightarrow \omega_{2}=\omega_{1} \frac{I_{1}}{I_{2}}=\left(\frac{2 \mathrm{rev}}{1.5 \mathrm{sec}}\right)\left(\frac{1}{3.5}\right)=0.38 \mathrm{rev} / \mathrm{s}
$$

6. The angular momentum is the total moment of inertia times the angular velocity.

$$
L=I \omega=\left[\frac{1}{12} M \ell^{2}+2 m\left(\frac{1}{2} \ell\right)^{2}\right] \omega=\left(\frac{1}{12} M+\frac{1}{2} m\right) \ell^{2} \omega
$$

7. (a) For the daily rotation about its axis, treat the Earth as a uniform sphere, with an angular frequency of one revolution per day.

$$
L_{\text {daily }}=I \omega_{\text {daily }}=\left(\frac{2}{5} M R_{\text {Earth }}^{2}\right) \omega_{\text {daily }}
$$

$$
=\frac{2}{5}\left(6.0 \times 10^{24} \mathrm{~kg}\right)\left(6.4 \times 10^{6} \mathrm{~m}\right)^{2}\left[\left(\frac{2 \pi \text { rad }}{1 \text { day }}\right)\left(\frac{1 \text { day }}{86,400 \mathrm{~s}}\right)\right]=7.1 \times 10^{33} \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}
$$

(b) For the yearly revolution about the Sun, treat the Earth as a particle, with an angular frequency of one revolution per year.

$$
\begin{aligned}
L_{\text {daily }}= & I \omega_{\text {daily }}=\left(M R_{\text {Sur- }}^{2}\right) \omega_{\text {Earth }} \\
& =\left(6.0 \times 10^{24} \mathrm{~kg}\right)\left(1.5 \times 10^{11} \mathrm{~m}\right)^{2}\left[\left(\frac{2 \pi \mathrm{rad}}{365 \text { day }}\right)\left(\frac{1 \text { day }}{86,400 \mathrm{~s}}\right)\right]=2.7 \times 10^{40} \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}
\end{aligned}
$$

8. (a) $L=I \omega=\frac{1}{2} M R^{2} \omega=\frac{1}{2}(48 \mathrm{~kg})(0.15 \mathrm{~m})^{2}\left(2.8 \frac{\mathrm{rev}}{\mathrm{s}}\right)\left(\frac{2 \pi \mathrm{rad}}{1 \mathrm{rev}}\right)=9.50 \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s} \approx 9.5 \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}$
(b) If the rotational inertia does not change, then the change in angular momentum is strictly due to a change in angular velocity.

$$
\tau=\frac{\Delta L}{\Delta t}=\frac{I \omega_{\text {final }}-I \omega_{0}}{\Delta t}=\frac{0-9.50 \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}}{5.0 \mathrm{~s}}=-1.9 \mathrm{~m} \cdot \mathrm{~N}
$$

The negative sign indicates that the torque is in the opposite direction as the initial angular momentum.
9. When the person and the platform rotate, they do so about the vertical axis. Initially there is no angular momentum pointing along the vertical axis, and so any change that the person-wheelplatform undergoes must result in no net angular momentum along the vertical axis.
(a) If the wheel is moved so that its angular momentum points upwards, then the person and platform must get an equal but opposite angular momentum, which will point downwards. Write the angular momentum conservation condition for the vertical direction to solve for the angular velocity of the platform.

$$
L_{i}=L_{f} \rightarrow 0=I_{\mathrm{w}} \omega_{\mathrm{w}}+I_{\mathrm{p}} \omega_{\mathrm{p}} \rightarrow \omega_{\mathrm{p}}=-\frac{I_{\mathrm{w}}}{I_{\mathrm{p}}} \omega_{\mathrm{w}}
$$

The negative sign means that the platform is rotating in the opposite direction of the wheel. If the wheel is spinning counterclockwise when viewed from above, the platform is spinning clockwise.
(b) If the wheel is pointing at a $60^{\circ}$ angle to the vertical, then the component of its angular momentum that is along the vertical direction is $I_{\mathrm{w}} \omega_{\mathrm{w}} \cos 60^{\circ}$. See the diagram. Write the angular momentum conservation condition for the vertical direction to solve for the angular
 velocity of the platform.

$$
L_{i}=L_{f} \rightarrow 0=I_{\mathrm{w}} \omega_{\mathrm{W}} \cos 60^{\circ}+I_{\mathrm{p}} \omega_{\mathrm{P}} \rightarrow \omega_{\mathrm{p}}=-\frac{I_{\mathrm{W}}}{2 I_{\mathrm{P}}} \omega_{\mathrm{W}}
$$

Again, the negative sign means that the platform is rotating in the opposite direction of the wheel.
(c) If the wheel is moved so that its angular momentum points downwards, then the person and platform must get an equal but opposite angular momentum, which will point upwards. Write the angular momentum conservation condition for the vertical direction to solve for the angular velocity of the platform.

$$
L_{i}=L_{f} \rightarrow 0=-I_{\mathrm{w}} \omega_{\mathrm{w}}+I_{\mathrm{p}} \omega_{\mathrm{p}} \rightarrow \omega_{\mathrm{p}}=\omega_{\mathrm{w}} I_{\mathrm{w}} / I_{\mathrm{p}}
$$

The platform is rotating in the same direction as the wheel. If the wheel is spinning counterclockwise when viewed from above, the platform is also spinning counterclockwise.
(d) Since the total angular momentum is 0 , if the wheel is stopped from rotating, the platform will also stop. Thus $\omega_{\mathrm{P}}=0$.
10. The angular momentum of the disk-rod combination will be conserved because there are no external torques on the combination. This situation is a totally inelastic collision, in which the final angular velocity is the same for both the disk and the rod. Subscript 1 represents before the collision, and subscript 2 represents after the collision. The rod has no initial angular momentum.

$$
\begin{aligned}
& L_{1}=L_{2} \rightarrow I_{1} \omega_{1}=I_{2} \omega_{2} \rightarrow \\
& \omega_{2}=\omega_{1} \frac{I_{1}}{I_{2}}=\omega_{1} \frac{I_{\mathrm{disk}}}{I_{\mathrm{disk}}+I_{\mathrm{rod}}}=\omega_{1}\left[\frac{\frac{1}{2} M R^{2}}{\frac{1}{2} M R^{2}+\frac{1}{12} M(2 R)^{2}}\right]=(3.7 \mathrm{rev} / \mathrm{s})\left(\frac{3}{5}\right)=2.2 \mathrm{rev} / \mathrm{s}
\end{aligned}
$$

11. Since the person is walking radially, no torques will be exerted on the person-platform system, and so angular momentum will be conserved. The person will be treated as a point mass. Since the person is initially at the center, they have no initial rotational inertia.
(a) $L_{i}=L_{f} \rightarrow I_{\text {platform }} \omega_{i}=\left(I_{\text {platform }}+I_{\text {person }}\right) \omega_{f}$

$$
\omega_{f}=\frac{I_{\text {platform }}}{I_{\text {platform }}+m R^{2}} \omega_{i}=\frac{920 \mathrm{~kg} \cdot \mathrm{~m}^{2}}{920 \mathrm{~kg} \cdot \mathrm{~m}^{2}+(75 \mathrm{~kg})(3.0 \mathrm{~m})^{2}}(0.95 \mathrm{rad} / \mathrm{s})=0.548 \mathrm{rad} / \mathrm{s} \approx 0.55 \mathrm{rad} / \mathrm{s}
$$

(b) $K E_{i}=\frac{1}{2} I_{\text {platform }} \omega_{i}^{2}=\frac{1}{2}\left(920 \mathrm{~kg} \cdot \mathrm{~m}^{2}\right)(0.95 \mathrm{rad} / \mathrm{s})^{2}=420 \mathrm{~J}$

$$
\begin{aligned}
K E_{f} & =\frac{1}{2}\left(I_{\text {platform }}+I_{\text {person }}\right) \omega_{f}^{2}=\frac{1}{2}\left(I_{\text {platform }}+m_{\text {person }} r_{\text {person }}^{2}\right) \omega_{f}^{2} \\
& =\frac{1}{2}\left[920 \mathrm{~kg} \cdot \mathrm{~m}^{2}+(75 \mathrm{~kg})(3.0 \mathrm{~m})^{2}\right](0.548 \mathrm{rad} / \mathrm{s})^{2}=239 \mathrm{~J} \approx 240 \mathrm{~J}
\end{aligned}
$$

12. Because there is no external torque applied to the wheel-clay system, the angular momentum will be conserved. We assume that the clay is thrown with no angular momentum so that its initial angular momentum is 0 . This situation is a totally inelastic collision, in which the final angular velocity is the same for both the clay and the wheel. Subscript 1 represents before the clay is thrown, and subscript 2 represents after the clay is thrown.

$$
\begin{aligned}
L_{1} & =L_{2} \rightarrow I_{1} \omega_{1}=I_{2} \omega_{2} \rightarrow \\
\omega_{2} & =\omega_{1} \frac{I_{1}}{I_{2}}=\frac{I_{\text {wheel }}}{I_{\text {wheel }}+I_{\text {clay }}}=\omega_{1}\left(\frac{\frac{1}{2} M_{\text {wheel }} R_{\text {wheel }}^{2}}{\frac{1}{2} M_{\text {wheel }} R_{\text {wheel }}^{2}+\frac{1}{2} M_{\text {clay }} R_{\text {clay }}^{2}}\right)=\omega_{1}\left(\frac{M_{\text {wheel }} R_{\text {wheel }}^{2}}{M_{\text {wheel }} R_{\text {wheel }}^{2}+M_{\text {clay }} R_{\text {clay }}^{2}}\right) \\
& =(1.5 \mathrm{rev} / \mathrm{s})\left[\frac{(5.0 \mathrm{~kg})(0.20 \mathrm{~m})^{2}}{(5.0 \mathrm{~kg})(0.20 \mathrm{~m})^{2}+(2.6 \mathrm{~kg})\left(8.0 \times 10^{-2} \mathrm{~m}\right)^{2}}\right]=1.385 \mathrm{rev} / \mathrm{s} \approx 1.4 \mathrm{rev} / \mathrm{s}
\end{aligned}
$$

13. The angular momentum of the merry-go-round and people combination will be conserved because there are no external torques on the combination. This situation is a totally inelastic collision, in which the final angular velocity is the same for both the merry-go-round and the people. Subscript 1 represents before the collision, and subscript 2 represents after the collision. The people have no initial angular momentum.

$$
L_{1}=L_{2} \rightarrow I_{1} \omega_{1}=I_{2} \omega_{2} \rightarrow
$$

$$
\begin{aligned}
\omega_{2} & =\omega_{1} \frac{I_{1}}{I_{2}}=\omega_{1} \frac{I_{\mathrm{m}-\mathrm{g}-\mathrm{r}}}{I_{\mathrm{m}-\mathrm{g}-\mathrm{r}}+I_{\text {people }}}=\omega_{1}\left[\frac{I_{\mathrm{m}-\mathrm{g}-\mathrm{r}}}{I_{\mathrm{m}-\mathrm{g}-\mathrm{r}}+4 M_{\text {person }} R^{2}}\right] \\
& =(0.80 \mathrm{rad} / \mathrm{s})\left[\frac{1760 \mathrm{~kg} \cdot \mathrm{~m}^{2}}{1760 \mathrm{~kg} \cdot \mathrm{~m}^{2}+4(65 \mathrm{~kg})(2.1 \mathrm{~m})^{2}}\right]=0.48 \mathrm{rad} / \mathrm{s}
\end{aligned}
$$

If the people jump off the merry-go-round radially, then they exert no torque on the merry-go-round, and thus cannot change the angular momentum of the merry-go-round. The merry-go-round would continue to rotate at $0.80 \mathrm{rad} / \mathrm{s}$.
14. (a) The angular momentum of the system will be conserved as the woman walks. The woman's distance from the axis of rotation is $r=R-v t$.

$$
\begin{aligned}
& L_{i}=L_{f} \rightarrow\left(I_{\text {platform }}+\underset{\text { woman }}{ }\right) \omega_{0}=\left(I_{\text {plaftorm }}+I_{\text {woman }}\right) \omega \rightarrow \\
& \left(\frac{1}{2} M R^{2}+m R^{2}\right) \omega_{0}=\left(\frac{1}{2} M R^{2}+m(R-v t)^{2}\right) \omega \rightarrow \\
& \omega=\frac{\left(\frac{1}{2} M R^{2}+m R^{2}\right) \omega_{0}}{\left(\frac{1}{2} M R^{2}+m(R-v t)^{2}\right)}=\frac{\left(\frac{1}{2} M+m\right) \omega_{0}}{\frac{1}{2} M+m\left(1-\frac{v t}{R}\right)^{2}}
\end{aligned}
$$

(b) Evaulate at $r=R-v t=0 \rightarrow R=v t$.

$$
\omega=\frac{\left(\frac{1}{2} M+m\right) \omega_{0}}{\frac{1}{2} M}=\left(1+\frac{2 m}{M}\right) \omega_{0}
$$

15. Since there are no external torques on the system, the angular momentum of the 2-disk system is conserved. The two disks have the same final angular velocity.

$$
L_{i}=L_{f} \rightarrow I \omega+I(0)=2 I \omega_{f} \rightarrow \omega_{f}=\frac{1}{2} \omega
$$

16. Since the lost mass carries away no angular momentum, the angular momentum of the remaining mass will be the same as the initial angular momentum.

$$
\begin{aligned}
& L_{i}=L_{f} \rightarrow I_{i} \omega_{i}=I_{f} \omega_{f} \rightarrow \frac{\omega_{f}}{\omega_{i}}=\frac{I_{i}}{I_{f}}=\frac{\frac{2}{5} M_{i} R_{i}^{2}}{\frac{2}{5} M_{f} R_{f}^{2}}=\frac{M_{i} R_{i}^{2}}{\left(0.5 M_{i}\right)\left(0.01 R_{f}\right)^{2}}=2.0 \times 10^{4} \\
& \omega_{f}=2.0 \times 10^{4} \omega_{i}=2.0 \times 10^{4}\left(\frac{2 \pi \mathrm{rad}}{30 \mathrm{day}}\right)\left(\frac{1 \mathrm{~d}}{86400 \mathrm{~s}}\right)=4.848 \times 10^{-2} \mathrm{rad} / \mathrm{s} \approx 5 \times 10^{-2} \mathrm{rad} / \mathrm{s}
\end{aligned}
$$

The period would be a factor of 20,000 smaller, which would make it about 130 seconds. The ratio of angular kinetic energies of the spinning mass would be as follows.

$$
\begin{aligned}
& \frac{K_{\text {final }}}{K_{\text {initial }}}=\frac{\frac{1}{2} I_{f} \omega_{f}^{2}}{\frac{1}{2} I_{i} \omega_{i}^{2}}=\frac{\frac{1}{2}\left[\frac{2}{5}\left(0.5 M_{i}\right)\left(0.01 R_{i}\right)^{2}\right]\left(2.0 \times 10^{4} \omega_{i}\right)^{2}}{\frac{1}{2}\left(\frac{2}{5} M_{i} R_{i}^{2}\right) \omega_{i}^{2}}=2.0 \times 10^{4} \rightarrow \\
& K_{\text {final }}=2 \times 10^{4} K_{\text {initial }}
\end{aligned}
$$

17. For our crude estimate, we model the hurricane as a rigid cylinder of air. Since the "cylinder" is rigid, each part of it has the same angular velocity. The mass of the air is the product of the density of air times the volume of the air cylinder.

$$
M=\rho V=\rho \pi R^{2} h=\left(1.3 \mathrm{~kg} / \mathrm{m}^{3}\right) \pi\left(8.5 \times 10^{4} \mathrm{~m}\right)^{2}\left(4.5 \times 10^{3} \mathrm{~m}\right)=1.328 \times 10^{14} \mathrm{~kg}
$$

(a) $K=\frac{1}{2} I \omega^{2}=\frac{1}{2}\left(\frac{1}{2} M R^{2}\right)\left(v_{\text {edge }} / R\right)^{2}=\frac{1}{4} M v_{\text {edge }}^{2}$

$$
=\frac{1}{4}\left(1.328 \times 10^{14} \mathrm{~kg}\right)\left[(120 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)\right]^{2}=3.688 \times 10^{16} \mathrm{~J} \approx 3.7 \times 10^{16} \mathrm{~J}
$$

(b)

$$
\begin{aligned}
L & =I \omega=\left(\frac{1}{2} M R^{2}\right)\left(v_{\text {edge }} / R\right)=\frac{1}{2} M R v_{\text {edge }} \\
& =\frac{1}{2}\left(1.328 \times 10^{14} \mathrm{~kg}\right)\left(8.5 \times 10^{4} \mathrm{~m}\right)\left[(120 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)\right]=2.213 \times 10^{20} \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s} \\
& \approx 1.9 \times 10^{20} \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}
\end{aligned}
$$

18. Angular momentum will be conserved in the Earth-asteroid system, since all forces and torques are internal to the system. The initial angular velocity of the satellite, just before collision, can be found from $\omega_{\text {asteroid }}=v_{\text {asteroid }} / R_{\text {Earth }}$. Assuming the asteroid becomes imbedded in the Earth at the surface, the Earth and the asteroid will have the same angular velocity after the collision. We model the Earth as a uniform sphere, and the asteroid as a point mass.

$$
L_{i}=L_{f} \rightarrow I_{\text {Earth }} \omega_{\text {Earth }}+I_{\text {asteroid }} \omega_{\text {asteroid }}=\left(I_{\text {Earth }}+I_{\text {asteroid }}\right) \omega_{f}
$$

The moment of inertia of the satellite can be ignored relative to that of the Earth on the right side of the above equation, and so the percent change in Earth's angular velocity is found as follows.

$$
\begin{aligned}
& I_{\text {Earth }} \omega_{\text {Earth }}+I_{\text {asteroid }} \omega_{\text {asteroid }}=I_{\text {Earth }} \omega_{f} \rightarrow \frac{\left(\omega_{f}-\omega_{\text {Earth }}\right)}{\omega_{\text {Earth }}}=\frac{I_{\text {asteroid }}}{I_{\text {Earth }}} \frac{\omega_{\text {asteroid }}}{\omega_{\text {Earth }}} \\
& \begin{aligned}
& \% \text { change }=\frac{\left(\omega_{f}-\omega_{\text {Earth }}\right)}{\omega_{\text {Earth }}}(100)=\frac{m_{\text {asteroid }} R_{\text {Earth }}^{2}}{\frac{2}{5} M_{\text {Earth }} R_{\text {Earth }}^{2}} \frac{v_{\text {asterocid }}}{R_{\text {Earth }}} \\
& \omega_{\text {Earth }} \frac{m_{\text {asteroid }}}{\frac{2}{5} M_{\text {Earth }}} \frac{v_{\text {asteroid }}}{\omega_{\text {Earth }} R_{\text {Earth }}}(100) \\
&=\frac{\left(1.0 \times 10^{5} \mathrm{~kg}\right)\left(3.5 \times 10^{4} \mathrm{~m} / \mathrm{s}\right)}{(0.4)\left(5.97 \times 10^{24} \mathrm{~kg}\right)\left(\frac{2 \pi \mathrm{rad}}{86400 \mathrm{~s}}\right)\left(6.38 \times 10^{6} \mathrm{~m}\right)}(100)=3.2 \times 10^{-16} \%
\end{aligned}
\end{aligned}
$$

19. The angular momentum of the person-turntable system will be conserved. Call the direction of the person's motion the positive rotation direction. Relative to the ground, the person's speed will be $v+v_{\mathrm{T}}$, where $v$ is the person's speed relative to the turntable, and $v_{\mathrm{T}}$ is the speed of the rim of the turntable with respect to the ground. The turntable's angular speed is $\omega_{\mathrm{T}}=v_{\mathrm{T}} / R$, and the person's angular speed relative to the ground is $\omega_{\mathrm{P}}=\frac{v+v_{\mathrm{T}}}{R}=\frac{v}{R}+\omega_{\mathrm{T}}$. The person is treated as a point particle for calculation of the moment of inertia.

$$
L_{i}=L_{f} \rightarrow 0=I_{\mathrm{T}} \omega_{\mathrm{T}}+I_{\mathrm{P}} \omega_{\mathrm{P}}=I_{\mathrm{T}} \omega_{\mathrm{T}}+m R^{2}\left(\omega_{\mathrm{T}}+\frac{v}{R}\right) \rightarrow
$$

$$
\omega_{\mathrm{T}}=-\frac{m R v}{I_{\mathrm{T}}+m R^{2}}=-\frac{(65 \mathrm{~kg})(3.25 \mathrm{~m})(3.8 \mathrm{~m} / \mathrm{s})}{1850 \mathrm{~kg} \cdot \mathrm{~m}^{2}+(65 \mathrm{~kg})(3.25 \mathrm{~m})^{2}}=-0.32 \mathrm{rad} / \mathrm{s}
$$

20. We use the determinant rule, Eq. 11-3b.
(a) $\quad \overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}}=\left|\begin{array}{ccc}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -A & 0 & 0 \\ 0 & 0 & B\end{array}\right|=\hat{\mathbf{i}}[(0)(B)-(0)(0)]+\hat{\mathbf{j}}[(0)(0)-(-A)(B)]+\hat{\mathbf{k}}[(-A)(0)-(0)(0)]$

$$
=A B \hat{\mathbf{j}}
$$

So the direction of $\overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}}$ is in the $\hat{\mathbf{j}}$ direction.
(b) Based on Eq. 11-4b, we see that interchanging the two vectors in a cross product reverses the direction. So the direction of $\overrightarrow{\mathbf{B}} \times \overrightarrow{\mathbf{A}}$ is in the $-\hat{\mathbf{j}}$ direction.
(c) Since $\overrightarrow{\mathbf{A}}$ and $\overrightarrow{\mathbf{B}}$ are perpendicular, we have $|\overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}}|=|\overrightarrow{\mathbf{B}} \times \overrightarrow{\mathbf{A}}|=A B \sin 90^{\circ}=A B$.
21. (a) For all three expressions, use the fact that $|\overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}}|=A B \sin \theta$. If both vectors in the cross product point in the same direction, then the angle between them is $\theta=0^{\circ}$. Since $\sin 0^{\circ}=0$, a vector crossed into itself will always give 0 . Thus $\hat{\hat{\mathbf{i}}} \times \hat{\mathbf{i}}=\hat{\mathbf{j}} \times \hat{\mathbf{j}}=\hat{\mathbf{k}} \times \hat{\mathbf{k}}=0$.
(b) We use the determinant rule (Eq. 11-3b) to evaluate the other expressions.

$$
\begin{aligned}
& \hat{\mathbf{i}} \times \hat{\mathbf{j}}=\left|\begin{array}{lll}
\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\
1 & 0 & 0 \\
0 & 1 & 0
\end{array}\right|=\hat{\mathbf{i}}[(0)(0)-(0)(1)]+\hat{\mathbf{j}}[(0)(0)-(1)(0)]+\hat{\mathbf{k}}[(1)(1)-(0)(0)]=\hat{\mathbf{k}} \\
& \hat{\mathbf{i}} \times \hat{\mathbf{k}}=\left|\begin{array}{lll}
\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\
1 & 0 & 0 \\
0 & 0 & 1
\end{array}\right|=\hat{\mathbf{i}}[(0)(1)-(0)(0)]+\hat{\mathbf{j}}[(0)(0)-(1)(1)]+\hat{\mathbf{k}}[(1)(0)-(0)(0)]=-\hat{\mathbf{j}} \\
& \hat{\mathbf{j}} \times \hat{\mathbf{k}}=\left|\begin{array}{lll}
\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right|=\hat{\mathbf{i}}[(1)(1)-(0)(0)]+\hat{\mathbf{j}}[(0)(0)-(0)(1)]+\hat{\mathbf{k}}[(0)(0)-(0)(1)]=\hat{\mathbf{i}}
\end{aligned}
$$

22. (a) East cross south is into the ground.
(b) East cross straight down is north.
(c) Straight up cross north is west.
(d) Straight up cross straight down is 0 (the vectors are anti-parallel).
23. Use the definitions of cross product and dot product, in terms of the angle between the two vectors.

$$
|\overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}}|=\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{B}} \rightarrow A B|\sin \theta|=A B \cos \theta \rightarrow|\sin \theta|=\cos \theta
$$

This is true only for angles with positive cosines, and so the angle must be in the first or fourth quadrant. Thus the solutions are $\theta=45^{\circ}, 315^{\circ}$. But the angle between two vectors is always taken to be the smallest angle possible, and so $\theta=45^{\circ}$.
24. We use the determinant rule, Eq. 11-3b, to evaluate the torque.

$$
\begin{aligned}
\overrightarrow{\boldsymbol{\tau}} & =\overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{F}}=\left|\begin{array}{ccc}
\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\
4.0 & 3.5 & 6.0 \\
0 & 9.0 & -4.0
\end{array}\right| \mathrm{m} \cdot \mathrm{~N} \\
& =\{\hat{\mathbf{i}}[(3.5)(-4.0)-(6)(9)]+\hat{\mathbf{j}}[(6)(0)-(-4)(4)]+\hat{\mathbf{k}}[(4)(9)-(3)(5)]\} \mathrm{m} \cdot \mathrm{~N} \\
& =(-68 \hat{\mathbf{i}}+16 \hat{\mathbf{j}}+36 \hat{\mathbf{k}}) \mathrm{m} \cdot \mathrm{~N}
\end{aligned}
$$

25. We choose coordinates so that the plane in which the particle rotates is the $x-y$ plane, and so the angular velocity is in the $z$ direction. The object is rotating in a circle of radius $r \sin \theta$, where $\theta$ is the angle between the position vector and the axis of rotation. Since the object is rigid and rotates about a fixed axis, the linear and angular velocities of the particle are related by $v=\omega r \sin \theta$. The magnitude of the tangential acceleration is $a_{\mathrm{tan}}=\alpha r \sin \theta$. The radial acceleration is given by
$a_{\mathrm{R}}=\frac{v^{2}}{r \sin \theta}=v \frac{v}{r \sin \theta}=v \omega$. We assume the object is gaining
 speed. See the diagram showing the various vectors involved.

The velocity and tangential acceleration are parallel to each other, and the angular velocity and angular acceleration are parallel to each other. The radial acceleration is perpendicular to the velocity, and the velocity is perpendicular to the angular velocity.

We see from the diagram that, using the right hand rule, the direction of $\overrightarrow{\mathbf{a}}_{\mathrm{R}}$ is in the direction of $\overrightarrow{\boldsymbol{\omega}} \times \overrightarrow{\mathbf{v}}$. Also, since $\overrightarrow{\boldsymbol{\omega}}$ and $\overrightarrow{\mathbf{v}}$ are perpendicular, we have $|\overrightarrow{\boldsymbol{\omega}} \times \overrightarrow{\mathbf{v}}|=\omega v$ which from above is $v \omega=a_{\mathrm{R}}$. Since both the magnitude and direction check out, we have $\overrightarrow{\overrightarrow{\mathbf{a}}_{\mathrm{R}}=\overrightarrow{\boldsymbol{\omega}} \times \overrightarrow{\mathbf{v}}}$.

We also see from the diagram that, using the right hand rule, the direction of $\overrightarrow{\mathbf{a}}_{\tan }$ is in the direction of $\overrightarrow{\boldsymbol{\alpha}} \times \overrightarrow{\mathbf{r}}$. The magnitude of $\overrightarrow{\boldsymbol{\alpha}} \times \overrightarrow{\mathbf{r}}$ is $|\overrightarrow{\boldsymbol{\alpha}} \times \overrightarrow{\mathbf{r}}|=\alpha r \sin \theta$, which from above is $\alpha r \sin \theta=a_{\tan }$. Since both the magnitude and direction check out, we have $\overrightarrow{\mathbf{a}}_{\mathrm{tan}}=\overrightarrow{\boldsymbol{\alpha}} \times \overrightarrow{\mathbf{r}}$.
26. (a) We use the distributive property, Eq. 11-4c, to obtain 9 single-term cross products.

$$
\begin{aligned}
\overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}}= & \left(A_{x} \hat{\mathbf{i}}+A_{y} \hat{\mathbf{j}}+A_{z} \hat{\mathbf{k}}\right) \times\left(B_{x} \hat{\mathbf{i}}+B_{y} \hat{\mathbf{j}}+B_{z} \hat{\mathbf{k}}\right) \\
= & A_{x} B_{x}(\hat{\mathbf{i}} \times \hat{\mathbf{i}})+A_{x} B_{y}(\hat{\mathbf{i}} \times \hat{\mathbf{j}})+A_{x} B_{z}(\hat{\mathbf{i}} \times \hat{\mathbf{k}})+A_{y} B_{x}(\hat{\mathbf{j}} \times \hat{\mathbf{i}})+A_{y} B_{y}(\hat{\mathbf{j}} \times \hat{\mathbf{j}})+A_{y} B_{z}(\hat{\mathbf{j}} \times \hat{\mathbf{k}}) \\
& +A_{z} B_{x}(\hat{\mathbf{k}} \times \hat{\mathbf{i}})+A_{z} B_{y}(\hat{\mathbf{k}} \times \hat{\mathbf{j}})+A_{z} B_{z}(\hat{\mathbf{k}} \times \hat{\mathbf{k}})
\end{aligned}
$$

Each of these cross products of unit vectors is evaluated using the results of Problem 21 and Eq. 11-4b.

$$
\begin{aligned}
& \overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}}=A_{x} B_{x}(0)+A_{x} B_{y} \hat{\mathbf{k}}+A_{x} B_{z}(-\hat{\mathbf{j}})+A_{y} B_{x}(-\hat{\mathbf{k}})+A_{y} B_{y}(0)+A_{y} B_{z} \hat{\mathbf{i}} \\
&+A_{z} B_{x} \hat{\mathbf{j}}+A_{z} B_{y}(-\hat{\mathbf{i}})+A_{z} B_{z}(0) \\
&=A_{x} B_{y} \hat{\mathbf{k}}-A_{x} B_{z} \hat{\mathbf{j}}-A_{y} B_{x} \hat{\mathbf{k}}+A_{y} B_{z} \hat{\mathbf{i}}+A_{z} B_{x} \hat{\mathbf{j}}-A_{z} B_{y} \hat{\mathbf{i}}
\end{aligned}
$$

$$
=\left(A_{y} B_{z}-A_{z} B_{y}\right) \hat{\mathbf{i}}+\left(A_{z} B_{x}-A_{x} B_{z}\right) \hat{\mathbf{j}}+\left(A_{x} B_{y}-A_{y} B_{x}\right) \hat{\mathbf{k}}
$$

(b) The rules for evaluating a literal determinant of a $3 \times 3$ matrix are as follows. The indices on the matrix elements identify the row and column of the element, respectively.

$$
\left|\begin{array}{lll}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{array}\right|=a_{11}\left(a_{22} a_{33}-a_{23} a_{32}\right)+a_{12}\left(a_{23} a_{31}-a_{21} a_{33}\right)+a_{13}\left(a_{21} a_{32}-a_{22} a_{31}\right)
$$

Apply this as a pattern for finding the cross product of two vectors.

$$
\overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}}=\left|\begin{array}{ccc}
\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\
A_{x} & A_{y} & A_{z} \\
B_{x} & B_{y} & B_{z}
\end{array}\right|=\hat{\mathbf{i}}\left(A_{y} B_{z}-A_{z} B_{y}\right)+\hat{\mathbf{j}}\left(A_{z} B_{x}-A_{x} B_{z}\right)+\hat{\mathbf{k}}\left(A_{x} B_{y}-A_{y} B_{x}\right)
$$

This is the same expression as found in part (a).
27. We use the determinant rule, Eq. 11-3b, to evaluate the torque.

$$
\begin{aligned}
\overrightarrow{\boldsymbol{\tau}}=\overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{F}} & =\left|\begin{array}{ccc}
\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\
0 & 8.0 & 6.0 \\
\pm 2.4 & -4.1 & 0
\end{array}\right| \mathrm{m} \cdot \mathrm{kN} \\
& =\{\hat{\mathbf{i}}[-(6.0)(-4.1)]+\hat{\mathbf{j}}[(6.0)( \pm 2.4)]+\hat{\mathbf{k}}[-(8.0)( \pm 2.4)]\} \mathrm{m} \cdot \mathrm{kN} \\
& =(24.6 \hat{\mathbf{i}} \pm 14.4 \hat{\mathbf{j}} \mp 19.2 \hat{\mathbf{k}}) \mathrm{m} \cdot \mathrm{kN} \approx(2.5 \hat{\mathbf{i}} \pm 1.4 \hat{\mathbf{j}} \mp 1.9 \hat{\mathbf{k}}) \times 10^{4} \mathrm{~m} \cdot \mathrm{~N}
\end{aligned}
$$

The magnitude of this maximum torque is also found.

$$
|\overrightarrow{\boldsymbol{\tau}}|=\sqrt{(2.46)^{2}+(1.44)^{2}+(1.92)^{2}} \times 10^{4} \mathrm{~m} \cdot \mathrm{~N}=3.4 \times 10^{4} \mathrm{~m} \cdot \mathrm{~N}
$$

28. We use the determinant rule, Eq. 11-3b, to evaluate the torque.

$$
\begin{aligned}
\overrightarrow{\boldsymbol{\tau}}=\overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{F}} & =\left|\begin{array}{ccc}
\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\
0.280 & 0.335 & 0 \\
215 \cos 33.0^{\circ} & 215 \sin 33.0^{\circ} & 0
\end{array}\right| \mathrm{m} \cdot \mathrm{~N} \\
& =\hat{\mathbf{k}}\left[(0.280)\left(215 \sin 33.0^{\circ}\right)-(0.335)\left(215 \cos 33.0^{\circ}\right)\right] \mathrm{m} \cdot \mathrm{~N} \\
& =-27.6 \mathrm{~m} \cdot \mathrm{~N} \hat{\mathbf{k}}=27.6 \mathrm{~m} \cdot \mathrm{~N} \text { in the }-z \text { direction }
\end{aligned}
$$

This could also be calculated by finding the magnitude and direction of $\overrightarrow{\mathbf{r}}$, and then using Eq. 11-3a and the right-hand rule.
29. (a) We use the determinant rule, Eq. 11-3b, to evaluate the cross product.

$$
\overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}}=\left|\begin{array}{ccc}
\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\
5.4 & -3.5 & 0 \\
-8.5 & 5.6 & 2.0
\end{array}\right|=-7.0 \hat{\mathbf{i}}-10.8 \hat{\mathbf{j}}+0.49 \hat{\mathbf{k}} \approx-7.0 \hat{\mathbf{i}}-11 \hat{\mathbf{j}}+0.5 \hat{\mathbf{k}}
$$

(b) Now use Eq. 11-3a to find the angle between the two vectors.

$$
|\overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}}|=\sqrt{(-7.0)^{2}+(-10.8)^{2}+(0.49)^{2}}=12.88
$$

$$
\begin{aligned}
& A=\sqrt{(5.4)^{2}+(3.5)^{2}}=6.435 ; B \sqrt{(-8.5)^{2}+(5.6)^{2}+(2.0)^{2}}=10.37 \\
& |\overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}}|=A B \sin \theta \rightarrow \theta=\sin ^{-1} \frac{|\overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}}|}{A B}=\sin ^{-1} \frac{12.88}{(6.435)(10.37)}=11.1^{\circ} \text { or } 168.9^{\circ}
\end{aligned}
$$

Use the dot product to resolve the ambiguity.

$$
\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{B}}=(5.4)(-8.5)+(3.5)(5.6)+0(2.0)=-26.3
$$

Since the dot product is negative, the angle between the vectors must be obtuse, and so $\theta=168.9^{\circ} \approx 170^{\circ}$.
30. We choose the $z$ axis to be the axis of rotation, and so $\overrightarrow{\boldsymbol{\omega}}=\omega \hat{\mathbf{k}}$. We describe the location of the point as $\overrightarrow{\mathbf{r}}=R \cos \omega \hat{\mathbf{i}}+R \sin \omega \hat{\mathbf{j}}+z_{0} \hat{\mathbf{k}}$. In this description, the point is moving counterclockwise in a circle of radius $R$ centered on the point $\left(0,0, z_{0}\right)$, and is located at $\left(R, 0, z_{0}\right)$ at $t=0$.

$$
\begin{aligned}
& \overrightarrow{\mathbf{v}}=\frac{d \overrightarrow{\mathbf{r}}}{d t}=-R \omega \sin \omega \hat{t}+R \omega \cos \omega \hat{\mathbf{i}} \hat{\mathbf{j}} \\
& \overrightarrow{\boldsymbol{\omega}} \times \overrightarrow{\mathbf{r}}=\left|\begin{array}{ccc}
\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\
0 & 0 & \omega \\
R \cos \omega t & R \sin \omega t & z_{0}
\end{array}\right|=-R \omega \sin \omega t \hat{\mathbf{i}}+R \omega \cos \omega t \hat{\mathbf{j}}=\overrightarrow{\mathbf{v}}
\end{aligned}
$$

And so we see that $\overrightarrow{\mathbf{v}}=\overrightarrow{\boldsymbol{\omega}} \times \overrightarrow{\mathbf{r}}$.
If the origin were moved to a different location on the axis of rotation (the $z$ axis) that would simply change the value of the $z$ coordinate of the point to some other value, say $z_{1}$. Changing that value will still lead to $\overrightarrow{\mathbf{v}}=\overrightarrow{\boldsymbol{\omega}} \times \overrightarrow{\mathbf{r}}$.

But if the origin is moved from the original point to something off the rotation axis, then the position vector will change. If the new origin is moved to $\left(x_{2}, y_{2}, z_{2}\right)$, then the position vector will change to $\overrightarrow{\mathbf{r}}=\left(R \cos \omega t-x_{2}\right) \hat{\mathbf{i}}+\left(R \sin \omega t-y_{2}\right) \hat{\mathbf{j}}+\left(z_{0}-z_{2}\right) \hat{\mathbf{k}}$. See how that affects the relationships.

$$
\overrightarrow{\mathbf{v}}=\frac{d \overrightarrow{\mathbf{r}}}{d t}=-R \omega \sin \omega t \hat{\mathbf{i}}+R \omega \cos \omega \hat{\mathbf{j}}
$$

$$
\overrightarrow{\boldsymbol{\omega}} \times \overrightarrow{\mathbf{r}}=\left|\begin{array}{ccc}
\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\
0 & 0 & \omega \\
R \cos \omega t-x_{2} & R \sin \omega t-y_{2} & z_{0}-z_{2}
\end{array}\right|=\left(-R \omega \sin \omega t+\omega y_{2}\right) \hat{\mathbf{i}}+\left(R \omega \cos \omega t-\omega x_{2}\right) \hat{\mathbf{j}}=\overrightarrow{\mathbf{v}}
$$

We see that with this new off-axis origin, $\overrightarrow{\mathbf{v}} \neq \overrightarrow{\boldsymbol{\omega}} \times \overrightarrow{\mathbf{r}}$.
31. Calculate the three "triple products" as requested.

$$
\begin{aligned}
& \overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}}=\left|\begin{array}{ccc}
\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\
A_{x} & A_{y} & A_{z} \\
B_{x} & B_{y} & B_{z}
\end{array}\right|=\hat{\mathbf{i}}\left(A_{y} B_{z}-A_{z} B_{y}\right)+\hat{\mathbf{j}}\left(A_{z} B_{x}-A_{x} B_{z}\right)+\hat{\mathbf{k}}\left(A_{x} B_{y}-A_{y} B_{x}\right) \\
& \overrightarrow{\mathbf{B}} \times \overrightarrow{\mathbf{C}}=\left|\begin{array}{ccc}
\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\
B_{x} & B_{y} & B_{z} \\
C_{x} & C_{y} & C_{z}
\end{array}\right|=\hat{\mathbf{i}}\left(B_{y} C_{z}-B_{z} C_{y}\right)+\hat{\mathbf{j}}\left(B_{z} C_{x}-B_{x} C_{z}\right)+\hat{\mathbf{k}}\left(B_{x} C_{y}-B_{y} C_{x}\right)
\end{aligned}
$$

$$
\begin{aligned}
& \left.\overrightarrow{\mathbf{C}} \times \overrightarrow{\mathbf{A}}=\left|\begin{array}{ccc}
\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\
C_{x} & C_{y} & C_{z} \\
A_{x} & A_{y} & A_{z}
\end{array}\right|=\hat{\mathbf{i}}\left(C_{y} A_{z}-C_{z} A_{y}\right)+\hat{\mathbf{j}}\left(C_{z} A_{x}-C_{x} A_{z}\right)+\hat{\mathbf{k}}\left(C_{x} A_{y}-C_{y} A_{x}\right)\right] \\
& \begin{aligned}
\overrightarrow{\mathbf{A}} \cdot(\overrightarrow{\mathbf{B}} \times \overrightarrow{\mathbf{C}}) & =\left(A_{x} \hat{\mathbf{i}}+A_{y} \hat{\mathbf{j}}+A_{z} \hat{\mathbf{k}}\right) \cdot\left[\hat{\mathbf{i}}\left(B_{y} C_{z}-B_{z} C_{y}\right)+\hat{\mathbf{j}}\left(B_{z} C_{x}-B_{x} C_{z}\right)+\hat{\mathbf{k}}\left(B_{x} C_{y}-B_{y} C_{x}\right)\right] \\
& =A_{x}\left(B_{y} C_{z}-B_{z} C_{y}\right)+A_{y}\left(B_{z} C_{x}-B_{x} C_{z}\right)+A_{z}\left(B_{x} C_{y}-B_{y} C_{x}\right) \\
& =A_{x} B_{y} C_{z}-A_{x} B_{z} C_{y}+A_{y} B_{z} C_{x}-A_{y} B_{x} C_{z}+A_{z} B_{x} C_{y}-A_{z} B_{y} C_{x} \\
\overrightarrow{\mathbf{B}} \cdot(\overrightarrow{\mathbf{C}} \times \overrightarrow{\mathbf{A}}) & =\left(B_{x} \hat{\mathbf{i}}+B_{y} \hat{\mathbf{j}}+B_{z} \hat{\mathbf{k}}\right) \cdot\left[\hat{\mathbf{i}}\left(C_{y} A_{z}-C_{z} A_{y}\right)+\hat{\mathbf{j}}\left(C_{z} A_{x}-C_{x} A_{z}\right)+\hat{\mathbf{k}}\left(C_{x} A_{y}-C_{y} A_{x}\right)\right] \\
& =B_{x}\left(C_{y} A_{z}-C_{z} A_{y}\right)+B_{y}\left(C_{z} A_{x}-C_{x} A_{z}\right)+B_{z}\left(C_{x} A_{y}-C_{y} A_{x}\right) \\
& =B_{x} C_{y} A_{z}-B_{x} C_{z} A_{y}+B_{y} C_{z} A_{x}-B_{y} C_{x} A_{z}+B_{z} C_{x} A_{y}-B_{z} C_{y} A_{x} \\
\overrightarrow{\mathbf{C}} \cdot(\overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}}) & =\left(C_{x} \hat{\mathbf{i}}+C_{y} \hat{\mathbf{j}}+C_{z} \hat{\mathbf{k}}\right) \cdot\left[\hat{\mathbf{i}}\left(A_{y} B_{z}-A_{z} B_{y}\right)+\hat{\mathbf{j}}\left(A_{z} B_{x}-A_{x} B_{z}\right)+\hat{\mathbf{k}}\left(A_{x} B_{y}-A_{y} B_{x}\right)\right] \\
& =C_{x}\left(A_{y} B_{z}-A_{z} B_{y}\right)+C_{y}\left(A_{z} B_{x}-A_{x} B_{z}\right)+C_{z}\left(A_{x} B_{y}-A_{y} B_{x}\right) \\
& =C_{x} A_{y} B_{z}-C_{x} A_{z} B_{y}+C_{y} A_{z} B_{x}-C_{y} A_{x} B_{z}+C_{z} A_{x} B_{y}-C_{z} A_{y} B_{x}
\end{aligned}
\end{aligned}
$$

A comparison of three results shows that they are all the same.
32. We use the determinant rule, Eq. 11-3b, to evaluate the angular momentum.

$$
\overrightarrow{\mathbf{L}}=\overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{p}}=\left|\begin{array}{ccc}
\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\
x & y & z \\
p_{x} & p_{y} & p_{z}
\end{array}\right|=\left(y p_{z}-z p_{y}\right) \hat{\mathbf{i}}+\left(z p_{x}-x p_{z}\right) \hat{\mathbf{j}}+\left(x p_{y}-y p_{x}\right) \hat{\mathbf{k}}
$$

33. The position vector and velocity vectors are at right angles to each other for circular motion. The angular momentum for a particle moving in a circle is $L=r p \sin \theta=r m v \sin 90^{\circ}=m r v$. The moment of inertia is $I=m r^{2}$.

$$
\frac{L^{2}}{2 I}=\frac{(m r v)^{2}}{2 m r^{2}}=\frac{m^{2} r^{2} v^{2}}{2 m r^{2}}=\frac{m v^{2}}{2}=\frac{1}{2} m v^{2}=K
$$

This is analogous to $K=\frac{p^{2}}{2 m}$ relating kinetic energy, linear momentum, and mass.
34. (a) See Figure 11-33 in the textbook. We have that $L=r_{\perp} p=d m v$. The direction is into the plane of the page.
(b) Since the velocity (and momentum) vectors pass through $\mathrm{O}^{\prime}, \overrightarrow{\mathbf{r}}$ and $\overrightarrow{\mathbf{p}}$ are parallel, and so $\overrightarrow{\mathbf{L}}=\overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{p}}=0$. Or, $r_{\perp}=0$, and so $L=0$.
35. See the diagram. Calculate the total angular momentum about the origin.

$$
\overrightarrow{\mathbf{L}}=\overrightarrow{\mathbf{r}}_{1} \times \overrightarrow{\mathbf{p}}+\overrightarrow{\mathbf{r}}_{2} \times(-\overrightarrow{\mathbf{p}})=\left(\overrightarrow{\mathbf{r}}_{1}-\overrightarrow{\mathbf{r}}_{2}\right) \times \overrightarrow{\mathbf{p}}
$$

The position dependence of the total angular momentum only depends on the difference in the two position vectors. That difference is the same no matter where the origin is chosen, because it is the relative distance between the two particles.

36. Use Eq. 11-6 to calculate the angular momentum.

$$
\begin{aligned}
\overrightarrow{\mathbf{L}} & =\overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{p}}=m(\overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{v}})=(0.075 \mathrm{~kg})\left|\begin{array}{ccc}
\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\
4.4 & -6.0 & 0 \\
3.2 & 0 & -8.0
\end{array}\right| \mathrm{m}^{2} / \mathrm{s} \\
& =(0.075)(48 \hat{\mathbf{i}}+35.2 \hat{\mathbf{j}}+19.2 \hat{\mathbf{k}}) \mathrm{kg} \cdot \mathrm{~m}^{2} / \mathrm{s}=(3.6 \hat{\mathbf{i}}+2.6 \hat{\mathbf{j}}+1.4 \hat{\mathbf{k}}) \mathrm{kg} \cdot \mathrm{~m}^{2} / \mathrm{s}
\end{aligned}
$$

37. Use Eq. 11-6 to calculate the angular momentum.

$$
\begin{aligned}
\overrightarrow{\mathbf{L}} & =\overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{p}}=m(\overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{v}})=(3.8 \mathrm{~kg})\left|\begin{array}{ccc}
\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\
1.0 & 2.0 & 3.0 \\
-5.0 & 2.8 & -3.1
\end{array}\right| \mathrm{m}^{2} / \mathrm{s} \\
& =(3.8)(-14.6 \hat{\mathbf{i}}-11.9 \hat{\mathbf{j}}+12.8 \hat{\mathbf{k}}) \mathrm{kg} \cdot \mathrm{~m}^{2} / \mathrm{s}=(-55 \hat{\mathbf{i}}-45 \hat{\mathbf{j}}+49 \hat{\mathbf{k}}) \mathrm{kg} \cdot \mathrm{~m}^{2} / \mathrm{s}
\end{aligned}
$$

38. (a) From Example 11-8, $a=\frac{\left(m_{\mathrm{B}}-m_{\mathrm{A}}\right) g}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}+I / R_{0}^{2}\right)}$.

$$
\begin{aligned}
a & =\frac{\left(m_{\mathrm{B}}-m_{\mathrm{A}}\right) g}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}+I / R_{0}^{2}\right)}=\frac{\left(m_{\mathrm{B}}-m_{\mathrm{A}}\right) g}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)+\frac{1}{2} m R_{0}^{2} / R_{0}^{2}}=\frac{\left(m_{\mathrm{B}}-m_{\mathrm{A}}\right) g}{m_{\mathrm{A}}+m_{\mathrm{B}}+\frac{1}{2} m} \\
& =\frac{(1.2 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{15.6 \mathrm{~kg}}=0.7538 \mathrm{~m} / \mathrm{s}^{2} \approx 0.75 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

(b) If the mass of the pulley is ignored, then we have the following.

$$
\begin{aligned}
& a=\frac{\left(m_{\mathrm{B}}-m_{\mathrm{A}}\right) g}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)}=\frac{(1.2 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{15.2 \mathrm{~kg}}=0.7737 \mathrm{~m} / \mathrm{s}^{2} \\
& \% \text { error }=\left(\frac{0.7737 \mathrm{~m} / \mathrm{s}^{2}-0.7538 \mathrm{~m} / \mathrm{s}^{2}}{0.7538 \mathrm{~m} / \mathrm{s}^{2}}\right) \times 100=2.6 \%
\end{aligned}
$$

39. The rotational inertia of the compound object is the sum of the individual moments of inertia.

$$
I=I_{\text {particles }}+I_{\text {rod }}=m(0)^{2}+m\left(\frac{1}{3} \ell\right)^{2}+m\left(\frac{2}{3} \ell\right)^{2}+m \ell^{2}+\frac{1}{3} M \ell^{2}=\left(\frac{14}{9} m+\frac{1}{3} M\right) \ell^{2}
$$

(a) $K=\frac{1}{2} I \omega^{2}=\frac{1}{2}\left(\frac{14}{9} m+\frac{1}{3} M\right) \ell^{2} \omega^{2}=\left(\frac{7}{9} m+\frac{1}{6} M\right) \ell^{2} \omega^{2}$
(b) $L=I \omega=\left(\frac{14}{9} m+\frac{1}{3} M\right) \ell^{2} \omega$
40. (a) We calculate the full angular momentum vector about the center of mass of the system. We take the instant shown in the diagram, with the positive x axis to the right, the positive y axis up along the axle, and the positive z axis out of the plane of the diagram towards the viewer. We take the upper mass as mass A and the lower mass as mass B. If we assume that the system is rotating counterclockwise when viewed from above along the rod, then the velocity of mass A is in the positive z direction, and the velocity of mass B is in the negative z direction. The speed is given by $v=\omega r=(4.5 \mathrm{rad} / \mathrm{s})(0.24 \mathrm{~m})=1.08 \mathrm{~m} / \mathrm{s}$.

$$
\overrightarrow{\mathbf{L}}=\overrightarrow{\mathbf{r}}_{\mathrm{A}} \times \overrightarrow{\mathbf{p}}_{\mathrm{A}}+\overrightarrow{\mathbf{r}}_{\mathrm{B}} \times \overrightarrow{\mathbf{p}}_{\mathrm{B}}=m\left\{\overrightarrow{\mathbf{r}}_{\mathrm{A}} \times \overrightarrow{\mathbf{v}}_{\mathrm{A}}+\overrightarrow{\mathbf{r}}_{\mathrm{B}} \times \overrightarrow{\mathbf{v}}_{\mathrm{B}}\right\}
$$

$$
\begin{aligned}
& =m\left\{\begin{array}{ccc}
\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\
-0.24 \mathrm{~m} & 0.21 \mathrm{~m} & 0 \\
0 & 0 & v
\end{array}\left|+\left|\begin{array}{ccc}
\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\
0.24 \mathrm{~m} & -0.21 \mathrm{~m} & 0 \\
0 & 0 & -v
\end{array}\right|\right\}\right. \\
& =m\{2 \hat{\mathbf{i}}(0.21 \mathrm{~m}) v+2 \hat{\mathbf{j}}(0.24 \mathrm{~m}) v\}=2 m v\{\hat{\mathbf{i}}(0.21 \mathrm{~m})+\hat{\mathbf{j}}(0.24 \mathrm{~m})\} \\
& =2(0.48 \mathrm{~kg})(1.08 \mathrm{~m} / \mathrm{s})\{\hat{\mathbf{i}}(0.21 \mathrm{~m})+\hat{\mathbf{j}}(0.24 \mathrm{~m})\}=[\hat{\mathbf{i}}(0.2177)+\hat{\mathbf{j}}(0.2488)] \mathrm{kg} \cdot \mathrm{~m}^{2} / \mathrm{s}
\end{aligned}
$$

The component along the axis is the $\hat{\mathbf{j}}$ component, $0.25 \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}$.
(b) The angular momentum vector will precess about the axle. The tip of the angular momentum vector traces out the dashed circle in the diagram.

$$
\theta=\tan ^{-1} \frac{L_{x}}{L_{y}}=\tan ^{-1} \frac{0.2177 \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}}{0.2488 \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}}=41^{\circ}
$$


41. (a) We assume the system is moving such that mass B is moving down, mass A is moving to the left, and the pulley is rotating counterclockwise. We take those as positive directions. The angular momentum of masses A and B is the same as that of a point mass. We assume the rope is moving without slipping, so $v=\omega_{\text {pulley }} R_{0}$.

$$
\begin{aligned}
L & =L_{\mathrm{A}}+L_{\mathrm{B}}+L_{\text {pulley }}=M_{\mathrm{A}} v R_{0}+M_{\mathrm{B}} v R_{0}+I \omega=M_{\mathrm{A}} v R_{0}+M_{\mathrm{B}} v R_{0}+I \frac{v}{R_{0}} \\
& =\left(\left(M_{\mathrm{A}}+M_{\mathrm{B}}\right) R_{0}+\frac{I}{R_{0}}\right) v
\end{aligned}
$$

(b) The net torque about the axis of the pulley is that provided by gravity, $M_{\mathrm{B}} g R_{0}$. Use Eq. 11-9, which is applicable since the axis is fixed.

$$
\begin{aligned}
& \sum \tau=\frac{d L}{d t} \rightarrow M_{\mathrm{B}} g R_{0}=\frac{d}{d t}\left(\left(M_{\mathrm{A}}+M_{\mathrm{B}}\right) R_{0}+\frac{I}{R_{0}}\right) v=\left(\left(M_{\mathrm{A}}+M_{\mathrm{B}}\right) R_{0}+\frac{I}{R_{0}}\right) a \rightarrow \\
& a=\frac{M_{\mathrm{B}} g R_{0}}{\left(\left(M_{\mathrm{A}}+M_{\mathrm{B}}\right) R_{0}+\frac{I}{R_{0}}\right)}=\frac{M_{\mathrm{B}} g}{M_{\mathrm{A}}+M_{\mathrm{B}}+\frac{I}{R_{0}^{2}}}
\end{aligned}
$$

42. Take the origin of coordinates to be at the rod's center, and the axis of rotation to be in the $z$ direction. Consider a differential element $d m=\frac{M}{\ell} d r$ of the rod, a distance $r$ from the center. That element rotates in a circle of radius $r \sin \phi$, at a height of $r \cos \phi$. The position and velocity of this point are given by the following.

$$
\begin{aligned}
\overrightarrow{\mathbf{r}} & =r \sin \phi \cos \omega t \hat{\mathbf{i}}+r \sin \phi \sin \omega t \hat{\mathbf{j}}+r \cos \phi \hat{\mathbf{k}} \\
& =r[\sin \phi \cos \omega t \hat{\mathbf{i}}+\sin \phi \sin \omega t \hat{\mathbf{j}}+\cos \phi \hat{\mathbf{k}}]
\end{aligned}
$$



$$
\begin{aligned}
\overrightarrow{\mathbf{v}} & =\frac{d \overrightarrow{\mathbf{r}}}{d t}=-r \omega \sin \phi \sin \omega t \hat{\mathbf{i}}+r \omega \sin \phi \cos \omega t \hat{\mathbf{j}} \\
& =r \omega[-\sin \phi \sin \omega t \hat{\mathbf{i}}+\sin \phi \cos \omega t \hat{\mathbf{j}}]
\end{aligned}
$$

Calculate the angular momentum of this element.

$$
\begin{aligned}
d \overrightarrow{\mathbf{L}} & =d m(\overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{v}})=r^{2} \omega \frac{M}{\ell} d r\left|\begin{array}{ccc}
\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\
\sin \phi \cos \omega t & \sin \phi \sin \omega t & \cos \phi \\
-\sin \phi \sin \omega t & \sin \phi \cos \omega t & 0
\end{array}\right| \\
& =\frac{M}{\ell} d r\left[(-\sin \phi \cos \phi \cos \omega t) \hat{\mathbf{i}}+(-\sin \phi \cos \phi \sin \omega t) \hat{\mathbf{j}}+\left(\sin ^{2} \phi \cos ^{2} \omega t+\sin ^{2} \phi \sin ^{2} \omega t\right) \hat{\mathbf{k}}\right] \\
& =\frac{M r^{2} \omega \sin \phi}{\ell} d r[(-\cos \phi \cos \omega t) \hat{\mathbf{i}}+(-\cos \phi \sin \omega t) \hat{\mathbf{j}}+\sin \phi \hat{\mathbf{k}}]
\end{aligned}
$$

Note that the directional portion has no $r$ dependence. Thus $d \overrightarrow{\mathbf{L}}$ for every piece of mass has the same direction. What is that direction? Consider the dot product $\overrightarrow{\mathbf{r}} \cdot d \overrightarrow{\mathbf{L}}$.

$$
\begin{aligned}
\overrightarrow{\mathbf{r}} \cdot d \overrightarrow{\mathbf{L}}= & r[\sin \phi \cos \omega t \hat{\mathbf{i}}+\sin \phi \sin \omega t \hat{\mathbf{j}}+\cos \phi \hat{\mathbf{k}}] \\
& \quad \cdot\left[\frac{M r^{2} \omega \sin \phi d r}{\ell}[(-\cos \phi \cos \omega t) \hat{\mathbf{i}}+(-\cos \phi \sin \omega t) \hat{\mathbf{j}}+\sin \phi \hat{\mathbf{k}}]\right] \\
= & \frac{M r^{3} \omega \sin \phi d r}{\ell}[\sin \phi \cos \omega t(-\cos \phi \cos \omega t)+\sin \phi \sin \omega t(-\cos \phi \sin \omega t)+\cos \phi \sin \phi]=0
\end{aligned}
$$

Thus $d \overrightarrow{\mathbf{L}} \perp \overrightarrow{\mathbf{r}}$ for every point on the rod. Also, if $\phi$ is an acute angle, the $z$ component of $d \overrightarrow{\mathbf{L}}$ is positive. The direction of $d \overrightarrow{\mathbf{L}}$ is illustrated in the diagram.
Integrate over the length of the rod to find the total angular momentum. And since the direction of $d \overrightarrow{\mathbf{L}}$ is not dependent on $r$, the direction of $\overrightarrow{\mathbf{L}}$ is the same as the direction of $d \overrightarrow{\mathbf{L}}$.

$$
\begin{aligned}
\overrightarrow{\mathbf{L}} & =\int d \overrightarrow{\mathbf{L}}=\frac{M \omega \sin \phi}{\ell}[(-\cos \phi \cos \omega t) \hat{\mathbf{i}}+(-\cos \phi \sin \omega t) \hat{\mathbf{j}}+\sin \phi \hat{\mathbf{k}}] \int_{-\ell / 2}^{\ell / 2} r^{2} d r \\
& =\frac{M \omega \ell^{2} \sin \phi}{12}[(-\cos \phi \cos \omega t) \hat{\mathbf{i}}+(-\cos \phi \sin \omega t) \hat{\mathbf{j}}+\sin \phi \hat{\mathbf{k}}]
\end{aligned}
$$

Find the magnitude using the Pythagorean theorem.

$$
L=\frac{M \omega \ell^{2} \sin \phi}{12}\left[(-\cos \phi \cos \omega t)^{2}+(-\cos \phi \sin \omega t)^{2}+\sin ^{2} \phi\right]^{1 / 2}=\frac{M \omega \ell^{2} \sin \phi}{12}
$$

$\overrightarrow{\mathbf{L}}$ is inclined upwards an angle of $\phi$ from the $x-y$ plane, perpendicular to the rod.
43. We follow the notation and derivation of Eq. 11-9b. Start with the general definition of angular momentum, $\overrightarrow{\mathbf{L}}=\sum_{i} \overrightarrow{\mathbf{r}}_{i} \times \overrightarrow{\mathbf{p}}_{i}$. Then express position and velocity with respect to the center of mass.
$\overrightarrow{\mathbf{r}}_{i}=\overrightarrow{\mathbf{r}}_{\mathrm{CM}}+\overrightarrow{\mathbf{r}}_{i}^{*}$, where $\overrightarrow{\mathbf{r}}_{i}^{*}$ is the position of the $i^{\text {th }}$ particle with respect to the center of mass
$\overrightarrow{\mathbf{v}}_{i}=\overrightarrow{\mathbf{v}}_{\mathrm{CM}}+\overrightarrow{\mathbf{v}}_{i}^{*}$, which comes from differentiating the above relationship for position
$\overrightarrow{\mathbf{L}}=\sum_{i} \overrightarrow{\mathbf{r}}_{i} \times \overrightarrow{\mathbf{p}}_{i}=\sum_{i} \overrightarrow{\mathbf{r}}_{i} \times m_{i} \overrightarrow{\mathbf{v}}_{i}=\sum_{i}\left(\overrightarrow{\mathbf{r}}_{\mathrm{CM}}+\overrightarrow{\mathbf{r}}_{i}^{*}\right) \times m_{i}\left(\overrightarrow{\mathbf{v}}_{\mathrm{CM}}+\overrightarrow{\mathbf{v}}_{i}^{*}\right)$

$$
=\sum_{i} m_{i} \overrightarrow{\mathbf{r}}_{\mathrm{CM}} \times \overrightarrow{\mathbf{v}}_{\mathrm{CM}}+\sum_{i} m_{i} \overrightarrow{\mathbf{r}}_{\mathrm{CM}} \times \overrightarrow{\mathbf{v}}_{i}^{*}+\sum_{i} m_{i} \overrightarrow{\mathbf{r}}_{i}^{*} \times \overrightarrow{\mathbf{v}}_{\mathrm{CM}}+\sum_{i} m_{i} \overrightarrow{\mathbf{r}}_{i}^{*} \times \overrightarrow{\mathbf{v}}_{i}^{*}
$$

Note that the center of mass quantities are not dependent on the summation subscript, and so they may be taken outside the summation process.

$$
\overrightarrow{\mathbf{L}}=\left(\overrightarrow{\mathbf{r}}_{\mathrm{CM}} \times \overrightarrow{\mathbf{v}}_{\mathrm{CM}}\right) \sum_{i} m_{i}+\overrightarrow{\mathbf{r}}_{\mathrm{CM}} \times \sum_{i} m_{i} \overrightarrow{\mathbf{v}}_{i}^{*}+\left(\sum_{i} m_{i} \overrightarrow{\mathbf{r}}_{i}^{*}\right) \times \overrightarrow{\mathbf{v}}_{\mathrm{CM}}+\sum_{i} m_{i} \overrightarrow{\mathbf{r}}_{i}^{*} \times \overrightarrow{\mathbf{v}}_{i}^{*}
$$

In the first term, $\sum m_{i}=M$. In the second term, we have the following.

$$
\sum_{i} m_{i} \overrightarrow{\mathbf{v}}_{i}^{*}=\sum_{i} m_{i}\left(\overrightarrow{\mathbf{v}}_{i}-\overrightarrow{\mathbf{v}}_{\mathrm{CM}}\right)=\sum_{i} m_{i} \overrightarrow{\mathbf{v}}_{i}-\sum_{i} m_{i} \overrightarrow{\mathbf{v}}_{\mathrm{CM}}=\sum_{i} m_{i} \overrightarrow{\mathbf{v}}_{i}-M \overrightarrow{\mathbf{v}}_{\mathrm{CM}}=0
$$

This is true from the definition of center of mass velocity: $\overrightarrow{\mathbf{v}}_{\mathrm{CM}}=\frac{1}{M} \sum_{i} m_{i} \overrightarrow{\mathbf{v}}_{i}$.
Likewise, in the third term, we have the following.

$$
\sum_{i} m_{i} \overrightarrow{\mathbf{r}}_{i}^{*}=\sum_{i} m_{i}\left(\overrightarrow{\mathbf{r}}_{i}-\overrightarrow{\mathbf{r}}_{\mathrm{CM}}\right)=\sum_{i} m_{i} \overrightarrow{\mathbf{r}}_{i}-\sum_{i} m_{i} \overrightarrow{\mathbf{r}}_{\mathrm{CM}}=\sum_{i} m_{i} \overrightarrow{\mathbf{r}}_{i}-M \overrightarrow{\mathbf{r}}_{\mathrm{CM}}=0
$$

This is true from the definition of center of mass: $\overrightarrow{\mathbf{r}}_{\mathrm{CM}}=\frac{1}{M} \sum_{i} m_{i} \overrightarrow{\mathbf{r}}_{i}$.
Thus $\overrightarrow{\mathbf{L}}=M\left(\overrightarrow{\mathbf{r}}_{\mathrm{CM}} \times \overrightarrow{\mathbf{v}}_{\mathrm{CM}}\right)+\sum_{i} m_{i} \overrightarrow{\mathbf{r}}_{i}^{*} \times \overrightarrow{\mathbf{v}}_{i}^{*}=\overrightarrow{\mathbf{L}}^{*}+\left(\overrightarrow{\mathbf{r}}_{\mathrm{CM}} \times M \overrightarrow{\mathbf{v}}_{\mathrm{CM}}\right)$ as desired.
44. The net torque to maintain the rotation is supplied by the forces at the bearings. From Figure 11-18 we see that the net torque is $2 F d$, where $d$ is the distance from the bearings to the center of the axle. The net torque is derived in Example 11-10.

$$
\tau_{\mathrm{net}}=\frac{I \omega^{2}}{\tan \phi}=2 F d \rightarrow F=\frac{I \omega^{2}}{2 d \tan \phi}=\frac{\left(m_{\mathrm{A}} r_{\mathrm{A}}^{2}+m_{\mathrm{B}} r_{\mathrm{B}}^{2}\right)\left(\omega^{2} \sin ^{2} \phi\right)}{2 d \tan \phi}
$$

45. As in problem 44, the bearings are taken to be a distance $d$ from point O. We choose the center of the circle in which $m_{\mathrm{A}}$ moves as the origin, and label it $\mathrm{O}^{\prime}$ in the diagram. This choice of origin makes the position vector and the velocity vector always perpendicular to each other, and so makes $\overrightarrow{\mathbf{L}}$ point along the axis of rotation at all times. So $\overrightarrow{\mathbf{L}}$ is parallel to $\overrightarrow{\boldsymbol{\omega}}$. The magnitude of the angular momentum is as follows.

$$
L=m_{\mathrm{A}} r_{\mathrm{A} \perp} v=m_{\mathrm{A}}\left(r_{\mathrm{A}} \sin \phi\right)\left(\omega r_{\mathrm{A}} \sin \phi\right)=m_{\mathrm{A}} r_{\mathrm{A}}^{2} \omega \sin ^{2} \phi
$$

$\overrightarrow{\mathbf{L}}$ is constant in both magnitude and direction, and so $\frac{d \overrightarrow{\mathbf{L}}}{d t}=\overrightarrow{\boldsymbol{\tau}}_{\text {net }}=0$.


Be careful to take torques about the same point used for the angular momentum.

$$
\tau_{\text {net }}=0=F_{\mathrm{A}}\left(d-r_{\mathrm{A}} \cos \phi\right)+F_{\mathrm{B}}\left(d+r_{\mathrm{A}} \cos \phi\right)=0 \rightarrow F_{\mathrm{B}}=-F_{\mathrm{A}} \frac{\left(d-r_{\mathrm{A}} \cos \phi\right)}{\left(d+r_{\mathrm{A}} \cos \phi\right)}
$$

The mass is moving in a circle and so must have a net centripetal force pulling in on the mass (if shown, it would point to the right in the diagram). This force is given by $F_{\mathrm{C}}=m_{\mathrm{A}} \omega^{2} r_{\mathrm{A}} \sin \phi$. By Newton's third law, there must be an equal but opposite force (to the left) on the rod and axle due to the mass. But the rod and axle are massless, and so the net force on it must be 0 .

$$
\begin{aligned}
& F_{\text {net }}^{\text {no axle }}=F_{\mathrm{A}}-F_{\mathrm{B}}-F_{\mathrm{C}}=F_{\mathrm{A}}-\left(-F_{\mathrm{A}} \frac{\left(d-r_{\mathrm{A}} \cos \phi\right)}{\left(d+r_{\mathrm{A}} \cos \phi\right)}\right)-m_{\mathrm{A}} \omega^{2} r_{\mathrm{A}} \sin \phi=0 \rightarrow \\
& F_{\mathrm{A}}=\frac{m_{\mathrm{A}} \omega^{2} r_{\mathrm{A}} \sin \phi\left(d+r_{\mathrm{A}} \cos \phi\right)}{2 d} \\
& F_{\mathrm{B}}=-F_{\mathrm{A}} \frac{\left(d-r_{\mathrm{A}} \cos \phi\right)}{\left(d+r_{\mathrm{A}} \cos \phi\right)}=-\frac{m_{\mathrm{A}} \omega^{2} r_{\mathrm{A}} \sin \phi\left(d-r_{\mathrm{A}} \cos \phi\right)}{2 d}
\end{aligned}
$$

We see that $\overrightarrow{\mathbf{F}}_{\text {B }}$ points in the opposite direction as shown in the free-body diagram.
46. We use the result from Problem 44,

$$
\begin{aligned}
F & =\frac{\left(m_{\mathrm{A}} r_{\mathrm{A}}^{2}+m_{\mathrm{B}} r_{\mathrm{B}}^{2}\right)\left(\omega^{2} \sin ^{2} \phi\right)}{2 d \tan \phi}=\frac{m r^{2} \omega^{2} \sin ^{2} \phi}{d \tan \phi}=\frac{(0.60 \mathrm{~kg})(0.30 \mathrm{~m})^{2}(11.0 \mathrm{rad} / \mathrm{s})^{2} \sin ^{2} 34.0^{\circ}}{(0.115 \mathrm{~m}) \tan 34.0^{\circ}} \\
& =26 \mathrm{~N}
\end{aligned}
$$

47. This is a variation on the ballistic pendulum problem. Angular momentum is conserved about the pivot at the upper end of the rod during the collision, and this is used to find the angular velocity of the system immediately after the collision. Mechanical energy is then conserved during the upward swing. Take the 0 position for gravitational potential energy to be the original location of the center of mass of the rod. The bottom of the rod will rise twice the distance of the center of mass of the system, since it is twice as far from the pivot.

$$
\begin{aligned}
& \underset{\substack{\text { before } \\
\text { collision }}}{\left.L_{\substack{\text { after } \\
\text { collision }}} \rightarrow m\left(\frac{1}{2} \ell\right) v=\left(I_{\text {rod }}+I_{\text {puty }}\right) \omega \rightarrow \omega=\frac{m \ell v}{2\left(I_{\text {rod }}+I_{\text {puty }}\right)}\right)} \\
& E_{\substack{\text { after } \\
\text { collision }}}=E_{\substack{\text { top of } \\
\text { swing }}} \rightarrow K_{\substack{\text { affer } \\
\text { collision }}}=U_{\substack{\text { top of } \\
\text { swing }}} \rightarrow \frac{1}{2}\left(I_{\text {rod }}+I_{\text {putty }}\right) \omega^{2}=(m+M) g h \rightarrow \\
& h_{\mathrm{CM}}=\frac{\left(I_{\text {rod }}+I_{\text {puty }}\right) \omega^{2}}{2(m+M) g}=\frac{\left(I_{\text {rod }}+I_{\text {puty }}\right)}{2(m+M) g}\left[\frac{m \ell v}{2\left(I_{\text {rod }}+I_{\text {putty }}\right)}\right]^{2}=\frac{m^{2} \ell^{2} v^{2}}{8 g(m+M)\left(I_{\text {rod }}+I_{\text {putty }}\right)} \\
& =\frac{m^{2} \ell^{2} v^{2}}{8 g(m+M)\left(\frac{1}{3} M \ell^{2}+m\left(\frac{1}{2} \ell\right)^{2}\right)}=\frac{m^{2} v^{2}}{2 g(m+M)\left(\frac{4}{3} M+m\right)} \\
& h_{\text {bottom }}=2 h_{\mathrm{CM}}=\frac{m^{2} v^{2}}{g(m+M)\left(\frac{4}{3} M+m\right)}
\end{aligned}
$$

48. Angular momentum about the pivot is conserved during this collision. Note that both objects have angular momentum after the collision.

$$
\begin{aligned}
& L_{\text {before }}=L_{\text {after }}^{\text {collision }} \text { collision }
\end{aligned} \rightarrow L_{\substack{\text { bullet } \\
\text { initial }}}=L_{\text {stick }}+L_{\text {bullet }} \rightarrow m_{\text {bullet }} v_{0}\left(\frac{1}{4} \ell\right)=I_{\text {stick }} \omega+m_{\text {bullet }} v_{\mathrm{f}}\left(\frac{1}{4} \ell\right) \rightarrow, ~ \begin{aligned}
\omega & =\frac{m_{\text {bullet }}\left(v_{0}-v_{\mathrm{f}}\right)\left(\frac{1}{4} \ell\right)}{I_{\text {final }}}=\frac{m_{\text {bullet }}\left(v_{0}-v_{\mathrm{f}}\right)\left(\frac{1}{4} \ell\right)}{\frac{1}{12} M_{\text {stick }} \ell_{\text {stick }}^{2}}=\frac{3 m_{\text {bullet }}\left(v_{0}-v_{\mathrm{f}}\right)}{M_{\text {stick }} \ell_{\text {stick }}}=\frac{3(0.0030 \mathrm{~kg})(110 \mathrm{~m} / \mathrm{s})}{(0.27 \mathrm{~kg})(1.0 \mathrm{~m})} \\
& =3.7 \mathrm{rad} / \mathrm{s}
\end{aligned}
$$

49. The angular momentum of the Earth-meteorite system is conserved in the collision. The Earth is spinning counterclockwise as viewed in the diagram. We take that direction as the positive direction for rotation about the Earth's axis, and so the initial angular momentum of the meteorite is negative.

$$
\left.\begin{array}{l}
L_{\text {initial }}=L_{\text {final }} \rightarrow I_{\text {Earth }} \omega_{0}-m R_{\mathrm{E}} v \sin 45^{\circ}=\left(I_{\text {Earth }}+I_{\text {meteorite }}\right) \omega \rightarrow \\
\omega
\end{array}=\frac{I_{\text {Earth }} \omega_{0}-m R_{\mathrm{E}} v \sin 45^{\circ}}{\left(I_{\text {Earth }}+I_{\text {meteorite }}\right)}=\frac{\frac{2}{5} M_{\mathrm{E}} R_{\mathrm{E}}^{2} \omega_{0}-m R_{\mathrm{E}} v \sin 45^{\circ}}{\left(\frac{2}{5} M_{\mathrm{E}} R_{\mathrm{E}}^{2}+m R_{\mathrm{E}}^{2}\right)}\right) \quad \begin{aligned}
& \frac{\omega}{\omega_{0}}=\frac{\frac{2}{5} M_{\mathrm{E}} R_{\mathrm{E}}^{2}-m R_{\mathrm{E}} \frac{v}{\omega_{0}} \frac{1}{\sqrt{2}}}{R_{\mathrm{E}}^{2}\left(\frac{2}{5} M_{\mathrm{E}}+m\right)}=\frac{R_{\mathrm{E}}^{2}\left(\frac{2}{5} M_{\mathrm{E}}-\frac{m v}{\sqrt{2} \omega_{0} R_{\mathrm{E}}}\right)}{R_{\mathrm{E}}^{2}\left(m+\frac{2}{5} M_{\mathrm{E}}\right)}=\frac{\left(\frac{2}{5} M_{\mathrm{E}}-\frac{m v}{\sqrt{2} \omega_{0} R_{\mathrm{E}}}\right)}{\left(m+\frac{2}{5} M_{\mathrm{E}}\right)} \\
& \frac{\Delta \omega}{\omega_{0}}=\frac{\omega-\omega_{0}}{\omega_{0}}=\frac{\omega}{\omega_{0}}-1=\frac{\left(\frac{2}{5} M_{\mathrm{E}}-\frac{m v}{\sqrt{2} \omega_{0} R_{\mathrm{E}}}\right)}{\left(m+\frac{2}{5} M_{\mathrm{E}}\right)}-1=\frac{-\left(\frac{v}{\sqrt{2} \omega_{0} R_{\mathrm{E}}}+1\right)}{\left(1+\frac{2}{5} \frac{M_{\mathrm{E}}}{m}\right)} \\
& \\
& \\
& =-\frac{2.2 \times 10^{4} \mathrm{~m} / \mathrm{s}}{\left(\frac{2 \pi}{\sqrt{2}\left(\frac{2 \pi}{86,400} \mathrm{rad} / \mathrm{s}\right)\left(6.38 \times 10^{6} \mathrm{~m}\right)}\right)} \begin{array}{l}
\left(1+\frac{2}{5} \frac{5.97 \times 10^{24} \mathrm{~kg}}{5.8 \times 10^{10} \mathrm{~kg}}\right)
\end{array}
\end{aligned}
$$

50. (a) Linear momentum of the center of mass is conserved in the totally inelastic collision.

$$
\begin{aligned}
& p_{\text {initial }}=p_{\text {final }} \rightarrow m_{\text {beam }} v_{0}=\left(m_{\text {beam }}+m_{\text {man }}\right) v_{\text {final }} \rightarrow \\
& v_{\text {final }}=\frac{m_{\text {beam }} v_{0}}{\left(m_{\text {beam }}+m_{\text {man }}\right)}=\frac{(230 \mathrm{~kg})(18 \mathrm{~m} / \mathrm{s})}{(295 \mathrm{~kg})}=14 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(b) Angular momentum about the center of mass of the system is conserved. First we find the center of mass, relative to the center of mass of the rod, taking down as the positive direction. See the diagram.

$$
y_{\text {CM }}=\frac{m_{\text {beam }}(0)+m_{\operatorname{man}}\left(\frac{1}{2} \ell\right)}{\left(m_{\text {beam }}+m_{\operatorname{man}}\right)}=\frac{(65 \mathrm{~kg})(1.35 \mathrm{~m})}{(295 \mathrm{~kg})}
$$

$$
=0.2975 \mathrm{~m} \text { below center of rod }
$$

We need the moment of inertia of the beam about the center of mass of the entire system. Use the parallel axis theorem.


$$
\begin{aligned}
& I_{\text {beam }}=\frac{1}{12} m_{\text {beam }} \ell^{2}+m_{\text {beam }} r_{\text {beam }}^{2} ; I_{\text {man }}=m_{\text {man }}\left(\frac{1}{2} \ell-r_{\text {beam }}\right)^{2} \\
& L_{\text {intital }}=L_{\text {final }} \rightarrow m_{\text {beam }} \nu_{0} r_{\text {beam }}=\left(I_{\text {beam }}+I_{\text {man }}\right) \omega_{\text {final }} \rightarrow \\
& \omega_{\text {final }}=\frac{m_{\text {beam }} v_{0} r_{\text {beam }}}{\left(I_{\text {beam }}+I_{\text {man }}\right)}=\frac{m_{\text {beam }} v_{0} r_{\text {beam }}}{\frac{1}{12} m_{\text {beam }}\left(\frac{1}{2} \ell\right)^{2}+m_{\text {beam }} r_{\text {beam }}^{2}+m_{\text {man }}\left(\frac{1}{2} \ell-r_{\text {beam }}\right)^{2}} \\
& =\frac{(230 \mathrm{~kg})(18 \mathrm{~m} / \mathrm{s})(0.2975 \mathrm{~m})}{\frac{1}{12}(230 \mathrm{~kg})(2.7 \mathrm{~m})^{2}+(230 \mathrm{~kg})(0.2975 \mathrm{~m})^{2}+(65 \mathrm{~kg})(1.0525 \mathrm{~m})^{2}} \\
& =5.307 \mathrm{rad} / \mathrm{s} \approx 5.3 \mathrm{rad} / \mathrm{s}
\end{aligned}
$$

51. Linear momentum of the center of mass is conserved in the totally inelastic collision.

$$
p_{\text {initial }}=p_{\text {final }} \rightarrow m v=(m+M) v_{\mathrm{CM}} \rightarrow \underset{\substack{\text { final }}}{v_{\mathrm{CM}}=\frac{m v}{\text { final }}} \frac{v^{\prime}}{m+M}
$$

Angular momentum about the center of mass of the system is conserved. First we find the center of mass, relative to the center of mass of the rod, taking up as the positive direction. See the diagram.

$$
y_{\mathrm{CM}}=\frac{m\left(\frac{1}{4} \ell\right)+M(0)}{(m+M)}=\frac{m \ell}{4(m+M)}
$$

The distance of the stuck clay ball from the system's center of mass is found.

$$
y_{\text {clay }}=\frac{1}{4} \ell-y_{\mathrm{CM}}=\frac{1}{4} \ell-\frac{m \ell}{4(m+M)}=\frac{M \ell}{4(m+M)}
$$

We need the moment of inertia of the rod about the center of mass of the entire system. Use the parallel axis theorem. Treat the clay as a point mass.

$$
I_{\mathrm{rod}}=\frac{1}{12} M \ell^{2}+M\left[\frac{m \ell}{4(m+M)}\right]^{2}
$$

Now express the conservation of angular momentum about the system's center of mass.

$$
\begin{aligned}
L_{\text {initial }} & =L_{\text {final }} \rightarrow m v y_{\text {clay }}=\left(I_{\text {rod }}+I_{\text {clay }}\right) \omega_{\text {final }} \rightarrow \\
\omega_{\text {final }} & =\frac{m v y_{\text {clay }}}{\left(I_{\text {rod }}+I_{\text {clay }}\right)}=\frac{m v y_{\text {clay }}}{\left(\frac{1}{12} M \ell^{2}+M\left[\frac{m \ell}{4(m+M)}\right]^{2}+m y_{\text {clay }}^{2}\right)} \\
& =\frac{m v \frac{M \ell}{4(m+M)}}{\left(\frac{1}{12} M \ell^{2}+M\left[\frac{m \ell}{4(m+M)}\right]^{2}+m\left[\frac{M \ell}{4(m+M)}\right]^{2}\right)}=\frac{12 m v(m+M)}{\ell\left(7 m^{2}+11 m M+4 M^{2}\right)} \\
& =\frac{12 m v}{\ell(7 m+4 M)}
\end{aligned}
$$

52. (a) See the free-body diagram for the ball, after it has moved away from the initial point. There are three forces on the ball. $\overrightarrow{\mathbf{F}}_{\mathrm{N}}$ and $m \overrightarrow{\mathbf{g}}$ are in opposite directions and each has the same lever arm about an axis passing through point O perpendicular to the plane of the paper. Thus they cause no net torque. $\overrightarrow{\mathbf{F}}_{\text {fr }}$ has a 0 lever arm about an axis through O , and so also produces no torque. Thus the net torque on the ball is 0 . Since we are calculating
 torques about a point fixed in an inertial reference frame, we may say that $\sum \overrightarrow{\boldsymbol{\tau}}=\frac{d \overrightarrow{\mathbf{L}}}{d t}=0$ and so $\overrightarrow{\mathbf{L}}$ is constant. Note that the ball is initially slipping while it rolls, and so we may NOT say that $v_{0}=R \omega_{0}$ at the initial motion of the ball.
(b) We follow the hint, and express the total angular momentum as a sum of two terms. We take clockwise as the positive rotational direction.

$$
\overrightarrow{\mathbf{L}}=\overrightarrow{\mathbf{L}}_{v_{\mathrm{cN}}}+\overrightarrow{\mathbf{L}}_{\omega}=m R v_{\mathrm{cm}}-I \omega
$$

The angular momentum is constant. We equate the angular momentum at the initial motion, with $v_{\mathrm{cm}}=v_{0}$ and $\omega=\omega_{0}=\omega_{\mathrm{C}}$, to the final angular momentum, with $v_{\mathrm{cm}}=0$ and $\omega=0$.

$$
\overrightarrow{\mathbf{L}}_{\text {initial }}=\overrightarrow{\mathbf{L}}_{\text {final } \rightarrow}=m R v_{0}-I \omega_{\mathrm{C}}=m R(0)-I(0)=0 \rightarrow \omega_{C}=\frac{m R v_{0}}{I_{\mathrm{CM}}}=\frac{m R v_{0}}{\frac{2}{5} m R^{2}}=\frac{5 v_{0}}{2 R}
$$

(c) Angular momentum is again conserved. In the initial motion, $v_{\mathrm{CM}}=v_{0}$ and $\omega_{0}=0.90 \omega_{\mathrm{C}}$. Note that in the final state, $\omega=v_{\mathrm{CM}} / R$, and the final angular momenta add to each other.

$$
\begin{aligned}
& L_{\text {initial }}=L_{\text {final }} \rightarrow m R v_{0}-0.90 I \omega_{\mathrm{C}}=m R v_{\mathrm{CM}}+I \frac{v_{\mathrm{CM}}}{R} \rightarrow \\
& m R v_{0}-0.90\left(\frac{2}{5} m R^{2}\right)\left(\frac{5 v_{0}}{2 R}\right)=m R v_{\mathrm{CM}}+\left(\frac{2}{5} m R^{2}\right) \frac{v_{\mathrm{CM}}}{R} \rightarrow \frac{1}{10} v_{0}=\frac{7}{5} v_{\mathrm{CM}} \rightarrow \\
& v_{\mathrm{CM}}=\frac{1}{14} v_{0}
\end{aligned}
$$

This answer is reasonable. There is not enough "backspin" since $\omega_{0}<\omega_{\mathrm{C}}$, and so the ball's final state is rolling forwards.
(d) Angular momentum is again conserved. In the initial motion, $v_{\mathrm{CM}}=v_{0}$ and $\omega_{0}=1.10 \omega_{\mathrm{C}}$. Note that in the final state, $\omega=v_{\mathrm{CM}} / R$, and the final angular momenta add to each other.

$$
\begin{aligned}
& L_{\text {initial }}=L_{\text {final }} \rightarrow m R v_{0}-1.10 I \omega_{\mathrm{C}}=m R v_{\mathrm{CM}}+I \frac{v_{\mathrm{CM}}}{R} \rightarrow \\
& m R v_{0}-1.10\left(\frac{2}{5} m R^{2}\right)\left(\frac{5 v_{0}}{2 R}\right)=m R v_{\mathrm{CM}}+\left(\frac{2}{5} m R^{2}\right) \frac{v_{\mathrm{CM}}}{R} \rightarrow-\frac{1}{10} v_{0}=\frac{7}{5} v_{\mathrm{CM}} \rightarrow \\
& v_{\mathrm{CM}}=-\frac{1}{14} v_{0}
\end{aligned}
$$

This answer is reasonable. There is more than enough "backspin" since $\omega_{0}>\omega_{\mathrm{C}}$, and so the ball's final state is rolling backwards.
53. Use Eq. 11-13c for the precessional angular velocity.

$$
\Omega=\frac{M g r}{I \omega} \rightarrow I=\frac{M g r}{\Omega \omega}=\frac{(0.22 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.035 \mathrm{~m})}{\left[\frac{1 \mathrm{rev}}{6.5 \mathrm{~s}}\left(\frac{2 \pi \mathrm{rad}}{\mathrm{rev}}\right)\right]\left[\frac{15 \mathrm{rev}}{1 \mathrm{~s}}\left(\frac{2 \pi \mathrm{rad}}{\mathrm{rev}}\right)\right]}=8.3 \times 10^{-4} \mathrm{~kg} \cdot \mathrm{~m}^{2}
$$

54. (a) The period of precession is related to the reciprocal of the angular precessional frequency.

$$
\begin{aligned}
T & =\frac{2 \pi}{\Omega}=\frac{2 \pi I \omega}{M g r}=\frac{2 \pi\left[\frac{1}{2} M r_{\text {disk }}^{2}\right] 2 \pi f}{M g r}=\frac{2 \pi^{2} f r_{\text {disk }}^{2}}{g r}=\frac{2 \pi^{2}(45 \mathrm{rev} / \mathrm{s})(0.055 \mathrm{~m})^{2}}{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.105 \mathrm{~m})} \\
& =2.611 \mathrm{~s} \approx 2.6 \mathrm{~s}
\end{aligned}
$$

(b) Use the relationship $T=\frac{2 \pi^{2} f r_{\text {disk }}^{2}}{g r}$ derived above to see the effect on the period.

$$
\left.\frac{T_{\text {new }}}{T_{\text {original }}}=\frac{\frac{2 \pi^{2} f r_{\text {disk }}^{2}}{\text { new }}}{\frac{g r_{\text {new }}}{2 \pi^{2} f r_{\text {disk }}^{2}}}=\frac{\begin{array}{l}
r_{\text {disk }}^{2} \\
\text { new }
\end{array}}{\frac{\begin{array}{l}
r_{\text {new }}
\end{array}}{r_{\text {disk }}^{2}}}=\binom{r_{\text {disk }}}{\text { new }}^{2} \frac{r}{r_{\text {disk }}}\right)_{\text {new }}^{r_{\text {new }}}=\left(\frac{2}{1}\right)^{2} \frac{1}{2}=2
$$

So the period would double, and thus be $T_{\text {new }}=2 T_{\text {original }}=2(2.611 \mathrm{~s})=5.222 \mathrm{~s} \approx 5.2 \mathrm{~s}$.
55. Use Eq. 11-13c for the precessional angular velocity.

$$
\Omega=\frac{M g r}{I \omega}=\frac{M g\left(\frac{1}{2} \ell_{\text {axle }}\right)}{\frac{1}{2} M r_{\text {wheel }}^{2} \omega}=\frac{g \ell_{\text {axle }}}{r_{\text {wheel }}^{2} \omega}=\frac{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.25 \mathrm{~m})}{(0.060 \mathrm{~m})^{2}(85 \mathrm{rad} / \mathrm{s})}=8.0 \mathrm{rad} / \mathrm{s} \quad(1.3 \mathrm{rev} / \mathrm{s})
$$

56. The mass is placed on the axis of rotation and so does not change the moment of inertia. The addition of the mass does change the center of mass position $r$, and it does change the total mass, $M$, to $\frac{3}{2} M$.

$$
\begin{aligned}
& r_{\text {new }}=\frac{M\left(\frac{1}{2} \ell_{\text {axle }}\right)+\frac{1}{2} M \ell_{\text {axle }}}{M+\frac{1}{2} M}=\frac{M \ell_{\text {axle }}}{\frac{3}{2} M}=\frac{2}{3} \ell_{\text {axle }} \\
& \frac{\Omega_{\text {nev }}}{\Omega_{\text {orignal }}}=\frac{\frac{M_{\text {new }} g r_{\text {new }}}{I \omega}}{\frac{M \text { orignal } r_{\text {original }}}{I \omega}=\frac{\frac{3}{2} M\left(\frac{2}{3} \ell_{\text {axle }}\right)}{M\left(\frac{1}{2} \ell_{\text {axle }}\right)}=2 \rightarrow} \\
& \Omega_{\text {new }}=2 \Omega_{\text {original }}=2(8.0 \mathrm{rad} / \mathrm{s})=16 \mathrm{rad} / \mathrm{s}
\end{aligned}
$$

57. The spinning bicycle wheel is a gyroscope. The angular frequency of precession is given by Eq. 11-13c.

$$
\begin{aligned}
\Omega & =\frac{M g r}{I \omega}=\frac{M g r}{M r_{\text {wheel }}^{2} \omega}=\frac{g r}{r_{\text {wheel }}^{2} \omega}=\frac{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.20 \mathrm{~m})}{(0.325 \mathrm{~m})^{2}(4.0 \pi \mathrm{rad} / \mathrm{s})} \\
& =1.477 \mathrm{rad} / \mathrm{s}\left(\frac{1 \mathrm{rev}}{2 \pi \mathrm{rad}}\right)\left(\frac{60 \mathrm{~s}}{1 \mathrm{~min}}\right)=14 \mathrm{rev} / \mathrm{min}
\end{aligned}
$$



In the figure, the torque from gravity is directed back into the paper. This gives the direction of precession. When viewed from above, the wheel will precess counterclockwise.
58. We assume that the plant grows in the direction of the local "normal" force. In the rotating frame of the platform, there is an outward fictitious force of magnitude $m \frac{v^{2}}{r}=m r \omega^{2}$. See the free body diagram for the rotating frame of reference. Since the object is not accelerated in that frame of reference, the "normal" force must be the vector sum of the other two forces. Write Newton's second law in this frame of reference.


$$
\begin{aligned}
& \sum F_{\text {vericial }}=F_{\mathrm{N}} \cos \theta-m g=0 \rightarrow F_{\mathrm{N}}=\frac{m g}{\cos \theta} \\
& \sum F_{\text {horizonal }}=F_{\mathrm{N}} \sin \theta-m r \omega^{2}=0 \rightarrow F_{\mathrm{N}}=\frac{m r \omega^{2}}{\sin \theta} \\
& \frac{m g}{\cos \theta}=\frac{m r \omega^{2}}{\sin \theta} \rightarrow \tan \theta=\frac{r \omega^{2}}{g} \rightarrow \theta=\tan ^{-1} \frac{r \omega^{2}}{g}
\end{aligned}
$$

In the inertial frame of reference, the "normal" force still must point inward. The horizontal component of that force is providing the centripetal acceleration, which points inward.
59. (a) At the North Pole, the factor $m \omega^{2} r$ is zero, and so there is no effect from the rotating reference frame.

$$
g^{\prime}=g-\omega^{2} r=g-0=9.80 \mathrm{~m} / \mathrm{s}^{2}, \text { inward along a radial line }
$$

(b) To find the direction relative to a radial line, we orient the coordinate system along the tangential $(x)$ and radial ( $y$, with inward as positive) directions. See the diagram. At a specific latitude $\phi$, the "true" gravity will point purely in the positive $y$ direction, $\overrightarrow{\mathbf{g}}=g \hat{\mathbf{j}}$. We label the effect of the rotating reference frame as $\overrightarrow{\mathbf{g}}_{\text {rot }}$. The effect of $\overrightarrow{\mathbf{g}}_{\text {rot }}$ can be found by decomposing it along the axes. Note that the radius of rotation is not the radius of the Earth, but $r=R_{\mathrm{E}} \cos \phi$.

$$
\begin{aligned}
\overrightarrow{\mathbf{g}}_{\text {rot }} & =r \omega^{2} \sin \phi \hat{\mathbf{i}}-r \omega^{2} \cos \phi \hat{\mathbf{j}} \\
& =R_{\mathrm{E}} \omega^{2} \cos \phi \sin \phi \hat{\mathbf{i}}-R_{\mathrm{E}} \omega^{2} \cos ^{2} \phi \hat{\mathbf{j}} \\
\overrightarrow{\mathbf{g}}^{\prime} & =\overrightarrow{\mathbf{g}}+\overrightarrow{\mathbf{g}}_{\text {rot }}=R_{\mathrm{E}} \omega^{2} \cos \phi \sin \phi \hat{\mathbf{i}}+\left(g-R_{\mathrm{E}} \omega^{2} \cos ^{2} \phi\right) \hat{\mathbf{j}}
\end{aligned}
$$



The angle of deflection from the vertical $\theta$ ) can be found from the components of $\overrightarrow{\mathbf{g}}^{\prime}$.

$$
\begin{aligned}
\theta & =\tan ^{-1} \frac{g_{x}^{\prime}}{g_{y}^{\prime}}=\tan ^{-1} \frac{R_{\mathrm{E}} \omega^{2} \cos \phi \sin \phi}{g-R_{\mathrm{E}} \omega^{2} \cos ^{2} \phi} \\
& =\tan ^{-1} \frac{\left(6.38 \times 10^{6} \mathrm{~m}\right)\left(\frac{2 \pi \mathrm{rad}}{86,400 \mathrm{~s}}\right)^{2} \cos 45^{\circ} \sin 45^{\circ}}{9.80 \mathrm{~m} / \mathrm{s}^{2}-\left(6.38 \times 10^{6} \mathrm{~m}\right)\left(\frac{2 \pi \mathrm{rad}}{86,400 \mathrm{~s}}\right)^{2} \cos ^{2} 45^{\circ}}=\tan ^{-1} \frac{1.687 \times 10^{-2} \mathrm{~m} / \mathrm{s}^{2}}{9.783 \mathrm{~m} / \mathrm{s}^{2}}=0.988^{\circ}
\end{aligned}
$$

The magnitude of $\overrightarrow{\mathbf{g}}^{\prime}$ is found from the Pythagorean theorem.

$$
g^{\prime}=\sqrt{g_{x}^{\prime 2}+g_{y}^{\prime 2}}=\sqrt{\left(1.687 \times 10^{-2} \mathrm{~m} / \mathrm{s}^{2}\right)^{2}+\left(9.783 \mathrm{~m} / \mathrm{s}^{2}\right)^{2}}=9.78 \mathrm{~m} / \mathrm{s}^{2}
$$

And so $g^{\prime}=9.78 \mathrm{~m} / \mathrm{s}^{2}, 0.0988^{\circ}$ south from an inward radial line.
(c) At the equator, the effect of the rotating reference frame is directly opposite to the "true" acceleration due to gravity. Thus the values simply subtract.

$$
\begin{aligned}
& g^{\prime}=g-\omega^{2} r=g-\omega^{2} R_{\text {Earth }}=9.80 \mathrm{~m} / \mathrm{s}^{2}-\left(\frac{2 \pi \mathrm{rad}}{86.400 \mathrm{~s}}\right)^{2}\left(6.38 \times 10^{6} \mathrm{~m}\right) \\
& =9.77 \mathrm{~m} / \mathrm{s}^{2}, \text { inward along a radial line }
\end{aligned}
$$

60. (a) In the inertial frame, the ball has the tangential speed of point $B$, $v_{\mathrm{B}}=r_{\mathrm{B}} \omega$. This is greater than the tangential speed of the women at
$\mathrm{A}, v_{\mathrm{A}}=r_{\mathrm{A}} \omega$, so the ball passes in front of the women. The ball deflects to the right of the intended motion. See the diagram.
(b) We follow a similar derivation to that given in section 11-9. In the inertial frame, the ball is given an inward radial velocity $v$ by the
 man at B . The ball moves radially inward a distance $r_{\mathrm{B}}-r_{\mathrm{A}}$ during a short time $t$, and so $r_{\mathrm{B}}-r_{\mathrm{A}}=v t$. During this time, the ball moves sideways a distance $s_{\mathrm{B}}=v_{\mathrm{B}} t$, while the woman moves a distance $s_{\mathrm{A}}=v_{\mathrm{A}} t$. The ball will pass in front of the woman a distance given by the following.

$$
s=s_{\mathrm{B}}-s_{\mathrm{A}}=\left(v_{\mathrm{B}}-v_{\mathrm{A}}\right) t=\left(r_{\mathrm{B}}-r_{\mathrm{A}}\right) \omega t=v \omega t^{2}
$$

This is the sideways displacement as seen from the noninertial frame, and so the deflection is $v \omega t^{2}$. This has the same form as motion at constant acceleration, with $s=v \omega t^{2}=\frac{1}{2} a_{\text {Cor }} t^{2}$.
Thus the Coriolis acceleration is $a_{\text {Cor }}=2 v \omega$.
61. The footnote on page 302 gives the Coriolis acceleration as $\overrightarrow{\mathbf{a}}_{\text {Cor }}=2 \overrightarrow{\boldsymbol{\omega}} \times \overrightarrow{\mathbf{v}}$. The angular velocity vector is parallel to the axis of rotation of the Earth. For the Coriolis acceleration to be 0 , then, the velocity must be parallel to the axis of rotation of the Earth. At the equator this means moving either due north or due south.
62. The Coriolis acceleration of the ball is modified to $a_{\text {Cor }}=2 \omega v_{\perp}=2 \omega v \cos \lambda$, where $v$ is the vertical speed of the ball. The vertical speed is not constant as the ball falls, but is given by $v=v_{0}+g t$.

Assuming the ball starts from rest, then $a_{\text {Cor }}=2 \omega g t \cos \lambda$. That is not a constant acceleration, and so to find the deflection due to this acceleration, we must integrate twice.

$$
\begin{aligned}
& a_{\text {Cor }}=2 \omega g t \cos \lambda=\frac{d v_{\text {Cor }}}{d t} \rightarrow d v_{\text {Cor }}=2 \omega g t \cos \lambda d t \rightarrow \int_{0}^{v_{\mathrm{Cor}}} d v_{\text {Cor }}=2 \omega g \cos \lambda \int_{0}^{t} t d t \rightarrow \\
& v_{\text {Cor }}=\omega g t^{2} \cos \lambda=\frac{d x_{\text {Cor }}}{d t} \rightarrow d x_{\text {Cor }}=\omega g t^{2} \cos \lambda d t \rightarrow \int_{0}^{x_{\mathrm{Cor}}} d x_{\text {Cor }}=\int_{0}^{t} \omega g t^{2} \cos \lambda d t \rightarrow \\
& x_{\text {Cor }}=\frac{1}{3} \omega g t^{3} \cos \lambda
\end{aligned}
$$

So to find the Coriolis deflection, we need the time of flight. The vertical motion is just uniform acceleration, for an object dropped from rest. Use that to find the time.

$$
y=y_{0}+v_{0 y} t+\frac{1}{2} g t^{2} \rightarrow t=\sqrt{\frac{2\left(y-y_{0}\right)}{g}}=\sqrt{\frac{2 h}{g}}
$$

$$
\begin{aligned}
x_{\text {Cor }} & =\frac{1}{3} \omega g t^{3} \cos \lambda=\frac{1}{3} \omega g \cos \lambda\left(\frac{2 h}{g}\right)^{3 / 2}=\frac{1}{3} \omega \cos \lambda\left(\frac{8 h^{3}}{g}\right)^{1 / 2} \\
& =\frac{1}{3}\left(\frac{2 \pi \mathrm{rad}}{86,400 \mathrm{~s}}\right)\left(\cos 44^{\circ}\right)\left(\frac{8(110 \mathrm{~m})^{3}}{9.80 \mathrm{~m} / \mathrm{s}^{2}}\right)^{1 / 2}=0.018 \mathrm{~m}
\end{aligned}
$$

The ball is deflected by about 2 cm in falling 110 meters.
63. The diagram is a view from above the wheel. The ant is moving in a curved path, and so there is a fictitious outward radial force of $m \omega^{2} r \hat{\mathbf{i}}$. The ant is moving away from the axis of rotation, and so there is a fictitious Coriolis force of $-2 m \omega v \hat{\mathbf{j}}$. The ant is moving with a constant speed, and so in the rotating reference frame the net force is 0 . Thus there must be forces that oppose these fictitious forces. The ant is in contact with the spoke, and so there can be components of that contact force in each of the coordinate axes. The force opposite to the local direction of motion is friction, and so is $-F_{\mathrm{fr}} \hat{\mathbf{i}}$. The spoke is also pushing in the opposite direction to
 the Coriolis force, and so we have $F_{\text {spoke }} \hat{\mathbf{j}}$. Finally, in the vertical direction, there is gravity $(-m g \hat{\mathbf{k}})$ and the usual normal force $\left(F_{\mathrm{N}} \hat{\mathbf{k}}\right)$. These forces are not shown on the diagram, since it is viewed from above.

$$
\begin{array}{|c}
\overrightarrow{\mathbf{F}}_{\text {rotaing }}^{\text {frame }}
\end{array}=\left(m \omega^{2} r-F_{\mathrm{Fr}}\right) \hat{\mathbf{i}}+\left(F_{\text {spoke }}-2 m \omega v\right) \hat{\mathbf{j}}+\left(F_{\mathrm{N}}-m g\right) \hat{\mathbf{k}}
$$

64. (a) Because the hoop is rolling without slipping, the acceleration of the center of the center of mass is related to the angular acceleration by $a_{\mathrm{CM}}=\alpha R$. From the free-body diagram, write Newton's second law for the vertical direction and for rotation. We call down and clockwise the positive directions. Combine those equations to find the angular acceleration.

$$
\begin{aligned}
& \sum F_{\text {vericial }}=M g-F_{\mathrm{T}}=M a_{\mathrm{CM}} \rightarrow F_{\mathrm{T}}=M\left(g-a_{\mathrm{CM}}\right) \\
& \sum \tau=F_{\mathrm{T}} R=I \alpha=M R^{2} \frac{a_{\mathrm{CM}}}{R}=M R a_{\mathrm{CM}} \\
& M\left(g-a_{\mathrm{CM}}\right) R=M R a_{\mathrm{CM}} \rightarrow\left(g-a_{\mathrm{CM}}\right)=a_{\mathrm{CM}} \rightarrow a_{\mathrm{CM}}=\frac{1}{2} g \rightarrow \alpha=\frac{1}{2} \frac{g}{R} \\
& \tau=I \alpha=M R^{2} \frac{1}{2} \frac{g}{R}=\frac{1}{2} M R g=\frac{d L}{d t} \rightarrow L=\frac{1}{2} M R g t
\end{aligned}
$$

(b) $\quad F_{\mathrm{T}}=M\left(g-a_{\text {См }}\right)=\frac{1}{2} M g$, and is constant in time.
65. (a) Use Eq. 11-6 to find the angular momentum.

$$
\overrightarrow{\mathbf{L}}=\overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{p}}=m(\overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{v}})=(1.00 \mathrm{~kg})\left|\begin{array}{ccc}
\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\
0 & 2.0 & 4.0 \\
7.0 & 6.0 & 0
\end{array}\right| \mathrm{kg} \cdot \mathrm{~m}^{2} / \mathrm{s}=(-24 \hat{\mathbf{i}}+28 \hat{\mathbf{j}}-14 \hat{\mathbf{k}}) \mathrm{kg} \cdot \mathrm{~m}^{2} / \mathrm{s}
$$

(b) $\overrightarrow{\boldsymbol{\tau}}=\overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{F}}=\left|\begin{array}{ccc}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & 2.0 & 4.0 \\ 4.0 & 0 & 0\end{array}\right| \mathrm{m} \cdot \mathrm{N}=(16 \hat{\mathbf{j}}-8.0 \hat{\mathbf{k}}) \mathrm{m} \cdot \mathrm{N}$
66. Angular momentum is conserved in the interaction between the child and the merry-go-round.

$$
\begin{aligned}
& L_{\text {initial }}=L_{\text {final }} \rightarrow \underset{\substack{\text { mgr }}}{L_{0}}=\underset{\substack{\mathrm{f} \\
\text { child }}}{L_{\mathrm{m}}}+\underset{\mathrm{mgr}}{L_{\mathrm{f}}} \rightarrow I_{\mathrm{mgr}} \omega_{0}=\left(I_{\mathrm{mgr}}+I_{\text {child }}\right) \omega=\left(I_{\mathrm{mgr}}+m_{\text {child }} R_{\mathrm{mgr}}^{2}\right) \omega \rightarrow \\
& m_{\text {child }}=\frac{I_{\mathrm{mgr}}\left(\omega_{0}-\omega\right)}{R_{\mathrm{mgr}}^{2} \omega}=\frac{\left(1260 \mathrm{~kg} \cdot \mathrm{~m}^{2}\right)(0.45 \mathrm{rad} / \mathrm{s})}{(2.5 \mathrm{~m})^{2}(1.25 \mathrm{rad} / \mathrm{s})}=73 \mathrm{~kg}
\end{aligned}
$$

67. (a) See the free-body diagram for the vehicle, tilted up on 2 wheels, on the verge of rolling over. The center of the curve is to the left in the diagram, and so the center of mass is accelerating to the left. The force of gravity acts through the center of mass, and so causes no torque about the center of mass, but the normal force and friction cause opposing torques about the center of mass. The amount of tilt is exaggerated. Write Newton's second laws for the horizontal and vertical directions and for torques, taking left, up, and counterclockwise as positive.


$$
\begin{aligned}
& \sum F_{\text {vericial }}=F_{\mathrm{N}}-M g=0 \rightarrow F_{\mathrm{N}}=M g \\
& \sum F_{\text {horizontal }}=F_{\mathrm{fr}}=M \frac{v_{\mathrm{C}}^{2}}{R} \\
& \sum \tau=F_{\mathrm{N}}\left(\frac{1}{2} w\right)-F_{\mathrm{fr}} h=0 \rightarrow F_{\mathrm{N}}\left(\frac{1}{2} w\right)=F_{\mathrm{fr}} h \\
& M g\left(\frac{1}{2} w\right)=M \frac{v_{\mathrm{C}}^{2}}{R} h \rightarrow v_{\mathrm{C}}=\sqrt{R g\left(\frac{w}{2 h}\right)}
\end{aligned}
$$

(b) From the above result, we see that $R=\frac{v_{\mathrm{C}}^{2}}{g} \frac{2 h}{w}=\frac{v_{\mathrm{C}}^{2}}{g(\mathrm{SSF})}$.

$$
\frac{R_{\mathrm{car}}}{R_{\mathrm{SUV}}}=\frac{\frac{v_{\mathrm{C}}^{2}}{g(\mathrm{SSF})_{\mathrm{car}}}}{\frac{v_{\mathrm{C}}^{2}}{g(\mathrm{SSF})_{\mathrm{Suv}}}}=\frac{(\mathrm{SSF})_{\mathrm{SUV}}}{(\mathrm{SSF})_{\mathrm{car}}}=\frac{1.05}{1.40}=0.750
$$

68. The force applied by the spaceship puts a torque on the asteroid which changes its angular momentum. We assume that the rocket ship's direction is adjusted to always be tangential to the surface. Thus the torque is always perpendicular to the angular momentum, and so will not change the magnitude of the angular momentum, but only its direction, similar to the action of a centripetal force on an object in circular motion. From the diagram, we make an approximation.


$$
\begin{aligned}
& \tau=\frac{d L}{d t} \approx \frac{\Delta L}{\Delta t} \approx \frac{L \Delta \theta}{\Delta t} \rightarrow \\
& \Delta t=\frac{L \Delta \theta}{\tau}=\frac{I \omega \Delta \theta}{F r}=\frac{\frac{2}{5} m r^{2} \omega \Delta \theta}{F r}=\frac{2 m r \omega \Delta \theta}{5 F}
\end{aligned}
$$

$$
\begin{aligned}
& =\frac{2\left(2.25 \times 10^{10} \mathrm{~kg}\right)(123 \mathrm{~m})\left[\left(\frac{4 \text { rev }}{1 \text { day }}\right)\left(\frac{2 \pi \mathrm{rad}}{1 \mathrm{rev}}\right)\left(\frac{1 \text { day }}{86400 \mathrm{~s}}\right)\right]\left[10.0^{\circ}\left(\frac{2 \pi \mathrm{rad}}{360^{\circ}}\right)\right]}{5(265 \mathrm{~N})} \\
& =\left(2.12 \times 10^{5} \mathrm{~s}\right) \frac{1 \mathrm{hr}}{3600 \mathrm{~s}}=58.9 \mathrm{hr}
\end{aligned}
$$

Note that, in the diagram in the book, the original angular momentum is "up" and the torque is into the page. Thus the planet's axis would actually tilt backwards into the plane of the paper, not rotate clockwise as shown in the figure above.
69. The velocity is the derivative of the position.

$$
\begin{aligned}
\overrightarrow{\mathbf{v}} & =\frac{d \overrightarrow{\mathbf{r}}}{d t}=\frac{d}{d t}[R \cos (\omega t) \hat{\mathbf{i}}+R \sin (\omega t) \hat{\mathbf{j}}]=-\omega R \sin (\omega t) \hat{\mathbf{i}}+\omega R \cos (\omega t) \hat{\mathbf{j}} \\
& =v[-\sin (\omega t) \hat{\mathbf{i}}+\cos (\omega t) \hat{\mathbf{j}}]
\end{aligned}
$$

From the right hand rule, a counterclockwise rotation in the $x-y$ plane produces an angular velocity in the $+\hat{\mathbf{k}}$-direction. Thus $\vec{\omega}=\left(\frac{v}{R}\right) \hat{\mathbf{k}}$. Now take the cross product $\vec{\omega} \times \overrightarrow{\mathbf{r}}$.

$$
\begin{aligned}
\vec{\omega} \times \overrightarrow{\mathbf{r}} & =\left[\frac{v}{R} \hat{\mathbf{k}}\right] \times[R \cos (\omega t) \hat{\mathbf{i}}+R \sin (\omega t) \hat{\mathbf{j}}]=\left|\begin{array}{ccc}
\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\
0 & 0 & \frac{v}{R} \\
R \cos (\omega t) & R \sin (\omega t) & 0
\end{array}\right| \\
& =-v \sin (\omega t) \hat{\mathbf{i}}+v \cos (\omega t) \hat{\mathbf{j}}=\overrightarrow{\mathbf{v}}
\end{aligned}
$$

Thus we see that $\overrightarrow{\mathbf{v}}=\overrightarrow{\boldsymbol{\omega}} \times \overrightarrow{\mathbf{r}}$.
70. Note that $z=v_{z} t$, and so $\frac{d z}{d t}=v_{z}$. To find the angular momentum, use Eq. 11-6, $\overrightarrow{\mathbf{L}}=\overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{p}}$.

$$
\begin{aligned}
& \overrightarrow{\mathbf{r}}=R \cos \left(\frac{2 \pi z}{d}\right) \hat{\mathbf{i}}+R \sin \left(\frac{2 \pi z}{d}\right) \hat{\mathbf{j}}+z \hat{\mathbf{k}}=R \cos \left(\frac{2 \pi v_{z} t}{d}\right) \hat{\mathbf{i}}+R \sin \left(\frac{2 \pi v_{z} t}{d}\right) \hat{\mathbf{j}}+v_{z} t \hat{\mathbf{k}} \\
& \overrightarrow{\mathbf{v}}=\frac{d \overrightarrow{\mathbf{r}}}{d t}=-R \frac{2 \pi v_{z}}{d} \sin \left(\frac{2 \pi v_{z} t}{d}\right) \hat{\mathbf{i}}+R \frac{2 \pi v_{z}}{d} \cos \left(\frac{2 \pi v_{z} t}{d}\right) \hat{\mathbf{j}}+v_{z} \hat{\mathbf{k}}
\end{aligned}
$$

To simplify the notation, let $\alpha \equiv \frac{2 \pi v_{z}}{d}$. Then the kinematical expressions are as follows.

$$
\begin{aligned}
\overrightarrow{\mathbf{r}}= & R \cos (\alpha t) \hat{\mathbf{i}}+R \sin (\alpha t) \hat{\mathbf{j}}+v_{z} t \hat{\mathbf{k}} ; \overrightarrow{\mathbf{v}}=-\alpha R \sin (\alpha t) \hat{\mathbf{i}}+\alpha R \cos (\alpha) \hat{\mathbf{j}}+v_{z} \hat{\mathbf{k}} \\
\overrightarrow{\mathbf{L}} & =\overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{p}}=m \overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{v}}=m\left|\begin{array}{ccc}
\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\
R \cos (\alpha t) & R \sin (\alpha t) & v_{z} t \\
-R \alpha \sin (\omega t) & R \alpha \cos (\alpha t) & v_{z}
\end{array}\right| \\
= & m\left[R v_{z} \sin (\alpha t)-R \alpha v_{z} t \cos (\alpha t)\right] \hat{\mathbf{i}}+m\left[-R \alpha v_{z} t \sin (\alpha t)-R v_{z} \cos (\alpha t)\right] \hat{\mathbf{j}} \\
& +m\left[R^{2} \alpha \cos ^{2}(\alpha t)+R^{2} \alpha \sin ^{2}(\alpha t)\right] \hat{\mathbf{k}}
\end{aligned}
$$

$$
\begin{aligned}
& =m R v_{z}[\sin (\alpha t)-\alpha t \cos (\alpha t)] \hat{\mathbf{i}}+m R v_{z}[-\alpha t \sin (\alpha t)-\cos (\alpha t)] \hat{\mathbf{j}}+m R^{2} \alpha \hat{\mathbf{k}} \\
& =m R v_{z}\left\{[\sin (\alpha t)-\alpha t \cos (\alpha t)] \hat{\mathbf{i}}+[-\alpha t \sin (\alpha t)-\cos (\alpha t)] \hat{\mathbf{j}}+\frac{R \alpha}{v_{z}} \hat{\mathbf{k}}\right\} \\
& =m R v_{z}\left\{\left[\sin \left(\frac{2 \pi z}{d}\right)-\frac{2 \pi z}{d} \cos \left(\frac{2 \pi z}{d}\right)\right] \hat{\mathbf{i}}+\left[-\frac{2 \pi z}{d} \sin \left(\frac{2 \pi z}{d}\right)-\cos \left(\frac{2 \pi z}{d}\right)\right] \hat{\mathbf{j}}+\frac{2 \pi R}{d} \mathbf{k}\right\}
\end{aligned}
$$

71. (a) From the free-body diagram, we see that the normal force will produce a torque about the center of mass. That torque, $\overrightarrow{\boldsymbol{\tau}}=\overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{F}}_{\mathrm{N}}$, is clockwise in the diagram and so points into the paper, and will cause a change $\Delta \overrightarrow{\mathbf{L}}=\overrightarrow{\boldsymbol{\tau}} \Delta t$ in the tire's original angular momentum. $\Delta \overrightarrow{\mathbf{L}}$ also points into the page, and so the angular momentum will change to have a component into the page. That means that the tire will turn to the right in the diagram.
(b) The original momentum is the moment of inertia times the angular velocity. We assume the wheel is rolling without slipping.

$$
\begin{aligned}
& \Delta L=\overrightarrow{\boldsymbol{\tau}} \Delta t=\left(r F_{\mathrm{N}} \sin \theta\right) \Delta t=r m g \sin \theta \Delta t ; L_{0}=I \omega=I v / r \\
& \frac{\Delta L}{L_{0}}=\frac{r^{2} m g \sin \theta \Delta t}{I v}=\frac{(0.32 \mathrm{~m})^{2}(8.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \sin 12^{\circ}(0.20 \mathrm{~s})}{\left(0.83 \mathrm{~kg} \cdot \mathrm{~m}^{2}\right)(2.1 \mathrm{~m} / \mathrm{s})}=0.19
\end{aligned}
$$


72. (a) See the diagram. The parallel axis theorem is used to find the moment of inertia of the arms.

$$
\begin{aligned}
& I_{a}=I_{\text {body }}+I_{\mathrm{arms}} \\
& =\frac{1}{2} M_{\mathrm{body}} R_{\mathrm{body}}^{2}+2\left[\frac{1}{12} M_{\mathrm{arm}} \ell_{\mathrm{arm}}^{2}+M_{\mathrm{arm}}\left(R_{\mathrm{body}}+\frac{1}{2} \ell_{\mathrm{arm}}\right)^{2}\right] \\
& =\frac{1}{2}(60 \mathrm{~kg})(0.12 \mathrm{~m})^{2} \\
& \text { (b) Now the arms can be treated like particles, since all of the mass of the } \\
& \text { arms is the same distance from the axis of rotation. } \\
& I_{b}=I_{\text {body }}+I_{\text {arms }}=\frac{1}{2} M_{\text {body }} R_{\text {body }}^{2}+2 M_{\text {arm }} R_{\text {body }}^{2} \\
& =\frac{1}{2}(60 \mathrm{~kg})(0.12 \mathrm{~m})^{2}+2(5.0 \mathrm{~kg})(0.12 \mathrm{~m})^{2}=0.576 \mathrm{~kg} \cdot \mathrm{~m}^{2} \\
& \approx 0.58 \mathrm{~kg} \cdot \mathrm{~m}^{2}
\end{aligned}
$$


(c) Angular momentum is conserved through the change in posture.

$$
\begin{aligned}
& L_{\text {initial }}=L_{\text {final }} \rightarrow I_{a} \omega_{a}=I_{b} \omega_{b} \rightarrow I_{a} \frac{2 \pi}{T_{a}}=I_{b} \frac{2 \pi}{T_{b}} \rightarrow \\
& T_{b}=\frac{I_{b}}{I_{a}} T_{a}=\frac{0.576 \mathrm{~kg} \cdot \mathrm{~m}^{2}}{2.496 \mathrm{~kg} \cdot \mathrm{~m}^{2}}(1.5 \mathrm{~s})=0.3462 \mathrm{~s} \approx 0.35 \mathrm{~s}
\end{aligned}
$$

(d) The change in kinetic energy is the final kinetic energy (arms horizontal) minus the initial kinetic energy (arms at sides).

$$
\begin{aligned}
\Delta K & =K_{a}-K_{b}=\frac{1}{2} I_{a} \omega_{a}^{2}-\frac{1}{2} I_{b} \omega_{b}^{2}=\frac{1}{2}\left(2.496 \mathrm{~kg} \cdot \mathrm{~m}^{2}\right)\left(\frac{2 \pi}{1.5 \mathrm{~s}}\right)^{2}-\frac{1}{2}\left(0.576 \mathrm{~kg} \cdot \mathrm{~m}^{2}\right)\left(\frac{2 \pi}{0.3462 \mathrm{~s}}\right)^{2} \\
& =-73 \mathrm{~J}
\end{aligned}
$$

(e) Because of the decrease in kinetic energy, it is easier to lift the arms when rotating. There is no corresponding change in kinetic energy if the person is at rest. In the rotating system, the arms tend to move away from the center of rotation. Another way to express this is that it takes work to bring the arms into the sides when rotating.
73. (a) The angular momentum delivered to the waterwheel is that lost by the water.

$$
\begin{aligned}
& \Delta L_{\text {wheel }}=-\Delta L_{\text {water }}=L_{\text {initial }}-L_{\text {final }}^{\text {water }} \\
& \text { water } \\
& \frac{\Delta L_{\text {wheel }}}{\Delta t}=\frac{m v_{1} R-m v_{1} R-m v_{2} R \rightarrow}{\Delta t}=\frac{m R}{\Delta t}\left(v_{1}-v_{2}\right)=(85 \mathrm{~kg} / \mathrm{s})(3.0 \mathrm{~m})(3.2 \mathrm{~m} / \mathrm{s})=816 \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}^{2} \\
& \approx 820 \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}^{2}
\end{aligned}
$$

(b) The torque is the rate of change of angular momentum, from Eq. 11-9.

$$
\tau_{\text {on }}=\frac{\Delta L_{\text {wheel }}}{\Delta t}=816 \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}^{2}=816 \mathrm{~m} \cdot \mathrm{~N} \approx 820 \mathrm{~m} \cdot \mathrm{~N}
$$

(c) Power is given by Eq. 10-21, $P=\tau \omega$.

$$
P=\tau \omega=(816 \mathrm{~m} \cdot \mathrm{~N})\left(\frac{2 \pi \mathrm{rev}}{5.5 \mathrm{~s}}\right)=930 \mathrm{~W}
$$

74. Due to the behavior of the Moon, the period for the Moon's rotation about its own axis is the same as the period for the Moon's rotation about the Earth. Thus the angular velocity is the same in both cases.

$$
\frac{L_{\text {spin }}}{L_{\text {orbit }}}=\frac{I_{\text {spin }} \omega}{I_{\text {orbit }} \omega}=\frac{I_{\text {spin }}}{I_{\text {orbit }}}=\left(\frac{\frac{2}{5} M R_{\text {Moon }}^{2}}{M R_{\text {orbit }}^{2}}\right)=\frac{2 R_{\text {Moon }}^{2}}{5 R_{\text {orbit }}^{2}}=\frac{2\left(1.74 \times 10^{6} \mathrm{~m}\right)^{2}}{5\left(384 \times 10^{6} \mathrm{~m}\right)^{2}}=8.21 \times 10^{-6}
$$

75. From problem 25, we have that $\overrightarrow{\mathbf{a}}_{\text {tan }}=\overrightarrow{\boldsymbol{\alpha}} \times \overrightarrow{\mathbf{r}}$. For this object, rotating counterclockwise and gaining angular speed, the angular acceleration is $\overrightarrow{\boldsymbol{\alpha}}=\alpha \hat{\mathbf{k}}$.

$$
\overrightarrow{\mathbf{a}}_{\mathrm{tan}}=\overrightarrow{\boldsymbol{\alpha}} \times \overrightarrow{\mathbf{r}}=\left|\begin{array}{ccc}
\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\
0 & 0 & \alpha \\
R \cos \theta & R \sin \theta & 0
\end{array}\right|=-\alpha R \sin \theta \hat{\mathbf{i}}+\alpha R \cos \theta \hat{\mathbf{j}}
$$

(a) We need the acceleration in order to calculate $\overrightarrow{\boldsymbol{\tau}}=\overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{F}}$. The force consists of two components, a radial (centripetal) component and a tangential component. There is no torque associated with the radial component since the angle between $\overrightarrow{\mathbf{r}}$ and $\overrightarrow{\mathbf{F}}_{\text {centrip }}$ is $180^{\circ}$. Thus $\overrightarrow{\boldsymbol{\tau}}=\overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{F}}=\overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{F}}$ tan $=\overrightarrow{\mathbf{r}} \times m \overrightarrow{\mathbf{a}}_{\mathrm{tan}}=m \overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{a}}_{\mathrm{tan}}$.

$$
\overrightarrow{\boldsymbol{\tau}}=m \overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{a}}_{\mathrm{tan}}=m\left|\begin{array}{ccc}
\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\
R \cos \theta & R \sin \theta & 0 \\
-\alpha R \sin \theta & \alpha R \cos \theta & 0
\end{array}\right|=m\left(R^{2} \alpha \cos ^{2} \theta+R^{2} \alpha \sin ^{2} \theta\right) \hat{\mathbf{k}}=m R^{2} \alpha \hat{\mathbf{k}}
$$

(b) The moment of inertia of the particle is $I=m R^{2}$.

$$
\overrightarrow{\boldsymbol{\tau}}=I \overrightarrow{\boldsymbol{\alpha}}=m R^{2} \alpha \hat{\mathbf{k}}
$$

76. (a) The acceleration is needed since $\overrightarrow{\mathbf{F}}=m \overrightarrow{\mathbf{a}}$.

$$
\begin{aligned}
& \overrightarrow{\mathbf{r}}=\left(v_{x 0} t\right) \hat{\mathbf{i}}+\left(v_{y 0} t-\frac{1}{2} g t^{2}\right) \hat{\mathbf{j}} ; \overrightarrow{\mathbf{v}}=\frac{d \overrightarrow{\mathbf{r}}}{d t}=v_{x 0} \hat{\mathbf{i}}+\left(v_{y 0}-g t\right) \hat{\mathbf{j}} ; \overrightarrow{\mathbf{a}}=\frac{d \overrightarrow{\mathbf{v}}}{d t}=-g \hat{\mathbf{j}} \text { (as expected) } \\
& \overrightarrow{\boldsymbol{\tau}}=\overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{F}}=\overrightarrow{\mathbf{r}} \times m \overrightarrow{\mathbf{a}}=m \overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{a}}=m\left|\begin{array}{ccc}
\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\
v_{x 0} t & v_{y 0} t-\frac{1}{2} g t^{2} & 0 \\
0 & -g & 0
\end{array}\right|=-g v_{x 0} t \hat{\mathbf{k}}
\end{aligned}
$$

(b) Find the angular momentum from $\overrightarrow{\mathbf{L}}=\overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{p}}=m(\overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{v}})$, and then differentiate with respect to time.

$$
\begin{aligned}
& \overrightarrow{\mathbf{L}}=\overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{p}}=m(\overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{v}})=m\left|\begin{array}{ccc}
\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\
v_{x 0} t & v_{y 0} t-\frac{1}{2} g t^{2} & 0 \\
v_{x 0} & v_{y 0}-g t & 0
\end{array}\right|=\left[v_{x 0} t\left(v_{y 0}-g t\right)-v_{x 0}\left(v_{y 0} t-\frac{1}{2} g t^{2}\right)\right] \hat{\mathbf{k}} \\
&=-\frac{1}{2} v_{x 0} g t^{2} \hat{\mathbf{k}} \\
& \frac{d \overrightarrow{\mathbf{L}}}{d t}=\frac{d}{d t}\left(-\frac{1}{2} v_{x 0} g t^{2} \hat{\mathbf{k}}\right)=-v_{x 0} g t \hat{\mathbf{k}}
\end{aligned}
$$

77. We calculate spin angular momentum for the Sun, and orbital angular momentum for the planets, treating them as particles relative to the size of their orbits. The angular velocities are calculated by $\omega=\frac{2 \pi}{T}$.

$$
\begin{aligned}
L_{\text {Sun }} & =I_{\text {Sun }} \omega_{\text {Sun }}=\frac{2}{5} M_{\text {Sun }} R_{\text {Sun }}^{2} \frac{2 \pi}{T_{\text {Sun }}}=\frac{2}{5}\left(1.99 \times 10^{30} \mathrm{~kg}\right)\left(6.96 \times 10^{8} \mathrm{~m}\right)^{2} \frac{2 \pi}{(25 \text { days })}\left(\frac{1 \text { day }}{86,400 \mathrm{~s}}\right) \\
& =1.1217 \times 10^{42} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s} \\
L_{\text {Jupiter }} & =M_{\text {Jupier }} R_{\text {Jupiter }}^{2} \frac{2 \pi}{\text { orbit }_{\text {Jupiter }}}=\left(190 \times 10^{25} \mathrm{~kg}\right)\left(778 \times 10^{9} \mathrm{~m}\right)^{2} \frac{2 \pi}{11.9 \mathrm{y}}\left(\frac{1 \mathrm{y}}{3.156 \times 10^{7} \mathrm{~s}}\right) \\
& =1.9240 \times 10^{43} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

In a similar fashion, we calculate the other planetary orbital angular momenta.

$$
\begin{aligned}
& L_{\text {Saturn }}=M_{\substack{\text { Saturn }}} R_{\substack{\text { Saturr } \\
\text { orbit }}}^{2} \frac{2 \pi}{T_{\text {Saturn }}}=7.806 \times 10^{42} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s} \\
& L_{\text {Uranus }}=M_{\text {Uranus }} R_{\substack{\text { Uranus } \\
\text { orbit }}}^{2} \frac{2 \pi}{T_{\text {Uramus }}}=1.695 \times 10^{42} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s} \\
& L_{\text {Neppune }}=M_{\text {Neppunec }} R_{\substack{\text { Neppune } \\
\text { orbit }}}^{2} \frac{2 \pi}{T_{\text {Neppune }}}=2.492 \times 10^{42} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

$$
f=\frac{L_{\text {planets }}}{L_{\text {planets }}+L_{\text {Sun }}}=\frac{(19.240+7.806+1.695+2.492) \times 10^{42} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}}{(19.240+7.806+1.695+2.492+1.122) \times 10^{42} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}}=0.965
$$

78. (a) In order not to fall over, the net torque on the cyclist about an axis through the CM and parallel to the ground must be zero. Consider the free-body diagram shown. Sum torques about the CM, with counterclockwise as positive, and set the sum equal to zero.

$$
\sum \tau=F_{\mathrm{N}} x-F_{\mathrm{fr}} y=0 \rightarrow \frac{F_{\mathrm{fr}}}{F_{\mathrm{N}}}=\frac{x}{y}=\tan \theta
$$

(b) The cyclist is not accelerating vertically, so $F_{\mathrm{N}}=m g$. The cyclist is accelerating horizontally, because he is traveling in a circle. Thus the
 frictional force must be supplying the centripetal force, so $F_{\mathrm{fr}}=m v^{2} / r$.

$$
\tan \theta=\frac{F_{\mathrm{fr}}}{F_{\mathrm{N}}}=\frac{m v^{2} / r}{m g}=\frac{v^{2}}{r g} \rightarrow \theta=\tan ^{-1} \frac{v^{2}}{r g}=\tan ^{-1} \frac{(9.2 \mathrm{~m} / \mathrm{s})^{2}}{(12 \mathrm{~m})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=35.74^{\circ} \approx 36^{\circ}
$$

(c) From $F_{\mathrm{ff}}=m v^{2} / r$, the smallest turning radius results in the maximum force. The maximum static frictional force is $F_{\mathrm{fr}}=\mu F_{\mathrm{N}}$. Use this to calculate the radius.

$$
m v^{2} / r_{\text {min }}=\mu_{\mathrm{s}} F_{\mathrm{N}}=\mu_{\mathrm{s}} m g \rightarrow r_{\min }=\frac{v^{2}}{\mu_{\mathrm{s}} g}=\frac{(9.2 \mathrm{~m} / \mathrm{s})^{2}}{(0.65)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=13 \mathrm{~m}
$$

79. (a) During the jump (while airborne), the only force on the skater is gravity, which acts through the skater's center of mass. Accordingly, there is no torque about the center of mass, and so angular momentum is conserved during the jump.
(b) For a single axel, the skater must have 1.5 total revolutions. The number of revolutions during each phase of the motion is the rotational frequency times the elapsed time. Note that the rate of rotation is the same for both occurrences of the "open" position.

$$
\begin{aligned}
& (1.2 \mathrm{rev} / \mathrm{s})(0.10 \mathrm{~s})+f_{\text {single }}(0.50 \mathrm{~s})+(1.2 \mathrm{rev} / \mathrm{s})(0.10 \mathrm{~s})=1.5 \mathrm{rev} \rightarrow \\
& f_{\text {single }}=\frac{1.5 \mathrm{rev}-2(1.2 \mathrm{rev} / \mathrm{s})(0.10 \mathrm{~s})}{(0.50 \mathrm{~s})}=2.52 \mathrm{rev} / \mathrm{s} \approx 2.5 \mathrm{rev} / \mathrm{s}
\end{aligned}
$$

The calculation is similar for the triple axel.

$$
\begin{aligned}
& (1.2 \mathrm{rev} / \mathrm{s})(0.10 \mathrm{~s})+f_{\text {triple }}(0.50 \mathrm{~s})+(1.2 \mathrm{rev} / \mathrm{s})(0.10 \mathrm{~s})=3.5 \mathrm{rev} \rightarrow \\
& f_{\text {triple }}=\frac{3.5 \mathrm{rev}-2(1.2 \mathrm{rev} / \mathrm{s})(0.10 \mathrm{~s})}{(0.50 \mathrm{~s})}=6.52 \mathrm{rev} / \mathrm{s} \approx 6.5 \mathrm{rev} / \mathrm{s}
\end{aligned}
$$

(c) Apply angular momentum conservation to relate the moments of inertia.

$$
\begin{aligned}
& L_{\substack{\text { single } \\
\text { open }}}=L_{\substack{\text { single } \\
\text { closed }}} \rightarrow \underset{\substack{\text { single } \\
\text { open }}}{\omega_{\text {single }}^{\text {open }}}<I_{\substack{\text { single } \\
\text { closed }}} \omega_{\text {singsed }} \rightarrow
\end{aligned}
$$

Thus the single axel moment of inertia must be reduced by a factor of about 2 .
For the triple axel, the calculation is similar.

$$
\left.\frac{\substack{\text { triple } \\ \text { closed }}}{I_{\text {triple }}} \text { open }\right\} \frac{f_{\text {single }} \text { open }}{f_{\text {single }}^{\text {cosed }}}=\frac{1.2 \mathrm{rev} / \mathrm{s}}{6.52 \mathrm{rev} / \mathrm{s}}=0.184 \approx \frac{1}{5}
$$

Thus the triple axel moment of inertia must be reduced by a factor of about 5 .
80. We assume that the tensions in the two unbroken cables immediately become zero, and so they have no effect on the motion. The forces on the tower are the forces at the base joint, and the weight. The axis of rotation is through the point of attachment to the ground. Since that axis is fixed in an inertial system, we may use Eq. 11-9 in one dimension, $\sum \tau=\frac{d L}{d t}$. See the free-body diagram in the text to express the torque.

$$
\sum \tau=\frac{d L}{d t} \rightarrow m g\left(\frac{1}{2} \ell\right) \sin \theta=\frac{d(I \omega)}{d t}=\frac{1}{3} m \ell^{2} \frac{d \omega}{d t} \quad \rightarrow \quad \frac{1}{2} g \sin \theta=\frac{1}{3} \ell \frac{d \omega}{d t}=\frac{1}{3} \ell \frac{d^{2} \theta}{d t^{2}}
$$

This equation could be considered, but it would yield $\theta$ as a function of time. Use the chain rule to eliminate the dependence on time.

$$
\begin{aligned}
& \frac{1}{2} g \sin \theta=\frac{1}{3} \ell \frac{d \omega}{d t}=\frac{1}{3} \ell \frac{d \omega}{d \theta} \frac{d \theta}{d t}=\frac{1}{3} \ell \omega \frac{d \omega}{d \theta} \rightarrow \frac{3}{2} \frac{g}{\ell} \sin \theta d \theta=\omega d \omega \rightarrow \\
& \frac{3}{2} \frac{g}{\ell} \int_{0}^{\theta} \sin \theta d \theta=\int_{0}^{\omega} \omega d \omega \rightarrow \frac{3}{2} \frac{g}{\ell}(1-\cos \theta)=\frac{1}{2} \omega^{2} \rightarrow \omega=\sqrt{3 \frac{g}{\ell}(1-\cos \theta)}=\frac{v}{\ell} \rightarrow \\
& v=\sqrt{3 g \ell(1-\cos \theta)}=\sqrt{3\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(12 \mathrm{~m})(1-\cos \theta)}=19 \sqrt{1-\cos \theta}
\end{aligned}
$$

Note that the same result can be obtained from conservation of energy, since the forces at the ground do no work.
81. (a) We assume that no angular momentum is in the thrown-off mass, so the final angular momentum of the neutron star is equal to the angular momentum before collapse.

$$
\begin{aligned}
L_{0} & =L_{\mathrm{f}} \rightarrow I_{0} \omega_{0}=I_{\mathrm{f}} \omega_{\mathrm{f}} \rightarrow\left[\frac{2}{5}\left(8.0 M_{\text {Sun }}\right) R_{\text {Sun }}^{2}\right] \omega_{0}=\left[\frac{2}{5}\left(\frac{1}{4} 8.0 M_{\text {Sun }}\right) R_{\mathrm{f}}^{2}\right] \omega_{f} \rightarrow \\
\omega_{f} & =\frac{\left[\frac{2}{5}\left(8.0 M_{\text {Sun }}\right) R_{\mathrm{Sun}}^{2}\right]}{\left[\frac{2}{5}\left(\frac{1}{4} 8.0 M_{\text {Sun }}\right) R_{\mathrm{f}}^{2}\right]} \omega_{0}=\frac{4 R_{\text {Sun }}^{2}}{R_{\mathrm{f}}^{2}} \omega_{0}=\frac{4\left(6.96 \times 10^{8} \mathrm{~m}\right)^{2}}{\left(12 \times 10^{3} \mathrm{~m}\right)^{2}}\left(\frac{1.0 \mathrm{rev}}{9.0 \text { days }}\right) \\
& =\left(1.495 \times 10^{9} \mathrm{rev} / \text { day }\right)\left(\frac{1 \text { day }}{86400 \mathrm{~s}}\right)=1.730 \times 10^{4} \mathrm{rev} / \mathrm{s} \approx 17,000 \mathrm{rev} / \mathrm{s}
\end{aligned}
$$

(b) Now we assume that the final angular momentum of the neutron star is only $1 / 4$ of the angular momentum before collapse. Since the rotation speed is directly proportional to angular momentum, the final rotation speed will be $1 / 4$ of that found in part $(a)$.

$$
\omega_{f}=\frac{1}{4}\left(1.730 \times 10^{4} \mathrm{rev} / \mathrm{s}\right)=4300 \mathrm{rev} / \mathrm{s}
$$

82. The desired motion is pure rotation about the handle grip. Since the grip is not to have any linear motion, an axis through the grip qualifies as an axis fixed in an inertial reference frame. The pure rotation condition is expressed by $a_{\mathrm{CM}}=\alpha_{\text {bat }}\left(d_{\mathrm{CM}}-d_{\text {grip }}\right)$, where $d_{\text {grip }}$ is the 0.050 m distance from the end of the bat to the grip. Apply Newton's second law for both the translational motion of the center of mass, and rotational motion about the handle grip.

$$
\sum F=F=m a_{\mathrm{CM}} ; \sum \tau=F d=I_{\mathrm{grip}} \alpha \rightarrow m a_{\mathrm{CM}} d=I_{\mathrm{grip}} \alpha \rightarrow
$$

$$
m \alpha\left(d_{\mathrm{CM}}-d_{\text {grip }}\right) d=I_{\text {grip }} \alpha \rightarrow d=\frac{I_{\text {gip }}}{m\left(d_{\mathrm{CM}}-d_{\text {gip }}\right)}
$$

So we must calculate the moment of inertia of the bat about an axis through the grip, the mass of the bat, and the location of the center of mass. An infinitesimal element of mass is given by $d m=\lambda d x$, where $\lambda$ is the linear mass density.

$$
\begin{aligned}
& I_{\text {gip }}=\int r^{2} d m=\int_{0}^{0.84 \mathrm{~m}}\left(x-d_{\text {gip }}\right)^{2} \lambda d x=\int_{0}^{0.84 \mathrm{~m}}(x-0.050)^{2}\left(0.61+3.3 x^{2}\right) d x \\
&=\int_{0}^{0.84 \mathrm{~m}}\left(3.3 x^{4}-0.33 x^{3}+0.61825 x^{2}-0.061 x+0.001525\right) d x \\
&=\left(\frac{1}{5} 3.3 x^{5}-\frac{1}{4} 0.33 x^{4}+\frac{1}{3} 0.61825 x^{3}-\frac{1}{2} 0.061 x^{2}+0.001525 x\right)_{0}^{0.84}=0.33685 \mathrm{~kg} \cdot \mathrm{~m}^{2} \\
& m=\int d m=\int \lambda d x=\int_{0}^{0.84 \mathrm{~m}}\left[\left(0.61+3.3 x^{2}\right) \mathrm{kg} / \mathrm{m}\right] d x=\left(0.61 x+1.1 x^{3}\right)_{0}^{0.84}=1.1644 \mathrm{~kg} \\
& x_{\mathrm{CM}}=\frac{1}{m} \int x d m=\frac{1}{m} \int x \lambda d x=\frac{1}{m} \int_{0}^{0.84 \mathrm{~m}}\left[\left(0.61 x+3.3 x^{3}\right) \mathrm{kg} / \mathrm{m}\right] d x=\frac{\left(\frac{1}{2} 0.61 x^{2}+\frac{1}{4} 3.3 x^{4}\right)_{0}^{0.84}}{1.1644 \mathrm{~kg}} \\
& \quad=0.53757 \mathrm{~m} \\
& d=\frac{0.33685 \mathrm{~kg} \cdot \mathrm{~m}^{2}}{(1.1644 \mathrm{~kg})(0.53757 \mathrm{~m}-0.050 \mathrm{~m})}=0.59333 \mathrm{~m} \approx 0.593 \mathrm{~m}
\end{aligned}
$$

So the distance from the end of the bat to the "sweet spot" is $d+0.050 \mathrm{~m}=0.643 \mathrm{~m} \approx 0.64 \mathrm{~m}$.
83. (a) Angular momentum about the pivot is conserved during this collision. Note that both objects have angular momentum after the collision.

$$
\begin{aligned}
& L_{\text {befire }}=L_{\text {after }}^{\text {colision }} \\
& \omega \\
& \omega=\frac{m_{\text {bullet }}\left(v_{0}-v_{\mathrm{f}}\right) x}{I_{\text {stisiok }}}=\frac{L_{\text {bullete }}=L_{\text {bullet }}\left(v_{0}-v_{\mathrm{f}}\right) x}{\frac{1}{\text { fitital }}}+L_{\text {bullet }} \rightarrow m_{\text {bullet }} v_{0} x=I_{\text {stick }} \omega+m_{\text {bullet }} v_{\mathrm{f}} x \rightarrow \\
& \\
& \\
& =\left(12 \frac{\mathrm{rad} / \mathrm{s}}{\mathrm{~m}}\right) x
\end{aligned}
$$

(b) The spreadsheet used for this problem can be found on the Media Manager, with filename
"PSE4_ISM_CH11.XLS," on tab "Problem 11.83b."

84. (a) Angular momentum about the center of mass of the system is conserved. First we find the center of mass, relative to the center of mass of the rod, taking up as the positive direction. See the diagram.

$$
x_{\mathrm{CM}}=\frac{m x+M(0)}{(m+M)}=\frac{m x}{m+M}
$$

The distance of the stuck clay ball from the system's center of mass is found.

$$
x_{\substack{\text { clay } \\ \text { from CM }}}=x-x_{\mathrm{CM}}=x-\frac{m x}{m+M}=\frac{M x}{m+M}
$$

Calculate the moment of inertia of the rod about the center of mass of the entire system. Use the parallel axis theorem. Treat the clay as a point mass.

$$
I_{\mathrm{rod}}=\frac{1}{12} M \boldsymbol{\ell}^{2}+M\left(\frac{m x}{m+M}\right)^{2}
$$

Now express the conservation of angular momentum about the system's center of mass.

$$
\begin{aligned}
& L_{\text {initial }}=L_{\text {final }} \rightarrow m v x_{\text {clay }}=\left(I_{\text {rod }}+I_{\text {clay }}\right) \omega_{\text {final }} \rightarrow \\
& \omega_{\text {final }}=\frac{m v x_{\text {clay }}}{\left(I_{\text {rod }}+I_{\text {clay }}\right)}=\frac{m v x_{\text {clay }}}{\left(\frac{1}{12} M \ell^{2}+M\left(\frac{m x}{m+M}\right)^{2}+m x_{\text {clay }}^{2}\right)} \\
& \\
& =\frac{m v \frac{M x}{m+M}}{\left(\frac{1}{12} M \ell^{2}+M\left(\frac{m x}{m+M}\right)^{2}+m\left(\frac{M x}{m+M}\right)^{2}\right)}=\frac{v x}{\frac{1}{12}\left(1+\frac{M}{m}\right) \ell^{2}+x^{2}}
\end{aligned}
$$

(b) Graph this function with the given values, from $x=$ 9 to $x=0.60 \mathrm{~m}$.

$$
\begin{aligned}
\omega_{\text {final }} & =\frac{v x}{\frac{1}{12}\left(1+\frac{M}{m}\right) \ell^{2}+x^{2}} \\
& =\frac{12 x}{3.72+x^{2}} \mathrm{rad} / \mathrm{s}
\end{aligned}
$$

The spreadsheet used for this problem can be found on the Media Manager,

with filename "PSE4_ISM_CH11.XLS," on tab "Problem 11.84b."
(c) Linear momentum of the center of mass is conserved in the totally inelastic collision.

$$
p_{\text {initial }}=p_{\text {final }} \rightarrow m v=(m+M) v_{\substack{\mathrm{CM} \\ \text { final }}} \rightarrow \underset{\substack{v_{\mathrm{CM}}=\frac{m v}{\text { final }}}}{m+M}
$$

We see that the translational motion (the velocity of the center of mass) is NOT dependent on $x$.

## CHAPTER 12: Static Equilibrium; Elasticity and Fracture

## Responses to Questions

Equilibrium requires both the net force and net torque on an object to be zero. One example is a meter stick with equal and opposite forces acting at opposite ends. The net force is zero but the net torque is not zero, because the forces are not co-linear. The meter stick will rotate about its center.
2. No. An object in equilibrium has zero acceleration. At the bottom of the dive, the bungee jumper momentarily has zero velocity, but not zero acceleration. There is a net upward force on the bungee jumper so he is not in equilibrium.
3. The meter stick is originally supported by both fingers. As you start to slide your fingers together, more of the weight of the meter stick is supported by the finger that is closest to the center of gravity, so that the torques produced by the fingers are equal and the stick is in equilibrium. The other finger feels a smaller normal force, and therefore a smaller frictional force, and so slides more easily and moves closer to the center of gravity. The roles switch back and forth between the fingers as they alternately move closer to the center of gravity. The fingers will eventually meet at the center of gravity.
4. The sliding weights on the movable scale arm are positioned much farther from the pivot point than is the force exerted by your weight. In this way, they can create a torque to balance the torque caused by your weight, even though they weigh less. When the torques are equal in magnitude and opposite in direction, the arm will be in rotational equilibrium.
5. (a) The wall remains upright if the counterclockwise and clockwise torques about the lower left corner of the wall are equal. The counterclockwise torque is produced by $\overrightarrow{\mathbf{F}}$. The clockwise torque is the sum of the torques produced by the normal force from the ground on the left side of the wall and the weight of the wall. $\overrightarrow{\mathbf{F}}$ and its lever arm are larger than the force and lever arm for the torque from the ground on the left. The lever arm for the torque generated by the weight is small, so the torque will be small, even if the wall is very heavy. Case ( $a$ ) is likely to be an unstable situation.
(b) In this case, the clockwise torque produced by the weight of the ground above the horizontal section of the wall and clockwise torque produced by the larger weight of the wall and its lever arm balance the counterclockwise torque produced by $\overrightarrow{\mathbf{F}}$.
6. Yes. For example, consider a meter stick lying along the $x$-axis. If you exert equal forces downward (in the negative $y$-direction) on the two ends of the stick, the torques about the center of the stick will be equal and opposite, so the net torque will be zero. However, the net force will not be zero; it will be in the negative $y$-direction. Also, any force through the pivot point will supply zero torque.
7. The ladder is more likely to slip when a person stands near the top of the ladder. The torque produced by the weight of the person about the bottom of the ladder increases as the person climbs the ladder, because the lever arm increases.
8. The mass of the meter stick is equal to the mass of the rock. Since the meter stick is uniform, its center of mass is at the $50-\mathrm{cm}$ mark, and in terms of rotational motion about a pivot at the $25-\mathrm{cm}$ mark, it can be treated as though its entire mass is concentrated at the center of mass. The meter stick's mass at the $50-\mathrm{cm}$ mark ( 25 cm from the pivot) balances the rock at the $0-\mathrm{cm}$ mark (also 25 cm from the pivot) so the masses must be equal.
9. You lean backward in order to keep your center of mass over your feet. If, due to the heavy load, your center of mass is in front of your feet, you will fall forward.
10. (a) The cone will be in stable equilibrium if it is placed flat on its base. If it is tilted slightly from this position and then released, it will return to the original position. (b) The cone will be in unstable equilibrium if it is balanced on its tip. A slight displacement in this case will cause the cone to topple over. (c) If the cone is placed on its side (as shown in Figure 12-42) it will be in neutral equilibrium. If the cone is displaced slightly while on its side, it will remain in its new position.
11. When you stand next to a door in the position described, your center of mass is over your heels. If you try to stand on your toes, your center of mass will not be over your area of support, and you will fall over backward.
12. Once you leave the chair, you are supported only by your feet. In order to keep from falling backward, your center of mass must be over your area of support, so you must lean forward so that your center of mass is over your feet.
13. When you do a sit-up, you generate a torque with your abdominal muscles to rotate the upper part of your body off the floor while keeping the lower part of your body on the floor. The weight of your legs helps produce the torque about your hips. When your legs are stretched out, they have a longer lever arm, and so produce a larger torque, than when they are bent at the knee. When your knees are bent, your abdominal muscles must work harder to do the sit-up.
14. Configuration $(b)$ is likely to be more stable. Because of the symmetry of the bricks, the center of mass of the entire system (the two bricks) is the midpoint between the individual centers of mass shown on the diagram. In figure (a), the center of mass of the entire system is not supported by the table.
15. A is a point of unstable equilibrium, $B$ is a point of stable equilibrium, and $C$ is a point of neutral equilibrium.
16. The Young's modulus for the bungee cord will be smaller than that for an ordinary rope. The Young's modulus for a material is the ratio of stress to strain. For a given stress (force per unit area), the bungee cord will have a greater strain (change in length divided by original length) than the rope, and therefore a smaller Young's modulus.
17. An object under shear stress has equal and opposite forces applied across its opposite faces. This is exactly what happens with a pair of scissors. One blade of the scissors pushes down on the cardboard, while the other blade pushes up with an equal and opposite force, at a slight displacement. This produces a shear stress in the cardboard, which causes it to fail.
18. Concrete or stone should definitely not be used for the support on the left. The left-hand support pulls downward on the beam, so the beam must pull upward on the support. Therefore, the support will be under tension and should not be made of ordinary concrete or stone, since these materials are weak under tension. The right-hand support pushes up on the beam and so the beam pushes down on it; it will therefore be under a compression force. Making this support of concrete or stone would be acceptable.

## Solutions to Problems

1. If the tree is not accelerating, then the net force in all directions is 0 .

$$
\begin{aligned}
& \sum F_{x}=F_{\mathrm{A}}+F_{\mathrm{B}} \cos 105^{\circ}+F_{\mathrm{C} x}=0 \rightarrow \\
& F_{\mathrm{C} x}=-F_{\mathrm{A}}-F_{\mathrm{B}} \cos 105^{\circ}=-385 \mathrm{~N}-(475 \mathrm{~N}) \cos 105^{\circ}=-262.1 \mathrm{~N} \\
& \sum F_{y}=F_{\mathrm{B}} \sin 105^{\circ}+F_{\mathrm{C} y}=0 \rightarrow \\
& F_{\mathrm{C} y}=-F_{\mathrm{B}} \sin 105^{\circ}=-(475 \mathrm{~N}) \sin 105^{\circ}=-458.8 \mathrm{~N} \\
& F_{\mathrm{C}}=\sqrt{F_{\mathrm{C} x}^{2}+F_{\mathrm{C} y}^{2}}=\sqrt{(-262.1 \mathrm{~N})^{2}+(-458.8 \mathrm{~N})^{2}}=528.4 \mathrm{~N} \approx 528 \mathrm{~N} \\
& \theta=\tan ^{-1} \frac{F_{\mathrm{C} y}}{F_{\mathrm{C} x}}=\tan ^{-1} \frac{-458.8 \mathrm{~N}}{-262.1 \mathrm{~N}}=60.3^{\circ}, \phi=180^{\circ}-60.3^{\circ}=120^{\circ}
\end{aligned}
$$



And so $\overrightarrow{\mathbf{F}}_{\mathrm{C}}$ is 528 N , at an angle of $120^{\circ}$ clockwise from $\overrightarrow{\mathbf{F}}_{\mathrm{A}}$. The angle has 3 sig. fig.
2. Calculate the torques about the elbow joint (the dot in the free body diagram). The arm is in equilibrium. Counterclockwise torques are positive.

$$
\begin{aligned}
\sum \tau & =F_{\mathrm{M}} d-m g D-M g L=0 \\
F_{\mathrm{M}} & =\frac{m D+M L}{d} g \\
& =\frac{(2.3 \mathrm{~kg})(0.12 \mathrm{~m})+(7.3 \mathrm{~kg})(0.300 \mathrm{~m})}{0.025 \mathrm{~m}}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=970 \mathrm{~N}
\end{aligned}
$$


3. Because the mass $m$ is stationary, the tension in the rope pulling up on the sling must be $m g$, and so the force of the sling on the leg must be $m g$, upward. Calculate torques about the hip joint, with counterclockwise torque taken as positive. See the free-body diagram for the leg. Note that the forces on the leg exerted by the hip joint are not drawn, because they do not exert a torque about the hip joint.

$$
\sum \tau=m g x_{2}-M g x_{1}=0 \rightarrow m=M \frac{x_{1}}{x_{2}}=(15.0 \mathrm{~kg}) \frac{(35.0 \mathrm{~cm})}{(78.0 \mathrm{~cm})}=6.73 \mathrm{~kg}
$$

4. (a) See the free-body diagram. Calculate torques about the pivot point $P$ labeled in the diagram. The upward force at the pivot will not have any torque. The total torque is zero since the crane is in equilibrium.

$$
\begin{aligned}
& \sum \tau=M g x-m g d=0 \rightarrow \\
& x=\frac{m d}{M}=\frac{(2800 \mathrm{~kg})(7.7 \mathrm{~m})}{(9500 \mathrm{~kg})}=2.3 \mathrm{~m}
\end{aligned}
$$


(b) Again we sum torques about the pivot point. Mass $m$ is the unknown in this case, and the counterweight is at its maximum distance from the pivot.

$$
\sum \tau=M g x_{\max }-m_{\max } g d=0 \rightarrow m_{\max }=\frac{M x_{\max }}{d}=\frac{(9500 \mathrm{~kg})(3.4 \mathrm{~m})}{(7.7 \mathrm{~kg})}=4200 \mathrm{~kg}
$$

5. (a) Let $m=0$. Calculate the net torque about the left end of the diving board, with counterclockwise torques positive. Since the board is in equilibrium, the net torque is zero.

$$
\begin{aligned}
& \sum \tau=F_{\mathrm{B}}(1.0 \mathrm{~m})-M g(4.0 \mathrm{~m})=0 \rightarrow \\
& F_{\text {в }}=4 M g=4(52 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=2038 \mathrm{~N} \approx 2.0 \times 10^{3} \mathrm{~N}, \mathrm{up}
\end{aligned}
$$



Use Newton's second law in the vertical direction to find $F_{\mathrm{A}}$.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{B}}-M g-F_{\mathrm{A}}=0 \rightarrow \\
& F_{\mathrm{A}}=F_{\mathrm{B}}-M g=4 M g-M g=3 M g=3(52 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=1529 \mathrm{~N} \approx 1500 \mathrm{~N}, \mathrm{down}
\end{aligned}
$$

(b) Repeat the basic process, but with $m=28 \mathrm{~kg}$. The weight of the board will add more clockwise torque.

$$
\begin{aligned}
& \sum \tau=F_{\mathrm{B}}(1.0 \mathrm{~m})-m g(2.0 \mathrm{~m})-M g(4.0 \mathrm{~m})=0 \rightarrow \\
& F_{\mathrm{B}}=4 M g+2 m g=[4(52 \mathrm{~kg})+2(28 \mathrm{~kg})]\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=2587 \mathrm{~N} \approx 2600 \mathrm{~N}, \mathrm{up} \\
& \sum F_{y}=F_{\mathrm{B}}-M g-m g-F_{\mathrm{A}} \rightarrow \\
& F_{\mathrm{A}}=F_{\mathrm{B}}-M g-m g=4 M g+2 m g-M g-m g=3 M g+m g \\
& \quad=[3(52 \mathrm{~kg})+28 \mathrm{~kg}]\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=1803 \mathrm{~N} \approx 1800 \mathrm{~N}, \text { down }
\end{aligned}
$$

6. Write Newton's second law for the junction, in both the $x$ and $y$ directions.

$$
\sum F_{x}=F_{\mathrm{B}}-F_{\mathrm{A}} \cos 45^{\circ}=0
$$

From this, we see that $F_{\mathrm{A}}>F_{\mathrm{B}}$. Thus set $F_{\mathrm{A}}=1660 \mathrm{~N}$.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{A}} \sin 45^{\circ}-m g=0 \\
& m g=F_{\mathrm{A}} \sin 45^{\circ}=(1660 \mathrm{~N}) \sin 45^{\circ}=1174 \mathrm{~N} \approx 1200 \mathrm{~N}
\end{aligned}
$$


7. Since the backpack is midway between the two trees, the angles in the diagram are equal. Write Newton's second law for the vertical direction for the point at which the backpack is attached to the cord, with the weight of the backpack being the downward vertical force. The angle is determined by the distance between the trees and the amount of sag at the midpoint, as illustrated in the second diagram.

(a)

$$
\begin{aligned}
& \theta=\tan ^{-1} \frac{y}{L / 2}=\tan ^{-1} \frac{1.5 \mathrm{~m}}{3.3 \mathrm{~m}}=24.4^{\circ} \\
& \sum F_{y}=2 F_{\mathrm{T}} \sin \theta_{1}-m g=0 \rightarrow \\
& F_{\mathrm{T}}=\frac{m g}{2 \sin \theta_{1}}=\frac{(19 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{2 \sin 24.4^{\circ}}=225.4 \mathrm{~N} \approx 230 \mathrm{~N}
\end{aligned}
$$

(b)

$$
\begin{aligned}
& \theta=\tan ^{-1} \frac{y}{L / 2}=\tan ^{-1} \frac{0.15 \mathrm{~m}}{3.3 \mathrm{~m}}=2.60^{\circ} \\
& F_{\mathrm{T}}=\frac{m g}{2 \sin \theta_{1}}=\frac{(19 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{2 \sin 2.60^{\circ}}=2052 \mathrm{~N} \approx 2100 \mathrm{~N}
\end{aligned}
$$

8. Let $m$ be the mass of the beam, and $M$ be the mass of the piano. Calculate torques about the left end of the beam, with counterclockwise torques positive. The conditions of equilibrium for the beam are used to find the forces that the support exerts on the beam.

$$
\begin{aligned}
& \sum \tau=F_{R} L-m g\left(\frac{1}{2} L\right)-M g\left(\frac{1}{4} L\right)=0 \\
& F_{R}=\left(\frac{1}{2} m+\frac{1}{4} M\right) g=\left[\frac{1}{2}(110 \mathrm{~kg})+\frac{1}{4}(320 \mathrm{~kg})\right]\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=1.32 \times 10^{3} \mathrm{~N} \\
& \sum F_{y}=F_{L}+F_{R}-m g-M g=0 \\
& F_{L}=(m+M) g-F_{R}=(430 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)-1.32 \times 10^{3} \mathrm{~N}=2.89 \times 10^{3} \mathrm{~N}
\end{aligned}
$$



The forces on the supports are equal in magnitude and opposite in direction to the above two results.

$$
\begin{array}{|l|l|}
\hline F_{\mathrm{R}}=1300 \mathrm{~N} \text { down } & F_{\mathrm{L}}=2900 \mathrm{~N} \text { down } \\
\hline
\end{array}
$$

9. Calculate torques about the left end of the beam, with counterclockwise torques positive. The conditions of equilibrium for the beam are used to find the forces that the support exerts on the beam.

$$
\begin{aligned}
& \sum \tau=F_{\mathrm{B}}(20.0 \mathrm{~m})-m g(25.0 \mathrm{~m})=0 \rightarrow \\
& F_{\mathrm{B}}=\frac{25.0}{20.0} m g=(1.25)(1200 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=1.5 \times 10^{4} \mathrm{~N} \\
& \sum F_{y}=F_{\mathrm{A}}+F_{\mathrm{B}}-m g=0 \\
& F_{\mathrm{A}}=m g-F_{\mathrm{B}}=m g-1.25 m g=-0.25 m g=-(0.25)(1200 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=-2900 \mathrm{~N}
\end{aligned}
$$



Notice that $\overrightarrow{\mathbf{F}}_{\mathrm{A}}$ points down.
10. The pivot should be placed so that the net torque on the board is zero. We calculate torques about the pivot point, with counterclockwise torques positive. The upward force $\overrightarrow{\mathbf{F}}_{\mathrm{p}}$ at the pivot point is shown, but it exerts no torque about the pivot point. The mass of the child is $m$, the mass of the adult is $M$, the mass of the board is $m_{\mathrm{B}}$, and the center of gravity is at the
 middle of the board.
(a) Ignore the force $m_{\mathrm{B}} g$.

$$
\begin{aligned}
& \sum \tau=M g x-m g(L-x)=0 \rightarrow \\
& x=\frac{m}{m+M} L=\frac{(25 \mathrm{~kg})}{(25 \mathrm{~kg}+75 \mathrm{~kg})}(9.0 \mathrm{~m})=2.25 \mathrm{~m} \approx 2.3 \mathrm{~m} \text { from adult }
\end{aligned}
$$

(b) Include the force $m_{\mathrm{B}} g$.

$$
\begin{aligned}
& \sum \tau=M g x-m g(L-x)-m_{\mathrm{B}} g(L / 2-x)=0 \\
& x=\frac{\left(m+m_{\mathrm{B}} / 2\right)}{\left(M+m+m_{\mathrm{B}}\right)} L=\frac{(25 \mathrm{~kg}+7.5 \mathrm{~kg})}{(75 \mathrm{~kg}+25 \mathrm{~kg}+15 \mathrm{~kg})}(9.0 \mathrm{~m})=2.54 \mathrm{~m} \approx 2.5 \mathrm{~m} \text { from adult }
\end{aligned}
$$

11. Using the free-body diagram, write Newton's second law for both the horizontal and vertical directions, with net forces of zero.

$$
\begin{aligned}
& \sum F_{x}=F_{\mathrm{T} 2}-F_{\mathrm{T} 1} \cos \theta=0 \rightarrow F_{\mathrm{T} 2}=F_{\mathrm{T} 1} \cos \theta \\
& \sum F_{y}=F_{\mathrm{T} 1} \sin \theta-m g=0 \rightarrow F_{\mathrm{T} 1}=\frac{m g}{\sin \theta} \\
& F_{\mathrm{T} 2}=F_{\mathrm{T} 1} \cos \theta=\frac{m g}{\sin \theta} \cos \theta=\frac{m g}{\tan \theta}=\frac{(190 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{\tan 33^{\circ}}=2867 \mathrm{~N} \approx 2900 \mathrm{~N} \\
& F_{\mathrm{T} 1}=\frac{m g}{\sin \theta}=\frac{(190 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{\sin 33^{\circ}}=3418 \mathrm{~N} \approx 3400 \mathrm{~N}
\end{aligned}
$$


12. Draw a free-body diagram of the junction of the three wires. The tensions can be found from the conditions for force equilibrium.

$$
\begin{aligned}
& \sum F_{x}=F_{\mathrm{T} 1} \cos 37^{\circ}-F_{\mathrm{T} 2} \cos 53^{\circ}=0 \rightarrow F_{\mathrm{T} 2}=\frac{\cos 37^{\circ}}{\cos 53^{\circ}} F_{\mathrm{T} 1} \\
& \sum F_{y}=F_{\mathrm{T} 1} \sin 37^{\circ}+F_{\mathrm{T} 2} \sin 53^{\circ}-m g=0 \\
& F_{\mathrm{T} 1} \sin 37^{\circ}+\frac{\cos 37^{\circ}}{\cos 53^{\circ}} F_{\mathrm{T} 1} \sin 53^{\circ}-m g=0 \rightarrow \\
& F_{\mathrm{T} 1}=\frac{(33 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{\sin 37^{\circ}+\frac{\cos 37^{\circ}}{\cos 53^{\circ}} \sin 53^{\circ}}=194.6 \mathrm{~N} \approx 190 \mathrm{~N} \\
& F_{\mathrm{T} 2}=\frac{\cos 37^{\circ}}{\cos 53^{\circ}} F_{\mathrm{T} 1}=\frac{\cos 37^{\circ}}{\cos 53^{\circ}}\left(1.946 \times 10^{2} \mathrm{~N}\right)=258.3 \mathrm{~N} \approx 260 \mathrm{~N}
\end{aligned}
$$


13. The table is symmetric, so the person can sit near either edge and the same distance will result. We assume that the person (mass $M)$ is on the right side of the table, and that the table (mass $m$ ) is on the verge of tipping, so that the left leg is on the verge of lifting off the floor. There will then be no normal force between the left leg of the table and the floor. Calculate torques about the right leg of the table, so that the normal force between the table
 and the floor causes no torque. Counterclockwise torques are taken to be positive. The conditions of equilibrium for the table are used to find the person's location.

$$
\sum \tau=m g(0.60 \mathrm{~m})-M g x=0 \rightarrow x=(0.60 \mathrm{~m}) \frac{m}{M}=(0.60 \mathrm{~m}) \frac{24.0 \mathrm{~kg}}{66.0 \mathrm{~kg}}=0.218 \mathrm{~m}
$$

Thus the distance from the edge of the table is $0.50 \mathrm{~m}-0.218 \mathrm{~m}=0.28 \mathrm{~m}$.
14. The cork screw will pull upward on the cork with a force of magnitude $F_{\text {cork }}$, and so there is a downward force on the opener of magnitude $F_{\text {cork }}$. We assume that there is no net torque on the opener, so that it does not have an angular acceleration. Calculate torques about the rim of the bottle where the opener is resting on the rim.

$$
\begin{aligned}
& \sum \tau=F(79 \mathrm{~mm})-F_{\text {cork }}(9 \mathrm{~mm})=0 \rightarrow \\
& F=\frac{9}{70} F_{\text {cork }}=\frac{9}{79}(200 \mathrm{~N}) \text { to } \frac{9}{79}(400 \mathrm{~N})=22.8 \mathrm{~N} \text { to } 45.6 \mathrm{~N} \approx 20 \mathrm{~N} \text { to } 50 \mathrm{~N}
\end{aligned}
$$

15. The beam is in equilibrium, and so both the net torque and net force on it must be zero. From the free-body diagram, calculate the net torque about the center of the left support, with counterclockwise torques as positive. Calculate the net force, with upward as positive. Use those two equations to find $F_{\mathrm{A}}$ and $F_{\mathrm{B}}$.


$$
\begin{aligned}
& \sum \tau=F_{\mathrm{B}}\left(x_{1}+x_{2}+x_{3}+x_{4}\right)-F_{1} x_{1}-F_{2}\left(x_{1}+x_{2}\right)-F_{3}\left(x_{1}+x_{2}+x_{3}\right)-m g x_{5} \\
& F_{\mathrm{B}}=\frac{F_{1} x_{1}+F_{2}\left(x_{1}+x_{2}\right)+F_{3}\left(x_{1}+x_{2}+x_{3}\right)+m g x_{5}}{\left(x_{1}+x_{2}+x_{3}+x_{4}\right)} \\
& \quad=\frac{(4300 \mathrm{~N})(2.0 \mathrm{~m})+(3100 \mathrm{~N})(6.0 \mathrm{~m})+(2200 \mathrm{~N})(9.0 \mathrm{~m})+(280 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(5.0 \mathrm{~m})}{10.0 \mathrm{~m}} \\
& \quad=6072 \mathrm{~N} \approx 6100 \mathrm{~N} \\
& \sum F=F_{\mathrm{A}}+F_{\mathrm{B}}-F_{1}-F_{2}-F_{3}-m g=0 \\
& F_{\mathrm{A}}=F_{1}+F_{2}+F_{3}+m g-F_{\mathrm{B}}=9600 \mathrm{~N}+(280 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)-6072 \mathrm{~N}=6272 \mathrm{~N} \approx 6300 \mathrm{~N}
\end{aligned}
$$

16. (a) Calculate the torques about the elbow joint (the dot in the freebody diagram). The arm is in equilibrium. Take counterclockwise torques as positive.

$$
\begin{aligned}
\sum \tau & =\left(F_{\mathrm{M}} \sin \theta\right) d-m g D=0 \rightarrow \\
F_{\mathrm{M}} & =\frac{m g D}{d \sin \theta}=\frac{(3.3 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.24 \mathrm{~m})}{(0.12 \mathrm{~m}) \sin 15^{\circ}}=249.9 \mathrm{~N} \\
& \approx 250 \mathrm{~N}
\end{aligned}
$$


(b) To find the components of $F_{\mathrm{J}}$, write Newton's second law for both the $x$ and $y$ directions. Then combine them to find the magnitude.

$$
\begin{aligned}
& \sum F_{x}=F_{\mathrm{J} x}-F_{\mathrm{M}} \cos \theta=0 \rightarrow F_{\mathrm{J} x}=F_{\mathrm{M}} \cos \theta=(249.9 \mathrm{~N}) \cos 15^{\circ}=241.4 \mathrm{~N} \\
& \sum F_{y}=F_{\mathrm{M}} \sin \theta-m g-F_{\mathrm{J} y}=0 \rightarrow \\
& \quad F_{\mathrm{J} y}=F_{\mathrm{M}} \sin \theta-m g=(249.9 \mathrm{~N}) \sin 15^{\circ}-(3.3 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=32.3 \mathrm{~N} \\
& \mathrm{~F}_{\mathrm{J}}=\sqrt{F_{\mathrm{J} x}^{2}+F_{\mathrm{J} y}^{2}}=\sqrt{(241.4 \mathrm{~N})^{2}+(32.3 \mathrm{~N})^{2}}=243.6 \mathrm{~N} \approx 240 \mathrm{~N}
\end{aligned}
$$

17. Calculate the torques about the shoulder joint, which is at the left end of the free-body diagram of the arm. Since the arm is in equilibrium, the sum of the torques will be zero. Take counterclockwise torques to be positive. The force due to the shoulder joint is drawn, but it does not exert any torque about the shoulder joint.

$$
\begin{aligned}
& \sum \tau=F_{m} d \sin \theta-m g D-M g L=0 \\
& F_{m}=\frac{m D+M L}{d \sin \theta} g=\frac{(3.3 \mathrm{~kg})(0.24 \mathrm{~cm})+(8.5 \mathrm{~kg})(0.52 \mathrm{~m})}{(0.12 \mathrm{~m}) \sin 15^{\circ}}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=1600 \mathrm{~N}
\end{aligned}
$$

18. From the free-body diagram, the conditions of equilibrium are used to find the location of the girl (mass $m_{\mathrm{C}}$ ). The 45kg boy is represented by $m_{\mathrm{A}}$, and the $35-\mathrm{kg}$ girl by $m_{\mathrm{B}}$. Calculate torques about the center of the see-saw, and take
 counterclockwise torques to be positive. The upward force
 of the fulcrum on the see-saw $(\overrightarrow{\mathbf{F}})$ causes no torque about the center.

$$
\begin{aligned}
& \sum \tau=m_{\mathrm{A}} g\left(\frac{1}{2} L\right)-m_{\mathrm{C}} g x-m_{\mathrm{B}} g\left(\frac{1}{2} L\right)=0 \\
& x=\frac{\left(m_{\mathrm{A}}-m_{\mathrm{B}}\right)}{m_{\mathrm{C}}}\left(\frac{1}{2} L\right)=\frac{(45 \mathrm{~kg}-35 \mathrm{~kg})}{25 \mathrm{~kg}} \frac{1}{2}(3.2 \mathrm{~m})=0.64 \mathrm{~m}
\end{aligned}
$$

19. There will be a normal force upwards at the ball of the foot, equal to the person's weight $\left(F_{\mathrm{N}}=m g\right)$. Calculate torques about a point on the floor directly below the leg bone (and so in line with the leg bone force, $\overrightarrow{\mathbf{F}}_{\mathrm{B}}$ ). Since the foot is in equilibrium, the sum of the torques will be zero. Take counterclockwise torques as
 positive.

$$
\begin{aligned}
& \sum \tau=F_{\mathrm{N}}(2 d)-F_{\mathrm{A}} d=0 \rightarrow \\
& F_{\mathrm{A}}=2 F_{\mathrm{N}}=2 m g=2(72 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=1400 \mathrm{~N}
\end{aligned}
$$

The net force in the $y$ direction must be zero. Use that to find $F_{\mathrm{B}}$.

$$
\sum F_{y}=F_{\mathrm{N}}+F_{\mathrm{A}}-F_{\mathrm{B}}=0 \rightarrow F_{\mathrm{B}}=F_{\mathrm{N}}+F_{\mathrm{A}}=2 m g+m g=3 m g=2100 \mathrm{~N}
$$

20. The beam is in equilibrium. Use the conditions of equilibrium to calculate the tension in the wire and the forces at the hinge. Calculate torques about the hinge, and take counterclockwise torques to be positive.

$$
\begin{aligned}
& \sum \tau=\left(F_{\mathrm{T}} \sin \theta\right) l_{2}-m_{1} g l_{1} / 2-m_{2} g l_{1}=0 \rightarrow \\
& F_{\mathrm{T}}=\frac{\frac{1}{2} m_{1} g l_{1}+m_{2} g l_{1}}{l_{2} \sin \theta}=\frac{\frac{1}{2}(155 \mathrm{~N})(1.70 \mathrm{~m})+(215 \mathrm{~N})(1.70 \mathrm{~m})}{(1.35 \mathrm{~m})\left(\sin 35.0^{\circ}\right)} \\
& \quad=642.2 \mathrm{~N} \approx 642 \mathrm{~N} \\
& \sum F_{x}=F_{\mathrm{H} x}-F_{\mathrm{T}} \cos \theta=0 \rightarrow F_{\mathrm{H} x}=F_{\mathrm{T}} \cos \theta=(642.2 \mathrm{~N}) \cos 35.0^{\circ}=526.1 \mathrm{~N} \approx 526 \mathrm{~N}
\end{aligned}
$$

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{H} y}+F_{\mathrm{T}} \sin \theta-m_{1} g-m_{2} g=0 \rightarrow \\
& F_{\mathrm{H} y}=m_{1} g+m_{2} g-F_{\mathrm{T}} \sin \theta=155 \mathrm{~N}+215 \mathrm{~N}-(642.2 \mathrm{~N}) \sin 35.0^{\circ}=1.649 \mathrm{~N} \approx 2 \mathrm{~N}
\end{aligned}
$$

21. (a) The pole is in equilibrium, and so the net torque on it must be zero. From the free-body diagram, calculate the net torque about the lower end of the pole, with counterclockwise torques as positive. Use that calculation to find the tension in the cable. The length of the pole is $\ell$.

$$
\begin{aligned}
\sum \tau & =F_{\mathrm{T}} h-m g(\ell / 2) \cos \theta-M g \ell \cos \theta=0 \\
F_{\mathrm{T}} & =\frac{(m / 2+M) g \ell \cos \theta}{h} \\
& =\frac{(6.0 \mathrm{~kg}+21.5 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(7.20 \mathrm{~m}) \cos 37^{\circ}}{3.80 \mathrm{~m}}=407.8 \mathrm{~N} \approx 410 \mathrm{~N}
\end{aligned}
$$

(b) The net force on the pole is also zero since it is in equilibrium. Write Newton's second law in both the $x$ and $y$ directions to solve for the forces at the pivot.

$$
\begin{aligned}
& \sum F_{x}=F_{\mathrm{P} x}-F_{\mathrm{T}}=0 \rightarrow F_{\mathrm{P} x}=F_{\mathrm{T}}=410 \mathrm{~N} \\
& \sum F_{y}=F_{\mathrm{P} y}-m g-M g=0 \rightarrow F_{\mathrm{P} y}=(m+M) g=(33.5 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=328 \mathrm{~N}
\end{aligned}
$$

22. The center of gravity of each beam is at its geometric center. Calculate torques about the left end of the beam, and take counterclockwise torques to be positive. The conditions of equilibrium for the beam are used to find the forces that the

$$
\begin{aligned}
& \text { support exerts on the beam. } \\
& \sum \tau=F_{\mathrm{B}} \ell-M g(\ell / 2)-\frac{1}{2} M g(\ell / 4)=0 \rightarrow \\
& F_{\mathrm{B}}=\frac{5}{8} M g=\frac{5}{8}(940 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=5758 \mathrm{~N} \approx 5800 \mathrm{~N} \\
& \sum F_{y}=F_{\mathrm{A}}+F_{\mathrm{B}}-M g-\frac{1}{2} M g=0 \rightarrow \\
& F_{\mathrm{A}}=\frac{3}{2} M g-F_{\mathrm{B}}=\frac{7}{8} M g=\frac{7}{8}(940 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=8061 \mathrm{~N} \approx 8100 \mathrm{~N}
\end{aligned}
$$

23. First consider the triangle made by the pole and one of the wires (first diagram). It has a vertical leg of 2.6 m , and a horizontal leg of 2.0 m . The angle that the tension (along the wire) makes with the vertical is
$\theta=\tan ^{-1} \frac{2.0}{2.6}=37.6^{\circ}$. The part of the tension that is parallel to the ground is therefore $F_{\mathrm{Th}}=F_{\mathrm{T}} \sin \theta$.

Now consider a top view of the pole, showing only force parallel to the ground (second diagram). The horizontal parts of the tension lie as the sides of an equilateral triangle, and so each make a $30^{\circ}$ angle with the tension force of the net. Write the equilibrium equation for the forces along the direction of the tension in the net.

$$
\begin{aligned}
& \sum F=F_{\text {net }}-2 F_{\mathrm{Th}} \cos 30^{\circ}=0 \rightarrow \\
& F_{\text {net }}=2 F_{\mathrm{T}} \sin \theta \cos 30^{\circ}=2(115 \mathrm{~N}) \sin 37.6^{\circ} \cos 30^{\circ}=121.5 \mathrm{~N} \approx 120 \mathrm{~N}
\end{aligned}
$$


24. See the free-body diagram. We assume that the board is at the edge of the door opposite the hinges, and that you are pushing at that same edge of the door. Then the width of the door does not enter into the problem. Force $\overrightarrow{\mathbf{F}}_{\text {push }}$ is the force of the door on the board, and is the same as the force the person exerts on the door. Take torques about the point A in the free-body diagram, where the board rests on the ground. The board is of length $\ell$.

$$
\begin{aligned}
& \sum \tau=F_{\text {push }} \ell \sin \theta-m g\left(\frac{1}{2} \ell\right) \cos \theta=0 \rightarrow \\
& F_{\text {push }}=\frac{m g}{2 \tan \theta}=\frac{(62.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{2 \tan 45^{\circ}}=303.8 \mathrm{~N} \approx 3.0 \times 10^{2} \mathrm{~N}
\end{aligned}
$$


25. Because the board is firmly set against the ground, the top of the board would move upwards as the door opened. Thus the frictional force on the board at the door must be down. We also assume that the static frictional force is a maximum, and so is given by $F_{\mathrm{fr}}=\mu F_{\mathrm{N}}=\mu F_{\text {push }}$. Take torques about the point A in the free-body diagram, where the board rests on the ground. The board is of length $\ell$.

$$
\begin{aligned}
& \sum \tau=F_{\text {push }} \ell \sin \theta-m g\left(\frac{1}{2} \ell\right) \cos \theta-F_{\text {fr }} \ell \cos \theta=0 \rightarrow \\
& F_{\text {push }} \ell \sin \theta-m g\left(\frac{1}{2} \ell\right) \cos \theta-\mu F_{\text {push }} \ell \cos \theta=0 \rightarrow \\
& F_{\text {push }}=\frac{m g}{2(\tan \theta-\mu)}=\frac{m g}{2(\tan \theta-\mu)}=\frac{(62.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{2\left(\tan 45^{\circ}-0.45\right)}=552.4 \mathrm{~N} \approx 550 \mathrm{~N}
\end{aligned}
$$

26. Draw the free-body diagram for the sheet, and write Newton's second law for the vertical direction. Note that the tension is the same in both parts of the clothesline.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{T}} \sin 3.5^{\circ}+F_{\mathrm{T}} \sin 3.5^{\circ}-m g=0 \rightarrow \\
& F_{\mathrm{T}}=\frac{m g}{2\left(\sin 3.5^{\circ}\right)}=\frac{(0.75 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{2\left(\sin 3.5^{\circ}\right)} \\
& \quad=60 \mathrm{~N}(2 \text { sig. fig. })
\end{aligned}
$$

The $60-\mathrm{N}$ tension is much higher than the $\sim 7.5-\mathrm{N}$ weight of the sheet because of the small angle. Only the vertical components of the tension are supporting the sheet, and since the angle is small, the tension has to be large to have a large enough vertical component to hold up the sheet.
27. (a) Choose the coordinates as shown in the free-body diagram.
(b) Write the equilibrium conditions for the horizontal and vertical forces.

$$
\begin{aligned}
& \sum F_{x}=F_{\text {rope }} \sin \phi-F_{\substack{\text { hinge } \\
\text { horiz }}}=0 \rightarrow \\
& F_{\text {hinge }}=F_{\text {rope }} \sin \phi=(85 \mathrm{~N}) \sin 37^{\circ}=51 \mathrm{~N}
\end{aligned}
$$



$$
\begin{aligned}
& \sum F_{y}=F_{\text {rope }} \cos \phi+F_{\substack{\text { hinge } \\
\text { vert }}}-m g-W=0 \rightarrow \\
& \begin{aligned}
F_{\text {hinge }} & =m g+W-F_{\text {rope }} \cos \phi=(3.8 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)+22 \mathrm{~N}-(85 \mathrm{~N}) \cos 37^{\circ} \\
& =-8.6 \mathrm{~N} \approx-9 \mathrm{~N}
\end{aligned}
\end{aligned}
$$

And so the vertical hinge force actually points downward.
(c) We take torques about the hinge point, with clockwise torques as positive.

$$
\begin{aligned}
& \sum \tau=W x \sin \theta+m g\left(\frac{1}{2} \ell\right) \sin \theta-F_{\text {rope }} \ell \sin (\theta-\phi)=0 \rightarrow \\
& x=\frac{F_{\text {rope }} \ell \sin (\theta-\phi)-m g\left(\frac{1}{2} \ell\right) \sin \theta}{W \sin \theta} \\
&=\frac{(85 \mathrm{~N})(5.0 \mathrm{~m}) \sin 16^{\circ}-(3.8 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(2.5 \mathrm{~m}) \sin 53^{\circ}}{(22 \mathrm{~N}) \sin 53^{\circ}}=2.436 \mathrm{~m} \approx 2.4 \mathrm{~m}
\end{aligned}
$$

28. (a) Consider the free-body diagram for each side of the ladder. Because the two sides are not identical, we must have both horizontal and vertical components to the hinge force of one side of the ladder on the other.
First determine the angle from $\cos \theta=\frac{\frac{1}{2} d}{\ell}=\frac{d}{2 \ell}$.

$$
\theta=\cos ^{-1} \frac{\frac{1}{2} d}{\ell}=\cos ^{-1} \frac{0.9 \mathrm{~m}}{2.5 \mathrm{~m}}=68.9^{\circ}
$$

Write equilibrium equations for the following conditions:
Vertical forces on total ladder:

$$
\begin{aligned}
& \sum F_{\text {vert }}=\underset{\substack{\text { left } \\
\text { left }}}{F_{\text {vert }}}-m g+\underset{\substack{\text { hinge } \\
\text { vert }}}{ }-F_{\text {hinge }}+\underset{\substack{\mathrm{N} \\
\text { right }}}{ }=0 \rightarrow \\
& F_{\mathrm{N}}+F_{\mathrm{N}}=m g \\
& \text { left }
\end{aligned}
$$

Torques on left side, about top, clockwise positive.

$$
\sum \tau=\underset{\substack{\text { Neff } \\ \text { le }}}{ }(\ell \cos \theta)-m g(0.2 \ell) \cos \theta-F_{\mathrm{T}}\left(\frac{1}{2} \ell\right) \sin \theta=0
$$

Torques on right side, about top, clockwise positive.

$$
\sum \tau=-F_{\substack{\mathrm{N} \\ \text { right }}}(\ell \cos \theta)+F_{\mathrm{T}}\left(\frac{1}{2} \ell\right) \sin \theta=0
$$

Subtract the second torque equation from the first.


$$
\left(F_{\substack{\mathrm{N} \\ \text { left }}}+\underset{\substack{\mathrm{N} \\ \text { right }}}{F_{\mathrm{T}}}\right)(\ell \cos \theta)-m g(0.2 \ell) \cos \theta-2 F_{\mathrm{T}}\left(\frac{1}{2} \ell\right) \sin \theta=0
$$

Substitute in from the vertical forces equation, and solve for the tension.

$$
\begin{aligned}
& m g(\ell \cos \theta)-m g(0.2 \ell) \cos \theta-2 F_{\mathrm{T}}\left(\frac{1}{2} \ell\right) \sin \theta=0 \rightarrow \\
& F_{\mathrm{T}}=\frac{m g}{\sin \theta}(0.8 \cos \theta)=\frac{0.8 m g}{\tan \theta}=\frac{0.8(56.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{\tan 68.9^{\circ}}=169.4 \mathrm{~N} \approx 170 \mathrm{~N}
\end{aligned}
$$

(b) To find the normal force on the right side, use the torque equation for the right side.

$$
\begin{aligned}
& -F_{\mathrm{N}}(\ell \cos \theta)+F_{\mathrm{T}}\left(\frac{1}{2} \ell\right) \sin \theta=0 \rightarrow \\
& F_{\mathrm{N} \text { ight }}=\frac{1}{2} F_{\mathrm{T}} \tan \theta=\frac{1}{2}(169.4 \mathrm{~N}) \tan 68.9^{\circ}=219.5 \mathrm{~N} \approx 220 \mathrm{~N}
\end{aligned}
$$

To find the normal force on the left side, use the vertical force equation for the entire ladder.

$$
\begin{aligned}
& F_{\mathrm{N}}+F_{\mathrm{N}}=m g \rightarrow \\
& \text { left } \\
& F_{\mathrm{N}}=m g-\underset{\substack{\mathrm{N} \\
\text { left }}}{ }=m g-\underset{\substack{\mathrm{N} \\
\text { right }}}{\mathrm{l}_{\mathrm{N}}}=(56.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)-219.5 \mathrm{~N}=329.3 \mathrm{~N} \approx 330 \mathrm{~N}
\end{aligned}
$$

(c) We find the hinge force components from the free-body diagram for the right side.

$$
\begin{aligned}
& \sum F_{\text {horiz }}=F_{\substack{\text { hinge } \\
\text { horiz }}}-F_{\mathrm{T}}=0 \rightarrow F_{\substack{\text { hinge } \\
\text { hooiz }}}=F_{\mathrm{T}}=169.4 \mathrm{~N} \\
& F_{\text {hinge }}=\sqrt{\begin{array}{c}
\text { hinge } \\
\text { horiz }
\end{array}}+F_{\substack{\text { hinge } \\
\text { vett }}}^{2}=\sqrt{(169.4 \mathrm{~N})^{2}+(219.5 \mathrm{~N})^{2}}=277.3 \mathrm{~N} \approx 280 \mathrm{~N} \\
& \phi_{\text {hinge }}=\tan ^{-1} \frac{F_{\text {hinge }}}{F_{\text {vert }}}=\tan ^{-1} \frac{219.5 \mathrm{~N}}{169.4 \mathrm{~N}}=52^{\circ}
\end{aligned}
$$

29. The forces on the door are due to gravity and the hinges. Since the door is in equilibrium, the net torque and net force must be zero. Write the three equations of equilibrium. Calculate torques about the bottom hinge, with counterclockwise torques as positive. From the statement of the problem, $F_{\mathrm{A} y}=F_{\mathrm{B} y}=\frac{1}{2} m g$.

$$
\begin{array}{ll}
\sum \tau=m g \frac{w}{2}-F_{A x}(h-2 d)=0 & \\
F_{A x}=\frac{m g w}{2(h-2 d)}=\frac{(13.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(1.30 \mathrm{~m})}{2(2.30 \mathrm{~m}-0.80 \mathrm{~m})}=55.2 \mathrm{~N} & m \overrightarrow{\mathbf{g}} \\
\sum F_{x}=F_{A x}-F_{B x}=0 \rightarrow F_{B x}=F_{A x}=55.2 \mathrm{~N} \\
\sum F_{y}=F_{A y}+F_{B y}-m g=0 \rightarrow F_{A y}=F_{B y}=\frac{1}{2} m g=\frac{1}{2}(13.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=63.7 \mathrm{~N}
\end{array}
$$


30. See the free-body diagram for the crate on the verge of tipping. From the textbook Figure 12-12 and the associated discussion, if a vertical line projected downward from the center of gravity falls outside the base of support, then the object will topple. So the limiting case is for the vertical line to intersect the edge of the base of support. Any more tilting and the gravity force would cause the block to tip over, with the axis of rotation through the lower corner of the crate.

$$
\tan \theta=\frac{1.00}{1.18} \rightarrow \theta=\tan ^{-1} \frac{1.00}{1.18}=40^{\circ}(2 \mathrm{sig} \mathrm{fig})
$$



The other forces on the block, the normal force and the frictional force, would be acting at the lower corner and so would not cause any torque about the lower corner. The gravity force causes the tipping. It wouldn't matter if the block were static or sliding, since the magnitude of the frictional force does not enter into the calculation.
31. We assume the truck is accelerating to the right. We want the refrigerator to not tip in the non-inertial reference frame of the truck. Accordingly, to analyze the refrigerator in the non-inertial reference frame, we must add a pseudoforce in the opposite direction of the actual acceleration. The free-body diagram is for a side view of the refrigerator, just ready to tip so that the normal force and frictional force are at the lower back corner of the refrigerator. The center of mass is in the geometric center of the refrigerator. Write the conditions for equilibrium, taking torques about an axis through the center of mass, perpendicular to the plane of the paper. The normal force and frictional force cause no torque about that axis.

$$
\begin{aligned}
& \sum F_{\text {horiz }}=F_{\text {fr }}-m a_{\text {tuck }}=0 \rightarrow F_{\mathrm{fr}}=m a_{\text {tuuck }} \\
& \sum F_{\text {vert }}=F_{\mathrm{N}}-m g=0 \rightarrow F_{\mathrm{N}}=m g \\
& \sum \tau=F_{\mathrm{N}}\left(\frac{1}{2} w\right)-F_{\mathrm{fr}}\left(\frac{1}{2} h\right)=0 \rightarrow \frac{F_{\mathrm{N}}}{F_{\mathrm{fr}}}=\frac{h}{w} \\
& \frac{F_{\mathrm{N}}}{F_{\mathrm{fr}}}=\frac{h}{w}=\frac{m g}{m a_{\text {tuck }}} \rightarrow a_{\mathrm{truck}}=g \frac{w}{h}=\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \frac{1.0 \mathrm{~m}}{1.9 \mathrm{~m}}=5.2 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$


32. Write the conditions of equilibrium for the ladder, with torques taken about the bottom of the ladder, and counterclockwise torques as positive.

$$
\begin{aligned}
& \sum \tau=F_{\mathrm{W}} \ell \sin \theta-m g\left(\frac{1}{2} \ell \cos \theta\right)=0 \rightarrow F_{\mathrm{W}}=\frac{1}{2} \frac{m g}{\tan \theta} \\
& \sum F_{x}=F_{\mathrm{G} x}-F_{\mathrm{W}}=0 \rightarrow F_{\mathrm{G} x}=F_{\mathrm{W}}=\frac{1}{2} \frac{m g}{\tan \theta} \\
& \sum F_{y}=F_{\mathrm{G} y}-m g=0 \rightarrow F_{\mathrm{G} y}=m g
\end{aligned}
$$

For the ladder to not slip, the force at the ground $F_{G x}$ must be less than
 or equal to the maximum force of static friction.

$$
F_{\mathrm{G} x} \leq \mu F_{\mathrm{N}}=\mu F_{\mathrm{G} y} \rightarrow \frac{1}{2} \frac{m g}{\tan \theta} \leq \mu m g \quad \rightarrow \frac{1}{2 \mu} \leq \tan \theta \rightarrow \theta \geq \tan ^{-1}\left(\frac{1}{2 \mu}\right)
$$

Thus the minimum angle is $\theta_{\text {min }}=\tan ^{-1}\left(\frac{1}{2 \mu}\right)$.
33. The tower can lean until a line projected downward through its center of gravity will fall outside its base of support. Since we are assuming that the tower is uniform, its center of gravity (or center of mass) will be at its geometric center. The center of mass can move a total of 3.5 m off of center and still be over the support base. It has currently moved 2.25 m off of center. So it can lean over another 1.25 m at the center, or 2.5 m at the top. Note that the diagram is NOT to scale. The tower should be twice as tall as shown to be to scale.

34. The amount of stretch can be found using the elastic modulus in Eq. 12-4.

$$
\Delta \ell=\frac{1}{E} \frac{F}{A} \ell_{0}=\frac{1}{5 \times 10^{9} \mathrm{~N} / \mathrm{m}^{2}} \frac{275 \mathrm{~N}}{\pi\left(5.00 \times 10^{-4}\right)^{2}}(0.300 \mathrm{~m})=2.10 \times 10^{-2} \mathrm{~m}
$$

35. (a) Stress $=\frac{F}{A}=\frac{m g}{A}=\frac{(25000 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{1.4 \mathrm{~m}^{2}}=175,000 \mathrm{~N} / \mathrm{m}^{2} \approx 1.8 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}$
(b) Strain $=\frac{\text { Stress }}{\text { Young's Modulus }}=\frac{175,000 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}}{50 \times 10^{9} \mathrm{~N} / \mathrm{m}^{2}}=3.5 \times 10^{-6}$
36. The change in length is found from the strain.

$$
\text { Strain }=\frac{\Delta \ell}{\ell_{0}} \rightarrow \Delta \ell=\ell_{0}(\text { Strain })=(8.6 \mathrm{~m})\left(3.5 \times 10^{-6}\right)=3.0 \times 10^{-5} \mathrm{~m}
$$

37. (a) Stress $=\frac{F}{A}=\frac{m g}{A}=\frac{(1700 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{0.012 \mathrm{~m}^{2}}=1.388 \times 10^{6} \mathrm{~N} / \mathrm{m}^{2} \approx 1.4 \times 10^{6} \mathrm{~N} / \mathrm{m}^{2}$
(b) Strain $=\frac{\text { Stress }}{\text { Young's Modulus }}=\frac{1.388 \times 10^{6} \mathrm{~N} / \mathrm{m}^{2}}{200 \times 10^{9} \mathrm{~N} / \mathrm{m}^{2}}=6.94 \times 10^{-6} \approx 6.9 \times 10^{-6}$
(c) $\Delta \boldsymbol{\ell}=($ Strain $)\left(\ell_{0}\right)=\left(6.94 \times 10^{-6}\right)(9.50 \mathrm{~m})=6.593 \times 10^{-5} \mathrm{~m} \approx 6.6 \times 10^{-5} \mathrm{~m}$
38. The relationship between pressure change and volume change is given by Eq. 12-7.

$$
\begin{aligned}
& \Delta V=-V_{0} \frac{\Delta P}{B} \rightarrow \Delta P=-\frac{\Delta V}{V_{0}} B=-\left(0.10 \times 10^{-2}\right)\left(90 \times 10^{9} \mathrm{~N} / \mathrm{m}^{2}\right)=9.0 \times 10^{7} \mathrm{~N} / \mathrm{m}^{2} \\
& \frac{\Delta P}{P_{\mathrm{atm}}}=\frac{9.0 \times 10^{7} \mathrm{~N} / \mathrm{m}^{2}}{1.0 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}}=9.0 \times 10^{2}, \text { or } 900 \text { atmospheres }
\end{aligned}
$$

39. The Young's Modulus is the stress divided by the strain.

$$
\text { Young's Modulus }=\frac{\text { Stress }}{\text { Strain }}=\frac{F / A}{\Delta \ell / \ell_{0}}=\frac{(13.4 \mathrm{~N}) /\left[\pi\left(\frac{1}{2} \times 8.5 \times 10^{-3} \mathrm{~m}\right)^{2}\right]}{\left(3.7 \times 10^{-3} \mathrm{~m}\right) /\left(15 \times 10^{-2} \mathrm{~m}\right)}=9.6 \times 10^{6} \mathrm{~N} / \mathrm{m}^{2}
$$

40. The percentage change in volume is found by multiplying the relative change in volume by 100 . The change in pressure is 199 times atmospheric pressure, since it increases from atmospheric pressure to 200 times atmospheric pressure. Use Eq. 12-7.

$$
100 \frac{\Delta V}{V_{o}}=-100 \frac{\Delta P}{B}=-100 \frac{199\left(1.0 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}\right)}{90 \times 10^{9} \mathrm{~N} / \mathrm{m}^{2}}=-2 \times 10^{-2} \%
$$

The negative sign indicates that the interior space got smaller.
41. (a) The torque due to the sign is the product of the weight of the sign and the distance of the sign from the wall.

$$
\tau=m g d=(6.1 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(2.2 \mathrm{~m})=130 \mathrm{~m} \cdot \mathrm{~N}, \text { clockwise }
$$

(b) Since the wall is the only other object that can put force on the pole
 (ignoring the weight of the pole), then the wall must put a torque on the pole. The torque due to the hanging sign is clockwise, so the torque due to the wall must be counterclockwise. See the diagram. Also note that the wall must put a net upward force on the pole as well, so that the net force on the pole will be zero.
(c) The torque on the rod can be considered as the wall pulling horizontally to the left on the top left corner of the rod and pushing horizontally to the right at the bottom left corner of the rod. The reaction forces to these put a shear on the wall at the point of contact. Also, since the wall is pulling upwards on the rod, the rod is pulling down on the wall at the top surface of contact, causing tension. Likewise the rod is pushing down on the wall at the bottom surface of contact, causing compression. Thus all three are present.
42. Set the compressive strength of the bone equal to the stress of the bone.

Compressive Strength $=\frac{F_{\max }}{A} \rightarrow F_{\max }=\left(170 \times 10^{6} \mathrm{~N} / \mathrm{m}^{2}\right)\left(3.0 \times 10^{-4} \mathrm{~m}^{2}\right)=5.1 \times 10^{4} \mathrm{~N}$
43. (a) The maximum tension can be found from the ultimate tensile strength of the material.

$$
\begin{aligned}
& \text { Tensile Strength }=\frac{F_{\max }}{A} \rightarrow \\
& F_{\max }=(\text { Tensile Strength }) A=\left(500 \times 10^{6} \mathrm{~N} / \mathrm{m}^{2}\right) \pi\left(5.00 \times 10^{-4} \mathrm{~m}\right)^{2}=393 \mathrm{~N}
\end{aligned}
$$

(b) To prevent breakage, thicker strings should be used, which will increase the cross-sectional area of the strings, and thus increase the maximum force. Breakage occurs because when the strings are hit by the ball, they stretch, increasing the tension. The strings are reasonably tight in the normal racket configuration, so when the tension is increased by a particularly hard hit, the tension may exceed the maximum force.
44. (a) Compare the stress on the bone to the compressive strength to see if the bone breaks.

$$
\begin{aligned}
\text { Stress } & =\frac{F}{A}=\frac{3.3 \times 10^{4} \mathrm{~N}}{3.6 \times 10^{-4} \mathrm{~m}^{2}} \\
& =9.167 \times 10^{7} \mathrm{~N} / \mathrm{m}^{2}<1.7 \times 10^{8} \mathrm{~N} / \mathrm{m}^{2}(\text { Compressive Strength of bone })
\end{aligned}
$$

## The bone will not break.

(b) The change in length is calculated from Eq. 12-4.

$$
\Delta \ell=\frac{\ell_{0}}{E} \frac{F}{A}=\left(\frac{0.22 \mathrm{~m}}{15 \times 10^{9} \mathrm{~N} / \mathrm{m}^{2}}\right)\left(9.167 \times 10^{7} \mathrm{~N} / \mathrm{m}^{2}\right)=1.3 \times 10^{-3} \mathrm{~m}
$$

45. (a) The area can be found from the ultimate tensile strength of the material.

$$
\frac{\text { Tensile Strength }}{\text { Safety Factor }}=\frac{F}{A} \rightarrow A=F\left(\frac{\text { Safety Factor }}{\text { Tensile Strength }}\right) \rightarrow
$$

$$
A=(270 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \frac{7.0}{500 \times 10^{6} \mathrm{~N} / \mathrm{m}^{2}}=3.704 \times 10^{-5} \mathrm{~m}^{2} \approx 3.7 \times 10^{-5} \mathrm{~m}^{2}
$$

(b) The change in length can be found from the stress-strain relationship, Eq. 12-5.

$$
\frac{F}{A}=E \frac{\Delta \ell}{\ell_{0}} \rightarrow \Delta \ell=\frac{\ell_{0} F}{A E}=\frac{(7.5 \mathrm{~m})(320 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{\left(3.704 \times 10^{-5} \mathrm{~m}^{2}\right)\left(200 \times 10^{9} \mathrm{~N} / \mathrm{m}^{2}\right)}=2.7 \times 10^{-3} \mathrm{~m}
$$

46. For each support, to find the minimum cross-sectional area with a safety factor means that $\frac{F}{A}=\frac{\text { Strength }}{\text { Safety Factor }}$, where either the tensile or compressive strength is used, as appropriate for each force. To find the force on each support, use the conditions of equilibrium for the beam.


Take torques about the left end of the beam, calling counterclockwise torques positive, and also sum the vertical forces, taking upward forces as positive.

$$
\begin{aligned}
& \sum \tau=F_{2}(20.0 \mathrm{~m})-m g(25.0 \mathrm{~m})=0 \rightarrow F_{2}=\frac{25.0}{20.0} m g=1.25 m g \\
& \sum F_{y}=F_{1}+F_{2}-m g=0 \rightarrow F_{1}=m g-F_{2}=m g-1.25 m g=-0.25 m g
\end{aligned}
$$

Notice that the forces on the supports are the opposite of $\overrightarrow{\mathbf{F}}_{1}$ and $\overrightarrow{\mathbf{F}}_{2}$. So the force on support $\# 1$ is directed upwards, which means that support \# 1 is in tension. The force on support \# 2 is directed downwards, so support \# 2 is in compression.

$$
\begin{aligned}
& \frac{F_{1}}{A_{1}}=\frac{\text { Tensile Strength }}{9.0} \rightarrow \\
& A_{1}=9.0 \frac{(0.25 \mathrm{mg})}{\text { Tensile Strength }}=9.0 \frac{(0.25)\left(2.9 \times 10^{3} \mathrm{~kg}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{40 \times 10^{6} \mathrm{~N} / \mathrm{m}^{2}}=1.6 \times 10^{-3} \mathrm{~m}^{2} \\
& \frac{F_{2}}{A_{2}}=\frac{\text { Compressive Strength }}{9.0} \rightarrow \\
& A_{1}=9.0 \frac{(1.25 \mathrm{mg})}{\text { Compressive Strength }}=9.0 \frac{(1.25)\left(2.9 \times 10^{3} \mathrm{~kg}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{35 \times 10^{6} \mathrm{~N} / \mathrm{m}^{2}}=9.1 \times 10^{-3} \mathrm{~m}^{2}
\end{aligned}
$$

47. The maximum shear stress is to be $1 / 7^{\text {th }}$ of the shear strength for iron. The maximum stress will occur for the minimum area, and thus the minimum diameter.

$$
\begin{aligned}
& \text { stress }_{\max }=\frac{F}{A_{\min }}=\frac{\text { shear strength }}{7.0} \rightarrow A_{1}=\pi\left(\frac{1}{2} d\right)^{2}=\frac{7.0 F}{\text { shear strength }} \rightarrow \\
& d=\sqrt{\frac{4(7.0) F}{\pi(\text { shear strength })}}=\sqrt{\frac{28(3300 \mathrm{~N})}{\pi\left(170 \times 10^{6} \mathrm{~N} / \mathrm{m}^{2}\right)}}=1.3 \times 10^{-2} \mathrm{~m}=1.3 \mathrm{~cm}
\end{aligned}
$$

48. From the free-body diagram, write Newton's second law for the vertical direction. Solve for the maximum tension required in the cable, which will occur for an upwards acceleration

$$
\sum F_{y}=F_{\mathrm{T}}-m g=m a \rightarrow F_{T}=m(g+a)
$$

The maximum stress is to be $1 / 8^{\text {th }}$ of the tensile strength for steel. The maximum stress will occur for the minimum area, and thus the minimum diameter.


$$
\begin{aligned}
& \text { stress }_{\max }=\frac{F_{\mathrm{T}}}{A_{\min }}=\frac{\text { tensile strength }}{8.0} \rightarrow A_{1}=\pi\left(\frac{1}{2} d\right)^{2}=\frac{8.0 F_{\mathrm{T}}}{\text { tensile strength }} \rightarrow \\
& d=\sqrt{\frac{4(8.0) m(g+a)}{\pi(\text { tensile strength })}}=\sqrt{\frac{32(3100 \mathrm{~kg})\left(11.0 \mathrm{~m} / \mathrm{s}^{2}\right)}{\pi\left(500 \times 10^{6} \mathrm{~N} / \mathrm{m}^{2}\right)}}=2.6 \times 10^{-2} \mathrm{~m}=2.6 \mathrm{~cm}
\end{aligned}
$$

49. (a) The three forces on the truss as a whole are the tension force at point B , the load at point E , and the force at point A . Since the truss is in equilibrium, these three forces must add to be 0 and must cause no net torque. Take torques about point A , calling clockwise torques positive. Each member is 3.0 m in length.

$$
\begin{aligned}
& \sum \tau=F_{\mathrm{T}}(3.0 \mathrm{~m}) \sin 60^{\circ}-M g(6.0 \mathrm{~m})=0 \rightarrow \\
& F_{T}=\frac{M g(6.0 \mathrm{~m})}{(3.0 \mathrm{~m}) \sin 60^{\circ}}=\frac{(66.0 \mathrm{kN})(6.0 \mathrm{~m})}{(3.0 \mathrm{~m}) \sin 60^{\circ}}=152 \mathrm{kN} \approx 150 \mathrm{kN}
\end{aligned}
$$



The components of $\overrightarrow{\mathbf{F}}_{\mathrm{A}}$ are found from the force equilibrium equations, and then the magnitude and direction can be found.

$$
\begin{aligned}
& \sum F_{\text {horiz }}=F_{\mathrm{T}}-F_{\mathrm{A} \text { horiz }}=0 \rightarrow F_{\mathrm{A} \text { horiz }}=F_{\mathrm{T}}=152 \mathrm{kN} \\
& \sum F_{\text {vert }}=F_{\mathrm{A} \text { vert }}-M g=0 \rightarrow F_{\mathrm{A} \text { vert }}=M g=66.0 \mathrm{kN} \\
& F_{\mathrm{A}}=\sqrt{F_{\mathrm{A} \text { horiz }}^{2}+F_{\mathrm{A} \text { vert }}^{2}}=\sqrt{(152 \mathrm{kN})^{2}+(66.0 \mathrm{kN})^{2}}=166 \mathrm{kN} \approx 170 \mathrm{kN} \\
& \theta_{\mathrm{A}}=\tan ^{-1} \frac{F_{\mathrm{A} \text { vert }}}{F_{\mathrm{A} \text { horiz }}}=\tan ^{-1} \frac{66.0 \mathrm{kN}}{152 \mathrm{kN}}=23.47^{\circ} \approx 23^{\circ} \text { above AC }
\end{aligned}
$$

(b) Analyze the forces on the pin at point E. See the second free-body diagram. Write equilibrium equations for the horizontal and vertical directions.

$$
\begin{aligned}
& \sum F_{\mathrm{vert}}=F_{\mathrm{DE}} \sin 60^{\circ}-M g=0 \rightarrow \\
& F_{\mathrm{DE}}=\frac{M g}{\sin 60^{\circ}}=\frac{66.0 \mathrm{kN}}{\sin 60^{\circ}}=76.2 \mathrm{kN} \approx 76 \mathrm{kN}, \text { in tension } \\
& \sum F_{\mathrm{horiz}}=F_{\mathrm{DE}} \cos 60^{\circ}-F_{\mathrm{CE}}=0 \rightarrow \\
& F_{\mathrm{CE}}=F_{\mathrm{DE}} \cos 60^{\circ}=(76.2 \mathrm{kN}) \cos 60^{\circ}=38.1 \mathrm{kN} \approx 38 \mathrm{kN}, \text { in compression }
\end{aligned}
$$



Analyze the forces on the pin at point D . See the third free-body diagram. Write equilibrium equations for the horizontal and vertical directions.

$$
\begin{aligned}
& \sum F_{\mathrm{vert}}=F_{\mathrm{DC}} \sin 60^{\circ}-F_{\mathrm{DE}} \sin 60^{\circ}=0 \rightarrow \\
& F_{\mathrm{DC}}=F_{\mathrm{DE}}=76.2 \mathrm{kN} \approx 76 \mathrm{kN}, \text { in compression }
\end{aligned}
$$



$$
\begin{aligned}
& \sum F_{\mathrm{horiz}}=F_{\mathrm{DB}}-F_{\mathrm{DE}} \cos 60^{\circ}-F_{\mathrm{DC}} \cos 60^{\circ}=0 \rightarrow \\
& F_{\mathrm{DB}}=\left(F_{\mathrm{DE}}+F_{\mathrm{DC}}\right) \cos 60^{\circ}=2(76.2 \mathrm{kN}) \cos 60^{\circ}=76.2 \mathrm{kN} \\
& \quad \approx 76 \mathrm{kN}, \text { in tension }
\end{aligned}
$$

Analyze the forces on the pin at point C. See the fourth free-body diagram. Write equilibrium equations for the horizontal and vertical directions.

$$
\begin{aligned}
& \sum F_{\mathrm{vert}}=F_{\mathrm{BC}} \sin 60^{\circ}-F_{\mathrm{DC}} \sin 60^{\circ}=0 \rightarrow \\
& F_{\mathrm{BC}}=F_{\mathrm{DC}}=76.2 \mathrm{kN} \approx 76 \mathrm{kN}, \text { in tension } \\
& \sum F_{\mathrm{horiz}}=F_{\mathrm{CE}}+F_{\mathrm{BC}} \cos 60^{\circ}+F_{\mathrm{DC}} \cos 60^{\circ}-F_{\mathrm{CA}}=0 \rightarrow \\
& F_{\mathrm{CA}}=F_{\mathrm{CE}}+\left(F_{\mathrm{BC}}+F_{\mathrm{DC}}\right) \cos 60^{\circ}=38.1 \mathrm{kN}+2(76.2 \mathrm{kN}) \cos 60^{\circ} \\
& \quad=114.3 \mathrm{kN} \approx 114 \mathrm{kN}, \text { in compression }
\end{aligned}
$$

Analyze the forces on the pin at point B. See the fifth free-body diagram. Write equilibrium equations for the horizontal and vertical directions.

$$
\begin{aligned}
& \sum F_{\mathrm{vert}}=F_{\mathrm{AB}} \sin 60^{\circ}-F_{\mathrm{BC}} \sin 60^{\circ}=0 \rightarrow \\
& F_{\mathrm{AB}}=F_{\mathrm{BC}}=76.2 \mathrm{kN} \approx 76 \mathrm{kN}, \text { in compression } \\
& \sum F_{\mathrm{horiz}}=F_{\mathrm{T}}-F_{\mathrm{BC}} \cos 60^{\circ}-F_{\mathrm{AB}} \cos 60^{\circ}-F_{\mathrm{DB}}=0 \rightarrow \\
& F_{\mathrm{T}}=\left(F_{\mathrm{BC}}+F_{\mathrm{AB}}\right) \cos 60^{\circ}+F_{\mathrm{DB}}=2(76.2 \mathrm{kN}) \cos 60^{\circ}+76.2 \mathrm{kN}=152 \mathrm{kN}
\end{aligned}
$$



This final result confirms the earlier calculation, so the results are consistent. We could also analyze point A to check for consistency.
50. There are upward forces at each support (points A and D) and a downward applied force at point C . Write the conditions for equilibrium for the entire truss by considering vertical forces and the torques about point A. Let clockwise torques be positive. Let each side of the equilateral triangle be of length $\ell$.

$$
\begin{aligned}
& \sum F_{\text {vert }}=F_{\mathrm{A}}+F_{\mathrm{D}}-F=0 \\
& \sum \tau=F\left(\frac{1}{2} \ell\right)-F_{\mathrm{D}} \ell=0 \rightarrow F_{\mathrm{D}}=\frac{1}{2} F=\frac{1}{2}\left(1.35 \times 10^{4} \mathrm{~N}\right)=6750 \mathrm{~N} \\
& F_{\mathrm{A}}=F-F_{\mathrm{D}}=1.35 \times 10^{4} \mathrm{~N}-6750 \mathrm{~N}=6750 \mathrm{~N}
\end{aligned}
$$


(a) Analyze the forces on the pin at point A. See the second free-body diagram. Write equilibrium equations for the horizontal and vertical directions.

$$
\begin{aligned}
& \sum F_{\text {vert }}=F_{\mathrm{A}}-F_{\mathrm{AB}} \sin 60^{\circ}=0 \rightarrow \\
& F_{\mathrm{AB}}=\frac{F_{\mathrm{A}}}{\sin 60^{\circ}}=\frac{6750 \mathrm{~N}}{\sin 60^{\circ}}=7794 \mathrm{~N} \approx 7790 \mathrm{~N}, \text { compression } \\
& \sum F_{\text {horiz }}=F_{\mathrm{AC}}-F_{\mathrm{AB}} \cos 60^{\circ}=0 \rightarrow \\
& F_{\mathrm{AC}}=F_{\mathrm{AB}} \cos 60^{\circ}=(7794 \mathrm{~N}) \cos 60^{\circ}=3897 \mathrm{~N} \approx 3900 \mathrm{~N}, \text { tension }
\end{aligned}
$$

By the symmetry of the structure, we also know that $F_{\mathrm{DB}}=7794 \mathrm{~N} \approx 7790 \mathrm{~N}$, compression and $F_{\mathrm{DC}}=3897 \mathrm{~N} \approx 3900 \mathrm{~N}$, tension. Finally, from consideration of the vertical forces on pin C, we
see that $F_{\mathrm{BC}}=1.35 \times 10^{4} \mathrm{~N}$, tension.
(b) As listed above, we have struts AB and DB under compression, and struts $\mathrm{AC}, \mathrm{DC}$, and BC under compression.
51. (a) We assume that all of the trusses are of the same cross-sectional area, and so to find the minimum area needed, we use the truss that has the highest force in it. That is $F_{\mathrm{AB}}=\frac{1}{\sqrt{3}} M g$. Apply the safety condition to find the area.

$$
\begin{aligned}
& \frac{F_{\mathrm{AB}}}{A}=\frac{\text { Ultimate strength }}{7.0} \rightarrow \\
& \begin{aligned}
A & =\frac{7.0 F_{\mathrm{AB}}}{\text { Ultimate strength }}=\frac{7.0 \mathrm{Mg}}{\sqrt{3}(\text { Ultimate strength })}=\frac{7.0\left(7.0 \times 10^{5} \mathrm{~kg}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{\sqrt{3}\left(500 \times 10^{6} \mathrm{~N} / \mathrm{m}^{2}\right)} \\
& =5.5 \times 10^{-2} \mathrm{~m}^{2}
\end{aligned}
\end{aligned}
$$

(b) Recall that each truss must carry half the load, and so we need to add in an additional mass equal to 30 trucks. As in Example 12-11, we will assume that the mass of the trucks acts entirely at the center, so the analysis of that example is still valid. Let $m$ represent the mass of a truck.

$$
\begin{aligned}
A & =\frac{7.0(M+30 \mathrm{~m}) g}{\sqrt{3}(\text { Ultimate strength })}=\frac{7.0\left[7.0 \times 10^{5} \mathrm{~kg}+30\left(1.3 \times 10^{4} \mathrm{~kg}\right)\right]\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{\sqrt{3}\left(500 \times 10^{6} \mathrm{~N} / \mathrm{m}^{2}\right)} \\
& =8.6 \times 10^{-2} \mathrm{~m}^{2}
\end{aligned}
$$

52. See the free-body diagram from Figure $12-29$, as modified to indicate the changes in the roadway mass distribution. As in that example, if the roadway mass is $1.40 \times 10^{6} \mathrm{~kg}$, then for one truss, we should use $M=7.0 \times 10^{5} \mathrm{~kg}$. Write the conditions for equilibrium for the entire truss by considering vertical forces and the torques about point A . Let clockwise torques be positive.

$$
\begin{aligned}
& \sum F_{\text {vert }}=F_{1}+F_{2}-M g=0 \\
& \sum \tau=\frac{1}{2} M g(32 \mathrm{~m})+\frac{1}{4} M g(64 \mathrm{~m})-F_{2}(64 \mathrm{~m})=0 \rightarrow F_{2}=\frac{1}{2} M g \\
& F_{1}=M g-F_{2}=\frac{1}{2} M g
\end{aligned}
$$



Note that the problem is still symmetric about a vertical line through pin C. Also note that the forces at the ends each bear half of the weight of that side of the structure.

Analyze the forces on the pin at point A. See the second free-body diagram.
Write equilibrium equations for the horizontal and vertical directions.

$$
\begin{aligned}
& \sum F_{\text {vert }}=\frac{1}{2} M g-\frac{1}{4} M g-F_{\mathrm{AB}} \sin 60^{\circ}=0 \rightarrow \\
& F_{\mathrm{AB}}=\frac{\frac{1}{4} M g}{\sin 60^{\circ}}=\frac{\frac{1}{4} M g}{\frac{1}{2} \sqrt{3}}=\frac{M g}{2 \sqrt{3}}, \text { in compression } \\
& \sum F_{\text {horiz }}=F_{\mathrm{AC}}-F_{\mathrm{AB}} \cos 60^{\circ}=0 \rightarrow F_{\mathrm{AC}}=F_{\mathrm{AB}} \cos 60^{\circ}=\frac{M g}{2 \sqrt{3}} \frac{1}{2}=\frac{M g}{4 \sqrt{3}}, \text { in tension }
\end{aligned}
$$

Analyze the forces on the pin at point B. See the third free-body diagram. Write equilibrium equations for the horizontal and vertical directions.

$$
\begin{aligned}
& \sum F_{\text {vert }}=F_{\mathrm{AB}} \sin 60^{\circ}-F_{\mathrm{BC}} \sin 60^{\circ}=0 \rightarrow \\
& F_{\mathrm{BC}}=F_{\mathrm{AB}}=\frac{M g}{2 \sqrt{3}}, \text { in tension } \\
& \sum F_{\text {horiz }}=F_{\mathrm{AB}} \cos 60^{\circ}+F_{\mathrm{BC}} \cos 60^{\circ}-F_{\mathrm{DB}}=0 \rightarrow \\
& F_{\mathrm{DB}}=\left(F_{\mathrm{AB}}+F_{\mathrm{BC}}\right) \cos 60^{\circ}=2\left(\frac{M g}{2 \sqrt{3}}\right) \cos 60^{\circ}=\frac{M g}{2 \sqrt{3}}, \text { in compression }
\end{aligned}
$$



By the symmetry of the geometry, we can determine the other forces.

$$
F_{\mathrm{DE}}=F_{\mathrm{AB}}=\frac{M g}{2 \sqrt{3}} \text {, in compression }, F_{\mathrm{DC}}=F_{\mathrm{BC}}=\frac{M g}{2 \sqrt{3}} \text {, in tension }, F_{\mathrm{CE}}=F_{\mathrm{AC}}=\frac{M g}{4 \sqrt{3}} \text {, in tension. } .
$$

Note that each force is reduced by a factor of 2 from the original solution given in Example 12-11.
53. See the free-body diagram from Figure 12-29. $M$ represents the mass of the train, and each member has a length of $\ell$. Write the conditions for equilibrium for the entire truss by considering vertical forces and the torques about point A. Let clockwise torques be positive.

$$
\begin{aligned}
& \sum F_{\text {vett }}=F_{1}+F_{2}-\frac{1}{2} M g=0 \\
& \sum \tau=\frac{1}{2} M g\left(\frac{1}{2} \ell\right)-F_{2}(2 \ell)=0 \rightarrow F_{2}=\frac{1}{8} M g \\
& F_{1}=\frac{1}{2} M g-F_{2}=\frac{3}{8} M g
\end{aligned}
$$



Analyze the forces on strut AC, using the free-body diagram given in Figure $12-29 b$. Note that the forces at the pins are broken up into components. See the second free-body diagram. Write equilibrium equations for the horizontal and vertical directions, and for torques about point A.

$$
\begin{aligned}
& \sum F_{\text {vert }}=F_{\mathrm{A} y}+F_{\mathrm{C} y}-\frac{1}{2} M g=0 \\
& \sum F_{\mathrm{horiz}}=-F_{\mathrm{A} x}+F_{\mathrm{C} x}=0 \rightarrow F_{\mathrm{C} x}=F_{\mathrm{A} x} \\
& \sum \tau=\frac{1}{2} M g\left(\frac{1}{2} \ell\right)-F_{\mathrm{C} y}(\ell)=\rightarrow F_{\mathrm{C} y}=\frac{1}{4} M g \\
& F_{\mathrm{A} y}=\frac{1}{2} M g-F_{\mathrm{C} y}=\frac{1}{4} M g
\end{aligned}
$$



Since their $x$ components are equal and their $y$ components are equal, $F_{\mathrm{A}}=F_{\mathrm{C}}=F_{\mathrm{AC}}$.
Analyze the forces on the pin at point A. The components found above are forces of the pin on the strut, so we put in the opposite forces, which are the forces of the strut on the pin. See the third free-body diagram. Write equilibrium equations for the horizontal and vertical directions.

$$
\begin{aligned}
& \sum F_{\text {vert }}=\frac{3}{8} M g-F_{\mathrm{AC} y}-F_{\mathrm{AB}} \sin 60^{\circ}=0 \rightarrow \\
& F_{\mathrm{AB}}=\frac{\frac{3}{8} M g-F_{\mathrm{AC} y}}{\sin 60^{\circ}}=\frac{\frac{3}{8} M g-\frac{1}{4} M g}{\frac{1}{2} \sqrt{3}}=\frac{M g}{4 \sqrt{3}}=\frac{\left(53 \times 10^{3} \mathrm{~kg}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{4 \sqrt{3}} \\
& \quad=7.497 \times 10^{4} \mathrm{~N} \approx 7.5 \times 10^{4} \mathrm{~N}, \text { in compression }
\end{aligned}
$$

$$
\begin{aligned}
& \sum F_{\text {horiz }}=F_{\mathrm{AC} x}-F_{\mathrm{AB}} \cos 60^{\circ}=0 \rightarrow \\
& F_{\mathrm{AC} x}=\frac{1}{2} F_{\mathrm{AB}}=\frac{M g}{8 \sqrt{3}}=3.7485 \times 10^{4} \mathrm{~N} \approx 3.7 \times 10^{4} \mathrm{~N}, \text { in tension }
\end{aligned}
$$

The actual force $F_{\mathrm{AC}}$ has both a tension component $F_{\mathrm{AC} x}$ and a shearing component $F_{\mathrm{AC} y}$. Since the problem asks for just the compressive or tension force, only $F_{\mathrm{ACx}}$ is included in the answer.

Analyze the forces on the pin at point B. See the fourth free-body diagram. Write equilibrium equations for the horizontal and vertical directions.

$$
\begin{aligned}
& \sum F_{\mathrm{vert}}=F_{\mathrm{AB}} \sin 60^{\circ}-F_{\mathrm{BC}} \sin 60^{\circ}=0 \rightarrow \\
& F_{\mathrm{BC}}=F_{\mathrm{AB}}=\frac{M g}{4 \sqrt{3}}=7.5 \times 10^{4} \mathrm{~N}, \text { in tension } \\
& \sum F_{\mathrm{horiz}}=F_{\mathrm{AB}} \cos 60^{\circ}+F_{\mathrm{BC}} \cos 60^{\circ}-F_{\mathrm{DB}}=0 \rightarrow \\
& F_{\mathrm{DB}}=\left(F_{\mathrm{AB}}+F_{\mathrm{BC}}\right) \cos 60^{\circ}=2\left(\frac{M g}{4 \sqrt{3}}\right) \cos 60^{\circ}=\frac{M g}{4 \sqrt{3}}=7.5 \times 10^{4} \mathrm{~N}, \text { in compression }
\end{aligned}
$$



Analyze the forces on the pin at point C. See the fifth free-body diagram. Write equilibrium equations for the horizontal and vertical directions.

$$
\begin{aligned}
& \sum F_{\text {vert }}=F_{\mathrm{BC}} \sin 60^{\circ}+F_{\mathrm{DC}} \sin 60^{\circ}-F_{\mathrm{AC} y}=0 \rightarrow \\
& F_{\mathrm{DC}}=\frac{F_{\mathrm{AC} y}}{\sin 60^{\circ}}-F_{\mathrm{BC}}=\frac{\frac{1}{4} M g}{\frac{1}{2} \sqrt{3}}-\frac{M g}{4 \sqrt{3}}=\frac{\frac{1}{4} M g}{\frac{1}{2} \sqrt{3}}-\frac{M g}{4 \sqrt{3}} \\
& \quad=\frac{M g}{4 \sqrt{3}} \approx 7.5 \times 10^{4} \mathrm{~N}, \text { in tension } \\
& \sum F_{\mathrm{horiz}}=F_{\mathrm{CE}}+F_{\mathrm{DC}} \cos 60^{\circ}-F_{\mathrm{BC}} \cos 60^{\circ}-F_{\mathrm{AC} x}=0 \rightarrow \\
& F_{\mathrm{CE}}=F_{\mathrm{ACx}}+\left(F_{\mathrm{BC}}-F_{\mathrm{DC}}\right) \cos 60^{\circ}=\frac{M g}{8 \sqrt{3}}+0 \approx 3.7 \times 10^{4} \mathrm{~N}, \text { in tension }
\end{aligned}
$$



Analyze the forces on the pin at point D . See the sixth free-body diagram. Write the equilibrium equation for the vertical direction.

$$
\begin{aligned}
& \sum F_{\mathrm{vert}}=F_{\mathrm{DE}} \sin 60^{\circ}-F_{\mathrm{DC}} \sin 60^{\circ}=0 \rightarrow \\
& F_{\mathrm{DE}}=F_{\mathrm{DC}}=\frac{M g}{4 \sqrt{3}} \approx 7.5 \times 10^{4} \mathrm{~N}, \text { in compression }
\end{aligned}
$$



This could be checked by considering the forces on pin $E$.
54. See the free-body diagram from Figure 12-29. We let $m$ be the mass of the truck, $x$ be the distance of the truck from the left end of the bridge, and $2 \ell$ be the length of the bridge. Write the conditions for equilibrium for the entire truss by considering vertical forces and the torques about point A. Let clockwise torques be positive. And we use half of the mass of the truck, because there are 2 trusses.

$$
\begin{aligned}
& \sum F_{\text {vert }}=F_{1}+F_{2}-\frac{1}{2} m g=0 \\
& \sum \tau=\frac{1}{2} m g x-F_{2}(2 \ell)=0 \rightarrow \\
& F_{2}=\frac{m g x}{4 \ell}=\frac{(23000 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(22 \mathrm{~m})}{4(32 \mathrm{~m})}=38740 \mathrm{~N}
\end{aligned}
$$



$$
F_{1}=\frac{1}{2} m g-F_{2}=\frac{1}{2}(23000 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)-38740=73960 \mathrm{~N}
$$

Analyze the forces on strut AC, using the free-body diagram given in Figure 12-29b. Note that the forces at the pins are broken up into components. See the second free-body diagram. Write equilibrium equations for the horizontal and vertical directions, and for torques about point A.

$$
\begin{aligned}
& \sum F_{\text {vert }}=F_{\mathrm{A} y}+F_{\mathrm{C} y}-\frac{1}{2} m g=0 \\
& \sum F_{\text {horiz }}=-F_{\mathrm{Ax}}+F_{\mathrm{C} x}=0 \rightarrow F_{\mathrm{C} x}=F_{\mathrm{A} x} \\
& \sum \tau=\frac{1}{2} m g x-F_{\mathrm{C} y}(\ell)=\rightarrow \\
& F_{\mathrm{C} y}=\frac{1}{2} m g \frac{x}{\ell}=\frac{1}{2}(23000 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \frac{22 \mathrm{~m}}{32 \mathrm{~m}}=77480 \mathrm{~N} \approx 77,000 \mathrm{~N} \\
& F_{\mathrm{A} y}=\frac{1}{2} m g-F_{\mathrm{C} y}=\frac{1}{2}(23000 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)-77480 \mathrm{~N}=35220 \mathrm{~N} \approx 35,000 \mathrm{~N}
\end{aligned}
$$



Since their $x$ components are equal, $F_{\mathrm{AC}}=F_{\mathrm{CA}}$ for tension or compression along the beams.
Analyze the forces on the pin at point A. The components found above are forces of the pin on the strut, so we put in the opposite forces, which are the forces of the strut on the pin. See the third free-body diagram. Write equilibrium equations for the horizontal and vertical directions.

$$
\begin{aligned}
& \sum F_{\text {vert }}=F_{1}-F_{\mathrm{A} y}-F_{\mathrm{AB}} \sin 60^{\circ}=0 \rightarrow \\
& F_{\mathrm{AB}}=\frac{F_{1}-F_{\mathrm{A} y}}{\sin 60^{\circ}}=\frac{73960 \mathrm{~N}-35220 \mathrm{~N}}{\sin 60^{\circ}}=44730 \mathrm{~N} \approx 4.5 \times 10^{4} \mathrm{~N}, \text { in compression } \\
& \sum F_{\text {horiz }}=F_{\mathrm{ACx}}-F_{\mathrm{AB}} \cos 60^{\circ}=0 \rightarrow \\
& F_{\mathrm{ACx}}=F_{\mathrm{AB}} \cos 60^{\circ}=(44730 \mathrm{~N}) \cos 60^{\circ}=22365 \mathrm{~N} \approx 2.2 \times 10^{4} \mathrm{~N}, \text { in tension }
\end{aligned}
$$



Analyze the forces on the pin at point B. See the fourth free-body diagram.
Write equilibrium equations for the horizontal and vertical directions.

$$
\begin{aligned}
& \sum F_{\text {vert }}=F_{\mathrm{AB}} \sin 60^{\circ}-F_{\mathrm{BC}} \sin 60^{\circ}=0 \rightarrow \\
& F_{\mathrm{BC}}=F_{\mathrm{AB}}=4.5 \times 10^{4} \mathrm{~N}, \text { in tension } \\
& \sum F_{\text {horiz }}=F_{\mathrm{AB}} \cos 60^{\circ}+F_{\mathrm{BC}} \cos 60^{\circ}-F_{\mathrm{DB}}=0 \rightarrow \\
& F_{\mathrm{DB}}=\left(F_{\mathrm{AB}}+F_{\mathrm{BC}}\right) \cos 60^{\circ}=F_{\mathrm{AB}}=4.5 \times 10^{4} \mathrm{~N}, \text { in compression }
\end{aligned}
$$



Analyze the forces on the pin at point C . See the fifth free-body diagram. Write equilibrium equations for the horizontal and vertical directions.

$$
\begin{aligned}
& \sum F_{\text {vert }}=F_{\mathrm{BC}} \sin 60^{\circ}+F_{\mathrm{DC}} \sin 60^{\circ}-F_{\mathrm{C} y}=0 \rightarrow \\
& F_{\mathrm{DC}}=\frac{F_{\mathrm{C} y}}{\sin 60^{\circ}}-F_{\mathrm{BC}}=\frac{77480 \mathrm{~N}}{\sin 60^{\circ}}-44730 \mathrm{~N}=44740 \mathrm{~N} \approx 4.5 \times 10^{4} \mathrm{~N}, \text { in tension } \\
& \sum F_{\mathrm{horiz}}=F_{\mathrm{CE}}+F_{\mathrm{DC}} \cos 60^{\circ}-F_{\mathrm{BC}} \cos 60^{\circ}-F_{\mathrm{ACx}}=0 \rightarrow \\
& F_{\mathrm{CE}}=F_{\mathrm{ACx}}+\left(F_{\mathrm{BC}}-F_{\mathrm{DC}}\right) \cos 60^{\circ}=F_{\mathrm{ACx}} \approx 2.2 \times 10^{4} \mathrm{~N}, \text { in tension }
\end{aligned}
$$



Analyze the forces on the pin at point $D$. See the sixth free-body diagram. Write the equilibrium equation for the vertical direction.

$$
\begin{aligned}
& \sum F_{\mathrm{vert}}=F_{\mathrm{DE}} \sin 60^{\circ}-F_{\mathrm{DC}} \sin 60^{\circ}=0 \rightarrow \\
& F_{\mathrm{DE}}=F_{\mathrm{DC}} \approx 4.5 \times 10^{4} \mathrm{~N}, \text { in compression }
\end{aligned}
$$



This could be checked by considering the forces on pin E.
55. We first show a free-body diagram for the entire structure. All acute angles in the structure are $45^{\circ}$. Write the conditions for equilibrium for the entire truss by considering vertical forces and the torques about point A. Let clockwise torques be positive.

$$
\begin{aligned}
& \sum F_{\text {vert }}=F_{1}+F_{2}-5 F=0 \\
& \sum \tau=F a+F(2 a)+F(3 a)+F(4 a)-F_{2}(4 a)=0 \\
& F_{2}=F \frac{10 a}{4 a}=2.5 F ; F_{1}=5 F-F_{2}=2.5 F
\end{aligned}
$$



Note that the forces at the ends each support half of the load. Analyze the forces on the pin at point A. See the second free-body diagram. Write equilibrium equations for the horizontal and vertical directions.

$$
\begin{aligned}
& \sum F_{\text {vert }}=F_{1}-F-F_{\mathrm{AB}} \sin 45^{\circ}=0 \rightarrow \\
& F_{\mathrm{AB}}=\frac{F_{1}-F}{\sin 45^{\circ}}=\frac{\frac{3}{2} F}{\frac{1}{2} \sqrt{2}}=\frac{3 F}{\sqrt{2}}, \text { in compression } \\
& \sum F_{\text {horiz }}=F_{\mathrm{AC}}-F_{\mathrm{AB}} \cos 45^{\circ}=0 \rightarrow F_{\mathrm{AC}}=F_{\mathrm{AB}} \cos 45^{\circ}=\frac{3 F}{\sqrt{2}} \frac{\sqrt{2}}{2}=\frac{3}{2} F, \text { in tension }
\end{aligned}
$$



Analyze the forces on the pin at point C. See the third free-body diagram. Write equilibrium equations for the horizontal and vertical directions.

$$
\begin{aligned}
& \sum F_{\text {vert }}=F_{\mathrm{BC}}-F=0 \rightarrow F_{\mathrm{BC}}=F, \text { tension } \\
& \sum F_{\text {horiz }}=F_{\mathrm{CE}}-F_{\mathrm{AC}}=0 \rightarrow F_{\mathrm{CE}}=F_{\mathrm{AC}}=\frac{3}{2} F, \text { in tension }
\end{aligned}
$$

Analyze the forces on the pin at point B. See the fourth free-body diagram.
 Write equilibrium equations for the horizontal and vertical directions.

$$
\begin{aligned}
& \sum F_{\text {vert }}=F_{\mathrm{AB}} \sin 45^{\circ}-F_{\mathrm{BE}} \sin 45^{\circ}-F_{\mathrm{BC}}=0 \rightarrow \\
& F_{\mathrm{BE}}=F_{\mathrm{AB}}-\frac{F_{\mathrm{BC}}}{\sin 45^{\circ}}=\frac{3 F}{\sqrt{2}}-\frac{F}{\frac{1}{2} \sqrt{2}}=\frac{F}{\sqrt{2}}, \text { tension } \\
& \sum F_{\text {horiz }}=F_{\mathrm{AB}} \cos 45^{\circ}+F_{\mathrm{BE}} \cos 45^{\circ}-F_{\mathrm{DB}}=0 \rightarrow \\
& F_{\mathrm{DB}}=\left(F_{\mathrm{AB}}+F_{\mathrm{BE}}\right) \cos 45^{\circ}=\left(\frac{3 F}{\sqrt{2}}+\frac{F}{\sqrt{2}}\right) \frac{\sqrt{2}}{2}=2 F, \text { in compression }
\end{aligned}
$$



Analyze the forces on the pin at point D. See the fifth free-body diagram. Write equilibrium equations for the vertical direction.

$$
\sum F_{\mathrm{vert}}=-F_{\mathrm{DE}} \rightarrow F_{\mathrm{DE}}=0
$$

All of the other forces can be found from the equilibrium of the structure.


$$
\begin{aligned}
& F_{\mathrm{DG}}=F_{\mathrm{DB}}=2 F, \text { in compression }, F_{\mathrm{GE}}=F_{\mathrm{BE}}=\frac{F}{\sqrt{2}}, \text { tension }, \\
& F_{\mathrm{EH}}=F_{\mathrm{CE}}=\frac{3}{2} F, \text { in tension }, F_{\mathrm{GH}}=F_{\mathrm{BC}}=F, \text { tension }, F_{\mathrm{HJ}}=F_{\mathrm{AC}}=\frac{3}{2} F, \text { in tension }, \\
& F_{\mathrm{GJ}}=F_{\mathrm{AB}}=\frac{3 F}{\sqrt{2}}, \text { in compression }
\end{aligned}
$$

56. Draw free-body diagrams similar to Figures 12-36(a) and 12-36(b) for the forces on the right half of a round arch and a pointed arch. The load force is placed at the same horizontal position on each arch. For each half-arch, take torques about the lower right hand corner, with counterclockwise as positive.
For the round arch:

$$
\sum \tau=F_{\text {Load }}(R-x)-F_{\mathrm{H}} \quad R=0 \rightarrow \underset{\substack{\mathrm{H} \\ \text { round }}}{F_{\mathrm{H}}}=F_{\text {Loadd }} \frac{R-x}{R}
$$



For the pointed arch:

$$
\sum \tau=F_{\text {Load }}(R-x)-\underset{\substack{\mathrm{H} \\ \text { pointed }}}{F_{\text {ded }}} y=0 \rightarrow \underset{\substack{\mathrm{H} \\ \text { pointed }}}{F_{\text {Load }}} \frac{R-x}{y}
$$

Solve for $y$, given that $\underset{\substack{\mathrm{H} \\ \text { pointed }}}{F_{3}}=\frac{1}{3} F_{\mathrm{H}}^{\text {round }}$.

$$
\begin{aligned}
& \underset{\substack{\text { poined }}}{F_{\text {p }}}=\frac{1}{3} F_{\text {round }} \rightarrow F_{\text {Load }} \frac{R-x}{y}=\frac{1}{3} F_{\text {Loodd }} \frac{R-x}{R} \rightarrow \\
& y=3 R=3\left(\frac{1}{2} 8.0 \mathrm{~m}\right)=12 \mathrm{~m}
\end{aligned}
$$


57. Each crossbar in the mobile is in equilibrium, and so the net torque about the suspension point for each crossbar must be 0 . Counterclockwise torques will be taken as positive. The suspension point is used so that the tension in the suspension string need not be known initially. The net vertical force must also be 0 .
The bottom bar:

$$
\begin{aligned}
& \sum \tau=m_{\mathrm{D}} g x_{\mathrm{D}}-m_{\mathrm{C}} g x_{\mathrm{C}}=0 \rightarrow \\
& m_{\mathrm{C}}=m_{\mathrm{D}} \frac{x_{\mathrm{D}}}{x_{\mathrm{C}}}=m_{\mathrm{D}} \frac{17.50 \mathrm{~cm}}{5.00 \mathrm{~cm}}=3.50 m_{\mathrm{D}} \\
& \sum F_{y}=F_{\mathrm{CD}}-m_{\mathrm{C}} g-m_{\mathrm{D}} g=0 \rightarrow F_{\mathrm{CD}}=\left(m_{\mathrm{C}}+m_{\mathrm{D}}\right) g=4.50 m_{\mathrm{D}} g
\end{aligned}
$$



The middle bar:

$$
\begin{aligned}
& \sum \tau=F_{\mathrm{CD}} x_{\mathrm{CD}}-m_{\mathrm{B}} g x_{\mathrm{B}}=0 \rightarrow F_{\mathrm{CD}}=m_{\mathrm{B}} g \frac{x_{\mathrm{B}}}{x_{\mathrm{CD}}} \rightarrow 4.50 m_{\mathrm{D}} g=m_{\mathrm{B}} g \frac{x_{\mathrm{B}}}{x_{\mathrm{CD}}} \\
& m_{\mathrm{D}}=\frac{m_{\mathrm{B}}}{4.50} \frac{x_{\mathrm{B}}}{x_{\mathrm{CD}}}=\frac{(0.748 \mathrm{~kg})(5.00 \mathrm{~cm})}{(4.50)(15.00 \mathrm{~cm})}=0.05541 \approx 5.54 \times 10^{-2} \mathrm{~kg} \\
& m_{\mathrm{C}}=3.50 m_{\mathrm{D}}=(3.50)(0.05541 \mathrm{~kg})=0.194 \mathrm{~kg} \\
& \sum F_{y}=F_{\mathrm{BCD}}-F_{\mathrm{CD}} c-m_{\mathrm{B}} g=0 \rightarrow F_{\mathrm{BCD}}=F_{\mathrm{CD}}+m_{\mathrm{B}} g=\left(4.50 m_{\mathrm{D}}+m_{\mathrm{B}}\right) g
\end{aligned}
$$



The top bar:

$$
\begin{aligned}
\sum \tau & =m_{\mathrm{A}} g x_{\mathrm{A}}-F_{\mathrm{BCD}} x_{\mathrm{BCD}}=0 \rightarrow \\
m_{\mathrm{A}} & =\frac{\left(4.50 m_{\mathrm{D}}+m_{\mathrm{B}}\right) g x_{\mathrm{BCD}}}{g x_{\mathrm{A}}}=\left(4.50 m_{\mathrm{D}}+m_{\mathrm{B}}\right) \frac{x_{\mathrm{BCD}}}{x_{\mathrm{A}}} \\
& =[(4.50)(0.05541 \mathrm{~kg})+0.748 \mathrm{~kg}] \frac{7.50 \mathrm{~cm}}{30.00 \mathrm{~cm}}=0.249 \mathrm{~kg}
\end{aligned}
$$


58. From the free-body diagram (not to scale), write the force equilibrium condition for the vertical direction.

$$
\begin{aligned}
\sum F_{y} & =2 F_{\mathrm{T}} \sin \theta-m g=0 \\
F_{T} & =\frac{m g}{2 \sin \theta} \approx \frac{m g}{2 \tan \theta}=\frac{(60.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{2\left(\frac{2.1 \mathrm{~m}}{18 \mathrm{~m}}\right)} \\
& =2500 \mathrm{~N}
\end{aligned}
$$



Note that the angle is small enough (about $7^{\circ}$ ) that we have made the substitution of $\sin \theta \approx \tan \theta$.
It is not possible to increase the tension so that there is no sag. There must always be a vertical component of the tension to balance the gravity force. The larger the tension gets, the smaller the sag angle will be, however.
59. (a) If the wheel is just lifted off the lowest level, then the only forces on the wheel are the horizontal pull, its weight, and the contact force $\overrightarrow{\mathbf{F}}_{\mathrm{N}}$ at the corner. Take torques about the corner point, for the wheel just barely off the ground, being held in equilibrium. The contact force at the corner exerts no torque and so does not enter the calculation. The pulling force has a lever arm of $R+R-h=2 R-h$, and gravity has a lever arm of $x$, found from the triangle shown.

$$
\begin{aligned}
& x=\sqrt{R^{2}-(R-h)^{2}}=\sqrt{h(2 R-h)} \\
& \sum \tau=M g x-F(2 R-h)=0 \rightarrow \\
& F=\frac{M g x}{2 R-h}=M g \frac{\sqrt{h(2 R-h)}}{2 R-h}=M g \sqrt{\frac{h}{2 R-h}}
\end{aligned}
$$

(b) The only difference is that now the pulling force has a lever arm of $R-h$.

$$
\begin{aligned}
& \sum \tau=M g x-F(R-h)=0 \rightarrow \\
& F=\frac{M g x}{R-h}=M g \frac{\sqrt{h(2 R-h)}}{R-h}
\end{aligned}
$$


60. The mass is to be placed symmetrically between two legs of the table. When enough mass is added, the table will rise up off of the third leg, and then the normal force on the table will all be on just two legs. Since the table legs are equally spaced, the angle marked in the diagram is $30^{\circ}$. Take torques about a line connecting the two legs that remain on the floor, so that the normal forces cause no torque. It is seen from the second diagram (a portion of the first diagram but enlarged) that the two forces are equidistant from the line joining the two legs on the floor. Since the lever arms are equal, then the torques will be equal if the forces are equal. Thus, to be in equilibrium, the two forces must be the same. If the force on the edge of the table is any bigger than the weight of the table, it will tip. Thus $M>28 \mathrm{~kg}$ will cause the table to tip.

61. (a) The weight of the shelf exerts a downward force and a clockwise torque about the point where the shelf touches the wall. Thus there must be an upward force and a counterclockwise torque exerted by the slot for the shelf to be in equilibrium. Since any force exerted by the slot will have a short lever arm relative to the point where the shelf touches the wall, the upward force
 must be larger than the gravity force. Accordingly, there then must be a downward force exerted by the slot at its left edge, exerting no torque, but balancing the vertical forces.
(b) Calculate the values of the three forces by first taking torques about the left end of the shelf, with the net torque being zero, and then sum the vertical forces, with the sum being zero.

$$
\begin{aligned}
& \sum \tau=F_{\text {Right }}\left(2.0 \times 10^{-2} \mathrm{~m}\right)-m g\left(17.0 \times 10^{-2} \mathrm{~m}\right)=0 \rightarrow \\
& F_{\text {Right }}=(6.6 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(\frac{17.0 \times 10^{-2} \mathrm{~m}}{2.0 \times 10^{-2} \mathrm{~m}}\right)=549.8 \mathrm{~N} \approx 550 \mathrm{~N} \\
& \sum F_{y}=F_{\text {Right }}-F_{\text {Left }}-m g \rightarrow \\
& F_{\text {Left }}=F_{\text {Right }}-m g=549.8 \mathrm{~N}-(6.6 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=490 \mathrm{~N} \\
& m g=(6.6 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=65 \mathrm{~N}
\end{aligned}
$$

(c) The torque exerted by the support about the left end of the rod is

$$
\tau=F_{\text {Right }}\left(2.0 \times 10^{-2} \mathrm{~m}\right)=(549.8 \mathrm{~N})\left(2.0 \times 10^{-2} \mathrm{~m}\right)=11 \mathrm{~m} \cdot \mathrm{~N}
$$

62. Assume that the building has just begun to tip, so that it is essentially vertical, but that all of the force on the building due to contact with the Earth is at the lower left corner, as shown in the figure. Take torques about that corner, with counterclockwise torques as positive.


$$
\begin{aligned}
\sum \tau & =F_{\mathrm{A}}(90.0 \mathrm{~m})-m g(23.0 \mathrm{~m}) \\
& =\left[\left(950 \mathrm{~N} / \mathrm{m}^{2}\right)(180.0 \mathrm{~m})(76.0 \mathrm{~m})\right](90.0 \mathrm{~m})-\left(1.8 \times 10^{7} \mathrm{~kg}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(23.0 \mathrm{~m}) \\
& =-2.9 \times 10^{9} \mathrm{~m} \cdot \mathrm{~N}
\end{aligned}
$$

Since this is a negative torque, the building will tend to rotate clockwise, which means it will rotate back down to the ground. Thus the building will not topple.
63. The truck will not tip as long as a vertical line down from the CG is between the wheels. When that vertical line is at the wheel, it is in unstable equilibrium and will tip if the road is inclined any more. See the diagram for the truck at the tipping angle, showing the truck's weight vector.

$$
\tan \theta=\frac{x}{h} \rightarrow \theta=\tan ^{-1} \frac{x}{h}=\tan ^{-1} \frac{1.2 \mathrm{~m}}{2.2 \mathrm{~m}}=29^{\circ}
$$


64. Draw a force diagram for the cable that is supporting the right-hand section. The forces will be the tension at the left end, $\overrightarrow{\mathbf{F}}_{\mathrm{T} 2}$, the tension at the right end, $\overrightarrow{\mathbf{F}}_{\mathrm{T} 1}$, and the weight of the section, $m \overrightarrow{\mathbf{g}}$. The weight acts at the midpoint of the horizontal span of the cable. The system is in equilibrium. Write Newton's second law in both the $x$ and $y$ directions to find the tensions.

$$
\begin{aligned}
& \sum F_{x}=F_{\mathrm{T} 1} \cos 19^{\circ}-F_{\mathrm{T} 2} \sin 60^{\circ}=0 \rightarrow \\
& F_{\mathrm{T} 2}=F_{\mathrm{T} 1} \frac{\cos 19^{\circ}}{\sin 60^{\circ}} \\
& \sum F_{y}=F_{\mathrm{T} 2} \cos 60^{\circ}-F_{\mathrm{T} 1} \sin 19^{\circ}-m g=0 \rightarrow \\
& F_{\mathrm{T} 1}=\frac{F_{\mathrm{T} 2} \cos 60^{\circ}-m g}{\sin 19^{\circ}}=\frac{F_{\mathrm{T} 1} \frac{\cos 19^{\circ}}{\sin 60^{\circ}} \cos 60^{\circ}-m g}{\sin 19^{\circ}} \rightarrow \\
& F_{\mathrm{T} 1}=m g \frac{\sin 60^{\circ}}{\left(\cos 19^{\circ} \cos 60^{\circ}-\sin 19^{\circ} \sin 60^{\circ}\right)}=4.539 m g \approx 4.5 m g \\
& F_{\mathrm{T} 2}=F_{\mathrm{T} 1} \frac{\cos 19^{\circ}}{\sin 60^{\circ}}=4.539 \frac{\cos 19^{\circ}}{\sin 60^{\circ}} m g=4.956 m g \approx 5.0 m g
\end{aligned}
$$



To find the height of the tower, take torques about the point where the roadway meets the ground, at the right side of the roadway. Note that then $\overrightarrow{\mathbf{F}}_{\mathrm{T} 1}$ will exert no torque. Take counterclockwise torques as positive. For purposes of calculating the torque due to $\overrightarrow{\mathbf{F}}_{\mathrm{T} 2}$, split it into $x$ and $y$ components.

$$
\begin{aligned}
& \sum \tau=m g\left(\frac{1}{2} d_{1}\right)+F_{\mathrm{T} 2 x} h-F_{\mathrm{T} 2 y} d_{1}=0 \rightarrow \\
& h
\end{aligned} \begin{aligned}
F_{\mathrm{T} 2 x} & \left(F_{\mathrm{T} 2 y}-\frac{1}{2} m g\right) \\
& d_{1}=\frac{\left(F_{\mathrm{T} 2} \cos 60^{\circ}-\frac{1}{2} m g\right)}{F_{\mathrm{T} 2} \sin 60^{\circ}} d_{1}=\frac{\left(4.956 m g \cos 60^{\circ}-0.50 m g\right)}{4.956 m g \sin 60^{\circ}}(343 \mathrm{~m}) \\
& =158 \mathrm{~m}
\end{aligned}
$$

65. We consider the right half of the bridge in the diagram in the book. We divide it into two segments of length $d_{1}$ and $\frac{1}{2} d_{2}$, and let the mass of those two segments be $M$. Since the roadway is uniform, the mass of each segment will be in proportion to the length of the section, as follows.

$$
\frac{m_{2}}{m_{1}}=\frac{\frac{1}{2} d_{2}}{d_{1}} \rightarrow \frac{d_{2}}{d_{1}}=2 \frac{m_{2}}{m_{1}}
$$

The net horizontal force on the right tower is to be 0 . From the force diagram for the tower, we write this.


$$
F_{\mathrm{T} 3} \sin \theta_{3}=F_{\mathrm{T} 2} \sin \theta_{2}
$$

From the force diagram for each segment of the cable, write Newton's second law for both the vertical and horizontal directions.

## Right segment:

$$
\begin{aligned}
& \sum F_{x}=F_{\mathrm{T} 1} \cos \theta_{1}-F_{\mathrm{T} 2} \sin \theta_{2}=0 \rightarrow \\
& F_{\mathrm{T} 1} \cos \theta_{1}=F_{\mathrm{T} 2} \sin \theta_{2} \\
& \sum F_{y}=F_{\mathrm{T} 2} \cos \theta_{2}-F_{\mathrm{T} 1} \sin \theta_{1}-m_{1} g=0 \rightarrow \\
& \quad m_{1} g=F_{\mathrm{T} 2} \cos \theta_{2}-F_{\mathrm{T} 1} \sin \theta_{1}
\end{aligned}
$$



Left segment:

$$
\begin{aligned}
& \sum F_{x}=F_{\mathrm{T} 3} \sin \theta_{3}-F_{\mathrm{T} 4}=0 \rightarrow F_{\mathrm{T} 3} \sin \theta_{3}=F_{\mathrm{T} 4} \\
& \sum F_{y}=F_{\mathrm{T} 3} \cos \theta_{3}-m_{2} g=0 \rightarrow \\
& \quad m_{2} g=F_{\mathrm{T} 3} \cos \theta_{3}
\end{aligned}
$$

We manipulate the relationships to solve for the ratio of the
 masses, which will give the ratio of the lengths.

$$
\begin{aligned}
& F_{\mathrm{T} 1} \cos \theta_{1}=F_{\mathrm{T} 2} \sin \theta_{2} \rightarrow F_{\mathrm{T} 1}=F_{\mathrm{T} 2} \frac{\sin \theta_{2}}{\cos \theta_{1}} \\
& m_{1} g=F_{\mathrm{T} 2} \cos \theta_{2}-F_{\mathrm{T} 1} \sin \theta_{1}=F_{\mathrm{T} 2} \cos \theta_{2}-F_{\mathrm{T} 2} \frac{\sin \theta_{2}}{\cos \theta_{1}} \sin \theta_{1}=F_{\mathrm{T} 2}\left(\cos \theta_{2}-\frac{\sin \theta_{2}}{\cos \theta_{1}} \sin \theta_{1}\right) \\
& F_{\mathrm{T} 3} \sin \theta_{3}=F_{\mathrm{T} 2} \sin \theta_{2} \rightarrow F_{\mathrm{T} 3}=F_{\mathrm{T} 2} \frac{\sin \theta_{2}}{\sin \theta_{3}} \rightarrow m_{2} g=F_{\mathrm{T} 3} \cos \theta_{3}=F_{\mathrm{T} 2} \frac{\sin \theta_{2}}{\sin \theta_{3}} \cos \theta_{3} \\
& \begin{array}{l}
\frac{d_{2}}{d_{1}}=2 \frac{m_{2}}{m_{1}}=2 \frac{m_{2} g}{m_{1} g}=\frac{2 F_{\mathrm{T} 2} \frac{\sin \theta_{2}}{\sin \theta_{3}} \cos \theta_{3}}{F_{\mathrm{T} 2}\left(\cos \theta_{2}-\frac{\sin \theta_{2}}{\cos \theta_{1}} \sin \theta_{1}\right)}=\frac{2 \sin \theta_{2} \cos \theta_{3} \cos \theta_{1}}{\left(\cos \theta_{2} \cos \theta_{1}-\sin \theta_{2} \sin \theta_{1}\right) \sin \theta_{3}} \\
\quad=\frac{2 \sin \theta_{2} \cos \theta_{1}}{\cos \left(\theta_{1}+\theta_{2}\right) \tan \theta_{3}}=\frac{2 \sin 60^{\circ} \cos 19^{\circ}}{\cos 79^{\circ} \tan 66^{\circ}}=3.821 \approx 3.8
\end{array}
\end{aligned}
$$

66. The radius of the wire can be determined from the relationship between stress and strain, expressed by Eq. 12-5.

$$
\frac{F}{A}=E \frac{\Delta \ell}{\ell_{0}} \rightarrow A=\frac{F \ell_{0}}{E \Delta \ell}=\pi r^{2} \rightarrow r=\sqrt{\frac{1}{\pi} \frac{F}{E} \frac{\ell_{0}}{\Delta \ell}}
$$



Use the free-body diagram for the point of connection of the mass to the wire to determine the tension force in the wire.

$$
\sum F_{y}=2 F_{\mathrm{T}} \sin \theta-m g=0 \rightarrow F_{\mathrm{T}}=\frac{m g}{2 \sin \theta}=\frac{(25 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{2 \sin 12^{\circ}}=589.2 \mathrm{~N}
$$

The fractional change in the length of the wire can be found from the geometry of the problem.

$$
\cos \theta=\frac{\ell_{0} / 2}{\frac{\ell_{0}+\Delta \ell}{2}} \rightarrow \frac{\Delta \ell}{\ell_{0}}=\frac{1}{\cos \theta}-1=\frac{1}{\cos 12^{\circ}}-1=2.234 \times 10^{-2}
$$



Thus the radius is

$$
r=\sqrt{\frac{1}{\pi} \frac{F_{\mathrm{T}}}{E} \frac{\ell_{0}}{\Delta \ell}}=\sqrt{\frac{1}{\pi} \frac{589.2 \mathrm{~N}}{70 \times 10^{9} \mathrm{~N} / \mathrm{m}^{2}} \frac{1}{\left(2.234 \times 10^{-2}\right)}}=3.5 \times 10^{-4} \mathrm{~m}
$$

67. The airplane is in equilibrium, and so the net force in each direction and the net torque are all equal to zero. First write Newton's second law for both the horizontal and vertical directions, to find the values of the forces.

$$
\begin{aligned}
& \sum F_{x}=F_{D}-F_{T}=0 \rightarrow F_{D}=F_{T}=5.0 \times 10^{5} \mathrm{~N} \\
& \sum F_{y}=F_{L}-m g=0 \\
& F_{L}=m g=\left(7.7 \times 10^{4} \mathrm{~kg}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=7.546 \times 10^{5} \mathrm{~N}
\end{aligned}
$$



Calculate the torques about the CM, calling counterclockwise torques positive.

$$
\begin{aligned}
& \sum \tau=F_{L} d-F_{D} h_{1}-F_{T} h_{2}=0 \\
& h_{1}=\frac{F_{L} d-F_{T} h_{2}}{F_{D}}=\frac{\left(7.546 \times 10^{5} \mathrm{~N}\right)(3.2 \mathrm{~m})-\left(5.0 \times 10^{5} \mathrm{~N}\right)(1.6 \mathrm{~m})}{\left(5.0 \times 10^{5} \mathrm{~N}\right)}=3.2 \mathrm{~m}
\end{aligned}
$$

68. Draw a free-body diagram for half of the cable. Write Newton's second law for both the vertical and horizontal directions, with the net force equal to 0 in each direction.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{T} 1} \sin 56^{\circ}-\frac{1}{2} m g=0 \rightarrow F_{\mathrm{T} 1}=\frac{1}{2} \frac{m g}{\sin 56^{\circ}}=0.603 \mathrm{mg} \\
& \sum F_{x}=F_{\mathrm{T} 2}-F_{\mathrm{T} 1} \cos 56^{\circ}=0 \rightarrow \\
& F_{\mathrm{T} 2}=0.603 \mathrm{mg}\left(\cos 56^{\circ}\right)=0.337 \mathrm{mg}
\end{aligned}
$$



So the results are:
(a) $F_{\mathrm{T} 2}=0.34 m g$
(b) $\quad F_{\mathrm{T} 1}=0.60 \mathrm{mg}$
(c) The direction of the tension force is tangent to the cable at all points on the cable. Thus the direction of the tension force is horizontal at the lowest point, and is

$$
56^{\circ} \text { above the horizontal at the attachment point. }
$$

69. (a) For the extreme case of the beam being ready to tip, there would be no normal force at point A from the support. Use the free-body diagram to write the equation of rotational equilibrium under that condition to find the weight of the person, with
 $F_{A}=0$. Take torques about the location of support
B , and call counterclockwise torques positive. $\overrightarrow{\mathbf{W}}$ is the weight of the person.

$$
\begin{aligned}
& \sum \tau=m_{B} g(5.0 \mathrm{~m})-W(5.0 \mathrm{~m})=0 \rightarrow \\
& W=m_{B} g=650 \mathrm{~N}
\end{aligned}
$$

(b) With the person standing at point D , we have already assumed that $F_{A}=0$. The net force in the vertical direction must also be zero.

$$
\sum F_{y}=F_{A}+F_{B}-m_{B} g-W=0 \rightarrow F_{B}=m_{B} g+W=650 \mathrm{~N}+650 \mathrm{~N}=1300 \mathrm{~N}
$$

(c) Now the person moves to a different spot, so the free-body diagram changes as shown. Again use the net torque about support $B$ and then use the net vertical force.

$$
\begin{aligned}
& \sum \tau=m_{B} g(5.0 \mathrm{~m})-W(2.0 \mathrm{~m})-F_{A}(12.0 \mathrm{~m})=0 \\
& F_{A}=\frac{m_{B} g(5.0 \mathrm{~m})-W(2.0 \mathrm{~m})}{12.0 \mathrm{~m}}=\frac{(650 \mathrm{~N})(3.0 \mathrm{~m})}{12.0 \mathrm{~m}} \\
& \quad=162.5 \mathrm{~N} \approx 160 \mathrm{~N} \\
& \sum F_{y}=F_{A}+F_{B}-m_{B} g-W=0 \rightarrow F_{B}=m_{B} g+W-F_{A}=1300 \mathrm{~N}-160 \mathrm{~N}=1140 \mathrm{~N}
\end{aligned}
$$

(d) Again the person moves to a different spot, so the free-body diagram changes again as shown. Again use the net torque about support B and then use the net vertical force.

$$
\begin{aligned}
& \sum \tau=m_{B} g(5.0 \mathrm{~m})+W(10.0 \mathrm{~m})-F_{A}(12.0 \mathrm{~m})=0 \quad \overrightarrow{\mathbf{F}}_{\mathrm{A}} \overrightarrow{\mathbf{W}} m_{\mathrm{B}} \overrightarrow{\mathbf{g}} \\
& F_{A}=\frac{m_{B} g(5.0 \mathrm{~m})+W(10.0 \mathrm{~m})}{12.0 \mathrm{~m}}=\frac{(650 \mathrm{~N})(5.0 \mathrm{~m})+(650 \mathrm{~N})(10.0 \mathrm{~m})}{12.0 \mathrm{~m}}=810 \mathrm{~N} \\
& \sum F_{y}=F_{A}+F_{B}-m_{B} g-W=0 \rightarrow F_{B}=m_{B} g+W-F_{A}=1300 \mathrm{~N}-810 \mathrm{~N}=490 \mathrm{~N}
\end{aligned}
$$

70. If the block is on the verge of tipping, the normal force will be acting at the lower right corner of the block, as shown in the free-body diagram. The block will begin to rotate when the torque caused by the pulling force is larger than the torque caused by gravity. For the block to be able to slide, the pulling force must be as large as the maximum static frictional force. Write the equations of equilibrium for forces in the $x$ and $y$ directions and for torque with the conditions as stated above.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{N}}-m g=0 \rightarrow F_{\mathrm{N}}=m g \\
& \sum F_{x}=F-F_{\mathrm{fr}}=0 \rightarrow F=F_{\mathrm{fr}}=\mu_{\mathrm{s}} F_{\mathrm{N}}=\mu_{\mathrm{s}} m g \\
& \sum \tau=m g \frac{\ell}{2}-F h=0 \rightarrow \frac{m g \ell}{2}=F h=\mu_{\mathrm{s}} m g h
\end{aligned}
$$

Solve for the coefficient of friction in this limiting case, to find $\mu_{\mathrm{s}}=\frac{\ell}{2 h}$.
(a) If $\mu_{\mathrm{s}}<\ell / 2 h$, then sliding will happen before tipping.
(b) If $\mu_{\mathrm{s}}>\ell / 2 h$, then tipping will happen before sliding.
71. The limiting condition for the safety of the painter is the tension in the ropes. The ropes can only exert an upward tension on the scaffold. The tension will be least in the rope that is farther from the painter. The mass of the pail is $m_{\mathrm{p}}$, the mass of the scaffold is $m$, and the mass of the painter is $M$.


Find the distance to the right that the painter can walk before the tension in the left rope becomes zero. Take torques about the point where the right-side rope is attached to the scaffold, so that its value need not be known. Take counterclockwise torques as positive.

$$
\begin{aligned}
& \sum \tau=m g(2.0 \mathrm{~m})+m_{\mathrm{p}} g(3.0 \mathrm{~m})-M g x=0 \rightarrow \\
& x=\frac{m(2.0 \mathrm{~m})+m_{\mathrm{p}}(3.0 \mathrm{~m})}{M}=\frac{(25 \mathrm{~kg})(2.0 \mathrm{~m})+(4.0 \mathrm{~kg})(3.0 \mathrm{~m})}{65.0 \mathrm{~kg}}=0.9538 \mathrm{~m} \approx 0.95 \mathrm{~m}
\end{aligned}
$$

The painter can walk to within 5 cm of the right edge of the scaffold.
Now find the distance to the left that the painter can walk before the tension in the right rope becomes zero. Take torques about the point where the left-side tension is attached to the scaffold, so that its value need not be known. Take counterclockwise torques as positive.

$$
\begin{aligned}
& \sum \tau=M g x-m_{\mathrm{p}} g(1.0 \mathrm{~m})-m g(2.0 \mathrm{~m})=0 \rightarrow \quad M \overrightarrow{\mathbf{g}} \quad m_{\mathrm{p}} \overrightarrow{\mathbf{g}} \quad m \overrightarrow{\mathbf{g}} \\
& x=\frac{m(2.0 \mathrm{~m})+m_{\mathrm{p}}(1.0 \mathrm{~m})}{M}=\frac{(25 \mathrm{~kg})(2.0 \mathrm{~m})+(4.0 \mathrm{~kg})(1.0 \mathrm{~m})}{65.0 \mathrm{~kg}}=0.8308 \mathrm{~m} \approx 0.83 \mathrm{~m}
\end{aligned}
$$



The painter can walk to within 17 cm of the left edge of the scaffold. We found that both ends are dangerous.
72. (a) The man is in equilibrium, so the net force and the net torque on him must be zero. We use half of his weight, and then consider the force just on one hand and one foot, considering him to be symmetric. Take torques about the point where the foot touches the ground, with counterclockwise as positive.

$$
\begin{aligned}
& \sum \tau=\frac{1}{2} m g d_{2}-F_{\mathrm{h}}\left(d_{1}+d_{2}\right)=0 \\
& F_{\mathrm{h}}=\frac{m g d_{2}}{2\left(d_{1}+d_{2}\right)}=\frac{(68 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.95 \mathrm{~m})}{2(1.37 \mathrm{~m})}=231 \mathrm{~N} \approx 230 \mathrm{~N}
\end{aligned}
$$


(b) Use Newton's second law for vertical forces to find the force on the feet.

$$
\begin{aligned}
& \sum F_{y}=2 F_{\mathrm{h}}+2 F_{\mathrm{f}}-m g=0 \\
& F_{\mathrm{f}}=\frac{1}{2} m g-F_{\mathrm{h}}=\frac{1}{2}(68 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)-231 \mathrm{~N}=103 \mathrm{~N} \approx 100 \mathrm{~N}
\end{aligned}
$$

The value of 100 N has 2 significant figures.
73. The force on the sphere from each plane is a normal force, and so is perpendicular to the plane at the point of contact. Use Newton's second law in both the horizontal and vertical directions to determine the magnitudes of the forces.

$$
\begin{aligned}
& \sum F_{x}=F_{\mathrm{L}} \sin \theta_{\mathrm{L}}-F_{\mathrm{R}} \sin \theta_{\mathrm{R}}=0 \rightarrow F_{\mathrm{R}}=F_{\mathrm{L}} \frac{\sin \theta_{\mathrm{L}}}{\sin \theta_{\mathrm{R}}}=F_{\mathrm{L}} \frac{\sin 67^{\circ}}{\sin 32^{\circ}} \\
& \sum F_{y}=F_{\mathrm{L}} \cos \theta_{\mathrm{L}}+F_{\mathrm{R}} \cos \theta_{\mathrm{R}}-m g=0 \rightarrow F_{\mathrm{L}}\left(\cos 67^{\circ}+\frac{\sin 67^{\circ}}{\sin 32^{\circ}} \cos 32^{\circ}\right)=m g \\
& F_{\mathrm{L}}=\frac{m g}{\left(\cos 67^{\circ}+\frac{\sin 67^{\circ}}{\sin 32^{\circ}} \cos 32^{\circ}\right)}=\frac{(23 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{\left(\cos 67^{\circ}+\frac{\sin 67^{\circ}}{\sin 32^{\circ}} \cos 32^{\circ}\right)}=120.9 \mathrm{~N} \approx 120 \mathrm{~N} \\
& F_{\mathrm{R}}=F_{\mathrm{L}} \frac{\sin 67^{\circ}}{\sin 32^{\circ}}=(120.9 \mathrm{~N}) \frac{\sin 67^{\circ}}{\sin 32^{\circ}}=210.0 \mathrm{~N} \approx 210 \mathrm{~N}
\end{aligned}
$$

74. See the free-body diagram. The ball is at rest, and so is in equilibrium. Write Newton's second law for the horizontal and vertical directions, and solve for the forces.

$$
\begin{aligned}
& \sum F_{\text {horiz }}=F_{\mathrm{B}} \sin \theta_{\mathrm{B}}-F_{\mathrm{A}} \sin \theta_{\mathrm{A}}=0 \rightarrow F_{\mathrm{B}}=F_{\mathrm{A}} \frac{\sin \theta_{\mathrm{A}}}{\sin \theta_{\mathrm{B}}} \\
& \sum F_{\text {vert }}=F_{\mathrm{A}} \cos \theta_{\mathrm{A}}-F_{\mathrm{B}} \cos \theta_{\mathrm{B}}-m g=0 \rightarrow F_{\mathrm{A}} \cos \theta_{\mathrm{A}}=F_{\mathrm{B}} \cos \theta_{\mathrm{B}}+m g \rightarrow \theta_{\mathrm{A}}\left(\cos \theta_{\mathrm{A}}-\frac{\sin \theta_{\mathrm{A}}}{\sin \theta_{\mathrm{B}}} \cos \theta_{\mathrm{B}}\right)=m g \rightarrow \\
& F_{\mathrm{A}} \cos \theta_{\mathrm{A}}=F_{\mathrm{A}} \frac{\sin \theta_{\mathrm{A}}}{\sin \theta_{\mathrm{B}}} \cos \theta_{\mathrm{B}}+m g \rightarrow \overrightarrow{\mathbf{g}} \\
& F_{\mathrm{A}}=m g \frac{\sin \theta_{\mathrm{B}}}{\left(\cos \theta_{\mathrm{A}} \sin \theta_{\mathrm{B}}-\sin \theta_{\mathrm{A}} \cos \theta_{\mathrm{B}}\right)}=m g \frac{\sin \theta_{\mathrm{B}}}{\sin \left(\theta_{\mathrm{B}}-\theta_{\mathrm{A}}\right)}=(15.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \frac{\sin 53^{\circ}}{\sin 31^{\circ}} \\
& \quad=228 \mathrm{~N} \approx 230 \mathrm{~N} \\
& F_{\mathrm{B}}=F_{\mathrm{A}} \frac{\sin \theta_{\mathrm{A}}}{\sin \theta_{\mathrm{B}}}=(228 \mathrm{~N}) \frac{\sin 22^{\circ}}{\sin 53^{\circ}}=107 \mathrm{~N} \approx 110 \mathrm{~N}
\end{aligned}
$$

75. Assume a constant acceleration as the person is brought to rest, with up as the positive direction. Use Eq. 2-12c to find the acceleration. From the acceleration, find the average force of the snow on the person, and compare the force per area to the strength of body tissue.

$$
\begin{aligned}
& v^{2}=v_{0}^{2}-2 a\left(x-x_{0}\right) \rightarrow a=\frac{v^{2}-v_{0}^{2}}{2\left(x-x_{0}\right)}=\frac{0-(55 \mathrm{~m} / \mathrm{s})^{2}}{2(-1.0 \mathrm{~m})}=1513 \mathrm{~m} / \mathrm{s}^{2} \\
& \frac{F}{A}=\frac{m a}{A}=\frac{(75 \mathrm{~kg})\left(1513 \mathrm{~m} / \mathrm{s}^{2}\right)}{0.30 \mathrm{~m}^{2}}=3.78 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}<\text { Tissue strength }=5 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}
\end{aligned}
$$

Since the average force on the person is less than the strength of body tissue, the person may escape serious injury. Certain parts of the body, such as the legs if landing feet first, may get more than the average force, though, and so still sustain injury.
76. The mass can be calculated from the equation for the relationship between stress and strain. The force causing the strain is the weight of the mass suspended from the wire. Use Eq. 12-4.

$$
\frac{\Delta \ell}{\ell_{0}}=\frac{1}{E} \frac{F}{A}=\frac{m g}{E A} \rightarrow m=\frac{E A}{g} \frac{\Delta \ell}{\ell_{0}}=\left(200 \times 10^{9} \mathrm{~N} / \mathrm{m}^{2}\right) \frac{\pi\left(1.15 \times 10^{-3} \mathrm{~m}\right)^{2}}{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)} \frac{0.030}{100}=25 \mathrm{~kg}
$$

77. To find the normal force exerted on the road by the trailer tires, take the torques about point B , with counterclockwise torques as positive.

$$
\begin{aligned}
\sum \tau & =m g(5.5 \mathrm{~m})-F_{\mathrm{A}}(8.0 \mathrm{~m})=0 \rightarrow \\
F_{\mathrm{A}} & =m g\left(\frac{5.5 \mathrm{~m}}{8.0 \mathrm{~m}}\right)=(2500 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(\frac{5.5 \mathrm{~m}}{8.0 \mathrm{~m}}\right)=16,844 \mathrm{~N} \\
& \approx 1.7 \times 10^{4} \mathrm{~N}
\end{aligned}
$$



The net force in the vertical direction must be zero.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{B}}+F_{\mathrm{A}}-m g=0 \rightarrow \\
& F_{\mathrm{B}}=m g-F_{\mathrm{A}}=(2500 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)-16,844 \mathrm{~N}=7656 \mathrm{~N} \approx 7.7 \times 10^{3} \mathrm{~N}
\end{aligned}
$$

78. The number of supports can be found from the compressive strength of the wood. Since the wood will be oriented longitudinally, the stress will be parallel to the grain.

$$
\begin{aligned}
& \frac{\text { Compressive Strength }}{\text { Safety Factor }}=\frac{\text { Load force on supports }}{\text { Area of supports }}=\frac{\text { Weight of roof }}{(\# \text { supports })(\text { area per support })} \\
& \begin{aligned}
(\# \text { supports }) & =\frac{\text { Weight of roof }}{(\text { area per support })} \frac{\text { Safety Factor }}{\text { Compressive Strength }} \\
& =\frac{\left(1.36 \times 10^{4} \mathrm{~kg}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{(0.040 \mathrm{~m})(0.090 \mathrm{~m})} \frac{12}{\left(35 \times 10^{6} \mathrm{~N} / \mathrm{m}^{2}\right)}=12.69 \text { supports }
\end{aligned}
\end{aligned}
$$

Since there are to be more than 12 supports, and to have the same number of supports on each side, there will be 14 supports, or 7 supports on each side. That means there will be 6 support-to-support spans, each of which would be given by Spacing $=\frac{10.0 \mathrm{~m}}{6 \text { gaps }}=1.66 \mathrm{~m} / \mathrm{gap}$.
79. The tension in the string when it breaks is found from the ultimate strength of nylon under tension, from Table 12-2.

$$
\begin{aligned}
\frac{F_{\mathrm{T}}}{A} & =\text { Tensile Strength } \rightarrow \\
F_{\mathrm{T}} & =A(\text { Tensile Strength }) \\
& =\pi\left[\frac{1}{2}\left(1.15 \times 10^{-3} \mathrm{~m}\right)\right]^{2}\left(500 \times 10^{6} \mathrm{~N} / \mathrm{m}^{2}\right)=519.3 \mathrm{~N}
\end{aligned}
$$



From the force diagram for the box, we calculate the angle of the rope relative to the horizontal from Newton's second law in the vertical direction. Note that since the tension is the same throughout the string, the angles must be the same so that the object does not accelerate horizontally.

$$
\begin{aligned}
& \sum F_{y}=2 F_{\mathrm{T}} \sin \theta-m g=0 \rightarrow \\
& \theta=\sin ^{-1} \frac{m g}{2 F_{\mathrm{T}}}=\sin ^{-1} \frac{(25 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{2(519.3 \mathrm{~N})}=13.64^{\circ}
\end{aligned}
$$



To find the height above the ground, consider the second diagram.

$$
\tan \theta=\frac{3.00 \mathrm{~m}-h}{2.00 \mathrm{~m}} \rightarrow h=3.00 \mathrm{~m}-2.00 \mathrm{~m}(\tan \theta)=3.00 \mathrm{~m}-2.00 \mathrm{~m}\left(\tan 13.64^{\circ}\right)=2.5 \mathrm{~m}
$$

80. See the free-body diagram. Assume that the ladder is just ready to slip, so the force of static friction is $F_{\mathrm{fr}}=\mu F_{\mathrm{N}}$. The ladder is of length $\ell$, and so $d_{1}=\frac{1}{2} \ell \sin \theta, d_{2}=\frac{3}{4} \ell \sin \theta$, and $d_{3}=\ell \cos \theta$. The ladder is in equilibrium, so the net vertical and horizontal forces are 0 , and the net torque is 0 . We express those three equilibrium conditions, along with the friction condition. Take torques about the point where the ladder rests on the ground, calling clockwise torques positive.

$$
\begin{aligned}
& \sum F_{\mathrm{vert}}=F_{\mathrm{G} y}-m g-M g=0 \rightarrow F_{\mathrm{G} y}=(m+M) g \\
& \sum F_{\mathrm{horiz}}=F_{\mathrm{Gx}}-F_{\mathrm{W}}=0 \rightarrow F_{\mathrm{Gx}}=F_{\mathrm{W}} \\
& \sum \tau=m g d_{1}+M g d_{2}-F_{\mathrm{w}} d_{3}=0 \rightarrow F_{\mathrm{W}}=\frac{m g d_{1}+M g d_{2}}{d_{3}} \\
& F_{\mathrm{fr}}=\mu F_{\mathrm{N}} \rightarrow F_{\mathrm{G} x}=\mu F_{\mathrm{G} y}
\end{aligned}
$$



These four equations may be solved for the coefficient of friction.

$$
\begin{aligned}
\mu & =\frac{F_{\mathrm{G} x}}{F_{\mathrm{G} y}}=\frac{F_{\mathrm{W}}}{(m+M) g}=\frac{\frac{m g d_{1}+M g d_{2}}{d_{3}}}{(m+M) g}=\frac{m d_{1}+M d_{2}}{d_{3}(m+M)}=\frac{m\left(\frac{1}{2} \ell \sin \theta\right)+M\left(\frac{3}{4} \ell \sin \theta\right)}{(\ell \cos \theta)(m+M)} \\
& =\frac{\left(\frac{1}{2} m+\frac{3}{4} M\right) \tan \theta}{(m+M)}=\frac{\left[\frac{1}{2}(16.0 \mathrm{~kg})+\frac{3}{4}(76.0 \mathrm{~kg})\right] \tan 20.0^{\circ}}{(92.0 \mathrm{~kg})}=0.257
\end{aligned}
$$

81. The maximum compressive force in a column will occur at the bottom. The bottom layer supports the entire weight of the column, and so the compressive force on that layer is $m g$. For the column to be on the verge of buckling, the weight divided by the area of the column will be the compressive strength of the material. The mass of the column is its volume (area $x$ height) times its density.

$$
\frac{m g}{A}=\text { Compressive Strength }=\frac{h A \rho g}{A} \rightarrow h=\frac{\text { Compressive Strength }}{\rho g}
$$

Note that the area of the column cancels out of the expression, and so the height does not depend on the cross-sectional area of the column.
(a)

$$
\begin{aligned}
& h_{\text {steel }}=\frac{\text { Compressive Strength }}{\rho g}=\frac{500 \times 10^{6} \mathrm{~N} / \mathrm{m}^{2}}{\left(7.8 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=6500 \mathrm{~m} \\
& h_{\text {granite }}=\frac{\text { Compressive Strength }}{\rho g}=\frac{170 \times 10^{6} \mathrm{~N} / \mathrm{m}^{2}}{\left(2.7 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=6400 \mathrm{~m}
\end{aligned}
$$

(b)
82. See the free-body diagram. Let $M$ represent the mass of the train, and $m$ represent the mass of the bridge. Write the equilibrium conditions for torques, taken about the left end, and for vertical forces. These two equations can be solved for the forces. Take counterclockwise torques as positive. Note that the position of the train is given by
 $x=v t$.

$$
\begin{aligned}
& \sum \tau=M g x+m g\left(\frac{1}{2} \ell\right)-F_{\mathrm{B}} \ell=0 \rightarrow \\
& F_{\mathrm{B}}=\left(M g \frac{x}{\ell}+\frac{1}{2} m g\right)=\left(\frac{M g v}{\ell} t+\frac{1}{2} m g\right) \\
& \quad=\left(\frac{(95000 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left((80.0 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h})}\right)\right.}{} \mathrm{\sum} t+\frac{1}{2}(23000 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\right] \\
& \\
& =\left(7.388 \times 10^{4} \mathrm{~N} / \mathrm{s}\right) t+1.127 \times 10^{5} \mathrm{~N} \approx\left(7.4 \times 10^{4} \mathrm{~N} / \mathrm{s}\right) t+1.1 \times 10^{5} \mathrm{~N} \\
& \sum F_{\text {vert }}=F_{\mathrm{A}}+F_{\mathrm{B}}-M g-m g=0 \rightarrow \\
& F_{\mathrm{A}}=(M+m) g-F_{\mathrm{B}}=\left(1.18 \times 10^{5} \mathrm{~kg}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)-\left[\left(7.388 \times 10^{4} \mathrm{~N} / \mathrm{s}\right) t+1.127 \times 10^{5} \mathrm{~N}\right] \\
& \quad=-\left(7.388 \times 10^{4} \mathrm{~N} / \mathrm{s}\right) t+1.044 \times 10^{6} \mathrm{~N} \approx-\left(7.4 \times 10^{4} \mathrm{~N} / \mathrm{s}\right) t+1.0 \times 10^{6} \mathrm{~N}
\end{aligned}
$$

83. Since the backpack is midway between the two trees, the angles in the free-body diagram are equal. Write Newton's second law for the vertical direction for the point at which the backpack is attached to the cord, with the weight of the backpack being the original downward vertical force.

$$
\sum F_{y}=2 F_{\mathrm{T} 0} \sin \theta_{0}-m g=0 \rightarrow F_{\mathrm{T} 0}=\frac{m g}{2 \sin \theta_{0}}
$$



Now assume the bear pulls down with an additional force, $F_{\text {bear }}$. The force equation would be modified as follows.

$$
\begin{aligned}
& \sum F_{y}=2 F_{\mathrm{T} \text { final }} \sin \theta_{\text {final }}-m g-F_{\text {bear }}=0 \rightarrow \\
& \begin{aligned}
F_{\text {bear }} & =2 F_{\mathrm{T} \text { final }} \sin \theta_{\text {final }}-m g=2\left(2 F_{\mathrm{T} 0}\right) \sin \theta_{\text {final }}-m g=4\left(\frac{m g}{2 \sin \theta_{0}}\right) \sin \theta_{\text {final }}-m g \\
& =m g\left(\frac{2 \sin \theta_{\text {final }}}{\sin \theta_{0}}-1\right)=(23.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(\frac{2 \sin 27^{\circ}}{\sin 15^{\circ}}-1\right)=565.3 \mathrm{~N} \approx 570 \mathrm{~N}
\end{aligned}
\end{aligned}
$$

84. (a) See the free-body diagram. To find the tension in the wire, take torques about the left edge of the beam, with counterclockwise as positive. The net torque must be 0 for the beam to be in equilibrium.

$$
\begin{aligned}
& \sum \tau=m g x+M g\left(\frac{1}{2} \ell\right)-F_{\mathrm{T}} \sin \theta \ell=0 \rightarrow \\
& F_{\mathrm{T}}=\frac{g(2 m x+M \ell)}{2 \ell \sin \theta}=\frac{m g}{\ell \sin \theta} x+\frac{M g}{2 \sin \theta}
\end{aligned}
$$



We see that the tension force is linear in $x$.
(b) Write the equilibrium condition for vertical and horizontal forces.

$$
\begin{aligned}
& \sum F_{x}=F_{\substack{\text { hinge } \\
\text { hoiz }}}-F_{\mathrm{T}} \cos \theta=0 \rightarrow F_{\substack{\text { hinge } \\
\text { horiz }}}=F_{\mathrm{T}} \cos \theta=\frac{g(2 m x+M \ell)}{2 \ell \sin \theta} \cos \theta=\frac{g(2 m x+M \ell)}{2 \ell \tan \theta} \\
& \sum F_{y}=F_{\substack{\text { hinge } \\
\text { vert }}}+F_{\mathrm{T}} \sin \theta-(m+M) g=0 \rightarrow \\
& F_{\text {hinge }}=(m+M) g-F_{\mathrm{T}} \sin \theta=(m+M) g-\frac{g(2 m x+M \ell)}{2 \ell \sin \theta} \sin \theta=m g\left(1-\frac{x}{\ell}\right)+\frac{1}{2} M g
\end{aligned}
$$

85. Draw a free-body diagram for one of the beams. By Newton's third law, if the right beam pushes down on the left beam, then the left beam pushes up on the right beam. But the geometry is symmetric for the two beams, and so the beam contact force must be horizontal. For the beam to be in equilibrium, $F_{\mathrm{N}}=m g$ and so $F_{\mathrm{fr}}=\mu_{\mathrm{s}} F_{\mathrm{N}}=\mu m g$ is the maximum friction force. Take torques about the top of the beam, so that $\overrightarrow{\mathbf{F}}_{\text {beam }}$ exerts no torque. Let clockwise torques be positive.

$$
\begin{aligned}
& \sum \tau=F_{\mathrm{N}} \ell \cos \theta-m g\left(\frac{1}{2} \ell\right) \cos \theta-F_{\mathrm{fr}} \ell \sin \theta=0 \rightarrow \\
& \theta=\tan ^{-1} \frac{1}{2 \mu_{\mathrm{s}}}=\tan ^{-1} \frac{1}{2(0.5)}=45^{\circ}
\end{aligned}
$$


86. Take torques about the elbow joint. Let clockwise torques be positive. Since the arm is in equilibrium, the total torque will be 0 .

$$
\begin{aligned}
& \sum \tau=(2.0 \mathrm{~kg}) g(0.15 \mathrm{~m})+(35 \mathrm{~kg}) g(0.35 \mathrm{~m})-F_{\max }(0.050 \mathrm{~m}) \sin 105^{\circ}=0 \rightarrow \\
& F_{\max }=\frac{(2.0 \mathrm{~kg}) g(0.15 \mathrm{~m})+(35 \mathrm{~kg}) g(0.35 \mathrm{~m})}{(0.050 \mathrm{~m}) \sin 105^{\circ}}=2547 \mathrm{~N} \approx 2500 \mathrm{~N}
\end{aligned}
$$

87. (a) Use the free-body diagram in the textbook. To find the magnitude of $\overrightarrow{\mathbf{F}}_{\mathrm{M}}$, take torques about an axis through point $S$ and perpendicular to the paper. The upper body is in equilibrium, so the net torque must be 0 . Take clockwise torques as positive.

$$
\begin{aligned}
\sum \tau & =\left[w_{\mathrm{T}}(0.36 \mathrm{~m})+w_{\mathrm{A}}(0.48 \mathrm{~m})+w_{\mathrm{H}}(0.72 \mathrm{~m})\right] \cos 30^{\circ}-F_{\mathrm{M}}(0.48 \mathrm{~m}) \sin 12^{\circ}=0 \rightarrow \\
F_{\mathrm{M}} & =\frac{\left[w_{\mathrm{T}}(0.36 \mathrm{~m})+w_{\mathrm{A}}(0.48 \mathrm{~m})+w_{\mathrm{H}}(0.72 \mathrm{~m})\right] \cos 30^{\circ}}{(0.48 \mathrm{~m}) \sin 12^{\circ}} \\
& =\frac{w[(0.46)(0.36 \mathrm{~m})+(0.12)(0.48 \mathrm{~m})+(0.07)(0.72 \mathrm{~m})] \cos 30^{\circ}}{(0.48 \mathrm{~m}) \sin 12^{\circ}}=2.374 w \approx 2.4 w
\end{aligned}
$$

(b) Write equilibrium conditions for the horizontal and vertical forces. Use those conditions to solve for the components of $\overrightarrow{\mathbf{F}}_{\mathrm{v}}$, and then find the magnitude and direction. Note the free-body diagram for determining the components of $\overrightarrow{\mathbf{F}}_{\mathrm{M}}$. The two dashed lines are parallel, and so both make an angle of $\theta$ with the heavy line representing the back.

$$
\begin{aligned}
& \sum F_{\mathrm{horiz}}=F_{\mathrm{V} \text { horiz }}-F_{\mathrm{M}} \cos \left(30^{\circ}-12^{\circ}\right)=0 \rightarrow \\
& F_{\mathrm{V} \text { horiz }}=F_{\mathrm{M}} \cos 18^{\circ}=(2.374 w) \cos 18^{\circ}=2.258 w \\
& \sum F_{\text {vert }}=F_{\mathrm{V} \text { vert }}-F_{\mathrm{M}} \sin \left(30^{\circ}-12^{\circ}\right)-w_{\mathrm{T}}-w_{\mathrm{A}}-w_{\mathrm{H}}=0 \rightarrow \\
& F_{\mathrm{V} \text { vert }}=F_{\mathrm{M}} \sin 18^{\circ}+w_{\mathrm{T}}+w_{\mathrm{A}}+w_{\mathrm{H}}=(2.374 w) \sin 18^{\circ}+0.65 w=1.384 w \\
& F_{\mathrm{V}}=\sqrt{F_{\mathrm{V} \text { horiz }}^{2}+F_{\mathrm{V} \text { vert }}^{2}}=\sqrt{(2.258 w)^{2}+(1.384 w)^{2}}=2.648 w \approx 2.6 w \\
& \theta_{\mathrm{V}}=\tan ^{-1} \frac{F_{\mathrm{V} \text { vert }}}{F_{\mathrm{V} \text { horiz }}}=\tan ^{-1} \frac{1.384 w}{2.258 w}=31.51^{\circ} \approx 32^{\circ} \text { above the horizontal }
\end{aligned}
$$

88. We are given that rod AB is under a compressive force $F$. Analyze the forces on the pin at point A. See the first free-body diagram. Write equilibrium equations for the horizontal and vertical directions.

$$
\begin{aligned}
& \sum F_{\text {horiz }}=F_{\mathrm{AD}} \cos 45^{\circ}-F_{\mathrm{AB}}=0 \rightarrow F_{\mathrm{AD}}=\frac{F_{\mathrm{AB}}}{\cos 45^{\circ}}=\sqrt{2} F, \text { in tension } \\
& \sum F_{\text {vert }}=F_{\mathrm{AC}}-F_{\mathrm{AD}} \sin 45^{\circ}=0 \rightarrow \\
& F_{\mathrm{AC}}=F_{\mathrm{AD}} \sin 45^{\circ}=\sqrt{2} F \frac{\sqrt{2}}{r}=F, \text { in compression }
\end{aligned}
$$



By symmetry, the other outer forces must all be the same magnitude as $F_{\mathrm{AB}}$, and the other diagonal force must be the same magnitude as $F_{\mathrm{AB}}$.

$$
F_{\mathrm{AC}}=F_{\mathrm{AB}}=F_{\mathrm{BD}}=F_{\mathrm{CD}}=F, \text { in compression } ; F_{\mathrm{AD}}=F_{\mathrm{BC}}=\sqrt{2} F, \text { in tension }
$$

89. (a) The fractional decrease in the rod's length is the strain Use Eq. 12-5. The force applied is the weight of the man.

$$
\frac{\Delta \ell}{\ell_{0}}=\frac{F}{A E}=\frac{m g}{\pi r^{2} E}=\frac{(65 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{\pi(0.15)^{2}\left(200 \times 10^{9} \mathrm{~N} / \mathrm{m}^{2}\right)}=4.506 \times 10^{-8}=\left(4.5 \times 10^{-6}\right) \%
$$

(b) The fractional change is the same for the atoms as for the macroscopic material. Let $d$ represent the interatomic spacing.

$$
\begin{aligned}
& \frac{\Delta d}{d_{0}}=\frac{\Delta \ell}{\ell_{0}}=4.506 \times 10^{-8} \rightarrow \\
& \Delta d=\left(4.506 \times 10^{-8}\right) d_{0}=\left(4.506 \times 10^{-8}\right)\left(2.0 \times 10^{-10} \mathrm{~m}\right)=9.0 \times 10^{-18} \mathrm{~m}
\end{aligned}
$$

90. (a) See the free-body diagram for the system, showing forces on the engine and the forces at the point on the rope where the mechanic is pulling (the point of analysis). Let $m$ represent the mass of the engine. The fact that the engine was raised a half-meter means that the part of the rope from the tree branch to the mechanic is 3.25 m , as well as the part from the mechanic to the bumper. From the free-body diagram for the engine, we know that the tension in the rope is equal to the weight of the engine. Use this, along with the equations of equilibrium at the point where the mechanic is pulling, to find the pulling force by the mechanic.


Angle: $\theta=\cos ^{-1} \frac{3.0 \mathrm{~m}}{3.25 \mathrm{~m}}=22.62^{\circ}$
Engine: $\sum F_{y}=F_{\mathrm{T}}-m g=0 \rightarrow F_{\mathrm{T}}=m g$
Point: $\quad \sum F_{x}=F-2 F_{\mathrm{T}} \sin \theta=0 \rightarrow$

$$
F=2 m g \sin \theta=2(280 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \sin 22.62^{\circ}=2111 \mathrm{~N} \approx 2100 \mathrm{~N}
$$

(b) Mechanical advantage $=\frac{\text { Load force }}{\text { Applied force }}=\frac{m g}{F}=\frac{(280 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{2111 \mathrm{~N}}=1.3 \mathrm{~N}$
91. Consider the free-body diagram for the box. The box is assumed to be in equilibrium, but just on the verge of both sliding and tipping. Since it is on the verge of sliding, the static frictional force is at its maximum value. Use the equations of equilibrium. Take torques about the lower right corner where the box touches the floor, and take clockwise torques as positive. We also assume that the box is just barely tipped up on its corner, so that the forces are still parallel and perpendicular to the edges of the box.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{N}}-W=0 \rightarrow F_{\mathrm{N}}=W \\
& \sum F_{x}=F-F_{\mathrm{fr}}=0 \rightarrow F=F_{\mathrm{fr}}=\mu W=(0.60)(250 \mathrm{~N})=150 \mathrm{~N} \\
& \sum \tau=F h-W(0.5 \mathrm{~m})=0 \rightarrow h=(0.5 \mathrm{~m}) \frac{W}{F}=(0.5 \mathrm{~m}) \frac{250 \mathrm{~N}}{150 \mathrm{~N}}=0.83 \mathrm{~m}
\end{aligned}
$$

92. See the free-body diagram. Take torques about the pivot point, with clockwise torques as positive. The plank is in equilibrium. Let $m$ represent the mass of the plank, and $M$ represent the mass of the person. The minimum nail force would occur if there was no normal force pushing up on the left end of the board.

$$
\begin{aligned}
\sum \tau= & m g(0.75 \mathrm{~m}) \cos \theta+M g(2.25 \mathrm{~m}) \cos \theta \\
& \quad-F_{\text {nails }}(0.75 \mathrm{~m}) \cos \theta=0 \rightarrow \\
F_{\text {nails }}= & \frac{m g(0.75 \mathrm{~m})+M g(2.25 \mathrm{~m})}{(0.75 \mathrm{~m})}=m g+3 M g \\
= & (45 \mathrm{~kg}+3(65 \mathrm{~kg}))\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=2352 \mathrm{~N} \approx 2400 \mathrm{~N}
\end{aligned}
$$

93. (a) Note that since the friction is static friction, we may NOT use $F_{\mathrm{fr}}=\mu F_{\mathrm{N}}$. It could be that $F_{\mathrm{fr}}<\mu F_{\mathrm{N}}$. So, we must determine $F_{\mathrm{fr}}$ by the equilibrium equations. Take an axis of rotation to be out of the paper, through the point of contact of the rope with the wall. Then neither $F_{\mathrm{T}}$ nor $F_{\mathrm{fr}}$ can cause any torque. The torque equilibrium equation is as follows.

$$
F_{\mathrm{N}} h=m g r_{0} \rightarrow F_{\mathrm{N}}=\frac{m g r_{0}}{h}
$$

Take the sum of the forces in the horizontal direction.


$$
F_{\mathrm{N}}=F_{\mathrm{T}} \sin \theta \rightarrow F_{\mathrm{T}}=\frac{F_{\mathrm{N}}}{\sin \theta}=\frac{m g r_{0}}{h \sin \theta}
$$

Take the sum of the forces in the vertical direction.

$$
\begin{aligned}
& F_{\mathrm{T}} \cos \theta+F_{\mathrm{fr}}=m g \rightarrow \\
& F_{\mathrm{fr}}=m g-F_{\mathrm{T}} \cos \theta=m g-\frac{m g r_{0} \cos \theta}{h \sin \theta}=m g\left(1-\frac{r_{0}}{h} \cot \theta\right)
\end{aligned}
$$

(b) Since the sphere is on the verge of slipping, we know that $F_{\mathrm{fr}}=\mu F_{\mathrm{N}}$.

$$
F_{\mathrm{fr}}=\mu F_{\mathrm{N}} \rightarrow m g\left(1-\frac{r_{0}}{h} \cot \theta\right)=\mu \frac{m g r_{0}}{h} \rightarrow\left(\frac{h}{r_{0}}-\cot \theta\right)=\mu=\frac{h}{r_{0}}-\cot \theta
$$

94. There are upward forces at each support (points A and D) and a downward applied force at point C . To find the angles of members AB and BD , see the free-body diagram for the whole truss.

$$
\theta_{\mathrm{A}}=\tan ^{-1} \frac{6.0}{4.0}=56.3^{\circ} ; \theta_{B}=\tan ^{-1} \frac{6.0}{6.0}=45^{\circ}
$$

Write the conditions for equilibrium for the entire truss by considering vertical forces and the torques about point $A$. Let clockwise torques be positive.

$$
\begin{aligned}
& \sum F_{\text {vert }}=F_{\mathrm{A}}+F_{\mathrm{D}}-F=0 \\
& \sum \tau=F(4.0 \mathrm{~m})-F_{\mathrm{D}}(10.0 \mathrm{~m})=0 \rightarrow F_{\mathrm{D}}=F\left(\frac{4.0}{10.0}\right)=(12,000 \mathrm{~N})\left(\frac{4.0}{10.0}\right)=4800 \mathrm{~N} \\
& F_{\mathrm{A}}=F-F_{\mathrm{D}}=12,000 \mathrm{~N}-4800 \mathrm{~N}=7200 \mathrm{~N}
\end{aligned}
$$

Analyze the forces on the pin at point A. See the second free-body diagram.
Write equilibrium equations for the horizontal and vertical directions.

$$
\begin{aligned}
& \sum F_{\text {vert }}=F_{\mathrm{A}}-F_{\mathrm{AB}} \sin \theta_{\mathrm{A}}=0 \rightarrow \\
& F_{\mathrm{AB}}=\frac{F_{\mathrm{A}}}{\sin \theta_{\mathrm{A}}}=\frac{7200 \mathrm{~N}}{\sin 56.3^{\circ}}=8654 \mathrm{~N} \approx 8700 \mathrm{~N}, \text { compression } \\
& \sum F_{\text {horiz }}=F_{\mathrm{AC}}-F_{\mathrm{AB}} \cos \theta_{\mathrm{A}}=0 \rightarrow \\
& F_{\mathrm{AC}}=F_{\mathrm{AB}} \cos \theta_{\mathrm{A}}=(8654 \mathrm{~N}) \cos 56.3^{\circ}=4802 \mathrm{~N} \approx 4800 \mathrm{~N}, \text { tension }
\end{aligned}
$$

Analyze the forces on the pin at point C. See the third free-body diagram. Write equilibrium equations for the horizontal and vertical directions.

$$
\begin{aligned}
& \sum F_{\text {vert }}=F_{\mathrm{BC}}-F=0 \rightarrow F_{\mathrm{BC}}=F=12,000 \mathrm{~N}, \text { tension } \\
& \sum F_{\text {horiz }}=F_{\mathrm{CD}}-F_{\mathrm{AC}}=0 \rightarrow F_{\mathrm{CD}}=F_{\mathrm{AC}}=4800 \mathrm{~N}, \text { tension }
\end{aligned}
$$

Analyze the forces on the pin at point D . See the fourth free-body diagram.
 Write the equilibrium equation for the horizontal direction.

$$
\begin{aligned}
& \sum F_{\text {ver }}=F_{\mathrm{BD}} \cos \theta_{\mathrm{D}}-F_{\mathrm{CD}}=0 \rightarrow \\
& F_{\mathrm{BD}}=\frac{F_{\mathrm{CD}}}{\cos \theta_{\mathrm{D}}}=\frac{4800 \mathrm{~N}}{\cos 45^{\circ}}=6788 \mathrm{~N} \approx 6800 \mathrm{~N}, \text { compression }
\end{aligned}
$$


95. (a) See the free-body diagram. We write the equilibrium conditions for horizontal and vertical forces, and for rotation. We also assume that both static frictional forces are at their maximum values. Take clockwise torques as positive. We solve for the smallest angle that makes the ladder be in equilibrium.

$$
\begin{aligned}
& \sum F_{\mathrm{horiz}}=F_{\mathrm{G} x}-F_{\mathrm{W} x}=0 \rightarrow F_{\mathrm{G} x}=F_{\mathrm{W} x} \\
& \sum F_{\mathrm{vert}}=F_{\mathrm{G} y}+F_{\mathrm{W} y}-m g=0 \rightarrow F_{\mathrm{G} y}+F_{\mathrm{W} y}=m g \\
& \sum \tau=m g\left(\frac{1}{2} \ell \cos \theta\right)-F_{\mathrm{W} x} \ell \sin \theta-F_{\mathrm{W} y} \ell \cos \theta=0 \\
& F_{\mathrm{G} x}=\mu_{\mathrm{G}} F_{\mathrm{G} y} ; F_{\mathrm{W} y}=\mu_{\mathrm{W}} F_{\mathrm{W} x}
\end{aligned}
$$

Substitute the first equation above into the fourth equation, and simplify
 the third equation, to give this set of equations.

$$
F_{\mathrm{G} y}+F_{\mathrm{W} y}=m g ; m g=2\left(F_{\mathrm{W} x} \tan \theta+F_{\mathrm{W} y}\right) ; F_{\mathrm{W} x}=\mu_{\mathrm{G}} F_{\mathrm{G} y} ; F_{\mathrm{w} y}=\mu_{\mathrm{W}} F_{\mathrm{W} x}
$$

Substitute the third equation into the second and fourth equations.

$$
F_{\mathrm{G} y}+F_{\mathrm{W} y}=m g ; m g=2\left(\mu_{\mathrm{G}} F_{\mathrm{G} y} \tan \theta+F_{\mathrm{W} y}\right) ; F_{\mathrm{W} y}=\mu_{\mathrm{W}} \mu_{\mathrm{G}} F_{\mathrm{G} y}
$$

Substitute the third equation into the first two equations.

$$
F_{\mathrm{G} y}+\mu_{\mathrm{w}} \mu_{\mathrm{G}} F_{\mathrm{G} y}=m g ; m g=2\left(\mu_{\mathrm{G}} F_{\mathrm{G} y} \tan \theta+\mu_{\mathrm{W}} \mu_{\mathrm{G}} F_{\mathrm{G} y}\right)
$$

Now equate the two expressions for $m g$, and simplify.

$$
F_{\mathrm{G} y}+\mu_{\mathrm{W}} \mu_{\mathrm{G}} F_{\mathrm{G} y}=2\left(\mu_{\mathrm{G}} F_{\mathrm{G} y} \tan \theta+\mu_{\mathrm{w}} \mu_{\mathrm{G}} F_{\mathrm{G} y}\right) \rightarrow \tan \theta_{\min }=\frac{1-\mu_{\mathrm{W}} \mu_{\mathrm{G}}}{2 \mu_{\mathrm{G}}}
$$

(b) For a frictional wall: $\theta_{\min }=\tan ^{-1} \frac{1-\mu_{\mathrm{w}} \mu_{\mathrm{G}}}{2 \mu_{\mathrm{G}}}=\tan ^{-1} \frac{1-(0.40)^{2}}{2(0.40)}=46.4^{\circ} \approx 46^{\circ}$

For a frictionless wall: $\theta_{\min }=\tan ^{-1} \frac{1-\mu_{\mathrm{w}} \mu_{\mathrm{G}}}{2 \mu_{\mathrm{G}}}=\tan ^{-1} \frac{1-(0)^{2}}{2(0.40)}=51.3^{\circ} \approx 51^{\circ}$
$\%$ diff $=\left(\frac{51.3^{\circ}-46.4^{\circ}}{46.4^{\circ}}\right) 100=10.6 \% \approx 11 \%$
96. (a) See the free-body diagram for the Tyrolean traverse technique. We analyze the point on the rope that is at the bottom of the "sag." To include the safety factor, the tension must be no more than 2900 N .

$$
\begin{aligned}
& \sum F_{\mathrm{vert}}=2 F_{\mathrm{T}} \sin \theta-m g=0 \rightarrow \\
& \theta_{\min }=\sin ^{-1} \frac{m g}{2 F_{\mathrm{T}}}=\sin ^{-1} \frac{(75 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{2(2900 \mathrm{~N})}=7.280^{\circ} \\
& \tan \theta_{\min }=\frac{x_{\min }}{12.5 \mathrm{~m}} \rightarrow x_{\min }=(12.5 \mathrm{~m}) \tan \left(7.280^{\circ}\right)=1.597 \mathrm{~m} \approx 1.6 \mathrm{~m}
\end{aligned}
$$

(b) Now the sag amount is $x=\frac{1}{4} x_{\min }=\frac{1}{4}(1.597 \mathrm{~m})=0.3992 \mathrm{~m}$. Use that distance to find the tension in the rope.

$$
\begin{aligned}
& \theta=\tan ^{-1} \frac{x}{12.5 \mathrm{~m}}=\tan ^{-1} \frac{0.3992 \mathrm{~m}}{12.5 \mathrm{~m}}=1.829^{\circ} \\
& F_{\mathrm{T}}=\frac{m g}{2 \sin \theta}=\frac{(75 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{2 \sin 1.829^{\circ}}=11,512 \mathrm{~N} \approx 12,000 \mathrm{~N}
\end{aligned}
$$

The rope will not break, but the safety factor will only be about 4 instead of 10 .
97. (a) The stress is given by $\frac{F}{A}$, the applied force divided by the cross-sectional area, and the strain is given by $\frac{\Delta \ell}{\ell_{0}}$, the elongation over the original length.


The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH12.XLS," on tab "Problem 12.97a."
(b) The elastic region is shown in the graph.

The slope of the stress vs. strain graph is the elastic modulus, and is

$$
2.02 \times 10^{11} \mathrm{~N} / \mathrm{m}^{2} \text {. }
$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH12.XLS," on tab "Problem 12.97b."
98. See the free-body diagram. We assume that point C is not accelerating, and so the net force at point C is 0 . That net force is the vector sum of applied force $\overrightarrow{\mathbf{F}}$ and two identical spring forces $\overrightarrow{\mathbf{F}}_{\text {elas }}$. The elastic forces are given by $F_{\text {elas }}=k$ (amount of stretch). If the springs are unstretched for $\theta=0$, then 2.0 m must be subtracted from the length of AC and BC to find the amount the
 springs have been stretched. Write Newton's second law for the vertical direction in order to obtain a relationship between $F$ and $\theta$. Note that $\cos \theta=\frac{2.0 \mathrm{~m}}{\ell}$.

$$
\begin{aligned}
& \sum F_{\text {vert }}=2 F_{\text {elas }} \sin \theta-F=0 \rightarrow F=2 F_{\text {elas }} \sin \theta \\
& F_{\text {clas }}=k(\ell-2.0 \mathrm{~m})=k\left(\frac{2.0 \mathrm{~m}}{\cos \theta}-2.0 \mathrm{~m}\right) \rightarrow \\
& F=2 F_{\text {elas }} \sin \theta=2 k\left(\frac{2.0 \mathrm{~m}}{\cos \theta}-2.0 \mathrm{~m}\right) \sin \theta=2(20.0 \mathrm{~N} / \mathrm{m})(2.0 \mathrm{~m})\left(\frac{1}{\cos \theta}-1\right) \sin \theta \\
& \quad=80 \mathrm{~N}(\tan \theta-\sin \theta)
\end{aligned}
$$

This gives $F$ as a function of $\theta$, but we require a graph of $\theta$ as a function of $F$. To graph this, we calculate $F$ for $0 \leq \theta \leq 75^{\circ}$, and then simply interchange the axes in the graph.

The spreadsheets used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH12.XLS", on tab "Problem 12.98".


## CHAPTER 13: Fluids

## Responses to Questions

1. No. If one material has a higher density than another, then the molecules of the first could be heavier than those of the second, or the molecules of the first could be more closely packed together than the molecules of the second.
2. The cabin of an airplane is maintained at a pressure lower than sea-level atmospheric pressure, and the baggage compartment is not pressurized. Atmospheric pressure is lower at higher altitudes, so when an airplane flies up to a high altitude, the air pressure outside a cosmetics bottle drops, compared to the pressure inside. The higher pressure inside the bottle forces fluid to leak out around the cap.
3. In the case of the two non-cylindrical containers, perpendicular forces from the sides of the containers on the fluid will contribute to the net force on the base. For the middle container, the forces from the sides (perpendicular to the sides) will have an upward component, which helps support the water and keeps the force on the base the same as the container on the left. For the container on the right, the forces from the sides will have a downward component, increasing the force on the base so that it is the same as the container on the left.
4. The pressure is what determines whether or not your skin will be cut. You can push both the pen and the pin with the same force, but the pressure exerted by the point of the pin will be much greater than the pressure exerted by the blunt end of the pen, because the area of the pin point is much smaller.
5. As the water boils, steam displaces some of the air in the can. When the lid is put on, and the water and the can cool, the steam that is trapped in the can condenses back into liquid water. This reduces the pressure in the can to less than atmospheric pressure, and the greater force from the outside air pressure crushes the can.
6. If the cuff is held below the level of the heart, the measured pressure will be the actual blood pressure from the pumping of the heart plus the pressure due to the height of blood above the cuff. This reading will be too high. Likewise, if the cuff is held above the level of the heart, the reported pressure measurement will be too low.
7. Ice floats in water, so ice is less dense than water. When ice floats, it displaces a volume of water that is equal to the weight of the ice. Since ice is less dense than water, the volume of water displaced is smaller than the volume of the ice, and some of the ice extends above the top of the water. When the ice melts and turns back into water, it will fill a volume exactly equal to the original volume of water displaced. The water will not overflow the glass as the ice melts.
8. No. Alcohol is less dense than ice, so the ice cube would sink. In order to float, the ice cube would need to displace a weight of alcohol equal to its own weight. Since alcohol is less dense than ice, this is impossible.
9. All carbonated drinks have gas dissolved in them, which reduces their density to less than that of water. However, Coke has a significant amount of sugar dissolved in it, making its density greater than that of water, so the can of Coke sinks. Diet Coke has no sugar, leaving its density, including the can, less that the density of water. The can of Diet Coke floats.
10. In order to float, a ship must displace an amount of water with a weight equal to its own weight. An iron block would sink, because it does not have enough volume to displace an amount of water equal to its weight. However, the iron of a ship is shaped more like a bowl, so it is able to displace more water. If you were to find the average density of the ship and all its contents, including the air it holds, you would find that this density would be less than the density of water.
11. The liquid in the vertical part of the tube over the lower container will fall into the container through the action of gravity. This action reduces the pressure in the top of the tube and draws liquid through the tube, and into the tube from the upper container. As noted, the tube must be full of liquid initially for this to work.
12. Sand must be added to the barge. If sand is removed, the barge will not need to displace as much water since its weight will be less, and it will rise up in the water, making it even less likely to fit under the bridge. If sand is added, the barge will sink lower into the water, making it more likely to fit under the bridge.
13. As the weather balloon rises into the upper atmosphere, atmospheric pressure on it decreases, allowing the balloon to expand as the gas inside it expands. If the balloon were filled to maximum capacity on the ground, then the balloon fabric would burst shortly after take-off, as the balloon fabric would be unable to expand any additional amount. Filling the balloon to a minimum value on take-off allows plenty of room for expansion as the balloon rises.
14. The water level will fall in all three cases.
(a) The boat, when floating in the pool, displaces water, causing an increase in the overall level of water in the pool. Therefore, when the boat is removed, the water returns to its original (lower) level.
(b) The boat and anchor together must displace an amount of water equal to their combined weight. If the anchor is removed, this water is no longer displaced and the water level in the pool will go down.
(c) If the anchor is removed and dropped in the pool, so that it rests on the bottom of the pool, the water level will again go down, but not by as much as when the anchor is removed from the boat and pool altogether. When the anchor is in the boat, the combination must displace an amount of water equal to their weight because they are floating. When the anchor is dropped overboard, it can only displace an amount of water equal to its volume, which is less than the amount of water equal to its weight. Less water is displaced so the water level in the pool goes down.
15. No. If the balloon is inflated, then the air inside the balloon is slightly compressed by the balloon fabric, making it more dense than the outside air. The increase in the buoyant force, present because the balloon is filled with air, is more than offset by the increase in weight due to the denser air filling the balloon. The apparent weight of the filled balloon will be slightly greater than that of the empty balloon.
16. In order to float, you must displace an amount of water equal to your own weight. Salt water is more dense than fresh water, so the volume of salt water you must displace is less than the volume of fresh water. You will float higher in the salt water because you are displacing a lower volume of water.
17. The papers will move toward each other. When you blow between the sheets of paper, you reduce the air pressure between them (Bernoulli's principle). The greater air pressure on the other side of each sheet will push the sheets toward each other.
18. As the water falls, it speeds up because of the acceleration due to gravity. Because the volume flow rate must remain constant, the faster-moving water must have a smaller cross-sectional area (equation of continuity). Therefore the water farther from the faucet will have a narrower stream than the water nearer the faucet.
19. As a high-speed train travels, it pulls some of the surrounding air with it, due to the viscosity of the air. The moving air reduces the air pressure around the train (Bernoulli's principle), which in turn creates a force toward the train from the surrounding higher air pressure. This force is large enough that it could push a light-weight child toward the train.
20. No. Both the cup and the water in it are in free fall and are accelerating downward because of gravity. There is no "extra" force on the water so it will not accelerate any faster than the cup; both will fall together and water will not flow out of the holes in the cup.
21. Taking off into the wind increases the velocity of the plane relative to the air, an important factor in the creation of lift. The plane will be able to take off with a slower ground speed, and a shorter runway distance.
22. As the ships move, they drag water with them. The moving water has a lower pressure than stationary water, as shown by Bernoulli's principle. If the ships are moving in parallel paths fairly close together, the water between them will have a lower pressure than the water to the outside of either one, since it is being dragged by both ships. The ships are in danger of colliding because the higher pressure of the water on the outsides will tend to push them towards each other.
23. Air traveling over the top of the car is moving quite fast when the car is traveling at high speed, and, due to Bernoulli's principle, will have a lower pressure than the air inside the car, which is stationary with respect to the car. The greater air pressure inside the car will cause the canvas top to bulge out.
24. The air pressure inside and outside a house is typically the same. During a hurricane or tornado, the outside air pressure may drop suddenly because of the high wind speeds, as shown by Bernoulli's principle. The greater air pressure inside the house may then push the roof off.

## Solutions to Problems

1. The mass is found from the density of granite (found in Table 13-1) and the volume of granite.

$$
m=\rho V=\left(2.7 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)\left(10^{8} \mathrm{~m}^{3}\right)=2.7 \times 10^{11} \mathrm{~kg} \approx 3 \times 10^{11} \mathrm{~kg}
$$

2. The mass is found from the density of air (found in Table 13-1) and the volume of air.

$$
m=\rho V=\left(1.29 \mathrm{~kg} / \mathrm{m}^{3}\right)(5.6 \mathrm{~m})(3.8 \mathrm{~m})(2.8 \mathrm{~m})=77 \mathrm{~kg}
$$

3. The mass is found from the density of gold (found in Table 13-1) and the volume of gold.

$$
m=\rho V=\left(19.3 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)(0.56 \mathrm{~m})(0.28 \mathrm{~m})(0.22 \mathrm{~m})=670 \mathrm{~kg} \quad(\approx 1500 \mathrm{lb})
$$

4. Assume that your density is that of water, and that your mass is 75 kg .

$$
V=\frac{m}{\rho}=\frac{75 \mathrm{~kg}}{1.00 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}}=7.5 \times 10^{-2} \mathrm{~m}^{3}=75 \mathrm{~L}
$$

5. To find the specific gravity of the fluid, take the ratio of the density of the fluid to that of water, noting that the same volume is used for both liquids.

$$
S J_{\text {fluid }}=\frac{\rho_{\text {fluid }}}{\rho_{\text {water }}}=\frac{(\mathrm{m} / \mathrm{V})_{\text {fluid }}}{(\mathrm{m} / \mathrm{V})_{\text {water }}}=\frac{m_{\text {fluid }}}{m_{\text {water }}}=\frac{89.22 \mathrm{~g}-35.00 \mathrm{~g}}{98.44 \mathrm{~g}-35.00 \mathrm{~g}}=0.8547
$$

6. The specific gravity of the mixture is the ratio of the density of the mixture to that of water. To find the density of the mixture, the mass of antifreeze and the mass of water must be known.

$$
\begin{aligned}
& m_{\text {antififecze }}=\rho_{\text {antififezec }} V_{\text {antifirezez }}=S G_{\text {antiffereze }} \rho_{\text {water }} V_{\text {antifirezeze }} \quad m_{\text {water }}=\rho_{\text {water }} V_{\text {water }} \\
& S G_{\text {mixture }}=\frac{\rho_{\text {mixture }}}{\rho_{\text {water }}}=\frac{m_{\text {mixture }} / V_{\text {mixture }}}{\rho_{\text {water }}}=\frac{m_{\text {antififeze }}+m_{\text {water }}}{\rho_{\text {water }} V_{\text {mixture }}}=\frac{S G_{\text {antiffereze }} \rho_{\text {water }} V_{\text {antififeeze }}+\rho_{\text {water }} V_{\text {water }}}{\rho_{\text {water }} V_{\text {mixture }}} \\
& =\frac{S G_{\text {antiffeceze }} V_{\text {aniffecece }}+V_{\text {water }}}{V_{\text {mixture }}}=\frac{(0.80)(5.0 \mathrm{~L})+4.0 \mathrm{~L}}{9.0 \mathrm{~L}}=0.89
\end{aligned}
$$

7. (a) The density from the three-part model is found from the total mass divided by the total volume.

Let subscript 1 represent the inner core, subscript 2 represent the outer core, and subscript 3 represent the mantle. The radii are then the outer boundaries of the labeled region.

$$
\begin{aligned}
\rho_{\text {three }} \text { hayers }
\end{aligned}=\frac{m_{1}+m_{2}+m_{3}}{V_{1}+V_{2}+V_{3}}=\frac{\rho_{1} m_{1}+\rho_{2} m_{2}+\rho_{3} m_{3}}{V_{1}+V_{2}+V_{3}}=\frac{\rho_{1} \frac{4}{3} \pi r_{1}^{3}+\rho_{2} \frac{4}{3} \pi\left(r_{2}^{3}-r_{1}^{3}\right)+\rho_{3} \frac{4}{3} \pi\left(r_{3}^{3}-r_{2}^{3}\right)}{\frac{4}{3} \pi r_{1}^{3}+\frac{4}{3} \pi\left(r_{2}^{3}-r_{1}^{3}\right)+\frac{4}{3} \pi\left(r_{3}^{3}-r_{2}^{3}\right)}, \frac{\rho_{1} r_{1}^{3}+\rho_{2}\left(r_{2}^{3}-r_{1}^{3}\right)+\rho_{3}\left(r_{3}^{3}-r_{2}^{3}\right)}{r_{3}^{3}}=\frac{r_{1}^{3}\left(\rho_{1}-\rho_{2}\right)+r_{2}^{3}\left(\rho_{2}-\rho_{3}\right)+r_{3}^{3} \rho_{3}}{r_{3}^{3}}, \begin{aligned}
& =\frac{(1220 \mathrm{~km})^{3}\left(1900 \mathrm{~kg} / \mathrm{m}^{3}\right)+(3480 \mathrm{~km})^{3}\left(6700 \mathrm{~kg} / \mathrm{m}^{3}\right)+(6371 \mathrm{~km})^{3}\left(4400 \mathrm{~kg} / \mathrm{m}^{3}\right)}{(6371 \mathrm{~km})^{3}} \\
& =5505.3 \mathrm{~kg} / \mathrm{m}^{3} \approx 5510 \mathrm{~kg} / \mathrm{m}^{3}
\end{aligned}
$$

(b)

$$
\begin{aligned}
& \rho_{\substack{\text { one } \\
\text { density }}}=\frac{M}{V}=\frac{M}{\frac{4}{3} \pi R^{3}}=\frac{5.98 \times 10^{24} \mathrm{~kg}}{\frac{4}{3} \pi\left(6371 \times 10^{3} \mathrm{~m}\right)^{3}}=5521 \mathrm{~kg} / \mathrm{m}^{3} \approx 5520 \mathrm{~kg} / \mathrm{m}^{3} \\
& \% \text { diff }=100\left(\frac{\begin{array}{c}
\rho_{\text {one }}-\rho_{\text {dhree }} \\
\text { density } \\
\text { layers } \\
\rho_{\text {drare }} \\
\text { layers }
\end{array}}{}\right)=100\left(\frac{5521 \mathrm{~kg} / \mathrm{m}^{3}-5505 \mathrm{~kg} / \mathrm{m}^{3}}{5505 \mathrm{~kg} / \mathrm{m}^{3}}\right)=0.2906 \approx 0.3 \%
\end{aligned}
$$

8. The pressure is given by Eq. 13-3.

$$
P=\rho g h=(1000)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(35 \mathrm{~m})=3.4 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2} \approx 3.4 \mathrm{~atm}
$$

9. (a) The pressure exerted on the floor by the chair leg is caused by the chair pushing down on the floor. That downward push is the reaction to the normal force of the floor on the leg, and the normal force on one leg is assumed to be one-fourth of the weight of the chair.

$$
P_{\text {chair }}=\frac{W_{\text {leg }}}{A}=\frac{\frac{1}{4}(66 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{\left(0.020 \mathrm{~cm}^{2}\right)\left(\frac{1 \mathrm{~m}}{100 \mathrm{~cm}}\right)^{2}}=8.085 \times 10^{7} \mathrm{~N} / \mathrm{m}^{2} \approx 8.1 \times 10^{7} \mathrm{~N} / \mathrm{m}^{2} \text {. }
$$

(b) The pressure exerted by the elephant is found in the same way, but with ALL of the weight being used, since the elephant is standing on one foot.

$$
P_{\text {elephant }}=\frac{W_{\text {elephant }}}{A}=\frac{(1300 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{\left(800 \mathrm{~cm}^{2}\right)\left(\frac{1 \mathrm{~m}}{100 \mathrm{~cm}}\right)^{2}}=1.59 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2} \approx 2 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2} \text {. }
$$

Note that the chair pressure is larger than the elephant pressure by a factor of about 400.
10. Use Eq. 13-3 to find the pressure difference. The density is found in Table 13-1.

$$
\begin{aligned}
P=\rho g h \rightarrow \Delta P & =\rho g \Delta h=\left(1.05 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(1.70 \mathrm{~m}) \\
& =1.749 \times 10^{4} \mathrm{~N} / \mathrm{m}^{2}\left(\frac{1 \mathrm{~mm}-\mathrm{Hg}}{133 \mathrm{~N} / \mathrm{m}^{2}}\right)=132 \mathrm{~mm}-\mathrm{Hg}
\end{aligned}
$$

11. The height is found from Eq. 13-3, using normal atmospheric pressure. The density is found in Table 13-1.

$$
P=\rho g h \rightarrow h=\frac{P}{\rho g}=\frac{1.013 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}}{\left(0.79 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=13 \mathrm{~m}
$$

That is so tall as to be impractical in many cases.
12. The pressure difference on the lungs is the pressure change from the depth of water.

$$
\Delta P=\rho g \Delta h \rightarrow \Delta h=\frac{\Delta P}{\rho g}=\frac{(85 \mathrm{~mm}-\mathrm{Hg})\left(\frac{133 \mathrm{~N} / \mathrm{m}^{2}}{1 \mathrm{~mm}-\mathrm{Hg}}\right)}{\left(1.00 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=1.154 \mathrm{~m} \approx 1.2 \mathrm{~m}
$$

13. The force exerted by the gauge pressure will be equal to the weight of the vehicle.

$$
\begin{aligned}
& m g=P A=P\left(\pi r^{2}\right) \rightarrow \\
& m=\frac{P \pi r^{2}}{g}=\frac{(17.0 \mathrm{~atm})\left(\frac{1.013 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}}{1 \mathrm{~atm}}\right) \pi\left[\frac{1}{2}(0.225 \mathrm{~m})\right]^{2}}{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=6990 \mathrm{~kg}
\end{aligned}
$$

14. The sum of the force exerted by the pressure in each tire is equal to the weight of the car.

$$
m g=4 P A \rightarrow m=\frac{4 P A}{g}=\frac{4\left(2.40 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}\right)\left(220 \mathrm{~cm}^{2}\right)\left(\frac{1 \mathrm{~m}^{2}}{10^{4} \mathrm{~cm}^{2}}\right)}{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=2200 \mathrm{~kg}
$$

15. (a) The absolute pressure is given by Eq. 13-6b, and the total force is the absolute pressure times the area of the bottom of the pool.

$$
\begin{aligned}
P & =P_{0}+\rho g h=1.013 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}+\left(1.00 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(1.8 \mathrm{~m}) \\
& =1.189 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2} \approx 1.2 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2} \\
F & =P A=\left(1.189 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}\right)(28.0 \mathrm{~m})(8.5 \mathrm{~m})=2.8 \times 10^{7} \mathrm{~N}
\end{aligned}
$$

(b) The pressure against the side of the pool, near the bottom, will be the same as the pressure at the bottom. Pressure is not directional. $P=1.2 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}$
16. (a) The gauge pressure is given by Eq. 13-3. The height is the height from the bottom of the hill to the top of the water tank.

$$
P_{\mathrm{G}}=\rho g h=\left(1.00 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left[5.0 \mathrm{~m}+(110 \mathrm{~m}) \sin 58^{\circ}\right]=9.6 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}
$$

(b) The water would be able to shoot up to the top of the tank (ignoring any friction).

$$
h=5.0 \mathrm{~m}+(110 \mathrm{~m}) \sin 58^{\circ}=98 \mathrm{~m}
$$

17. The pressure at points $a$ and $b$ are equal since they are the same height in the same fluid. If they were unequal, the fluid would flow. Calculate the pressure at both $a$ and $b$, starting with atmospheric pressure at the top surface of each liquid, and then equate those pressures.

$$
\begin{aligned}
& P_{a}=P_{b} \rightarrow P_{0}+\rho_{\text {oil }} g h_{\text {oil }}=P_{0}+\rho_{\text {water }} g h_{\text {water }} \rightarrow \rho_{\text {oil }} h_{\text {oil }}=\rho_{\text {water }} h_{\text {water }} \rightarrow \\
& \rho_{\text {oil }}=\frac{\rho_{\text {water }} h_{\text {water }}}{h_{\text {oil }}}=\frac{\left(1.00 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)(0.272 \mathrm{~m}-0.0862 \mathrm{~m})}{(0.272 \mathrm{~m})}=683 \mathrm{~kg} / \mathrm{m}^{3}
\end{aligned}
$$

18. (a) The mass of water in the tube is the volume of the tube times the density of water.

$$
m=\rho V=\rho \pi r^{2} h=\left(1.00 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right) \pi\left(0.30 \times 10^{-2} \mathrm{~m}\right)^{2}(12 \mathrm{~m})=0.3393 \mathrm{~kg} \approx 0.34 \mathrm{~kg}
$$

(b) The net force exerted on the lid is the gauge pressure of the water times the area of the lid. The gauge pressure is found from Eq. 13-3.

$$
F=P_{\text {gauge }} A=\rho g h \pi R^{2}=\left(1.00 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(12 \mathrm{~m}) \pi(0.21 \mathrm{~m})^{2}=1.6 \times 10^{4} \mathrm{~N}
$$

19. We use the relationship developed in Example 13-5.

$$
P=P_{0} e^{-\left(\rho_{0} g / P_{0}\right) y}=\left(1.013 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}\right) e^{-\left(1.25 \times 10^{-4} \mathrm{~m}^{-1}\right)(8850 \mathrm{~m})}=3.35 \times 10^{4} \mathrm{~N} / \mathrm{m}^{2} \approx 0.331 \mathrm{~atm}
$$

Note that if we used the constant density approximation, $P=P_{0}+\rho g h$, a negative pressure would result.
20. Consider the lever (handle) of the press. The net torque on that handle is 0 . Use that to find the force exerted by the hydraulic fluid upwards on the small cylinder (and the lever). Then Pascal's principle can be used to find the upwards force on the large cylinder, which is the same as the force on the sample.

$$
\begin{array}{ll}
\sum \tau=F(2 \ell)-F_{1} \ell=0 \rightarrow F_{1}=2 F \\
P_{1}=P_{2} \rightarrow \frac{F_{1}}{\pi\left(\frac{1}{2} d_{1}\right)^{2}}=\frac{F_{2}}{\pi\left(\frac{1}{2} d_{2}\right)^{2}} \rightarrow \\
F_{2}=F_{1}\left(d_{2} / d_{1}\right)^{2}=2 F\left(d_{2} / d_{1}\right)^{2}=F_{\text {sample }} \rightarrow \\
P_{\text {sample }}=\frac{F_{\text {sample }}}{A_{\text {sample }}}=\frac{2 F\left(d_{2} / d_{1}\right)^{2}}{A_{\text {sample }}}=\frac{2(350 \mathrm{~N})(5)^{2}}{4.0 \times 10^{-4} \mathrm{~m}^{2}}=4.4 \times 10^{7} \mathrm{~N} / \mathrm{m}^{2} & \overrightarrow{\mathbf{F}}_{2}
\end{array}
$$


21. The pressure in the tank is atmospheric pressure plus the pressure difference due to the column of mercury, as given in Eq. 13-6b.
(a) $P=P_{0}+\rho g h=1.04 \mathrm{bar}+\rho_{\mathrm{Hg}} g h$

$$
=(1.04 \text { bar })\left(\frac{1.00 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}}{1 \text { bar }}\right)+\left(13.6 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.210 \mathrm{~m})=1.32 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}
$$

(b)

$$
P=(1.04 \operatorname{bar})\left(\frac{1.00 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}}{1 \mathrm{bar}}\right)+\left(13.6 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(-0.052 \mathrm{~m})=9.7 \times 10^{4} \mathrm{~N} / \mathrm{m}^{2}
$$

22. (a) See the diagram. In the accelerated frame of the beaker, there is a pseudoforce opposite to the direction of the acceleration, and so there is a pseudo acceleration as shown on the diagram. The effective acceleration, $\overrightarrow{\mathbf{g}}^{\prime}$, is given by $\overrightarrow{\mathbf{g}}^{\prime}=\overrightarrow{\mathbf{g}}+\overrightarrow{\mathbf{a}}$. The surface of the water will be perpendicular to the effective acceleration, and thus makes an angle $\theta=\tan ^{-1} \frac{a}{g}$.

(b) The left edge of the water surface, opposite to the direction of the acceleration, will be higher.
(c) Constant pressure lines will be parallel to the surface. From the second diagram, we see that a vertical depth of $h$ corresponds to a depth of $h^{\prime}$ perpendicular to the surface, where $h^{\prime}=h \cos \theta$, and so we have the following.

$$
\begin{aligned}
P & =P_{0}+\rho g^{\prime} h^{\prime}=P_{0}+\rho \sqrt{g^{2}+a^{2}}(h \cos \theta) \\
& =P_{0}+\rho \sqrt{g^{2}+a^{2}}\left(h \frac{g}{\sqrt{g^{2}+a^{2}}}\right)=P_{0}+\rho h g
\end{aligned}
$$

And so $P=P_{0}+\rho h g$, as in the unaccelerated case.
23. (a) Because the pressure varies with depth, the force on the wall will also vary with depth. So to find the total force on the wall, we will have to integrate. Measure vertical distance $y$ downward from the top level of the water behind the dam. Then at a depth $y$, choose an infinitesimal area of width $b$ and height $d \underline{y}$. The pressure due to the water at that depth is $P=\rho g y$.

$$
\begin{aligned}
& P=\rho g y ; d F=P d A=(\rho g y)(b d y) \rightarrow \\
& F=\int d F=\int_{0}^{h}(\rho g y)(b d y)=\frac{1}{2} \rho g b h^{2}
\end{aligned}
$$

(b) The lever arm for the force $d F$ about the bottom of the dam is $h-y$, and so the torque caused by that force is
$d \tau=(h-y) d F$. Integrate to find the total torque.

$$
\begin{aligned}
\tau & =\int d \tau=\int_{0}^{h}(h-y)(\rho g y)(b d y)=\rho g b \int_{0}^{h}\left(h y-y^{2}\right) d y \\
& =\rho g b\left(\frac{1}{2} h y^{2}-\frac{1}{3} y^{3}\right)_{0}^{h}=\frac{1}{6} \rho g b h^{3}
\end{aligned}
$$



Consider that torque as caused by the total force, applied at a single distance from the bottom $d$.

$$
\tau=\frac{1}{6} \rho g b h^{3}=F d=\frac{1}{2} \rho g b h^{2} d \rightarrow d=\frac{1}{3} h
$$

(c) To prevent overturning, the torque caused by gravity about the lower right front corner in the diagram must be at least as big as the torque caused by the water. The lever arm for gravity is half the thickness of the dam.

$$
\begin{aligned}
& m g\left(\frac{1}{2} t\right) \geq \frac{1}{6} \rho g b h^{3} \rightarrow \rho_{\text {conerete }}(h b t) g\left(\frac{1}{2} t\right) \geq \frac{1}{6} \rho_{\text {water }} g b h^{3} \rightarrow \\
& \frac{t}{h} \geq \sqrt{\frac{1}{3} \frac{\rho_{\text {water }}}{\rho_{\text {conerete }}}}=\sqrt{\frac{1}{3} \frac{1.00 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}}{2.3 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}}}=0.38
\end{aligned}
$$

So we must have $t \geq 0.38 h$ to prevent overturning. Atmospheric pressure need not be added in because it is exerted on BOTH sides of the dam, and so causes no net force or torque. In part (a), the actual pressure at a depth $y$ is $P=P_{0}+\rho g y$, and of course air pressure acts on the exposed side of the dam as well.
24. From section $9-5$, the change in volume due to pressure change is $\frac{\Delta V}{V_{0}}=-\frac{\Delta P}{B}$, where $B$ is the bulk modulus of the water, given in Table 12-1. The pressure increase with depth for a fluid of constant density is given by $\Delta P=\rho g \Delta h$, where $\Delta h$ is the depth of descent. If the density change is small, then we can use the initial value of the density to calculate the pressure change, and so $\Delta P \approx \rho_{0} g \Delta h$. Finally, consider a constant mass of water. That constant mass will relate the volume and density at the two locations by $M=\rho V=\rho_{0} V_{0}$. Combine these relationships and solve for the density deep in the sea, $\rho$.

$$
\begin{aligned}
& \rho V=\rho_{0} V_{0} \rightarrow \\
& \begin{aligned}
\rho & =\frac{\rho_{0} V_{0}}{V}=\frac{\rho_{0} V_{0}}{V_{0}+\Delta V}=\frac{\rho_{0} V_{0}}{V_{0}+\left(-V_{0} \frac{\Delta P}{B}\right)}=\frac{\rho_{0}}{1-\frac{\rho_{0} g h}{B}}=\frac{1025 \mathrm{~kg} / \mathrm{m}^{3}}{1-\frac{\left(1025 \mathrm{~kg} / \mathrm{m}^{3}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(5.4 \times 10^{3} \mathrm{~m}\right)}{2.0 \times 10^{9} \mathrm{~N} / \mathrm{m}^{2}}} \\
& =1054 \mathrm{~kg} / \mathrm{m}^{3} \approx 1.05 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}
\end{aligned} \\
& \rho / \rho_{0}=\frac{1054}{1025}=1.028
\end{aligned}
$$

The density at the 6 km depth is about $3 \%$ larger than the density at the surface.
25. Consider a layer of liquid of (small) height $\Delta h$, and ignore the pressure variation due to height in that layer. Take a cylindrical ring of water of height $\Delta h$, radius $r$, and thickness $d r$. See the diagram (the height is not shown). The volume of the ring of liquid is $(2 \pi r \Delta h) d r$, and so has a mass of $d m=(2 \pi \rho r \Delta h) d r$. That mass of water has a net centripetal force on it of magnitude $d F_{\text {radial }}=\omega^{2} r(d m)=\omega^{2} r \rho(2 \pi r \Delta h) d r$. That force comes from a pressure difference across the surface area of the liquid. Let the pressure at the inside surface be $P$, which causes an outward force, and the pressure at the outside surface be $P+d P$, which causes an inward force. The surface area over which these
 pressures act is $2 \pi r \Delta h$, the "walls" of the cylindrical ring. Use Newton's second law.

$$
d F_{\text {radial }}=d F_{\substack{\text { outer } \\ \text { wall }}}-d F_{\substack{\text { inere } \\ \text { wall }}} \rightarrow \omega^{2} r \rho(2 \pi r \Delta h) d r=(P+d P) 2 \pi r \Delta h-(P) 2 \pi r \Delta h \rightarrow
$$

$$
d P=\omega^{2} r \rho d r \rightarrow \int_{P_{0}}^{P} d P=\int_{0}^{r} \omega^{2} r \rho d r \rightarrow P-P_{0}=\frac{1}{2} \rho \omega^{2} r^{2} \rightarrow P=P_{0}+\frac{1}{2} \rho \omega^{2} r^{2}
$$

26. If the iron is floating, then the net force on it is zero. The buoyant force on the iron must be equal to its weight. The buoyant force is equal to the weight of the mercury displaced by the submerged iron.

$$
\begin{aligned}
& F_{\text {buoyant }}=m_{\mathrm{Fe}} g \rightarrow \rho_{\mathrm{Hg}} g V_{\text {submerged }}=\rho_{\mathrm{Fe}} g V_{\text {total }} \rightarrow \\
& \frac{V_{\text {submerged }}}{V_{\text {total }}}=\frac{\rho_{\mathrm{Fe}}}{\rho_{\mathrm{Hg}}}=\frac{7.8 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}}{13.6 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}}=0.57 \approx 57 \%
\end{aligned}
$$

27. The difference in the actual mass and the apparent mass is the mass of the water displaced by the rock. The mass of the water displaced is the volume of the rock times the density of water, and the volume of the rock is the mass of the rock divided by its density. Combining these relationships yields an expression for the density of the rock.

$$
\begin{aligned}
& m_{\text {actual }}-m_{\text {apparent }}=\Delta m=\rho_{\text {water }} V_{\text {rock }}=\rho_{\text {water }} \frac{m_{\text {rock }}}{\rho_{\text {rock }}} \rightarrow \\
& \rho_{\text {rock }}=\rho_{\text {water }} \frac{m_{\text {rock }}}{\Delta m}=\left(1.00 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right) \frac{9.28 \mathrm{~kg}}{9.28 \mathrm{~kg}-6.18 \mathrm{~kg}}=2990 \mathrm{~kg} / \mathrm{m}^{3}
\end{aligned}
$$

28. (a) When the hull is submerged, both the buoyant force and the tension force act upward on the hull, and so their sum is equal to the weight of the hull, if the hill is not accelerated as it is lifted. The buoyant force is the weight of the water displaced.

$$
\begin{aligned}
& T+F_{\text {buoyant }}=m g \quad \rightarrow \\
& T=m g-F_{\text {buoyant }}=m_{\text {hull }} g-\rho_{\text {water }} V_{\text {sub }} g=m_{\text {hull }} g-\rho_{\text {water }} \frac{m_{\text {hull }}}{\rho_{\text {hull }}} g=m_{\text {hull }} g\left(1-\frac{\rho_{\text {water }}}{\rho_{\text {hull }}}\right) \\
& =\left(1.6 \times 10^{4} \mathrm{~kg}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(1-\frac{1.00 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}}{7.8 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}}\right)=1.367 \times 10^{5} \mathrm{~N} \approx 1.4 \times 10^{5} \mathrm{~N}
\end{aligned}
$$

(b) When the hull is completely out of the water, the tension in the crane's cable must be equal to the weight of the hull.

$$
T=m g=\left(1.6 \times 10^{4} \mathrm{~kg}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=1.568 \times 10^{5} \mathrm{~N} \approx 1.6 \times 10^{5} \mathrm{~N}
$$

29. The buoyant force of the balloon must equal the weight of the balloon plus the weight of the helium in the balloon plus the weight of the load. For calculating the weight of the helium, we assume it is at $0^{\circ} \mathrm{C}$ and 1 atm pressure. The buoyant force is the weight of the air displaced by the volume of the balloon.

$$
\begin{aligned}
& F_{\text {buoyant }}=\rho_{\text {air }} V_{\text {balloon }} g=m_{\text {He }} g+m_{\text {balloon }} g+m_{\text {cargo }} g \rightarrow \\
& \begin{aligned}
m_{\text {cargo }} & =\rho_{\text {air }} V_{\text {balloon }}-m_{\text {He }}-m_{\text {balloon }}=\rho_{\text {air }} V_{\text {balloon }}-\rho_{\mathrm{He}} V_{\text {balloon }}-m_{\text {balloon }}=\left(\rho_{\text {air }}-\rho_{\text {He }}\right) V_{\text {balloon }}-m_{\text {balloon }} \\
& =\left(1.29 \mathrm{~kg} / \mathrm{m}^{3}-0.179 \mathrm{~kg} / \mathrm{m}^{3}\right) \frac{4}{3} \pi(7.35 \mathrm{~m})^{3}-930 \mathrm{~kg}=920 \mathrm{~kg}=9.0 \times 10^{3} \mathrm{~N}
\end{aligned}
\end{aligned}
$$

30. The difference in the actual mass and the apparent mass is the mass of the water displaced by the legs. The mass of the water displaced is the volume of the legs times the density of water, and the volume of the legs is the mass of the legs divided by their density. The density of the legs is assumed to be the same as that of water. Combining these relationships yields an expression for the mass of the legs.

$$
\begin{aligned}
& m_{\text {actual }}-m_{\text {apparent }}=\Delta m=\rho_{\text {water }} V_{\text {legs }}=\rho_{\text {water }} \frac{m_{\text {legs }}}{\rho_{\operatorname{legs}}}=2 m_{\operatorname{leg}} \rightarrow \\
& m_{\operatorname{leg}}=\frac{1}{2} \Delta m=\frac{1}{2}(74 \mathrm{~kg}-54 \mathrm{~kg})=10 \mathrm{~kg}(2 \text { sig. fig. })
\end{aligned}
$$

31. The apparent weight is the actual weight minus the buoyant force. The buoyant force is weight of a mass of water occupying the volume of the metal sample.

$$
\begin{aligned}
& m_{\text {apparent }} g=m_{\text {metal }} g-F_{\mathrm{B}}=m_{\text {metal }} g-V_{\text {metal }} \rho_{\mathrm{H}_{2} \mathrm{o}} g=m_{\text {metal }} g-\frac{m_{\text {metal }}}{\rho_{\text {metal }}} \rho_{\mathrm{H}_{2} \mathrm{O}} g \rightarrow \\
& m_{\text {apparent }}=m_{\text {metal }}-\frac{m_{\text {metal }}}{\rho_{\text {metal }}} \rho_{\mathrm{H}_{2} \mathrm{O}} \rightarrow \\
& \rho_{\text {metal }}=\frac{m_{\text {metal }}}{\left(m_{\text {meal }}-m_{\text {apparent }}\right)} \rho_{\mathrm{H}_{2} \mathrm{O}}=\frac{63.5 \mathrm{~g}}{(63.5 \mathrm{~g}-55.4 \mathrm{~g})}\left(1000 \mathrm{~kg} / \mathrm{m}^{3}\right)=7840 \mathrm{~kg} / \mathrm{m}^{3}
\end{aligned}
$$

Based on the density value, the metal is probably iron or steel.
32. The difference in the actual mass and the apparent mass of the aluminum is the mass of the air displaced by the aluminum. The mass of the air displaced is the volume of the aluminum times the density of air, and the volume of the aluminum is the actual mass of the aluminum divided by the density of aluminum. Combining these relationships yields an expression for the actual mass.

$$
\begin{aligned}
& m_{\text {actual }}-m_{\text {apparent }}=\rho_{\text {air }} V_{\mathrm{Al}}=\rho_{\text {air }} \frac{m_{\text {actual }}}{\rho_{\mathrm{Al}}} \rightarrow \\
& m_{\text {actual }}=\frac{m_{\text {apparent }}}{1-\frac{\rho_{\text {air }}}{\rho_{\mathrm{Al}}}}=\frac{3.0000 \mathrm{~kg}}{1-\frac{1.29 \mathrm{~kg} / \mathrm{m}^{3}}{2.70 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}}}=3.0014 \mathrm{~kg}
\end{aligned}
$$

33. The buoyant force on the drum must be equal to the weight of the steel plus the weight of the gasoline. The weight of each component is its respective volume times density. The buoyant force is the weight of total volume of displaced water. We assume that the drum just "barely" floats - in other words, the volume of water displaced is equal to the total volume of gasoline and steel.

$$
\begin{aligned}
& F_{\mathrm{B}}=W_{\text {stell }}+W_{\text {gasoline }} \rightarrow\left(V_{\text {gasoline }}+V_{\text {steel }}\right) \rho_{\text {water }} g=V_{\text {steel }} \rho_{\text {stel }} g+V_{\text {gasoline }} \rho_{\text {gasolinec }} g \rightarrow \\
& V_{\text {gasoline }} \rho_{\text {water }}+V_{\text {steel }} \rho_{\text {water }}=V_{\text {steel }} \rho_{\text {steel }}+V_{\text {gasoline }} \rho_{\text {gasoline }} \rightarrow \\
& V_{\text {stel }}=V_{\text {gasoline }}\left(\frac{\rho_{\text {water }}-\rho_{\text {gasoline }}}{\rho_{\text {steel }}-\rho_{\text {water }}}\right)=(230 \mathrm{~L})\left(\frac{1000 \mathrm{~kg} / \mathrm{m}^{3}-680 \mathrm{~kg} / \mathrm{m}^{3}}{7800 \mathrm{~kg} / \mathrm{m}^{3}-1000 \mathrm{~kg} / \mathrm{m}^{3}}\right)=10.82 \mathrm{~L} \approx 1.1 \times 10^{-2} \mathrm{~m}^{3}
\end{aligned}
$$

34. (a) The buoyant force is the weight of the water displaced, using the density of sea water.

$$
\begin{aligned}
F_{\text {buyant }} & =m_{\text {water }}^{\text {displaced }} \\
& g=\rho_{\text {water }} V_{\text {displaced }} g \\
& =\left(1.025 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)(65.0 \mathrm{~L})\left(\frac{1 \times 10^{-3} \mathrm{~m}^{3}}{1 \mathrm{~L}}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=653 \mathrm{~N}
\end{aligned}
$$

(b) The weight of the diver is $m_{\text {diver }} g=(68.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=666 \mathrm{~N}$. Since the buoyant force is not as large as her weight, she will sink , although it will be very gradual since the two forces are almost the same.
35. The buoyant force on the ice is equal to the weight of the ice, since it floats.

$$
\begin{aligned}
& F_{\text {buoyant }}=W_{\text {ice }} \rightarrow m_{\text {seawater }}^{\text {submerged }} \\
& \rho_{\text {seawater }} V_{\text {seawater }}=\rho_{\text {ice }} V_{\text {ice }} \rightarrow(S G)_{\text {seawater }} \rho_{\text {water }} V_{\text {submerged }}^{\text {ice }}=(S G)_{\text {ice }} \rho_{\text {water }} V_{\text {ice }} \rightarrow \\
& (S G)_{\text {seawawater }}=V_{\text {ice }} \rightarrow \\
& V_{\text {submerged }}=(S G)_{\text {ice }} V_{\text {ice }} \rightarrow \\
& V_{\text {submerged }}=\frac{(S G)_{\text {ice }}}{(S G)_{\text {seawater }}} V_{\text {ice }}=\frac{0.917}{1.025} V_{\text {ice }}=0.895 V_{\text {ice }}
\end{aligned}
$$

Thus the fraction above the water is $V_{\text {above }}=V_{\text {ice }}-V_{\text {submerged }}=0.105 V_{\text {ice }}$ or $10.5 \%$
36. (a) The difference in the actual mass and the apparent mass of the aluminum ball is the mass of the liquid displaced by the ball. The mass of the liquid displaced is the volume of the ball times the density of the liquid, and the volume of the ball is the mass of the ball divided by its density.
Combining these relationships yields an expression for the density of the liquid.

$$
\begin{gathered}
m_{\text {actual }}-m_{\text {apparent }}=\Delta m=\rho_{\text {liquid }} V_{\text {ball }}=\rho_{\text {liquid }} \frac{m_{\text {ball }}}{\rho_{\text {Al }}} \rightarrow \\
\rho_{\text {liquid }}=\frac{\Delta m}{m_{\text {ball }}} \rho_{\text {Al }}=\frac{(3.80 \mathrm{~kg}-2.10 \mathrm{~kg})}{3.80 \mathrm{~kg}}\left(2.70 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)=1210 \mathrm{~kg} / \mathrm{m}^{3} \\
\text { (b) Generalizing the relation from above, we have } \rho_{\text {liquid }}=\left(\frac{m_{\text {object }}-m_{\text {apparent }}}{m_{\text {object }}}\right) \rho_{\text {object }} .
\end{gathered}
$$

37. (a) The buoyant force on the object is equal to the weight of the fluid displaced. The force of gravity of the fluid can be considered to act at the center of gravity of the fluid (see section 9-8). If the object were removed from the fluid and that space re-filled with an equal volume of fluid, that fluid would be in equilibrium. Since there are only two forces on that volume of fluid, gravity and the buoyant force, they must be equal in magnitude and act at the same point. Otherwise they would be a couple (see Figure 12-4), exert a non-zero torque, and cause rotation of the fluid. Since the fluid does not rotate, we may conclude that the buoyant force acts at the center of gravity.
(b) From the diagram, if the center of buoyancy (the point where the buoyancy force acts) is above the center of gravity (the point where gravity acts) of the entire ship, when the ship tilts, the net torque about the center of mass will tend to reduce the tilt. If the center of buoyancy is below the center of gravity of the entire ship, when the
 ship tilts, the net torque about the center of mass will tend to increase the tilt. Stability is achieved when the center of buoyancy is above the center of gravity.
38. The weight of the object must be balanced by the two buoyant forces, one from the water and one from the oil. The buoyant force is the density of the liquid, times the volume in the liquid, times the acceleration due to gravity. We represent the edge length of the cube by $\ell$.

$$
\begin{aligned}
m g & =F_{\substack{\mathrm{B} \\
\text { oil }}}^{F_{\mathrm{B}}} F_{\text {water }}=\rho_{\text {oil }} V_{\text {oil }} g+\rho_{\text {water }} V_{\text {water }} g=\rho_{\text {oil }} \ell^{2}(0.28 \ell) g+\rho_{\text {water }} \ell^{2}(0.72 \ell) g \rightarrow \\
\text { l }^{\prime} & =\ell^{3}\left(0.28 \rho_{\text {oil }}+0.72 \rho_{\text {water }}\right)=(0.100 \mathrm{~m})^{3}\left[0.28\left(810 \mathrm{~kg} / \mathrm{m}^{2}\right)+0.72\left(1000 \mathrm{~kg} / \mathrm{m}^{2}\right)\right] \\
& =0.9468 \mathrm{~kg} \approx 0.95 \mathrm{~kg}
\end{aligned}
$$

The buoyant force is the weight of the object, $m g=(0.9468 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=9.3 \mathrm{~N}$
39. The buoyant force must be equal to the combined weight of the helium balloons and the person. We ignore the buoyant force due to the volume of the person, and we ignore the mass of the balloon material.

$$
\begin{aligned}
& F_{\mathrm{B}}=\left(m_{\text {person }}+m_{\mathrm{He}}\right) g \rightarrow \rho_{\text {air }} V_{\mathrm{He}} g=\left(m_{\text {person }}+\rho_{\mathrm{He}} V_{\mathrm{He}}\right) g \rightarrow V_{\mathrm{He}}=N \frac{4}{3} \pi r^{3}=\frac{m_{\text {person }}}{\left(\rho_{\text {air }}-\rho_{\mathrm{He}}\right)} \rightarrow \\
& N=\frac{3(75 \mathrm{~kg})}{4 \pi r^{3}\left(\rho_{\text {person }}-\rho_{\mathrm{He}}\right)}=\frac{3(0.165 \mathrm{~m})^{3}\left(1.29 \mathrm{~kg} / \mathrm{m}^{3}-0.179 \mathrm{~kg} / \mathrm{m}^{3}\right)}{4 \pi(3587 \approx 3600 \text { balloons }}
\end{aligned}
$$

40. There will be a downward gravity force and an upward buoyant force on the fully submerged tank. The buoyant force is constant, but the gravity force will decrease as the air is removed. Take upwards to be positive.

$$
\begin{aligned}
& F_{\text {full }}=F_{\mathrm{B}}-m_{\text {totala }} g=\rho_{\text {water }} V_{\text {tank }} g-\left(m_{\text {tank }}+m_{\text {air }}\right) g \\
& \quad=\left[\left(1025 \mathrm{~kg} / \mathrm{m}^{3}\right)\left(0.0157 \mathrm{~m}^{3}\right)-17.0 \mathrm{~kg}\right]\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=-8.89 \mathrm{~N} \approx 9 \mathrm{~N} \text { downward } \\
& F_{\text {empty }}=F_{\mathrm{B}}-m_{\text {totala }} g=\rho_{\text {water }} V_{\text {tank }} g-\left(m_{\text {tank }}+m_{\text {air }}\right) g \\
& \quad=\left[\left(1025 \mathrm{~kg} / \mathrm{m}^{3}\right)\left(0.0157 \mathrm{~m}^{3}\right)-14.0 \mathrm{~kg}\right]\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=20.51 \mathrm{~N} \approx 21 \mathrm{~N} \text { upward }
\end{aligned}
$$

41. The apparent weight is the force required to hold the system in equilibrium. In the first case, the object is held above the water. In the second case, the object is allowed to be pulled under the water. Consider the free-body diagram for each case.
Case 1: $\quad \sum F=w_{1}-w+F_{\substack{\text { buoy } \\ \text { sinker }}}-w_{\text {sinker }}=0$
Case 2: $\quad \sum F=w_{2}+F_{\substack{\text { buoy } \\ \text { object }}}-w+F_{\substack{\text { buoy } \\ \text { sinker }}}-w_{\text {sinker }}=0$
Since both add to 0 , equate them. Also note that the specific gravity can be expressed in terms of the buoyancy force.


$$
\begin{aligned}
& F_{\text {buoy }}^{\text {object }}=V_{\text {object }} \rho_{\text {water }} g=\frac{m_{\text {object }}}{\rho_{\text {object }}} \rho_{\text {water }} g=m_{\text {object1 }} g \frac{\rho_{\text {water }}}{\rho_{\text {object }}}=\frac{w}{\text { S.G. }}
\end{aligned}
$$

$$
\begin{aligned}
& w_{1}=w_{2}+F_{\substack{\text { buay } \\
\text { object }}}=w_{2}+\frac{w}{\text { S.G. }} \rightarrow \text { S.G. }=\frac{w}{\left(w_{1}-w_{2}\right)}
\end{aligned}
$$

42. For the combination to just barely sink, the total weight of the wood and lead must be equal to the total buoyant force on the wood and the lead.

$$
\begin{aligned}
& F_{\text {weight }}=F_{\text {buyant }} \rightarrow m_{\text {wood }} g+m_{\mathrm{Pb}} g=V_{\text {wood }} \rho_{\text {water }} g+V_{\mathrm{Pb}} \rho_{\text {water }} g \rightarrow \\
& m_{\text {wood }}+m_{\mathrm{Pb}}=\frac{m_{\text {wood }}}{\rho_{\text {wood }}} \rho_{\text {water }}+\frac{m_{\mathrm{Pb}}}{\rho_{\mathrm{Pb}}} \rho_{\text {water }} \rightarrow m_{\mathrm{Pb}}\left(1-\frac{\rho_{\text {water }}}{\rho_{\mathrm{Pb}}}\right)=m_{\text {wood }}\left(\frac{\rho_{\text {water }}}{\rho_{\text {wood }}}-1\right) \rightarrow
\end{aligned}
$$

$$
m_{\mathrm{Pb}}=m_{\text {wood }} \frac{\left(\frac{\rho_{\text {water }}}{\rho_{\text {wood }}}-1\right)}{\left(1-\frac{\rho_{\text {water }}}{\rho_{\mathrm{Pb}}}\right)}=m_{\text {wood }} \frac{\left(\frac{1}{S G_{\text {wood }}}-1\right)}{\left(1-\frac{1}{S G_{\mathrm{Pb}}}\right)}=(3.25 \mathrm{~kg}) \frac{\left(\frac{1}{0.50}-1\right)}{\left(1-\frac{1}{11.3}\right)}=3.57 \mathrm{~kg}
$$

43. We apply the equation of continuity at constant density, Eq. 13-7b.

## Flow rate out of duct = Flow rate into room

$$
A_{\text {duct }} v_{\text {duct }}=\pi r^{2} v_{\text {duct }}=\frac{V_{\text {room }}}{t_{\substack{\text { to fill } \\ \text { room }}} \rightarrow v_{\text {duct }}=\frac{V_{\text {room }}}{\pi r^{2} t_{\substack{\text { to fill } \\ \text { room }}}}=\frac{(8.2 \mathrm{~m})(5.0 \mathrm{~m})(3.5 \mathrm{~m})}{\pi(0.15 \mathrm{~m})^{2}(12 \mathrm{~min})\left(\frac{60 \mathrm{~s}}{1 \mathrm{~min}}\right)}=2.8 \mathrm{~m} / \mathrm{s} .}
$$

44. Use Eq. 13-7b, the equation of continuity for an incompressible fluid, to compare blood flow in the aorta and in the major arteries.

$$
\begin{aligned}
& (A v)_{\text {aorta }}=(A v)_{\text {arteries }} \rightarrow \\
& v_{\text {arteries }}=\frac{A_{\text {aorta }}}{A_{\text {arteries }}} v_{\text {aorta }}=\frac{\pi(1.2 \mathrm{~cm})^{2}}{2.0 \mathrm{~cm}^{2}}(40 \mathrm{~cm} / \mathrm{s})=90.5 \mathrm{~cm} / \mathrm{s} \approx 0.9 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

45. We may apply Torricelli's theorem, Eq. 13-9.

$$
\left.v_{1}=\sqrt{2 g\left(y_{2}-y_{1}\right)}=\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(5.3 \mathrm{~m})}=10.2 \mathrm{~m} / \mathrm{s} \approx 10 \mathrm{~m} / \mathrm{s} \text { ( } 2 \text { sig. fig. }\right)
$$

46. The flow speed is the speed of the water in the input tube. The entire volume of the water in the tank is to be processed in 4.0 h . The volume of water passing through the input tube per unit time is the volume rate of flow, as expressed in the text immediately after Eq. 13-7b.

$$
\frac{V}{\Delta t}=A v \rightarrow v=\frac{V}{A \Delta t}=\frac{\ell w h}{\pi r^{2} \Delta t}=\frac{(0.36 \mathrm{~m})(1.0 \mathrm{~m})(0.60 \mathrm{~m})}{\pi(0.015 \mathrm{~m})^{2}(4.0 \mathrm{~h})\left(\frac{3600 \mathrm{~s}}{1 \mathrm{~h}}\right)}=0.02122 \mathrm{~m} / \mathrm{s} \approx 2.1 \mathrm{~cm} / \mathrm{s}
$$

47. Apply Bernoulli's equation with point 1 being the water main, and point 2 being the top of the spray. The velocity of the water will be zero at both points. The pressure at point 2 will be atmospheric pressure. Measure heights from the level of point 1 .

$$
\begin{aligned}
& P_{1}+\frac{1}{2} \rho v_{1}^{2}+\rho g y_{1}=P_{2}+\frac{1}{2} \rho v_{2}^{2}+\rho g y_{2} \rightarrow \\
& P_{1}-P_{\mathrm{atm}}=\rho g y_{2}=\left(1.00 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(18 \mathrm{~m})=1.8 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}
\end{aligned}
$$

48. The volume flow rate of water from the hose, multiplied times the time of filling, must equal the volume of the pool.

$$
\begin{aligned}
& (A v)_{\text {hose }}=\frac{V_{\text {pool }}}{t} \rightarrow t=\frac{V_{\text {pool }}}{A_{\text {hose }} v_{\text {hose }}}=\frac{\pi(3.05 \mathrm{~m})^{2}(1.2 \mathrm{~m})}{\pi\left[\frac{1}{2}\left(\frac{5}{8}\right)^{\prime}\left(\frac{1 \mathrm{~m}}{39.37^{\prime \prime}}\right)\right]^{2}(0.40 \mathrm{~m} / \mathrm{s})}=4.429 \times 10^{5} \mathrm{~s} \\
& 4.429 \times 10^{5} \mathrm{~s}\left(\frac{1 \text { day }}{60 \times 60 \times 24 \mathrm{~s}}\right)=5.1 \text { days }
\end{aligned}
$$

49. We assume that there is no appreciable height difference between the two sides of the roof. Then the net force on the roof due to the air is the difference in pressure on the two sides of the roof, times the area of the roof. The difference in pressure can be found from Bernoulli's equation.

$$
\begin{aligned}
& P_{\text {inside }}+\frac{1}{2} \rho v_{\text {inside }}^{2}+\rho g y_{\text {inside }}=P_{\text {outside }}+\frac{1}{2} \rho v_{\text {ousside }}^{2}+\rho g y_{\text {ouside }} \rightarrow \\
& P_{\text {inside }}-P_{\text {outside }}=\frac{1}{2} \rho_{\text {air }} v_{\text {outside }}^{2}=\frac{F_{\text {air }}}{A_{\text {roof }}} \rightarrow \\
& F_{\text {air }}=\frac{1}{2} \rho_{\text {air }} \text { outside }_{2} A_{\text {roof }}=\frac{1}{2}\left(1.29 \mathrm{~kg} / \mathrm{m}^{3}\right)\left[(180 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)\right]^{2}(6.2 \mathrm{~m})(12.4 \mathrm{~m}) \\
& \quad=1.2 \times 10^{5} \mathrm{~N}
\end{aligned}
$$

50. Use the equation of continuity (Eq. 13-7b) to relate the volume flow of water at the two locations, and use Bernoulli's equation (Eq. 13-8) to relate the pressure conditions at the two locations. We assume that the two locations are at the same height. Express the pressures as atmospheric pressure plus gauge pressure. Use subscript " 1 " for the larger diameter, and " 2 " for the smaller diameter.

$$
\begin{aligned}
& A_{1} v_{1}=A_{2} v_{2} \rightarrow v_{2}=v_{1} \frac{A_{1}}{A_{2}}=v_{1} \frac{\pi r_{1}^{2}}{\pi r_{2}^{2}}=v_{1} \frac{r_{1}^{2}}{r_{2}^{2}} \\
& P_{0}+P_{1}+\frac{1}{2} \rho v_{1}^{2}+\rho g y_{1}=P_{0}+P_{2}+\frac{1}{2} \rho v_{2}^{2}+\rho g y_{2} \rightarrow \\
& P_{1}+\frac{1}{2} \rho v_{1}^{2}=P_{2}+\frac{1}{2} \rho v_{2}^{2}=P_{2}+\frac{1}{2} \rho v_{1}^{2} \frac{r_{1}^{4}}{r_{2}^{4}} \rightarrow v_{1}=\sqrt{\frac{2\left(P_{1}-P_{2}\right)}{\rho\left(\frac{r_{1}^{4}}{r_{2}^{4}}-1\right)}} \rightarrow \\
& A_{1} v_{1}=\pi r_{1}^{2} \sqrt{\frac{2\left(P_{1}-P_{2}\right)}{\rho\left(\frac{r_{1}^{4}}{r_{2}^{4}}-1\right)}}=\pi\left(3.0 \times 10^{-2} \mathrm{~m}\right)^{2} \sqrt{\frac{2\left(32.0 \times 10^{3} \mathrm{~Pa}-24.0 \times 10^{3} \mathrm{~Pa}\right)}{\left(1.0 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)\left(\frac{\left(3.0 \times 10^{-2} \mathrm{~m}\right)^{4}}{\left(2.25 \times 10^{-2} \mathrm{~m}\right)^{4}}-1\right)}} \\
& =7.7 \times 10^{-3} \mathrm{~m}^{3} / \mathrm{s}
\end{aligned}
$$

51. The air pressure inside the hurricane can be estimated using Bernoulli's equation. Assume the pressure outside the hurricane is air pressure, the speed of the wind outside the hurricane is 0 , and that the two pressure measurements are made at the same height.

$$
\begin{aligned}
& P_{\text {inside }}+\frac{1}{2} \rho v_{\text {inside }}^{2}+\rho g y_{\text {inside }}=P_{\text {outside }}+\frac{1}{2} \rho v_{\text {outside }}^{2}+\rho g y_{\text {outside }} \rightarrow \\
& P_{\text {inside }}=P_{\text {outside }}-\frac{1}{2} \rho_{\text {air }} v_{\text {inside }}^{2} \\
& \quad=1.013 \times 10^{5} \mathrm{~Pa}-\frac{1}{2}\left(1.29 \mathrm{~kg} / \mathrm{m}^{3}\right)\left[(300 \mathrm{~km} / \mathrm{h})\left(\frac{1000 \mathrm{~m}}{\mathrm{~km}}\right)\left(\frac{1 \mathrm{~h}}{3600 \mathrm{~s}}\right)\right]^{2} \\
& \quad=9.7 \times 10^{4} \mathrm{~Pa} \approx 0.96 \mathrm{~atm}
\end{aligned}
$$

52. The lift force would be the difference in pressure between the two wing surfaces, times the area of the wing surface. The difference in pressure can be found from Bernoulli's equation. We consider the two surfaces of the wing to be at the same height above the ground. Call the bottom surface of the wing point 1 , and the top surface point 2 .

$$
\begin{aligned}
P_{1} & +\frac{1}{2} \rho v_{1}^{2}+\rho g y_{1}=P_{2}+\frac{1}{2} \rho v_{2}^{2}+\rho g y_{2} \rightarrow P_{1}-P_{2}=\frac{1}{2} \rho\left(v_{2}^{2}-v_{1}^{2}\right) \\
F_{\text {lift }} & =\left(P_{1}-P_{2}\right)(\text { Area of wing })=\frac{1}{2} \rho\left(v_{2}^{2}-v_{1}^{2}\right) A \\
& =\frac{1}{2}\left(1.29 \mathrm{~kg} / \mathrm{m}^{3}\right)\left[(280 \mathrm{~m} / \mathrm{s})^{2}-(150 \mathrm{~m} / \mathrm{s})^{2}\right]\left(88 \mathrm{~m}^{2}\right)=3.2 \times 10^{6} \mathrm{~N}
\end{aligned}
$$

53. Consider the volume of fluid in the pipe. At each end of the pipe there is a force towards the contained fluid, given by $F=P A$. Since the area of the pipe is constant, we have that $F_{\text {net }}=\left(P_{1}-P_{2}\right) A$. Then, since the power required is the force on the fluid times its velocity, and $A V=Q=$ volume rate of flow, we have $P=F_{\text {net }} v=\left(P_{1}-P_{2}\right) A v=\left(P_{1}-P_{2}\right) Q$.
54. Use the equation of continuity (Eq. 13-7b) to relate the volume flow of water at the two locations, and use Bernoulli's equation (Eq. 13-8) to relate the conditions at the street to those at the top floor. Express the pressures as atmospheric pressure plus gauge pressure.

$$
\begin{aligned}
& A_{\text {street }} v_{\text {street }}=A_{\text {top }} v_{\text {top }} \rightarrow \\
& \begin{aligned}
& v_{\text {top }}= v_{\text {street }} \frac{A_{\text {street }}}{A_{\text {top }}}=(0.68 \mathrm{~m} / \mathrm{s}) \frac{\pi\left[\frac{1}{2}\left(5.0 \times 10^{-2} \mathrm{~m}\right)\right]^{2}}{\pi\left[\frac{1}{2}\left(2.8 \times 10^{-2} \mathrm{~m}\right)\right]^{2}}=2.168 \mathrm{~m} / \mathrm{s} \approx 2.2 \mathrm{~m} / \mathrm{s} \\
& P_{0}+P_{\substack{\text { gauge } \\
\text { street }}}+\frac{1}{2} \rho v_{\text {street }}^{2}+\rho g y_{\text {street }}=P_{0}+P_{\substack{\text { gauge } \\
\text { top }}}+\frac{1}{2} \rho v_{\text {top }}^{2}+\rho g y_{\text {top }} \rightarrow \\
& P_{\substack{\text { gauge } \\
\text { top }}}=P_{\substack{\text { gauge } \\
\text { street }}}+\frac{1}{2} \rho\left(v_{\text {street }}^{2}-v_{\text {top }}^{2}\right)+\rho g y\left(y_{\text {street }}-y_{\text {top }}\right) \\
&=(3.8 \mathrm{~atm})\left(\frac{1.013 \times 10^{5} \mathrm{~Pa}}{\mathrm{~atm}}\right)+\frac{1}{2}\left(1.00 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)\left[(0.68 \mathrm{~m} / \mathrm{s})^{2}-(2.168 \mathrm{~m} / \mathrm{s})^{2}\right] \\
&+\left(1.00 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(-18 \mathrm{~m}) \\
&=2.064 \times 10^{5} \mathrm{~Pa}\left(\frac{1 \mathrm{~atm}}{1.013 \times 10^{5} \mathrm{~Pa}}\right) \approx 2.0 \mathrm{~atm}
\end{aligned}
\end{aligned}
$$

Apply both Bernoulli's equation and the equation of continuity between the two openings of the tank. Note that the pressure at each opening will be atmospheric pressure.

$$
\begin{aligned}
& A_{2} v_{2}=A_{1} v_{1} \rightarrow v_{2}=v_{1} \frac{A_{1}}{A_{2}} \\
& P_{1}+\frac{1}{2} \rho v_{1}^{2}+\rho g y_{1}=P_{2}+\frac{1}{2} \rho v_{2}^{2}+\rho g y_{2} \rightarrow v_{1}^{2}-v_{2}^{2}=2 g\left(y_{2}-y_{1}\right)=2 g h \\
& v_{1}^{2}-\left(v_{1} \frac{A_{1}}{A_{2}}\right)=2 g h \rightarrow v_{1}^{2}\left(1-\frac{A_{1}^{2}}{A_{2}^{2}}\right)=2 g h \rightarrow v_{1}=\sqrt{\frac{2 g h}{\left(1-A_{1}^{2} / A_{2}^{2}\right)}}
\end{aligned}
$$

56. (a) Relate the conditions at the top surface and at the opening by Bernoulli's equation.

$$
\begin{aligned}
& P_{\text {top }}+\frac{1}{2} \rho v_{2}^{2}+\rho g y_{2}=P_{\text {opening }}+\frac{1}{2} \rho v_{1}^{2}+\rho g y_{1} \rightarrow P_{2}+P_{0}+\rho g\left(y_{2}-y_{1}\right)=P_{0}+\frac{1}{2} \rho v_{1}^{2} \rightarrow \\
& v_{1}=\sqrt{\frac{2 P_{2}}{\rho}+2 g\left(y_{2}-y_{1}\right)}
\end{aligned}
$$

$$
\begin{equation*}
v_{1}=\sqrt{\frac{2 P_{2}}{\rho}+2 g\left(y_{2}-y_{1}\right)}=\sqrt{\frac{2(0.85 \mathrm{~atm})\left(\frac{1.013 \times 10^{5} \mathrm{~Pa}}{\mathrm{~atm}}\right)}{\left(1.00 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)}+2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(2.4 \mathrm{~m})}=15 \mathrm{~m} / \mathrm{s} \tag{b}
\end{equation*}
$$

57. We assume that the water is launched from the same level at which it lands. Then the level range formula, derived in Example 3-10, applies. That formula is $R=\frac{v_{0}^{2} \sin 2 \theta_{0}}{g}$. If the range has increased by a factor of 4 , then the initial speed has increased by a factor of 2 . The equation of continuity is then applied to determine the change in the hose opening. The water will have the same volume rate of flow, whether the opening is large or small.

$$
(A v)_{\substack{\text { fully } \\ \text { open }}}=(A v)_{\substack{\text { partly } \\ \text { open }}}^{\rightarrow} \rightarrow A_{\substack{\text { partly } \\ \text { open }}}=A_{\substack{\text { fully } \\ \text { open }}}^{\substack{v_{\text {fully }} \\ v_{\text {ppent }} \\ \text { open }}}=A_{\substack{\text { fully } \\ \text { open }}}\left(\frac{1}{2}\right)
$$

Thus $1 / 2$ of the hose opening was blocked.
58. Use Bernoulli's equation to find the speed of the liquid as it leaves the opening, assuming that the speed of the liquid at the top is 0 , and that the pressure at each opening is air pressure.

$$
P_{1}+\frac{1}{2} \rho v_{1}^{2}+\rho g y_{1}=P_{2}+\frac{1}{2} \rho v_{2}^{2}+\rho g y_{2} \rightarrow v_{1}=\sqrt{2 g\left(h_{2}-h_{1}\right)}
$$

(a) Since the liquid is launched horizontally, the initial vertical speed is zero. Use Eq. 2-12b for constant acceleration to find the time of fall, with upward as the positive direction. Then multiply the time of fall times $v_{1}$, the (constant) horizontal speed.

$$
\begin{aligned}
& y=y_{0}+v_{0 y} t+\frac{1}{2} a_{y} t^{2} \rightarrow 0=h_{1}+0-\frac{1}{2} g t^{2} \rightarrow t=\sqrt{\frac{2 h_{1}}{g}} \\
& \Delta x=v_{1} t=\sqrt{2 g\left(h_{2}-h_{1}\right)} \sqrt{\frac{2 h_{1}}{g}}=2 \sqrt{\left(h_{2}-h_{1}\right) h_{1}}
\end{aligned}
$$

(b) We seek some height $h_{1}^{\prime}$ such that $2 \sqrt{\left(h_{2}-h_{1}\right) h_{1}}=2 \sqrt{\left(h_{2}-h_{1}^{\prime}\right) h_{1}^{\prime}}$.

$$
\begin{aligned}
& 2 \sqrt{\left(h_{2}-h_{1}\right) h_{1}}=2 \sqrt{\left(h_{2}-h_{1}^{\prime}\right) h_{1}^{\prime}} \rightarrow\left(h_{2}-h_{1}\right) h_{1}=\left(h_{2}-h_{1}^{\prime}\right) h_{1}^{\prime} \rightarrow \\
& h_{1}^{\prime 2}-h_{2} h_{1}^{\prime}+\left(h_{2}-h_{1}\right) h_{1}=0 \rightarrow \\
& h_{1}^{\prime}=\frac{h_{2} \pm \sqrt{h_{2}^{2}-4\left(h_{2}-h_{1}\right) h_{1}}}{2}=\frac{h_{2} \pm \sqrt{h_{2}^{2}-4 h_{1} h_{2}+4 h_{1}^{2}}}{2}=\frac{h_{2} \pm\left(h_{2}-2 h_{1}\right)}{2}=\frac{2 h_{2}-2 h_{1}}{2}, \frac{2 h_{1}}{2} \\
& h_{1}^{\prime}=h_{2}-h_{1}
\end{aligned}
$$

59. (a) Apply Bernoulli's equation to point 1, the exit hole, and point 2, the top surface of the liquid in the tank. Note that both points are open to the air and so the pressure is atmospheric pressure.
Also apply the equation of continuity $\left(A_{1} v_{1}=A_{2} v_{2}\right)$ to the same two points.

$$
\begin{aligned}
& P_{1}+\frac{1}{2} \rho v_{1}^{2}+\rho g y_{1}=P_{2}+\frac{1}{2} \rho v_{2}^{2}+\rho g y_{2} \rightarrow P_{\mathrm{atm}}+\frac{1}{2} \rho\left(v_{1}^{2}-v_{2}^{2}\right)=P_{\mathrm{atm}}+\rho g\left(y_{2}-y_{1}\right) \rightarrow \\
& \frac{1}{2} \rho\left(v_{1}^{2}-v_{2}^{2}\right)=\rho g h \rightarrow\left(v_{1}^{2}-v_{2}^{2}\right)=2 g h \rightarrow\left(\frac{A_{2}^{2}}{A_{1}^{2}}-1\right) v_{2}^{2}=2 g h \rightarrow
\end{aligned}
$$

$$
v_{2}=\sqrt{\frac{2 g h}{\left(\frac{A_{2}^{2}}{A_{1}^{2}}-1\right)}}=\sqrt{\frac{2 g h A_{1}^{2}}{\left(A_{2}^{2}-A_{1}^{2}\right)}}
$$

Note that since the water level is decreasing, we have $v_{2}=-\frac{d h}{d t}$, and so $\frac{d h}{d t}=-\sqrt{\frac{2 g h A_{1}^{2}}{\left(A_{2}^{2}-A_{1}^{2}\right)}}$.
(b) Integrate to find the height as a function of time.

$$
\begin{aligned}
& \frac{d h}{d t}=-\sqrt{\frac{2 g h A_{1}^{2}}{\left(A_{2}^{2}-A_{1}^{2}\right)}} \rightarrow \frac{d h}{\sqrt{h}}=-\sqrt{\frac{2 g A_{1}^{2}}{\left(A_{2}^{2}-A_{1}^{2}\right)}} d t \rightarrow \int_{h_{0}}^{h} \frac{d h}{\sqrt{h}}=-\sqrt{\frac{2 g A_{1}^{2}}{\left(A_{2}^{2}-A_{1}^{2}\right)}} \int_{0}^{t} d t \rightarrow \\
& 2\left(\sqrt{h}-\sqrt{h_{0}}\right)=-\sqrt{\frac{2 g A_{1}^{2}}{\left(A_{2}^{2}-A_{1}^{2}\right)}} t \rightarrow h=\left[\sqrt{h_{0}}-t \sqrt{\frac{g A_{1}^{2}}{2\left(A_{2}^{2}-A_{1}^{2}\right)}}\right]^{2}
\end{aligned}
$$

(c) We solve for the time at which $h=0$, given the other parameters. In particular,

$$
\begin{aligned}
& A_{1}=\pi\left(0.25 \times 10^{-2} \mathrm{~m}\right)^{2}=1.963 \times 10^{-5} \mathrm{~m}^{2} ; A_{2}=\frac{1.3 \times 10^{-3} \mathrm{~m}^{3}}{0.106 \mathrm{~m}}=1.226 \times 10^{-2} \mathrm{~m}^{2} \\
& {\left[\sqrt{h_{0}}-t \sqrt{\frac{g A_{1}^{2}}{2\left(A_{2}^{2}-A_{1}^{2}\right)}}\right]^{2}=0 \rightarrow} \\
& t=\sqrt{\frac{2 h_{0}\left(A_{2}^{2}-A_{1}^{2}\right)}{g A_{1}^{2}}}=\sqrt{\frac{2(0.106 \mathrm{~m})\left[\left(1.226 \times 10^{-2} \mathrm{~m}^{2}\right)^{2}-\left(1.963 \times 10^{-5} \mathrm{~m}^{2}\right)^{2}\right]}{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(1.963 \times 10^{-5} \mathrm{~m}^{2}\right)^{2}}}=92 \mathrm{~s}
\end{aligned}
$$

60. (a) Apply the equation of continuity and Bernoulli's equation at the same height to the wide and narrow portions of the tube.

$$
\begin{aligned}
& A_{2} v_{2}=A_{1} v_{1} \rightarrow v_{2}=v_{1} \frac{A_{1}}{A_{2}} \\
& P_{1}+\frac{1}{2} \rho v_{1}^{2}=P_{2}+\frac{1}{2} \rho v_{2}^{2} \rightarrow \frac{2\left(P_{1}-P_{2}\right)}{\rho}=v_{2}^{2}-v_{1}^{2} \rightarrow \\
& \left(v_{1} \frac{A_{1}}{A_{2}}\right)^{2}-v_{1}^{2}=\frac{2\left(P_{1}-P_{2}\right)}{\rho} \rightarrow v_{1}^{2}\left(\frac{A_{1}^{2}}{A_{2}^{2}}-\frac{A_{2}^{2}}{A_{2}^{2}}\right)=\frac{2\left(P_{1}-P_{2}\right)}{\rho} \rightarrow \\
& v_{1}^{2}=\frac{2 A_{2}^{2}\left(P_{1}-P_{2}\right)}{\rho\left(A_{1}^{2}-A_{2}^{2}\right)} \rightarrow v_{1}=A_{2} \sqrt{\frac{2\left(P_{1}-P_{2}\right)}{\rho\left(A_{1}^{2}-A_{2}^{2}\right)}} \\
& \text { (b) } v_{1}=A_{2} \sqrt{\frac{2\left(P_{1}-P_{2}\right)}{\rho\left(A_{1}^{2}-A_{2}^{2}\right)}} \\
& =\pi\left[\frac{1}{2}(0.010 \mathrm{~m})\right]^{2} \sqrt{\frac{2(18 \mathrm{~mm} \mathrm{Hg})\left(\frac{133 \mathrm{~N} / \mathrm{m}^{2}}{\mathrm{~mm} \mathrm{Hg}}\right)}{\left(1000 \mathrm{~kg} / \mathrm{m}^{3}\right)\left(\pi^{2}\left[\frac{1}{2}(0.030 \mathrm{~m})\right]^{4}-\pi^{2}\left[\frac{1}{2}(0.010 \mathrm{~m})\right]^{4}\right)}}=0.24 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

61. (a) Relate the conditions inside the rocket and just outside the exit orifice by means of Bernoulli's equation and the equation of continuity. We ignore any height difference between the two locations.

$$
\begin{aligned}
& P_{\text {in }}+\frac{1}{2} \rho v_{\text {in }}^{2}+\rho g y_{\text {in }}=P_{\text {out }}+\frac{1}{2} \rho v_{\text {out }}^{2}+\rho g y_{\text {out }} \rightarrow P+\frac{1}{2} \rho v_{\text {in }}^{2}=P_{0}+\frac{1}{2} \rho v_{\text {out }}^{2} \rightarrow \\
& \frac{2\left(P-P_{0}\right)}{\rho}=v_{\text {out }}^{2}-v_{\text {in }}^{2}=v_{\text {out }}^{2}\left[1-\left(\frac{v_{\text {in }}}{v_{\text {out }}}\right)^{2}\right] \\
& A_{\text {in }} v_{\text {in }}=A_{\text {out }} v_{\text {out }} \rightarrow A v_{\text {in }}=A_{0} v_{\text {out }} \rightarrow \frac{v_{\text {in }}}{v_{\text {out }}}=\frac{A_{0}}{A} \ll 1 \rightarrow \\
& \frac{2\left(P-P_{0}\right)}{\rho}=v_{\text {out }}^{2}\left[1-\left(\frac{v_{\text {in }}}{v_{\text {out }}}\right)^{2}\right] \approx v_{\text {out }}^{2} \rightarrow v_{\text {out }}=v=\sqrt{\frac{2\left(P-P_{0}\right)}{\rho}}
\end{aligned}
$$

(b) Thrust is defined in section 9-10, by $F_{\text {thrust }}=v_{\text {rel }} \frac{d m}{d t}$, and is interpreted as the force on the rocket due to the ejection of mass.

$$
\begin{aligned}
F_{\text {troust }} & =v_{\text {rel }} \frac{d m}{d t}=v_{\text {out }} \frac{d(\rho V)}{d t}=v_{\text {out }} \rho \frac{d V}{d t}=v_{\text {out }} \rho\left(v_{\text {out }} A_{\text {out }}\right)=\rho v^{2} A_{0}=\rho \frac{2\left(P-P_{0}\right)}{\rho} A_{0} \\
& =2\left(P-P_{0}\right) A_{0}
\end{aligned}
$$

62. There is a forward force on the exiting water, and so by Newton's third law there is an equal force pushing backwards on the hose. To keep the hose stationary, you push forward on the hose, and so the hose pushes backwards on you. So the force on the exiting water is the same magnitude as the force on the person holding the hose. Use Newton's second law and the equation of continuity to find the force. Note that the $450 \mathrm{~L} / \mathrm{min}$ flow rate is the volume of water being accelerated per unit time. Also, the flow rate is the product of the cross-sectional area of the moving fluid, times the speed of the fluid, and so $\frac{V}{t}=A_{1} v_{1}=A_{2} v_{2}$.

$$
\begin{aligned}
F & =m \frac{\Delta v}{\Delta t}=m \frac{v_{2}-v_{1}}{t}=\rho\left(\frac{V}{t}\right)\left(v_{2}-v_{1}\right)=\rho\left(\frac{V}{t}\right)\left(\frac{A_{2} v_{2}}{A_{2}}-\frac{A_{1} v_{1}}{A_{1}}\right)=\rho\left(\frac{V}{t}\right)^{2}\left(\frac{1}{A_{2}}-\frac{1}{A_{1}}\right) \\
& =\rho\left(\frac{V}{t}\right)^{2}\left(\frac{1}{\pi r_{2}^{2}}-\frac{1}{\pi r_{1}^{2}}\right) \\
& =\left(1.00 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)\left(\frac{450 \mathrm{~L}}{\min } \times \frac{1 \mathrm{~min}}{60 \mathrm{~s}} \times \frac{1 \mathrm{~m}^{3}}{1000 \mathrm{~L}}\right)^{2}\left(\frac{1}{\pi \frac{1}{2}\left(0.75 \times 10^{-2} \mathrm{~m}\right)^{2}}-\frac{1}{\pi \frac{1}{2}\left(7.0 \times 10^{-2} \mathrm{~m}\right)^{2}}\right) \\
& =1259 \mathrm{~N} \approx 1300 \mathrm{~N}
\end{aligned}
$$

63. Apply Eq. 13-11 for the viscosity force. Use the average radius to calculate the plate area.

$$
F=\eta A \frac{v}{\ell} \rightarrow \eta=\frac{F \ell}{A v}=\frac{\left(\frac{\tau}{r_{\text {inner }}}\right)\left(r_{\text {outer }}-r_{\text {imer }}\right)}{\left(2 \pi r_{\text {avg }} h\right)\left(\omega r_{\text {iner }}\right)}
$$

$$
=\frac{\left(\frac{0.024 \mathrm{~m} \cdot \mathrm{~N}}{0.0510 \mathrm{~m}}\right)\left(0.20 \times 10^{-2} \mathrm{~m}\right)}{2 \pi(0.0520 \mathrm{~m})(0.120 \mathrm{~m})\left(57 \frac{\mathrm{rev}}{\min } \times \frac{2 \pi \mathrm{rad}}{\mathrm{rev}} \times \frac{1 \mathrm{~min}}{60 \mathrm{~s}}\right)(0.0510 \mathrm{~m})}=7.9 \times 10^{-2} \mathrm{~Pa} \cdot \mathrm{~s}
$$

64. The relationship between velocity and the force of viscosity is given by Eq. 13-11, $F_{\text {vis }}=\eta A \frac{v}{\ell}$. The variable $A$ is the area of contact between the moving surface and the liquid. For a cylinder, $A=2 \pi r h$. The variable $\ell$ is the thickness of the fluid layer between the two surfaces. See the diagram. If the object falls with terminal velocity, then the net force must be 0 , and so the viscous force will equal the weight. Note that

$$
\begin{aligned}
& \ell=\frac{1}{2}(1.00 \mathrm{~cm}-0.900 \mathrm{~cm})=0.05 \mathrm{~cm} . \\
& \\
& F_{\text {weight }}=F_{\text {vis }} \rightarrow m g=\eta A \frac{v}{\ell} \rightarrow \\
& v=\frac{m g \ell}{\eta A}=\frac{m g \ell}{2 \pi r h \eta}=\frac{(0.15 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(0.050 \times 10^{-2} \mathrm{~m}\right)}{2 \pi\left(0.450 \times 10^{-2} \mathrm{~m}\right)(0.300 \mathrm{~m})\left(200 \times 10^{-3} \mathrm{~N} \cdot \mathrm{~s} / \mathrm{m}^{2}\right)} \\
& \quad=0.43 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

65. Use Poiseuille's equation (Eq. 13-12) to find the pressure difference.

$$
\begin{aligned}
& Q=\frac{\pi R^{4}\left(P_{2}-P_{1}\right)}{8 \eta \ell} \rightarrow \\
& \left(P_{2}-P_{1}\right)=\frac{8 Q \eta \ell}{\pi R^{4}}=\frac{8\left[\frac{6.2 \times 10^{-3} \mathrm{~L}}{\min } \times \frac{1 \mathrm{~min}}{60 \mathrm{~s}} \times \frac{1 \times 10^{-3} \mathrm{~m}^{3}}{1 \mathrm{~L}}\right](0.2 \mathrm{~Pa} \cdot \mathrm{~s})\left(8.6 \times 10^{-2} \mathrm{~m}\right)}{\pi\left(0.9 \times 10^{-3} \mathrm{~m}\right)^{4}} \\
& \quad=6900 \mathrm{~Pa}
\end{aligned}
$$

66. From Poiseuille's equation, the volume flow rate $Q$ is proportional to $R^{4}$ if all other factors are the same. Thus $Q / R^{4}=\frac{V}{t} \frac{1}{R^{4}}$ is constant. If the volume of water used to water the garden is to be the same in both cases, then $t R^{4}$ is constant.

$$
t_{1} R_{1}^{4}=t_{2} R_{2}^{4} \rightarrow t_{2}=t_{1}\left(\frac{R_{1}}{R_{2}}\right)^{4}=t_{1}\left(\frac{3 / 8}{5 / 8}\right)^{4}=0.13 t_{1}
$$

Thus the time has been cut by $87 \%$.
67. Use Poiseuille's equation to find the radius, and then double the radius to the diameter.

$$
Q=\frac{\pi R^{4}\left(P_{2}-P_{1}\right)}{8 \eta \ell} \rightarrow
$$

$$
d=2 R=2\left[\frac{8 \eta \ell Q}{\pi\left(P_{2}-P_{1}\right)}\right]^{1 / 4}=2\left[\frac{8\left(1.8 \times 10^{-5} \mathrm{~Pa} \cdot \mathrm{~s}\right)(15.5 \mathrm{~m})\left(\frac{8.0 \times 14.0 \times 4.0 \mathrm{~m}^{3}}{720 \mathrm{~s}}\right)}{\pi\left(0.71 \times 10^{-3} \mathrm{~atm}\right)\left(1.013 \times 10^{5} \mathrm{~Pa} / \mathrm{atm}\right)}\right]^{1 / 4}=0.10 \mathrm{~m}
$$

68. Use Poiseuille's equation to find the pressure difference.

$$
\begin{aligned}
& Q=\frac{\pi R^{4}\left(P_{2}-P_{1}\right)}{8 \eta \ell} \rightarrow \\
& \begin{aligned}
\left(P_{2}-P_{1}\right) & =\frac{8 Q \eta \ell}{\pi R^{4}}=\frac{8\left(650 \mathrm{~cm}^{3} / \mathrm{s}\right)\left(10^{-6} \mathrm{~m}^{3} / \mathrm{cm}^{3}\right)(0.20 \mathrm{~Pa} \cdot \mathrm{~s})\left(1.9 \times 10^{3} \mathrm{~m}\right)}{\pi(0.145 \mathrm{~m})^{4}} \\
= & 1423 \mathrm{~Pa} \approx 1400 \mathrm{~Pa}
\end{aligned}
\end{aligned}
$$

69. (a) $R e=\frac{2 \bar{v} r \rho}{\eta}=\frac{2(0.35 \mathrm{~m} / \mathrm{s})\left(0.80 \times 10^{-2} \mathrm{~m}\right)\left(1.05 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)}{4 \times 10^{-3} \mathrm{~Pa} \cdot \mathrm{~s}}=1470$

The flow is laminar at this speed.
(b) Since the velocity is doubled the Reynolds number will double to 2940. The flow is turbulent at this speed.
70. From Poiseuille's equation, Eq. 13-12, the volume flow rate $Q$ is proportional to $R^{4}$ if all other factors are the same. Thus $Q / R^{4}$ is constant.

$$
\frac{Q_{\text {final }}}{R_{\text {final }}^{4}}=\frac{Q_{\text {initial }}}{R_{\text {initial }}^{4}} \rightarrow R_{\text {final }}=\left(\frac{Q_{\text {final }}}{Q_{\text {initial }}}\right)^{1 / 4} R_{\text {initial }}=(0.15)^{1 / 4} R_{\text {initial }}=0.622 R_{\text {initial }}, \text { a } 38 \% \text { reduction. }
$$

71. The fluid pressure must be 78 torr higher than air pressure as it exits the needle, so that the blood will enter the vein. The pressure at the entrance to the needle must be higher than 78 torr, due to the viscosity of the blood. To produce that excess pressure, the blood reservoir is placed above the level of the needle. Use Poiseuille's equation to calculate the excess pressure needed due to the viscosity, and then use Eq. 13-6b to find the height of the blood reservoir necessary to produce that excess pressure.

$$
\begin{aligned}
Q & =\frac{\pi R^{4}\left(P_{2}-P_{1}\right)}{8 \eta_{\text {blood }} \ell} \rightarrow P_{2}=P_{1}+\frac{8 \eta_{\text {blood }} \ell Q}{\pi R^{4}}=\rho_{\text {blood }} g \Delta h \rightarrow \\
\Delta h & =\frac{1}{\rho_{\text {blood }} g}\left(P_{1}+\frac{8 \eta_{\text {blood }} \ell Q}{\pi R^{4}}\right) \\
& =\frac{(78 \mathrm{~mm}-\mathrm{Hg})\left(\frac{133 \mathrm{~N} / \mathrm{m}^{2}}{1 \mathrm{~mm}-\mathrm{Hg}}\right)+}{\left(1.05 \times 10^{3} \frac{\mathrm{~kg}}{\mathrm{~m}^{3}}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}\left(\frac{8\left(4 \times 10^{-3} \mathrm{~Pa} \cdot \mathrm{~s}\right)\left(2.5 \times 10^{-2} \mathrm{~m}\right)\left(\frac{2.0 \times 10^{-6} \mathrm{~m}^{3}}{60 \mathrm{~s}}\right)}{\pi\left(0.4 \times 10^{-3} \mathrm{~m}\right)^{4}}\right) \\
& =1.04 \mathrm{~m} \approx 1.0 \mathrm{~m}
\end{aligned}
$$

72. In Figure 13-35, we have $\gamma=F / 2 \ell$. Use this to calculate the force.

$$
\gamma=\frac{F}{2 \ell}=\frac{3.4 \times 10^{-3} \mathrm{~N}}{2(0.070 \mathrm{~m})}=2.4 \times 10^{-2} \mathrm{~N} / \mathrm{m}
$$

73. In Figure 13-35, we have $\gamma=F / 2 \ell$. Use this relationship to calculate the force.

$$
\gamma=F / 2 \ell \rightarrow F=2 \gamma \ell=2(0.025 \mathrm{~N} / \mathrm{m})(0.245 \mathrm{~m})=1.2 \times 10^{-2} \mathrm{~N}
$$

74. (a) We assume that the weight of the platinum ring is negligible. Then the surface tension is the force to lift the ring, divided by the length of surface that is being pulled. Surface tension will act at both edges of the ring, as in Figure 13-35b. Thus $\gamma=\frac{F}{2(2 \pi r)}=\frac{F}{4 \pi r}$
(b)

$$
\gamma=\frac{F}{4 \pi r}=\frac{5.80 \times 10^{-3} \mathrm{~N}}{4 \pi\left(2.8 \times 10^{-2} \mathrm{~m}\right)}=1.6 \times 10^{-2} \mathrm{~N} / \mathrm{m}
$$

75. As an estimate, we assume that the surface tension force acts vertically. We assume that the freebody diagram for the cylinder is similar to Figure 13-37(a) in the text. The weight must equal the total surface tension force. The needle is of length $\ell$.

$$
\begin{aligned}
& m g=2 F_{\mathrm{T}} \rightarrow \rho_{\text {needle }} \pi\left(\frac{1}{2} d_{\text {needle }}\right)^{2} \ell g=2 \gamma \ell \rightarrow \\
& d_{\text {needle }}=\sqrt{\frac{8 \gamma}{\rho_{\text {needle }} \pi g}}=\sqrt{\frac{8(0.072 \mathrm{~N} / \mathrm{m})}{\left(7800 \mathrm{~kg} / \mathrm{m}^{3}\right) \pi\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}}=1.55 \times 10^{-3} \mathrm{~m} \approx 1.5 \mathrm{~mm}
\end{aligned}
$$

76. Consider half of the soap bubble - a hemisphere. The forces on the hemisphere will be the surface tensions on the two circles and the net force from the excess pressure between the inside and the outside of the bubble. This net force is the sum of all the forces perpendicular to the surface of the hemisphere, but must be parallel to the surface tension. Therefore we can find it by finding the force on the circle that is the base of the hemisphere. The total force must be zero. Note that the forces $\overrightarrow{\mathbf{F}}_{\text {Touter }}$ and $\overrightarrow{\mathbf{F}}_{\mathrm{T} \text { inner }}$ act over the entire length of the circles to which they are applied. The diagram may look like there are 4 tension forces, but there are only 2 . Likewise, there is only 1 pressure force, $\overrightarrow{\mathbf{F}}_{\mathrm{p}}$, but it acts over the area of the hemisphere.

$$
2 F_{\mathrm{T}}=F_{\mathrm{P}} \rightarrow 2(2 \pi r \gamma)=\pi r^{2} \Delta P \rightarrow \Delta P=\frac{4 \gamma}{r}
$$

77. The mass of liquid that rises in the tube will have the force of gravity acting down on it, and the force of surface tension acting upwards. The two forces must be equal for the liquid to be in equilibrium. The surface tension force is the surface tension times the circumference of the tube, since the tube circumference is the length of the "cut" in the liquid surface. The mass of the risen liquid is the density times the volume.

$$
F_{\mathrm{T}}=m g \rightarrow \gamma 2 \pi r=\rho \pi r^{2} h g \rightarrow h=2 \gamma / \rho g r
$$

78. (a) The fluid in the needle is confined, and so Pascal's principle may be applied.

$$
\begin{aligned}
P_{\text {plunger }} & =P_{\text {needle }} \rightarrow \frac{F_{\text {plunger }}}{A_{\text {plunger }}}=\frac{F_{\text {needle }}}{A_{\text {needle }}} \rightarrow \\
F_{\text {needle }} & =F_{\text {plunger }} \frac{A_{\text {needle }}}{A_{\text {plunger }}}=F_{\text {plunger }} \frac{\pi r_{\text {needle }}^{2}}{\pi r_{\text {plunger }}^{2}}=F_{\text {plunger }} \frac{r_{\text {necdle }}^{2}}{r_{\text {plunger }}^{2}}=(2.8 \mathrm{~N}) \frac{\left(0.10 \times 10^{-3} \mathrm{~m}\right)^{2}}{\left(0.65 \times 10^{-2} \mathrm{~m}\right)^{2}} \\
& =6.627 \times 10^{-4} \mathrm{~N} \approx 6.6 \times 10^{-4} \mathrm{~N} \\
\text { (b) } \quad F_{\text {plunger }} & =P_{\text {plunger }} A_{\text {plunger }}=(75 \mathrm{~mm}-\mathrm{Hg})\left(\frac{133 \mathrm{~N} / \mathrm{m}^{2}}{1 \mathrm{~mm}-\mathrm{Hg}}\right) \pi\left(0.65 \times 10^{-2} \mathrm{~m}\right)^{2}=1.3 \mathrm{~N}
\end{aligned}
$$

79. The pressures for parts $(a)$ and $(b)$ stated in this problem are gauge pressures, relative to atmospheric pressure. The pressure change due to depth in a fluid is given by $\Delta P=\rho g \Delta h$.
(a) $\Delta h=\frac{\Delta P}{\rho g}=\frac{(55 \mathrm{~mm}-\mathrm{Hg})\left(\frac{133 \mathrm{~N} / \mathrm{m}^{2}}{1 \mathrm{~mm}-\mathrm{Hg}}\right)}{\left(1.00 \frac{\mathrm{~g}}{\mathrm{~cm}^{3}} \times \frac{1 \mathrm{~kg}}{1000 \mathrm{~g}} \times \frac{10^{6} \mathrm{~cm}^{3}}{1 \mathrm{~m}^{3}}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=0.75 \mathrm{~m}$
(b) $\Delta h=\frac{\Delta P}{\rho g}=\frac{\left(650 \mathrm{~mm}-\mathrm{H}_{2} \mathrm{O}\right)\left(\frac{9.81 \mathrm{~N} / \mathrm{m}^{2}}{1 \mathrm{~mm}-\mathrm{H}_{2} \mathrm{O}}\right)}{\left(1.00 \frac{\mathrm{~g}}{\mathrm{~cm}^{3}} \times \frac{1 \mathrm{~kg}}{1000 \mathrm{~g}} \times \frac{10^{6} \mathrm{~cm}^{3}}{1 \mathrm{~m}^{3}}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=0.65 \mathrm{~m}$
(c) For the fluid to just barely enter the vein, the fluid pressure must be the same as the blood pressure.

$$
\Delta h=\frac{\Delta P}{\rho g}=\frac{(78 \mathrm{~mm}-\mathrm{Hg})\left(\frac{133 \mathrm{~N} / \mathrm{m}^{2}}{1 \mathrm{~mm}-\mathrm{Hg}}\right)}{\left(1.00 \frac{\mathrm{~g}}{\mathrm{~cm}^{3}} \times \frac{1 \mathrm{~kg}}{1000 \mathrm{~g}} \times \frac{10^{6} \mathrm{~cm}^{3}}{1 \mathrm{~m}^{3}}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=1.059 \mathrm{~m} \approx 1.1 \mathrm{~m}
$$

80. The ball has three vertical forces on it - string tension, buoyant force, and gravity. See the free-body diagram for the ball. The net force must be 0 .

$$
\begin{aligned}
& F_{\text {net }}=F_{\mathrm{T}}+F_{\mathrm{B}}-m g=0 \rightarrow \\
& F_{\mathrm{T}}=m g-F_{\mathrm{B}}=\frac{4}{3} \pi r^{3} \rho_{\mathrm{Cu}} g-\frac{4}{3} \pi r^{3} \rho_{\text {water }} g=\frac{4}{3} \pi r^{3} g\left(\rho_{\mathrm{Cu}}-\rho_{\text {water }}\right) \\
& \qquad=\frac{4}{3} \pi(0.013 \mathrm{~m})^{3}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(8900 \mathrm{~kg} / \mathrm{m}^{3}-1000 \mathrm{~kg} / \mathrm{m}^{3}\right)=0.7125 \mathrm{~N} \approx 0.71 \mathrm{~N}
\end{aligned}
$$



Since the water pushes up on the ball via the buoyant force, there is a downward force on the water due to the ball, equal in magnitude to the buoyant force. That mass-equivalent of that force (indicated by $m_{\mathrm{B}}=F_{\mathrm{B}} / g$ ) will show up as an increase in the balance reading.

$$
\begin{aligned}
& F_{\mathrm{B}}=\frac{4}{3} \pi r^{3} \rho_{\text {water }} g \rightarrow \\
& m_{\mathrm{B}}=\frac{F_{\mathrm{B}}}{g}=\frac{4}{3} \pi r^{3} \rho_{\text {water }}=\frac{4}{3} \pi(0.013 \mathrm{~m})^{3}\left(1000 \mathrm{~kg} / \mathrm{m}^{3}\right)=9.203 \times 10^{-3} \mathrm{~kg}=9.203 \mathrm{~g}
\end{aligned}
$$

Balance reading $=998.0 \mathrm{~g}+9.2 \mathrm{~g}=1007.2 \mathrm{~g}$
81. The change in pressure with height is given by $\Delta P=\rho g \Delta h$.

$$
\begin{aligned}
& \Delta P=\rho g \Delta h \rightarrow \frac{\Delta P}{P_{0}}=\frac{\rho g \Delta h}{P_{0}}=\frac{\left(1.29 \mathrm{~kg} / \mathrm{m}^{3}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(380 \mathrm{~m})}{1.013 \times 10^{5} \mathrm{~Pa}}=0.047 \rightarrow \\
& \Delta P=0.047 \mathrm{~atm}
\end{aligned}
$$

82. (a) The input pressure is equal to the output pressure.

$$
\begin{aligned}
P_{\text {input }} & =P_{\text {output }} \rightarrow \frac{F_{\text {input }}}{A_{\text {input }}}=\frac{F_{\text {output }}}{A_{\text {output }}} \rightarrow \\
A_{\text {input }} & =A_{\text {output }} \frac{F_{\text {input }}}{F_{\text {output }}}=\pi\left(9.0 \times 10^{-2} \mathrm{~m}\right)^{2} \frac{350 \mathrm{~N}}{(920 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=9.878 \times 10^{-4} \mathrm{~m}^{2} \\
& \approx 9.9 \times 10^{-4} \mathrm{~m}^{2}
\end{aligned}
$$

(b) The work is the force needed to lift the car (its weight) times the vertical distance lifted.

$$
W=m g h=(920 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.42 \mathrm{~m})=3787 \mathrm{~J} \approx 3800 \mathrm{~J}
$$

(c) The work done by the input piston is equal to the work done in lifting the car.

$$
\begin{aligned}
& W_{\text {input }}=W_{\text {output }} \rightarrow F_{\text {input }} d_{\text {ipput }}=F_{\text {oupput }} d_{\text {ouptut }}=m g h \rightarrow \\
& h=\frac{F_{\text {input }} d_{\text {input }}}{m g}=\frac{(350 \mathrm{~N})(0.13 \mathrm{~m})}{(920 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=5.047 \times 10^{-3} \mathrm{~m} \approx 5.0 \times 10^{-3} \mathrm{~m}
\end{aligned}
$$

(d) The number of strokes is the full distance divided by the distance per stroke.

$$
h_{\text {full }}=N h_{\text {stroke }} \rightarrow N=\frac{h_{\text {full }}}{h_{\text {stroke }}}=\frac{0.42 \mathrm{~m}}{5.047 \times 10^{-3} \mathrm{~m}}=83 \text { strokes }
$$

(e) The work input is the input force times the total distance moved by the input piston.

$$
W_{\text {input }}=N F_{\text {input }} d_{\text {input }} \rightarrow 83(350 \mathrm{~N})(0.13 \mathrm{~m})=3777 \mathrm{~J} \approx 3800 \mathrm{~J}
$$

Since the work input is equal to the work output, energy is conserved.
83. The pressure change due to a change in height is given by $\Delta P=\rho g \Delta h$. That pressure is the excess force on the eardrum, divided by the area of the eardrum.

$$
\begin{aligned}
& \Delta P=\rho g \Delta h=\frac{F}{A} \rightarrow \\
& F=\rho g \Delta h A=\left(1.29 \mathrm{~kg} / \mathrm{m}^{3}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(950 \mathrm{~m})\left(0.20 \times 10^{-4} \mathrm{~m}^{2}\right)=0.24 \mathrm{~N}
\end{aligned}
$$

84. The change in pressure with height is given by $\Delta P=\rho g \Delta h$.

$$
\begin{aligned}
& \Delta P=\rho g \Delta h \rightarrow \frac{\Delta P}{P_{0}}=\frac{\rho g \Delta h}{P_{0}}=\frac{\left(1.05 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(6 \mathrm{~m})}{1.013 \times 10^{5} \mathrm{~Pa}}=0.609 \rightarrow \\
& \Delta P=0.6 \mathrm{~atm}
\end{aligned}
$$

85. The pressure difference due to the lungs is the pressure change in the column of water.

$$
\Delta P=\rho g \Delta h \rightarrow \Delta h=\frac{\Delta P}{\rho g}=\frac{(75 \mathrm{~mm}-\mathrm{Hg})\left(\frac{133 \mathrm{~N} / \mathrm{m}^{2}}{1 \mathrm{~mm}-\mathrm{Hg}}\right)}{\left(1.00 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=1.018 \mathrm{~m} \approx 1.0 \mathrm{~m}
$$

86. We use the relationship developed in Example 13-5.

$$
P=P_{0} e^{-\left(\rho_{0} g / P_{0}\right) y}=(1.0 \mathrm{~atm}) e^{-\left(1.25 \times 10^{-4} \mathrm{~m}^{-1}\right)(2400 \mathrm{~m})}=0.74 \mathrm{~atm}
$$

87. The buoyant force, equal to the weight of mantle displaced, must be equal to the weight of the continent. Let $h$ represent the full height of the continent, and $y$ represent the height of the continent above the surrounding rock.

$$
\begin{aligned}
& W_{\text {continent }}=W_{\substack{\text { displaced } \\
\text { mantee }}} \rightarrow A h \rho_{\text {continent }} g=A(h-y) \rho_{\text {mantle }} g \rightarrow \\
& y=h\left(1-\frac{\rho_{\text {continent }}}{\rho_{\text {mante }}}\right)=(35 \mathrm{~km})\left(1-\frac{2800 \mathrm{~kg} / \mathrm{m}^{3}}{3300 \mathrm{~kg} / \mathrm{m}^{3}}\right)=5.3 \mathrm{~km}
\end{aligned}
$$

88. The "extra" buoyant force on the ship, due to the loaded fresh water, is the weight of "extra" displaced seawater, as indicated by the ship floating lower in the sea. This buoyant force is given by $F_{\text {buogant }}=V_{\text {displaced }} \rho_{\text {sea }}^{\text {water }}$. But this "extra" buoyant force is what holds up the fresh water, and so must also be equal to the weight of the fresh water.

$$
F_{\text {buoyant }}=V_{\text {displaced }} \rho_{\text {sea }} g=m_{\text {wrester }} g \rightarrow m_{\text {fresh }}=\left(2240 \mathrm{~m}^{2}\right)(8.50 \mathrm{~m})\left(1025 \mathrm{~kg} / \mathrm{m}^{3}\right)=1.95 \times 10^{7} \mathrm{~kg}
$$

This can also be expressed as a volume.

$$
V_{\text {fresh }}=\frac{m_{\text {fresh }}}{\rho_{\text {fresh }}}=\frac{1.95 \times 10^{7} \mathrm{~kg}}{1.00 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}}=1.95 \times 10^{4} \mathrm{~m}^{3}=1.95 \times 10^{7} \mathrm{~L}
$$

89. (a) We assume that the one descending is close enough to the surface of the Earth that constant density may be assumed. Take Eq. 13-6b, modify it for rising, and differentiate it with respect to time.

$$
\begin{aligned}
& P=P_{0}-\rho g y \rightarrow \\
& \frac{d P}{d t}=-\rho g \frac{d y}{d t}=-\left(1.29 \mathrm{~kg} / \mathrm{m}^{3}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(-7.0 \mathrm{~m} / \mathrm{s})=88.49 \mathrm{~Pa} / \mathrm{s} \approx 88 \mathrm{~Pa} / \mathrm{s}
\end{aligned}
$$

(b) $\Delta y=v t \rightarrow t=\frac{\Delta y}{v}=\frac{350 \mathrm{~m}}{7.0 \mathrm{~m} / \mathrm{s}}=50 \mathrm{~s}$ (2 sig. fig.)
90. The buoyant force must be equal to the weight of the water displaced by the full volume of the logs, and must also be equal to the full weight of the raft plus the passengers. Let $N$ represent the number of passengers.
weight of water displaced by $\log s=$ weight of people + weight of logs

$$
\begin{aligned}
& 12\left(V_{\log } \rho_{\text {water }} g\right)=N m_{\text {person }} g+12\left(V_{\log } \rho_{\log } g\right) \rightarrow \\
& N=\frac{12 V_{\log }\left(\rho_{\text {water }}-\rho_{\log }\right)}{m_{\text {person }}}=\frac{12 \pi r_{\log }^{2} l_{\log }\left(\rho_{\text {water }}-S G_{\log } \rho_{\text {water }}\right)}{m_{\text {person }}}=\frac{12 \pi r_{\log }^{2} l_{\log } \rho_{\text {water }}\left(1-S G_{\log }\right)}{m_{\text {person }}}
\end{aligned}
$$

$$
=\frac{12 \pi(0.225 \mathrm{~m})^{2}(6.1 \mathrm{~m})\left(1000 \mathrm{~kg} / \mathrm{m}^{3}\right)(1-0.60)}{68 \mathrm{~kg}}=68.48
$$

Thus 68 people can stand on the raft without getting wet. When the $69^{\text {th }}$ person gets on, the raft will go under the surface.
91. We assume that the air pressure is due to the weight of the atmosphere, with the area equal to the surface area of the Earth.

$$
\begin{aligned}
& P=\frac{F}{A} \rightarrow F=P A=m g \rightarrow \\
& m=\frac{P A}{g}=\frac{4 \pi R_{\text {Earth }}^{2} P}{g}=\frac{4 \pi\left(6.38 \times 10^{6} \mathrm{~m}\right)^{2}\left(1.013 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}\right)}{9.80 \mathrm{~m} / \mathrm{s}^{2}}=5.29 \times 10^{18} \mathrm{~kg} \approx 5 \times 10^{18} \mathrm{~kg}
\end{aligned}
$$

92. The work done during each heartbeat is the force on the fluid times the distance that the fluid moves in the direction of the force.

$$
W=F \Delta l=P A \Delta l=P V \quad \rightarrow
$$

$$
\text { Power }=\frac{W}{t}=\frac{P V}{t}=\frac{(105 \mathrm{~mm}-\mathrm{Hg})\left(\frac{133 \mathrm{~N} / \mathrm{m}^{2}}{1 \mathrm{~mm}-\mathrm{Hg}}\right)\left(70 \times 10^{-6} \mathrm{~m}^{3}\right)}{\left(\frac{1}{70} \min \right)\left(\frac{60 \mathrm{~s}}{\min }\right)}=1.1 \mathrm{~W} \approx 1 \mathrm{~W}
$$

93. (a) We assume that the water is launched at ground level. Since it also lands at ground level, the level range formula from Example 3-10 may be used.

$$
R=\frac{v_{0}^{2} \sin 2 \theta}{g} \rightarrow v_{0}=\sqrt{\frac{R g}{\sin 2 \theta}}=\sqrt{\frac{(7.0 \mathrm{~m})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{\sin 70^{\circ}}}=8.544 \mathrm{~m} / \mathrm{s} \approx 8.5 \mathrm{~m} / \mathrm{s}
$$

(b) The volume rate of flow is the area of the flow times the speed of the flow. Multiply by 4 for the 4 heads.

$$
\begin{aligned}
& \text { Volume flow rate }=A v=4 \pi r^{2} v=4 \pi\left(1.5 \times 10^{-3} \mathrm{~m}\right)^{2}(8.544 \mathrm{~m} / \mathrm{s}) \\
& =2.416 \times 10^{-4} \mathrm{~m}^{3} / \mathrm{s}\left(\frac{1 \mathrm{~L}}{1.0 \times 10^{-3} \mathrm{~m}^{3}}\right) \approx 0.24 \mathrm{~L} / \mathrm{s}
\end{aligned}
$$

(c) Use the equation of continuity to calculate the flow rate in the supply pipe.

$$
(A v)_{\text {supply }}=(A v)_{\text {heads }} \rightarrow v_{\text {supply }}=\frac{(A v)_{\text {heads }}}{A_{\text {supply }}}=\frac{2.416 \times 10^{-4} \mathrm{~m}^{3} / \mathrm{s}}{\pi\left(0.95 \times 10^{-2} \mathrm{~m}\right)^{2}}=0.85 \mathrm{~m} / \mathrm{s}
$$

94. The buoyant force on the rock is the force that would be on a mass of water with the same volume as the rock. Since the equivalent mass of water is accelerating upward, that same acceleration must be taken into account in the calculation of the buoyant force.

$$
\begin{aligned}
& F_{\text {buoyant }}-m_{\text {waterg }} g=m_{\text {water }} a \rightarrow \\
& \begin{aligned}
F_{\text {buovant }} & =m_{\text {water }}(g+a)=V_{\text {water }} \rho_{\text {water }}(g+a)=V_{\text {rock }} \rho_{\text {water }}(g+a)=\frac{m_{\text {rock }}}{\rho_{\text {rock }}} \rho_{\text {water }}(g+a) \\
& =\frac{m_{\text {rock }}}{S G_{\text {rock }}}(g+1.8 g)=\frac{(3.0 \mathrm{~kg}) 2.8\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{2.7}=30.49 \mathrm{~N} \approx 30 \mathrm{~N}(2 \text { sig. fig. })
\end{aligned}
\end{aligned}
$$

For the rock to not sink, the upward buoyant force on the rock minus the weight of the rock must be equal to the net force on the rock.

$$
F_{\text {buoyant }}-m_{\text {rock }} g=m_{\text {rock }} a \rightarrow F_{\text {buoyant }}=m_{\text {rock }}(g+a)=(3.0 \mathrm{~kg}) 2.8\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)=82 \mathrm{~N}
$$

The rock will sink, because the buoyant force is not large enough to "float" the rock.
95. Apply both Bernoulli's equation and the equation of continuity at the two locations of the stream, with the faucet being location 0 and the lower position being location 1. The pressure will be air pressure at both locations. The lower location has $y_{1}=0$ and the faucet is at height $y_{0}=y$.

$$
\begin{aligned}
& A_{0} v_{0}=A_{1} v_{1} \rightarrow v_{1}=v_{0} \frac{A_{0}}{A_{1}}=v_{0} \frac{\pi\left(d_{0} / 2\right)^{2}}{\pi\left(d_{1} / 2\right)^{2}}=v_{0} \frac{d_{0}^{2}}{d_{1}^{2}} \rightarrow \\
& P_{0}+\frac{1}{2} \rho v_{0}^{2}+\rho g y_{0}=P_{1}+\frac{1}{2} \rho v_{1}^{2}+\rho g y_{1} \rightarrow v_{0}^{2}+2 g y=v_{1}^{2}=v_{0}^{2} \frac{d_{0}^{4}}{d_{1}^{4}} \rightarrow \\
& d_{1}=d_{0}\left(\frac{v_{0}^{2}}{v_{0}^{2}+2 g y}\right)^{1 / 4}
\end{aligned}
$$

96. (a) Apply Bernoulli's equation between the surface of the water in the sink and the lower end of the siphon tube. Note that both are open to the air, and so the pressure at both is air pressure.

$$
\begin{aligned}
& P_{\text {top }}+\frac{1}{2} \rho v_{\text {top }}^{2}+\rho g y_{\text {top }}=P_{\text {bottom }}+\frac{1}{2} \rho v_{\text {botom }}^{2}+\rho g y_{\text {bottom }} \rightarrow \\
& v_{\text {botom }}=\sqrt{2 g\left(y_{\text {top }}-y_{\text {bottom }}\right)}=\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.44 \mathrm{~m})}=2.937 \mathrm{~m} / \mathrm{s} \approx 2.9 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(b) The volume flow rate (at the lower end of the tube) times the elapsed time must equal the volume of water in the sink.

$$
(A v)_{\text {Iower }} \Delta t=V_{\text {sink }} \rightarrow \Delta t=\frac{V_{\text {sink }}}{(A v)_{\text {lower }}}=\frac{\left(0.38 \mathrm{~m}^{2}\right)\left(4.0 \times 10^{-2} \mathrm{~m}\right)}{\pi\left(1.0 \times 10^{-2} \mathrm{~m}\right)^{2}(2.937 \mathrm{~m} / \mathrm{s})}=16.47 \mathrm{~s} \approx 16 \mathrm{~s}
$$

97. The upward force due to air pressure on the bottom of the wing must be equal to the weight of the airplane plus the downward force due to air pressure on the top of the wing. Bernoulli's equation can be used to relate the forces due to air pressure. We assume that there is no appreciable height difference between the top and the bottom of the wing.

$$
\begin{aligned}
& P_{\text {top }} A+m g=P_{\text {botom }} A \rightarrow\left(P_{\text {bottom }}-P_{\text {top }}\right)=\frac{m g}{A} \\
& P_{0}+P_{\text {bottom }}+\frac{1}{2} \rho v_{\text {botom }}^{2}+\rho g y_{\text {bottom }}=P_{0}+P_{\text {top }}+\frac{1}{2} \rho v_{\text {top }}^{2}+\rho g y_{\text {top }} \\
& v_{\text {top }}^{2}=\frac{2\left(P_{\text {botom }}-P_{\text {top }}\right)}{\rho}+v_{\text {bottom }}^{2} \rightarrow \\
& v_{\text {top }}=\sqrt{\frac{2\left(P_{\text {bottom }}-P_{\text {top }}\right)}{\rho}+v_{\text {botom }}^{2}}=\sqrt{\frac{2 m g}{\rho A}+v_{\text {bottom }}^{2}}=\sqrt{\frac{2\left(1.7 \times 10^{6} \mathrm{~kg}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{\left(1.29 \mathrm{~kg} / \mathrm{m}^{3}\right)\left(1200 \mathrm{~m}^{2}\right)}+(95 \mathrm{~m} / \mathrm{s})^{2}} \\
& \quad=174.8 \mathrm{~m} / \mathrm{s} \approx 170 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

98. We label three vertical levels. Level 0 is at the pump, and the supply tube has a radius of $r_{0}$ at that location. Level 1 is at the nozzle, and the nozzle has a radius of $r_{1}$. Level 1 is a height $h_{1}$ above level 0 . Level 2 is the highest point reached by the water. Level 2 is a height $h_{2}$ above level 1 . We may write Bernoulli's equation relating any 2 of the levels, and we may write the equation of continuity relating any 2 of the levels. The desired result is the gauge pressure of the pump, which would be $P_{0}-P_{\text {atm }}$. Start by using Bernoulli's equation to relate level 0 to level 1.

$$
P_{0}+\rho g h_{0}+\frac{1}{2} \rho v_{0}^{2}=P_{1}+\rho g h_{\cdot 1}+\frac{1}{2} \rho v_{1}^{2}
$$

Since level 1 is open to the air, $P_{1}=P_{\text {atm }}$. Use that in the above equation.


$$
P_{0}-P_{\mathrm{atm}}=\rho g h_{1}+\frac{1}{2} \rho v_{1}^{2}-\frac{1}{2} \rho v_{0}^{2}
$$

Use the equation of continuity to relate level 0 to level 1 , and then use that result in the Bernoulli expression above.

$$
\begin{aligned}
& A_{0} v_{0}=A_{1} v_{1} \rightarrow \pi r_{0}^{2} v_{0}=\pi r_{1}^{2} v_{1} \rightarrow v_{0}=\frac{r_{1}^{2}}{r_{0}^{2}} v_{1}=\frac{d_{1}^{2}}{d_{0}^{2}} v_{1} \\
& P_{0}-P_{\mathrm{atm}}=\rho g h_{\cdot 1}+\frac{1}{2} \rho v_{1}^{2}-\frac{1}{2} \rho\left(\frac{d_{1}^{2}}{d_{0}^{2}} v_{1}\right)^{2}=\rho g h_{\cdot 1}+\frac{1}{2} \rho v_{1}^{2}\left(1-\frac{d_{1}^{4}}{d_{0}^{4}}\right)
\end{aligned}
$$

Use Bernoulli's equation to relate levels 1 and 2. Since both levels are open to the air, the pressures are the same. Also note that the speed at level 2 is zero. Use that result in the Bernoulli expression above.

$$
\begin{aligned}
& P_{1}+\rho g h_{1}+\frac{1}{2} \rho v_{1}^{2}=P_{2}+\rho g\left(h_{1}+h_{2}\right)+\frac{1}{2} \rho v_{2}^{2} \rightarrow v_{1}^{2}=2 g h_{2} \\
& \begin{aligned}
P_{0}-P_{\mathrm{atm}} & =\rho g h_{.1}+\frac{1}{2} \rho v_{1}^{2}\left(1-\frac{d_{1}^{4}}{d_{0}^{4}}\right)=\rho g\left[h_{\cdot 1}+h_{2}\left(1-\frac{d_{1}^{4}}{d_{0}^{4}}\right)\right] \\
& =\left(1.00 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left[1.1 \mathrm{~m}+(0.14 \mathrm{~m})\left(1-0.5^{4}\right)\right] \\
& =12066 \mathrm{~N} / \mathrm{m}^{2} \approx 1.2 \times 10^{4} \mathrm{~N} / \mathrm{m}^{2}
\end{aligned}
\end{aligned}
$$

99. We assume that there is no appreciable height difference to be considered between the two sides of the window. Then the net force on the window due to the air is the difference in pressure on the two sides of the window, times the area of the window. The difference in pressure can be found from Bernoulli's equation.

$$
\begin{aligned}
& P_{\text {inside }}+\frac{1}{2} \rho v_{\text {inside }}^{2}+\rho g y_{\text {inside }}=P_{\text {outside }}+\frac{1}{2} \rho v_{\text {outside }}^{2}+\rho g y_{\text {outside }} \rightarrow \\
& P_{\text {inside }}-P_{\text {outside }}=\frac{1}{2} \rho_{\text {air }} v_{\text {outside }}^{2}=\frac{F_{\text {air }}}{A_{\text {roof }}} \rightarrow \\
& F_{\text {air }}=\frac{1}{2} \rho_{\text {air }} v_{\text {outside }}^{2} A_{\text {roof }}=\frac{1}{2}\left(1.29 \mathrm{~kg} / \mathrm{m}^{3}\right)\left[(200 \mathrm{~km} / \mathrm{h})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)\right]^{2}\left(6.0 \mathrm{~m}^{2}\right)=1.2 \times 10^{4} \mathrm{~N}
\end{aligned}
$$

100. From Poiseuille's equation, the viscosity can be found from the volume flow rate, the geometry of the tube, and the pressure difference. The pressure difference over the length of the tube is the same as the pressure difference due to the height of the reservoir, assuming that the open end of the needle is at atmospheric pressure.

$$
\begin{aligned}
& Q=\frac{\pi R^{4}\left(P_{2}-P_{1}\right)}{8 \eta \ell} ; P_{2}-P_{1}=\rho_{\text {blood }} g h \rightarrow \\
& \eta=\frac{\pi R^{4}\left(P_{2}-P_{1}\right)}{8 Q \ell}=\frac{\pi R^{4} \rho_{\text {blood }} g h}{8 Q \ell}=\frac{\pi\left(0.20 \times 10^{-3} \mathrm{~m}\right)^{4}\left(1.05 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(1.30 \mathrm{~m})}{8\left[4.1 \frac{\mathrm{~cm}^{3}}{\min } \times \frac{1 \mathrm{~min}}{60 \mathrm{~s}} \times \frac{10^{-6} \mathrm{~m}^{3}}{\mathrm{~cm}^{3}}\right]\left(3.8 \times 10^{-2} \mathrm{~m}\right)} \\
& =3.2 \times 10^{-3} \mathrm{~Pa} \cdot \mathrm{~s}
\end{aligned}
$$

101. The net force is 0 if the balloon is moving at terminal velocity. Therefore the upwards buoyancy force (equal to the weight of the displaced air) must be equal to the net downwards force of the weight of the balloon material plus the weight of the helium plus the drag force at terminal velocity. Find the terminal velocity, and use that to find the time to rise 12 m .

$$
\begin{aligned}
& F_{\mathrm{B}}=m_{\text {balloon }} g+m_{\text {Helium }} g+F_{\mathrm{D}} \rightarrow \frac{4}{3} \pi r^{3} \rho_{\text {air }} g=m_{\text {balloon }} g+\frac{4}{3} \pi r^{3} \rho_{\text {He }} g+\frac{1}{2} C_{\mathrm{D}} \rho_{\text {air }} \pi r^{2} v_{\mathrm{T}}^{2} \\
& v_{\mathrm{T}}=\sqrt{\frac{2\left[\frac{4}{3} \pi r^{3}\left(\rho_{\text {air }}-\rho_{\text {He }}\right)-m\right] g}{C_{\mathrm{D}} \rho_{\text {air }} \pi r^{2}}}=\frac{h}{t} \rightarrow t=h \sqrt{\frac{C_{\mathrm{D}} \rho_{\text {air }} \pi r^{2}}{2\left[\frac{4}{3} \pi r^{3}\left(\rho_{\text {air }}-\rho_{\text {He }}\right)-m\right] g}} \rightarrow \\
& t=(12 \mathrm{~m}) \sqrt{\frac{(0.47)\left(1.29 \mathrm{~kg} / \mathrm{m}^{3}\right) \pi(0.15 \mathrm{~m})^{2}}{2\left[\frac{4}{3} \pi(0.15 \mathrm{~m})^{3}\left(1.29 \mathrm{~kg} / \mathrm{m}^{3}-0.179 \mathrm{~kg} / \mathrm{m}^{3}\right)-(0.0028 \mathrm{~kg})\right]\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}}=4.9 \mathrm{~s}
\end{aligned}
$$

102. From Poiseuille's equation, the volume flow rate $Q$ is proportional to $R^{4}$ if all other factors are the same. Thus $Q / R^{4}$ is constant. Also, if the diameter is reduced by $15 \%$, so is the radius.

$$
\frac{Q_{\text {final }}}{R_{\text {final }}^{4}}=\frac{Q_{\text {initial }}}{R_{\text {initial }}^{4}} \rightarrow \frac{Q_{\text {final }}}{Q_{\text {intitial }}}=\frac{R_{\text {final }}^{4}}{R_{\text {initial }}^{4}}=(0.85)^{4}=0.52
$$

The flow rate is $52 \%$ of the original value.
103. Use the definition of density and specific gravity, and then solve for the fat fraction, $f$.

$$
\begin{aligned}
& m_{\text {fat }}=m f=V_{\text {fat }} \rho_{\text {fat }} ; m_{\substack{\text { fat } \\
\text { free }}}=m(1-f)=V_{\substack{\text { fat } \\
\text { fiee }}} \rho_{\text {fat }} \\
& \rho_{\text {body }}=X \rho_{\text {water }}=\frac{m_{\text {total }}}{V_{\text {total }}}=\frac{m_{\text {fat }}+m_{\text {fat }}}{\text { friee }}=\frac{m}{V_{\text {fat }}+V_{\substack{\text { fat } \\
\text { free }}}^{\frac{m f}{\rho_{\text {fat }}}+\frac{m(1-f)}{\rho_{\text {fat }}}}=\frac{1}{\frac{f}{\text { free }}}+\frac{(1-f)}{\rho_{\text {fat }}}+\frac{(1}{\rho_{\text {fat }}} \text { free }} \rightarrow \\
& f=\frac{\rho_{\text {fat }} \rho_{\text {fat }}}{\text { free }^{\text {free }}} \underset{X \rho_{\text {water }}\left(\rho_{\substack{\text { fat } \\
\text { free }}}-\rho_{\text {fat }}\right)}{ }-\frac{\rho_{\text {fat }}}{\left(\rho_{\text {fat }}-\rho_{\text {fat }}\right)}=\frac{\left(0.90 \mathrm{~g} / \mathrm{cm}^{3}\right)\left(1.10 \mathrm{~g} / \mathrm{cm}^{3}\right)}{X\left(1.0 \mathrm{~g} / \mathrm{cm}^{3}\right)\left(0.20 \mathrm{~g} / \mathrm{cm}^{3}\right)}-\frac{\left(0.90 \mathrm{~g} / \mathrm{cm}^{3}\right)}{\left(0.20 \mathrm{~g} / \mathrm{cm}^{3}\right)} \\
& =\frac{4.95}{X}-4.5 \rightarrow \% \text { Body fat }=100 f=100\left(\frac{4.95}{X}-4.5\right)=\frac{495}{X}-450
\end{aligned}
$$

104. The graph is shown. The best-fit equations as calculated by Excel are also shown. Let $P$ represent the pressure in kPa and $y$ the altitude in m .

The spreadsheet used for this problem can be found on the Media Manager, with filename
"PSE4_ISM_CH13.XLS," on tab "Problem 13.104."

(a) Quadratic fit: $\quad P_{\text {quad }}=\left(3.9409 \times 10^{-7}\right) y^{2}-\left(1.1344 \times 10^{-2}\right) y+100.91$,
(b) Exponential fit: $\quad P_{\exp }=103.81 e^{-\left(1.3390 \times 10^{-4}\right) y}$
(c) $P_{\text {quad }}=\left(3.9409 \times 10^{-7}\right)(8611)^{2}-\left(1.1344 \times 10^{-2}\right)(8611)+100.91=32.45 \mathrm{kPa}$
$P_{\text {exp }}=103.81 e^{-\left(1.3300 \times 10^{-4}\right)(8611)}=32.77 \mathrm{kPa}$
$\%$ diff $=\frac{100\left(P_{\exp }-P_{\text {quad }}\right)}{\frac{1}{2}\left(P_{\exp }+P_{\text {quad }}\right)}=\frac{200\left(P_{\exp }-P_{\text {quad }}\right)}{\left(P_{\exp }+P_{\text {quad }}\right)}=\frac{200(32.77 \mathrm{kPa}-32.45 \mathrm{kPa})}{(32.77 \mathrm{kPa}+32.45 \mathrm{kPa})}=0.98 \%$

## CHAPTER 14: Oscillations

## Responses to Questions

1. Examples are: a child's swing (SHM, for small oscillations), stereo speakers (complicated motion, the addition of many SHMs), the blade on a jigsaw (approximately SHM), the string on a guitar (complicated motion, the addition of many SHMs).
2. The acceleration of a simple harmonic oscillator is momentarily zero as the mass passes through the equilibrium point. At this point, there is no force on the mass and therefore no acceleration.
3. When the engine is running at constant speed, the piston will have a constant period. The piston has zero velocity at the top and bottom of its path. Both of these properties are also properties of SHM. In addition, there is a large force exerted on the piston at one extreme of its motion, from the combustion of the fuel-air mixture, and in SHM the largest forces occur at the extremes of the motion.
4. The true period will be larger and the true frequency will be smaller. The spring needs to accelerate not only the mass attached to its end, but also its own mass. As a mass on a spring oscillates, potential energy is converted into kinetic energy. The maximum potential energy depends on the displacement of the mass. This maximum potential energy is converted into the maximum kinetic energy, but if the mass being accelerated is larger then the velocity will be smaller for the same amount of energy. A smaller velocity translates into a longer period and a smaller frequency.
5. The maximum speed of a simple harmonic oscillator is given by $v=A \sqrt{\frac{k}{m}}$. The maximum speed can be doubled by doubling the amplitude, $A$.
6. Before the trout is released, the scale reading is zero. When the trout is released, it will fall downward, stretching the spring to beyond its equilibrium point so that the scale reads something over 5 kg . Then the spring force will pull the trout back up, again to a point beyond the equilibrium point, so that the scale will read something less than 5 kg . The spring will undergo damped oscillations about equilibrium and eventually come to rest at equilibrium. The corresponding scale readings will oscillate about the $5-\mathrm{kg}$ mark, and eventually come to rest at 5 kg .
7. At high altitude, $g$ is slightly smaller than it is at sea level. If $g$ is smaller, then the period $T$ of the pendulum clock will be longer, and the clock will run slow (or lose time).
8. The tire swing is a good approximation of a simple pendulum. Pull the tire back a short distance and release it, so that it oscillates as a pendulum in simple harmonic motion with a small amplitude. Measure the period of the oscillations and calculate the length of the pendulum from the expression $T=2 \pi \sqrt{\frac{\ell}{g}}$. The length, $\ell$, is the distance from the center of the tire to the branch. The height of the branch is $\ell$ plus the height of the center of the tire above the ground.
9. The displacement and velocity vectors are in the same direction while the oscillator is moving away from its equilibrium position. The displacement and acceleration vectors are never in the same direction.
10. The period will be unchanged, so the time will be (c), two seconds. The period of a simple pendulum oscillating with a small amplitude does not depend on the mass.
11. The two masses reach the equilibrium point simultaneously. The angular frequency is independent of amplitude and will be the same for both systems.
12. Empty. The period of the oscillation of a spring increases with increasing mass, so when the car is empty the period of the harmonic motion of the springs will be shorter, and the car will bounce faster.
13. When walking at a normal pace, about 1 s (timed). The faster you walk, the shorter the period. The shorter your legs, the shorter the period.
14. When you rise to a standing position, you raise your center of mass and effectively shorten the length of the swing. The period of the swing will decrease.
15. The frequency will decrease. For a physical pendulum, the period is proportional to the square root of the moment of inertia divided by the mass. When the small sphere is added to the end of the rod, both the moment of inertia and the mass of the pendulum increase. However, the increase in the moment of inertia will be greater because the added mass is located far from the axis of rotation. Therefore, the period will increase and the frequency will decrease.
16. When the $264-\mathrm{Hz}$ fork is set into vibration, the sound waves generated are close enough in frequency to the resonance frequency of the $260-\mathrm{Hz}$ fork to cause it to vibrate. The $420-\mathrm{Hz}$ fork has a resonance frequency far from 264 Hz and far from the harmonic at 528 Hz , so it will not begin to vibrate.
17. If you shake the pan at a resonant frequency, standing waves will be set up in the water and it will slosh back and forth. Shaking the pan at other frequencies will not create large waves. The individual water molecules will move but not in a coherent way.
18. Examples of resonance are: pushing a child on a swing (if you push at one of the limits of the oscillation), blowing across the top of a bottle, producing a note from a flute or organ pipe.
19. Yes. Rattles which occur only when driving at certain speeds are most likely resonance phenomena.
20. Building with lighter materials doesn't necessarily make it easier to set up resonance vibrations, but it does shift the fundamental frequency and decrease the ability of the building to dampen oscillations. Resonance vibrations will be more noticeable and more likely to cause damage to the structure.

## Solutions to Problems

1. The particle would travel four times the amplitude: from $x=A$ to $x=0$ to $x=-A$ to $x=0$ to $x=A$. So the total distance $=4 A=4(0.18 \mathrm{~m})=0.72 \mathrm{~m}$.
2. The spring constant is the ratio of external applied force to displacement.

$$
=\frac{F_{\text {ext }}}{x}=\frac{180 \mathrm{~N}-75 \mathrm{~N}}{0.85 \mathrm{~m}-0.65 \mathrm{~m}}=\frac{105 \mathrm{~N}}{0.20 \mathrm{~m}}=525 \mathrm{~N} / \mathrm{m} \approx 530 \mathrm{~N} / \mathrm{m}
$$

3. The spring constant is found from the ratio of applied force to displacement.

$$
k=\frac{F_{\text {ext }}}{x}=\frac{m g}{x}=\frac{(68 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{5.0 \times 10^{-3} \mathrm{~m}}=1.333 \times 10^{5} \mathrm{~N} / \mathrm{m}
$$

The frequency of oscillation is found from the total mass and the spring constant.

$$
f=\frac{1}{2 \pi} \sqrt{\frac{k}{m}}=\frac{1}{2 \pi} \sqrt{\frac{1.333 \times 10^{5} \mathrm{~N} / \mathrm{m}}{1568 \mathrm{~kg}}}=1.467 \mathrm{~Hz} \approx 1.5 \mathrm{~Hz}
$$

4. (a) The motion starts at the maximum extension, and so is a cosine. The amplitude is the displacement at the start of the motion.

$$
\begin{aligned}
x & =A \cos (\omega t)=A \cos \left(\frac{2 \pi}{T} t\right)=(8.8 \mathrm{~cm}) \cos \left(\frac{2 \pi}{0.66} t\right)=(8.8 \mathrm{~cm}) \cos (9.520 t) \\
& \approx(8.8 \mathrm{~cm}) \cos (9.5 t)
\end{aligned}
$$

(b) Evaluate the position function at $t=1.8 \mathrm{~s}$.

$$
x=(8.8 \mathrm{~cm}) \cos \left(9.520 \mathrm{~s}^{-1}(1.8 \mathrm{~s})\right)=-1.252 \mathrm{~cm} \approx-1.3 \mathrm{~cm}
$$

5. The period is 2.0 seconds, and the mass is 35 kg . The spring constant can be calculated from Eq. 147 b .

$$
T=2 \pi \sqrt{\frac{m}{k}} \rightarrow T^{2}=4 \pi^{2} \frac{m}{k} \rightarrow k=4 \pi^{2} \frac{m}{T^{2}}=4 \pi^{2} \frac{35 \mathrm{~kg}}{(2.0 \mathrm{~s})^{2}}=350 \mathrm{~N} / \mathrm{m}
$$

6. (a) The spring constant is found from the ratio of applied force to displacement.

$$
k=\frac{F_{\text {ext }}}{x}=\frac{m g}{x}=\frac{(2.4 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{0.036 \mathrm{~m}}=653 \mathrm{~N} / \mathrm{m} \approx 650 \mathrm{~N} / \mathrm{m}
$$

(b) The amplitude is the distance pulled down from equilibrium, so $A=2.5 \mathrm{~cm}$

The frequency of oscillation is found from the oscillating mass and the spring constant.

$$
f=\frac{1}{2 \pi} \sqrt{\frac{k}{m}}=\frac{1}{2 \pi} \sqrt{\frac{653 \mathrm{~N} / \mathrm{m}}{2.4 \mathrm{~kg}}}=2.625 \mathrm{~Hz} \approx 2.6 \mathrm{~Hz}
$$

7. The maximum velocity is given by Eq. 14-9a.

$$
\nu_{\max }=\omega A=\frac{2 \pi A}{T}=\frac{2 \pi(0.15 \mathrm{~m})}{7.0 \mathrm{~s}}=0.13 \mathrm{~m} / \mathrm{s}
$$

The maximum acceleration is given by Eq. $14-9$ b.

$$
\begin{aligned}
& a_{\max }=\omega^{2} A=\frac{4 \pi^{2} A}{T^{2}}=\frac{4 \pi^{2}(0.15 \mathrm{~m})}{(7.0 \mathrm{~s})^{2}}=0.1209 \mathrm{~m} / \mathrm{s}^{2} \approx 0.12 \mathrm{~m} / \mathrm{s}^{2} \\
& \frac{a_{\max }}{g}=\frac{0.1209 \mathrm{~m} / \mathrm{s}^{2}}{9.80 \mathrm{~m} / \mathrm{s}^{2}}=1.2 \times 10^{-2}=1.2 \%
\end{aligned}
$$

8. The table of data is shown, along with the smoothed graph. Every quarter of a period, the mass moves from an extreme point to the equilibrium. The

| time | position |
| :---: | :---: |
| 0 | -A |
| $\mathrm{T} / 4$ | 0 |
| $\mathrm{~T} / 2$ | A |
| $3 \mathrm{~T} / 4$ | 0 |
| T | -A |
| $5 \mathrm{~T} / 4$ | 0 | graph resembles a cosine wave (actually, the opposite of a cosine wave).


9. The relationship between frequency, mass, and spring constant is Eq. 14-7a, $f=\frac{1}{2 \pi} \sqrt{\frac{k}{m}}$.
(a) $f=\frac{1}{2 \pi} \sqrt{\frac{k}{m}} \rightarrow k=4 \pi^{2} f^{2} m=4 \pi^{2}(4.0 \mathrm{~Hz})^{2}\left(2.5 \times 10^{-4} \mathrm{~kg}\right)=0.1579 \mathrm{~N} / \mathrm{m} \approx 0.16 \mathrm{~N} / \mathrm{m}$
(b) $f=\frac{1}{2 \pi} \sqrt{\frac{k}{m}}=\frac{1}{2 \pi} \sqrt{\frac{0.1579 \mathrm{~N} / \mathrm{m}}{5.0 \times 10^{-4} \mathrm{~kg}}}=2.8 \mathrm{~Hz}$
10. The spring constant is the same regardless of what mass is attached to the spring.

$$
\begin{aligned}
& f=\frac{1}{2 \pi} \sqrt{\frac{k}{m}} \rightarrow \frac{k}{4 \pi^{2}}=m f^{2}=\text { constant } \rightarrow m_{1} f_{1}^{2}=m_{2} f_{1}^{2} \rightarrow \\
& (m \mathrm{~kg})(0.83 \mathrm{~Hz})^{2}=(m \mathrm{~kg}+0.68 \mathrm{~kg})(0.60 \mathrm{~Hz})^{2} \rightarrow m=\frac{(0.68 \mathrm{~kg})(0.60 \mathrm{~Hz})^{2}}{(0.83 \mathrm{~Hz})^{2}-(0.60 \mathrm{~Hz})^{2}}=0.74 \mathrm{~kg}
\end{aligned}
$$

11. We assume that the spring is stretched some distance $y_{0}$ while the rod is in equilibrium and horizontal. Calculate the net torque about point A while the object is in equilibrium, with clockwise torques as positive.

$$
\sum \tau=M g\left(\frac{1}{2} \ell\right)-F_{\mathrm{s}} \ell=\frac{1}{2} M g \ell-k y_{0} \ell=0
$$



Now consider the rod being displaced an additional distance $y$
below the horizontal, so that the rod makes a small angle of $\theta$ as shown in the free-body diagram. Again write the net torque about point A . If the angle is small, then there has been no appreciable horizontal displacement of the rod.

$$
\sum \tau=M g\left(\frac{1}{2} \ell\right)-F_{\mathrm{s}} \ell=\frac{1}{2} M g \ell-k\left(y+y_{0}\right) \ell=I \alpha=\frac{1}{3} M \ell^{2} \frac{d^{2} \theta}{d t^{2}}
$$

Include the equilibrium condition, and the approximation that $y=\ell \sin \theta \approx \ell \theta$.

$$
\begin{aligned}
& \frac{1}{2} M g \ell-k y \ell-k y_{0} \ell=\frac{1}{3} M \ell^{2} \frac{d^{2} \theta}{d t^{2}} \rightarrow \frac{1}{2} M g \ell-k y \ell-\frac{1}{2} M g \ell=\frac{1}{3} M \ell^{2} \frac{d^{2} \theta}{d t^{2}} \rightarrow \\
& -k \ell^{2} \theta=\frac{1}{3} M \ell^{2} \frac{d^{2} \theta}{d t^{2}} \rightarrow \frac{d^{2} \theta}{d t^{2}}+\frac{3 k}{M} \theta=0
\end{aligned}
$$

This is the equation for simple harmonic motion, corresponding to Eq. $14-3$, with $\omega^{2}=\frac{3 k}{M}$.

$$
\omega^{2}=4 \pi^{2} f^{2}=\frac{3 k}{M} \rightarrow f=\frac{1}{2 \pi} \sqrt{\frac{3 k}{M}}
$$

12. (a) We find the effective spring constant from the mass and the frequency of oscillation.

$$
\begin{aligned}
& f=\frac{1}{2 \pi} \sqrt{\frac{k}{m}} \rightarrow \\
& k=4 \pi^{2} m f^{2}=4 \pi^{2}(0.055 \mathrm{~kg})(3.0 \mathrm{~Hz})^{2}=19.54 \mathrm{~N} / \mathrm{m} \approx 20 \mathrm{~N} / \mathrm{m}(2 \mathrm{sig} \mathrm{fig})
\end{aligned}
$$

(b) Since the objects are the same size and shape, we anticipate that the spring constant is the same.

$$
f=\frac{1}{2 \pi} \sqrt{\frac{k}{m}}=\frac{1}{2 \pi} \sqrt{\frac{19.54 \mathrm{~N} / \mathrm{m}}{0.25 \mathrm{~kg}}}=1.4 \mathrm{~Hz}
$$

13. (a) For A, the amplitude is $A_{\mathrm{A}}=2.5 \mathrm{~m}$. For B , the amplitude is $A_{\mathrm{B}}=3.5 \mathrm{~m}$.
(b) For A, the frequency is 1 cycle every 4.0 seconds, so $f_{\mathrm{A}}=0.25 \mathrm{~Hz}$. For B , the frequency is 1 cycle every 2.0 seconds, so $f_{\mathrm{B}}=0.50 \mathrm{~Hz}$.
(c) For C, the period is $T_{\mathrm{A}}=4.0 \mathrm{~s}$. For B, the period is $T_{\mathrm{B}}=2.0 \mathrm{~s}$
(d) Object A has a displacement of 0 when $t=0$, so it is a sine function.

$$
x_{\mathrm{A}}=A_{\mathrm{A}} \sin \left(2 \pi f_{\mathrm{A}} t\right) \rightarrow x_{\mathrm{A}}=(2.5 \mathrm{~m}) \sin \left(\frac{1}{2} \pi t\right)
$$

Object B has a maximum displacement when $t=0$, so it is a cosine function.

$$
x_{\mathrm{B}}=A_{\mathrm{B}} \cos \left(2 \pi f_{\mathrm{B}} t\right) \rightarrow x_{\mathrm{B}}=(3.5 \mathrm{~m}) \cos (\pi t)
$$

14. Eq. 14-4 is $x=A \cos (\omega t+\phi)$.
(a) If $x(0)=-A$, then $-A=A \cos \phi \rightarrow \phi=\cos ^{-1}(-1) \rightarrow \phi=\pi$.
(b) If $x(0)=0$, then $0=A \cos \phi \rightarrow \phi=\cos ^{-1}(0) \rightarrow \phi= \pm \frac{1}{2} \pi$.
(c) If $x(0)=A$, then $A=A \cos \phi \rightarrow \phi=\cos ^{-1}(1) \rightarrow \phi=0$.
(d) If $x(0)=\frac{1}{2} A$, then $\frac{1}{2} A=A \cos \phi \rightarrow \phi=\cos ^{-1}\left(\frac{1}{2}\right) \rightarrow \phi= \pm \frac{1}{3} \pi$.
(e) If $x(0)=-\frac{1}{2} A$, then $-\frac{1}{2} A=A \cos \phi \rightarrow \phi=\cos ^{-1}\left(-\frac{1}{2}\right) \rightarrow \phi= \pm \frac{2}{3} \pi$.
(f) If $x(0)=A / \sqrt{2}$, then $A / \sqrt{2}=A \cos \phi \rightarrow \phi=\cos ^{-1}\left(\frac{1}{\sqrt{2}}\right) \rightarrow \phi= \pm \frac{1}{4} \pi$.

The ambiguity in the answers is due to not knowing the direction of motion at $t=0$.
15. We assume that downward is the positive direction of motion. For this motion, we have
$k=305 \mathrm{~N} / \mathrm{m}, A=0.280 \mathrm{~m}, m=0.260 \mathrm{~kg}$, and $\omega=\sqrt{k / m}=\sqrt{(305 \mathrm{~N} / \mathrm{m}) / 0.260 \mathrm{~kg}}=34.250 \mathrm{rad} / \mathrm{s}$.
(a) Since the mass has a zero displacement and a positive velocity at $t=0$, the equation is a sine function.

$$
y(t)=(0.280 \mathrm{~m}) \sin [(34.3 \mathrm{rad} / \mathrm{s}) t]
$$

(b) The period of oscillation is given by $T=\frac{2 \pi}{\omega}=\frac{2 \pi}{34.25 \mathrm{rad} / \mathrm{s}}=0.18345 \mathrm{~s}$. The spring will have its maximum extension at times given by the following.

$$
t_{\max }=\frac{T}{4}+n T=4.59 \times 10^{-2} \mathrm{~s}+n(0.183 \mathrm{~s}), n=0,1,2, \cdots
$$

The spring will have its minimum extension at times given by the following.

$$
t_{\min }=\frac{3 T}{4}+n T=1.38 \times 10^{-1} \mathrm{~s}+n(0.183 \mathrm{~s}), n=0,1,2, \cdots
$$

16. (a) From the graph, the period is 0.69 s . The period and the mass can be used to find the spring constant.

$$
T=2 \pi \sqrt{\frac{m}{k}} \rightarrow k=4 \pi^{2} \frac{m}{T^{2}}=4 \pi^{2} \frac{0.0095 \mathrm{~kg}}{(0.69 \mathrm{~s})^{2}}=0.7877 \mathrm{~N} / \mathrm{m} \approx 0.79 \mathrm{~N} / \mathrm{m}
$$

(b) From the graph, the amplitude is 0.82 cm . The phase constant can be found from the initial conditions.

$$
\begin{aligned}
& x=A \cos \left(\frac{2 \pi}{T} t+\phi\right)=(0.82 \mathrm{~cm}) \cos \left(\frac{2 \pi}{0.69} t+\phi\right) \\
& x(0)=(0.82 \mathrm{~cm}) \cos \phi=0.43 \mathrm{~cm} \rightarrow \phi=\cos ^{-1} \frac{0.43}{0.82}= \pm 1.02 \mathrm{rad}
\end{aligned}
$$

Because the graph is shifted to the RIGHT from the 0-phase cosine, the phase constant must be subtracted.

$$
x=(0.82 \mathrm{~cm}) \cos \left(\frac{2 \pi}{0.69} t-1.0\right) \text { or }(0.82 \mathrm{~cm}) \cos (9.1 t-1.0)
$$

17. (a) The period and frequency are found from the angular frequency.

$$
\omega=2 \pi f \rightarrow f=\frac{1}{2 \pi} \omega=\frac{1}{2 \pi} \frac{5 \pi}{4}=\frac{5}{8} \mathrm{~Hz} \quad T=\frac{1}{f}=1.6 \mathrm{~s}
$$

(b) The velocity is the derivative of the position.

$$
\begin{aligned}
& x=(3.8 \mathrm{~m}) \cos \left(\frac{5 \pi}{4} t+\frac{\pi}{6}\right) \quad v=\frac{d x}{d t}=-(3.8 \mathrm{~m})\left(\frac{5 \pi}{4}\right) \sin \left(\frac{5 \pi}{4} t+\frac{\pi}{6}\right) \\
& x(0)=(3.8 \mathrm{~m}) \cos \left(\frac{\pi}{6}\right)=3.3 \mathrm{~m}
\end{aligned} \quad v(0)=-(3.8 \mathrm{~m})\left(\frac{5 \pi}{4}\right) \sin \left(\frac{\pi}{6}\right)=-7.5 \mathrm{~m} / \mathrm{s} .
$$

(c) The acceleration is the derivative of the velocity.

$$
\begin{aligned}
& v=-(3.8 \mathrm{~m})\left(\frac{5 \pi}{4}\right) \sin \left(\frac{5 \pi}{4} t+\frac{\pi}{6}\right) \quad a=\frac{d v}{d t}=-(3.8 \mathrm{~m})\left(\frac{5 \pi}{4}\right)^{2} \cos \left(\frac{5 \pi}{4} t+\frac{\pi}{6}\right) \\
& v(2.0)=-(3.8 \mathrm{~m})\left(\frac{5 \pi}{4}\right) \sin \left(\frac{5 \pi}{4}(2.0)+\frac{\pi}{6}\right)=-13 \mathrm{~m} / \mathrm{s} \\
& a(2.0)=-(3.8 \mathrm{~m})\left(\frac{5 \pi}{4}\right)^{2} \cos \left(\frac{5 \pi}{4}(2.0)+\frac{\pi}{6}\right)=29 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

18. (a) The maximum speed is given by Eq. 14-9a.

$$
v_{\text {max }}=2 \pi f A=2 \pi(441 \mathrm{~Hz})\left(1.5 \times 10^{-3} \mathrm{~m}\right)=4.2 \mathrm{~m} / \mathrm{s} \text {. }
$$

(b) The maximum acceleration is given by Eq. 14-9b.

$$
a_{\max }=4 \pi^{2} f^{2} A=4 \pi^{2}(441 \mathrm{~Hz})^{2}\left(1.5 \times 10^{-3} \mathrm{~m}\right)=1.2 \times 10^{4} \mathrm{~m} / \mathrm{s}^{2} .
$$

19. When the object is at rest, the magnitude of the spring force is equal to the force of gravity. This determines the spring constant. The period can then be found.

$$
\begin{aligned}
& \sum F_{\text {vertical }}=k x_{0}-m g \rightarrow k=\frac{m g}{x_{0}} \\
& T=2 \pi \sqrt{\frac{m}{k}}=2 \pi \sqrt{\frac{m}{\frac{m g}{x_{0}}}}=2 \pi \sqrt{\frac{x_{0}}{g}}=2 \pi \sqrt{\frac{0.14 \mathrm{~m}}{9.80 \mathrm{~m} / \mathrm{s}^{2}}}=0.75 \mathrm{~s}
\end{aligned}
$$

20. The spring constant can be found from the stretch distance corresponding to the weight suspended on the spring.

$$
k=\frac{F_{\text {ext }}}{x}=\frac{m g}{x}=\frac{(1.62 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{0.215 \mathrm{~m}}=73.84 \mathrm{~N} / \mathrm{m}
$$

After being stretched further and released, the mass will oscillate. It takes one-quarter of a period for the mass to move from the maximum displacement to the equilibrium position.

$$
\frac{1}{4} T=\frac{1}{4} 2 \pi \sqrt{m / k}=\frac{\pi}{2} \sqrt{\frac{1.62 \mathrm{~kg}}{73.84 \mathrm{~N} / \mathrm{m}}}=0.233 \mathrm{~s}
$$

21. Each object will pass through the origin at the times when the argument of its sine function is a multiple of $\pi$.

A: $2.0 t_{\mathrm{A}}=n_{\mathrm{A}} \pi \rightarrow t_{\mathrm{A}}=\frac{1}{2} n_{\mathrm{A}} \pi, n_{\mathrm{A}}=1,2,3, \ldots$ so $t_{\mathrm{A}}=\frac{1}{2} \pi, \pi, \frac{3}{2} \pi, 2 \pi, \frac{5}{2} \pi, 3 \pi, \frac{7}{2} \pi, 4 \pi, \ldots$
B: $3.0 t_{\mathrm{B}}=n_{\mathrm{B}} \pi \rightarrow t_{\mathrm{B}}=\frac{1}{3} n_{\mathrm{B}} \pi, n_{\mathrm{B}}=1,2,3, \ldots$ so $t_{\mathrm{B}}=\frac{1}{3} \pi, \frac{2}{3} \pi, \pi, \frac{4}{3} \pi, \frac{5}{3} \pi, 2 \pi, \frac{7}{3} \pi, \frac{8}{3} \pi, 3 \pi, \ldots$
Thus we see the first three times are $\pi \mathrm{s}, 2 \pi \mathrm{~s}, 3 \pi \mathrm{~s}$ or $3.1 \mathrm{~s}, 6.3 \mathrm{~s}, 9.4 \mathrm{~s}$.
22. (a) The object starts at the maximum displacement in the positive direction, and so will be represented by a cosine function. The mass, period, and amplitude are given.

$$
A=0.16 \mathrm{~m} ; \omega=\frac{2 \pi}{T}=\frac{2 \pi}{0.55 \mathrm{~s}}=11.4 \mathrm{rad} / \mathrm{s} \rightarrow y=(0.16 \mathrm{~m}) \cos (11 t)
$$

(b) The time to reach the equilibrium is one-quarter of a period, so $\frac{1}{4}(0.55 \mathrm{~s})=0.14 \mathrm{~s}$.
(c) The maximum speed is given by Eq. 14-9a.

$$
v_{\max }=\omega A=(11.4 \mathrm{rad} / \mathrm{s})(0.16 \mathrm{~m})=1.8 \mathrm{~m} / \mathrm{s}
$$

(d) The maximum acceleration is given by Eq. 14-9b.

$$
a_{\text {max }}=\omega^{2} A=(11.4 \mathrm{rad} / \mathrm{s})^{2}(0.16 \mathrm{~m})=2.1 \mathrm{~m} / \mathrm{s}^{2}
$$

The maximum acceleration occurs at the endpoints of the motion, and is first attained at the release point.
23. The period of the jumper's motion is $T=\frac{43.0 \mathrm{~s}}{8 \text { cycles }}=5.375 \mathrm{~s}$. The spring constant can then be found from the period and the jumper's mass.

$$
T=2 \pi \sqrt{\frac{m}{k}} \rightarrow k=\frac{4 \pi^{2} m}{T^{2}}=\frac{4 \pi^{2}(65.0 \mathrm{~kg})}{(5.375 \mathrm{~s})^{2}}=88.821 \mathrm{~N} / \mathrm{m} \approx 88.8 \mathrm{~N} / \mathrm{m}
$$

The stretch of the bungee cord needs to provide a force equal to the weight of the jumper when he is at the equilibrium point.

$$
k \Delta x=m g \rightarrow \Delta x=\frac{m g}{k}=\frac{(65.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{88.821 \mathrm{~N} / \mathrm{m}}=7.17 \mathrm{~m}
$$

Thus the unstretched bungee cord must be $25.0 \mathrm{~m}-7.17 \mathrm{~m}=17.8 \mathrm{~m}$.
24. Consider the first free-body diagram for the block while it is at equilibrium, so that the net force is zero. Newton's second law for vertical forces, with up as positive, gives this.

$$
\sum F_{y}=F_{\mathrm{A}}+F_{\mathrm{B}}-m g=0 \rightarrow F_{\mathrm{A}}+F_{\mathrm{B}}=m g
$$

Now consider the second free-body diagram, in which the block is displaced a distance $x$ from the equilibrium point.
 Each upward force will have increased by an amount $-k x$, since $x<0$. Again write Newton's second law for vertical forces.

$$
\sum F_{y}=F_{n e t}=F_{\mathrm{A}}^{\prime}+F_{\mathrm{B}}^{\prime}-m g=F_{\mathrm{A}}-k x+F_{\mathrm{B}}-k x-m g=-2 k x+\left(F_{\mathrm{A}}+F_{\mathrm{B}}-m g\right)=-2 k x
$$

This is the general form of a restoring force that produces SHM, with an effective spring constant of $2 k$. Thus the frequency of vibration is as follows.

$$
f=\frac{1}{2 \pi} \sqrt{k_{\text {effective }} / m}=\frac{1}{2 \pi} \sqrt{\frac{2 k}{m}}
$$

25. (a) If the block is displaced a distance $x$ to the right in Figure 14-32a, then the length of spring \# 1 will be increased by a distance $x_{1}$ and the length of spring \# 2 will be increased by a distance $x_{2}$, where $x=x_{1}+x_{2}$. The force on the block can be written $F=-k_{\text {eff }} x$. Because the springs are massless, they act similar to a rope under tension, and the same force $F$ is exerted by each spring. Thus $F=-k_{\text {eff }} x=-k_{1} x_{1}=-k_{2} x_{2}$.

$$
\begin{aligned}
& x=x_{1}+x_{2}=-\frac{F}{k_{1}}-\frac{F}{k_{2}}=-F\left(\frac{1}{k_{1}}+\frac{1}{k_{2}}\right)=-\frac{F}{k_{\mathrm{eff}}} \rightarrow \frac{1}{k_{\mathrm{eff}}}=\frac{1}{k_{1}}+\frac{1}{k_{2}} \\
& T=2 \pi \sqrt{\frac{m}{k_{\mathrm{eff}}}}=2 \pi \sqrt{m\left(\frac{1}{k_{1}}+\frac{1}{k_{2}}\right)}
\end{aligned}
$$

(b) The block will be in equilibrium when it is stationary, and so the net force at that location is zero. Then, if the block is displaced a distance $x$ to the right in the diagram, then spring \# 1 will exert an additional force of $F_{1}=-k_{1} x$, in the opposite direction to $x$. Likewise, spring \# 2 will exert an additional force $F_{2}=-k_{2} x$, in the same direction as $F_{1}$. Thus the net force on the
displaced block is $F=F_{1}+F_{2}=-k_{1} x-k_{2} x=-\left(k_{1}+k_{2}\right) x$. The effective spring constant is thus $k=k_{1}+k_{2}$, and the period is given by $T=2 \pi \sqrt{\frac{m}{k}}=2 \pi \sqrt{\frac{m}{k_{1}+k_{2}}}$.
26. The impulse, which acts for a very short time, changes the momentum of the mass, giving it an initial velocity $v_{0}$. Because this occurs at the equilibrium position, this is the maximum velocity of the mass. Since the motion starts at the equilibrium position, we represent the motion by a sine function.

$$
\begin{aligned}
& J=\Delta p=m \Delta v=m v_{0}-0=m v_{0} \rightarrow v_{0}=\frac{J}{m}=v_{\max }=A \omega=A \sqrt{\frac{k}{m}} \rightarrow \\
& \frac{J}{m}=A \sqrt{\frac{k}{m}} \rightarrow A=\frac{J}{\sqrt{k m}} \rightarrow x=A \sin \omega t=\frac{J}{\sqrt{k m}} \sin \left(\sqrt{\frac{k}{m}} t\right)
\end{aligned}
$$

27. The various values can be found from the equation of motion, $x=A \cos \omega t=0.650 \cos 7.40 t$.
(a) The amplitude is the maximum value of $x$, and so $A=0.650 \mathrm{~m}$.
(b) The frequency is $f=\frac{\omega}{2 \pi}=\frac{7.40 \mathrm{rad} / \mathrm{s}}{2 \pi \mathrm{rad}}=1.18 \mathrm{~Hz}$.
(c) The total energy can be found from the maximum potential energy.

$$
E=U_{\text {max }}=\frac{1}{2} k A^{2}=\frac{1}{2} m \omega^{2} A^{2}=\frac{1}{2}(1.15 \mathrm{~kg})(7.40 \mathrm{rad} / \mathrm{s})^{2}(0.650 \mathrm{~m})^{2}=13.303 \mathrm{~J} \approx 13.3 \mathrm{~J}
$$

(d) The potential energy can be found from $U=\frac{1}{2} k x^{2}$, and the kinetic energy from $E=U+K$.

$$
\begin{aligned}
& U=\frac{1}{2} k x^{2}=\frac{1}{2} m \omega^{2} x^{2}=\frac{1}{2}(1.15 \mathrm{~kg})(7.40 \mathrm{rad} / \mathrm{s})^{2}(0.260 \mathrm{~m})^{2}=2.1 \mathrm{~J} \\
& K=E-U=13.3 \mathrm{~J}-2.1 \mathrm{~J}=11.2 \mathrm{~J}
\end{aligned}
$$

28. (a) The total energy is the maximum potential energy.

$$
U=\frac{1}{2} E \rightarrow \frac{1}{2} k x^{2}=\frac{1}{2}\left(\frac{1}{2} k A^{2}\right) \rightarrow \quad x=A / \sqrt{2} \approx 0.707 A
$$

(b) Now we are given that $x=\frac{1}{3} A$.

$$
\frac{U}{E}=\frac{\frac{1}{2} k x^{2}}{\frac{1}{2} k A^{2}}=\frac{x^{2}}{A^{2}}=\frac{1}{9}
$$

Thus the energy is divided up into $\frac{1}{9}$ potential and $\frac{8}{9}$ kinetic.
29. The total energy can be found from the spring constant and the amplitude.

$$
E=\frac{1}{2} k A^{2}=\frac{1}{2}(95 \mathrm{~N} / \mathrm{m})(0.020 \mathrm{~m})^{2}=0.019 \mathrm{~J}
$$

That is represented by the horizontal line on the graph.
(a) From the graph, at $x=1.5 \mathrm{~cm}$, we have $U \approx 0.011 \mathrm{~J}$.
(b) From energy conservation, at $x=1.5 \mathrm{~cm}$, we have $K=E-U=0.008 \mathrm{~J}$.
(c) Find the speed from the estimated kinetic energy.

$$
\begin{aligned}
K & =\frac{1}{2} m v^{2} \rightarrow \\
v & =\sqrt{\frac{2 K}{m}}=\sqrt{\frac{2(0.008 \mathrm{~J})}{0.055 \mathrm{~kg}}} \\
& =0.5 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

The spreadsheet used for this problem can be found on the Media Manager,

with filename
"PSE4_ISM_CH14.XLS," on tab "Problem 14.29."
30. (a) At equilibrium, the velocity is its maximum. Use Eq. 14-9a, and realize that the object can be moving in either direction.

$$
v_{\max }=\omega A=2 \pi f A=2 \pi(2.5 \mathrm{~Hz})(0.15 \mathrm{~m})=2.356 \mathrm{~m} / \mathrm{s} \rightarrow v_{\text {equib }} \approx \pm 2.4 \mathrm{~m} / \mathrm{s}
$$

(b) From Eq. 14-11b, we find the velocity at any position.

$$
v= \pm v_{\max } \sqrt{1-\frac{x^{2}}{A^{2}}}= \pm(2.356 \mathrm{~m} / \mathrm{s}) \sqrt{1-\frac{(0.10 \mathrm{~m})^{2}}{(0.15 \mathrm{~m})^{2}}}= \pm 1.756 \mathrm{~m} / \mathrm{s} \approx \pm 1.8 \mathrm{~m} / \mathrm{s}
$$

(c) $E_{\text {total }}=\frac{1}{2} m v_{\max }^{2}=\frac{1}{2}(0.35 \mathrm{~kg})(2.356 \mathrm{~m} / \mathrm{s})^{2}=0.9714 \mathrm{~J} \approx 0.97 \mathrm{~J}$
(d) Since the object has a maximum displacement at $t=0$, the position will be described by the cosine function.

$$
x=(0.15 \mathrm{~m}) \cos (2 \pi(2.5 \mathrm{~Hz}) t) \rightarrow x=(0.15 \mathrm{~m}) \cos (5.0 \pi t)
$$

31. The spring constant is found from the ratio of applied force to displacement.

$$
k=\frac{F}{x}=\frac{95.0 \mathrm{~N}}{0.175 \mathrm{~m}}=542.9 \mathrm{~N} / \mathrm{m}
$$

Assuming that there are no dissipative forces acting on the ball, the elastic potential energy in the loaded position will become kinetic energy of the ball.

$$
E_{i}=E_{f} \rightarrow \frac{1}{2} k x_{\max }^{2}=\frac{1}{2} m v_{\max }^{2} \rightarrow v_{\max }=x_{\max } \sqrt{\frac{k}{m}}=(0.175 \mathrm{~m}) \sqrt{\frac{542.9 \mathrm{~N} / \mathrm{m}}{0.160 \mathrm{~kg}}}=10.2 \mathrm{~m} / \mathrm{s}
$$

32. The energy of the oscillator will be conserved after the collision.

$$
E=\frac{1}{2} k A^{2}=\frac{1}{2}(m+M) v_{\max }^{2} \rightarrow v_{\max }=A \sqrt{k /(m+M)}
$$

This speed is the speed that the block and bullet have immediately after the collision. Linear momentum in one dimension will have been conserved during the (assumed short time) collision, and so the initial speed of the bullet can be found.

$$
\begin{aligned}
& p_{\text {before }}=p_{\text {affer }} \rightarrow m v_{o}=(m+M) v_{\max } \\
& v_{o}=\frac{m+M}{m} A \sqrt{\frac{k}{m+M}}=\frac{0.2525 \mathrm{~kg}}{0.0125 \mathrm{~kg}}(0.124 \mathrm{~m}) \sqrt{\frac{2250 \mathrm{~N} / \mathrm{m}}{0.2525 \mathrm{~kg}}}=236 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

33. To compare the total energies, we can compare the maximum potential energies. Since the frequencies and the masses are the same, the spring constants are the same.
34. (a) The spring constant can be found from the mass and the frequency of oscillation.

$$
\omega=\sqrt{\frac{k}{m}}=2 \pi f \rightarrow k=4 \pi^{2} f^{2} m=4 \pi^{2}(3.0 \mathrm{~Hz})^{2}(0.24 \mathrm{~kg})=85.27 \mathrm{~N} / \mathrm{m} \approx 85 \mathrm{~N} / \mathrm{m}
$$

(b) The energy can be found from the maximum potential energy.

$$
E=\frac{1}{2} k A^{2}=\frac{1}{2}(85.27 \mathrm{~N} / \mathrm{m})(0.045 \mathrm{~m})^{2}=8.634 \times 10^{-2} \mathrm{~J} \approx 0.086 \mathrm{~J}
$$

35. (a) The work done in compressing the spring is stored as potential energy. The compressed location corresponds to the maximum potential energy and the amplitude of the ensuing motion.

$$
W=\frac{1}{2} k A^{2} \rightarrow k=\frac{2 W}{A^{2}}=\frac{2(3.6 \mathrm{~J})}{(0.13 \mathrm{~m})^{2}}=426 \mathrm{~N} / \mathrm{m} \approx 430 \mathrm{~N} / \mathrm{m}
$$

(b) The maximum acceleration occurs at the compressed location, where the spring is exerting the maximum force. If the compression distance is positive, then the acceleration is negative.

$$
F=-k x=m a \rightarrow m=-\frac{k x}{a}=-\frac{(426 \mathrm{~N} / \mathrm{m})(0.13 \mathrm{~m})}{15 \mathrm{~m} / \mathrm{s}^{2}}=3.7 \mathrm{~kg}
$$

36. (a) The total energy of an object in SHM is constant. When the position is at the amplitude, the speed is zero. Use that relationship to find the amplitude.

$$
\begin{aligned}
& E_{\mathrm{tot}}=\frac{1}{2} m v^{2}+\frac{1}{2} k x^{2}=\frac{1}{2} k A^{2} \rightarrow \\
& A=\sqrt{\frac{m}{k} v^{2}+x^{2}}=\sqrt{\frac{2.7 \mathrm{~kg}}{280 \mathrm{~N} / \mathrm{m}}(0.55 \mathrm{~m} / \mathrm{s})^{2}+(0.020 \mathrm{~m})^{2}}=5.759 \times 10^{-2} \mathrm{~m} \approx 5.8 \times 10^{-2} \mathrm{~m}
\end{aligned}
$$

(b) Again use conservation of energy. The energy is all kinetic energy when the object has its maximum velocity.

$$
\begin{aligned}
& E_{\text {tot }}=\frac{1}{2} m v^{2}+\frac{1}{2} k x^{2}=\frac{1}{2} k A^{2}=\frac{1}{2} m v_{\text {max }}^{2} \rightarrow \\
& v_{\text {max }}=A \sqrt{\frac{k}{m}}=\left(5.759 \times 10^{-2} \mathrm{~m}\right) \sqrt{\frac{280 \mathrm{~N} / \mathrm{m}}{2.7 \mathrm{~kg}}}=0.5865 \mathrm{~m} / \mathrm{s} \approx 0.59 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

37. We assume that the collision of the bullet and block is so quick that there is no significant motion of the large mass or spring during the collision. Linear momentum is conserved in this collision. The speed that the combination has right after the collision is the maximum speed of the oscillating system. Then, the kinetic energy that the combination has right after the collision is stored in the spring when it is fully compressed, at the amplitude of its motion.

$$
\begin{aligned}
& p_{\text {before }}=p_{\text {after }} \rightarrow m v_{0}=(m+M) v_{\max } \rightarrow v_{\max }=\frac{m}{m+M} v_{0} \\
& \frac{1}{2}(m+M) v_{\max }^{2}=\frac{1}{2} k A^{2} \rightarrow \frac{1}{2}(m+M)\left(\frac{m}{m+M} v_{0}\right)^{2}=\frac{1}{2} k A^{2} \rightarrow
\end{aligned}
$$

$$
\begin{aligned}
v_{0} & =\frac{A}{m} \sqrt{k(m+M)}=\frac{\left(9.460 \times 10^{-2} \mathrm{~m}\right)}{\left(7.870 \times 10^{-3} \mathrm{~kg}\right)} \sqrt{(142.7 \mathrm{~N} / \mathrm{m})\left(7.870 \times 10^{-3} \mathrm{~kg}+4.648 \mathrm{~kg}\right)} \\
& =309.8 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

38. The hint says to integrate Eq. 14-11a, which comes from the conservation of energy. Let the initial position of the oscillator be $x_{0}$.

$$
\begin{aligned}
& v= \pm \sqrt{\frac{k}{m}\left(A^{2}-x^{2}\right)}=\frac{d x}{d t} \rightarrow \frac{d x}{\sqrt{\left(A^{2}-x^{2}\right)}}= \pm \sqrt{\frac{k}{m}} d t \rightarrow \int_{x_{0}}^{x} \frac{d x}{\sqrt{\left(A^{2}-x^{2}\right)}}= \pm \sqrt{\frac{k}{m}} \int_{0}^{t} d t \rightarrow \\
& -\cos ^{-1}\left(\frac{x}{A}\right)_{x_{0}}^{x}=-\cos ^{-1} \frac{x}{A}+\cos ^{-1} \frac{x_{0}}{A}= \pm \sqrt{\frac{k}{m}} t
\end{aligned}
$$

Make these definitions: $\sqrt{\frac{k}{m}} \equiv \omega ; \cos ^{-1} \frac{x_{0}}{A} \equiv \phi$. Then we have the following.

$$
-\cos ^{-1} \frac{x}{A}+\cos ^{-1} \frac{x_{0}}{A}= \pm \sqrt{\frac{k}{m}} t \rightarrow-\cos ^{-1} \frac{x}{A}+\phi= \pm \omega t \rightarrow x=A \cos ( \pm \omega t+\phi)
$$

The phase angle definition could be changed so that the function is a sine instead of a cosine. And the $\pm$ sign can be resolved if the initial velocity is known.
39. (a) Find the period and frequency from the mass and the spring constant.

$$
\begin{aligned}
& T=2 \pi \sqrt{\frac{m}{k}}=2 \pi \sqrt{\frac{0.785 \mathrm{~kg}}{184 \mathrm{~N} / \mathrm{m}}}=0.4104 \mathrm{~s} \approx 0.410 \mathrm{~s} \\
& f=\frac{1}{T}=\frac{1}{2 \pi} \sqrt{\frac{k}{m}}=\frac{1}{2 \pi} \sqrt{\frac{184 \mathrm{~N} / \mathrm{m}}{0.785 \mathrm{~kg}}}=2.437 \mathrm{~Hz} \approx 2.44 \mathrm{~Hz}
\end{aligned}
$$

(b) The initial speed is the maximum speed, and that can be used to find the amplitude.

$$
\begin{aligned}
& v_{\max }=A \sqrt{k / m} \rightarrow \\
& A=v_{\max } \sqrt{m / k}=(2.26 \mathrm{~m} / \mathrm{s}) \sqrt{0.785 \mathrm{~kg} /(184 \mathrm{~N} / \mathrm{m})}=0.1476 \mathrm{~m} \approx 0.148 \mathrm{~m}
\end{aligned}
$$

(c) The maximum acceleration can be found from the mass, spring constant, and amplitude

$$
a_{\text {max }}=A k / m=(0.1476 \mathrm{~m})(184 \mathrm{~N} / \mathrm{m}) /(0.785 \mathrm{~kg})=34.6 \mathrm{~m} / \mathrm{s}^{2}
$$

(d) Because the mass started at the equilibrium position of $x=0$, the position function will be proportional to the sine function.

$$
x=(0.148 \mathrm{~m}) \sin [2 \pi(2.437 \mathrm{~Hz}) t] \rightarrow x=(0.148 \mathrm{~m}) \sin (4.87 \pi t)
$$

(e) The maximum energy is the kinetic energy that the object has when at the equilibrium position.

$$
E=\frac{1}{2} m v_{\text {max }}^{2}=\frac{1}{2}(0.785 \mathrm{~kg})(2.26 \mathrm{~m} / \mathrm{s})^{2}=2.00 \mathrm{~J}
$$

( $f$ ) Use the conservation of mechanical energy for the oscillator.

$$
\begin{aligned}
& E=\frac{1}{2} k x^{2}+\frac{1}{2} m v^{2}=\frac{1}{2} k A^{2} \rightarrow \frac{1}{2} k(0.40 A)^{2}+K=\frac{1}{2} k A^{2} \rightarrow \\
& K=\frac{1}{2} k A^{2}\left(1-0.40^{2}\right)=(2.00 \mathrm{~J})(0.84)=1.68 \mathrm{~J}
\end{aligned}
$$

40. We solve this using conservation of energy, equating the energy at the compressed point with the energy as the ball leaves the launcher. Take the 0 location for gravitational potential energy to be at the level where the ball is on the compressed spring. The 0 location for elastic potential energy is the uncompressed position of the spring. Initially, the ball has only elastic potential energy. At the point where the spring is uncompressed and the ball just leaves the spring, there will be gravitational potential energy, translational kinetic energy, and rotational kinetic energy. The ball is rolling without slipping.

$$
\begin{aligned}
E_{\mathrm{i}} & =E_{\mathrm{f}} \rightarrow \frac{1}{2} k x^{2}=m g h+\frac{1}{2} m v^{2}+\frac{1}{2} I \omega^{2}=m g x \sin \theta+\frac{1}{2} m v^{2}+\frac{1}{2}\left(\frac{2}{5}\right) m r^{2} \frac{v^{2}}{r^{2}} \rightarrow \\
k & =2 \frac{m}{x^{2}}\left(g x \sin \theta+\frac{7}{10} v^{2}\right)=2 \frac{0.025 \mathrm{~kg}}{(0.060 \mathrm{~m})^{2}}\left[\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.060 \mathrm{~m}) \sin 15^{\circ}+\frac{7}{10}(3.0 \mathrm{~m} / \mathrm{s})^{2}\right] \\
& =89.61 \mathrm{~N} / \mathrm{m} \approx 90 \mathrm{~N} / \mathrm{m}(2 \text { sig. fig. })
\end{aligned}
$$

41. The period of a pendulum is given by $T=2 \pi \sqrt{L / g}$. The length is assumed to be the same for the pendulum both on Mars and on Earth.

$$
\begin{aligned}
& T=2 \pi \sqrt{L / g} \rightarrow \frac{T_{\text {Mars }}}{T_{\text {Earth }}}=\frac{2 \pi \sqrt{L / g_{\text {Mars }}}}{2 \pi \sqrt{L / g_{\text {Earth }}}}=\sqrt{\frac{g_{\text {Earth }}}{g_{\text {Mars }}}} \rightarrow \\
& T_{\text {Mars }}=T_{\text {Earth }} \sqrt{\frac{g_{\text {Earth }}}{g_{\text {Mars }}}}=(1.35 \mathrm{~s}) \sqrt{\frac{1}{0.37}}=2.2 \mathrm{~s}
\end{aligned}
$$

42. (a) The period is given by $T=\frac{50 \mathrm{~s}}{32 \text { cycles }}=1.6 \mathrm{~s}$.
(b) The frequency is given by $f=\frac{32 \text { cycles }}{50 \mathrm{~s}}=0.64 \mathrm{~Hz}$.
43. We consider this a simple pendulum. Since the motion starts at the amplitude position at $t=0$, we may describe it by a cosine function with no phase angle, $\theta=\theta_{\max } \cos \omega t$. The angular velocity can be written as a function of the length, $\theta=\theta_{\max } \cos \left(\sqrt{\frac{g}{\ell}} t\right)$.
(a) $\theta(t=0.35 \mathrm{~s})=13^{\circ} \cos \left(\sqrt{\frac{9.80 \mathrm{~m} / \mathrm{s}^{2}}{0.30 \mathrm{~m}}}(0.35 \mathrm{~s})\right)=-5.4^{\circ}$
(b) $\theta(t=3.45 \mathrm{~s})=13^{\circ} \cos \left(\sqrt{\frac{9.80 \mathrm{~m} / \mathrm{s}^{2}}{0.30 \mathrm{~m}}}(3.45 \mathrm{~s})\right)=8.4^{\circ}$
(c) $\theta(t=6.00 \mathrm{~s})=13^{\circ} \cos \left(\sqrt{\frac{9.80 \mathrm{~m} / \mathrm{s}^{2}}{0.30 \mathrm{~m}}}(6.00 \mathrm{~s})\right)=-13^{\circ}$
44. The period of a pendulum is given by $T=2 \pi \sqrt{L / g}$.
(a) $T=2 \pi \sqrt{L / g}=2 \pi \sqrt{\frac{0.53 \mathrm{~m}}{9.80 \mathrm{~m} / \mathrm{s}^{2}}}=1.5 \mathrm{~s}$
(b) If the pendulum is in free fall, there is no tension in the string supporting the pendulum bob, and so no restoring force to cause oscillations. Thus there will be no period - the pendulum will not oscillate and so no period can be defined.
45. If we consider the pendulum as starting from its maximum displacement, then the equation of motion can be written as $\theta=\theta_{0} \cos \omega t=\theta_{0} \cos \frac{2 \pi t}{T}$. Solve for the time for the position to decrease to half the amplitude.

$$
\theta_{1 / 2}=\frac{1}{2} \theta_{0}=\theta_{0} \cos \frac{2 \pi t_{1 / 2}}{T} \rightarrow \frac{2 \pi t_{1 / 2}}{T}=\cos ^{-1} \frac{1}{2}=\frac{\pi}{3} \rightarrow t_{1 / 2}=\frac{1}{6} T
$$

It takes $\frac{1}{6} T$ for the position to change from $+10^{\circ}$ to $+5^{\circ}$. It takes $\frac{1}{4} T$ for the position to change from $+10^{\circ}$ to 0 . Thus it takes $\frac{1}{4} T-\frac{1}{6} T=\frac{1}{12} T$ for the position to change from $+5^{\circ}$ to 0 . Due to the symmetric nature of the cosine function, it will also take $\frac{1}{12} T$ for the position to change from 0 to $-5^{\circ}$, and so from $+5^{\circ}$ to $-5^{\circ}$ takes $\frac{1}{6} T$. The second half of the cycle will be identical to the first, and so the total time spent between $+5^{\circ}$ and $-5^{\circ}$ is $\frac{1}{3} T$. So the pendulum spends one-third of its time between $+5^{\circ}$ and $-5^{\circ}$.
46. There are $(24 \mathrm{~h})(60 \mathrm{~min} / \mathrm{h})(60 \mathrm{~s} / \mathrm{min})=86,400 \mathrm{~s}$ in a day. The clock should make one cycle in exactly two seconds (a "tick" and a "tock"), and so the clock should make 43,200 cycles per day. After one day, the clock in question is 26 seconds slow, which means that it has made 13 less cycles than required for precise timekeeping. Thus the clock is only making 43,187 cycles in a day.
Accordingly, the period of the clock must be decreased by a factor of $\frac{43,187}{43,200}$.

$$
\begin{aligned}
& T_{\text {new }}=\frac{43,187}{43,200} T_{\text {old }} \rightarrow 2 \pi \sqrt{\ell_{\text {new }} / g}=\left(\frac{43,187}{43,200}\right) 2 \pi \sqrt{\ell_{\text {old }} / g} \rightarrow \\
& \ell_{\text {new }}=\left(\frac{43,187}{43,200}\right)^{2} \ell_{\text {old }}=\left(\frac{43,187}{43,200}\right)^{2}(0.9930 \mathrm{~m})=0.9924 \mathrm{~m}
\end{aligned}
$$

Thus the pendulum should be shortened by 0.6 mm .
47. Use energy conservation to relate the potential energy at the maximum height of the pendulum to the kinetic energy at the lowest point of the swing. Take the lowest point to be the zero location for gravitational potential energy. See the diagram.

$$
\begin{aligned}
& E_{\text {top }}=E_{\text {bottom }} \rightarrow K_{\text {top }}+U_{\text {top }}=K_{\text {bottom }}+U_{\text {bottom }} \rightarrow \\
& 0+m g h=\frac{1}{2} m v_{\max }^{2} \rightarrow v_{\max }=\sqrt{2 g h}=\sqrt{2 g \ell(1-\cos \theta)}
\end{aligned}
$$


48. (a) For a physical pendulum with the small angle approximation, we may apply Eq. 1414. We need the moment of inertia and the distance from the suspension point to the center of mass. We approximate the cord as a rod, and find the center of mass relative to the stationary end of the cord.

$$
\begin{aligned}
& I=I_{\mathrm{bob}}+I_{\mathrm{cord}}=M \ell^{2}+\frac{1}{3} m \ell^{2}=\left(M+\frac{1}{3} m\right) \ell^{2} \\
& h=x_{\mathrm{CM}}=\frac{M \ell+m\left(\frac{1}{2} \ell\right)}{M+m}=\left(\frac{M+\frac{1}{2} m}{M+m}\right) \ell \\
& T=2 \pi \sqrt{\frac{I}{m_{\text {total }} g h}}=2 \pi \sqrt{\frac{\left(M+\frac{1}{3} m\right) \ell^{2}}{(M+m) g\left(\frac{M+\frac{1}{2} m}{M+m}\right) \ell}}=2 \pi \sqrt{\frac{\left(M+\frac{1}{3} m\right) \ell}{\left(M+\frac{1}{2} m\right) g}}
\end{aligned}
$$

(b) If we use the expression for a simple pendulum we would have $T_{\text {simple }}=2 \pi \sqrt{\ell / g}$. Find the fractional error.

$$
\text { error } \left.=\frac{T-T_{\text {simple }}}{T}=\frac{2 \pi \sqrt{\frac{\left(M+\frac{1}{3} m\right) \ell}{\left(M+\frac{1}{2} m\right) g}}-2 \pi \sqrt{\frac{\ell}{g}}}{2 \pi \sqrt{\frac{\left(M+\frac{1}{3} m\right) \ell}{\left(M+\frac{1}{2} m\right) g}}}=\frac{\sqrt{\frac{\left(M+\frac{1}{3} m\right)}{\left(M+\frac{1}{2} m\right)}}}{\sqrt{\frac{\left(M+\frac{1}{3} m\right)}{\left(M+\frac{1}{2} m\right)}}}=1-\sqrt{\frac{\left(M+\frac{1}{2} m\right)}{\left(M+\frac{1}{3} m\right)}}\right)
$$

Note that this is negative, indicating that the simple pendulum approximation is too large.
49. The balance wheel of the watch is a torsion pendulum, described by $\tau=-K \theta$. A specific torque and angular displacement are given, and so the torsional constant can be determined. The angular frequency is given by $\omega=\sqrt{K / I}$. Use these relationships to find the mass.

$$
\begin{aligned}
& \tau=-K \theta \rightarrow K=\left|\frac{\theta}{\tau}\right|=\frac{1.1 \times 10^{-5} \mathrm{~m} \cdot \mathrm{~N}}{\pi / 4 \mathrm{rad}} \\
& \omega=2 \pi f=\sqrt{\frac{K}{I}}=\sqrt{\frac{K}{m r^{2}}} \rightarrow \\
& m=\frac{K}{4 \pi^{2} f^{2} r^{2}}=\frac{1.1 \times 10^{-5} \mathrm{~m} \cdot \mathrm{~N}}{\pi / 4 \mathrm{rad}} \\
& 4 \pi^{2}(3.10 \mathrm{~Hz})^{2}\left(0.95 \times 10^{-2} \mathrm{~m}\right)^{2}
\end{aligned}=4.1 \times 10^{-4} \mathrm{~kg}=0.41 \mathrm{~g}
$$

50. (a) We call the upper mass $M$ and the lower mass $m$. Both masses have length $\ell$. The period of the physical pendulum is given by Eq. 14-14. Note that we must find both the moment of inertia of the system about the uppermost point, and the center of mass of the system. The parallel axis theorem is used to find the moment of inertia.

$$
\begin{aligned}
& I=I_{\text {upper }}+I_{\text {lower }}=\frac{1}{3} M \ell^{2}+\frac{1}{12} m \ell^{2}+m\left(\frac{3}{2} \ell\right)^{2}=\left(\frac{1}{3} M+\frac{7}{3} m\right) \ell^{2} \\
& h=x_{\mathrm{CM}}=\frac{M\left(\frac{1}{2} \ell\right)+m\left(\frac{3}{2} \ell\right)}{M+m}=\left(\frac{\frac{1}{2} M+\frac{3}{2} m}{M+m}\right) \ell \\
& T=2 \pi \sqrt{\frac{I}{m_{\text {total }} g h}}=2 \pi \sqrt{\frac{\left(\frac{1}{3} M+\frac{7}{3} m\right) \ell^{2}}{(M+m) g\left(\frac{\frac{1}{2} M+\frac{3}{2} m}{M+m}\right) \ell}}=2 \pi \sqrt{\frac{\left(\frac{1}{3} M+\frac{7}{3} m\right) \ell}{\left(\frac{1}{2} M+\frac{3}{2} m\right) g}}
\end{aligned}
$$



$$
=2 \pi \sqrt{\frac{\left[\frac{1}{3}(7.0 \mathrm{~kg})+\frac{7}{3}(4.0 \mathrm{~kg})\right](0.55 \mathrm{~m})}{\left[\frac{1}{2}(7.0 \mathrm{~kg})+\frac{3}{2}(4.0 \mathrm{~kg})\right]\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}}=1.6495 \mathrm{~s} \approx 1.6 \mathrm{~s}
$$

(b) It took 7.2 seconds for 5 swings, which gives a period of 1.4 seconds. That is reasonable qualitative agreement.
51. (a) In the text, we are given that $\tau=-K \theta$. Newton's second law for rotation, Eq. $10-14$, says that $\sum \tau=I \alpha=I \frac{d^{2} \theta}{d t^{2}}$. We assume that the torque applied by the twisting of the wire is the only torque.

$$
\sum \tau=I \alpha=I \frac{d^{2} \theta}{d t^{2}}=-K \theta \rightarrow \frac{d^{2} \theta}{d t^{2}}=-\frac{K}{I} \theta=-\omega^{2} \theta
$$

This is the same form as Eq. 14-3, which is the differential equation for simple harmonic oscillation. We exchange variables with Eq. 14-4, and write the equation for the angular motion.

$$
x=A \cos (\omega t+\phi) \rightarrow \theta=\theta_{0} \cos (\omega t+\phi), \omega^{2}=\frac{K}{I}
$$

(b) The period of the motion is found from the angular velocity $\omega$.

$$
\omega^{2}=\frac{K}{I} \rightarrow \omega=\sqrt{\frac{K}{I}}=\frac{2 \pi}{T} \rightarrow T=2 \pi \sqrt{\frac{I}{K}}
$$

52. The meter stick used as a pendulum is a physical pendulum. The period is given by Eq. 14-14, $T=2 \pi \sqrt{\frac{I}{m g h}}$. Use the parallel axis theorem to find the moment of inertia about the pin. Express the distances from the center of mass.

$$
\begin{aligned}
& I=I_{\mathrm{CM}}+m h^{2}=\frac{1}{12} m \ell^{2}+m h^{2} \rightarrow T=2 \pi \sqrt{\frac{I}{m g h}}=2 \pi \sqrt{\frac{\frac{1}{12} m \ell^{2}+m h^{2}}{m g h}}=\frac{2 \pi}{\sqrt{g}}\left(\frac{1}{12} \frac{\ell^{2}}{h}+h\right)^{1 / 2} \\
& \frac{d T}{d h}=2 \pi\left(\frac{1}{2}\right)\left(\frac{1}{12} \frac{\ell^{2}}{h}+h\right)^{-1 / 2}\left(-\frac{1}{12} \frac{\ell^{2}}{h^{2}}+1\right)=0 \rightarrow h=\sqrt{\frac{1}{12}} \ell=0.2887 \mathrm{~m} \\
& x=\frac{1}{2} \ell-h=0.500-0.2887 \approx 0.211 \mathrm{~m} \text { from the end }
\end{aligned}
$$

Use the distance for $h$ to calculate the period.

$$
T=\frac{2 \pi}{\sqrt{g}}\left(\frac{1}{12} \frac{\ell^{2}}{h}+h\right)^{1 / 2}=\frac{2 \pi}{\sqrt{9.80 \mathrm{~m} / \mathrm{s}^{2}}}\left(\frac{1}{12} \frac{(1.00 \mathrm{~m})^{2}}{0.2887 \mathrm{~m}}+0.2887 \mathrm{~m}\right)^{1 / 2}=1.53 \mathrm{~s}
$$

53. This is a torsion pendulum. The angular frequency is given in the text as $\omega=\sqrt{K / I}$, where $K$ is the torsion constant (a property of the wire, and so a constant in this problem). The rotational inertia of a rod about its center is $\frac{1}{12} M \ell^{2}$.

$$
\omega=\sqrt{\frac{K}{I}}=\frac{2 \pi}{T} \rightarrow T=2 \pi \sqrt{\frac{I}{K}} \rightarrow
$$

$$
\begin{aligned}
& \frac{T}{T_{0}}=\frac{2 \pi \sqrt{\frac{I}{K}}}{2 \pi \sqrt{\frac{I_{0}}{K}}}=\sqrt{\frac{I}{I_{0}}}=\sqrt{\frac{\frac{1}{12} M \ell^{2}}{\frac{1}{12} M_{0} \ell_{0}^{2}}}=\sqrt{\frac{\left(0.700 M_{0}\right)\left(0.700 \ell_{0}\right)^{2}}{M_{0} \ell_{0}^{2}}}=0.58566 \\
& T=(0.58566) T_{0}=(0.58566)(5.0 \mathrm{~s})=2.9 \mathrm{~s}
\end{aligned}
$$

54. The torsional constant is related to the period through the relationship given in problem 51. The rotational inertia of a disk in this configuration is $I=\frac{1}{2} M R^{2}$.

$$
\begin{aligned}
T=2 \pi \sqrt{\frac{I}{K}} \rightarrow K & =\frac{4 \pi^{2} I}{T^{2}}=\frac{4 \pi^{2} \frac{1}{2} M R^{2}}{T^{2}}=2 \pi^{2} M R^{2} f^{2}=2 \pi^{2}(0.375 \mathrm{~kg})(0.0625 \mathrm{~m})^{2}(0.331 \mathrm{~Hz})^{2} \\
& =3.17 \times 10^{-3} \mathrm{~m} \cdot \mathrm{~N} / \mathrm{rad}
\end{aligned}
$$

55. This is a physical pendulum. Use the parallel axis theorem to find the moment of inertia about the pin at point A, and then use Eq. 14-14 to find the period.

$$
\begin{aligned}
& I_{\mathrm{pin}}=I_{\mathrm{CM}}+M h^{2}=\frac{1}{2} M R^{2}+M h^{2}=M\left(\frac{1}{2} R^{2}+h^{2}\right) \\
& T
\end{aligned} \begin{aligned}
\frac{I}{M g h} & =2 \pi \sqrt{\frac{M\left(\frac{1}{2} R^{2}+h^{2}\right)}{M g h}}=2 \pi \sqrt{\frac{\left(\frac{1}{2} R^{2}+h^{2}\right)}{g h}} \\
& =2 \pi \sqrt{\frac{\frac{1}{2}(0.200 \mathrm{~m})^{2}+(0.180 \mathrm{~m})^{2}}{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.180 \mathrm{~m})}}=1.08 \mathrm{~s}
\end{aligned}
$$


56. (a) The period of the motion can be found from Eq. 14-18, giving the angular frequency for the damped motion.

$$
\begin{aligned}
& \omega^{\prime}=\sqrt{\frac{k}{m}-\frac{b^{2}}{4 m^{2}}}=\sqrt{\frac{(41.0 \mathrm{~N} / \mathrm{m})}{(0.835 \mathrm{~kg})}-\frac{(0.662 \mathrm{~N} \cdot \mathrm{~s} / \mathrm{m})^{2}}{4(0.835 \mathrm{~kg})^{2}}}=6.996 \mathrm{rad} / \mathrm{s} \\
& T=\frac{2 \pi}{\omega}=\frac{2 \pi}{6.996 \mathrm{rad} / \mathrm{s}}=0.898 \mathrm{~s}
\end{aligned}
$$

(b) If the amplitude at some time is $A$, then one cycle later, the amplitude will be $A e^{-\gamma T}$. Use this to find the fractional change.

$$
\text { fractional change }=\frac{A e^{-\gamma T}-A}{A}=e^{-\gamma T}-1=e^{-\frac{b}{2 m} T}-1=e^{-\frac{(0.662 \mathrm{~N} \cdot \mathrm{~s} / \mathrm{m})}{2(0.83 \mathrm{~kg})}(0.898 \mathrm{~s})}-1=-0.300
$$

And so the amplitude decreases by $30 \%$ from the previous amplitude, every cycle.
(c) Since the object is at the origin at $t=0$, we will use a sine function to express the equation of motion.

$$
\begin{aligned}
& x=A e^{-\gamma t} \sin \left(\omega^{\prime} t\right) \rightarrow 0.120 \mathrm{~m}=A e^{-\frac{(0.662 \mathrm{~N} \cdot \mathrm{~s} / \mathrm{m})}{2(0.835 \mathrm{~kg})}(1.00 \mathrm{~s})} \sin (6.996 \mathrm{rad}) \rightarrow \\
& A=\frac{0.120 \mathrm{~m}}{e^{-\frac{(0.662 \mathrm{~N} \cdot \mathrm{~s} / \mathrm{m})}{2(0.835 \mathrm{~kg})}(1.00 \mathrm{~s})} \sin (6.996 \mathrm{rad})}=0.273 \mathrm{~m} ; \gamma=\frac{b}{2 m}=\frac{(0.662 \mathrm{~N} \cdot \mathrm{~s} / \mathrm{m})}{2(0.835 \mathrm{~kg})}=0.396 \mathrm{~s}^{-1}
\end{aligned}
$$

$$
x=(0.273 \mathrm{~m}) e^{-\left(0.396 \mathrm{~s}^{-1}\right) t} \sin [(7.00 \mathrm{rad} / \mathrm{s}) t]
$$

57. We assume that initially, the system is critically damped, so $b_{\text {critical }}^{2}=4 m k$. Then, after aging, we assume that after 3 cycles, the car's oscillatory amplitude has dropped to $5 \%$ of its original amplitude. That is expressed by $A=A_{0} e^{-\frac{b t}{2 m}}$.

$$
\begin{aligned}
& A=A_{0} e^{-\frac{b t}{2 m}} \rightarrow 0.05 A_{0}=A e^{-\frac{b(3 T)}{2 m}}=A e^{-\frac{b}{2 m f}} \rightarrow \ln (0.05)=-\frac{3 b}{2 m} \frac{1}{f} \rightarrow \\
& \ln (0.05)=-\frac{3 b}{2 m} \frac{1}{\frac{1}{2 \pi} \sqrt{\frac{k}{m}-\frac{b^{2}}{4 m^{2}}}}=-\frac{3 b}{2 m} \frac{2 \pi}{\sqrt{\frac{b_{\text {critical }}^{2}-\frac{b^{2}}{4 m^{2}}}{4 m^{2}}}}=-\frac{6 \pi b}{\sqrt{b_{\text {critical }}^{2}-b^{2}}} \rightarrow \\
& \frac{b}{b_{\text {critical }}}=\left(1+\frac{36 \pi^{2}}{[\ln (0.05)]^{2}}\right)^{-1 / 2}=0.16
\end{aligned}
$$

And so $b$ has decreased to about $16 \%$ of its original value, or decreased by a factor of 6 . If we used $2 \%$ instead of $5 \%$, we would have found that $b$ decreased to about $20 \%$ of its original value. And if we used $10 \%$ instead of $5 \%$, we would have found that $b$ decreased to about $6 \%$ of its original value.
58. (a) Since the angular displacement is given as $\theta=A e^{-\gamma t} \cos \left(\omega^{\prime} t\right)$, we see that the displacement at $t$ $=0$ is the initial amplitude, so $A=15^{\circ}$. We evaluate the amplitude 8.0 seconds later.

$$
5.5^{\circ}=15^{\circ} e^{-\gamma(8.0 \mathrm{~s})} \rightarrow \gamma=\frac{-1}{8.0 \mathrm{~s}} \ln \left(\frac{5.5}{15}\right)=0.1254 \mathrm{~s}^{-1} \approx 0.13 \mathrm{~s}^{-1}
$$

(b) The approximate period can be found from the damped angular frequency. The undamped angular frequency is also needed for the calculation.

$$
\begin{aligned}
& \omega_{0}=\sqrt{\frac{m g h}{I}}=\sqrt{\frac{m g\left(\frac{1}{2} \ell\right)}{\frac{1}{3} m \ell^{2}}}=\sqrt{\frac{3 g}{2 \ell}} \\
& \omega^{\prime}=\sqrt{\omega_{0}^{2}-\gamma^{2}}=\sqrt{\frac{3 g}{2 \ell}-\gamma^{2}}=\sqrt{\frac{3\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{2(0.85 \mathrm{~m})}-\left(0.1254 \mathrm{~s}^{-1}\right)^{2}}=4.157 \mathrm{rad} / \mathrm{s} \\
& T^{\prime}=\frac{2 \pi}{\omega^{\prime}}=\frac{2 \pi \mathrm{rad}}{4.157 \mathrm{rad} / \mathrm{s}}=1.5 \mathrm{~s}
\end{aligned}
$$

(c) We solve the equation of motion for the time when the amplitude is half the original amplitude.

$$
7.5^{\circ}=15^{\circ} e^{-\gamma t} \rightarrow t_{1 / 2}=\frac{\ln 2}{\gamma}=\frac{\ln 2}{0.1254 \mathrm{~s}^{-1}}=5.5 \mathrm{~s}
$$

59. (a) The energy of the oscillator is all potential energy when the cosine (or sine) factor is 1 , and so $E=\frac{1}{2} k A^{2}=\frac{1}{2} k A_{0}^{2} e^{-\frac{b t}{m}}$. The oscillator is losing $6.0 \%$ of its energy per cycle. Use this to find the actual frequency, and then compare to the natural frequency.

$$
E(t+T)=0.94 E(t) \rightarrow \frac{1}{2} k A_{0}^{2} e^{\frac{b(t+T)}{m}}=0.94\left(\frac{1}{2} k A_{0}^{2} e^{-\frac{b t}{m}}\right) \rightarrow e^{\frac{b T}{m}}=0.94 \rightarrow
$$

$$
\begin{aligned}
& \frac{b}{2 m}=-\frac{1}{2 T} \ln (0.94)=-\frac{\omega_{0}}{4 \pi} \ln (0.94) \\
& \frac{f^{\prime}-f_{0}}{f_{0}}=\frac{\frac{1}{2 \pi} \sqrt{\omega_{0}^{2}-\frac{b^{2}}{4 m^{2}}}-\frac{\omega_{0}}{2 \pi}}{\frac{\omega_{0}}{2 \pi}}=\sqrt{1-\frac{b^{2}}{4 m^{2} \omega_{0}^{2}}}-1=\sqrt{1-\frac{[\ln (0.94)]^{2}}{16 \pi^{2}}}-1 \approx 1-\frac{1}{2} \frac{[\ln (0.94)]^{2}}{16 \pi^{2}}-1 \\
& \\
& =-\frac{1}{2} \frac{[\ln (0.94)]^{2}}{16 \pi^{2}}=-1.2 \times 10^{-5} \rightarrow \% \text { diff }=\left(\frac{f^{\prime}-f_{0}}{f_{0}}\right) 100=\left(-1.2 \times 10^{-3}\right) \%
\end{aligned}
$$

(b) The amplitude's decrease in time is given by $A=A_{0} e^{-\frac{b t}{2 m}}$. Find the decrease at a time of $n T$, and solve for $n$. The value of $\frac{b}{2 m}$ was found in part (a).

$$
\begin{aligned}
& A=A_{0} e^{-\frac{b t}{2 m}} \rightarrow A_{0} e^{-1}=A_{0} e^{-\frac{b n T}{2 m}} \rightarrow 1=\frac{b}{2 m} n T=-\frac{1}{2 T} \ln (0.94) n T \rightarrow \\
& n=-\frac{2}{\ln (0.94)}=32.32 \approx 32 \text { periods }
\end{aligned}
$$

60. The amplitude of a damped oscillator decreases according to $A=A_{0} e^{-\gamma t}=A_{0} e^{-\frac{b t}{2 \eta}}$. The data can be used to find the damping constant.

$$
A=A_{0} e^{-\frac{b t}{2 m}} \rightarrow b=\frac{2 m}{t} \ln \left(\frac{A_{0}}{A}\right)=\frac{2(0.075 \mathrm{~kg})}{(3.5 \mathrm{~s})} \ln \left(\frac{5.0}{2.0}\right)=0.039 \mathrm{~kg} / \mathrm{s}
$$

61. (a) For the "lightly damped" harmonic oscillator, we have $b^{2} \ll 4 m k \rightarrow \frac{b^{2}}{4 m^{2}} \ll \frac{k}{m} \rightarrow \omega^{\prime} \approx \omega_{0}$.

We also assume that the object starts to move from maximum displacement, and so

$$
\begin{aligned}
& x=A_{0} e^{-\frac{b t}{2 m}} \cos \omega^{\prime} t \text { and } v=\frac{d x}{d t}=-\frac{b}{2 m} A_{0} e^{-\frac{b t}{2 m}} \cos \omega^{\prime} t-\omega^{\prime} A_{0} e^{-\frac{b t}{2 m}} \sin \omega^{\prime} t \approx-\omega_{0} A_{0} e^{-\frac{b t}{2 m}} \sin \omega^{\prime} t . \\
& E=\frac{1}{2} k x^{2}+\frac{1}{2} m v^{2}=\frac{1}{2} k A_{0}^{2} e^{-\frac{b t}{m}} \cos ^{2} \omega^{\prime} t+\frac{1}{2} m \omega_{0}^{2} A_{0}^{2} e^{-\frac{b t}{m}} \sin ^{2} \omega^{\prime} t \\
&=\frac{1}{2} k A_{0}^{2} e^{-\frac{b t}{m}} \cos ^{2} \omega^{\prime} t+\frac{1}{2} k A_{0} e^{-\frac{b t}{m}} \sin ^{2} \omega^{\prime} t=\frac{1}{2} k A_{0}^{2} e^{-\frac{b t}{m}}=E_{0} e^{-\frac{b t}{m}}
\end{aligned}
$$

(b) The fractional loss of energy during one period is as follows. Note that we use the

$$
\begin{aligned}
& \text { approximation that } \frac{b}{2 m} \ll \omega_{0}=\frac{2 \pi}{T} \rightarrow \frac{b T}{m}<4 \pi \rightarrow \frac{b T}{m} \ll 1 . \\
& \Delta E=E(t)-E(t+T)=E_{0} e^{\frac{b t}{m}}-E_{0} e^{-\frac{b(t+T)}{m}}=E_{0} e^{-\frac{b t}{m}}\left(1-e^{-\frac{b T}{m}}\right) \rightarrow \\
& \frac{\Delta E}{E}=\frac{E_{0} e^{\frac{b t}{m}}\left(1-e^{-\frac{b T}{m}}\right)}{E_{0} e^{\frac{b t}{m}}}=1-e^{\frac{b T}{m}} \approx 1-\left(1-\frac{b T}{m}\right)=\frac{b T}{m}=\frac{b 2 \pi}{m \omega_{0}}=\frac{2 \pi}{Q}
\end{aligned}
$$

62. (a) From problem 25 (b), we can calculate the frequency of the undamped motion.

$$
\begin{aligned}
& T=2 \pi \sqrt{\frac{m}{k_{1}+k_{2}}}=2 \pi \sqrt{\frac{m}{2 k}} \rightarrow \\
& f=\frac{1}{2 \pi} \sqrt{\frac{2 k}{m}}=\sqrt{\frac{k}{2 \pi^{2} m}}=\sqrt{\frac{125 \mathrm{~N} / \mathrm{s}}{2 \pi^{2}(0.215 \mathrm{~kg})}}=5.43 \mathrm{~Hz}
\end{aligned}
$$

(b) Eq. 14-16 says $x=A e^{-\gamma t} \cos \omega^{\prime} t$, which says the amplitude follows the relationship $x_{\max }=A e^{-\gamma t}$. Use the fact that $x_{\max }=\frac{1}{2} A$ after 55 periods have elapsed, and assume that the damping is light enough that the damped frequency is the same as the natural frequency.

$$
\frac{1}{2} A=A e^{-\gamma(55 T)} \rightarrow \gamma=\frac{\ln 2}{55 T}=\frac{f}{55} \ln 2=\frac{5.43 \mathrm{~Hz}}{55} \ln 2=0.06843 \mathrm{~s}^{-1} \approx 0.0684 \mathrm{~s}^{-1}
$$

(c) Again use $x_{\max }=A e^{-\gamma t}$.

$$
x_{\text {max }}=A e^{-\gamma t} \rightarrow \frac{1}{4} A=A e^{-\gamma t} \rightarrow t=\frac{\ln 4}{\gamma}=\frac{\ln 4}{0.06843 \mathrm{~s}^{-1}}=20.3 \mathrm{~s}
$$

This is the time for 110 oscillations, since 55 oscillations corresponds to a "half-life."
63. (a) Eq. 14-24 is used to calculate $\phi_{0}$.

$$
\phi_{0}=\tan ^{-1} \frac{\omega_{0}^{2}-\omega^{2}}{\omega(b / m)} \rightarrow \text { if } \omega=\omega, \phi_{0}=\tan ^{-1} \frac{\omega_{0}^{2}-\omega_{0}^{2}}{\omega(b / m)}=0
$$

(b) With $\omega=\omega_{0}$, we have $F_{\text {ext }}=F_{0} \cos \omega_{0} t$ and $x=A_{0} \sin \omega_{0} t$. The displacement and the driving force are one-quarter cycle $\left(\frac{1}{2} \pi \mathrm{rad}\right.$ or $\left.90^{\circ}\right)$ out of phase with each other. The displacement is 0 when the driving force is a maximum, and the displacement is a maximum $(+A$ or $-A)$ when the driving force is 0 .
(c) As mentioned above, the phase difference is $90^{\circ}$.
64. Eq. 14-23 gives the amplitude $A_{0}$ as a function of driving frequency $\omega$. To find the frequency for maximum amplitude, we set $\frac{d A_{0}}{d \omega}=0$ and solve for $\omega$.

$$
\begin{aligned}
& A_{0}=\frac{F_{0}}{m \sqrt{\left(\omega^{2}-\omega_{0}^{2}\right)^{2}+b^{2} \omega^{2} / m^{2}}}=\frac{F_{0}}{m}\left[\left(\omega^{2}-\omega_{0}^{2}\right)^{2}+b^{2} \omega^{2} / m^{2}\right]^{-1 / 2} \\
& \frac{d A_{0}}{d \omega}=\frac{F_{0}}{m}\left(-\frac{1}{2}\right)\left[\left(\omega^{2}-\omega_{0}^{2}\right)^{2}+b^{2} \omega^{2} / m^{2}\right]^{-3 / 2}\left[2\left(\omega^{2}-\omega_{0}^{2}\right) 2 \omega+2 b^{2} \omega / m^{2}\right]=0 \rightarrow \\
& -\frac{1}{2} \frac{F_{0}}{m} \frac{\left[2\left(\omega^{2}-\omega_{0}^{2}\right) 2 \omega+2 b^{2} \omega / m^{2}\right]}{\left[\left(\omega^{2}-\omega_{0}^{2}\right)^{2}+b^{2} \omega^{2} / m^{2}\right]^{3 / 2}}=0 \rightarrow 2\left(\omega^{2}-\omega_{0}^{2}\right) 2 \omega+2 b^{2} \omega / m^{2}=0 \rightarrow \\
& \omega^{2}=\omega_{0}^{2}-\frac{b^{2}}{2 m^{2}} \rightarrow \omega=\sqrt{\omega_{0}^{2}-\frac{b^{2}}{2 m^{2}}}
\end{aligned}
$$

65. We approximate that each spring of the car will effectively support one-fourth of the mass. The rotation of the improperly-balanced car tire will force the spring into oscillation. The shaking will be most prevalent at resonance, where the frequency of the tire matches the frequency of the spring. At resonance, the angular velocity of the car tire, $\omega=\frac{v}{r}$, will be the same as the angular frequency of the spring system, $\omega=\sqrt{\frac{k}{m}}$.

$$
\omega=\frac{v}{r}=\sqrt{\frac{k}{m}} \rightarrow v=r \sqrt{\frac{k}{m}}=(0.42 \mathrm{~m}) \sqrt{\frac{16,000 \mathrm{~N} / \mathrm{m}}{\frac{1}{4}(1150 \mathrm{~kg})}}=3.1 \mathrm{~m} / \mathrm{s}
$$

66. First, we put Eq. 14-23 into a form that explicitly shows $A_{0}$ as a function of $Q$ and has the ratio $\omega / \omega_{0}$.

$$
\begin{aligned}
A_{0} & =\frac{F_{0}}{m \sqrt{\left(\omega^{2}-\omega_{0}^{2}\right)^{2}+b^{2} \omega^{2} / m^{2}}}=\frac{F_{0}}{m \sqrt{\left(\omega^{2} \frac{\omega_{0}^{2}}{\omega_{0}^{2}}-\omega_{0}^{2}\right)^{2}+\frac{b^{2} \omega^{2}}{m^{2}} \frac{\omega_{0}^{2}}{\omega_{0}^{2}}}} \\
& =\frac{F_{0}}{m \sqrt{\omega_{0}^{4}\left(\frac{\omega^{2}}{\omega_{0}^{2}}-1\right)^{2}+\omega_{0}^{4} \frac{b^{2}}{m^{2} \omega_{0}^{2}} \frac{\omega^{2}}{\omega_{0}^{2}}}}=\frac{F_{0}}{m \omega_{0}^{2} \sqrt{\left(\frac{\omega^{2}}{\omega_{0}^{2}}-1\right)^{2}+\frac{1}{Q^{2} \frac{\omega^{2}}{\omega_{0}^{2}}}}}=\frac{F_{0} /\left(m \omega_{0}^{2}\right)}{\sqrt{\left(\frac{\omega^{2}}{\omega_{0}^{2}}-1\right)^{2}+\frac{1}{Q^{2}} \frac{\omega^{2}}{\omega_{0}^{2}}}} \\
& =\frac{F_{0} / k}{\sqrt{\left(\omega^{2} / \omega_{0}^{2}-1\right)^{2}+\frac{1}{Q^{2}} \omega^{2} / \omega_{0}^{2}}} \rightarrow \frac{A_{0}}{F_{0} / k}=\frac{\sqrt{\left(\left(\omega / \omega_{0}\right)^{2}-1\right)^{2}+\left(\omega / \omega_{0}\right)^{2} \frac{1}{Q^{2}}}}{}
\end{aligned}
$$

For a value of $Q=6.0$, the following graph is obtained.
The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH14.XLS," on tab "Problem 14.66."

67. Apply the resonance condition, $\omega=\omega_{0}$, to Eq. 14-23, along with the given condition of $A_{0}=23.7 \frac{F_{0}}{m}$. Note that for this condition to be true, the value of 23.7 must have units of $\mathrm{s}^{2}$.

$$
A_{0}=\frac{F_{0}}{m \sqrt{\left(\omega^{2}-\omega_{0}^{2}\right)^{2}+b^{2} \omega^{2} / m^{2}}} \rightarrow
$$

$$
A_{0}\left(\omega=\omega_{0}\right)=\frac{F_{0}}{m \sqrt{b^{2} \omega_{0}^{2} / m^{2}}}=\frac{F_{0}}{m \frac{b \omega_{0}}{m}}=\frac{F_{0}}{m \frac{b \omega_{0}^{2}}{m \omega_{0}}}=\frac{F_{0}}{\frac{m \omega_{0}^{2}}{Q}}=Q \frac{F_{0}}{k}=23.7 \frac{F_{0}}{k} \rightarrow Q=23.7
$$

68. We are to show that $x=A_{0} \sin \left(\omega t+\phi_{0}\right)$ is a solution of $m \frac{d x^{2}}{d t^{2}}+b \frac{d x}{d t}+k x=F_{0} \cos \omega t$ by direct substitution.

$$
\begin{aligned}
& x=A_{0} \sin \left(\omega t+\phi_{0}\right) ; \frac{d x}{d t}=\omega A_{0} \cos \left(\omega t+\phi_{0}\right) ; \frac{d^{2} x}{d t^{2}}=-\omega^{2} A_{0} \sin \left(\omega t+\phi_{0}\right) \\
& m \frac{d x^{2}}{d t^{2}}+b \frac{d x}{d t}+k x=F_{0} \cos \omega t \rightarrow \\
& m\left[-\omega^{2} A_{0} \sin \left(\omega t+\phi_{0}\right)\right]+b\left[\omega A_{0} \cos \left(\omega t+\phi_{0}\right)\right]+k\left[A_{0} \sin \left(\omega t+\phi_{0}\right)\right]=F_{0} \cos \omega t
\end{aligned}
$$

Expand the trig functions.

$$
\left(k A_{0}-m \omega^{2} A_{0}\right)\left[\sin \omega t \cos \phi_{0}+\cos \omega t \sin \phi_{0}\right]+b \omega A_{0}\left[\cos \omega t \cos \phi_{0}-\sin \omega t \sin \phi_{0}\right]=F_{0} \cos \omega t
$$

Group by function of time.

$$
\begin{aligned}
& {\left[\left(k A_{0}-m \omega^{2} A_{0}\right) \cos \phi_{0}-b \omega A_{0} \sin \phi_{0}\right] \sin \omega t+\left[\left(k A_{0}-m \omega^{2} A_{0}\right) \sin \phi_{0}+b \omega A_{0} \cos \phi_{0}\right] \cos \omega t} \\
& \quad \quad=F_{0} \cos \omega t
\end{aligned}
$$

The equation has to be valid for all times, which means that the coefficients of the functions of time must be the same on both sides of the equation. Since there is no $\sin \omega t$ on the right side of the equation, the coefficient of $\sin \omega t$ must be 0 .

$$
\begin{aligned}
& \left(k A_{0}-m \omega^{2} A_{0}\right) \cos \phi_{0}-b \omega A_{0} \sin \phi_{0}=0 \rightarrow \\
& \frac{\sin \phi_{0}}{\cos \phi_{0}}=\frac{k A_{0}-m \omega^{2} A_{0}}{b \omega A_{0}}=\frac{k-m \omega^{2}}{b \omega}=\frac{m \omega_{0}^{2}-m \omega^{2}}{b \omega}=\frac{\omega_{0}^{2}-\omega^{2}}{\omega b / m}=\tan \phi_{0} \rightarrow \phi_{0}=\tan ^{-1} \frac{\omega_{0}^{2}-\omega^{2}}{\omega b / m}
\end{aligned}
$$

Thus we see that Eq. 14-24 is necessary for $x=A_{0} \sin \left(\omega t+\phi_{0}\right)$ to be the solution. This can be illustrated with the diagram shown.

Equate the coefficients of $\cos \omega t$.

$$
\sqrt{\left(\omega_{0}^{2}-\omega^{2}\right)^{2}+\frac{\omega^{2} b^{2}}{m^{2}}} \omega_{0}^{2}-\omega^{2}
$$

Thus we see that Eq. 14-23 is also necessary for $x=A_{0} \sin \left(\omega t+\phi_{0}\right)$ to be the solution.

$$
\begin{aligned}
& \left(k A_{0}-m \omega^{2} A_{0}\right) \sin \phi_{0}+b \omega A_{0} \cos \phi_{0}=F_{0} \rightarrow \\
& A_{0}\left[\left(k-m \omega^{2}\right) \frac{\left(\omega_{0}^{2}-\omega^{2}\right)}{\sqrt{\left(\omega_{0}^{2}-\omega^{2}\right)^{2}+\frac{\omega^{2} b^{2}}{m^{2}}}}+b \omega \frac{\frac{\omega b}{m}}{\sqrt{\left(\omega_{0}^{2}-\omega^{2}\right)^{2}+\frac{\omega^{2} b^{2}}{m^{2}}}}\right]=F_{0} \rightarrow \\
& \left.A_{0} m\left[\frac{\left(\omega_{0}^{2}-\omega^{2}\right)^{2}}{\sqrt{\left(\omega_{0}^{2}-\omega^{2}\right)^{2}+\frac{\omega^{2} b^{2}}{m^{2}}}}+\frac{\frac{\omega^{2} b^{2}}{m^{2}}}{\sqrt{\left(\omega_{0}^{2}-\omega^{2}\right)^{2}+\frac{\omega^{2} b^{2}}{m^{2}}}}\right]=F_{0} \rightarrow A_{0}=\frac{F_{0}}{m\left[\sqrt{\left(\omega_{0}^{2}-\omega^{2}\right)^{2}+\frac{\omega^{2} b^{2}}{m^{2}}}\right]}\right]
\end{aligned}
$$

69. (a) For the damped oscillator, the amplitude decays according to $A=A_{0} e^{-\frac{b t}{2 m}}$. We are also given the $Q$ value, and $Q=\frac{m \omega_{0}}{b}$. We use these relationships to find the time for the amplitude to decrease to one-third of its original value.

$$
\begin{aligned}
& Q=\frac{m \omega_{0}}{b} \rightarrow \frac{b}{m}=\frac{\omega_{0}}{Q}=\frac{\sqrt{g / \ell}}{Q} ; A=A_{0} e^{-\frac{b b_{1 / 3}}{2 m}}=\frac{1}{3} A_{0} \rightarrow \\
& t_{1 / 3}=\frac{2 m}{b} \ln 3=\frac{2 Q}{\omega_{0}} \ln 3=\frac{2 Q}{\sqrt{g / \ell}} \ln 3=\frac{2(350)}{\sqrt{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) /(0.50 \mathrm{~m})}} \ln 3=173.7 \mathrm{~s} \approx 170 \mathrm{~s}
\end{aligned}
$$

(b) The energy is all potential energy when the displacement is at its maximum value, which is the amplitude. We assume that the actual angular frequency is very nearly the same as the natural angular frequency.

$$
\begin{aligned}
& E=\frac{1}{2} k A^{2}=\frac{1}{2} \omega^{2} m\left(A_{0} e^{-\frac{b t}{2 m}}\right)^{2}=\frac{m g}{2 \ell} A_{0}^{2} e^{-\frac{b t}{m}} ; \frac{d E}{d t}=-\frac{b}{m} \frac{m g}{2 \ell} A_{0}^{2} e^{-\frac{b t}{m}} \rightarrow \\
& \left.\frac{d E}{d t}\right|_{t=0}=-\frac{\sqrt{g} / \ell}{Q} \frac{m g}{2 \ell}=\frac{m A_{0}^{2}}{2 Q}\left(\frac{g}{\ell}\right)^{3 / 2}=\frac{(0.27 \mathrm{~kg})(0.020 \mathrm{~m})^{2}}{2(350)}\left(\frac{9.80 \mathrm{~m} / \mathrm{s}^{2}}{0.50 \mathrm{~m}}\right)^{3 / 2}=1.3 \times 10^{-5} \mathrm{~W}
\end{aligned}
$$

(c) Use Eq. 14-26 to find the frequency spread.

$$
\begin{aligned}
& \frac{\Delta \omega}{\omega_{0}}=Q \rightarrow \frac{\Delta 2 \pi f}{2 \pi f_{0}}=\frac{\Delta f}{f_{0}}=\frac{1}{Q} \rightarrow \\
& \Delta f=\frac{f_{0}}{Q}=\frac{\omega_{0}}{2 \pi Q}=\frac{\sqrt{g / \ell}}{2 \pi Q}=\frac{\sqrt{\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) /(0.50 \mathrm{~m})}}{2 \pi(350)}=2.0 \times 10^{-3} \mathrm{~Hz}
\end{aligned}
$$

Since this is the total spread about the resonance frequency, the driving frequency must be within $1.0 \times 10^{-3} \mathrm{~Hz}$ on either side of the resonance frequency.
70. Consider the conservation of energy for the person. Call the unstretched position of the fire net the zero location for both elastic potential energy and gravitational potential energy. The amount of stretch of the fire net is given by $x$, measured positively in the downward direction. The vertical displacement for gravitational potential energy is given by the variable $y$, measured positively for the upward direction. Calculate the spring constant by conserving energy between the window height and the lowest location of the person. The person has no kinetic energy at either location.

$$
\begin{aligned}
& E_{\text {top }}=E_{\text {botom }} \rightarrow m g y_{\text {top }}=m g y_{\text {bottom }}+\frac{1}{2} k x_{\text {botom }}^{2} \\
& k=2 m g \frac{\left(y_{\text {top }}-y_{\text {botom }}\right)}{x_{\text {bottom }}^{2}}=2(62 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \frac{[20.0 \mathrm{~m}-(-1.1 \mathrm{~m})]}{(1.1 \mathrm{~m})^{2}}=2.119 \times 10^{4} \mathrm{~N} / \mathrm{m}
\end{aligned}
$$

(a) If the person were to lie on the fire net, they would stretch the net an amount such that the upward force of the net would be equal to their weight.

$$
F_{\text {ext }}=k x=m g \rightarrow x=\frac{m g}{k}=\frac{(62 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{2.1198 \times 10^{4} \mathrm{~N} / \mathrm{m}}=2.9 \times 10^{-2} \mathrm{~m}
$$

(b) To find the amount of stretch given a starting height of 38 m , again use conservation of energy. Note that $y_{\text {botom }}=-x$, and there is no kinetic energy at the top or bottom positions.

$$
\begin{aligned}
& E_{\text {top }}=E_{\text {bottom }} \rightarrow m g y_{\text {top }}=m g y_{\text {bottom }}+\frac{1}{2} k x^{2} \rightarrow x^{2}-2 \frac{m g}{k} x-2 \frac{m g}{k} y_{\text {top }}=0 \\
& x^{2}-2 \frac{(62 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{2.1198 \times 10^{4} \mathrm{~N} / \mathrm{m}} x-2 \frac{(62 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{2.1198 \times 10^{4} \mathrm{~N} / \mathrm{m}}(38 \mathrm{~m})=0 \rightarrow \\
& x^{2}-0.057326 x-2.1784=0 \rightarrow x=1.5049 \mathrm{~m},-1.4476 \mathrm{~m}
\end{aligned}
$$

This is a quadratic equation. The solution is the positive root, since the net must be below the unstretched position. The result is 1.5 m .
71. Apply the conservation of mechanical energy to the car, calling condition $\# 1$ to be before the collision and condition \# 2 to be after the collision. Assume that all of the kinetic energy of the car is converted to potential energy stored in the bumper. We know that $x_{1}=0$ and $v_{2}=0$.

$$
\begin{aligned}
& E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+\frac{1}{2} k x_{1}^{2}=\frac{1}{2} m v_{2}^{2}+\frac{1}{2} k x_{2}^{2} \rightarrow \frac{1}{2} m v_{1}^{2}=\frac{1}{2} k x_{2}^{2} \rightarrow \\
& x_{2}=\sqrt{\frac{m}{k}} v_{1}=\sqrt{\frac{1300 \mathrm{~kg}}{430 \times 10^{3} \mathrm{~N} / \mathrm{m}}}(2.0 \mathrm{~m} / \mathrm{s})=0.11 \mathrm{~m}
\end{aligned}
$$

72. (a) The frequency can be found from the length of the pendulum, and the acceleration due to gravity.

$$
f=\frac{1}{2 \pi} \sqrt{\frac{g}{\ell}}=\frac{1}{2 \pi} \sqrt{\frac{9.80 \mathrm{~m} / \mathrm{s}^{2}}{0.63 \mathrm{~m}}}=0.6277 \mathrm{~Hz} \approx 0.63 \mathrm{~Hz}
$$

(b) To find the speed at the lowest point, use the conservation of energy relating the lowest point to the release point of the pendulum. Take the lowest point to be the zero level of
 gravitational potential energy.

$$
\begin{aligned}
& E_{\text {top }}=E_{\text {bottom }} \rightarrow K E_{\text {top }}+P E_{\text {top }}=K E_{\text {bottom }}+P E_{\text {botom }} \\
& 0+m g(L-L \cos \theta)=\frac{1}{2} m v_{\text {botom }}^{2}+0 \\
& v_{\text {botoom }}=\sqrt{2 g L(1-\cos \theta)}=\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.63 \mathrm{~m})\left(1-\cos 15^{\circ}\right)}=0.6487 \mathrm{~m} / \mathrm{s} \approx 0.65 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(c) The total energy can be found from the kinetic energy at the bottom of the motion.

$$
E_{\text {total }}=\frac{1}{2} m v_{\text {bottom }}^{2}=\frac{1}{2}(0.295 \mathrm{~kg})(0.6487 \mathrm{~m} / \mathrm{s})^{2}=6.2 \times 10^{-2} \mathrm{~J}
$$

73. The frequency of a simple pendulum is given by $f=\frac{1}{2 \pi} \sqrt{\frac{g}{L}}$. The pendulum is accelerating vertically which is equivalent to increasing (or decreasing) the acceleration due to gravity by the acceleration of the pendulum.
(a)

$$
\begin{aligned}
& f_{\text {new }}=\frac{1}{2 \pi} \sqrt{\frac{g+a}{L}}=\frac{1}{2 \pi} \sqrt{\frac{1.50 g}{L}}=\sqrt{1.50} \frac{1}{2 \pi} \sqrt{\frac{g}{L}}=\sqrt{1.50} f=1.22 f \\
& f_{\text {new }}=\frac{1}{2 \pi} \sqrt{\frac{g+a}{L}}=\frac{1}{2 \pi} \sqrt{\frac{0.5 g}{L}}=\sqrt{0.5} \frac{1}{2 \pi} \sqrt{\frac{g}{L}}=\sqrt{0.5} f=0.71 f
\end{aligned}
$$

74. The equation of motion is $x=0.25 \sin 5.50 t=A \sin \omega t$.
(a) The amplitude is $A=x_{\text {max }}=0.25 \mathrm{~m}$.
(b) The frequency is found by $\omega=2 \pi f=5.50 \mathrm{~s}^{-1} \rightarrow f=\frac{5.50 \mathrm{~s}^{-1}}{2 \pi}=0.875 \mathrm{~Hz}$
(c) The period is the reciprocal of the frequency. $T=1 / f=\frac{2 \pi}{5.50 \mathrm{~s}^{-1}}=1.14 \mathrm{~s}$.
(d) The total energy is given by

$$
E_{\text {total }}=\frac{1}{2} m v_{\max }^{2}=\frac{1}{2} m(\omega A)^{2}=\frac{1}{2}(0.650 \mathrm{~kg})\left[\left(5.50 \mathrm{~s}^{-1}\right)(0.25 \mathrm{~m})\right]^{2}=0.6145 \mathrm{~J} \approx 0.61 \mathrm{~J} .
$$

(e) The potential energy is given by

$$
E_{\text {potential }}=\frac{1}{2} k x^{2}=\frac{1}{2} m \omega^{2} x^{2}=\frac{1}{2}(0.650 \mathrm{~kg})\left(5.50 \mathrm{~s}^{-1}\right)^{2}(0.15 \mathrm{~m})^{2}=0.2212 \mathrm{~J} \approx 0.22 \mathrm{~J} .
$$

The kinetic energy is given by

$$
E_{\text {kinetic }}=E_{\text {total }}-E_{\text {potential }}=0.6145 \mathrm{~J}-0.2212 \mathrm{~J}=0.3933 \mathrm{~J} \approx 0.39 \mathrm{~J} \text {. }
$$

75. (a) The car on the end of the cable produces tension in the cable, and stretches the cable according to Equation (12-4), $\Delta \ell=\frac{1}{E} \frac{F}{A} \ell_{o}$, where $E$ is Young's modulus. Rearrange this equation to see that the tension force is proportional to the amount of stretch, $F=\frac{E A}{\ell_{o}} \Delta \ell$, and so the effective spring constant is $k=\frac{E A}{\ell_{o}}$. The period of the bouncing can be found from the spring constant and the mass on the end of the cable.

$$
T=2 \pi \sqrt{\frac{m}{k}}=2 \pi \sqrt{\frac{m \ell_{o}}{E A}}=2 \pi \sqrt{\frac{(1350 \mathrm{~kg})(20.0 \mathrm{~m})}{\left(200 \times 10^{9} \mathrm{~N} / \mathrm{m}^{2}\right) \pi\left(3.2 \times 10^{-3} \mathrm{~m}\right)^{2}}}=0.407 \mathrm{~s} \approx 0.41 \mathrm{~s}
$$

(b) The cable will stretch some due to the load of the car, and then the amplitude of the bouncing will make it stretch even farther. The total stretch is to be used in finding the maximum amplitude. The tensile strength is found in Table 12-2.

$$
\begin{aligned}
\frac{F}{A} & =\frac{k\left(x_{\text {sataic }}+x_{\text {ampliude }}\right)}{\pi r^{2}}=\text { tensile strength }(\text { abbrev T.S. }) \rightarrow \\
x_{\text {amplitude }} & =\frac{(\text { T.S. }) \pi r^{2}}{k}-\Delta \ell=\frac{(\text { T.S. }) \pi r^{2}}{\frac{E \pi r^{2}}{\ell_{0}}}-\frac{m g \ell_{0}}{E \pi r^{2}}=\frac{\ell_{0}}{E}\left[(\text { T.S. })-\frac{m g}{\pi r^{2}}\right] \\
& \left.=\frac{(20.0 \mathrm{~m})}{\left(200 \times 10^{9} \mathrm{~N} / \mathrm{m}^{2}\right)}\left[500 \times 10^{6} \mathrm{~N} / \mathrm{m}^{2}-\frac{(1350 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{\pi(0.0032 \mathrm{~m})^{2}}\right]=9 \times 10^{-3} \mathrm{~m}\right]=9 \mathrm{~mm}
\end{aligned}
$$

76. The spring constant does not change, but the mass does, and so the frequency will change. Use Eq. 14-7 a to relate the spring constant, the mass, and the frequency.

$$
\begin{aligned}
& f=\frac{1}{2 \pi} \sqrt{\frac{k}{m}} \rightarrow \frac{k}{4 \pi^{2}}=f^{2} m=\text { constant } \rightarrow f_{\mathrm{o}}^{2} m_{\mathrm{o}}=f_{\mathrm{s}}^{2} m_{\mathrm{s}} \rightarrow \\
& f_{\mathrm{s}}=f_{\mathrm{o}} \sqrt{\frac{m_{\mathrm{o}}}{m_{\mathrm{s}}}}=\left(3.7 \times 10^{13} \mathrm{~Hz}\right) \sqrt{\frac{16.0}{32.0}}=2.6 \times 10^{13} \mathrm{~Hz}
\end{aligned}
$$

77. The period of a pendulum is given by $T=2 \pi \sqrt{\ell / g}$, and so the length is $\ell=\frac{T^{2} g}{4 \pi^{2}}$.
(a) $\ell_{\text {Austin }}=\frac{T^{2} g_{\text {Austin }}}{4 \pi^{2}}=\frac{(2.000 \mathrm{~s})^{2}\left(9.793 \mathrm{~m} / \mathrm{s}^{2}\right)}{4 \pi^{2}}=0.992238 \mathrm{~m} \approx 0.9922 \mathrm{~m}$
(b) $\ell_{\text {Paris }}=\frac{T^{2} g_{\text {Pais }}}{4 \pi^{2}}=\frac{(2.000 \mathrm{~s})^{2}\left(9.809 \mathrm{~m} / \mathrm{s}^{2}\right)}{4 \pi^{2}}=0.993859 \mathrm{~m} \approx 0.9939 \mathrm{~m}$

$$
\ell_{\text {Paris }}-\ell_{\text {Austin }}=0.993859 \mathrm{~m}-0.992238 \mathrm{~m}=0.001621 \mathrm{~m} \approx 1.6 \mathrm{~mm}
$$

(c) $\ell_{\text {Moon }}=\frac{T^{2} g_{\text {Moon }}}{4 \pi^{2}}=\frac{(2.00 \mathrm{~s})^{2}\left(1.62 \mathrm{~m} / \mathrm{s}^{2}\right)}{4 \pi^{2}}=0.164 \mathrm{~m}$
78. The force of the man's weight causes the raft to sink, and that causes the water to put a larger upward force on the raft. This extra buoyant force is a restoring force, because it is in the opposite direction of the force put on the raft by the man. This is analogous to pulling down on a mass-spring system that is in equilibrium, by applying an extra force. Then when the man steps off, the restoring force pushes upward on the raft, and thus the raft-water system acts like a spring, with a spring constant found as follows.

$$
k=\frac{F_{\text {ext }}}{x}=\frac{(75 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{3.5 \times 10^{-2} \mathrm{~m}}=2.1 \times 10^{4} \mathrm{~N} / \mathrm{m}
$$

(a) The frequency of vibration is determined by the "spring constant" and the mass of the raft.

$$
f_{n}=\frac{1}{2 \pi} \sqrt{\frac{k}{m}}=\frac{1}{2 \pi} \sqrt{\frac{2.1 \times 10^{4} \mathrm{~N} / \mathrm{m}}{320 \mathrm{~kg}}}=1.289 \mathrm{~Hz} \approx 1.3 \mathrm{~Hz}
$$

(b) As explained in the text, for a vertical spring the gravitational potential energy can be ignored if the displacement is measured from the oscillator's equilibrium position. The total energy is thus

$$
E_{\text {toal }}=\frac{1}{2} k A^{2}=\frac{1}{2}\left(2.1 \times 10^{4} \mathrm{~N} / \mathrm{m}\right)\left(3.5 \times 10^{-2} \mathrm{~m}\right)^{2}=12.86 \mathrm{~J} \approx 13 \mathrm{~J} \text {. }
$$

79. The relationship between the velocity and the position of a SHO is given by Eq. 14-11b. Set that expression equal to half the maximum speed, and solve for the displacement.

$$
\begin{aligned}
& v= \pm v_{\max } \sqrt{1-x^{2} / A^{2}}=\frac{1}{2} v_{\max } \rightarrow \pm \sqrt{1-x^{2} / A^{2}}=\frac{1}{2} \rightarrow 1-x^{2} / A^{2}=\frac{1}{4} \rightarrow x^{2} / A^{2}=\frac{3}{4} \rightarrow \\
& x= \pm \sqrt{3} A / 2 \approx \pm 0.866 A
\end{aligned}
$$

80. For the pebble to lose contact with the board means that there is no normal force of the board on the pebble. If there is no normal force on the pebble, then the only force on the pebble is the force of gravity, and the acceleration of the pebble will be $g$ downward, the acceleration due to gravity. This is the maximum downward acceleration that the pebble can have. Thus if the board's downward acceleration exceeds $g$, then the pebble will lose contact. The maximum acceleration and the amplitude are related by $a_{\text {max }}=4 \pi^{2} f^{2} A$.

$$
a_{\max }=4 \pi^{2} f^{2} A \leq g \rightarrow A \leq \frac{g}{4 \pi^{2} f^{2}} \leq \frac{9.80 \mathrm{~m} / \mathrm{s}^{2}}{4 \pi^{2}(2.5 \mathrm{~Hz})^{2}} \leq 4.0 \times 10^{-2} \mathrm{~m}
$$

81. Assume the block has a cross-sectional area of $A$. In the equilibrium position, the net force on the block is zero, and so $F_{\text {buoy }}=m g$. When the block is pushed into the water (downward) an additional distance $\Delta x$, there is an increase in the buoyancy force ( $F_{\text {extra }}$ ) equal to the weight of the additional water displaced. The weight of the extra water displaced is the density of water times the volume displaced.

$$
F_{\text {extra }}=m_{\substack{\text { add. } \\ \text { water }}} g=\rho_{\text {water }}^{\substack{\text { add } \\ \text { water }}} V_{\text {vater }} g=\rho_{\text {water }} g A \Delta x=\left(\rho_{\text {water }} g A\right) \Delta x
$$

This is the net force on the displaced block. Note that if the block is pushed down, the additional force is upwards. And if the block were to be displaced upwards by a distance $\Delta x$, the buoyancy force would actually be less than the weight of the block by the amount $F_{\text {exta }}$, and so there would be a net force downwards of magnitude $F_{\text {exta }}$. So in both upward and downward displacement, there is a net force of magnitude $\left(\rho_{\text {water }} g A\right) \Delta x$ but opposite to the direction of displacement. As a vector, we can write the following.

$$
\overrightarrow{\mathbf{F}}_{\text {net }}=-\left(\rho_{\text {water }} g A\right) \Delta \overrightarrow{\mathbf{x}}
$$

This is the equation of simple harmonic motion, with a "spring constant" of $k=\rho_{\text {water }} g A$
82. (a) From conservation of energy, the initial kinetic energy of the car will all be changed into elastic potential energy by compressing the spring.

$$
\begin{aligned}
& E_{1}=E_{2} \rightarrow \frac{1}{2} m v_{1}^{2}+\frac{1}{2} k x_{1}^{2}=\frac{1}{2} m v_{2}^{2}+\frac{1}{2} k x_{2}^{2} \rightarrow \frac{1}{2} m v_{1}^{2}=\frac{1}{2} k x_{2}^{2} \rightarrow \\
& k=m \frac{v_{1}^{2}}{x_{2}^{2}}=(950 \mathrm{~kg}) \frac{(25 \mathrm{~m} / \mathrm{s})^{2}}{(5.0 \mathrm{~m})^{2}}=2.375 \times 10^{4} \mathrm{~N} / \mathrm{m} \approx 2.4 \times 10^{4} \mathrm{~N} / \mathrm{m}
\end{aligned}
$$

(b) The car will be in contact with the spring for half a period, as it moves from the equilibrium location to maximum displacement and back to equilibrium.

$$
\frac{1}{2} T=\frac{1}{2} 2 \pi \sqrt{\frac{m}{k}}=\pi \sqrt{\frac{(950 \mathrm{~kg})}{2.375 \times 10^{4} \mathrm{~N} / \mathrm{m}}}=0.63 \mathrm{~s}
$$

83. (a) The effective spring constant is found from the final displacement caused by the additional mass on the table. The weight of the mass will equal the upward force exerted by the compressed springs.

$$
\begin{aligned}
& F_{\text {grav }}=F_{\text {springs }} \rightarrow m g=k \Delta y \rightarrow \\
& k=\frac{m g}{\Delta y}=\frac{(0.80 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{(0.060 \mathrm{~m})}=130.67 \mathrm{~N} / \mathrm{m} \approx 130 \mathrm{~N} / \mathrm{m}
\end{aligned}
$$

(b) We assume the collision takes place in such a short time that the springs do not compress a significant amount during the collision. Use momentum conservation to find the speed immediately after the collision.

$$
\begin{aligned}
& p_{\text {before }}=p_{\text {after }} \rightarrow m_{\text {clay }} v_{\text {clay }}=\left(m_{\text {clay }}+m_{\text {table }}\right) v_{\text {after }} \rightarrow \\
& v_{\text {affer }}=\frac{m_{\text {clay }}}{\left(m_{\text {clay }}+m_{\text {table }}\right)} v_{\text {clay }}=\frac{0.80 \mathrm{~kg}}{2.40 \mathrm{~kg}}(1.65 \mathrm{~m} / \mathrm{s})=0.55 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

As discussed in the text, if we measure displacements from the new equilibrium position, we may use an energy analysis of the spring motion without including the effects of gravity. The tal elastic and kinetic energy immediately after the collision will be the maximum elastic energy, at the amplitude location.

$$
\begin{aligned}
& E_{1}=E_{2} \rightarrow \frac{1}{2} m_{\text {total }} v_{\text {after }}^{2}+\frac{1}{2} k x_{\text {after }}^{2}=\frac{1}{2} k A^{2} \rightarrow \\
& A=\sqrt{\frac{m_{\text {total }}}{k} v_{\text {after }}^{2}+x_{\text {after }}^{2}}=\sqrt{\left(\frac{2.40 \mathrm{~kg}}{130.67 \mathrm{~N} / \mathrm{m}}\right)(0.55 \mathrm{~m} / \mathrm{s})^{2}+(0.060 \mathrm{~m})^{2}}=0.096 \mathrm{~m}=9.6 \mathrm{~cm}
\end{aligned}
$$

84. (a) The graph is shown. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH14.XLS," on tab "Problem 14.84a."
(b) Equilibrium occurs at the location where the force is 0 . Set the force equal to 0 and solve for the separation distance $r$.


$$
\begin{aligned}
& F\left(r_{0}\right)=-\frac{C}{r_{0}^{2}}+\frac{D}{r_{0}^{3}}=0 \rightarrow \\
& \frac{C}{r_{0}^{2}}=\frac{D}{r_{0}^{3}} \rightarrow C r_{0}^{3}=D r_{0}^{2} \rightarrow r_{0}=\frac{D}{C}
\end{aligned}
$$

This does match with the graph, which shows $F=0$ at $r=D / C$.
(c) We find the net force at $r=r_{0}+\Delta r$. Use the binomial expansion.

$$
\begin{aligned}
F\left(r_{0}+\Delta r\right) & =-C\left(r_{0}+\Delta r\right)^{-2}+D\left(r_{0}+\Delta r\right)^{-3}=-C r_{0}^{-2}\left(1+\frac{\Delta r}{r_{0}}\right)^{-2}+D r_{0}^{-3}\left(1+\frac{\Delta r}{r_{0}}\right)^{-3} \\
& \approx-\frac{C}{r_{0}^{2}}\left(1-2 \frac{\Delta r}{r_{0}}\right)+\frac{D}{r_{0}^{3}}\left(1-3 \frac{\Delta r}{r_{0}}\right)=\frac{C}{r_{0}^{3}}\left[-r_{0}\left(1-2 \frac{\Delta r}{r_{0}}\right)+\frac{D}{C}\left(1-3 \frac{\Delta r}{r_{0}}\right)\right] \\
& =\frac{C}{r_{0}^{3}}\left[-r_{0}+2 \Delta r+r_{0}-3 \Delta r\right]=\frac{C}{r_{0}^{3}}[-\Delta r] \rightarrow F\left(r_{0}+\Delta r\right)=-\frac{C}{r_{0}^{3}} \Delta r
\end{aligned}
$$

We see that the net force is proportional to the displacement and in the opposite direction to the displacement. Thus the motion is simple harmonic.
(d) Since for simple harmonic motion, the general form is $F=-k x$, we see that for this situation, the spring constant is given by $k=\frac{C}{r_{0}^{3}}=\frac{C^{4}}{D^{3}}$.
(e) The period of the motion can be found from Eq. 14-7b.

$$
T=2 \pi \sqrt{\frac{m}{k}}=2 \pi \sqrt{\frac{m D^{3}}{C^{4}}}
$$

85. (a) The relationship between the velocity and the position of a SHO is given by Eq. 14-11b. Set that expression equal to half the maximum speed, and solve for the displacement.

$$
\begin{aligned}
& v= \pm v_{\max } \sqrt{1-x^{2} / x_{0}^{2}}=\frac{1}{2} v_{\max } \rightarrow \pm \sqrt{1-x^{2} / x_{0}^{2}}=\frac{1}{2} \rightarrow 1-x^{2} / x_{0}^{2}=\frac{1}{4} \rightarrow \\
& x^{2} / x_{0}^{2}=\frac{3}{4} \rightarrow x= \pm \sqrt{3} x_{0} / 2 \approx \pm 0.866 x_{0}
\end{aligned}
$$

(b) Since $F=-k x=m a$ for an object attached to a spring, the acceleration is proportional to the displacement (although in the opposite direction), as $a=-x k / m$. Thus the acceleration will have half its maximum value where the displacement has half its maximum value, at $\pm \frac{1}{2} x_{0}$
86. The effective spring constant is determined by the frequency of vibration and the mass of the oscillator. Use Eq. 14-7a.

$$
\begin{aligned}
& f=\frac{1}{2 \pi} \sqrt{\frac{k}{m}} \rightarrow \\
& k=4 \pi^{2} f^{2} m=4 \pi^{2}\left(2.83 \times 10^{13} \mathrm{~Hz}\right)(16.00 \mathrm{u})\left(\frac{1.66 \times 10^{-27} \mathrm{~kg}}{1 \mathrm{u}}\right)=840 \mathrm{~N} / \mathrm{m} \text { (3 sig. fig.) }
\end{aligned}
$$

87. We quote from the next to last paragraph of Appendix D: "... we see that at points within a solid sphere, say 100 km below the Earth's surface, only the mass up to that radius contributes to the net force. The outer shells beyond the point in question contribute zero net gravitational effect." So when the mass is a distance $r$ from the center of the Earth, there will be a force toward the center, opposite to $r$, due only to the mass within a sphere of radius $r$. We call that mass $m_{r}$. It is the density of the (assumed uniform) Earth, times the volume within a sphere of radius $r$.


$$
\begin{aligned}
& m_{r}=\rho V_{r}=\frac{M_{\text {Earth }}}{V_{\text {Earth }}} V_{r}=\frac{M_{\text {Earth }}}{\frac{4}{3} \pi R_{\text {Earth }}^{3}} \pi r^{3}=M_{\text {Earth }} \frac{r^{3}}{R_{\text {Earth }}^{3}} \\
& F=-\frac{G m m_{r}}{r^{2}}=-\frac{G m M_{\text {Earth }} \frac{r^{3}}{R_{\text {Earth }}^{3}}}{r^{2}}=-\frac{G m M_{\text {Earth }}}{R_{\text {Earth }}^{3}} r
\end{aligned}
$$

The force on the object is opposite to and proportional to the displacement, and so will execute simple harmonic motion, with a "spring constant" of $k=\frac{G m M_{\text {Earth }}}{R_{\text {Earth }}^{3}}$. The time for the apple to return is the period, found from the "spring constant."

$$
\begin{aligned}
T & =2 \pi \sqrt{\frac{m}{k}}=2 \pi \sqrt{\frac{m}{\frac{G m M_{\text {Earth }}}{R_{\text {Earth }}^{3}}}}=2 \pi \sqrt{\frac{R_{\text {Earth }}^{3}}{G M_{\text {Earth }}}}=2 \pi \sqrt{\frac{\left(6.38 \times 10^{6} \mathrm{~m}\right)^{3}}{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(5.98 \times 10^{24} \mathrm{~kg}\right)}} \\
& =507 \mathrm{~s} \text { or } 84.5 \mathrm{~min}
\end{aligned}
$$

88. (a) The rod is a physical pendulum. Use Eq. 14-14 for the period of a physical pendulum.

$$
T=2 \pi \sqrt{\frac{I}{m g h}}=2 \pi \sqrt{\frac{\frac{1}{3} m \ell^{2}}{m g\left(\frac{1}{2} \ell\right)}}=2 \pi \sqrt{\frac{2 \ell}{3 g}}=2 \pi \sqrt{\frac{2(1.00 \mathrm{~m})}{3\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}}=1.64 \mathrm{~s}
$$

(b) The simple pendulum has a period given by $T=2 \pi \sqrt{\ell / g}$. Use this to find the length.

$$
T=2 \pi \sqrt{\frac{\ell_{\text {simple }}}{g}}=2 \pi \sqrt{\frac{2 \ell}{3 g}} \rightarrow \ell_{\text {simple }}=\frac{2}{3} \ell=\frac{2}{3}(1.00 \mathrm{~m})=0.667 \mathrm{~m}
$$

89. Consider energy conservation for the mass over the range of motion from "letting go" (the highest point) to the lowest point. The mass falls the same distance that the spring is stretched, and has no kinetic energy at either endpoint. Call the lowest point the zero of gravitational potential energy. The variable " $x$ " represents the amount that the spring is stretched from the equilibrium position.

$$
\begin{aligned}
& E_{\text {top }}=E_{\text {bottom }} \rightarrow \frac{1}{2} m v_{\text {top }}^{2}+m g y_{\text {top }}+\frac{1}{2} k x_{\text {top }}^{2}=\frac{1}{2} m v_{\text {bottom }}^{2}+m g y_{\text {botom }}+\frac{1}{2} k x_{\text {bottom }}^{2} \\
& \frac{1}{2} m v_{\text {top }}^{2}+m g y_{\text {top }}+\frac{1}{2} k x_{\text {top }}^{2}=\frac{1}{2} m v_{\text {bottom }}^{2}+m g y_{\text {bottom }}+\frac{1}{2} k x_{\text {bottom }}^{2} \\
& 0+m g H+0=0+0+\frac{1}{2} k H^{2} \rightarrow \frac{k}{m}=\frac{2 g}{H}=\omega^{2} \rightarrow \omega=\sqrt{\frac{2 g}{H}} \\
& f=\frac{\omega}{2 \pi}=\frac{1}{2 \pi} \sqrt{\frac{2 g}{H}}=\frac{1}{2 \pi} \sqrt{\frac{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{0.320 \mathrm{~m}}}=1.25 \mathrm{~Hz}
\end{aligned}
$$

90. For there to be no slippage, the child must have the same acceleration as the slab. This will only happen if the force of static friction is big enough to provide the child with an acceleration at least as large as the maximum acceleration of the slab. The maximum force of static friction is given by $\underset{\max }{F_{\mathrm{fr}}}=\mu_{\mathrm{s}} F_{\mathrm{N}}$. Since the motion is horizontal and there are not other vertical forces besides gravity
and the normal force, we know that $F_{\mathrm{N}}=m g$. Finally, the maximum acceleration of the slab will occur at the endpoints, and is given by Eq. 14-9b. The mass to use in Eq. $14-9 b$ is the mass of the oscillating system, $m+M$.

$$
\begin{aligned}
& a_{\mathrm{fr}} \geq a_{\text {elastic }} \rightarrow \frac{\mu_{\mathrm{s}} F_{\mathrm{N}}}{m}=\frac{\mu_{\mathrm{s}} m g}{m}=\mu_{\mathrm{s}} g \geq \frac{k}{m+M} A \rightarrow \\
& m \geq \frac{k}{\mu_{\mathrm{s}} g} A-M=\frac{430 \mathrm{~N} / \mathrm{m}}{(0.40)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}(0.50 \mathrm{~m})-35 \mathrm{~kg}=19.8 \mathrm{~kg} \approx 20 \mathrm{~kg} \text { (2 sig. fig.) }
\end{aligned}
$$

And so the child must have a minimum mass of 20 kg (about 44 lbs ) in order to ride safely.
91. We must make several assumptions. Consider a static displacement of the trampoline, by someone sitting on the trampoline mat. The upward elastic force of the trampoline must equal the downward force of gravity. We estimate that a $75-\mathrm{kg}$ person will depress the trampoline about 25 cm at its midpoint.

$$
k x=m g \rightarrow k=\frac{m g}{x}=\frac{(75 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{0.25 \mathrm{~m}}=2940 \mathrm{~N} / \mathrm{m} \approx 3000 \mathrm{~N} / \mathrm{m}
$$

92. We may use Eq. $10-14, \sum \tau=I \alpha$, as long as the axis of rotation is fixed in an inertial frame. We choose the axis to be at the point of support, perpendicular to the plane of motion of the pendulum. There are two forces on the pendulum bob, but only gravity causes any torque. Note that if the pendulum is displaced in the counterclockwise direction (as shown in Fig. 14-46), then the torque caused by gravity will be in the clockwise direction, and vice versa. See the free-body diagram in order to write Newton's second law for rotation, with counterclockwise as the positive rotational direction.


$$
\sum \tau=-m g \ell \sin \theta=I \alpha=I \frac{d^{2} \theta}{d t^{2}}
$$

If the angular displacement is limited to about $15^{\circ}$, then $\sin \theta \approx \theta$.

$$
-m g \ell \theta=I \frac{d^{2} \theta}{d t^{2}} \rightarrow \frac{d^{2} \theta}{d t^{2}}=-\frac{m g \ell}{I} \theta=-\frac{m g \ell}{m \ell^{2}} \theta=-\frac{g}{\ell} \theta
$$

This is the equation of simple harmonic motion, with $\omega^{2}=g / \ell$. Thus we can write the displacement of the pendulum as follows, imitating Eq. 14-4.

$$
\theta=\theta_{\max } \cos (\omega t+\phi) \rightarrow \theta=\theta_{\max } \cos \left(\sqrt{\frac{g}{\ell}} t+\phi\right)
$$

93. (a) Start with Eq. 14-7b, $T=2 \pi \sqrt{\frac{m}{k}} \rightarrow T^{2}=\frac{4 \pi^{2}}{k} m$. This fits the straight-line equation form of $y=($ slope $) x+(y$-intercept $)$, if we plot $T^{2}$ vs. $m$. The slope is $4 \pi^{2} / k$, and so $k=\frac{4 \pi^{2}}{\text { slope }}$. The $y$-intercept is expected to be 0 .
(b) The graph is included on the next page. The slope is $0.1278 \mathrm{~s}^{2} / \mathrm{kg} \approx 0.13 \mathrm{~s}^{2} / \mathrm{kg}$, and the $y$ intercept is $0.1390 \mathrm{~s}^{2} \approx 0.14 \mathrm{~s}^{2}$. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH14.XLS," on tab "Problem 14.93b."
(c) Start with the modified Eq. 14-7b.

$$
\begin{aligned}
& T=2 \pi \sqrt{\frac{m+m_{0}}{k}} \rightarrow \\
& T^{2}=\frac{4 \pi^{2}}{k} m+\frac{4 \pi^{2} m_{0}}{k}
\end{aligned}
$$

The spring constant is still given by $k=\frac{4 \pi^{2}}{\text { slope }}$ and the $y$-intercept is expected to be $\frac{4 \pi^{2} m_{0}}{k}$.


$$
\begin{aligned}
& k=\frac{4 \pi^{2}}{0.1278 \mathrm{~s}^{2} / \mathrm{kg}}=308.9 \mathrm{~N} / \mathrm{m} \approx 310 \mathrm{~N} / \mathrm{m} \\
& \frac{4 \pi^{2} m_{0}}{k}=y_{0}=y \text {-intercept } \rightarrow m_{0}=\frac{k y_{0}}{4 \pi^{2}}=\frac{y_{0}}{\text { slope }}=\frac{0.1390 \mathrm{~s}^{2}}{0.1278 \mathrm{~s}^{2} / \mathrm{kg}}=1.088 \mathrm{~kg} \approx 1.1 \mathrm{~kg}
\end{aligned}
$$

(d) The mass $m_{0}$ can be interpreted as the effective mass of the spring. The mass of the spring does oscillate, but not all of the mass has the same amplitude of oscillation, and so $m_{0}$ is likely less than the mass of the spring. One straightforward analysis predicts that $m_{0}=\frac{1}{3} M_{\text {spring }}$.
94. There is a subtle point in the modeling of this problem. It would be easy to assume that the net force on the spring is given by $F_{\text {net }}=-k x-c v^{2}=m a$. But then the damping force would always be in the negative direction, since $c v^{2} \geq 0$. So to model a damping force that is in the opposite direction of the velocity, we instead must use $F_{\text {net }}=-k x-c v|v|=m a$. Then the damping force will be in the
opposite direction of the velocity, and have a magnitude of $c v^{2}$. We find the acceleration as a function of velocity, and then use numeric integration with a constant acceleration approximation to estimate the speed and position of the oscillator at later times. We take the downward direction to be positive, and the starting position to be $y=0$.

$$
F=-k x-c v|v|=m a \rightarrow a=-\frac{k}{m} x-\frac{c}{m} v|v|
$$

From Example 14-5, we have $x(0)=x_{0}=-0.100 \mathrm{~m}$ and $v(0)=v_{0}=0$. We calculate the initial acceleration, $a_{0}=-\frac{k}{m} x_{0}-\frac{c}{m} v_{0}\left|v_{0}\right|$, and assume that acceleration is constant over the next time interval. Then $x_{1}=x_{0}+v_{0} \Delta t+\frac{1}{2} a_{0}(\Delta t)^{2}, v_{1}=v_{0}+a_{0} \Delta t$, and $a_{1}=-\frac{k}{m} x_{1}-\frac{c}{m} v_{1}\left|v_{1}\right|$. This continues for each successive interval. We apply this method first for a time interval of 0.01 s , and record the position, velocity, and acceleration $t=2.00 \mathrm{~s}$. Then we reduce the interval to 0.005 s and again find the position, velocity, and acceleration at $t=2.00 \mathrm{~s}$. We compare the results from the smaller time interval with those of the larger time interval to see if they agree within $2 \%$. If not, a smaller interval is used, and the process repeated. For this problem, the results for position, velocity, and acceleration for time intervals of 0.001 s and 0.0005 s agree to within $2 \%$. Here are the results for various intervals.

$$
\begin{array}{llll}
\Delta t=0.01 \mathrm{~s}: & x(2.00 \mathrm{~s})=0.0713 \mathrm{~m} & v(2.00 \mathrm{~s})=-0.291 \mathrm{~m} / \mathrm{s} & a(2.00 \mathrm{~s})=-4.58 \mathrm{~m} / \mathrm{s}^{2} \\
\Delta t=0.005 \mathrm{~s}: & x(2.00 \mathrm{~s})=0.0632 \mathrm{~m} & v(2.00 \mathrm{~s})=-0.251 \mathrm{~m} / \mathrm{s} & a(2.00 \mathrm{~s})=-4.07 \mathrm{~m} / \mathrm{s}^{2} \\
\Delta t=0.001 \mathrm{~s}: & x(2.00 \mathrm{~s})=0.0574 \mathrm{~m} & v(2.00 \mathrm{~s})=-0.222 \mathrm{~m} / \mathrm{s} & a(2.00 \mathrm{~s})=-3.71 \mathrm{~m} / \mathrm{s}^{2} \\
\Delta t=0.0005 \mathrm{~s}: & x(2.00 \mathrm{~s})=0.0567 \mathrm{~m} & v(2.00 \mathrm{~s})=-0.218 \mathrm{~m} / \mathrm{s} & a(2.00 \mathrm{~s})=-3.66 \mathrm{~m} / \mathrm{s}^{2}
\end{array}
$$

The graphs of position, velocity, and acceleration are shown below. The spreadsheet used can be found on the Media Manager, with filename "PSE4_ISM_CH14.XLS", on tab "Problem 14.94".



## CHAPTER 15: Wave Motion

## Responses to Questions

1. Yes. A simple periodic wave travels through a medium, which must be in contact with or connected to the source for the wave to be generated. If the medium changes, the wave speed and wavelength can change but the frequency remains constant.
2. The speed of the transverse wave is the speed at which the wave disturbance propagates down the cord. The individual tiny pieces of cord will move perpendicular to the cord with an average speed of four times the amplitude divided by the period. The average velocity of the individual pieces of cord is zero, but the average speed is not the same as the wave speed.
3. The maximum climb distance $(4.3 \mathrm{~m})$ occurs when the tall boat is at a crest and the short boat is in a trough. If we define the height difference of the boats on level seas as $\Delta h$ and the wave amplitude as $A$, then $\Delta h+2 A=4.3 \mathrm{~m}$. The minimum climb distance ( 2.5 m ) occurs when the tall boat is in a trough and the short boat is at a crest. Then $\Delta h-2 A=2.5 \mathrm{~m}$. Solving these two equations for $A$ gives a wave amplitude of 0.45 m .
4. (a) Striking the rod vertically from above will displace particles in a direction perpendicular to the rod and will set up primarily transverse waves.
(b) Striking the rod horizontally parallel to its length will give the particles an initial displacement parallel to the rod and will set up primarily longitudinal waves.
5. The speed of sound in air obeys the equation $v=\sqrt{B / \rho}$. If the bulk modulus is approximately constant and the density of air decreases with temperature, then the speed of sound in air should increase with increasing temperature.
6. First, estimate the number of wave crests that pass a given point per second. This is the frequency of the wave. Then, estimate the distance between two successive crests, which is the wavelength. The product of the frequency and the wavelength is the speed of the wave.
7. The speed of sound is defined as $v=\sqrt{B / \rho}$, where $B$ is the bulk modulus and $\rho$ is the density of the material. The bulk modulus of most solids is at least $10^{6}$ times as great as the bulk modulus of air. This difference overcomes the larger density of most solids, and accounts for the greater speed of sound in most solids than in air.
8. One reason is that the wave energy is spread out over a larger area as the wave travels farther from the source, as can be seen by the increasing diameter of the circular wave. The wave does not gain energy as it travels, so if the energy is spread over a larger area, the amplitude of the wave must be smaller. Secondly, the energy of the wave dissipates due to damping, and the amplitude decreases.
9. If two waves have the same speed but one has half the wavelength of the other, the wave with the shorter wavelength must have twice the frequency of the other. The energy transmitted by a wave depends on the wave speed and the square of the frequency. The wave with the shorter wavelength will transmit four times the energy transmitted by the other wave.
10. Y ny function of $(x-v t)$ will represent wave motion because it will satisfy the wave equation, $\mathrm{E}_{1}$ - 16 .
11. The frequency does not change at the boundary because the two sections of cord are tied to each other and they must oscillate together. The wavelength and wave speed can be different, but the frequency must remain constant across the boundary.
12. The transmitted wave has a shorter wavelength. If the wave is inverted upon reflection at the boundary between the two sections of rope, then the second section of rope must be heavier. Therefore, the transmitted wave (traveling in the heavier rope) will have a lower velocity than the incident wave or the reflected wave. The frequency does not change at the boundary, so the wavelength of the transmitted wave must also be smaller.
13. Yes, total energy is always conserved. The particles in the medium, which are set into motion by the wave, have both kinetic and potential energy. At the instant in which two waves interfere destructively, the displacement of the medium may be zero, but the particles of the medium will have velocity, and therefore kinetic energy.
14. Yes. If you touch the string at any node you will not disturb the motion. There will be nodes at each end as well as at the points one-third and two-thirds of the distance along the length of the string.
15. No. The energy of the incident and reflected wave is distributed around the antinodes, which exhibit large oscillations. The energy is a property of the wave as a whole, not of one particular point on the wave.
16. Yes. A standing wave is an example of a resonance phenomenon, caused by constructive interference between a traveling wave and its reflection. The wave energy is distributed around the antinodes, which exhibit large amplitude oscillations, even when the generating oscillations from the hand are small.
17. When a hand or mechanical oscillator vibrates a string, the motion of the hand or oscillator is not exactly the same for each vibration. This variation in the generation of the wave leads to nodes which are not quite "true" nodes. In addition, real cords have damping forces which tend to reduce the energy of the wave. The reflected wave will have a smaller amplitude than the incident wave, so the two waves will not completely cancel, and the node will not be a true node.
18. AM radio waves have a much longer wavelength than FM radio waves. How much waves bend, or diffract, around obstacles depends on the wavelength of the wave in comparison to the size of the obstacle. A hill is much larger than the wavelength of FM waves, and so there will be a "shadow" region behind the hill. However, the hill is not large compared to the wavelength of AM signals, so the AM radio waves will bend around the hill.
19. Waves exhibit diffraction. If a barrier is placed between the energy source and the energy receiver, and energy is still received, it is a good indication that the energy is being carried by waves. If placement of the barrier stops the energy transfer, it may be because the energy is being transferred by particles or that the energy is being transferred by waves with wavelengths smaller than the barrier.

## Solutions to Problems

1. The wave speed is given by $v=\lambda f$. The period is 3.0 seconds, and the wavelength is 8.0 m .

$$
v=\lambda f=\lambda / T=(8.0 \mathrm{~m}) /(3.0 \mathrm{~s})=2.7 \mathrm{~m} / \mathrm{s}
$$

2. The distance between wave crests is the wavelength of the wave.

$$
\lambda=v / f=343 \mathrm{~m} / \mathrm{s} / 262 \mathrm{~Hz}=1.31 \mathrm{~m}
$$

3. The elastic and bulk moduli are taken from Table 12-1. The densities are taken from Table 13-1.
(a) For water: $v=\sqrt{B / \rho}=\sqrt{\frac{2.0 \times 10^{9} \mathrm{~N} / \mathrm{m}^{2}}{1.00 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}}}=1400 \mathrm{~m} / \mathrm{s}$
(b) For granite: $v=\sqrt{E / \rho}=\sqrt{\frac{45 \times 10^{9} \mathrm{~N} / \mathrm{m}^{2}}{2.7 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}}}=4100 \mathrm{~m} / \mathrm{s}$
(c) For steel: $v=\sqrt{E / \rho}=\sqrt{\frac{200 \times 10^{9} \mathrm{~N} / \mathrm{m}^{2}}{7.8 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}}}=5100 \mathrm{~m} / \mathrm{s}$
4. To find the wavelength, use $\lambda=v / f$.

AM: $\quad \lambda_{1}=\frac{v}{f_{1}}=\frac{3.00 \times 10^{8} \mathrm{~m} / \mathrm{s}}{550 \times 10^{3} \mathrm{~Hz}}=545 \mathrm{~m} \quad \lambda_{2}=\frac{v}{f_{2}}=\frac{3.00 \times 10^{8} \mathrm{~m} / \mathrm{s}}{1600 \times 10^{3} \mathrm{~Hz}}=188 \mathrm{~m} \quad$ AM: 190 m to 550 m
FM: $\quad \lambda_{1}=\frac{v}{f_{1}}=\frac{3.00 \times 10^{8} \mathrm{~m} / \mathrm{s}}{88 \times 10^{6} \mathrm{~Hz}}=3.41 \mathrm{~m} \quad \lambda_{2}=\frac{v}{f_{2}}=\frac{3.00 \times 10^{8} \mathrm{~m} / \mathrm{s}}{108 \times 10^{6} \mathrm{~Hz}}=2.78 \mathrm{~m} \quad$ FM: 2.8 m to 3.4 m
5. The speed of the longitudinal wave is given by Eq. $15-3, v=\sqrt{E / \rho}$. The speed and the frequency are used to find the wavelength. The bulk modulus is found in Table 12-1, and the density is found in Table 13-1.

$$
\lambda=\frac{v}{f}=\frac{\sqrt{\frac{E}{\rho}}}{f}=\frac{\sqrt{\frac{100 \times 10^{9} \mathrm{~N} / \mathrm{m}^{2}}{7.8 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}}}}{5800 \mathrm{~Hz}}=0.62 \mathrm{~m}
$$

6. To find the time for a pulse to travel from one end of the cord to the other, the velocity of the pulse on the cord must be known. For a cord under tension, we have Eq. 15-2, $v=\sqrt{F_{T} / \mu}$.

$$
v=\frac{\Delta x}{\Delta t}=\sqrt{\frac{F_{T}}{\mu}} \rightarrow \Delta t=\frac{\Delta x}{\sqrt{\frac{F_{T}}{\mu}}}=\frac{8.0 \mathrm{~m}}{\sqrt{\frac{140 \mathrm{~N}}{(0.65 \mathrm{~kg}) /(8.0 \mathrm{~m})}}}=0.19 \mathrm{~s}
$$

7. For a cord under tension, we have from Eq. $15-2$ that $v=\sqrt{F_{T} / \mu}$. The speed is also the displacement divided by the elapsed time, $v=\frac{\Delta x}{\Delta t}$. The displacement is the length of the cord.

$$
v=\sqrt{\frac{F_{T}}{\mu}}=\frac{\Delta x}{\Delta t} \rightarrow F_{T}=\mu \frac{\ell^{2}}{(\Delta t)^{2}}=\frac{m}{\ell} \frac{\ell^{2}}{(\Delta t)^{2}}=\frac{m \ell}{(\Delta t)^{2}}=\frac{(0.40 \mathrm{~kg})(7.8 \mathrm{~m})}{(0.85 \mathrm{~s})^{2}}=4.3 \mathrm{~N}
$$

8. The speed of the water wave is given by $v=\sqrt{B / \rho}$, where $B$ is the bulk modulus of water, from Table 12-1, and $\rho$ is the density of sea water, from Table 13-1. The wave travels twice the depth of the ocean during the elapsed time.

$$
v=\frac{2 \ell}{t} \rightarrow \ell=\frac{v t}{2}=\frac{t}{2} \sqrt{\frac{B}{\rho}}=\frac{2.8 \mathrm{~s}}{2} \sqrt{\frac{2.0 \times 10^{9} \mathrm{~N} / \mathrm{m}^{2}}{1.025 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}}}=2.0 \times 10^{3} \mathrm{~m}
$$

9. (a) The speed of the pulse is given by

$$
v=\frac{\Delta x}{\Delta t}=\frac{2(660 \mathrm{~m})}{17 \mathrm{~s}}=77.65 \mathrm{~m} / \mathrm{s} \approx 78 \mathrm{~m} / \mathrm{s}
$$

(b) The tension is related to the speed of the pulse by $v=\sqrt{F_{\mathrm{T}} / \mu}$. The mass per unit length of the cable can be found from its volume and density.

$$
\begin{aligned}
& \rho=\frac{m}{V}=\frac{m}{\pi(d / 2)^{2} \ell} \rightarrow \\
& \mu=\frac{m}{\ell}=\pi \rho\left(\frac{d}{2}\right)^{2}=\pi\left(7.8 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)\left(\frac{1.5 \times 10^{-2} \mathrm{~m}}{2}\right)^{2}=1.378 \mathrm{~kg} / \mathrm{m} \\
& v=\sqrt{F_{\mathrm{T}} / \mu} \rightarrow F_{\mathrm{T}}=v^{2} \mu=(77.65 \mathrm{~m} / \mathrm{s})^{2}(1.378 \mathrm{~kg} / \mathrm{m})=8300 \mathrm{~N}
\end{aligned}
$$

10. (a) Both waves travel the same distance, so $\Delta x=v_{1} t_{1}=v_{2} t_{2}$. We let the smaller speed be $v_{1}$, and the larger speed be $v_{2}$. The slower wave will take longer to arrive, and so $t_{1}$ is more than $t_{2}$.

$$
\begin{aligned}
& t_{1}=t_{2}+1.7 \mathrm{~min}=t_{2}+102 \mathrm{~s} \rightarrow v_{1}\left(t_{2}+102 \mathrm{~s}\right)=v_{2} t_{2} \rightarrow \\
& t_{2}=\frac{v_{1}}{v_{2}-v_{1}}(102 \mathrm{~s})=\frac{5.5 \mathrm{~km} / \mathrm{s}}{8.5 \mathrm{~km} / \mathrm{s}-5.5 \mathrm{~km} / \mathrm{s}}(102 \mathrm{~s})=187 \mathrm{~s} \\
& \Delta x=v_{2} t_{2}=(8.5 \mathrm{~km} / \mathrm{s})(187 \mathrm{~s})=1600 \mathrm{~km}
\end{aligned}
$$

(b) This is not enough information to determine the epicenter. All that is known is the distance of the epicenter from the seismic station. The direction is not known, so the epicenter lies on a circle of radius $1.9 \times 10^{3} \mathrm{~km}$ from the seismic station. Readings from at least two other seismic stations are needed to determine the epicenter's position.
11. (a) The shape will not change. The wave will move 1.10 meters to the right in 1.00 seconds. See the graph. The parts of the string that are moving up or down are indicated.

(b) At the instant shown, the string at point A will be moving down. As the wave moves to the right, the string at point A will move down by 1 cm in the time it takes the "valley" between 1 m and 2 m to move to the right by about 0.25 m .

$$
v=\frac{\Delta y}{\Delta t}=\frac{-1 \mathrm{~cm}}{0.25 \mathrm{~m} / 1.10 \mathrm{~m} / \mathrm{s}} \approx-4 \mathrm{~cm} / \mathrm{s}
$$

This answer will vary depending on the values read from the graph.
12. We assume that the wave will be transverse. The speed is given by Eq. 15-2. The tension in the wire is equal to the weight of the hanging mass. The linear mass density is the volume mass density times the cross-sectional area of the wire. The volume mass density is found in Table 13-1.

$$
v=\sqrt{\frac{F_{T}}{\mu}}=\sqrt{\frac{m_{\text {ball }} g}{\frac{\rho V}{\ell}}}=\sqrt{\frac{m_{\text {ball }} g}{\rho \frac{A \ell}{\ell}}}=\sqrt{\frac{(5.0 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{\left(7800 \mathrm{~kg} / \mathrm{m}^{3}\right) \pi\left(0.50 \times 10^{-3} \mathrm{~m}\right)^{2}}}=89 \mathrm{~m} / \mathrm{s}
$$

13. The speed of the waves on the cord can be found from Eq. 15-2, $v=\sqrt{F_{\mathrm{T}} / \mu}$. The distance between the children is the wave speed times the elapsed time.

$$
\Delta x=v \Delta t=\Delta t \sqrt{\frac{F_{\mathrm{T}}}{m / \Delta x}} \rightarrow \Delta x=(\Delta t)^{2} \frac{F_{\mathrm{T}}}{m}=(0.50 \mathrm{~s})^{2} \frac{35 \mathrm{~N}}{0.50 \mathrm{~kg}}=18 \mathrm{~m}
$$

14. (a) We are told that the speed of the waves only depends on the acceleration due to gravity and the wavelength.

$$
\begin{aligned}
& v=k g^{\alpha} \lambda^{\gamma} \rightarrow\left[\frac{L}{T}\right]=\left[\frac{L}{T^{2}}\right]^{\alpha}[L]^{\gamma} \\
& L:-1=-2 \alpha \rightarrow \alpha=1 / 2 \\
& L: 1=\alpha+\gamma \rightarrow \gamma=1-\alpha=1 / 2
\end{aligned}
$$

(b) Here the speed of the waves depends only on the acceleration due to gravity and the depth of the water.

$$
\begin{array}{ll}
v=k g^{\alpha} h^{\beta} \rightarrow\left[\frac{L}{T}\right]=\left[\frac{L}{T^{2}}\right]^{\alpha}[L]^{\beta} & T:-1=-2 \alpha \rightarrow \alpha=1 / 2 \\
L: 1=\alpha+\beta \rightarrow \beta=1-\alpha=1 / 2 & v=k \sqrt{g h}
\end{array}
$$

15. From Eq. 15-7, if the speed, medium density, and frequency of the two waves are the same, then the intensity is proportional to the square of the amplitude.

$$
I_{2} / I_{1}=E_{2} / E_{1}=A_{2}^{2} / A_{1}^{2}=3 \rightarrow A_{2} / A_{1}=\sqrt{3}=1.73
$$

The more energetic wave has the larger amplitude.
16. (a) Assume that the earthquake waves spread out spherically from the source. Under those conditions, Eq. (15-8ab) applies, stating that intensity is inversely proportional to the square of the distance from the source of the wave.

$$
I_{45 \mathrm{~km}} / I_{15 \mathrm{~km}}=(15 \mathrm{~km})^{2} /(45 \mathrm{~km})^{2}=0.11
$$

(b) The intensity is proportional to the square of the amplitude, and so the amplitude is inversely proportional to the distance from the source of the wave.

$$
A_{45 \mathrm{~km}} / A_{15 \mathrm{~km}}=15 \mathrm{~km} / 45 \mathrm{~km}=0.33
$$

17. We assume that all of the wave motion is outward along the surface of the water - no waves are propagated downwards. Consider two concentric circles on the surface of the water, centered on the place where the circular waves are generated. If there is no damping, then the power (energy per unit time) being transferred across the boundary of each of those circles must be the same. Or, the power associated with the wave must be the same at each circular boundary. The intensity depends on the amplitude squared, so for the power we have this.

$$
P=I(2 \pi r)=k A^{2} 2 \pi r=\mathrm{constant} \rightarrow A^{2}=\frac{\text { constant }}{2 \pi r k} \rightarrow A \propto \frac{1}{\sqrt{r}}
$$

18. (a) Assuming spherically symmetric waves, the intensity will be inversely proportional to the square of the distance from the source. Thus $I r^{2}$ will be constant.

$$
\begin{aligned}
& I_{\text {near }} r_{\text {near }}^{2}=I_{\text {far }} r_{\text {far }}^{2} \rightarrow \\
& I_{\text {near }}=I_{\text {far }} \frac{r_{\text {far }}^{2}}{r_{\text {near }}^{2}}=\left(3.0 \times 10^{6} \mathrm{~W} / \mathrm{m}^{2}\right) \frac{(48 \mathrm{~km})^{2}}{(1.0 \mathrm{~km})^{2}}=6.912 \times 10^{9} \mathrm{~W} / \mathrm{m}^{2} \approx 6.9 \times 10^{9} \mathrm{~W} / \mathrm{m}^{2}
\end{aligned}
$$

(b) The power passing through an area is the intensity times the area.

$$
P=I A=\left(6.912 \times 10^{9} \mathrm{~W} / \mathrm{m}^{2}\right)\left(2.0 \mathrm{~m}^{2}\right)=1.4 \times 10^{10} \mathrm{~W}
$$

19. (a) The power transmitted by the wave is assumed to be the same as the output of the oscillator. That power is given by Eq. 15-6. The wave speed is given by Eq. 15-2. Note that the mass per unit length can be expressed as the volume mass density times the cross sectional area.

$$
\begin{aligned}
\bar{P} & =2 \pi^{2} \rho S v f^{2} A^{2}=2 \pi^{2} \rho S \sqrt{\frac{F_{\mathrm{T}}}{\mu}} f^{2} A^{2}=2 \pi^{2} \rho S \sqrt{\frac{F_{\mathrm{T}}}{\rho S}} f^{2} A^{2}=2 \pi^{2} f^{2} A^{2} \sqrt{S \rho F_{\mathrm{T}}} \\
& =2 \pi^{2}(60.0 \mathrm{~Hz})^{2}(0.0050 \mathrm{~m})^{2} \sqrt{\pi\left(5.0 \times 10^{-3} \mathrm{~m}\right)^{2}\left(7800 \mathrm{~kg} / \mathrm{m}^{3}\right)(7.5 \mathrm{~N})}=0.38 \mathrm{~W}
\end{aligned}
$$

(b) The frequency and amplitude are both squared in the equation. Thus is the power is constant, and the frequency doubles, the amplitude must be halved, and so be 0.25 cm .
20. Consider a wave traveling through an area $S$ with speed $v$, much like Figure 15-11. Start with Eq. 15-7, and use Eq. 15-6.

$$
I=\frac{\bar{P}}{S}=\frac{E}{S t}=\frac{E \ell}{S \ell t}=\frac{E}{S \ell} \frac{\ell}{t}=\frac{\text { energy }}{\text { volume }} \times v
$$

21. (a) We start with Eq. 15-6. The linear mass density is the mass of a given volume of the cord divided by the cross-sectional area of the cord.

$$
\bar{P}=2 \pi^{2} \rho S v f^{2} A^{2} ; \mu=\frac{m}{\ell}=\frac{\rho V}{\ell}=\frac{\rho S \ell}{\ell}=\rho S \rightarrow \bar{P}=2 \pi^{2} \mu v f^{2} A^{2}
$$

(b) The speed of the wave is found from the given tension and mass density, according to Eq. 15-2.

$$
\begin{aligned}
\bar{P} & =2 \pi^{2} \mu v f^{2} A^{2}=2 \pi^{2} f^{2} A^{2} \mu \sqrt{F_{\mathrm{T}} / \mu}=2 \pi^{2} f^{2} A^{2} \sqrt{\mu F_{\mathrm{T}}} \\
& =2 \pi^{2}(120 \mathrm{~Hz})^{2}(0.020 \mathrm{~m})^{2} \sqrt{(0.10 \mathrm{~kg} / \mathrm{m})(135 \mathrm{~N})}=420 \mathrm{~W}
\end{aligned}
$$

22. (a) The only difference is the direction of motion.

$$
D(x, t)=0.015 \sin (25 x+1200 t)
$$

(b) The speed is found from the wave number and the angular frequency, Eq. 15-12.

$$
v=\frac{\omega}{k}=\frac{1200 \mathrm{rad} / \mathrm{s}}{25 \mathrm{rad} / \mathrm{m}}=48 \mathrm{~m} / \mathrm{s}
$$

23. To represent a wave traveling to the left, we replace $x$ by $x+v t$. The resulting expression can be given in various forms.

$$
\begin{aligned}
D & =A \sin [2 \pi(x+v t) / \lambda+\phi]=A \sin \left[2 \pi\left(\frac{x}{\lambda}+\frac{v t}{\lambda}\right)+\phi\right]=A \sin \left[2 \pi\left(\frac{x}{\lambda}+\frac{t}{T}\right)+\phi\right] \\
& =A \sin (k x+\omega t+\phi)
\end{aligned}
$$

24. The traveling wave is given by $D=0.22 \sin (5.6 x+34 t)$.
(a) The wavelength is found from the coefficient of $x$.

$$
5.6 \mathrm{~m}^{-1}=\frac{2 \pi}{\lambda} \rightarrow \lambda=\frac{2 \pi}{5.6 \mathrm{~m}^{-1}}=1.122 \mathrm{~m} \approx 1.1 \mathrm{~m}
$$

(b) The frequency is found from the coefficient of $t$.

$$
34 \mathrm{~s}^{-1}=2 \pi f \rightarrow f=\frac{34 \mathrm{~s}^{-1}}{2 \pi}=5.411 \mathrm{~Hz} \approx 5.4 \mathrm{~Hz}
$$

(c) The velocity is the ratio of the coefficients of $t$ and $x$.

$$
v=\lambda f=\frac{2 \pi}{5.6 \mathrm{~m}^{-1}} \frac{34 \mathrm{~s}^{-1}}{2 \pi}=6.071 \mathrm{~m} / \mathrm{s} \approx 6.1 \mathrm{~m} / \mathrm{s}
$$

Because both coefficients are positive, the velocity is in the negative $x$ direction.
(d) The amplitude is the coefficient of the sine function, and so is 0.22 m .
(e) The particles on the cord move in simple harmonic motion with the same frequency as the wave. From Chapter $14, v_{\max }=D \omega=2 \pi f D$.

$$
v_{\max }=2 \pi f D=2 \pi\left(\frac{34 \mathrm{~s}^{-1}}{2 \pi}\right)(0.22 \mathrm{~m})=7.5 \mathrm{~m} / \mathrm{s}
$$

The minimum speed is when a particle is at a turning point of its motion, at which time the speed is 0 .

$$
v_{\min }=0
$$

25. The traveling wave is given by $D(x, t)=(0.026 \mathrm{~m}) \sin \left[\left(45 \mathrm{~m}^{-1}\right) x-\left(1570 \mathrm{~s}^{-1}\right) t+0.66\right]$.
(a) $v_{x}=\frac{\partial D(x, t)}{\partial t}=-\left(1570 \mathrm{~s}^{-1}\right)(0.026 \mathrm{~m}) \cos \left[\left(45 \mathrm{~m}^{-1}\right) x-\left(1570 \mathrm{~s}^{-1}\right) t+0.66\right] \rightarrow$ $\left(v_{x}\right)_{\max }=\left(1570 \mathrm{~s}^{-1}\right)(0.026 \mathrm{~m})=41 \mathrm{~m} / \mathrm{s}$
(b) $\quad a_{x}=\frac{\partial^{2} D(x, t)}{\partial t^{2}}=-\left(1570 \mathrm{~s}^{-1}\right)^{2}(0.026 \mathrm{~m}) \sin \left[\left(45 \mathrm{~m}^{-1}\right) x-\left(1570 \mathrm{~s}^{-1}\right) t+0.66\right] \rightarrow$
$\left(a_{x}\right)_{\max }=\left(1570 \mathrm{~s}^{-1}\right)^{2}(0.026 \mathrm{~m})=6.4 \times 10^{4} \mathrm{~m} / \mathrm{s}^{2}$

$$
\text { (c) } \begin{aligned}
v_{x}(1.00 \mathrm{~m}, 2.50 \mathrm{~s}) & =-\left(1570 \mathrm{~s}^{-1}\right)(0.026 \mathrm{~m}) \cos \left[\left(45 \mathrm{~m}^{-1}\right)(1.00 \mathrm{~m})-\left(1570 \mathrm{~s}^{-1}\right)(2.50 \mathrm{~s})+0.66\right] \\
& =35 \mathrm{~m} / \mathrm{s} \\
a_{x}(1.00 \mathrm{~m}, 2.50 \mathrm{~s}) & =-\left(1570 \mathrm{~s}^{-1}\right)^{2}(0.026 \mathrm{~m}) \sin \left[\left(45 \mathrm{~m}^{-1}\right)(1.00 \mathrm{~m})-\left(1570 \mathrm{~s}^{-1}\right)(2.50 \mathrm{~s})+0.66\right] \\
& =3.2 \times 10^{4} \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

26. The displacement of a point on the cord is given by the wave, $D(x, t)=0.12 \sin (3.0 x-15.0 t)$. The velocity of a point on the cord is given by $\frac{\partial D}{\partial t}$.

$$
\begin{aligned}
& D(0.60 \mathrm{~m}, 0.20 \mathrm{~s})=(0.12 \mathrm{~m}) \sin \left[\left(3.0 \mathrm{~m}^{-1}\right)(0.60 \mathrm{~m})-\left(15.0 \mathrm{~s}^{-1}\right)(0.20 \mathrm{~s})\right]=-0.11 \mathrm{~m} \\
& \frac{\partial D}{\partial t}=(0.12 \mathrm{~m})\left(-15.0 \mathrm{~s}^{-1}\right) \cos (3.0 x-15.0 t) \\
& \frac{\partial D}{\partial t}(0.60 \mathrm{~m}, 0.20 \mathrm{~s})=(0.12 \mathrm{~m})\left(-15.0 \mathrm{~s}^{-1}\right) \cos \left[\left(3.0 \mathrm{~m}^{-1}\right)(0.60 \mathrm{~m})-\left(15.0 \mathrm{~s}^{-1}\right)(0.20 \mathrm{~s})\right]=-0.65 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

27. (́a) The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.27a."

(b) For motion to the right, replace $x$ by $x-v t$.

$$
D(x, t)=(0.45 \mathrm{~m}) \cos [2.6(x-2.0 t)+1.2]
$$

(c) See the graph above.
(d) For motion to the left, replace $x$ by $x+v t$. Also see the graph above.

$$
D(x, t)=(0.45 \mathrm{~m}) \cos [2.6(x+2.0 t)+1.2]
$$

28. (a) The wavelength is the speed divided by the frequency.

$$
\lambda=\frac{v}{f}=\frac{345 \mathrm{~m} / \mathrm{s}}{524 \mathrm{~Hz}}=0.658 \mathrm{~m}
$$

(b) In general, the phase change in degrees due to a time difference is given by $\frac{\Delta \phi}{360^{\circ}}=\frac{\Delta t}{T}$.

$$
\frac{\Delta \phi}{360^{\circ}}=\frac{\Delta t}{T}=f \Delta t \rightarrow \Delta t=\frac{1}{f} \frac{\Delta \phi}{360^{\circ}}=\frac{1}{524 \mathrm{~Hz}}\left(\frac{90^{\circ}}{360^{\circ}}\right)=4.77 \times 10^{-4} \mathrm{~s}
$$

(c) In general, the phase change in degrees due to a position difference is given by $\frac{\Delta \phi}{360^{\circ}}=\frac{\Delta x}{\lambda}$.

$$
\frac{\Delta \phi}{360^{\circ}}=\frac{\Delta x}{\lambda} \rightarrow \Delta \phi=\frac{\Delta x}{\lambda}\left(360^{\circ}\right)=\frac{0.044 \mathrm{~m}}{0.658 \mathrm{~m}}\left(360^{\circ}\right)=24.1^{\circ}
$$

29. The amplitude is 0.020 cm , the wavelength is 0.658 m , and the frequency is 524 Hz . The displacement is at its most negative value at $x=0, t=0$, and so the wave can be represented by a cosine that is phase shifted by half of a cycle.

$$
\begin{aligned}
& D(x, t)=A \cos (k x-\omega t+\phi) \\
& A=0.020 \mathrm{~cm} ; k=\frac{2 \pi}{\lambda}=\frac{2 \pi f}{v}=\frac{2 \pi(524 \mathrm{~Hz})}{345 \mathrm{~m} / \mathrm{s}}=9.54 \mathrm{~m}^{-1} ; \omega=2 \pi f=2 \pi(524 \mathrm{~Hz})=3290 \mathrm{rad} / \mathrm{s} \\
& D(x, t)=(0.020 \mathrm{~cm}) \cos \left[\left(9.54 \mathrm{~m}^{-1}\right) x-(3290 \mathrm{rad} / \mathrm{s}) t+\pi\right], x \text { in m,t in s}
\end{aligned}
$$

Other equivalent expressions include the following.

$$
\begin{aligned}
& D(x, t)=-(0.020 \mathrm{~cm}) \cos \left[\left(9.54 \mathrm{~m}^{-1}\right) x-(3290 \mathrm{rad} / \mathrm{s}) t\right] \\
& D(x, t)=(0.020 \mathrm{~cm}) \sin \left[\left(9.54 \mathrm{~m}^{-1}\right) x-(3290 \mathrm{rad} / \mathrm{s}) t+\frac{3}{2} \pi\right]
\end{aligned}
$$

30. (a) For the particle of string at $x=0$, the displacement is not at the full amplitude at $t=0$. The particle is moving upwards, and so a maximum is approaching from the right. The general form of the wave is given by
$D(x, t)=A \sin (k x+\omega t+\phi)$. At
$x=0$ and $t=0, D(0,0)=A \sin \phi$
and so we can find the phase angle.


$$
D(0,0)=A \sin \phi \rightarrow 0.80 \mathrm{~cm}=(1.00 \mathrm{~cm}) \sin \phi \rightarrow \phi=\sin ^{-1}(0.80)=0.93
$$

So we have $D(x, 0)=A \sin \left(\frac{2 \pi}{3.0} x+0.93\right), x$ in cm. See the graph. It matches the description given earlier. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.30a."
(b) We use the given data to write the wave function. Note that the wave is moving to the right, and that the phase angle has already been determined.

$$
\begin{aligned}
& D(x, t)=A \sin (k x+\omega t+\phi) \\
& A=1.00 \mathrm{~cm} ; k=\frac{2 \pi}{3.00 \mathrm{~cm}}=2.09 \mathrm{~cm}^{-1} ; \omega=2 \pi f=2 \pi(245 \mathrm{~Hz})=1540 \mathrm{rad} / \mathrm{s} \\
& D(x, t)=(1.00 \mathrm{~cm}) \sin \left[\left(2.09 \mathrm{~cm}^{-1}\right) x+(1540 \mathrm{rad} / \mathrm{s}) t+0.93\right], x \text { in } \mathrm{cm}, t \text { in s }
\end{aligned}
$$

31. To be a solution of the wave equation, the function must satisfy Eq. $15-16, \frac{\partial^{2} D}{\partial x^{2}}=\frac{1}{v^{2}} \frac{\partial^{2} D}{\partial t^{2}}$.

$$
\begin{aligned}
& D=A \sin k x \cos \omega t \\
& \frac{\partial D}{\partial x}=k A \cos k x \cos \omega t ; \quad \frac{\partial^{2} D}{\partial x^{2}}=-k^{2} A \sin k x \cos \omega t \\
& \frac{\partial D}{\partial t}=-\omega A \sin k x \sin \omega t ; \quad \frac{\partial^{2} D}{\partial t^{2}}=-\omega^{2} A \sin k x \cos \omega t
\end{aligned}
$$

This gives $\frac{\partial^{2} D}{\partial x^{2}}=\frac{k^{2}}{\omega^{2}} \frac{\partial^{2} D}{\partial t^{2}}$, and since $v=\frac{\omega}{k}$ from Eq. 15-12, we have $\frac{\partial^{2} D}{\partial x^{2}}=\frac{1}{v^{2}} \frac{\partial^{2} D}{\partial t^{2}}$.
Yes, the function is a solution.
32. To be a solution of the wave equation, the function must satisfy Eq. 15-16, $\frac{\partial^{2} D}{\partial x^{2}}=\frac{1}{v^{2}} \frac{\partial^{2} D}{\partial t^{2}}$.
(a) $D=A \ln (x+v t)$

$$
\frac{\partial D}{\partial x}=\frac{A}{x+v t} ; \frac{\partial^{2} D}{\partial x^{2}}=-\frac{A}{(x+v t)^{2}} ; \frac{\partial D}{\partial t}=\frac{A v}{x+v t} ; \frac{\partial^{2} D}{\partial t^{2}}=-\frac{A v^{2}}{(x+v t)^{2}}
$$

This gives $\frac{\partial^{2} D}{\partial x^{2}}=\frac{1}{v^{2}} \frac{\partial^{2} D}{\partial t^{2}}$, and so yes, the function is a solution.
(b) $D=(x-v t)^{4}$

$$
\frac{\partial D}{\partial x}=4(x-v t)^{3} ; \frac{\partial^{2} D}{\partial x^{2}}=12(x-v t)^{2} ; \quad \frac{\partial D}{\partial t}=-4 v(x-v t)^{3} ; \quad \frac{\partial^{2} D}{\partial t^{2}}=12 v^{2}(x-v t)^{2}
$$

This gives $\frac{\partial^{2} D}{\partial x^{2}}=\frac{1}{v^{2}} \frac{\partial^{2} D}{\partial t^{2}}$, and so yes, the function is a solution.
33. We find the various derivatives for the function from Eq. 15-13c.

$$
\begin{aligned}
D(x, t) & =A \sin (k x+\omega t) ; \frac{\partial D}{\partial x}=A k \cos (k x+\omega t) ; \frac{\partial^{2} D}{\partial x^{2}}=-A k^{2} \sin (k x+\omega t) ; \\
\frac{\partial D}{\partial t} & =A \omega \cos (k x+\omega t) ; \frac{\partial^{2} D}{\partial t^{2}}=-A \omega^{2} \sin (k x+\omega t)
\end{aligned}
$$

To satisfy the wave equation, we must have $\frac{\partial^{2} D}{\partial x^{2}}=\frac{1}{v^{2}} \frac{\partial^{2} D}{\partial t^{2}}$.

$$
\frac{\partial^{2} D}{\partial x^{2}}=\frac{1}{v^{2}} \frac{\partial^{2} D}{\partial t^{2}} \rightarrow-A k^{2} \sin (k x+\omega t)=\frac{1}{v^{2}}\left(-A \omega^{2} \sin (k x+\omega t)\right) \rightarrow k^{2}=\frac{\omega^{2}}{v^{2}}
$$

Since $v=\omega / k$, the wave equation is satisfied.
We find the various derivatives for the function from Eq. 15-15. Make the substitution that $u=x+v t$, and then use the chain rule.

$$
\begin{aligned}
& D(x, t)=D(x+v t)=D(u) ; \quad \frac{\partial D}{\partial x}=\frac{d D}{d u} \frac{\partial u}{\partial x}=\frac{d D}{d u} ; \frac{\partial^{2} D}{\partial x^{2}}=\frac{\partial}{\partial x} \frac{d D}{d u}=\left(\frac{d}{d x} \frac{d D}{d u}\right) \frac{\partial u}{\partial x}=\frac{d^{2} D}{d u^{2}} \\
& \frac{\partial D}{\partial t}=\frac{d D}{d u} \frac{\partial u}{\partial t}=v \frac{d D}{d u} ; \frac{\partial^{2} D}{\partial t^{2}}=\frac{\partial}{\partial t}\left(v \frac{d D}{d u}\right)=v \frac{\partial}{\partial t} \frac{d D}{d u}=v\left(\frac{d}{d u} \frac{d D}{d u}\right) \frac{\partial u}{\partial t}=v \frac{d^{2} D}{d u^{2}} v=v^{2} \frac{d^{2} D}{d u^{2}}
\end{aligned}
$$

To satisfy the wave equation, we must have $\frac{\partial^{2} D}{\partial x^{2}}=\frac{1}{v^{2}} \frac{\partial^{2} D}{\partial t^{2}}$.

$$
\frac{\partial^{2} D}{\partial x^{2}}=\frac{1}{v^{2}} \frac{\partial^{2} D}{\partial t^{2}} \rightarrow \frac{d^{2} D}{d u^{2}}=\frac{1}{v^{2}} v^{2} \frac{d^{2} D}{d u^{2}}=\frac{d^{2} D}{d u^{2}}
$$

Since we have an identity, the wave equation is satisfied.
34. Find the various derivatives for the linear combination.

$$
\begin{aligned}
& D(x, t)=C_{1} D_{1}+C_{2} D_{2}=C_{1} f_{1}(x, t)+C_{2} f_{2}(x, t) \\
& \frac{\partial D}{\partial x}=C_{1} \frac{\partial f_{1}}{\partial x}+C_{2} \frac{\partial f_{2}}{\partial x} ; \frac{\partial^{2} D}{\partial x^{2}}=C_{1} \frac{\partial^{2} f_{1}}{\partial x^{2}}+C_{2} \frac{\partial^{2} f_{2}}{\partial x^{2}} \\
& \frac{\partial D}{\partial t}=C_{1} \frac{\partial f_{1}}{\partial t}+C_{2} \frac{\partial f_{2}}{\partial t} ; \frac{\partial^{2} D}{\partial t^{2}}=C_{1} \frac{\partial^{2} f_{1}}{\partial t^{2}}+C_{2} \frac{\partial^{2} f_{2}}{\partial t^{2}}
\end{aligned}
$$

To satisfy the wave equation, we must have $\frac{\partial^{2} D}{\partial x^{2}}=\frac{1}{v^{2}} \frac{\partial^{2} D}{\partial t^{2}}$. Use the fact that both $f_{1}$ and $f_{2}$ satisfy the wave equation.

$$
\frac{\partial^{2} D}{\partial x^{2}}=C_{1} \frac{\partial^{2} f_{1}}{\partial x^{2}}+C_{2} \frac{\partial^{2} f_{2}}{\partial x^{2}}=C_{1}\left[\frac{1}{v^{2}} \frac{\partial^{2} f_{1}}{\partial t^{2}}\right]+C_{2}\left[\frac{1}{v^{2}} \frac{\partial^{2} f_{2}}{\partial t^{2}}\right]=\frac{1}{v^{2}}\left[C_{1} \frac{\partial^{2} f_{1}}{\partial t^{2}}+C_{2} \frac{\partial^{2} f_{2}}{\partial t^{2}}\right]=\frac{1}{v^{2}} \frac{\partial^{2} D}{\partial t^{2}}
$$

Thus we see that $\frac{\partial^{2} D}{\partial x^{2}}=\frac{1}{v^{2}} \frac{\partial^{2} D}{\partial t^{2}}$, and so $D$ satisfies the wave equation.
35. To be a solution of the wave equation, the function must satisfy Eq. 15-16, $\frac{\partial^{2} D}{\partial x^{2}}=\frac{1}{v^{2}} \frac{\partial^{2} D}{\partial t^{2}}$.

$$
\begin{aligned}
& D=e^{-(k x-\omega t)^{2}} ; \frac{\partial D}{\partial x}=-2 k(k x-\omega t) e^{-(k x-\omega t)^{2}} \\
& \frac{\partial^{2} D}{\partial x^{2}}=-2 k(k x-\omega t)\left[-2 k(k x-\omega t) e^{-(k x-\omega t)^{2}}\right]+\left(-2 k^{2}\right) e^{-(k x-\omega t)^{2}}=2 k^{2}\left[2(k x-\omega t)^{2}-1\right] e^{-(k x-\omega t)^{2}} \\
& \frac{\partial D}{\partial t}=2 \omega(k x-\omega t) e^{-(k x-\omega t)^{2}} \\
& \frac{\partial^{2} D}{\partial t^{2}}=2 \omega(k x-\omega t)\left[2 \omega(k x-\omega t) e^{-(k x-\omega t)^{2}}\right]+\left(-2 \omega^{2}\right) e^{-(k x-\omega t)^{2}}=2 \omega^{2}\left[2(k x-\omega t)^{2}-1\right] e^{-(k x-\omega t)^{2}} \\
& \frac{\partial^{2} D}{\partial x^{2}}=\frac{1}{v^{2}} \frac{\partial^{2} D}{\partial t^{2}} \rightarrow 2 k^{2}\left[2(k x-\omega t)^{2}-1\right] e^{-(k x-\omega t)^{2}}=\frac{1}{v^{2}} 2 \omega^{2}\left[2(k x-\omega t)^{2}-1\right] e^{-(k x-\omega t)^{2}} \rightarrow \\
& k^{2}=\frac{\omega^{2}}{v^{2}}
\end{aligned}
$$

Since $v=\frac{\omega}{k}$, we have an identity. Yes, the function is a solution.
36. We assume that $A \ll \lambda$ for the wave given by $D=A \sin (k x-\omega t)$.

$$
\begin{aligned}
& D=A \sin (k x-\omega t) \rightarrow v^{\prime}=\frac{\partial D}{\partial t}=-\omega A \cos (k x-\omega t) \rightarrow v_{\max }^{\prime}=\omega A \\
& A \ll \lambda \rightarrow \frac{v_{\max }^{\prime}}{\omega} \ll \lambda \rightarrow v_{\max }^{\prime}<\omega \lambda=v_{\text {wave }} \rightarrow v_{\max }^{\prime}<v_{\text {wave }} \\
& \frac{v_{\max }^{\prime}}{v}=\frac{\omega A}{v}=\frac{2 \pi f A}{v}=\frac{2 \pi f \frac{\lambda}{100}}{f \lambda}=\frac{\pi}{50} \approx 0.063
\end{aligned}
$$

37. (a) For the wave in the lighter cord, $D(x, t)=(0.050 \mathrm{~m}) \sin \left[\left(7.5 \mathrm{~m}^{-1}\right) x-\left(12.0 \mathrm{~s}^{-1}\right) t\right]$.

$$
\lambda=\frac{2 \pi}{k}=\frac{2 \pi}{\left(7.5 \mathrm{~m}^{-1}\right)}=0.84 \mathrm{~m}
$$

(b) The tension is found from the velocity, using Eq. 15-2.

$$
v=\sqrt{\frac{F_{\mathrm{T}}}{\mu}} \rightarrow F_{\mathrm{T}}=\mu v^{2}=\mu \frac{\omega^{2}}{k^{2}}=(0.10 \mathrm{~kg} / \mathrm{m}) \frac{\left(12.0 \mathrm{~s}^{-1}\right)^{2}}{\left(7.5 \mathrm{~m}^{-1}\right)^{2}}=0.26 \mathrm{~N}
$$

(c) The tension and the frequency do not change from one section to the other.

$$
F_{\mathrm{T} 1}=F_{\mathrm{T} 2} \rightarrow=\mu_{1} \frac{\omega_{1}^{2}}{k_{1}^{2}}=\mu_{2} \frac{\omega_{2}^{2}}{k_{2}^{2}} \rightarrow \lambda_{2}=\lambda_{1} \sqrt{\frac{\mu_{1}}{\mu_{2}}}=\frac{2 \pi}{k_{1}} \sqrt{\frac{\mu_{1}}{\mu_{2}}}=\frac{2 \pi}{\left(7.5 \mathrm{~m}^{-1}\right)} \sqrt{0.5}=0.59 \mathrm{~m}
$$

38. (a) The speed of the wave in a stretched cord is given by Eq. 15-2, $v=\sqrt{F_{\mathrm{T}} / \mu}$. The tensions must be the same in both parts of the cord. If they were not the same, then the net longitudinal force on the joint between the two parts would not be zero, and the joint would have to accelerate along the length of the cord.

$$
v=\sqrt{F_{\mathrm{T}} / \mu} \rightarrow \frac{v_{\mathrm{H}}}{v_{\mathrm{L}}}=\frac{\sqrt{F_{\mathrm{T}} / \mu_{\mathrm{H}}}}{\sqrt{F_{\mathrm{T}} / \mu_{\mathrm{L}}}}=\sqrt{\frac{\mu_{\mathrm{L}}}{\mu_{\mathrm{H}}}}
$$

(b) The frequency must be the same in both sections. If it were not, then the joint between the two sections would not be able to keep the two sections together. The ends could not stay in phase with each other if the frequencies were different.

$$
f=\frac{v}{\lambda} \rightarrow \frac{v_{\mathrm{H}}}{\lambda_{\mathrm{H}}}=\frac{v_{\mathrm{L}}}{\lambda_{\mathrm{L}}} \rightarrow \frac{\lambda_{\mathrm{H}}}{\lambda_{\mathrm{L}}}=\frac{v_{\mathrm{H}}}{v_{\mathrm{L}}}=\sqrt{\frac{\mu_{\mathrm{L}}}{\mu_{\mathrm{H}}}}
$$

(c) The ratio under the square root sign is less than 1 , and so the lighter cord has the greater wavelength.
39. (a) The distance traveled by the reflected sound wave is found from the Pythagorean theorem.

$$
d=2 \sqrt{D^{2}+\left(\frac{1}{2} x\right)^{2}}=v t \rightarrow t=\frac{2}{v} \sqrt{D^{2}+\left(\frac{1}{2} x\right)^{2}}
$$

(b) Solve for $t^{2}$.

$$
t^{2}=\frac{4}{v^{2}}\left[D^{2}+\left(\frac{1}{2} x\right)^{2}\right]=\frac{x^{2}}{v^{2}}+\frac{4}{v^{2}} D^{2}
$$

A plot of $t^{2}$ vs $x^{2}$ would have a slope of $1 / v^{2}$, which can be used to determine the value of $v$. The $y$ intercept of that plot is $\frac{4}{v^{2}} D^{2}$. Knowing the $y$ intercept and the value of $v$, the value of $D$ can be determined.
40. The tension and the frequency do not change from one side of the knot to the other.
(a) We force the cord to be continuous at $x=0$ for all times. This is done by setting the initial wave plus the reflected wave (the displacement of a point infinitesimally to the LEFT of $x=0$ ) equal to the transmitted wave (the displacement of a point infinitesimally to the RIGHT of $x=0$ ) for all times. We also use the facts that $\sin (-\theta)=-\sin \theta$ and $k_{1} v_{1}=k_{2} v_{2}$.

$$
\begin{aligned}
& D(0, t)+D_{\mathrm{R}}(0, t)=D_{\mathrm{T}}(0, t) \rightarrow A \sin \left(-k_{1} v_{1} t\right)+A_{\mathrm{R}} \sin \left(k_{1} v_{1} t\right)=A_{\mathrm{T}} \sin \left(-k_{2} v_{2} t\right) \rightarrow \\
& -A \sin \left(k_{1} v_{1} t\right)+A_{\mathrm{R}} \sin \left(k_{1} v_{1} t\right)=-A_{\mathrm{T}} \sin \left(k_{2} v_{2} t\right)=-A_{\mathrm{T}} \sin \left(k_{1} v_{1} t\right) \rightarrow \\
& -A+A_{\mathrm{R}}=-A_{\mathrm{T}} \rightarrow A=A_{\mathrm{T}}+A_{\mathrm{R}}
\end{aligned}
$$

(b) To make the slopes match for all times, we must have $\frac{\partial}{\partial x}\left[D(x, t)+D_{\mathrm{R}}(x, t)\right]=\frac{\partial}{\partial x}\left[D_{\mathrm{T}}(x, t)\right]$ when evaluated at the origin. We also use the result of the above derivation, and the facts that $\cos (-\theta)=\cos \theta$ and $k_{1} v_{1}=k_{2} v_{2}$.

$$
\begin{aligned}
& \left.\frac{\partial}{\partial x}\left[D(x, t)+D_{\mathrm{R}}(x, t)\right]\right|_{x=0}=\left.\frac{\partial}{\partial x}\left[D_{\mathrm{T}}(x, t)\right]\right|_{x=0} \rightarrow \\
& k_{1} A \cos \left(-k_{1} v_{1} t\right)+k_{1} A_{\mathrm{R}} \cos \left(k_{1} v_{1} t\right)=k_{2} A_{\mathrm{T}} \cos \left(-k_{2} v_{2} t\right) \rightarrow \\
& k_{1} A \cos \left(k_{1} v_{1} t\right)+k_{1} A_{\mathrm{R}} \cos \left(k_{1} v_{1} t\right)=k_{2} A_{\mathrm{T}} \cos \left(k_{2} v_{2} t\right) \rightarrow \\
& k_{1} A+k_{1} A_{\mathrm{R}}=k_{2} A_{\mathrm{T}}=k_{2}\left(A-A_{\mathrm{R}}\right) \rightarrow A_{\mathrm{R}}=\left(\frac{k_{2}-k_{1}}{k_{2}+k_{1}}\right) A
\end{aligned}
$$

Use $k_{2}=k_{1} \frac{v_{1}}{v_{2}}$.

$$
A_{\mathrm{R}}=\left(\frac{k_{2}-k_{1}}{k_{2}+k_{1}}\right) A=\left(\frac{k_{1} \frac{v_{1}}{v_{2}}-k_{1}}{k_{1} \frac{v_{1}}{v_{2}}+k_{1}}\right) A=\frac{k_{1}}{k_{1}}\left(\frac{\frac{v_{1}}{v_{2}}-1}{\frac{v_{1}}{v_{2}}+1}\right) A=\left(\frac{\frac{v_{1}}{v_{2}}-\frac{v_{2}}{v_{2}}}{\frac{v_{1}}{v_{2}}+\frac{v_{2}}{v_{2}}}\right) A=\left(\frac{v_{1}-v_{2}}{v_{1}+v_{2}}\right) A
$$

(c) Combine the results from the previous two parts.

$$
\begin{aligned}
A_{\mathrm{T}} & =A-A_{\mathrm{R}}=A-\left(\frac{k_{2}-k_{1}}{k_{2}+k_{1}}\right) A=A\left[1-\left(\frac{k_{2}-k_{1}}{k_{2}+k_{1}}\right)\right]=A\left[\left(\frac{k_{2}+k_{1}}{k_{2}+k_{1}}\right)-\left(\frac{k_{2}-k_{1}}{k_{2}+k_{1}}\right)\right]=\left(\frac{2 k_{1}}{k_{2}+k_{1}}\right) A \\
& =\left(\frac{2 k_{1}}{k_{1} \frac{v_{1}}{v_{2}}+k_{1}}\right) A=\left(\frac{2 v_{2}}{v_{1}+v_{2}}\right) A
\end{aligned}
$$

41. (a)

(b)

(c) The energy is all kinetic energy at the moment when the string has no displacement. There is no elastic potential energy at that moment. Each piece of the string has speed but no displacement.
42. (a) The resultant wave is the algebraic sum of the two component waves.

$$
\begin{aligned}
& D=D_{1}+D_{2}=A \sin (k x-\omega t)+A \sin (k x-\omega t+\phi)=A[\sin (k x-\omega t)+A \sin (k x-\omega t+\phi)] \\
& =A\left\{2 \sin \frac{1}{2}[(k x-\omega t)+(k x-\omega t+\phi)]\right\}\left\{\cos \frac{1}{2}[(k x-\omega t)-(k x-\omega t+\phi)]\right\} \\
& =2 A\left\{\sin \frac{1}{2}(2 k x-2 \omega t+\phi)\right\}\left\{\cos \frac{1}{2}(\phi)\right\}=\left(2 A \cos \frac{\phi}{2}\right) \sin \left(k x-\omega t+\frac{\phi}{2}\right)
\end{aligned}
$$

(b) The amplitude is the absolute value of the coefficient of the sine function, $2 A \cos \frac{\phi}{2}$. The wave is purely sinusoidal because the dependence on $x$ and $t$ is $\sin \left(k x-\omega t+\frac{\phi}{2}\right)$.
(c) If $\phi=0,2 \pi, 4 \pi, \cdots, 2 n \pi$, then the amplitude is $\left|2 A \cos \frac{\phi}{2}\right|=\left|2 A \cos \frac{2 n \pi}{2}\right|=|2 A \cos n \pi|=|2 A( \pm 1)|$
$=2 A$, which is constructive interference. If $\phi=\pi, 3 \pi, 5 \pi, \cdots,(2 n+1) \pi$, then the amplitude is $\left|2 A \cos \frac{\phi}{2}\right|=\left|2 A \cos \frac{(2 n+1) \pi}{2}\right|=\left|2 A \cos \left[\left(n+\frac{1}{2}\right) \pi\right]\right|=0$, which is destructive interference.
(d) If $\phi=\frac{\pi}{2}$, then the resultant wave is as follows.

$$
D=\left(2 A \cos \frac{\phi}{2}\right) \sin \left(k x-\omega t+\frac{\phi}{2}\right)=\left(2 A \cos \frac{\pi}{4}\right) \sin \left(k x-\omega t+\frac{\pi}{4}\right)=\sqrt{2} A \sin \left(k x-\omega t+\frac{\pi}{4}\right)
$$

This wave has an amplitude of $\sqrt{2} A$, is traveling in the positive $x$ direction, and is shifted to the left by an eighth of a cycle. This is "halfway" between the two original waves. The displacement is $\frac{1}{2} A$ at the origin at $t=0$.
43. The fundamental frequency of the full string is given by $f_{\text {unfingered }}=\frac{v}{2 \ell}=441 \mathrm{~Hz}$. If the length is reduced to $2 / 3$ of its current value, and the velocity of waves on the string is not changed, then the new frequency will be as follows.

$$
f_{\text {fingered }}=\frac{v}{2\left(\frac{2}{3} \ell\right)}=\frac{3}{2} \frac{v}{2 \ell}=\left(\frac{3}{2}\right) f_{\text {unfingered }}=\left(\frac{3}{2}\right)(441 \mathrm{~Hz})=662 \mathrm{~Hz}
$$

44. The frequencies of the harmonics of a string that is fixed at both ends are given by $f_{n}=n f_{1}$, and so the first four harmonics are $f_{1}=294 \mathrm{~Hz}, f_{2}=588 \mathrm{~Hz}, f_{3}=882 \mathrm{~Hz}, f_{4}=1176 \mathrm{~Hz}$.
45. The oscillation corresponds to the fundamental. The frequency of that oscillation is $f_{1}=\frac{1}{T}=\frac{1}{1.5 \mathrm{~s}}=\frac{2}{3} \mathrm{~Hz}$. The bridge, with both ends fixed, is similar to a vibrating string, and so $f_{n}=n f_{1}=\frac{2 n}{3} \mathrm{~Hz}, n=1,2,3 \ldots$. The periods are the reciprocals of the frequency, and so $T_{n}=\frac{1.5 \mathrm{~s}}{n}, n=1,2,3 \ldots$.
46. Four loops is the standing wave pattern for the $4^{\text {th }}$ harmonic, with a frequency given by $f_{4}=4 f_{1}=280 \mathrm{~Hz}$. Thus $f_{1}=70 \mathrm{~Hz}, f_{2}=140 \mathrm{~Hz}, f_{3}=210 \mathrm{~Hz}$, and $f_{5}=350 \mathrm{~Hz}$ are all other resonant frequencies.
47. Each half of the cord has a single node, at the center of the cord. Thus each half of the cord is a half of a wavelength, assuming that the ends of the cord are also nodes. The tension is the same in both halves of the cord, and the wavelengths are the same based on the location of the node. Let subscript 1 represent the lighter density, and subscript 2 represent the heavier density.

$$
\begin{aligned}
& v_{1}=\sqrt{\frac{F_{\mathrm{T} 1}}{\mu_{1}}}=\lambda_{1} f_{1} ; v_{2}=\sqrt{\frac{F_{\mathrm{T} 2}}{\mu_{2}}}=\lambda_{2} f_{2} ; \lambda_{1}=\lambda_{2} ; F_{\mathrm{T} 1}=F_{\mathrm{T} 2} \\
& \frac{f_{1}}{f_{2}}=\frac{\frac{1}{\lambda_{1}} \sqrt{\frac{F_{\mathrm{T} 1}}{\mu_{1}}}}{\frac{1}{\lambda_{2}} \sqrt{\frac{F_{\mathrm{T} 2}}{\mu_{2}}}}=\sqrt{\frac{\mu_{2}}{\mu_{1}}}=\sqrt{2}
\end{aligned}
$$

The frequency is higher on the lighter portion.
48. Adjacent nodes are separated by a half-wavelength, as examination of Figure 15-26 will show.

$$
\lambda=\frac{v}{f} \rightarrow \Delta x_{\text {node }}=\frac{1}{2} \lambda=\frac{v}{2 f}=\frac{96 \mathrm{~m} / \mathrm{s}}{2(445 \mathrm{~Hz})}=0.11 \mathrm{~m}
$$

49. Since $f_{n}=n f_{1}$, two successive overtones differ by the fundamental frequency, as shown below.

$$
\Delta f=f_{n+1}-f_{n}=(n+1) f_{1}-n f_{1}=f_{1}=320 \mathrm{~Hz}-240 \mathrm{~Hz}=80 \mathrm{~Hz}
$$

50. The speed of waves on the string is given by Eq. 15-2, $v=\sqrt{F_{\mathrm{T}} / \mu}$. The resonant frequencies of a string with both ends fixed are given by Eq. 15-17b, $f_{n}=\frac{n v}{2 \ell_{\text {vib }}}$, where $\ell_{\text {vib }}$ is the length of the portion that is actually vibrating. Combining these relationships allows the frequencies to be calculated.

$$
\begin{aligned}
& f_{n}=\frac{n}{2 \ell_{\text {vib }}} \sqrt{\frac{F_{\mathrm{T}}}{\mu}} \quad f_{1}=\frac{1}{2(0.600 \mathrm{~m})} \sqrt{\frac{520 \mathrm{~N}}{\left(3.16 \times 10^{-3} \mathrm{~kg}\right) /(0.900 \mathrm{~m})}}=320.7 \mathrm{~Hz} \\
& f_{2}=2 f_{1}=641.4 \mathrm{~Hz} \quad f_{3}=3 f_{1}=962.1 \mathrm{~Hz}
\end{aligned}
$$

So the three frequencies are $320 \mathrm{~Hz}, 640 \mathrm{~Hz}, 960 \mathrm{~Hz}$, to 2 significant figures.
51. The speed of the wave is given by Eq. 15-2, $v=\sqrt{F_{\mathrm{T}} / \mu}$. The wavelength of the fundamental is $\lambda_{1}=2 \ell$. Thus the frequency of the fundamental is $f_{1}=\frac{v}{\lambda_{1}}=\frac{1}{2 \ell} \sqrt{\frac{F_{\mathrm{T}}}{\mu}}$. Each harmonic is present in a vibrating string, and so $f_{n}=n f_{1}=\frac{n}{2 \ell} \sqrt{\frac{F_{\mathrm{T}}}{\mu}}, n=1,2,3, \ldots$
52. The string must vibrate in a standing wave pattern to have a certain number of loops. The frequency of the standing waves will all be 120 Hz , the same as the vibrator. That frequency is also expressed by Eq. 15-17b, $f_{n}=\frac{n v}{2 \ell}$. The speed of waves on the string is given by Eq. $15-2, v=\sqrt{F_{\mathrm{T}} / \mu}$. The tension in the string will be the same as the weight of the masses hung from the end of the string, $F_{\mathrm{T}}=m g$, ignoring the mass of the string itself. Combining these relationships gives an expression for the masses hung from the end of the string.
(a)

$$
\begin{aligned}
& f_{n}=\frac{n v}{2 \ell}=\frac{n}{2 \ell} \sqrt{\frac{F_{\mathrm{T}}}{\mu}}=\frac{n}{2 \ell} \sqrt{\frac{m g}{\mu}} \rightarrow m=\frac{4 \ell^{2} f_{n}^{2} \mu}{n^{2} g} \\
& m_{1}=\frac{4(1.50 \mathrm{~m})^{2}(120 \mathrm{~Hz})^{2}\left(6.6 \times 10^{-4} \mathrm{~kg} / \mathrm{m}\right)}{1^{2}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=8.728 \mathrm{~kg} \approx 8.7 \mathrm{~kg}
\end{aligned}
$$

(b) $m_{2}=\frac{m_{1}}{2^{2}}=\frac{8.728 \mathrm{~kg}}{4}=2.2 \mathrm{~kg}$
(c) $m_{5}=\frac{m_{1}}{5^{2}}=\frac{8.728 \mathrm{~kg}}{25}=0.35 \mathrm{~kg}$
53. The tension in the string is the weight of the hanging mass, $F_{\mathrm{T}}=m g$. The speed of waves on the string can be found by $v=\sqrt{\frac{F_{\mathrm{T}}}{\mu}}=\sqrt{\frac{m g}{\mu}}$, and the frequency is given as $f=120 \mathrm{~Hz}$. The wavelength of waves created on the string will thus be given by

$$
\lambda=\frac{v}{f}=\frac{1}{f} \sqrt{\frac{m g}{\mu}}=\frac{1}{120 \mathrm{~Hz}} \sqrt{\frac{(0.070 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{\left(6.6 \times 10^{-4} \mathrm{~kg} / \mathrm{m}\right)}}=0.2687 \mathrm{~m} .
$$

The length of the string must be an integer multiple of half of the wavelength for there to be nodes at both ends and thus form a standing wave. Thus $\ell=\lambda / 2, \lambda, 3 \lambda / 2, \cdots n \lambda / 2$. The number of standing wave patterns is given by the number of integers that satisfy $0.10 \mathrm{~m}<n \lambda / 2<1.5 \mathrm{~m}$.

$$
0.10 \mathrm{~m}<n \lambda / 2 \rightarrow n>\frac{2(0.10 \mathrm{~m})}{\lambda}=\frac{2(0.10 \mathrm{~m})}{0.2687 \mathrm{~m}}=0.74
$$

$$
n \lambda / 2<1.5 \mathrm{~m} \rightarrow n<\frac{2(1.5 \mathrm{~m})}{\lambda}=\frac{2(1.5 \mathrm{~m})}{0.2687 \mathrm{~m}}=11.1
$$

Thus we see that we must have $n$ from 1 to 11 , and so there are 11 standing wave patterns that may be achieved.
54. The standing wave is given by $D=(2.4 \mathrm{~cm}) \sin (0.60 x) \cos (42 t)$.
(a) The distance between nodes is half of a wavelength.

$$
d=\frac{1}{2} \lambda=\frac{1}{2} \frac{2 \pi}{k}=\frac{\pi}{0.60 \mathrm{~cm}^{-1}}=5.236 \mathrm{~cm} \approx 5.2 \mathrm{~cm}
$$

(b) The component waves travel in opposite directions. Each has the same frequency and speed, and each has half the amplitude of the standing wave.

$$
\begin{aligned}
& A=\frac{1}{2}(2.4 \mathrm{~cm})=1.2 \mathrm{~cm} ; f=\frac{\omega}{2 \pi}=\frac{42 \mathrm{~s}^{-1}}{2 \pi}=6.685 \mathrm{~Hz} \approx 6.7 \mathrm{~Hz} \\
& v=\lambda f=2 d_{\text {node }} f=2(5.236 \mathrm{~cm})(6.685 \mathrm{~Hz})=70.01 \mathrm{~cm} / \mathrm{s} \approx 70 \mathrm{~cm} / \mathrm{s}(2 \text { sig. fig. })
\end{aligned}
$$

(c) The speed of a particle is given by $\frac{\partial D}{\partial t}$.

$$
\begin{aligned}
& \frac{\partial D}{\partial t}=\frac{\partial}{\partial t}[(2.4 \mathrm{~cm}) \sin (0.60 x) \cos (42 t)]=(-42 \mathrm{rad} / \mathrm{s})(2.4 \mathrm{~cm}) \sin (0.60 x) \sin (42 t) \\
& \begin{aligned}
\frac{\partial D}{\partial t}(3.20 \mathrm{~cm}, 2.5 \mathrm{~s}) & =(-42 \mathrm{rad} / \mathrm{s})(2.4 \mathrm{~cm}) \sin \left[\left(0.60 \mathrm{~cm}^{-1}\right)(3.20 \mathrm{~cm})\right] \sin [(42 \mathrm{rad} / \mathrm{s})(2.5 \mathrm{~s})] \\
= & 92 \mathrm{~cm} / \mathrm{s}
\end{aligned}
\end{aligned}
$$

55. (a) The given wave is $D_{1}=4.2 \sin (0.84 x-47 t+2.1)$. To produce a standing wave, we simply need to add a wave of the same characteristics but traveling in the opposite direction. This is the appropriate wave.

$$
D_{2}=4.2 \sin (0.84 x+47 t+2.1)
$$

(b) The standing wave is the sum of the two component waves. We use the trigonometric identity that $\sin \theta_{1}+\sin \theta_{2}=2 \sin \frac{1}{2}\left(\theta_{1}+\theta_{2}\right) \cos \frac{1}{2}\left(\theta_{1}-\theta_{2}\right)$.

$$
\begin{aligned}
D= & D_{1}+D_{2}=4.2 \sin (0.84 x-47 t+2.1)+4.2 \sin (0.84 x+47 t+2.1) \\
& =4.2(2)\left\{\sin \frac{1}{2}[(0.84 x-47 t+2.1)+(0.84 x+47 t+2.1)]\right\} \\
& \quad\left\{\cos \frac{1}{2}[(0.84 x-47 t+2.1)-(0.84 x+47 t+2.1)]\right\} \\
= & 8.4 \sin (0.84 x+2.1) \cos (-47 t)=8.4 \sin (0.84 x+2.1) \cos (47 t)
\end{aligned}
$$

We note that the origin is NOT a node.
56. From the description of the water's behavior, there is an antinode at each end of the tub, and a node in the middle. Thus one wavelength is twice the tub length.

$$
v=\lambda f=\left(2 \ell_{\text {tub }}\right) f=2(0.45 \mathrm{~m})(0.85 \mathrm{~Hz})=0.77 \mathrm{~m} / \mathrm{s}
$$

57. The frequency is given by $f=\frac{v}{\lambda}=\frac{1}{\lambda} \sqrt{\frac{F}{\mu}}$. The wavelength and the mass density do not change when the string is tightened.

$$
f=\frac{v}{\lambda}=\frac{1}{\lambda} \sqrt{\frac{F}{\mu}} \rightarrow \frac{f_{2}}{f_{1}}=\frac{\frac{1}{\lambda} \sqrt{\frac{F_{2}}{\mu}}}{\frac{1}{\lambda} \sqrt{\frac{F_{1}}{\mu}}}=\sqrt{\frac{F_{2}}{F_{1}}} \rightarrow f_{2}=f_{1} \sqrt{\frac{F_{2}}{F_{1}}}=(294 \mathrm{~Hz}) \sqrt{1.15}=315 \mathrm{~Hz}
$$

58. (a) Plotting one full wavelength means from $x=0$ to
$x=\lambda=\frac{2 \pi}{k}=\frac{2 \pi}{3.5 \mathrm{~m}^{-1}}=1.795 \mathrm{~m}$
$\approx 1.8 \mathrm{~m}$. The functions to be plotted are given below. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on
 tab "Problem 15.58."

$$
D_{1}=(0.15 \mathrm{~m}) \sin \left[\left(3.5 \mathrm{~m}^{-1}\right) x-1.8\right] \text { and } D_{2}=(0.15 \mathrm{~m}) \sin \left[\left(3.5 \mathrm{~m}^{-1}\right) x+1.8\right]
$$

(b) The sum $D_{1}+D_{2}$ is plotted, and the nodes and antinodes are indicated. The analytic result is given below. The spreadsheet used for this problem can be found on the Media Manager, with filename
"PSE4_ISM_CH15.XLS," on tab "Problem 15.58."


$$
\begin{aligned}
D_{1}+D_{2} & =(0.15 \mathrm{~m}) \sin \left[\left(3.5 \mathrm{~m}^{-1}\right) x-1.8\right]+(0.15 \mathrm{~m}) \sin \left[\left(3.5 \mathrm{~m}^{-1}\right) x-1.8\right] \\
& =(0.30 \mathrm{~m}) \sin \left(3.5 \mathrm{~m}^{-1} x\right) \cos (1.8)
\end{aligned}
$$

This expression should have nodes and antinodes at positions given by the following.

$$
\begin{aligned}
& 3.5 \mathrm{~m}^{-1} x_{\text {node }}=n \pi, n=0,1,2 \ldots \rightarrow x=\frac{n \pi}{3.5}=0,0.90 \mathrm{~m}, 1.80 \mathrm{~m} \\
& 3.5 \mathrm{~m}^{-1} x_{\text {antinode }}=\left(n+\frac{1}{2}\right) \pi, n=0,1,2 \ldots \rightarrow x=\frac{\left(n+\frac{1}{2}\right) \pi}{3.5}=0.45 \mathrm{~m}, 1.35 \mathrm{~m}
\end{aligned}
$$

The graph agrees with the calculations.
59. The standing wave formed from the two individual waves is given below. The period is given by $T=2 \pi / \omega=2 \pi / 1.8 \mathrm{~s}^{-1}=3.5 \mathrm{~s}$.

$$
\begin{aligned}
D_{1}+D_{2} & =(0.15 \mathrm{~m}) \sin \left[\left(3.5 \mathrm{~m}^{-1}\right) x-\left(1.8 \mathrm{~s}^{-1}\right) t\right]+(0.15 \mathrm{~m}) \sin \left[\left(3.5 \mathrm{~m}^{-1}\right) x-\left(1.8 \mathrm{~s}^{-1}\right) t\right] \\
& =(0.30 \mathrm{~m}) \sin \left(3.5 \mathrm{~m}^{-1} x\right) \cos \left(1.8 \mathrm{~s}^{-1} t\right)
\end{aligned}
$$

(a) For the point $x=$ 0 , we see that the sum of the two waves is identically 0 . This means that the point $x=0$ is a node of the standing wave. The spreadsheet used for this problem can be found on the


Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.59."
(b) For the point $x=\lambda / 4$, we see that the amplitude of that point is twice the amplitude of either wave. Thus this point is an antinode of the standing wave. The spreadsheet used
 for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.59."
60. (a) The maximum swing is twice the amplitude of the standing wave. Three loops is 1.5 wavelengths, and the frequency is given.

$$
\begin{aligned}
& A=\frac{1}{2}(8.00 \mathrm{~cm})=4.00 \mathrm{~cm} ; \omega=2 \pi f=2 \pi(120 \mathrm{~Hz})=750 \mathrm{rad} / \mathrm{s} \\
& k=\frac{2 \pi}{\lambda} \rightarrow \quad ; \frac{3}{2} \lambda=1.64 \mathrm{~m} \rightarrow \lambda=1.09 \mathrm{~m} ; k=\frac{2 \pi}{1.09 \mathrm{~m}}=5.75 \mathrm{~m}^{-1} \\
& D=A \sin (k x) \cos (\omega t)=(4.00 \mathrm{~cm}) \sin \left[\left(5.75 \mathrm{~m}^{-1}\right) x\right] \cos [(750 \mathrm{rad} / \mathrm{s}) t]
\end{aligned}
$$

(b) Each component wave has the same wavelength, the same frequency, and half the amplitude of the standing wave.

$$
\begin{aligned}
& D_{1}=(2.00 \mathrm{~cm}) \sin \left[\left(5.75 \mathrm{~m}^{-1}\right) x-(750 \mathrm{rad} / \mathrm{s}) t\right] \\
& D_{2}=(2.00 \mathrm{~cm}) \sin \left[\left(5.75 \mathrm{~m}^{-1}\right) x+(750 \mathrm{rad} / \mathrm{s}) t\right]
\end{aligned}
$$

61. Any harmonic with a node directly above the pickup will NOT be "picked up" by the pickup. The pickup location is exactly $1 / 4$ of the string length from the end of the string, so a standing wave with a frequency corresponding to 4 (or 8 or 12 etc.) loops will not excite the pickup. So $n=4,8$, and 12 will not excite the pickup.
62. The gap between resonant frequencies is the fundamental frequency (which is thus 300 Hz for this problem), and the wavelength of the fundamental is twice the string length.

$$
v=\lambda f=(2 \ell)\left(f_{n+1}-f_{n}\right)=2(0.65 \mathrm{~m})(300 \mathrm{~Hz})=390 \mathrm{~m} / \mathrm{s}
$$

63. The standing wave is the sum of the two individual standing waves. We use the trigonometric identities for the cosine of a difference and a sum.

$$
\begin{aligned}
\cos \left(\theta_{1}-\theta_{2}\right) & =\cos \theta_{1} \cos \theta_{2}+\sin \theta_{1} \sin \theta_{2} ; \cos \left(\theta_{1}+\theta_{2}\right)=\cos \theta_{1} \cos \theta_{2}-\sin \theta_{1} \sin \theta_{2} \\
D & =D_{1}+D_{2}=A \cos (k x-\omega t)+A \cos (k x+\omega t)=A[\cos (k x-\omega t)+\cos (k x+\omega t)] \\
& =A[\cos k x \cos \omega t+\sin k x \sin \omega t+\cos k x \cos \omega t-\sin k x \sin \omega t] \\
& =2 A \cos k x \cos \omega t
\end{aligned}
$$

Thus the standing wave is $D=2 A \cos k x \cos \omega t$. The nodes occur where the position term forces
$D=2 A \cos k x \cos \omega t=0$ for all time. Thus $\cos k x=0 \rightarrow k x= \pm(2 n+1) \frac{\pi}{2}, n=0,1,2, \cdots$. Thus,
since $k=2.0 \mathrm{~m}^{-1}$, we have $x= \pm\left(n+\frac{1}{2}\right) \frac{\pi}{2} \mathrm{~m}, n=0,1,2, \cdots$.
64. The frequency for each string must be the same, to ensure continuity of the string at its junction. Each string will obey these relationships: $\lambda f=v, v=\sqrt{\frac{F_{\mathrm{T}}}{\mu}}, \lambda=\frac{2 \ell}{n}$. Combine these to find the nodes. Note that $n$ is the number of "loops" in the string segment, and that $n$ loops requires $n+1$ nodes.

$$
\begin{aligned}
& \lambda f=v, v=\sqrt{\frac{F_{\mathrm{T}}}{\mu}}, \lambda=\frac{2 \ell}{n} \rightarrow \frac{2 \ell}{n} f=\sqrt{\frac{F_{\mathrm{T}}}{\mu}} \rightarrow f=\frac{n}{2 \ell} \sqrt{\frac{F_{\mathrm{T}}}{\mu}} \\
& \frac{n_{\mathrm{Al}}}{2 \ell_{\mathrm{Al}}} \sqrt{\frac{F_{\mathrm{T}}}{\mu_{\mathrm{Al}}}}=\frac{n_{\mathrm{Fe}}}{2 \ell_{\mathrm{Fe}}} \sqrt{\frac{F_{\mathrm{T}}}{\mu_{\mathrm{Fe}}}} \rightarrow \frac{n_{\mathrm{Al}}}{n_{\mathrm{Fe}}}=\frac{\ell_{\mathrm{Al}}}{\ell_{\mathrm{Fe}}} \sqrt{\frac{\mu_{\mathrm{Al}}}{\mu_{\mathrm{Fe}}}}=\frac{0.600 \mathrm{~m}}{0.882 \mathrm{~m}} \sqrt{\frac{2.70 \mathrm{~g} / \mathrm{m}}{7.80 \mathrm{~g} / \mathrm{m}}}=0.400=\frac{2}{5}
\end{aligned}
$$

Thus there are 3 nodes on the aluminum, since $n_{\mathrm{Al}}=2$, and 6 nodes on the steel, since $n_{\mathrm{Fe}}=5$, but one node is shared so there are 8 total nodes. Use the formula derived above to find the lower frequency.

$$
f=\frac{n_{\mathrm{Al}}}{2 \ell_{\mathrm{Al}}} \sqrt{\frac{F_{\mathrm{Al}}}{\mu_{\mathrm{Al}}}}=\frac{2}{2(0.600 \mathrm{~m})} \sqrt{\frac{135 \mathrm{~N}}{2.70 \times 10^{-3} \mathrm{~kg} / \mathrm{m}}}=373 \mathrm{~Hz}
$$

65. The speed in the second medium can be found from the law of refraction, Eq. 15-19.

$$
\frac{\sin \theta_{2}}{\sin \theta_{1}}=\frac{v_{2}}{v_{1}} \rightarrow v_{2}=v_{1} \frac{\sin \theta_{2}}{\sin \theta_{1}}=(8.0 \mathrm{~km} / \mathrm{s})\left(\frac{\sin 31^{\circ}}{\sin 52^{\circ}}\right)=5.2 \mathrm{~km} / \mathrm{s}
$$

66. The angle of refraction can be found from the law of refraction, Eq. 15-19.

$$
\frac{\sin \theta_{2}}{\sin \theta_{1}}=\frac{v_{2}}{v_{1}} \rightarrow \sin \theta_{2}=\sin \theta_{1} \frac{v_{2}}{v_{1}}=\sin 35^{\circ} \frac{2.5 \mathrm{~m} / \mathrm{s}}{2.8 \mathrm{~m} / \mathrm{s}}=0.512 \rightarrow \theta_{2}=\sin ^{-1} 0.419=31^{\circ}
$$

67. The angle of refraction can be found from the law of refraction, Eq. 15-19. The relative velocities can be found from the relationship given in the problem.

$$
\begin{aligned}
& \frac{\sin \theta_{2}}{\sin \theta_{1}}=\frac{v_{2}}{v_{1}}=\frac{331+0.60 T_{2}}{331+0.60 T_{1}} \rightarrow \sin \theta_{2}=\sin 33^{\circ} \frac{331+0.60(-15)}{331+0.60(25)}=\sin 33^{\circ} \frac{322}{346}=0.5069 \\
& \theta_{2}=\sin ^{-1} 0.5069=30^{\circ}(2 \text { sig. fig. })
\end{aligned}
$$

68. (a) Eq. 15-19 gives the relationship between the angles and the speed of sound in the two media. For total internal reflection (for no sound to enter the water), $\theta_{\text {water }}=90^{\circ}$ or $\sin \theta_{\text {water }}=1$. The air is the "incident" media. Thus the incident angle is given by the following.

$$
\frac{\sin \theta_{\text {air }}}{\sin \theta_{\text {water }}}=\frac{v_{\text {air }}}{v_{\text {water }}} ; \theta_{\text {air }}=\theta_{\mathrm{i}}=\sin ^{-1}\left[\sin \theta_{\text {water }} \frac{v_{\text {air }}}{v_{\text {water }}}\right] \rightarrow \theta_{\mathrm{iM}}=\sin ^{-1}\left[\frac{v_{\text {air }}}{v_{\text {water }}}\right]=\sin ^{-1}\left[\frac{v_{\mathrm{i}}}{v_{\mathrm{r}}}\right]
$$

(b) From the angle of incidence, the distance is found. See the diagram.

$$
\begin{aligned}
& \theta_{\text {air } \mathrm{M}}=\sin ^{-1} \frac{v_{\text {air }}}{v_{\text {water }}}=\sin ^{-1} \frac{343 \mathrm{~m} / \mathrm{s}}{1440 \mathrm{~m} / \mathrm{s}}=13.8^{\circ} \\
& \tan \theta_{\text {air } \mathrm{M}}=\frac{x}{1.8 \mathrm{~m}} \rightarrow x=(1.8 \mathrm{~m}) \tan 13.8^{\circ}=0.44 \mathrm{~m}
\end{aligned}
$$


69. The angle of refraction can be found from the law of refraction, Eq. 15-19. The relative velocities can be found from Eq. 15-3.

$$
\begin{aligned}
& \frac{\sin \theta_{2}}{\sin \theta_{1}}=\frac{v_{2}}{v_{1}}=\frac{\sqrt{E / \rho_{2}}}{\sqrt{E / \rho_{1}}}=\sqrt{\frac{\rho_{1}}{\rho_{2}}}=\sqrt{\frac{S G_{1} \rho_{\text {water }}}{S G_{2} \rho_{\text {water }}}}=\sqrt{\frac{S G_{1}}{S G_{2}}} \\
& \sin \theta_{2}=\sin \theta_{1} \sqrt{\frac{S G_{1}}{S G_{2}}}=\sin 38^{\circ} \sqrt{\frac{3.6}{2.8}}=0.70 \rightarrow \theta_{2}=\sin ^{-1} 0.70=44^{\circ}
\end{aligned}
$$

70. The error of $2^{\circ}$ is allowed due to diffraction of the waves. If the waves are incident at the "edge" of the dish, they can still diffract into the dish if the relationship $\theta \approx \lambda / \ell$ is satisfied.

$$
\theta \approx \frac{\lambda}{\ell} \rightarrow \lambda=\ell \theta=(0.5 \mathrm{~m})\left(2^{\circ} \times \frac{\pi \mathrm{rad}}{180^{\circ}}\right)=1.745 \times 10^{-2} \mathrm{~m} \approx 2 \times 10^{-2} \mathrm{~m}
$$

If the wavelength is longer than that, there will not be much diffraction, but "shadowing" instead.
71. The frequency is 880 Hz and the phase velocity is $440 \mathrm{~m} / \mathrm{s}$, so the wavelength is
$\lambda=\frac{v}{f}=\frac{440 \mathrm{~m} / \mathrm{s}}{880 \mathrm{~Hz}}=0.50 \mathrm{~m}$.
(a) Use the ratio of distance to wavelength to define the phase difference.

$$
\frac{x}{\lambda}=\frac{\pi / 6}{2 \pi} \rightarrow x=\frac{\lambda}{12}=\frac{0.50 \mathrm{~m}}{12}=0.042 \mathrm{~m}
$$

(b) Use the ratio of time to period to define the phase difference.
$\frac{t}{T}=\frac{\phi}{2 \pi} \rightarrow \phi=\frac{2 \pi t}{T}=2 \pi t f=2 \pi\left(1.0 \times 10^{-4} \mathrm{~s}\right)(880 \mathrm{~Hz})=0.55 \mathrm{rad}$
72. The frequency at which the water is being shaken is about 1 Hz . The sloshing coffee is in a standing wave mode, with antinodes at each edge of the cup. The cup diameter is thus a half-wavelength, or $\lambda=16 \mathrm{~cm}$. The wave speed can be calculated from the frequency and the wavelength.

$$
v=\lambda f=(16 \mathrm{~cm})(1 \mathrm{~Hz})=16 \mathrm{~cm} / \mathrm{s}
$$

73. The speed of a longitudinal wave in a solid is given by Eq. $15-3, v=\sqrt{E / \rho}$. Let the density of the less dense material be $\rho_{1}$, and the density of the more dense material be $\rho_{2}$. The less dense material will have the higher speed, since the speed is inversely proportional to the square root of the density.

$$
\frac{v_{1}}{v_{2}}=\frac{\sqrt{E / \rho_{1}}}{\sqrt{E / \rho_{2}}}=\sqrt{\frac{\rho_{2}}{\rho_{1}}}=\sqrt{2.5} \approx 1.6
$$

74. From Eq. 15-7, if the speed, medium density, and frequency of the two waves are the same, then the intensity is proportional to the square of the amplitude.

$$
I_{2} / I_{1}=P_{2} / P_{1}=A_{2}^{2} / A_{1}^{2}=2.5 \rightarrow A_{2} / A_{1}=\sqrt{2.5}=1.6
$$

The more energetic wave has the larger amplitude.
75. (a) The amplitude is half the peak-to-peak distance, so 0.05 m .
(b) The maximum kinetic energy of a particle in simple harmonic motion is the total energy, which is given by $E_{\text {total }}=\frac{1}{2} k A^{2}$.

Compare the two kinetic energy maxima.

$$
\frac{K_{2 \max }}{K_{1 \max }}=\frac{\frac{1}{2} k A_{2}^{2}}{\frac{1}{2} k A_{1}^{2}}=\left(\frac{A_{2}}{A_{1}}\right)^{2}=\left(\frac{0.075 \mathrm{~m}}{0.05 \mathrm{~m}}\right)^{2}=2.25
$$

76. From Eq. $15-17 \mathrm{~b}, f_{n}=\frac{n v}{2 L}$, we see that the frequency is proportional to the wave speed on the stretched string. From Eq. $15-2, v=\sqrt{F_{\mathrm{T}} / \mu}$, we see that the wave speed is proportional to the square root of the tension. Thus the frequency is proportional to the square root of the tension.

$$
\sqrt{\frac{F_{\mathrm{T} 2}}{F_{\mathrm{T} 1}}}=\frac{f_{2}}{f_{1}} \rightarrow F_{\mathrm{T} 2}=\left(\frac{f_{2}}{f_{1}}\right)^{2} F_{\mathrm{T} 1}=\left(\frac{247 \mathrm{~Hz}}{255 \mathrm{~Hz}}\right)^{2} F_{\mathrm{T} 1}=0.938 F_{\mathrm{T} 1}
$$

Thus the tension should be decreased by $6.2 \%$.
77. We assume that the earthquake wave is moving the ground vertically, since it is a transverse wave. An object sitting on the ground will then be moving with SHM, due to the two forces on it - the normal force upwards from the ground and the weight downwards due to gravity. If the object loses contact with the ground, then the normal force will be zero, and the only force on the object will be its weight. If the only force is the weight, then the object will have an acceleration of $g$ downwards. Thus the limiting condition for beginning to lose contact with the ground is when the maximum acceleration caused by the wave is greater than $g$. Any larger downward acceleration and the ground would "fall" quicker than the object. The maximum acceleration is related to the amplitude and the frequency as follows.

$$
a_{\max }=\omega^{2} A>g \rightarrow A>\frac{g}{\omega^{2}}=\frac{g}{4 \pi^{2} f^{2}}=\frac{9.80 \mathrm{~m} / \mathrm{s}^{2}}{4 \pi^{2}(0.60 \mathrm{~Hz})^{2}}=0.69 \mathrm{~m}
$$

78. (a) The speed of the wave at a point $h$ above the lower end depends on the tension at that point and the linear mass density of the cord. The tension must equal the mass of the lower segment if the lower segment is in equilibrium. Use Eq. 15-2 for the wave speed.

$$
F_{\mathrm{T}}=m_{\text {segment }} g=\frac{h}{\ell} m g ; v=\sqrt{\frac{F_{\mathrm{T}}}{\mu}}=\sqrt{\frac{\frac{h}{\ell} m g}{\frac{\ell}{\ell}}}=\sqrt{h g}
$$

(b) We treat $h$ as a variable, measured from the bottom of the cord. The wave speed at that point is given above as $v=\sqrt{h g}$. The distance a wave would travel up the cord during a time $d t$ is then $d h=v d t=\sqrt{h g} d t$. To find the total time for a wave to travel up the cord, integrate over the length of the cord.

$$
\begin{aligned}
& d h=v d t=\sqrt{h g} d t \rightarrow d t=\frac{d h}{\sqrt{h g}} \rightarrow \int_{0}^{t_{\text {toal }}} d t=\int_{0}^{L} \frac{d h}{\sqrt{h g}} \rightarrow \\
& t_{\text {total }}=\int_{0}^{L} \frac{d h}{\sqrt{h g}}=\left.2 \sqrt{\frac{h}{g}}\right|_{0} ^{L}=2 \sqrt{\frac{L}{g}}
\end{aligned}
$$

79. (a) The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.79."

(b) The wave function is found by replacing $x$ in the pulse by $x-v t$.

$$
D(x, t)=\frac{4.0 \mathrm{~m}^{3}}{[x-(2.4 \mathrm{~m} / \mathrm{s}) t]^{2}+2.0 \mathrm{~m}^{2}}
$$

(c) The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.79."

(d) The wave function is found by replacing $x$ in the pulse by $x+v t$. The spreadsheet used for this problem can be found on the Media Manager, with filename
"PSE4_ISM_CH15.XLS," on tab "Problem 15.79."
$D=\frac{4.0 \mathrm{~m}^{3}}{[x+(2.4 \mathrm{~m} / \mathrm{s}) t]^{2}+2.0 \mathrm{~m}^{2}}$

80. (a) The frequency is related to the tension by Eqs. 15-1 and 15-2.

$$
\begin{gathered}
f=\frac{v}{\lambda}=\frac{1}{\lambda} \sqrt{\frac{F_{\mathrm{T}}}{\mu}} \rightarrow \frac{d f}{d F_{\mathrm{T}}}=\frac{1}{2 \lambda} \sqrt{\frac{1}{\mu F_{\mathrm{T}}}}=\frac{1}{2 F_{\mathrm{T}}} \frac{1}{\lambda} \sqrt{\frac{F_{\mathrm{T}}}{\mu}}=\frac{f}{2 F_{\mathrm{T}}} \\
\frac{\Delta f}{\Delta F_{\mathrm{T}}} \approx \frac{f}{2 F_{\mathrm{T}}} \rightarrow \Delta f \approx \frac{\Delta}{2}\left(\frac{\Delta F_{\mathrm{T}}}{F_{\mathrm{T}}}\right) f \\
\text { (b) } \frac{\Delta f}{\Delta F_{\mathrm{T}}} \approx \frac{f}{2 F_{\mathrm{T}}} \rightarrow \frac{\Delta F_{\mathrm{T}}}{F_{\mathrm{T}}} \approx 2 \frac{\Delta f}{f}=2\left(\frac{6}{436}\right)=0.0275=3 \%
\end{gathered}
$$

(c) The only change in the expression $\frac{1}{\lambda} \sqrt{\frac{F_{\mathrm{T}}}{\mu}}$ as the overtone changes is the wavelength, and the wavelength does not influence the final result. So yes, the formula still applies.
81. (a) The overtones are given by $f_{n}=n f_{1}, n=2,3,4 \ldots$

$$
\begin{array}{lll}
\mathrm{G}: & f_{2}=2(392 \mathrm{~Hz})=784 \mathrm{~Hz} & f_{3}=3(392 \mathrm{~Hz})=1176 \mathrm{~Hz} \approx 1180 \mathrm{~Hz} \\
\mathrm{~B}: & f_{2}=2(494 \mathrm{~Hz})=988 \mathrm{~Hz} & f_{3}=3(440 \mathrm{~Hz})=1482 \mathrm{~Hz} \approx 1480 \mathrm{~Hz}
\end{array}
$$

(b) If the two strings have the same length, they have the same wavelength. The frequency difference is then due to a difference in wave speed caused by different masses for the strings.

$$
\frac{f_{\mathrm{G}}}{f_{\mathrm{A}}}=\frac{v_{\mathrm{G}} / \lambda}{v_{\mathrm{A}} / \lambda}=\frac{v_{\mathrm{G}}}{v_{\mathrm{A}}}=\frac{\sqrt{\frac{F_{\mathrm{T}}}{m_{\mathrm{G}} / \ell}}}{\sqrt{\frac{F_{\mathrm{T}}}{m_{\mathrm{A}} / \ell}}}=\sqrt{\frac{m_{\mathrm{A}}}{m_{\mathrm{G}}}} \rightarrow \frac{m_{\mathrm{G}}}{m_{\mathrm{A}}}=\left(\frac{f_{\mathrm{A}}}{f_{\mathrm{G}}}\right)^{2}=\left(\frac{494}{392}\right)^{2}=1.59
$$

(c) If the two strings have the same mass per unit length and the same tension, then the wave speed on both strings is the same. The frequency difference is then due to a difference in wavelength. For the fundamental, the wavelength is twice the length of the string.

$$
\frac{f_{\mathrm{G}}}{f_{\mathrm{B}}}=\frac{v / \lambda_{\mathrm{G}}}{v / \lambda_{\mathrm{B}}}=\frac{\lambda_{\mathrm{B}}}{\lambda_{\mathrm{G}}}=\frac{2 \ell_{\mathrm{B}}}{2 \ell_{\mathrm{G}}} \rightarrow \frac{\ell_{\mathrm{G}}}{\ell_{\mathrm{B}}}=\frac{f_{\mathrm{B}}}{f_{\mathrm{G}}}=\frac{494}{392}=1.26
$$

(d) If the two strings have the same length, they have the same wavelength. The frequency difference is then due to a difference in wave speed caused by different tensions for the strings.

$$
\frac{f_{\mathrm{B}}}{f_{\mathrm{A}}}=\frac{v_{\mathrm{B}} / \lambda}{v_{\mathrm{A}} / \lambda}=\frac{v_{\mathrm{B}}}{v_{\mathrm{A}}}=\frac{\sqrt{\frac{F_{\mathrm{TB}}}{m / L}}}{\sqrt{\frac{F_{\mathrm{TA}}}{m / L}}}=\sqrt{\frac{F_{\mathrm{TB}}}{F_{\mathrm{TA}}}} \rightarrow \frac{F_{\mathrm{TB}}}{F_{\mathrm{TA}}}=\left(\frac{f_{\mathrm{B}}}{f_{\mathrm{A}}}\right)^{2}=\left(\frac{392}{494}\right)^{2}=0.630
$$

82. Relative to the fixed needle position, the ripples are moving with a linear velocity given by

$$
v=\left(33 \frac{\mathrm{rev}}{\mathrm{~min}}\right)\left(\frac{1 \mathrm{~min}}{60 \mathrm{~s}}\right)\left(\frac{2 \pi(0.108 \mathrm{~m})}{1 \mathrm{rev}}\right)=0.3732 \mathrm{~m} / \mathrm{s}
$$

This speed is the speed of the ripple waves moving past the needle. The frequency of the waves is

$$
f=\frac{v}{\lambda}=\frac{0.3732 \mathrm{~m} / \mathrm{s}}{1.55 \times 10^{-3} \mathrm{~m}}=240.77 \mathrm{~Hz} \approx 240 \mathrm{~Hz}
$$

83. The speed of the pulses is found from the tension and mass per unit length of the wire.

$$
v=\sqrt{\frac{F_{\mathrm{T}}}{\mu}}=\sqrt{\frac{255 \mathrm{~N}}{0.152 \mathrm{~kg} / 10.0 \mathrm{~m}}}=129.52 \mathrm{~m} / \mathrm{s}
$$

The total distance traveled by the two pulses will be the length of the wire. The second pulse has a shorter time of travel than the first pulse, by 20.0 ms .

$$
\begin{aligned}
& \ell=d_{1}+d_{2}=v t_{1}+v t_{2}=v t_{1}+v\left(t_{1}-2.00 \times 10^{-2}\right) \\
& t_{1}=\frac{\ell+2.00 \times 10^{-2} v}{2 v}=\frac{(10.0 \mathrm{~m})+2.00 \times 10^{-2}(129.52 \mathrm{~m} / \mathrm{s})}{2(129.52 \mathrm{~m} / \mathrm{s})}=4.8604 \times 10^{-2} \mathrm{~s} \\
& d_{1}=v t_{1}=(129.52 \mathrm{~m} / \mathrm{s})\left(4.8604 \times 10^{-2} \mathrm{~s}\right)=6.30 \mathrm{~m}
\end{aligned}
$$

The two pulses meet 6.30 m from the end where the first pulse originated.
84. We take the wave function to be $D(x, t)=A \sin (k x-\omega t)$. The wave speed is given by $v=\frac{\omega}{k}=\frac{\lambda}{f}$, while the speed of particles on the cord is given by $\frac{\partial D}{\partial t}$.

$$
\begin{aligned}
& \frac{\partial D}{\partial t}=-\omega A \cos (k x-\omega t) \rightarrow\left(\frac{\partial D}{\partial t}\right)_{\max }=\omega A \\
& \omega A=v=\frac{\omega}{k} \rightarrow A=\frac{1}{k}=\frac{\lambda}{2 \pi}=\frac{10.0 \mathrm{~cm}}{2 \pi}=1.59 \mathrm{~cm}
\end{aligned}
$$

For a resonant condition, the free end of the string will be an antinode, and the fixed end of the string will be a node. The
 minimum distance
from a node to an antinode is $\lambda / 4$. Other wave patterns that fit the boundary conditions of a node at
one end and an antinode at the other end include $3 \lambda / 4,5 \lambda / 4, \ldots$. See the diagrams. The general relationship is $\ell=(2 n-1) \lambda / 4, n=1,2,3, \cdots$. Solving for the wavelength gives
$\lambda=\frac{4 \ell}{2 n-1}, n=1,2,3, \cdots$.
86. The addition of the support will force the bridge to have its lowest mode of oscillation to have a node at the center of the span, which would be the first overtone of the fundamental frequency. If the wave speed in the bridge material remains constant, then the resonant frequency will double, to 6.0 Hz . Since earthquakes don't do significant shaking at that frequency, the modifications would be effective at keeping the bridge from having large oscillations during an earthquake.
87. From the figure, we can see that the amplitude is 3.5 cm , and the wavelength is 20 cm . The maximum of the wave at $x=0$ has moved to $x=12 \mathrm{~cm}$ at $t=0.80 \mathrm{~s}$, which is used to find the velocity. The wave is moving to the right. Finally, since the displacement is a maximum at $x=0$ and $t=0$, we can use a cosine function without a phase angle.

$$
\begin{aligned}
& A=3.5 \mathrm{~cm} ; \lambda=20 \mathrm{~cm} \rightarrow k=\frac{2 \pi}{\lambda}=0.10 \pi \mathrm{~cm}^{-1} ; v=\frac{12 \mathrm{~cm}}{0.80 \mathrm{~s}}=15 \mathrm{~cm} / \mathrm{s} ; \omega=v k=1.5 \pi \mathrm{rad} / \mathrm{s} \\
& D(x, t)=A \cos (k x-\omega t)=(3.5 \mathrm{~cm}) \cos (0.10 \pi x-1.5 \pi t), x \text { in } \mathrm{cm}, t \text { in s }
\end{aligned}
$$

88. From the given data, $A=0.50 \mathrm{~m}$ and $v=2.5 \mathrm{~m} / 4.0 \mathrm{~s}=0.625 \mathrm{~m} / \mathrm{s}$. We use Eq. $15-6$ for the average power, with the density of sea water from Table 13-1. We estimate the area of the chest as $(0.30 \mathrm{~m})^{2}$. Answers may vary according to the approximation used for the area of the chest.

$$
\begin{aligned}
\bar{P} & =2 \pi^{2} \rho S v f^{2} A^{2}=2 \pi^{2}\left(1025 \mathrm{~kg} / \mathrm{m}^{3}\right)(0.30 \mathrm{~m})^{2}(0.625 \mathrm{~m} / \mathrm{s})(0.25 \mathrm{~Hz})^{2}(0.50 \mathrm{~m})^{2} \\
& =18 \mathrm{~W}
\end{aligned}
$$

89. The unusual decrease of water corresponds to a trough in Figure 15-4. The crest or peak of the wave is then one-half wavelength from the shore. The peak is 107.5 km away, traveling at $550 \mathrm{~km} / \mathrm{hr}$.

$$
\Delta x=v t \rightarrow t=\frac{\Delta x}{v}=\frac{\frac{1}{2}(215 \mathrm{~km})}{550 \mathrm{~km} / \mathrm{hr}}\left(\frac{60 \mathrm{~min}}{1 \mathrm{hr}}\right)=11.7 \mathrm{~min} \approx 12 \mathrm{~min}
$$

90. At $t=1.0 \mathrm{~s}$, the leading edge of each wave is 1.0 cm from the other wave. They have not yet interfered. The leading edge of the wider wave is at 22 cm , and the leading edge of the narrower wave is at 23 cm .


At $t=2.0 \mathrm{~s}$, the waves are overlapping. The diagram uses dashed lines to show the parts of the original waves that are undergoing interference.


At $t=3.0 \mathrm{~s}$, the waves have "passed through" each other, and are no longer interfering.

91. Because the radiation is uniform, the same energy must pass through every spherical surface, which has the surface area $4 \pi r^{2}$. Thus the intensity must decrease as $1 / r^{2}$. Since the intensity is proportional to the square of the amplitude, the amplitude will decrease as $1 / r$. The radial motion will be sinusoidal, and so we have $D=\left(\frac{A}{r}\right) \sin (k r-\omega t)$.
92. The wavelength is to be 1.0 m . Use Eq. 15-1.

$$
v=f \lambda \rightarrow f=\frac{v}{\lambda}=\frac{344 \mathrm{~m} / \mathrm{s}}{1.0 \mathrm{~m}}=340 \mathrm{~Hz}
$$

There will be significant diffraction only for wavelengths larger than the width of the window, and so waves with frequencies lower than 340 Hz would diffract when passing through this window.
93. The value of $k$ was taken to be $1.0 \mathrm{~m}^{-1}$ for this problem. The peak of the wave moves to the right by 0.50 m during each second that elapses. This can be seen qualitatively from the graph, and quantitatively from the spreadsheet data. Thus the wave speed is given by the constant $c, 0.50 \mathrm{~m} / \mathrm{s}$. The direction of motion is in the positive $x$ direction. The wavelength is seen to be $\lambda=\pi \mathrm{m}$. Note that this doesn't agree with the relationship $\lambda=\frac{2 \pi}{k}$. The period of the function $\sin ^{2} \theta$ is $\pi$, not $2 \pi$ as is the case for $\sin \theta$. In a similar fashion the period of this function is $T=2 \pi \mathrm{~s}$. Note that this
doesn't agree with the relationship $k v=\omega=\frac{2 \pi}{T}$, again because of the behavior of the $\sin ^{2} \theta$ function. But the relationship $\frac{\lambda}{T}=v$ is still true for this wave function. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.93."


Further insight is gained by re-writing the function using the trigonometric identity $\sin ^{2} \theta=\frac{1}{2}-\frac{1}{2} \cos 2 \theta$, because function $\cos 2 \theta$ has a period of $\pi$.
94. (a) The graph shows the wave moving 3.0 m to the right each second, which is the expected amount since the speed of the wave is $3.0 \mathrm{~m} / \mathrm{s}$ and the form of the wave function says the wave is moving to the right. The spreadsheet used for this problem can be found on the Media Manager, with filename
"PSE4_ISM_CH15.XLS," on tab "Problem 15.94a."

(b) The graph shows the wave moving 3.0 m to the left each second, which is the expected amount since the speed of the wave is $3.0 \mathrm{~m} / \mathrm{s}$ and the form of the wave function says the wave is moving to the left. The spreadsheet used for this problem can be found on the Media Manager, with filename
"PSE4_ISM_CH15.XLS," on tab "Problem 15.94b."


## CHAPTER 16: Sound

## Responses to Questions

1. Sound exhibits diffraction, refraction, and interference effects that are characteristic of waves. Sound also requires a medium, a characteristic of mechanical waves.
2. Sound can cause objects to vibrate, which is evidence that sound is a form of energy. In extreme cases, sound waves can even break objects. (See Figure 14-24 showing a goblet shattering from the sound of a trumpet.)
3. Sound waves generated in the first cup cause the bottom of the cup to vibrate. These vibrations excite vibrations in the stretched string which are transmitted down the string to the second cup, where they cause the bottom of the second cup to vibrate, generating sound waves which are heard by the second child.
4. The wavelength will change. The frequency cannot change at the boundary since the media on both sides of the boundary are oscillating together. If the frequency were to somehow change, there would be a "pile-up" of wave crests on one side of the boundary.
5. If the speed of sound in air depended significantly on frequency, then the sounds that we hear would be separated in time according to frequency. For example, if a chord were played by an orchestra, we would hear the high notes at one time, the middle notes at another, and the lower notes at still another. This effect is not heard for a large range of distances, indicating that the speed of sound in air does not depend significantly on frequency.
6. Helium is much less dense than air, so the speed of sound in the helium is higher than in air. The wavelength of the sound produced does not change, because it is determined by the length of the vocal cords and other properties of the resonating cavity. The frequency therefore increases, increasing the pitch of the voice.
7. The speed of sound in a medium is equal to $v=\sqrt{B / \rho}$, where $B$ is the bulk modulus and $\rho$ is the density of the medium. The bulk moduli of air and hydrogen are very nearly the same. The density of hydrogen is less than the density of air. The reduced density is the main reason why sound travels faster in hydrogen than in air.
8. The intensity of a sound wave is proportional to the square of the frequency, so the higher-frequency tuning fork will produce more intense sound.
9. Variations in temperature will cause changes in the speed of sound and in the length of the pipes. As the temperature rises, the speed of sound in air increases, increasing the resonance frequency of the pipes, and raising the pitch of the sound. But the pipes get slightly longer, increasing the resonance wavelength and decreasing the resonance frequency of the pipes and lowering the pitch. As the temperature decreases, the speed of sound decreases, decreasing the resonance frequency of the pipes, and lowering the pitch of the sound. But the pipes contract, decreasing the resonance wavelength and increasing the resonance frequency of the pipes and raising the pitch. These effects compete, but the effect of temperature change on the speed of sound dominates.
10. A tube will have certain resonance frequencies associated with it, depending on the length of the tube and the temperature of the air in the tube. Sounds at frequencies far from the resonance
frequencies will not undergo resonance and will not persist. By choosing a length for the tube that isn't resonant for specific frequencies you can reduce the amplitude of those frequencies.
11. As you press on frets closer to the bridge, you are generating higher frequency (and shorter wavelength) sounds. The difference in the wavelength of the resonant standing waves decreases as the wavelengths decrease, so the frets must be closer together as you move toward the bridge.
12. Sound waves can diffract around obstacles such as buildings if the wavelength of the wave is large enough in comparison to the size of the obstacle. Higher frequency corresponds to shorter wavelength. When the truck is behind the building, the lower frequency (longer wavelength) waves bend around the building and reach you, but the higher frequency (shorter wavelength) waves do not. Once the truck has emerged from behind the building, all the different frequencies can reach you.
13. Standing waves are generated by a wave and its reflection. The two waves have a constant phase relationship with each other. The interference depends only on where you are along the string, on your position in space. Beats are generated by two waves whose frequencies are close but not equal. The two waves have a varying phase relationship, and the interference varies with time rather than position.
14. The points would move farther apart. A lower frequency corresponds to a longer wavelength, so the distance between points where destructive and constructive interference occur would increase.
15. According to the principle of superposition, adding a wave and its inverse produces zero displacement of the medium. Adding a sound wave and its inverse effectively cancels out the sound wave and substantially reduces the sound level heard by the worker.
16. (a) The closer the two component frequencies are to each other, the longer the wavelength of the beat. If the two frequencies are very close together, then the waves very nearly overlap, and the distance between a point where the waves interfere constructively and a point where they interfere destructively will be very large.
17. No. The Doppler shift is caused by relative motion between the source and observer.
18. No. The Doppler shift is caused by relative motion between the source and observer. If the wind is blowing, both the wavelength and the velocity of the sound will change, but the frequency of the sound will not.
19. The child will hear the highest frequency at position C , where her speed toward the whistle is the greatest.
20. The human ear can detect frequencies from about 20 Hz to about $20,000 \mathrm{~Hz}$. One octave corresponds to a doubling of frequency. Beginning with 20 Hz , it takes about 10 doublings to reach $20,000 \mathrm{~Hz}$. So, there are approximately 10 octaves in the human audible range.
21. If the frequency of the sound is halved, then the ratio of the frequency of the sound as the car recedes to the frequency of the sound as the car approaches is equal to $1 / 2$. Substituting the appropriate Doppler shift equations in for the frequencies yields a speed for the car of $1 / 3$ the speed of sound.

## Solutions to Problems

In these solutions, we usually treat frequencies as if they are significant to the whole number of units. For example, 20 Hz is taken as to the nearest Hz , and 20 kHz is taken as to the nearest kHz . We also treat all decibel values as good to whole number of decibels. So 120 dB is good to the nearest decibel.

1. The round trip time for sound is 2.0 seconds, so the time for sound to travel the length of the lake is 1.0 seconds. Use the time and the speed of sound to determine the length of the lake.

$$
d=v t=(343 \mathrm{~m} / \mathrm{s})(1.0 \mathrm{~s})=343 \mathrm{~m} \approx 340 \mathrm{~m}
$$

2. The round trip time for sound is 2.5 seconds, so the time for sound to travel the length of the lake is 1.25 seconds. Use the time and the speed of sound in water to determine the depth of the lake.

$$
d=v t=(1560 \mathrm{~m} / \mathrm{s})(1.25 \mathrm{~s})=1950 \mathrm{~m}=2.0 \times 10^{3} \mathrm{~m}
$$

3. (a) $\lambda_{20 \mathrm{~Hz}}=\frac{v}{f}=\frac{343 \mathrm{~m} / \mathrm{s}}{20 \mathrm{~Hz}}=17 \mathrm{~m} \quad \lambda_{20 \mathrm{kHz}}=\frac{v}{f}=\frac{343 \mathrm{~m} / \mathrm{s}}{2.0 \times 10^{4} \mathrm{~Hz}}=1.7 \times 10^{-2} \mathrm{~m}$

So the range is from 1.7 cm to 17 m .
(b) $\lambda=\frac{v}{f}=\frac{343 \mathrm{~m} / \mathrm{s}}{15 \times 10^{6} \mathrm{~Hz}}=2.3 \times 10^{-5} \mathrm{~m}$
4. The distance that the sounds travels is the same on both days. That distance is equal to the speed of sound times the elapsed time. Use the temperature-dependent relationships for the speed of sound in air.

$$
\begin{aligned}
& d=v_{1} t_{1}=v_{2} t_{2} \rightarrow[(331+0.6(27)) \mathrm{m} / \mathrm{s}](4.70 \mathrm{~s})=\left[\left(331+0.6\left(T_{2}\right)\right) \mathrm{m} / \mathrm{s}\right](5.20 \mathrm{~s}) \rightarrow \\
& T_{2}=-29^{\circ} \mathrm{C}
\end{aligned}
$$

5. (a) The ultrasonic pulse travels at the speed of sound, and the round trip distance is twice the distance $d$ to the object.

$$
2 d_{\min }=v t_{\min } \rightarrow d_{\min }=\frac{1}{2} v t_{\min }=\frac{1}{2}(343 \mathrm{~m} / \mathrm{s})\left(1.0 \times 10^{-3} \mathrm{~s}\right)=0.17 \mathrm{~m}
$$

(b) The measurement must take no longer than $1 / 15 \mathrm{~s}$. Again, the round trip distance is twice the distance to the object.

$$
2 d_{\max }=v t_{\max } \rightarrow d_{\max }=\frac{1}{2} v t_{\max }=\frac{1}{2}(343 \mathrm{~m} / \mathrm{s})\left(\frac{1}{15} \mathrm{~s}\right)=11 \mathrm{~m}
$$

(c) The distance is proportional to the speed of sound. So the percentage error in distance is the same as the percentage error in the speed of sound. We assume the device is calibrated to work at $20^{\circ} \mathrm{C}$.

$$
\frac{\Delta d}{d}=\frac{\Delta v}{v}=\frac{v_{23^{\circ} \mathrm{C}}-v_{20^{\circ} \mathrm{C}}}{v_{20^{\circ} \mathrm{C}}}=\frac{[331+0.60(23)] \mathrm{m} / \mathrm{s}-343 \mathrm{~m} / \mathrm{s}}{343 \mathrm{~m} / \mathrm{s}}=0.005248 \approx 0.5 \%
$$

6. (a) For the fish, the speed of sound in seawater must be used.

$$
d=v t \rightarrow t=\frac{d}{v}=\frac{1350 \mathrm{~m}}{1560 \mathrm{~m} / \mathrm{s}}=0.865 \mathrm{~s}
$$

(b) For the fishermen, the speed of sound in air must be used.

$$
d=v t \rightarrow t=\frac{d}{v}=\frac{1350 \mathrm{~m}}{343 \mathrm{~m} / \mathrm{s}}=3.94 \mathrm{~s}
$$

7. The total time $T$ is the time for the stone to fall $\left(t_{\text {down }}\right)$ plus the time for the sound to come back to the top of the cliff $\left(t_{\text {up }}\right): T=t_{\text {up }}+t_{\text {down }}$. Use constant acceleration relationships for an object dropped from rest that falls a distance $h$ in order to find $t_{\text {down }}$, with down as the positive direction. Use the constant speed of sound to find $t_{\text {up }}$ for the sound to travel a distance $h$.

$$
\begin{aligned}
& \text { down: } y=y_{0}+v_{0} t_{\text {down }}+\frac{1}{2} a t_{\text {down }}^{2} \rightarrow h=\frac{1}{2} g t_{\text {down }}^{2} \quad \text { up: } h=v_{\text {snd }} t_{\text {up }} \rightarrow t_{\text {up }}=\frac{h}{v_{\text {snd }}} \\
& h=\frac{1}{2} g t_{\text {down }}^{2}=\frac{1}{2} g\left(T-t_{\text {up }}\right)^{2}=\frac{1}{2} g\left(T-\frac{h}{v_{\text {snd }}}\right)^{2} \rightarrow h^{2}-2 v_{\text {snd }}\left(\frac{v_{\text {snd }}}{g}+T\right) h+T^{2} v_{\text {snd }}^{2}=0
\end{aligned}
$$

This is a quadratic equation for the height. This can be solved with the quadratic formula, but be sure to keep several significant digits in the calculations.

$$
\begin{aligned}
& h^{2}-2(343 \mathrm{~m} / \mathrm{s})\left(\frac{343 \mathrm{~m} / \mathrm{s}}{9.80 \mathrm{~m} / \mathrm{s}^{2}}+3.0 \mathrm{~s}\right) h+(3.0 \mathrm{~s})^{2}(343 \mathrm{~m} / \mathrm{s})^{2}=0 \rightarrow \\
& h^{2}-(26068 \mathrm{~m}) h+1.0588 \times 10^{6} \mathrm{~m}^{2}=0 \rightarrow h=26028 \mathrm{~m}, 41 \mathrm{~m}
\end{aligned}
$$

The larger root is impossible since it takes more than 3.0 sec for the rock to fall that distance, so the correct result is $h=41 \mathrm{~m}$.
8. The two sound waves travel the same distance. The sound will travel faster in the concrete, and thus take a shorter time.

$$
\begin{aligned}
& d=v_{\text {air }} t_{\text {air }}=v_{\text {concretec }} t_{\text {concrete }}=v_{\text {concrete }}\left(t_{\text {air }}-0.75 \mathrm{~s}\right) \rightarrow t_{\text {air }}=\frac{v_{\text {concrete }}}{v_{\text {concrecte }}-v_{\text {air }}} 0.75 \mathrm{~s} \\
& d=v_{\text {air }} t_{\text {air }}=v_{\text {air }}\left(\frac{v_{\text {concrecte }}}{v_{\text {concrete }}-v_{\text {air }}} 0.75 \mathrm{~s}\right)
\end{aligned}
$$

The speed of sound in concrete is obtained from Table $16-1$ as $3000 \mathrm{~m} / \mathrm{s}$.

$$
d=(343 \mathrm{~m} / \mathrm{s})\left(\frac{3000 \mathrm{~m} / \mathrm{s}}{3000 \mathrm{~m} / \mathrm{s}-343 \mathrm{~m} / \mathrm{s}}(0.75 \mathrm{~s})\right)=290 \mathrm{~m}
$$

9. The " 5 second rule" says that for every 5 seconds between seeing a lightning strike and hearing the associated sound, the lightning is 1 mile distant. We assume that there are 5 seconds between seeing the lightning and hearing the sound.
(a) At $30^{\circ} \mathrm{C}$, the speed of sound is $[331+0.60(30)] \mathrm{m} / \mathrm{s}=349 \mathrm{~m} / \mathrm{s}$. The actual distance to the lightning is therefore $d=v t=(349 \mathrm{~m} / \mathrm{s})(5 \mathrm{~s})=1745 \mathrm{~m}$. A mile is 1610 m .

$$
\% \text { error }=\frac{1745-1610}{1745}(100) \approx 8 \%
$$

(b) At $10^{\circ} \mathrm{C}$, the speed of sound is $[331+0.60(10)] \mathrm{m} / \mathrm{s}=337 \mathrm{~m} / \mathrm{s}$. The actual distance to the lightning is therefore $d=v t=(337 \mathrm{~m} / \mathrm{s})(5 \mathrm{~s})=1685 \mathrm{~m}$. A mile is 1610 m .

$$
\% \text { error }=\frac{1685-1610}{1685}(100) \approx 4 \%
$$

10. The relationship between the pressure and displacement amplitudes is given by Eq. 16-5.
(a) $\Delta P_{\mathrm{M}}=2 \pi \rho v A f \rightarrow A=\frac{\Delta P_{\mathrm{M}}}{2 \pi \rho v f}=\frac{3.0 \times 10^{-3} \mathrm{~Pa}}{2 \pi\left(1.29 \mathrm{~kg} / \mathrm{m}^{3}\right)(331 \mathrm{~m} / \mathrm{s})(150 \mathrm{~Hz})}=7.5 \times 10^{-9} \mathrm{~m}$
(b) $\quad A=\frac{\Delta P_{\mathrm{M}}}{2 \pi \rho v f}=\frac{3.0 \times 10^{-3} \mathrm{~Pa}}{2 \pi\left(1.29 \mathrm{~kg} / \mathrm{m}^{3}\right)(331 \mathrm{~m} / \mathrm{s})\left(15 \times 10^{3} \mathrm{~Hz}\right)}=7.5 \times 10^{-11} \mathrm{~m}$
11. The pressure amplitude is found from Eq. 16-5. The density of air is $1.29 \mathrm{~kg} / \mathrm{m}^{3}$.
(a) $\Delta P_{\mathrm{M}}=2 \pi \rho v A f=2 \pi\left(1.29 \mathrm{~kg} / \mathrm{m}^{3}\right)(331 \mathrm{~m} / \mathrm{s})\left(3.0 \times 10^{-10} \mathrm{~m}\right)(55 \mathrm{~Hz})=4.4 \times 10^{-5} \mathrm{~Pa}$
(b) $\Delta P_{\mathrm{M}}=2 \pi \rho v A f=2 \pi\left(1.29 \mathrm{~kg} / \mathrm{m}^{3}\right)(331 \mathrm{~m} / \mathrm{s})\left(3.0 \times 10^{-10} \mathrm{~m}\right)(5500 \mathrm{~Hz})=4.4 \times 10^{-3} \mathrm{~Pa}$
12. The pressure wave can be written as Eq. 16-4.
(a) $\Delta P=-\Delta P_{\mathrm{M}} \cos (k x-\omega t)$

$$
\begin{aligned}
& \Delta P_{\mathrm{M}}=4.4 \times 10^{-5} \mathrm{~Pa} ; \omega=2 \pi f=2 \pi(55 \mathrm{~Hz})=110 \pi \mathrm{rad} / \mathrm{s} ; k=\frac{\omega}{v}=\frac{110 \pi \mathrm{rad} / \mathrm{s}}{331 \mathrm{~m} / \mathrm{s}}=0.33 \pi \mathrm{~m}^{-1} \\
& \Delta P=-\left(4.4 \times 10^{-5} \mathrm{~Pa}\right) \cos \left[\left(0.33 \pi \mathrm{~m}^{-1}\right) x-(110 \pi \mathrm{rad} / \mathrm{s}) t\right]
\end{aligned}
$$

(b) All is the same except for the amplitude and $\omega=2 \pi f=2 \pi(5500 \mathrm{~Hz})=1.1 \times 10^{4} \pi \mathrm{rad} / \mathrm{s}$.

$$
\Delta P=-\left(4.4 \times 10^{-3} \mathrm{~Pa}\right) \cos \left[\left(0.33 \pi \mathrm{~m}^{-1}\right) x-\left(1.1 \times 10^{4} \pi \mathrm{rad} / \mathrm{s}\right) t\right]
$$

13. The pressure wave is $\Delta P=(0.0035 \mathrm{~Pa}) \sin \left[\left(0.38 \pi \mathrm{~m}^{-1}\right) x-\left(1350 \pi \mathrm{~s}^{-1}\right) t\right]$.
(a) $\lambda=\frac{2 \pi}{k}=\frac{2 \pi}{0.38 \pi \mathrm{~m}^{-1}}=5.3 \mathrm{~m}$
(b) $f=\frac{\omega}{2 \pi}=\frac{1350 \pi \mathrm{~s}^{-1}}{2 \pi}=675 \mathrm{~Hz}$
(c) $v=\frac{\omega}{k}=\frac{1350 \pi \mathrm{~s}^{-1}}{0.38 \pi \mathrm{~m}^{-1}}=3553 \mathrm{~m} / \mathrm{s} \approx 3600 \mathrm{~m} / \mathrm{s}$
(d) Use Eq. 16-5 to find the displacement amplitude.

$$
\begin{aligned}
& \Delta P_{\mathrm{M}}=2 \pi \rho v A f \rightarrow \\
& A=\frac{\Delta P_{\mathrm{M}}}{2 \pi \rho v f}=\frac{(0.0035 \mathrm{~Pa})}{2 \pi\left(2300 \mathrm{~kg} / \mathrm{m}^{3}\right)(3553 \mathrm{~m} / \mathrm{s})(675 \mathrm{~Hz})}=1.0 \times 10^{-13} \mathrm{~m}
\end{aligned}
$$

14. $120 \mathrm{~dB}=10 \log \frac{I_{120}}{I_{0}} \rightarrow I_{120}=10^{12} I_{0}=10^{12}\left(1.0 \times 10^{-12} \mathrm{~W} / \mathrm{m}^{2}\right)=1.0 \mathrm{~W} / \mathrm{m}^{2}$
$20 \mathrm{~dB}=10 \log \frac{I_{20}}{I_{0}} \rightarrow I_{20}=10^{2} I_{0}=10^{2}\left(1.0 \times 10^{-12} \mathrm{~W} / \mathrm{m}^{2}\right)=1.0 \times 10^{-10} \mathrm{~W} / \mathrm{m}^{2}$
The pain level is $10^{10}$ times more intense than the whisper.
15. $\beta=10 \log \frac{I}{I_{0}}=10 \log \frac{2.0 \times 10^{-6} \mathrm{~W} / \mathrm{m}^{2}}{1.0 \times 10^{-12} \mathrm{~W} / \mathrm{m}^{2}}=63 \mathrm{~dB}$
16. From Figure 16-6, at 40 dB the low frequency threshold of hearing is about $70-80 \mathrm{~Hz}$. There is no intersection of the threshold of hearing with the 40 dB level on the high frequency side of the chart, and so a 40 dB signal can be heard all the way up to the highest frequency that a human can hear, $20,000 \mathrm{~Hz}$.
17. (a) From Figure $16-6$, at 100 Hz , the threshold of hearing (the lowest detectable intensity by the ear) is approximately $5 \times 10^{-9} \mathrm{~W} / \mathrm{m}^{2}$. The threshold of pain is about $5 \mathrm{~W} / \mathrm{m}^{2}$. The ratio of highest to lowest intensity is thus $\frac{5 \mathrm{~W} / \mathrm{m}^{2}}{5 \times 10^{-9} \mathrm{~W} / \mathrm{m}^{2}}=10^{9}$.
(b) At 5000 Hz , the threshold of hearing is about $10^{-13} \mathrm{~W} / \mathrm{m}^{2}$, and the threshold of pain is about $10^{-1} \mathrm{~W} / \mathrm{m}^{2}$. The ratio of highest to lowest intensity is $\frac{10^{-1} \mathrm{~W} / \mathrm{m}^{2}}{10^{-13} \mathrm{~W} / \mathrm{m}^{2}}=10^{12}$.
Answers may vary due to estimation in the reading of the graph.
18. Compare the two power output ratings using the definition of decibels.

$$
\beta=10 \log \frac{P_{150}}{P_{100}}=10 \log \frac{150 \mathrm{~W}}{100 \mathrm{~W}}=1.8 \mathrm{~dB}
$$

This would barely be perceptible.
19. The intensity can be found from the decibel value.

$$
\beta=10 \log \frac{I}{I_{0}} \rightarrow I=10^{\beta / 10} I_{0}=10^{12}\left(10^{-12} \mathrm{~W} / \mathrm{m}^{2}\right)=1.0 \mathrm{~W} / \mathrm{m}^{2}
$$

Consider a square perpendicular to the direction of travel of the sound wave. The intensity is the energy transported by the wave across a unit area perpendicular to the direction of travel, per unit time. So $I=\frac{\Delta E}{S \Delta t}$, where $S$ is the area of the square. Since the energy is "moving" with the wave, the "speed" of the energy is $v$, the wave speed. In a time $\Delta t$, a volume equal to $\Delta V=S v \Delta t$ would contain all of the energy that had been transported across the area $S$. Combine these relationships to find the energy in the volume.

$$
I=\frac{\Delta E}{S \Delta t} \rightarrow \Delta E=I S \Delta t=\frac{I \Delta V}{v}=\frac{\left(1.0 \mathrm{~W} / \mathrm{m}^{2}\right)(0.010 \mathrm{~m})^{3}}{343 \mathrm{~m} / \mathrm{s}}=2.9 \times 10^{-9} \mathrm{~J}
$$

20. From Example 12-4, we see that a sound level decrease of 3 dB corresponds to a halving of intensity. Thus the sound level for one firecracker will be $95 \mathrm{~dB}-3 \mathrm{~dB}=92 \mathrm{~dB}$.
21. From Example 16-4, we see that a sound level decrease of 3 dB corresponds to a halving of intensity. Thus, if two engines are shut down, the intensity will be cut in half, and the sound level will be 127 dB . Then, if one more engine is shut down, the intensity will be cut in half again, and the sound level will drop by 3 more $d B$, to a final value of 124 dB .
22. $62 \mathrm{~dB}=10 \log \left(I_{\text {Signal }} / I_{\text {Noise }}\right)_{\text {tape }} \rightarrow\left(I_{\text {Signal }} / I_{\text {Noise }}\right)_{\text {tape }}=10^{6.2}=1.6 \times 10^{6}$
$98 \mathrm{~dB}=10 \log \left(I_{\text {Signal }} / I_{\text {Noise }}\right)_{\text {tape }} \rightarrow\left(I_{\text {Signal }} / I_{\text {Noise }}\right)_{\text {tape }}=10^{9.8}=6.3 \times 10^{9}$
23. (a) According to Table 16-2, the intensity in normal conversation, when about 50 cm from the speaker, is about $3 \times 10^{-6} \mathrm{~W} / \mathrm{m}^{2}$. The intensity is the power output per unit area, and so the power output can be found. The area is that of a sphere.

$$
I=\frac{P}{A} \rightarrow P=I A=I\left(4 \pi r^{2}\right)=\left(3 \times 10^{-6} \mathrm{~W} / \mathrm{m}^{2}\right) 4 \pi(0.50 \mathrm{~m})^{2}=9.425 \times 10^{-6} \mathrm{~W} \approx 9.4 \times 10^{-6} \mathrm{~W}
$$

(b) $75 \mathrm{~W}\left(\frac{1 \text { person }}{9.425 \times 10^{-6} \mathrm{~W}}\right)=7.96 \times 10^{6} \approx 8.0 \times 10^{6}$ people
24. (a) The energy absorbed per second is the power of the wave, which is the intensity times the area.

$$
\begin{aligned}
& 50 \mathrm{~dB}=10 \log \frac{I}{I_{0}} \rightarrow I=10^{5} I_{0}=10^{5}\left(1.0 \times 10^{-12} \mathrm{~W} / \mathrm{m}^{2}\right)=1.0 \times 10^{-7} \mathrm{~W} / \mathrm{m}^{2} \\
& P=I A=\left(1.0 \times 10^{-7} \mathrm{~W} / \mathrm{m}^{2}\right)\left(5.0 \times 10^{-5} \mathrm{~m}^{2}\right)=5.0 \times 10^{-12} \mathrm{~W}
\end{aligned}
$$

(b)
$1 \mathrm{~J}\left(\frac{1 \mathrm{~s}}{5.0 \times 10^{-12} \mathrm{~J}}\right)\left(\frac{1 \mathrm{yr}}{3.16 \times 10^{7} \mathrm{~s}}\right)=6.3 \times 10^{3} \mathrm{yr}$
25. The intensity of the sound is defined to be the power per unit area. We assume that the sound spreads out spherically from the loudspeaker.
(a) $I_{250}=\frac{250 \mathrm{~W}}{4 \pi(3.5 \mathrm{~m})^{2}}=1.624 \mathrm{~W} / \mathrm{m}^{2} \quad \beta_{250}=10 \log \frac{I_{250}}{I_{0}}=10 \log \frac{1.624 \mathrm{~W} / \mathrm{m}^{2}}{1.0 \times 10^{-12} \mathrm{~W} / \mathrm{m}^{2}}=122 \mathrm{~dB}$

$$
I_{45}=\frac{45 \mathrm{~W}}{4 \pi(3.5 \mathrm{~m})^{2}}=0.2923 \mathrm{~W} / \mathrm{m}^{2} \quad \beta_{45}=10 \log \frac{I_{45}}{I_{0}}=10 \log \frac{0.2923 \mathrm{~W} / \mathrm{m}^{2}}{1.0 \times 10^{-12} \mathrm{~W} / \mathrm{m}^{2}}=115 \mathrm{~dB}
$$

(b) According to the textbook, for a sound to be perceived as twice as loud as another means that the intensities need to differ by a factor of 10 . That is not the case here - they differ only by a factor of $\frac{1.624}{0.2598} \approx 6$. The expensive amp will not sound twice as loud as the cheaper one.
26. (a) Find the intensity from the 130 dB value, and then find the power output corresponding to that intensity at that distance from the speaker.

$$
\begin{aligned}
& \beta=130 \mathrm{~dB}=10 \log \frac{I_{2.8 \mathrm{~m}}}{I_{0}} \rightarrow I_{2.8 \mathrm{~m}}=10^{13} I_{0}=10^{13}\left(1.0 \times 10^{-12} \mathrm{~W} / \mathrm{m}^{2}\right)=10 \mathrm{~W} / \mathrm{m}^{2} \\
& P=I A=4 \pi r^{2} I=4 \pi(2.2 \mathrm{~m})^{2}\left(10 \mathrm{~W} / \mathrm{m}^{2}\right)=608 \mathrm{~W} \approx 610 \mathrm{~W}
\end{aligned}
$$

(b) Find the intensity from the 85 dB value, and then from the power output, find the distance corresponding to that intensity.

$$
\begin{aligned}
& \beta=85 \mathrm{~dB}=10 \log \frac{I}{I_{0}} \rightarrow I=10^{8.5} I_{0}=10^{8.5}\left(1.0 \times 10^{-12} \mathrm{~W} / \mathrm{m}^{2}\right)=3.16 \times 10^{-4} \mathrm{~W} / \mathrm{m}^{2} \\
& P=4 \pi r^{2} I \rightarrow r=\sqrt{\frac{P}{4 \pi I}}=\sqrt{\frac{608 \mathrm{~W}}{4 \pi\left(3.16 \times 10^{-4} \mathrm{~W} / \mathrm{m}^{2}\right)}}=390 \mathrm{~m}
\end{aligned}
$$

27. The first person is a distance of $r_{1}=100 \mathrm{~m}$ from the explosion, while the second person is a distance $r_{2}=\sqrt{5}(100 \mathrm{~m})$ from the explosion. The intensity detected away from the explosion is inversely proportional to the square of the distance from the explosion.

$$
\frac{I_{1}}{I_{2}}=\frac{r_{2}^{2}}{r_{1}^{2}}=\left[\frac{\sqrt{5}(100 \mathrm{~m})}{100 \mathrm{~m}}\right]^{2}=5 ; \beta=10 \log \frac{I_{1}}{I_{2}}=10 \log 5=7.0 \mathrm{~dB}
$$

28. (a) The intensity is proportional to the square of the amplitude, so if the amplitude is 2.5 times greater, the intensity will increase by a factor of $6.25 \approx 6.3$.
(b) $\beta=10 \log I / I_{0}=10 \log 6.25=8 \mathrm{~dB}$
29. (a) The pressure amplitude is seen in Eq. 16-5 to be proportional to the displacement amplitude and to the frequency. Thus the higher frequency wave has the larger pressure amplitude, by a factor of 2.6.
(b) The intensity is proportional to the square of the frequency. Thus the ratio of the intensities is the square of the frequency ratio.

$$
\frac{I_{2.6 f}}{I_{f}}=\frac{(2.6 f)^{2}}{f^{2}}=6.76 \approx 6.8
$$

30. The intensity is given by Eq. $15-7, I=2 \pi^{2} v \rho f^{2} A^{2}$, using the density of air and the speed of sound in air.

$$
\begin{aligned}
& I=2 \rho v \pi^{2} f^{2} A^{2}=2\left(1.29 \mathrm{~kg} / \mathrm{m}^{3}\right)(343 \mathrm{~m} / \mathrm{s}) \pi^{2}(380 \mathrm{~Hz})^{2}\left(1.3 \times 10^{-4} \mathrm{~m}\right)^{2}=21.31 \mathrm{~W} / \mathrm{m}^{2} \\
& \beta=10 \log \frac{I}{I_{0}}=10 \log \frac{21.31 \mathrm{~W} / \mathrm{m}^{2}}{1.0 \times 10^{-12} \mathrm{~W} / \mathrm{m}^{2}}=133 \mathrm{~dB} \approx 130 \mathrm{~dB}
\end{aligned}
$$

Note that this is above the threshold of pain.
31. (a) We find the intensity of the sound from the decibel value, and then calculate the displacement amplitude from Eq. 15-7.

$$
\begin{aligned}
& \beta=10 \log \frac{I}{I_{0}} \rightarrow I=10^{\beta / 10} I_{0}=10^{12}\left(10^{-12} \mathrm{~W} / \mathrm{m}^{2}\right)=1.0 \mathrm{~W} / \mathrm{m}^{2} \\
& I=2 \pi^{2} v \rho f^{2} A^{2} \rightarrow \\
& A=\frac{1}{\pi f} \sqrt{\frac{I}{2 \rho v}}=\frac{1}{\pi(330 \mathrm{~Hz})} \sqrt{\frac{1.0 \mathrm{~W} / \mathrm{m}^{2}}{2\left(1.29 \mathrm{~kg} / \mathrm{m}^{3}\right)(343 \mathrm{~m} / \mathrm{s})}}=3.2 \times 10^{-5} \mathrm{~m}
\end{aligned}
$$

(b) The pressure amplitude can be found from Eq. 16-7.

$$
\begin{aligned}
& I=\frac{\left(\Delta P_{\mathrm{M}}\right)^{2}}{2 v \rho} \rightarrow \\
& \Delta P_{\mathrm{M}}=\sqrt{2 v \rho I}=\sqrt{2(343 \mathrm{~m} / \mathrm{s})\left(1.29 \mathrm{~kg} / \mathrm{m}^{3}\right)\left(1.0 \mathrm{~W} / \mathrm{m}^{2}\right)}=30 \mathrm{~Pa}(2 \text { sig. fig. })
\end{aligned}
$$

32. (a) We assume that there has been no appreciable absorption in this 25 meter distance. The tensity is the power divide by the area of a sphere of radius 25 meters. We express the sound revel in dB.

$$
I=\frac{P}{4 \pi r^{2}} ; \beta=10 \log \frac{I}{I_{0}}=10 \log \frac{P}{4 \pi r^{2} I_{0}}=10 \log \frac{\left(5.0 \times 10^{5} \mathrm{~W}\right)}{4 \pi(25 \mathrm{~m})^{2}\left(10^{-12} \mathrm{~W} / \mathrm{m}^{2}\right)}=138 \mathrm{~dB}
$$

(b) We find the intensity level at the new distance, and subtract due to absorption.

$$
\begin{aligned}
& \beta=10 \log \frac{P}{4 \pi r^{2} I_{0}}=10 \log \frac{\left(5.0 \times 10^{5} \mathrm{~W}\right)}{4 \pi(1000 \mathrm{~m})^{2}\left(10^{-12} \mathrm{~W} / \mathrm{m}^{2}\right)}=106 \mathrm{~dB} \\
& \beta_{\text {with }}=106 \mathrm{~dB}-(1.00 \mathrm{~km})(7.0 \mathrm{~dB} / \mathrm{km})=99 \mathrm{~dB}
\end{aligned}
$$

(c) We find the intensity level at the new distance, and subtract due to absorption.

$$
\begin{aligned}
& \beta=10 \log \frac{P}{4 \pi r^{2} I_{0}}=10 \log \frac{\left(5.0 \times 10^{5} \mathrm{~W}\right)}{4 \pi(7500 \mathrm{~m})^{2}\left(10^{-12} \mathrm{~W} / \mathrm{m}^{2}\right)}=88.5 \mathrm{~dB} \\
& \beta_{\text {with }}=88.5 \mathrm{~dB}-(7.50 \mathrm{~km})(7.0 \mathrm{~dB} / \mathrm{km})=36 \mathrm{~dB}
\end{aligned}
$$

33. For a closed tube, Figure 16-12 indicates that $f_{1}=\frac{v}{4 \ell}$. We assume the bass clarinet is at room temperature.

$$
f_{1}=\frac{v}{4 \ell} \rightarrow \ell=\frac{v}{4 f_{1}}=\frac{343 \mathrm{~m} / \mathrm{s}}{4(69.3 \mathrm{~Hz})}=1.24 \mathrm{~m}
$$

34. For a vibrating string, the frequency of the fundamental mode is given by $f=\frac{v}{2 L}=\frac{1}{2 L} \sqrt{\frac{F_{\mathrm{T}}}{m / L}}$.

$$
f=\frac{1}{2 L} \sqrt{\frac{F_{\mathrm{T}}}{m / L}} \rightarrow F_{\mathrm{T}}=4 L f^{2} m=4(0.32 \mathrm{~m})(440 \mathrm{~Hz})^{2}\left(3.5 \times 10^{-4} \mathrm{~kg}\right)=87 \mathrm{~N}
$$

35. (a) If the pipe is closed at one end, only the odd harmonic frequencies are present, and are given by

$$
\begin{aligned}
f_{n}=\frac{n v}{4 L} & =n f_{1}, n=1,3,5 \cdots . \\
f_{1} & =\frac{v}{4 L}=\frac{343 \mathrm{~m} / \mathrm{s}}{4(1.24 \mathrm{~m})}=69.2 \mathrm{~Hz} \\
f_{3} & =3 f_{1}=207 \mathrm{~Hz} \quad f_{5}=5 f_{1}=346 \mathrm{~Hz} \quad f_{7}=7 f_{1}=484 \mathrm{~Hz}
\end{aligned}
$$

(b) If the pipe is open at both ends, all the harmonic frequencies are present, and are given by

$$
\begin{aligned}
f_{n}=\frac{n v}{2 \ell} & =n f_{1} . \\
f_{1} & =\frac{v}{2 \ell}=\frac{343 \mathrm{~m} / \mathrm{s}}{2(1.24 \mathrm{~m})}=138.3 \mathrm{~Hz} \approx 138 \mathrm{~Hz} \\
f_{2} & =2 f_{1}=\frac{v}{\ell}=277 \mathrm{~Hz} \quad f_{3}=3 f_{1}=\frac{3 v}{2 \ell}=415 \mathrm{~Hz} \quad f_{4}=4 f_{1}=\frac{2 v}{\ell}=553 \mathrm{~Hz}
\end{aligned}
$$

36. (a) The length of the tube is one-fourth of a wavelength for this (one end closed) tube, and so the wavelength is four times the length of the tube.

$$
f=\frac{v}{\lambda}=\frac{343 \mathrm{~m} / \mathrm{s}}{4(0.21 \mathrm{~m})}=410 \mathrm{~Hz}
$$

(b) If the bottle is one-third full, then the effective length of the air column is reduced to 14 cm .

$$
f=\frac{v}{\lambda}=\frac{343 \mathrm{~m} / \mathrm{s}}{4(0.14 \mathrm{~m})}=610 \mathrm{~Hz}
$$

37. For a pipe open at both ends, the fundamental frequency is given by $f_{1}=\frac{v}{2 \ell}$, and so the length for a given fundamental frequency is $\ell=\frac{v}{2 f_{1}}$.

$$
\ell_{20 \mathrm{~Hz}}=\frac{343 \mathrm{~m} / \mathrm{s}}{2(20 \mathrm{~Hz})}=8.6 \mathrm{~m} \quad \ell_{20 \mathrm{kHz}}=\frac{343 \mathrm{~m} / \mathrm{s}}{2(20,000 \mathrm{~Hz})}=8.6 \times 10^{-3} \mathrm{~m}
$$

38. We approximate the shell as a closed tube of length 20 cm , and calculate the fundamental frequency.

$$
f=\frac{v}{4 \ell}=\frac{343 \mathrm{~m} / \mathrm{s}}{4(0.20 \mathrm{~m})}=429 \mathrm{~Hz} \approx 430 \mathrm{~Hz}
$$

39. (a) We assume that the speed of waves on the guitar string does not change when the string is fretted. The fundamental frequency is given by $f=\frac{v}{2 \ell}$, and so the frequency is inversely proportional to the length.

$$
\begin{aligned}
& f \propto \frac{1}{\ell} \rightarrow f \ell=\text { constant } \\
& f_{\mathrm{E}} \ell_{\mathrm{E}}=f_{\mathrm{A}} \ell_{\mathrm{A}} \rightarrow \ell_{\mathrm{A}}=\ell_{\mathrm{E}} \frac{f_{\mathrm{E}}}{f_{\mathrm{A}}}=(0.73 \mathrm{~m})\left(\frac{330 \mathrm{~Hz}}{440 \mathrm{~Hz}}\right)=0.5475 \mathrm{~m}
\end{aligned}
$$

The string should be fretted a distance $0.73 \mathrm{~m}-0.5475 \mathrm{~m}=0.1825 \mathrm{~m} \approx 0.18 \mathrm{~m}$ from the nut of the guitar.
(b) The string is fixed at both ends and is vibrating in its fundamental mode. Thus the wavelength is twice the length of the string (see Fig. 16-7).

$$
\lambda=2 \ell=2(0.5475 \mathrm{~m})=1.095 \mathrm{~m} \approx 1.1 \mathrm{~m}
$$

(c) The frequency of the sound will be the same as that of the string, 440 Hz . The wavelength is given by the following.

$$
\lambda=\frac{v}{f}=\frac{343 \mathrm{~m} / \mathrm{s}}{440 \mathrm{~Hz}}=0.78 \mathrm{~m}
$$

40. (a) At $T=15^{\circ} \mathrm{C}$, the speed of sound is given by $v=(331+0.60(15)) \mathrm{m} / \mathrm{s}=340 \mathrm{~m} / \mathrm{s}$ (with 3 significant figures). For an open pipe, the fundamental frequency is given by $f=\frac{v}{2 \ell}$.

$$
f=\frac{v}{2 \ell} \rightarrow \ell=\frac{v}{2 f}=\frac{340 \mathrm{~m} / \mathrm{s}}{2(262 \mathrm{~Hz})}=0.649 \mathrm{~m}
$$

(b) The frequency of the standing wave in the tube is 262 Hz . The wavelength is twice the length of the pipe, 1.30 m .
(c) The wavelength and frequency are the same in the air, because it is air that is resonating in the organ pipe. The frequency is 262 Hz and the wavelength is 1.30 m .
41. The speed of sound will change as the temperature changes, and that will change the frequency of the organ. Assume that the length of the pipe (and thus the resonant wavelength) does not change.

$$
\begin{aligned}
& f_{22}=\frac{v_{22}}{\lambda} \quad f_{5.0}=\frac{v_{5.0}}{\lambda} \quad \Delta f=f_{5.0}-f_{22}=\frac{v_{5.0}-v_{22}}{\lambda} \\
& \frac{\Delta f}{f}=\frac{\frac{v_{5.0}-v_{22}}{\lambda}}{\frac{v_{22}}{\lambda}}=\frac{v_{5.0}}{v_{22}}-1=\frac{331+0.60(5.0)}{331+0.60(22)}-1=-2.96 \times 10^{-2}=-3.0 \%
\end{aligned}
$$

42. A flute is a tube that is open at both ends, and so the fundamental frequency is given by $f=\frac{v}{2 \ell}$, where $\ell$ is the distance from the mouthpiece (antinode) to the first open side hole in the flute tube (antinode).

$$
f=\frac{v}{2 \ell} \rightarrow \ell=\frac{v}{2 f}=\frac{343 \mathrm{~m} / \mathrm{s}}{2(349 \mathrm{~Hz})}=0.491 \mathrm{~m}
$$

43. For a tube open at both ends, all harmonics are allowed, with $f_{n}=n f_{1}$. Thus consecutive harmonics differ by the fundamental frequency. The four consecutive harmonics give the following values for the fundamental frequency.

$$
f_{1}=523 \mathrm{~Hz}-392 \mathrm{~Hz}=131 \mathrm{~Hz}, 659 \mathrm{~Hz}-523 \mathrm{~Hz}=136 \mathrm{~Hz}, 784 \mathrm{~Hz}-659 \mathrm{~Hz}=125 \mathrm{~Hz}
$$

The average of these is $f_{1}=\frac{1}{3}(131 \mathrm{~Hz}+136 \mathrm{~Hz}+125 \mathrm{~Hz}) \approx 131 \mathrm{~Hz}$. We use that for the fundamental frequency.
(a) $f_{1}=\frac{v}{2 \ell} \rightarrow \ell=\frac{v}{2 f_{1}}=\frac{343 \mathrm{~m} / \mathrm{s}}{2(131 \mathrm{~Hz})}=1.31 \mathrm{~m}$

Note that the bugle is coiled like a trumpet so that the full length fits in a smaller distance.
(b)

$$
\begin{aligned}
& f_{n}=n f_{1} \rightarrow n_{\mathrm{G} 4}=\frac{f_{\mathrm{G} 4}}{f_{1}}=\frac{392 \mathrm{~Hz}}{131 \mathrm{~Hz}}=2.99 ; n_{\mathrm{C} 5}=\frac{f_{\mathrm{C} 5}}{f_{1}}=\frac{523 \mathrm{~Hz}}{131 \mathrm{~Hz}}=3.99 ; \\
& n_{\mathrm{E} 5}=\frac{f_{\mathrm{E} 5}}{f_{1}}=\frac{659 \mathrm{~Hz}}{131 \mathrm{~Hz}}=5.03 ; n_{\mathrm{G} 5}=\frac{f_{\mathrm{G} 5}}{f_{1}}=\frac{784 \mathrm{~Hz}}{131 \mathrm{~Hz}}=5.98
\end{aligned}
$$

The harmonics are $3,4,5$, and 6 .
44. (a) The difference between successive overtones for this pipe is 176 Hz . The difference between successive overtones for an open pipe is the fundamental frequency, and each overtone is an integer multiple of the fundamental. Since 264 Hz is not a multiple of $176 \mathrm{~Hz}, 176 \mathrm{~Hz}$ cannot be the fundamental, and so the pipe cannot be open. Thus it must be a closed pipe.
(b) For a closed pipe, the successive overtones differ by twice the fundamental frequency. Thus 176 Hz must be twice the fundamental, so the fundamental is 88 Hz . This is verified since 264 Hz is 3 times the fundamental, 440 Hz is 5 times the fundamental, and 616 Hz is 7 times the fundamental.
45. The tension and mass density of the string do not change, so the wave speed is constant. The frequency ratio for two adjacent notes is to be $2^{1 / 12}$. The frequency is given by $f=\frac{v}{2 \ell}$.

$$
\begin{aligned}
& f=\frac{v}{2 \ell} \rightarrow \frac{f_{\text {list }}}{\text { fret }}<\frac{\frac{v}{2 \ell_{1 \text { ts }}}}{f_{\text {unfingered }}}=\frac{v}{\frac{v}{2 \ell_{\text {unfingered }}}}=2^{1 / 12} \rightarrow \ell_{\substack{\text { stt } \\
\text { fret }}}=\frac{\ell_{\text {unfingered }}}{2^{1 / 12}}=\frac{65.0 \mathrm{~cm}}{2^{1 / 12}}=61.35 \mathrm{~cm}
\end{aligned}
$$

$$
\begin{aligned}
& x_{1}=(65.0 \mathrm{~cm})\left(1-2^{-1 / 12}\right)=3.6 \mathrm{~cm} ; x_{2}=(65.0 \mathrm{~cm})\left(1-2^{-2 / 12}\right)=7.1 \mathrm{~cm} \\
& x_{3}=(65.0 \mathrm{~cm})\left(1-2^{-3 / 12}\right)=10.3 \mathrm{~cm} ; x_{4}=(65.0 \mathrm{~cm})\left(1-2^{-4 / 12}\right)=13.4 \mathrm{~cm} \\
& x_{5}=(65.0 \mathrm{~cm})\left(1-2^{-5 / 12}\right)=16.3 \mathrm{~cm} ; x_{6}=(65.0 \mathrm{~cm})\left(1-2^{-6 / 12}\right)=19.0 \mathrm{~cm}
\end{aligned}
$$

46. (a) The difference between successive overtones for an open pipe is the fundamental frequency.

$$
f_{1}=330 \mathrm{~Hz}-275 \mathrm{~Hz}=55 \mathrm{~Hz}
$$

(b) The fundamental frequency is given by $f_{1}=\frac{v}{2 \ell}$. Solve this for the speed of sound.

$$
v=2 \ell f_{1}=2(1.80 \mathrm{~m})(55 \mathrm{~Hz})=198 \mathrm{~m} / \mathrm{s} \approx 2.0 \times 10^{2} \mathrm{~m} / \mathrm{s}
$$

47. The difference in frequency for two successive harmonics is 40 Hz . For an open pipe, two successive harmonics differ by the fundamental, so the fundamental could be 40 Hz , with 240 Hz being the $6^{\text {th }}$ harmonic and 280 Hz being the $7^{\text {th }}$ harmonic. For a closed pipe, two successive harmonics differ by twice the fundamental, so the fundamental could be 20 Hz . But the overtones of a closed pipe are odd multiples of the fundamental, and both overtones are even multiples of 30 Hz . So the pipe must be an open pipe.

$$
f=\frac{v}{2 \ell} \rightarrow \ell=\frac{v}{2 f}=\frac{[331+0.60(23.0)] \mathrm{m} / \mathrm{s}}{2(40 \mathrm{~Hz})}=4.3 \mathrm{~m}
$$

48. (a) The harmonics for the open pipe are $f_{n}=\frac{n v}{2 \ell}$. To be audible, they must be below 20 kHz .

$$
\frac{n v}{2 \ell}<2 \times 10^{4} \mathrm{~Hz} \rightarrow n<\frac{2(2.48 \mathrm{~m})\left(2 \times 10^{4} \mathrm{~Hz}\right)}{343 \mathrm{~m} / \mathrm{s}}=289.2
$$

Since there are 289 harmonics, there are 288 overtones.
(b) The harmonics for the closed pipe are $f_{n}=\frac{n v}{4 \ell}, n$ odd. Again, they must be below 20 kHz .

$$
\frac{n v}{4 \ell}<2 \times 10^{4} \mathrm{~Hz} \rightarrow n<\frac{4(2.48 \mathrm{~m})\left(2 \times 10^{4} \mathrm{~Hz}\right)}{343 \mathrm{~m} / \mathrm{s}}=578.4
$$

The values of $n$ must be odd, so $n=1,3,5, \ldots, 577$. There are 289 harmonics, and so there are 288 overtones.
49. A tube closed at both ends will have standing waves with displacement nodes at each end, and so has the same harmonic structure as a string that is fastened at both ends. Thus the wavelength of the fundamental frequency is twice the length of the hallway, $\lambda_{1}=2 \ell=16.0 \mathrm{~m}$.

$$
f_{1}=\frac{v}{\lambda_{1}}=\frac{343 \mathrm{~m} / \mathrm{s}}{16.0 \mathrm{~m}}=21.4 \mathrm{~Hz} ; f_{2}=2 f_{1}=42.8 \mathrm{~Hz}
$$

50. To operate with the first harmonic, we see from the figure that the thickness must be half of a wavelength, so the wavelength is twice the thickness. The speed of sound in the quartz is given by $v=\sqrt{G / \rho}$, analogous to Eqs. 15-3 and 15-4.

$$
t=\frac{1}{2} \lambda=\frac{1}{2} \frac{v}{f}=\frac{1}{2} \frac{\sqrt{G / \rho}}{f}=\frac{1}{2} \frac{\sqrt{\left(2.95 \times 10^{10} \mathrm{~N} / \mathrm{m}^{2}\right) /\left(2650 \mathrm{~kg} / \mathrm{m}^{2}\right)}}{12.0 \times 10^{6} \mathrm{~Hz}}=1.39 \times 10^{-4} \mathrm{~m}
$$

51. The ear canal can be modeled as a closed pipe of length 2.5 cm . The resonant frequencies are given by $f_{n}=\frac{n v}{4 \ell}, n$ odd. The first several frequencies are calculated here.

$$
\begin{aligned}
& f_{n}=\frac{n v}{4 \ell}=\frac{n(343 \mathrm{~m} / \mathrm{s})}{4\left(2.5 \times 10^{-2} \mathrm{~m}\right)}=n(3430 \mathrm{~Hz}), n \text { odd } \\
& f_{1}=3430 \mathrm{~Hz} \quad f_{3}=10,300 \mathrm{~Hz} \quad f_{5}=17,200 \mathrm{~Hz}
\end{aligned}
$$

In the graph, the most sensitive frequency is between 3000 and 4000 Hz . This corresponds to the fundamental resonant frequency of the ear canal. The sensitivity decrease above 4000 Hz , but is seen to "flatten out" around $10,000 \mathrm{~Hz}$ again, indicating higher sensitivity near $10,000 \mathrm{~Hz}$ than at surrounding frequencies. This $10,000 \mathrm{~Hz}$ relatively sensitive region corresponds to the first overtone resonant frequency of the ear canal.
52. From Eq. 15-7, the intensity is proportional to the square of the amplitude and the square of the frequency. From Fig. 16-14, the relative amplitudes are $\frac{A_{2}}{A_{1}} \approx 0.4$ and $\frac{A_{3}}{A_{1}} \approx 0.15$.

$$
\begin{aligned}
& I=2 \pi^{2} v \rho f^{2} A^{2} \rightarrow \frac{I_{2}}{I_{1}}=\frac{2 \pi^{2} v \rho f_{2}^{2} A_{2}^{2}}{2 \pi^{2} v \rho f_{1}^{2} A_{1}^{2}}=\frac{f_{2}^{2} A_{2}^{2}}{f_{1}^{2} A_{1}^{2}}=\left(\frac{f_{2}}{f_{1}}\right)^{2}\left(\frac{A_{2}}{A_{1}}\right)^{2}=2^{2}(0.4)^{2}=0.64 \\
& \frac{I_{3}}{I_{1}}=\left(\frac{f_{3}}{f_{1}}\right)^{2}\left(\frac{A_{3}}{A_{1}}\right)^{2}=3^{2}(0.15)^{2}=0.20 \\
& \beta_{2-1}=10 \log \frac{I_{2}}{I_{1}}=10 \log 0.64=-2 \mathrm{~dB} ; \beta_{3-1}=10 \log \frac{I_{3}}{I_{1}}=10 \log 0.24=-7 \mathrm{~dB}
\end{aligned}
$$

53. The beat period is 2.0 seconds, so the beat frequency is the reciprocal of that, 0.50 Hz . Thus the other string is off in frequency by $\pm 0.50 \mathrm{~Hz}$. The beating does not tell the tuner whether the second string is too high or too low.
54. The beat frequency is the difference in the two frequencies, or $277 \mathrm{~Hz}-262 \mathrm{~Hz}=15 \mathrm{~Hz}$. If the $\mathrm{fr} \leftrightarrows$ ncies are both reduced by a factor of 4 , then the difference between the two frequencies will also ve reduced by a factor of 4 , and so the beat frequency will be $\frac{1}{4}(15 \mathrm{~Hz})=3.75 \mathrm{~Hz} \approx 3.8 \mathrm{~Hz}$.
55. Since there are 4 beats/s when sounded with the 350 Hz tuning fork, the guitar string must have a frequency of either 346 Hz or 354 Hz . Since there are 9 beats/s when sounded with the 355 Hz tuning fork, the guitar string must have a frequency of either 346 Hz or 364 Hz . The common value
is 346 Hz .
56. (a) Since the sounds are initially $180^{\circ}$ out of phase, another $180^{\circ}$ of phase must be added by a path
 length difference. Thus the difference of the distances from the speakers to the point of constructive interference must be half of a wavelength. See the diagram.

$$
(d-x)-x=\frac{1}{2} \lambda \rightarrow d=2 x+\frac{1}{2} \lambda \rightarrow d_{\min }=\frac{1}{2} \lambda=\frac{v}{2 f}=\frac{343 \mathrm{~m} / \mathrm{s}}{2(294 \mathrm{~Hz})}=0.583 \mathrm{~m}
$$

This minimum distance occurs when the observer is right at one of the speakers. If the speakers are separated by more than 0.583 m , the location of constructive interference will be moved away from the speakers, along the line between the speakers.
(b) Since the sounds are already $180^{\circ}$ out of phase, as long as the listener is equidistant from the speakers, there will be completely destructive interference. So even if the speakers have a tiny separation, the point midway between them will be a point of completely destructive interference. The minimum separation between the speakers is 0 .
57. Beats will be heard because the difference in the speed of sound for the two flutes will result in two different frequencies.

$$
\begin{aligned}
& f_{1}=\frac{v_{1}}{2 \ell}=\frac{[331+0.60(28)] \mathrm{m} / \mathrm{s}}{2(0.66 \mathrm{~m})}=263.4 \mathrm{~Hz} \\
& f_{2}=\frac{v_{2}}{2 \ell}=\frac{[331+0.60(5.0)] \mathrm{m} / \mathrm{s}}{2(0.66 \mathrm{~m})}=253.0 \mathrm{~Hz} \quad \Delta f=263.4 \mathrm{~Hz}-253.0 \mathrm{~Hz}=10 \mathrm{beats} / \mathrm{sec}
\end{aligned}
$$

58. (a) The microphone must be moved to the right until the difference in distances from the two sources is half a wavelength. See the diagram. We square the expression, collect terms, isolate the

$$
\begin{aligned}
& \text { remaining square root, and square again. } \\
& \begin{array}{l}
S_{2}-S_{1}=\frac{1}{2} \lambda \rightarrow \\
\sqrt{\left(\frac{1}{2} D+x\right)^{2}+\ell^{2}}-\sqrt{\left(\frac{1}{2} D-x\right)^{2}+\ell^{2}}=\frac{1}{2} \lambda \rightarrow \\
\sqrt{\left(\frac{1}{2} D+x\right)^{2}+\ell^{2}}=\frac{1}{2} \lambda+\sqrt{\left(\frac{1}{2} D-x\right)^{2}+\ell^{2}} \rightarrow \\
\left(\frac{1}{2} D+x\right)^{2}+\ell^{2}=\frac{1}{4} \lambda^{2}+2\left(\frac{1}{2} \lambda\right) \sqrt{\left(\frac{1}{2} D-x\right)^{2}+\ell^{2}}+\left(\frac{1}{2} D-x\right)^{2}+\ell^{2} \\
2 D x-\frac{1}{4} \lambda^{2}=\lambda \sqrt{\left(\frac{1}{2} D-x\right)^{2}+\ell^{2}} \rightarrow 4 D^{2} x^{2}-2(2 D x) \frac{1}{4} \lambda^{2}+\frac{1}{16} \lambda^{4}=\lambda^{2}\left[\left(\frac{1}{2} D-x\right)^{2}+\ell^{2}\right] \\
4 D^{2} x^{2}-D x \lambda^{2}+\frac{1}{16} \lambda^{4}=\frac{1}{4} D^{2} \lambda^{2}-D x \lambda^{2}+x^{2} \lambda^{2}+\lambda^{2} \ell^{2} \rightarrow x=\lambda \sqrt{\frac{\left(\frac{1}{4} D^{2}+\ell^{2}-\frac{1}{16} \lambda^{2}\right)}{\left(4 D^{2}-\lambda^{2}\right)}}
\end{array}
\end{aligned}
$$

The values are $D=3.00 \mathrm{~m}, \ell=3.20 \mathrm{~m}$, and $\lambda=v / f=(343 \mathrm{~m} / \mathrm{s}) /(494 \mathrm{~Hz})=0.694 \mathrm{~m}$.

$$
x=(0.694 \mathrm{~m}) \sqrt{\frac{\frac{1}{4}(3.00 \mathrm{~m})^{2}+(3.20 \mathrm{~m})^{2}-\frac{1}{16}(0.694 \mathrm{~m})^{2}}{4(3.00 \mathrm{~m})^{2}-(0.694 \mathrm{~m})^{2}}}=0.411 \mathrm{~m}
$$

(b) When the speakers are exactly out of phase, the maxima and minima will be interchanged. The intensity maxima are 0.411 m to the left or right of the midpoint, and the intensity minimum is at the midpoint.
59. The beat frequency is 3 beats per 2 seconds, or 1.5 Hz . We assume the strings are the same length and the same mass density.
(a) The other string must be either $220.0 \mathrm{~Hz}-1.5 \mathrm{~Hz}=218.5 \mathrm{~Hz}$ or $220.0 \mathrm{~Hz}+1.5 \mathrm{~Hz}$ $=221.5 \mathrm{~Hz}$.
(b) Since $f=\frac{v}{2 \ell}=\frac{1}{2 \ell} \sqrt{\frac{F_{\mathrm{T}}}{\mu}}$, we have $f \propto \sqrt{F_{\mathrm{T}}} \rightarrow \frac{f}{\sqrt{F_{\mathrm{T}}}}=\frac{f^{\prime}}{\sqrt{F_{\mathrm{T}}^{\prime}}} \rightarrow F_{\mathrm{T}}^{\prime}=F_{\mathrm{T}}\left(\frac{f^{\prime}}{f}\right)^{2}$.

To change 218.5 Hz to $220.0 \mathrm{~Hz}: \quad F^{\prime}=F_{\mathrm{T}}\left(\frac{220.0}{218.5}\right)^{2}=1.014,1.4 \%$ increase.
To change 221.5 Hz to $220.0 \mathrm{~Hz}: \quad F^{\prime}=F_{\mathrm{T}}\left(\frac{220.0}{221.5}\right)^{2}=0.9865,1.3 \%$ decrease .
60. (a) To find the beat frequency, calculate the frequency of each sound, and then subtract the two frequencies.

$$
f_{\text {beat }}=\left|f_{1}-f_{2}\right|=\left|\frac{v}{\lambda_{1}}-\frac{v}{\lambda_{2}}\right|=(343 \mathrm{~m} / \mathrm{s})\left|\frac{1}{2.64 \mathrm{~m}}-\frac{1}{2.72 \mathrm{~m}}\right|=3.821 \mathrm{~Hz} \approx 4 \mathrm{~Hz}
$$

(b) The speed of sound is $343 \mathrm{~m} / \mathrm{s}$, and the beat frequency is 3.821 Hz . The regions of maximum intensity are one "beat wavelength" apart.

$$
\lambda=\frac{v}{f}=\frac{343 \mathrm{~m} / \mathrm{s}}{3.821 \mathrm{~Hz}}=89.79 \mathrm{~m} \approx 90 \mathrm{~m} \text { (2 sig. fig.) }
$$

61. (a) Observer moving towards stationary source.

$$
f^{\prime}=\left(1+\frac{v_{\text {obs }}}{v_{\text {snd }}}\right) f=\left(1+\frac{30.0 \mathrm{~m} / \mathrm{s}}{343 \mathrm{~m} / \mathrm{s}}\right)(1350 \mathrm{~Hz})=1470 \mathrm{~Hz}
$$

(b) Observer moving away from stationary source.

$$
f^{\prime}=\left(1-\frac{v_{\mathrm{obs}}}{v_{\mathrm{snd}}}\right) f=\left(1-\frac{30.0 \mathrm{~m} / \mathrm{s}}{343 \mathrm{~m} / \mathrm{s}}\right)(1350 \mathrm{~Hz})=1230 \mathrm{~Hz}
$$

62. The moving object can be treated as a moving "observer" for calculating the frequency it receives and reflects. The bat (the source) is stationary.

$$
f_{\text {object }}^{\prime}=f_{\text {bat }}\left(1-\frac{v_{\text {object }}}{v_{\text {snd }}}\right)
$$

Then the object can be treated as a moving source emitting the frequency $f_{\text {object }}^{\prime}$, and the bat as a stationary observer.

$$
f_{\text {bat }}^{\prime \prime}=\frac{f_{\text {object }}^{\prime}}{\left(1+\frac{v_{\text {object }}}{v_{\text {snd }}}\right)}=f_{\text {bat }} \frac{\left(1-\frac{v_{\text {object }}}{v_{\text {snd }}}\right)}{\left(1+\frac{v_{\text {object }}}{v_{\text {snd }}}\right)}=f_{\text {bat }} \frac{\left(v_{\text {snd }}-v_{\text {object }}\right)}{\left(v_{\text {snd }}+v_{\text {object }}\right)}
$$

$$
=\left(5.00 \times 10^{4} \mathrm{~Hz}\right) \frac{343 \mathrm{~m} / \mathrm{s}-30.0 \mathrm{~m} / \mathrm{s}}{343 \mathrm{~m} / \mathrm{s}+30.0 \mathrm{~m} / \mathrm{s}}=4.20 \times 10^{4} \mathrm{~Hz}
$$

63. (a) For the $18 \mathrm{~m} / \mathrm{s}$ relative velocity:

$$
\begin{aligned}
& f_{\substack{\text { source } \\
\text { moving }}}^{\prime}=f \frac{1}{\left(1-\frac{v_{\text {src }}}{v_{\text {snd }}}\right)}=(2300 \mathrm{~Hz}) \frac{1}{\left(1-\frac{18 \mathrm{~m} / \mathrm{s}}{343 \mathrm{~m} / \mathrm{s}}\right)}=2427 \mathrm{~Hz} \approx 2430 \mathrm{~Hz} \\
& f_{\substack{\text { observer } \\
\text { moving }}}^{\prime}=f\left(1+\frac{v_{\text {sre }}}{v_{\text {snd }}}\right)=(2300 \mathrm{~Hz})\left(1+\frac{18 \mathrm{~m} / \mathrm{s}}{343 \mathrm{~m} / \mathrm{s}}\right)=2421 \mathrm{~Hz} \approx 2420 \mathrm{~Hz}
\end{aligned}
$$

The frequency shifts are slightly different, with $f_{\substack{\text { source } \\ \text { moving }}}^{\prime}>f_{\substack{\text { observer } \\ \text { moving }}}^{\prime}$. The two frequencies are close, but they are not identical. As a means of comparison, calculate the spread in frequencies divided by the original frequency.

$$
\frac{f_{\text {source }}^{\prime}-f_{\text {oubing }}^{\prime}}{\substack{\text { moverer } \\ \text { moving }}}=\frac{2427 \mathrm{~Hz}-2421 \mathrm{~Hz}}{f_{\text {source }}}=0.0026=0.26 \%
$$

(b) For the $160 \mathrm{~m} / \mathrm{s}$ relative velocity:

$$
\begin{aligned}
& f_{\substack{\text { source } \\
\text { moving }}}^{\prime}=f \frac{1}{\left(1-\frac{v_{\text {src }}}{v_{\text {snd }}}\right)}=(2300 \mathrm{~Hz}) \frac{1}{\left(1-\frac{160 \mathrm{~m} / \mathrm{s}}{343 \mathrm{~m} / \mathrm{s}}\right)}=4311 \mathrm{~Hz} \approx 4310 \mathrm{~Hz} \\
& f_{\text {osserver }}^{\text {moving }}
\end{aligned}=f\left(1+\frac{v_{\text {scr }}}{v_{\text {snd }}}\right)=(2300 \mathrm{~Hz})\left(1+\frac{160 \mathrm{~m} / \mathrm{s}}{343 \mathrm{~m} / \mathrm{s}}\right)=3372 \mathrm{~Hz} \approx 3370 \mathrm{~Hz} .
$$

The difference in the frequency shifts is much larger this time, still with $f_{\substack{\text { source } \\ \text { moving }}}^{\prime}>f_{\substack{\text { observer } \\ \text { moving }}}^{\prime}$.

$$
\frac{\substack{f_{\text {source }}^{\prime}-f_{\text {oboserver }}^{\prime} \\ \text { moving }}}{\prime}=\frac{4311 \mathrm{~Hz}-3372 \mathrm{~Hz}}{2300 \mathrm{~Hz}}=0.4083=41 \%
$$

(c) For the $320 \mathrm{~m} / \mathrm{s}$ relative velocity:

$$
\begin{aligned}
& f_{\substack{\text { source } \\
\text { moving }}}^{\prime}=f \frac{1}{\left(1-\frac{v_{\text {scc }}}{v_{\text {snd }}}\right)}=(2300 \mathrm{~Hz}) \frac{1}{\left(1-\frac{320 \mathrm{~m} / \mathrm{s}}{343 \mathrm{~m} / \mathrm{s}}\right)}=34,300 \mathrm{~Hz} \\
& f_{\substack{\text { observer } \\
\text { moving }}}^{\prime}=f\left(1+\frac{v_{\text {src }}}{v_{\text {snd }}}\right)=(2300 \mathrm{~Hz})\left(1+\frac{320 \mathrm{~m} / \mathrm{s}}{343 \mathrm{~m} / \mathrm{s}}\right)=4446 \mathrm{~Hz} \approx 4450 \mathrm{~Hz}
\end{aligned}
$$

The difference in the frequency shifts is quite large, still with $f_{\substack{\text { source } \\ \text { moving }}}^{\prime}>f_{\substack{\text { observer } \\ \text { moving }}}^{\prime}$.

$$
\left.\frac{f_{\text {surce }}^{\prime}-f_{\text {moving }}^{\prime}}{\prime} \begin{array}{l}
\text { moserver } \\
\text { moving }
\end{array}\right)=\frac{34,300 \mathrm{~Hz}-4446 \mathrm{~Hz}}{2300 \mathrm{~Hz}}=12.98=1300 \%
$$

(d) The Doppler formulas are asymmetric, with a larger shift for the moving source than for the moving observer, when the two are getting closer to each other. In the following derivation, assume $v_{\text {src }} \ll v_{\text {snd }}$, and use the binomial expansion.

$$
\underset{\substack{\text { source } \\ \text { moving }}}{\prime}=f \frac{1}{\left(1-\frac{v_{\text {src }}}{v_{\text {snd }}}\right)}=f\left(1-\frac{v_{\text {src }}}{v_{\text {snd }}}\right)^{-1} \approx f\left(1+\frac{v_{\text {src }}}{v_{\text {snd }}}\right)=f_{\substack{\text { observer } \\ \text { moving }}}^{\prime}
$$

64. The frequency received by the stationary car is higher than the frequency emitted by the stationary car, by $\Delta f=4.5 \mathrm{~Hz}$.

$$
\begin{aligned}
& f_{\text {obs }}=f_{\text {source }}+\Delta f=\frac{f_{\text {source }}}{\left(1-\frac{v_{\text {source }}}{v_{\text {snd }}}\right)} \rightarrow \\
& f_{\text {source }}=\Delta f\left(\frac{v_{\text {snd }}}{v_{\text {source }}}-1\right)=(4.5 \mathrm{~Hz})\left(\frac{343 \mathrm{~m} / \mathrm{s}}{15 \mathrm{~m} / \mathrm{s}}-1\right)=98 \mathrm{~Hz}
\end{aligned}
$$

65. (a) The observer is stationary, and the source is moving. First the source is approaching, then the source is receding.

$$
\begin{aligned}
& 120.0 \mathrm{~km} / \mathrm{h}\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)=33.33 \mathrm{~m} / \mathrm{s} \\
& f_{\substack{\text { sourve } \\
\text { nownd } \\
\text { towards }}}^{\prime}=f \frac{1}{\left(1-\frac{v_{\text {src }}}{v_{\text {snd }}}\right)}=(1280 \mathrm{~Hz}) \frac{1}{\left(1-\frac{33.33 \mathrm{~m} / \mathrm{s}}{343 \mathrm{~m} / \mathrm{s}}\right)}=1420 \mathrm{~Hz} \\
& f_{\substack{\text { source } \\
\text { moving } \\
\text { away }}}^{\prime}=f \frac{1}{\left(1+\frac{v_{\text {src }}}{v_{\text {snd }}}\right)}=(1280 \mathrm{~Hz}) \frac{1}{\left(1+\frac{33.33 \mathrm{~m} / \mathrm{s}}{343 \mathrm{~m} / \mathrm{s}}\right)}=1170 \mathrm{~Hz}
\end{aligned}
$$

(b) Both the observer and the source are moving, and so use Eq. 16-11.

$$
\begin{aligned}
& 90.0 \mathrm{~km} / \mathrm{h}\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)=25 \mathrm{~m} / \mathrm{s} \\
& f_{\text {approaching }}^{\prime}=f \frac{\left(v_{\text {snd }}+v_{\text {obs }}\right)}{\left(v_{\text {snd }}-v_{\text {src }}\right)}=(1280 \mathrm{~Hz}) \frac{(343 \mathrm{~m} / \mathrm{s}+25 \mathrm{~m} / \mathrm{s})}{(343 \mathrm{~m} / \mathrm{s}-33.33 \mathrm{~m} / \mathrm{s})}=1520 \mathrm{~Hz} \\
& f_{\text {receding }}^{\prime}=f \frac{\left(v_{\text {snd }}-v_{\text {oss }}\right)}{\left(v_{\text {snd }}+v_{\text {src }}\right)}=(1280 \mathrm{~Hz}) \frac{(343 \mathrm{~m} / \mathrm{s}-25 \mathrm{~m} / \mathrm{s})}{(343 \mathrm{~m} / \mathrm{s}+33.33 \mathrm{~m} / \mathrm{s})}=1080 \mathrm{~Hz}
\end{aligned}
$$

(c) Both the observer and the source are moving, and so again use Eq. 16-11.

$$
\begin{aligned}
& 80.0 \mathrm{~km} / \mathrm{h}\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{h}}\right)=22.22 \mathrm{~m} / \mathrm{s} \\
& f_{\substack{\text { police } \\
\text { approaching }}}^{\prime}=f \frac{\left(v_{\text {snd }}-v_{\text {obs }}\right)}{\left(v_{\text {snd }}-v_{\text {stc }}\right)}=(1280 \mathrm{~Hz}) \frac{(343 \mathrm{~m} / \mathrm{s}-22.22 \mathrm{~m} / \mathrm{s})}{(343 \mathrm{~m} / \mathrm{s}-33.33 \mathrm{~m} / \mathrm{s})}=1330 \mathrm{~Hz} \\
& f_{\substack{\text { police } \\
\text { car } \\
\text { receding }}}^{\prime}=f \frac{\left(v_{\text {snd }}+v_{\text {obs }}\right)}{\left(v_{\text {snd }}+v_{\text {src }}\right)}=(1280 \mathrm{~Hz}) \frac{(343 \mathrm{~m} / \mathrm{s}+22.22 \mathrm{~m} / \mathrm{s})}{(343 \mathrm{~m} / \mathrm{s}+33.33 \mathrm{~m} / \mathrm{s})}=1240 \mathrm{~Hz}
\end{aligned}
$$

66. The wall can be treated as a stationary "observer" for calculating the frequency it receives. The bat is flying toward the wall.

$$
f_{\text {wall }}^{\prime}=f_{\text {bat }} \frac{1}{\left(1-\frac{v_{\text {bat }}}{v_{\text {snd }}}\right)}
$$

Then the wall can be treated as a stationary source emitting the frequency $f_{\text {wall }}^{\prime}$, and the bat as a moving observer, flying toward the wall.

$$
\begin{aligned}
f_{\text {bat }}^{\prime \prime} & =f_{\text {wall }}^{\prime}\left(1+\frac{v_{\text {bat }}}{v_{\text {snd }}}\right)=f_{\text {bat }} \frac{1}{\left(1-\frac{v_{\text {bat }}}{v_{\text {snd }}}\right)}\left(1+\frac{v_{\text {bat }}}{v_{\text {snd }}}\right)=f_{\text {bat }} \frac{\left(v_{\text {snd }}+v_{\text {bat }}\right)}{\left(v_{\text {snd }}-v_{\text {bat }}\right)} \\
& =\left(3.00 \times 10^{4} \mathrm{~Hz}\right) \frac{343 \mathrm{~m} / \mathrm{s}+7.0 \mathrm{~m} / \mathrm{s}}{343 \mathrm{~m} / \mathrm{s}-7.0 \mathrm{~m} / \mathrm{s}}=3.13 \times 10^{4} \mathrm{~Hz}
\end{aligned}
$$

67. We assume that the comparison is to be made from the frame of reference of the stationary tuba. The stationary observers would observe a frequency from the moving tuba of

$$
f_{\text {obs }}=\frac{f_{\text {source }}}{\left(1-\frac{v_{\text {source }}}{v_{\text {snd }}}\right)}=\frac{75 \mathrm{~Hz}}{\left(1-\frac{12.0 \mathrm{~m} / \mathrm{s}}{343 \mathrm{~m} / \mathrm{s}}\right)}=78 \mathrm{~Hz} \quad f_{\text {beat }}=78 \mathrm{~Hz}-75 \mathrm{~Hz}=3 \mathrm{~Hz} .
$$

68. For the sound to be shifted up by one note, we must have $f_{\substack{\text { souree } \\ \text { moving }}}^{\prime}=f\left(2^{1 / 12}\right)$.

$$
\begin{aligned}
& f_{\substack{\text { source } \\
\text { moving }}}^{\prime}=f \frac{1}{\left(1-\frac{v_{\text {src }}}{v_{\text {snd }}}\right)}=f\left(2^{1 / 12}\right) \rightarrow \\
& v_{\text {src }}=\left(1-\frac{1}{2^{1 / 12}}\right) v_{\text {snd }}=\left(1-\frac{1}{2^{1 / 12}}\right)(343 \mathrm{~m} / \mathrm{s})=19.25 \mathrm{~m} / \mathrm{s}\left(\frac{3.6 \mathrm{~km} / \mathrm{h}}{\mathrm{~m} / \mathrm{s}}\right)=69.3 \mathrm{~km} / \mathrm{h}
\end{aligned}
$$

69. The ocean wave has $\lambda=44 \mathrm{~m}$ and $v=18 \mathrm{~m} / \mathrm{s}$ relative to the ocean floor. The frequency of the ocean wave is then $f=\frac{v}{\lambda}=\frac{18 \mathrm{~m} / \mathrm{s}}{44 \mathrm{~m}}=0.409 \mathrm{~Hz}$.
(a) For the boat traveling west, the boat will encounter a Doppler shifted frequency, for an observer moving towards a stationary source. The speed $v=18 \mathrm{~m} / \mathrm{s}$ represents the speed of the waves in the stationary medium, and so corresponds to the speed of sound in the Doppler formula. The time between encountering waves is the period of the Doppler shifted frequency.

$$
\begin{aligned}
& f_{\substack{\text { obserer } \\
\text { moving }}}^{\prime}=\left(1+\frac{v_{\text {obs }}}{v_{\text {snd }}}\right) f=\left(1+\frac{15 \mathrm{~m} / \mathrm{s}}{18 \mathrm{~m} / \mathrm{s}}\right)(0.409 \mathrm{~Hz})=0.750 \mathrm{~Hz} \rightarrow \\
& T=\frac{1}{f}=\frac{1}{0.750 \mathrm{~Hz}}=1.3 \mathrm{~s}
\end{aligned}
$$

(b) For the boat traveling east, the boat will encounter a Doppler shifted frequency, for an observer moving away from a stationary source.

$$
\begin{aligned}
& f_{\substack{\text { observer } \\
\text { moving }}}^{\prime}=\left(1-\frac{v_{\text {obs }}}{v_{\text {snd }}}\right) f=\left(1-\frac{15 \mathrm{~m} / \mathrm{s}}{18 \mathrm{~m} / \mathrm{s}}\right)(0.409 \mathrm{~Hz})=0.0682 \mathrm{~Hz} \rightarrow \\
& T=\frac{1}{f}=\frac{1}{0.0682 \mathrm{~Hz}}=15 \mathrm{~s}
\end{aligned}
$$

70. The Doppler effect occurs only when there is relative motion of the source and the observer along the line connecting them. In the first four parts of this problem, the whistle and the observer are not moving relative to each other and so there is no Doppler shift. The wind speed increases (or decreases) the velocity of the waves in the direction of the wind, as if the speed of sound were different, but the frequency of the waves doesn't change. We do a detailed analysis of this claim in part (a).
(a) The wind velocity is a movement of the medium, and so adds or subtracts from the speed of sound in the medium. Because the wind is blowing away from the observer, the effective speed of sound is $v_{\text {snd }}-v_{\text {wind }}$. The wavelength of the waves traveling towards the observer is $\lambda_{a}=\left(v_{\text {snd }}-v_{\text {wind }}\right) / f_{0}$, where $f_{0}$ is the frequency emitted by the factory whistle. This wavelength approaches the observer at a relative speed of $v_{\text {snd }}-v_{\text {wind }}$. Thus the observer hears the frequency calculated here.

$$
f_{a}=\frac{v_{\text {snd }}-v_{\text {wind }}}{\lambda_{a}}=\frac{v_{\text {snd }}-v_{\text {wind }}}{\frac{v_{\text {snd }}-v_{\text {wind }}}{f_{0}}}=f_{0}=720 \mathrm{~Hz}
$$

(b) Because the wind is blowing towards the observer, the effective speed of sound is $v_{\text {snd }}+v_{\text {wind }}$. The same kind of analysis as applied in part (a) gives that $f_{b}=720 \mathrm{~Hz}$.
(c) Because the wind is blowing perpendicular to the line towards the observer, the effective speed of sound along that line is $v_{\text {snd }}$. Since there is no relative motion of the whistle and observer, there will be no change in frequency, and so $f_{c}=720 \mathrm{~Hz}$.
(d) This is just like part (c), and so $f_{d}=720 \mathrm{~Hz}$.
(e) Because the wind is blowing toward the cyclist, the effective speed of sound is $v_{\text {snd }}+v_{\text {wind }}$. The wavelength traveling toward the cyclist is $\lambda_{e}=\left(v_{\text {snd }}+v_{\text {wind }}\right) / f_{0}$. This wavelength approaches the cyclist at a relative speed of $v_{\text {snd }}+v_{\text {wind }}+v_{\text {cycle }}$. The cyclist will hear the following frequency.

$$
\begin{aligned}
f_{e} & =\frac{\left(v_{\text {snd }}+v_{\text {wind }}+v_{\text {cycle }}\right)}{\lambda_{e}}=\frac{\left(v_{\text {snd }}+v_{\text {wind }}+v_{\text {cycle }}\right)}{\left(v_{\text {snd }}+v_{\text {wind }}\right)} f_{0}=\frac{(343+15.0+12.0) \mathrm{m} / \mathrm{s}}{(343+15.0)}(720 \mathrm{~Hz}) \\
& =744 \mathrm{~Hz}
\end{aligned}
$$

(f) Since the wind is not changing the speed of the sound waves moving towards the cyclist, the speed of sound is $343 \mathrm{~m} / \mathrm{s}$. The observer is moving towards a stationary source with a speed of $12.0 \mathrm{~m} / \mathrm{s}$.

$$
f^{\prime}=f\left(1+\frac{v_{\text {obs }}}{v_{\text {sns }}}\right)=(720 \mathrm{~Hz})\left(1+\frac{12.0 \mathrm{~m} / \mathrm{s}}{343 \mathrm{~m} / \mathrm{s}}\right)=745 \mathrm{~Hz}
$$

71. The maximum Doppler shift occurs when the heart has its maximum velocity. Assume that the heart is moving away from the original source of sound. The beats arise from the combining of the original 2.25 MHz frequency with the reflected signal which has been Doppler shifted. There are two Doppler shifts - one for the heart receiving the original signal (observer moving away from stationary source) and one for the detector receiving the reflected signal (source moving away from stationary observer).

$$
\begin{aligned}
& f_{\text {heart }}^{\prime}=f_{\text {original }}\left(1-\frac{v_{\text {heart }}}{v_{\text {snd }}}\right) \quad f_{\text {detector }}^{\prime \prime}=\frac{f_{\text {heart }}^{\prime}}{\left(1+\frac{v_{\text {heart }}}{v_{\text {snd }}}\right)}=f_{\text {original }} \frac{\left(1-\frac{v_{\text {heat }}}{v_{\text {snd }}}\right)}{\left(1+\frac{v_{\text {heart }}}{v_{\text {snd }}}\right)}=f_{\text {original }} \frac{\left(v_{\text {snd }}-v_{\text {heart }}\right)}{\left(v_{\text {snd }}+v_{\text {heart }}\right)} \\
& \Delta f=f_{\text {original }}-f_{\text {detector }}^{\prime \prime}=f_{\text {original }}-f_{\text {original }} \frac{\left(v_{\text {snd }} v_{\text {blood }}\right)}{\left(v_{\text {snd }}+v_{\text {blood }}\right)}=f_{\text {original }} \frac{2 v_{\text {blood }}}{\left(v_{\text {snd }}+v_{\text {blood }}\right)} \rightarrow \\
& v_{\text {blood }}=v_{\text {snd }} \frac{\Delta f}{2 f_{\text {original }}-\Delta f}=\left(1.54 \times 10^{3} \mathrm{~m} / \mathrm{s}\right) \frac{260 \mathrm{~Hz}}{2\left(2.25 \times 10^{6} \mathrm{~Hz}\right)-260 \mathrm{~Hz}}=8.9 \times 10^{-2} \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

If instead we had assumed that the heart was moving towards the original source of sound, we would get $v_{\text {blood }}=v_{\text {snd }} \frac{\Delta f}{2 f_{\text {original }}+\Delta f}$. Since the beat frequency is much smaller than the original frequency, the $\Delta f$ term in the denominator does not significantly affect the answer.
72. (a) The angle of the shock wave front relative to the direction of motion is given by Eq. 16-12.

$$
\sin \theta=\frac{v_{\text {snd }}}{v_{\text {obj }}}=\frac{v_{\text {snd }}}{2.0 v_{\text {snd }}}=\frac{1}{2.0} \rightarrow \theta=\sin ^{-1} \frac{1}{2.0}=30^{\circ} \text { (2 sig. fig.) }
$$

(b) The displacement of the plane $\left(v_{\text {obj }} t\right)$ from the time it passes overhead to the time the shock wave reaches the observer is shown, along with the shock wave front. From the displacement and height of the plane, the time is found.

$$
\begin{aligned}
& \tan \theta=\frac{h}{v_{\text {obj }} t} \rightarrow t=\frac{h}{v_{\text {obj }} \tan \theta} \\
& =\frac{6500 \mathrm{~m}}{(2.0)(310 \mathrm{~m} / \mathrm{s}) \tan 30^{\circ}}=18 \mathrm{~s}
\end{aligned}
$$

73. (a) The Mach number is the ratio of the object's speed to the speed of sound.

$$
M=\frac{v_{\text {obs }}}{v_{\text {sound }}}=\frac{\left(1.5 \times 10^{4} \mathrm{~km} / \mathrm{hr}\right)\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{hr}}\right)}{45 \mathrm{~m} / \mathrm{s}}=92.59 \approx 93
$$

(b) Use Eq. 16-125 to find the angle.

$$
\theta=\sin ^{-1} \frac{v_{\text {snd }}}{v_{\text {obj }}}=\sin ^{-1} \frac{1}{M}=\sin ^{-1} \frac{1}{92.59}=0.62^{\circ}
$$

74. From Eq. 16-12, $\sin \theta=\frac{v_{\text {snd }}}{v_{\text {obj }}}$.
(a) $\theta=\sin ^{-1} \frac{v_{\text {snd }}}{v_{\text {obj }}}=\sin ^{-1} \frac{343 \mathrm{~m} / \mathrm{s}}{8800 \mathrm{~m} / \mathrm{s}}=2.2^{\circ}$
(b) $\theta=\sin ^{-1} \frac{v_{\text {snd }}}{v_{\text {obj }}}=\sin ^{-1} \frac{1560 \mathrm{~m} / \mathrm{s}}{8800 \mathrm{~m} / \mathrm{s}}=10^{\circ}$ (2 sig. fig.)
75. Consider one particular wave as shown in the diagram, created at the location of the black dot. After a time $t$ has elapsed from the creation of that wave, the supersonic source has moved a distance $v_{\text {obj }} t$, and the wave front has moved a distance $v_{\text {snd }} t$. The line from the position of the source at time $t$ is tangent to all of the wave fronts, showing the
 location of the shock wave. A tangent to a circle at a point is perpendicular to the radius connecting that point to the center, and so a right angle is formed. From the right triangle, the angle $\theta$ can be defined.

$$
\sin \theta=\frac{v_{\mathrm{snd}} t}{v_{\mathrm{obj}} t}=\frac{v_{\mathrm{snd}}}{v_{\mathrm{obj}}}
$$

76. (a) The displacement of the plane from the time it passes overhead to the time the shock wave reaches the listener is shown, along with the shock wave front. From the displacement and height of the plane, the angle of the shock wave front relative to the direction of motion can be found. Then use Eq. 16-12.

$$
\begin{align*}
\tan \theta & =\frac{1.25 \mathrm{~km}}{2.0 \mathrm{~km}} \rightarrow \theta=\tan ^{-1} \frac{1.25}{2.0}=32^{\circ} \\
M= & \frac{v_{\text {obj }}}{v_{\text {snd }}} \tag{b}
\end{align*}=\frac{1}{\sin \theta}=\frac{1}{\sin 32^{\circ}}=1.9 .
$$

77. Find the angle of the shock wave, and then find the distance the plane has traveled when the shock wave reaches the observer. Use Eq. 16-12.

$$
\begin{aligned}
& \theta=\sin ^{-1} \frac{v_{\text {snd }}}{v_{\text {obj }}}=\sin ^{-1} \frac{v_{\text {snd }}}{2.2 v_{\text {snd }}}=\sin ^{-1} \frac{1}{2.2}=27^{\circ} \\
& \tan \theta=\frac{9500 \mathrm{~m}}{D} \rightarrow D=\frac{9500 \mathrm{~m}}{\tan 27^{\circ}}=18616 \mathrm{~m}=19 \mathrm{~km}
\end{aligned}
$$


78. The minimum time between pulses would be the time for a pulse to travel from the boat to the maximum distance and back again. The total distance traveled by the pulse will be 150 m , at the speed of sound in fresh water, $1440 \mathrm{~m} / \mathrm{s}$.

$$
d=v t \rightarrow t=\frac{d}{v}=\frac{150 \mathrm{~m}}{1440 \mathrm{~m} / \mathrm{s}}=0.10 \mathrm{~s}
$$

79. Assume that only the fundamental frequency is heard. The fundamental frequency of an open pipe is given by $f=\frac{v}{2 L}$.
(a) $f_{3.0}=\frac{v}{2 L}=\frac{343 \mathrm{~m} / \mathrm{s}}{2(3.0 \mathrm{~m})}=57 \mathrm{~Hz} \quad f_{2.5}=\frac{v}{2 L}=\frac{343 \mathrm{~m} / \mathrm{s}}{2(2.5 \mathrm{~m})}=69 \mathrm{~Hz}$
$f_{2.0}=\frac{v}{2 L}=\frac{343 \mathrm{~m} / \mathrm{s}}{2(2.0 \mathrm{~m})}=86 \mathrm{~Hz} \quad f_{1.5}=\frac{v}{2 L}=\frac{343 \mathrm{~m} / \mathrm{s}}{2(1.5 \mathrm{~m})}=114.3 \mathrm{~Hz} \approx 110 \mathrm{~Hz}$
$f_{1.0}=\frac{v}{2 L}=\frac{343 \mathrm{~m} / \mathrm{s}}{2(1.0 \mathrm{~m})}=171.5 \mathrm{~Hz} \approx 170 \mathrm{~Hz}$
(b) On a noisy day, there are a large number of component frequencies to the sounds that are being made - more people walking, more people talking, etc. Thus it is more likely that the frequencies listed above will be a component of the overall sound, and then the resonance will be more prominent to the hearer. If the day is quiet, there might be very little sound at the desired frequencies, and then the tubes will not have any standing waves in them to detect.
80. The single mosquito creates a sound intensity of $I_{0}=1 \times 10^{-12} \mathrm{~W} / \mathrm{m}^{2}$. Thus 100 mosquitoes will create a sound intensity of 100 times that of a single mosquito.

$$
I=100 I_{0} \quad \beta=10 \log \frac{100 I_{0}}{I_{0}}=10 \log 100=20 \mathrm{~dB} .
$$

81. The two sound level values must be converted to intensities, then the intensities added, and then converted back to sound level.

$$
\begin{aligned}
& I_{82}: 82 \mathrm{~dB}=10 \log \frac{I_{82}}{I_{0}} \rightarrow I_{82}=10^{8.2} I_{0}=1.585 \times 10^{8} I_{0} \\
& I_{89}: 89 \mathrm{~dB}=10 \log \frac{I_{87}}{I_{0}} \rightarrow I_{89}=10^{8.9} I_{0}=7.943 \times 10^{8} I_{0} \\
& I_{\text {toata }}=I_{82}+I_{89}=\left(9.528 \times 10^{8}\right) I_{0} \rightarrow \\
& \beta_{\text {total }}=10 \log \frac{9.528 \times 10^{8} I_{0}}{I_{0}}=10 \log 6.597 \times 10^{8}=89.8 \mathrm{~dB} \approx 90 \mathrm{~dB}(2 \text { sig. fig. })
\end{aligned}
$$

82. The power output is found from the intensity, which is the power radiated per unit area.

$$
\begin{aligned}
& 115 \mathrm{~dB}=10 \log \frac{I}{I_{0}} \rightarrow I=10^{11.5} I_{0}=10^{11.5}\left(1.0 \times 10^{-12} \mathrm{~W} / \mathrm{m}^{2}\right)=3.162 \times 10^{-1} \mathrm{~W} / \mathrm{m}^{2} \\
& I=\frac{P}{A}=\frac{P}{4 \pi r^{2}} \rightarrow P=4 \pi r^{2} I=4 \pi(9.00 \mathrm{~m})^{2}\left(3.162 \times 10^{-1} \mathrm{~W} / \mathrm{m}^{2}\right)=322 \mathrm{~W}
\end{aligned}
$$

83. Relative to the 1000 Hz output, the 15 kHz output is -12 dB .

$$
-12 \mathrm{~dB}=10 \log \frac{P_{15 \mathrm{kHz}}}{175 \mathrm{~W}} \rightarrow-1.2=\log \frac{P_{15 \mathrm{kHz}}}{175 \mathrm{~W}} \rightarrow 10^{-1.2}=\frac{P_{15 \mathrm{kHz}}}{175 \mathrm{~W}} \rightarrow P_{15 \mathrm{kHz}}=11 \mathrm{~W}
$$

84. The 130 dB level is used to find the intensity, and the intensity is used to find the power. It is assumed that the jet airplane engine radiates equally in all directions.

$$
\begin{aligned}
& \beta=130 \mathrm{~dB}=10 \log \frac{I}{I_{0}} \rightarrow I=10^{13} I_{0}=10^{13}\left(1.0 \times 10^{-12} \mathrm{~W} / \mathrm{m}^{2}\right)=1.0 \times 10^{1} \mathrm{~W} / \mathrm{m}^{2} \\
& P=I A=I \pi r^{2}=\left(1.0 \times 10^{1} \mathrm{~W} / \mathrm{m}^{2}\right) \pi\left(2.0 \times 10^{-2}\right)^{2}=0.013 \mathrm{~W}
\end{aligned}
$$

85. The gain is given by $\beta=10 \log \frac{P_{\text {out }}}{P_{\text {in }}}=10 \log \frac{125 \mathrm{~W}}{1.0 \times 10^{-3} \mathrm{~W}}=51 \mathrm{~dB}$.
86. It is desired that the sound from the speaker arrives at a listener 30 ms after the sound from the singer arrives. The fact that the speakers are 3.0 m behind the singer adds in a delay of $\frac{3.0 \mathrm{~m}}{343 \mathrm{~m} / \mathrm{s}}=$ $8.7 \times 10^{-3} \mathrm{~s}$, or about 9 ms . Thus there must be 21 ms of delay added into the electronic circuitry.
87. The strings are both tuned to the same frequency, and both have the same length. The mass per unit length is the density times the cross sectional area. The frequency is related to the tension by Eqs. $15-1$ and 15-2.

$$
\begin{aligned}
& f=\frac{v}{2 \ell} ; v=\sqrt{\frac{F_{\mathrm{T}}}{\mu}} \rightarrow f=\frac{1}{2 \ell} \sqrt{\frac{F_{\mathrm{T}}}{\mu}}=\frac{1}{2 \ell} \sqrt{\frac{F_{\mathrm{T}}}{\rho \pi r^{2}}} \rightarrow F_{\mathrm{T}}=4 \ell^{2} \rho f^{2} \pi r^{2} \rightarrow \\
& \frac{F_{\text {Thigh }}}{F_{\text {Tlow }}}=\frac{4 \ell^{2} \rho f^{2} \pi r_{\text {high }}^{2}}{4 \ell^{2} \rho f^{2} \pi r_{\text {low }}^{2}}=\left(\frac{r_{\text {ligh }}}{r_{\text {low }}}\right)^{2}=\left(\frac{\frac{1}{2} d_{\text {high }}}{\frac{1}{2} d_{\text {low }}}\right)^{2}=\left(\frac{0.724 \mathrm{~mm}}{0.699 \mathrm{~mm}}\right)^{2}=1.07
\end{aligned}
$$

88. The strings are both tuned to the same frequency, and both have the same length. The mass per unit length is the density times the cross sectional area. The frequency is related to the tension by Eqs. 15-1 and 15-2.

$$
\begin{aligned}
& f=\frac{v}{2 \ell} ; v=\sqrt{\frac{F_{\mathrm{T}}}{\mu}} \rightarrow f=\frac{1}{2 \ell} \sqrt{\frac{F_{\mathrm{T}}}{\mu}}=\frac{1}{2 \ell} \sqrt{\frac{F_{\mathrm{T}}}{\rho \pi r^{2}}} \rightarrow F_{\mathrm{T}}=4 \ell^{2} \rho f^{2} \pi r^{2} \rightarrow \\
& \frac{F_{\text {Tacostic }}}{F_{\text {Telectic }}}=\frac{4 \ell^{2} \rho_{\text {acoustic }} f^{2} \pi r_{\text {acoustic }}^{2}}{4 \ell^{2} \rho_{\text {electric }} f^{2} \pi r_{\text {electric }}^{2}}=\frac{\rho_{\text {acoustic }} r_{\text {acoustic }}^{2}}{\rho_{\text {electric }}^{2} r_{\text {lectric }}^{2}}=\left(\frac{\rho_{\text {acoustic }}}{\rho_{\text {electric }}}\right)\left(\frac{d_{\text {acoustic }}}{d_{\text {electric }}}\right)^{2} \\
& =\left(\frac{7760 \mathrm{~kg} / \mathrm{m}^{3}}{7990 \mathrm{~kg} / \mathrm{m}^{3}}\right)\left(\frac{0.33 \mathrm{~m}}{0.25 \mathrm{~m}}\right)^{2}=1.7
\end{aligned}
$$

89. (a) The wave speed on the string can be found from the length and the fundamental frequency.

$$
f=\frac{v}{2 \ell} \rightarrow v=2 \ell f=2(0.32 \mathrm{~m})(440 \mathrm{~Hz})=281.6 \mathrm{~m} / \mathrm{s} \approx 280 \mathrm{~m} / \mathrm{s}
$$

The tension is found from the wave speed and the mass per unit length.

$$
v=\sqrt{\frac{F_{\mathrm{T}}}{\mu}} \rightarrow F_{\mathrm{T}}=\mu v^{2}=\left(7.21 \times 10^{-4} \mathrm{~kg} / \mathrm{m}\right)(281.6 \mathrm{~m} / \mathrm{s})^{2}=57 \mathrm{~N}
$$

(b) The length of the pipe can be found from the fundamental frequency and the speed of sound.

$$
f=\frac{v}{4 \ell} \rightarrow \ell=\frac{v}{4 f}=\frac{343 \mathrm{~m} / \mathrm{s}}{4(440 \mathrm{~Hz})}=0.1949 \mathrm{~m} \approx 0.19 \mathrm{~m}
$$

(c) The first overtone for the string is twice the fundamental. 880 Hz The first overtone for the open pipe is 3 times the fundamental. 1320 Hz
90. The apparatus is a closed tube. The water level is the closed end, and so is a node of air displacement. As the water level lowers, the distance from one resonance level to the next corresponds to the distance between adjacent nodes, which is one-half wavelength.

$$
\begin{aligned}
& \Delta \ell=\frac{1}{2} \lambda \rightarrow \lambda=2 \Delta \ell=2(0.395 \mathrm{~m}-0.125 \mathrm{~m})=0.540 \mathrm{~m} \\
& f=\frac{v}{\lambda}=\frac{343 \mathrm{~m} / \mathrm{s}}{0.540 \mathrm{~m}}=635 \mathrm{~Hz}
\end{aligned}
$$

91. The fundamental frequency of a tube closed at one end is given by $f_{1}=\frac{v}{4 \ell}$. The change in air temperature will change the speed of sound, resulting in two different frequencies.

$$
\begin{aligned}
& \frac{f_{30.0^{\circ} \mathrm{C}}}{f_{25.0^{\circ} \mathrm{C}}}=\frac{\frac{v_{30.0^{\circ} \mathrm{C}}}{4 \ell}}{\frac{v_{25.0^{\circ} \mathrm{C}}}{4 \ell}}=\frac{v_{30.0^{\circ} \mathrm{C}}}{v_{25.0^{\circ} \mathrm{C}}} \rightarrow f_{30.0^{\circ} \mathrm{C}}=f_{25.0^{\circ} \mathrm{C}}\left(\frac{v_{30.0^{\circ} \mathrm{C}}}{v_{25.0^{\circ} \mathrm{C}}}\right) \\
& \Delta f=f_{30.0^{\circ} \mathrm{C}}-f_{25.0^{\circ} \mathrm{C}}=f_{25.0^{\circ} \mathrm{C}}\left(\frac{v_{30.0^{\circ} \mathrm{C}}}{v_{25.0^{\circ} \mathrm{C}}}-1\right)=(349 \mathrm{~Hz})\left(\frac{331+0.60(30.0)}{331+0.60(25.0)}-1\right)=3 \mathrm{~Hz}
\end{aligned}
$$

92. Call the frequencies of four strings of the violin $f_{\mathrm{A}}, f_{\mathrm{B}}, f_{\mathrm{C}}, f_{\mathrm{D}}$ with $f_{\mathrm{A}}$ the lowest pitch. The mass per unit length will be named $\mu$. All strings are the same length and have the same tension. For a string with both ends fixed, the fundamental frequency is given by $f_{1}=\frac{v}{2 \ell}=\frac{1}{2 \ell} \sqrt{\frac{F_{\mathrm{T}}}{\mu}}$.

$$
\begin{aligned}
& f_{\mathrm{B}}=1.5 f_{\mathrm{A}} \rightarrow \frac{1}{2 \ell} \sqrt{\frac{F_{\mathrm{T}}}{\mu_{\mathrm{B}}}}=1.5 \frac{1}{2 \ell} \sqrt{\frac{F_{\mathrm{T}}}{\mu_{\mathrm{A}}}} \rightarrow \mu_{\mathrm{B}}=\frac{\mu_{\mathrm{A}}}{(1.5)^{2}}=0.44 \mu_{\mathrm{A}} \\
& f_{\mathrm{C}}=1.5 f_{\mathrm{B}}=(1.5)^{2} f_{\mathrm{A}} \rightarrow \frac{1}{2 \ell} \sqrt{\frac{F_{\mathrm{T}}}{\mu_{\mathrm{C}}}}=(1.5)^{2} \frac{1}{2 \ell} \sqrt{\frac{F_{\mathrm{T}}}{\mu_{\mathrm{A}}}} \rightarrow \mu_{\mathrm{C}}=\frac{\mu_{\mathrm{A}}}{(1.5)^{4}}=0.20 \mu_{\mathrm{A}} \\
& f_{\mathrm{D}}=1.5 f_{\mathrm{C}}=(1.5)^{3} f_{\mathrm{A}} \rightarrow \frac{1}{2 \ell} \sqrt{\frac{F_{\mathrm{T}}}{\mu_{\mathrm{D}}}}=(1.5)^{3} \frac{1}{2 \ell} \sqrt{\frac{F_{\mathrm{T}}}{\mu_{\mathrm{A}}}} \rightarrow \mu_{\mathrm{D}}=\frac{\mu_{\mathrm{A}}}{(1.5)^{6}}=0.088 \mu_{\mathrm{A}}
\end{aligned}
$$

93. The effective length of the tube is $\ell_{\text {eff. }}=\ell+\frac{1}{3} D=0.60 \mathrm{~m}+\frac{1}{3}(0.030 \mathrm{~m})=0.61 \mathrm{~m}$.

Uncorrected frequencies: $\quad f_{n}=\frac{(2 n-1) v}{4 \ell}, n=1,2,3 \ldots \rightarrow$

$$
f_{1-4}=(2 n-1) \frac{343 \mathrm{~m} / \mathrm{s}}{4(0.60 \mathrm{~m})}=143 \mathrm{~Hz}, 429 \mathrm{~Hz}, 715 \mathrm{~Hz}, 1000 \mathrm{~Hz}
$$

Corrected frequencies: $\quad f_{n}=\frac{(2 n-1) v}{4 \ell_{\text {eff }}}, n=1,2,3 \ldots \rightarrow$

$$
f_{1-4}=(2 n-1) \frac{343 \mathrm{~m} / \mathrm{s}}{4(0.61 \mathrm{~m})}=141 \mathrm{~Hz}, 422 \mathrm{~Hz}, 703 \mathrm{~Hz}, 984 \mathrm{~Hz}
$$

94. Since the sound is loudest at points equidistant from the two sources, the two sources must be in phase. The difference in distance from the two sources must be an odd number of half-wavelengths for destructive interference.

$$
\begin{array}{ll}
0.28 \mathrm{~m}=\lambda / 2 \rightarrow \lambda=0.56 \mathrm{~m} & f=v / \lambda=343 \mathrm{~m} / \mathrm{s} / 0.56 \mathrm{~m}=610 \mathrm{~Hz} \\
0.28 \mathrm{~m}=3 \lambda / 2 \rightarrow \lambda=0.187 \mathrm{~m} & f=v / \lambda=343 \mathrm{~m} / \mathrm{s} / 0.187 \mathrm{~m}=1840 \mathrm{~Hz} \text { (out of range) }
\end{array}
$$

95. As the train approaches, the observed frequency is given by $f_{\text {appoach }}^{\prime}=f /\left(1-\frac{v_{\text {train }}}{v_{\text {snd }}}\right)$. As the train recedes, the observed frequency is given by $f_{\text {recede }}^{\prime}=f /\left(1+\frac{v_{\text {train }}}{v_{\text {snd }}}\right)$. Solve each expression for $f$, equate them, and then solve for $v_{\text {train }}$.

$$
\begin{aligned}
& f_{\text {approach }}^{\prime}\left(1-\frac{v_{\text {train }}}{v_{\text {snd }}}\right)=f_{\text {recede }}^{\prime}\left(1+\frac{v_{\text {train }}}{v_{\text {snd }}}\right) \rightarrow \\
& v_{\text {train }}=v_{\text {snd }} \frac{\left(f_{\text {approach }}^{\prime}-f_{\text {recede }}^{\prime}\right)}{\left(f_{\text {approach }}^{\prime}+f_{\text {recede }}^{\prime}\right)}=(343 \mathrm{~m} / \mathrm{s}) \frac{(552 \mathrm{~Hz}-486 \mathrm{~Hz})}{(552 \mathrm{~Hz}+486 \mathrm{~Hz})}=22 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

96. The Doppler shift is 3.5 Hz , and the emitted frequency from both trains is 516 Hz . Thus the frequency received by the conductor on the stationary train is 519.5 Hz . Use this to find the moving train's speed.

$$
f^{\prime}=f \frac{v_{\text {snd }}}{\left(v_{\text {snd }}-v_{\text {source }}\right)} \rightarrow v_{\text {source }}=\left(1-\frac{f}{f^{\prime}}\right) v_{\text {snd }}=\left(1-\frac{516 \mathrm{~Hz}}{519.5 \mathrm{~Hz}}\right)(343 \mathrm{~m} / \mathrm{s})=2.31 \mathrm{~m} / \mathrm{s}
$$

97. (a) Since both speakers are moving towards the observer at the same speed, both frequencies have the same Doppler shift, and the observer hears no beats .
(b) The observer will detect an increased frequency from the speaker moving towards him and a decreased frequency from the speaker moving away. The difference in those two frequencies will be the beat frequency that is heard.

$$
\begin{aligned}
& f_{\text {towards }}^{\prime}=f \frac{1}{\left(1-\frac{v_{\text {train }}}{v_{\text {snd }}}\right)} \quad f_{\text {away }}^{\prime}=f \frac{1}{\left(1+\frac{v_{\text {train }}}{v_{\text {snd }}}\right)} \\
& f_{\text {towards }}^{\prime}-f_{\text {away }}^{\prime}=f \frac{1}{\left(1-\frac{v_{\text {train }}}{v_{\text {snd }}}\right)}-f \frac{1}{\left(1+\frac{v_{\text {train }}}{v_{\text {snd }}}\right)}=f\left[\frac{v_{\text {snd }}}{\left(v_{\text {snd }}-v_{\text {train }}\right)}-\frac{v_{\text {snd }}}{\left(v_{\text {snd }}+v_{\text {train }}\right)}\right] \\
& (348 \mathrm{~Hz})\left[\frac{343 \mathrm{~m} / \mathrm{s}}{(343 \mathrm{~m} / \mathrm{s}-10.0 \mathrm{~m} / \mathrm{s})}-\frac{343 \mathrm{~m} / \mathrm{s}}{(343 \mathrm{~m} / \mathrm{s}+10.0 \mathrm{~m} / \mathrm{s})}\right]=20 \mathrm{~Hz}(2 \text { sig. fig. })
\end{aligned}
$$

(c) Since both speakers are moving away from the observer at the same speed, both frequencies have the same Doppler shift, and the observer hears no beats .
98. For each pipe, the fundamental frequency is given by $f=\frac{v}{2 \ell}$. Find the frequency of the shortest pipe.

$$
f=\frac{v}{2 \ell}=\frac{343 \mathrm{~m} / \mathrm{s}}{2(2.40 \mathrm{~m})}=71.46 \mathrm{~Hz}
$$

The longer pipe has a lower frequency. Since the beat frequency is 8.0 Hz , the frequency of the longer pipe must be 63.46 Hz . Use that frequency to find the length of the longer pipe.

$$
f=\frac{v}{2 \ell} \rightarrow \ell=\frac{v}{2 f}=\frac{343 \mathrm{~m} / \mathrm{s}}{2(63.46 \mathrm{~Hz})}=2.70 \mathrm{~m}
$$

99. Use Eq. 16-11, which applies when both source and observer are in motion. There will be two Doppler shifts in this problem - first for the emitted sound with the bat as the source and the moth as the observer, and then the reflected sound with the moth as the source and the bat as the observer.

$$
\begin{aligned}
f_{\text {moth }}^{\prime}=f_{\text {bat }} \frac{\left(v_{\text {snd }}+v_{\text {moth }}\right)}{\left(v_{\text {snd }}-v_{\text {bat }}\right)} \quad f_{\text {bat }}^{\prime \prime} & =f_{\text {moth }}^{\prime} \frac{\left(v_{\text {snd }}+v_{\text {bat }}\right)}{\left(v_{\text {snd }}-v_{\text {moth }}\right)}=f_{\text {bat }} \frac{\left(v_{\text {snd }}+v_{\text {moth }}\right)}{\left(v_{\text {snd }}-v_{\text {bat }}\right)} \frac{\left(v_{\text {snd }}+v_{\text {bat }}\right)}{\left(v_{\text {snd }}-v_{\text {moth }}\right)} \\
& =(51.35 \mathrm{kHz}) \frac{(343+5.0)}{(343-7.5)} \frac{(343+7.5)}{(343-5.0)}=55.23 \mathrm{kHz}
\end{aligned}
$$

100. The beats arise from the combining of the original 3.80 MHz frequency with the reflected signal which has been Doppler shifted. There are two Doppler shifts - one for the blood cells receiving the original frequency (observer moving away from stationary source) and one for the detector receiving the reflected frequency (source moving away from stationary observer).

$$
\begin{aligned}
& f_{\text {blood }}^{\prime}=f_{\text {original }}\left(1-\frac{v_{\text {blood }}}{v_{\text {snd }}}\right) \quad f_{\text {detector }}^{\prime \prime}=\frac{f_{\text {blood }}^{\prime}}{\left(1+\frac{v_{\text {blood }}}{v_{\text {snd }}}\right)}=f_{\text {original }} \frac{\left(1-\frac{v_{\text {blood }}}{v_{\text {snd }}}\right)}{\left(1+\frac{v_{\text {blood }}}{v_{\text {snd }}}\right)}=f_{\text {original }} \frac{\left(v_{\text {snd }}-v_{\text {blood }}\right)}{\left(v_{\text {snd }}+v_{\text {blood }}\right)} \\
& \Delta f=f_{\text {original }}-f_{\text {detector }}^{\prime \prime}=f_{\text {original }}-f_{\text {original }} \frac{\left(v_{\text {snd }}-v_{\text {blood }}\right)}{\left(v_{\text {snd }}+v_{\text {blood }}\right)}=f_{\text {original }} \frac{2 v_{\text {blood }}}{\left(v_{\text {snd }}+v_{\text {blood }}\right)} \\
& =\left(3.80 \times 10^{6} \mathrm{~Hz}\right) \frac{2(0.32 \mathrm{~m} / \mathrm{s})}{\left(1.54 \times 10^{3} \mathrm{~m} / \mathrm{s}+0.32 \mathrm{~m} / \mathrm{s}\right)}=1600 \mathrm{~Hz}
\end{aligned}
$$

101. It is 70.0 ms from the start of one chirp to the start of the next. Since the chirp itself is 3.0 ms long, it is 67.0 ms from the end of a chirp to the start of the next. Thus the time for the pulse to travel to the moth and back again is 67.0 ms . The distance to the moth is half the distance that the sound can travel in 67.0 ms , since the sound must reach the moth and return during the 67.0 ms .

$$
d=v_{\text {snd }} t=(343 \mathrm{~m} / \mathrm{s}) \frac{1}{2}\left(67.0 \times 10^{-3} \mathrm{~s}\right)=11.5 \mathrm{~m}
$$

102. (a) We assume that $v_{\text {src }} \ll v_{\text {snd }}$, and use the binomial expansion.

$$
f_{\substack{\text { source } \\ \text { moving }}}^{\prime}=f \frac{1}{\left(1-\frac{v_{\text {sro }}}{v_{\text {snd }}}\right)}=f\left(1-\frac{v_{\text {src }}}{v_{\text {snd }}}\right)^{-1} \approx f\left(1+\frac{v_{\text {src }}}{v_{\text {snd }}}\right)=f_{\substack{\text { observer } \\ \text { moving }}}^{\prime}
$$

(b) We calculate the percent error in general, and then substitute in the given relative velocity.

$$
\begin{aligned}
& \% \text { error }=\left(\frac{\text { approx. }- \text { exact }}{\text { exact }}\right) 100=100\left(\frac{f\left(1+\frac{v_{\text {src }}}{v_{\text {snd }}}\right)-f \frac{1}{\left(1-\frac{v_{\text {src }}}{v_{\text {snd }}}\right)}}{f \frac{1}{\left(1-\frac{v_{\text {src }}}{v_{\text {snd }}}\right)}}\right) \\
& \quad=100\left[\left(1+\frac{v_{\text {src }}}{v_{\text {snd }}}\right)\left(1-\frac{v_{\text {src }}}{v_{\text {snd }}}\right)-1\right]=-100\left(\frac{v_{\text {src }}}{v_{\text {snd }}}\right)^{2}=-100\left(\frac{18.0 \mathrm{~m} / \mathrm{s}}{343 \mathrm{~m} / \mathrm{s}}\right)^{2}=-0.28 \%
\end{aligned}
$$

The negative sign indicates that the approximate value is less than the exact value.
103. The person will hear a frequency $f_{\text {towards }}^{\prime}=f\left(1+\frac{v_{\text {walk }}}{v_{\text {snd }}}\right)$ from the speaker that they walk towards.

The person will hear a frequency $f_{\text {away }}^{\prime}=f\left(1-\frac{v_{\text {walk }}}{v_{\text {snd }}}\right)$ from the speaker that they walk away from.
The beat frequency is the difference in those two frequencies.

$$
f_{\text {towards }}^{\prime}-f_{\text {away }}^{\prime}=f\left(1+\frac{v_{\text {walk }}}{v_{\text {snd }}}\right)-f\left(1-\frac{v_{\text {walk }}}{v_{\text {snd }}}\right)=2 f \frac{v_{\text {walk }}}{v_{\text {snd }}}=2(282 \mathrm{~Hz}) \frac{1.4 \mathrm{~m} / \mathrm{s}}{343 \mathrm{~m} / \mathrm{s}}=2.3 \mathrm{~Hz}
$$

104. There will be two Doppler shifts in this problem - first for a stationary source with a moving "observer" (the blood cells), and then for a moving source (the blood cells) and a stationary "observer" (the receiver). Note that the velocity component of the blood parallel to the sound transmission is $v_{\text {blood }} \cos 45^{\circ}=\frac{1}{\sqrt{2}} v_{\text {blood }}$. It is that component that causes the Doppler shift.

$$
\begin{aligned}
& f_{\text {blood }}^{\prime}=f_{\text {original }}\left(1-\frac{\frac{1}{\sqrt{2}} v_{\text {blood }}}{v_{\text {snd }}}\right) \\
& f_{\text {detector }}^{\prime \prime}=\frac{f_{\text {blood }}^{\prime}}{\left(1+\frac{\frac{1}{\sqrt{2}} v_{\text {blood }}}{v_{\text {snd }}}\right)}=f_{\text {original }} \frac{\left(1-\frac{\frac{1}{\sqrt{2}} v_{\text {blood }}}{v_{\text {snd }}}\right)}{\left(1+\frac{\frac{1}{\sqrt{2}} v_{\text {blood }}}{v_{\text {snd }}}\right)}=f_{\text {original }} \frac{\left(v_{\text {snd }}-\frac{1}{\sqrt{2}} v_{\text {blood }}\right)}{\left(v_{\text {snd }}+\frac{1}{\sqrt{2}} v_{\text {blood }}\right)} \rightarrow \\
& v_{\text {blood }}=\sqrt{2} \frac{\left(f_{\text {original }}-f_{\text {detector }}^{\prime \prime}\right)}{\left(f_{\text {detector }}^{\prime \prime}+f_{\text {original }}\right)} v_{\text {snd }}
\end{aligned}
$$

Since the cells are moving away from the transmitter / receiver combination, the final frequency received is less than the original frequency, by 780 Hz . Thus $f_{\text {detector }}^{\prime \prime}=f_{\text {original }}-780 \mathrm{~Hz}$.

$$
\begin{aligned}
v_{\text {blood }} & =\sqrt{2} \frac{\left(f_{\text {original }}-f_{\text {detector }}^{\prime \prime}\right)}{\left(f_{\text {detector }}^{\prime \prime}+f_{\text {original }}\right)} v_{\text {snd }}=\sqrt{2} \frac{(780 \mathrm{~Hz})}{\left(2 f_{\text {original }}-780 \mathrm{~Hz}\right)} v_{\text {snd }} \\
& =\sqrt{2} \frac{(780 \mathrm{~Hz})}{\left[2\left(5.0 \times 10^{6} \mathrm{~Hz}\right)-780 \mathrm{~Hz}\right]}(1540 \mathrm{~m} / \mathrm{s})=0.17 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

105. The apex angle is $15^{\circ}$, so the shock wave angle is $7.5^{\circ}$. The angle of the shock wave is also given by $\sin \theta=v_{\text {wave }} / v_{\text {object }}$.

$$
\sin \theta=v_{\text {wave }} / v_{\text {object }} \rightarrow v_{\text {object }}=v_{\text {wave }} / \sin \theta=2.2 \mathrm{~km} / \mathrm{h} / \sin 7.5^{\circ}=17 \mathrm{~km} / \mathrm{h}
$$

106. First, find the path difference in the original configuration. Then move the obstacle to the right by $\Delta d$ so that the path difference increases by $\frac{1}{2} \lambda$. Note that the path difference change must be on the same order as the wavelength, and so $\Delta d \ll d, \ell$ since $\lambda \ll \ell, d$.

$$
\begin{aligned}
& (\Delta D)_{\text {initial }}=2 \sqrt{d^{2}+\left(\frac{1}{2} \ell\right)^{2}}-\ell ;(\Delta D)_{\text {final }}=2 \sqrt{(d+\Delta d)^{2}+\left(\frac{1}{2} \ell\right)^{2}}-\ell \\
& (\Delta D)_{\text {final }}-(\Delta D)_{\text {initial }}=\frac{1}{2} \lambda=\left(2 \sqrt{(d+\Delta d)^{2}+\left(\frac{1}{2} \ell\right)^{2}}-\ell\right)-\left(2 \sqrt{d^{2}+\left(\frac{1}{2} \ell\right)^{2}}-\ell\right) \rightarrow \\
& 2 \sqrt{(d+\Delta d)^{2}+\left(\frac{1}{2} \ell\right)^{2}}=\frac{1}{2} \lambda+2 \sqrt{d^{2}+\left(\frac{1}{2} \ell\right)^{2}}
\end{aligned}
$$

Square the last equation above.

$$
4\left[d^{2}+2 d \Delta d+(\Delta d)^{2}+\left(\frac{1}{2} \ell\right)^{2}\right]=\frac{1}{4} \lambda^{2}+2\left(\frac{1}{2} \lambda\right) 2 \sqrt{d^{2}+\left(\frac{1}{2} \ell\right)^{2}}+4\left[d^{2}+\left(\frac{1}{2} \ell\right)^{2}\right]
$$

We delete terms that are second order in the small quantities $\Delta d$ and $\lambda$.

$$
8 d \Delta d=2 \lambda \sqrt{d^{2}+\left(\frac{1}{2} \ell\right)^{2}} \rightarrow \Delta d=\frac{\lambda}{4 d} \sqrt{d^{2}+\left(\frac{1}{2} \ell\right)^{2}}
$$

107. (a) The "singing" rod is manifesting standing waves. By holding the rod at its midpoint, it has a node at its midpoint, and antinodes at its ends. Thus the length of the rod is a half wavelength. The speed of sound in aluminum is found in Table 16-1.

$$
f=\frac{v}{\lambda}=\frac{v}{2 L}=\frac{5100 \mathrm{~m} / \mathrm{s}}{1.50 \mathrm{~m}}=3400 \mathrm{~Hz}
$$

(b) The wavelength of sound in the rod is twice the length of the rod, 1.50 m .
(c) The wavelength of the sound in air is determined by the frequency and the speed of sound in air.

$$
\lambda=\frac{v}{f}=\frac{343 \mathrm{~m} / \mathrm{s}}{3400 \mathrm{~Hz}}=0.10 \mathrm{~m}
$$

108. The displacement amplitude is related to the intensity by Eq. 15-7. The intensity can be calculated from the decibel value. The medium is air.

$$
\begin{aligned}
\beta & =10 \log \frac{I}{I_{0}} \rightarrow I=\left(10^{\beta / 10}\right) I_{0}=10^{10.5}\left(10^{-12} \mathrm{~W} / \mathrm{m}^{2}\right)=0.0316 \mathrm{~W} / \mathrm{m}^{2} \\
\text { (a) } \quad I & =2 \pi^{2} v \rho f^{2} A^{2} \rightarrow \\
A & =\frac{1}{\pi f} \sqrt{\frac{I}{2 v \rho}}=\frac{1}{\pi\left(8.0 \times 10^{3} \mathrm{~Hz}\right)} \sqrt{\frac{0.0316 \mathrm{~W} / \mathrm{m}^{2}}{2(343 \mathrm{~m} / \mathrm{s})\left(1.29 \mathrm{~kg} / \mathrm{m}^{3}\right)}}=2.4 \times 10^{-7} \mathrm{~m}
\end{aligned}
$$

(b)

$$
A=\frac{1}{\pi f} \sqrt{\frac{I}{2 v \rho}}=\frac{1}{\pi(35 \mathrm{~Hz})} \sqrt{\frac{0.0316 \mathrm{~W} / \mathrm{m}^{2}}{2(343 \mathrm{~m} / \mathrm{s})\left(1.29 \mathrm{~kg} / \mathrm{m}^{3}\right)}}=5.4 \times 10^{-5} \mathrm{~m}
$$

109. (a) The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH16.XLS," on tab "Problem 16.109a."

(b) The spreadsheet used for this problem can be found on the Media Manager, with filename $\backslash$ "PSE4_ISM_CH16.XLS," on tab "Problem 16.109b."


## CHAPTER 21: Electric Charges and Electric Field

## Responses to Questions

1. Rub a glass rod with silk and use it to charge an electroscope. The electroscope will end up with a net positive charge. Bring the pocket comb close to the electroscope. If the electroscope leaves move farther apart, then the charge on the comb is positive, the same as the charge on the electroscope. If the leaves move together, then the charge on the comb is negative, opposite the charge on the electroscope.
2. The shirt or blouse becomes charged as a result of being tossed about in the dryer and rubbing against the dryer sides and other clothes. When you put on the charged object (shirt), it causes charge separation within the molecules of your skin (see Figure 21-9), which results in attraction between the shirt and your skin.
3. Fog or rain droplets tend to form around ions because water is a polar molecule, with a positive region and a negative region. The charge centers on the water molecule will be attracted to the ions (positive to negative).
4. See also Figure 21-9 in the text. The negatively charged electrons in the paper are attracted to the positively charged rod and move towards it within their molecules. The attraction occurs because the negative charges in the paper are closer to the positive rod than are the positive charges in the paper, and therefore the attraction between the unlike charges is greater than the repulsion
 between the like charges.
5. A plastic ruler that has been rubbed with a cloth is charged. When brought near small pieces of paper, it will cause separation of charge in the bits of paper, which will cause the paper to be attracted to the ruler. On a humid day, polar water molecules will be attracted to the ruler and to the separated charge on the bits of paper, neutralizing the charges and thus eliminating the attraction.
6. The net charge on a conductor is the difference between the total positive charge and the total negative charge in the conductor. The "free charges" in a conductor are the electrons that can move about freely within the material because they are only loosely bound to their atoms. The "free electrons" are also referred to as "conduction electrons." A conductor may have a zero net charge but still have substantial free charges.

Most of the electrons are strongly bound to nuclei in the metal ions. Only a few electrons per atom (usually one or two) are free to move about throughout the metal. These are called the "conduction electrons." The rest are bound more tightly to the nucleus and are not free to move. Furthermore, in the cases shown in Figures 21-7 and 21-8, not all of the conduction electrons will move. In Figure 21-7, electrons will move until the attractive force on the remaining conduction electrons due to the incoming charged rod is balanced by the repulsive force from electrons that have already gathered at the left end of the neutral rod. In Figure 21-8, conduction electrons will be repelled by the incoming rod and will leave the stationary rod through the ground connection until the repulsive force on the remaining conduction electrons due to the incoming charged rod is balanced by the attractive force from the net positive charge on the stationary rod.
8. The electroscope leaves are connected together at the top. The horizontal component of this tension force balances the electric force of repulsion. (Note: The vertical component of the tension force balances the weight of the leaves.)
9. Coulomb's law and Newton's law are very similar in form. The electrostatic force can be either attractive or repulsive; the gravitational force can only be attractive. The electrostatic force constant is also much larger than the gravitational force constant. Both the electric charge and the gravitational mass are properties of the material. Charge can be positive or negative, but the gravitational mass only has one form.
10. The gravitational force between everyday objects on the surface of the Earth is extremely small. (Recall the value of G: $6.67 \times 10^{-11} \mathrm{Nm}^{2} / \mathrm{kg}^{2}$.) Consider two objects sitting on the floor near each other. They are attracted to each other, but the force of static fiction for each is much greater than the gravitational force each experiences from the other. Even in an absolutely frictionless environment, the acceleration resulting from the gravitational force would be so small that it would not be noticeable in a short time frame. We are aware of the gravitational force between objects if at least one of them is very massive, as in the case of the Earth and satellites or the Earth and you.

The electric force between two objects is typically zero or close to zero because ordinary objects are typically neutral or close to neutral. We are aware of electric forces between objects when the objects are charged. An example is the electrostatic force (static cling) between pieces of clothing when you pull the clothes out of the dryer.
11. Yes, the electric force is a conservative force. Energy is conserved when a particle moves under the influence of the electric force, and the work done by the electric force in moving an object between two points in space is independent of the path taken.
12. Coulomb observed experimentally that the force between two charged objects is directly proportional to the charge on each one. For example, if the charge on either object is tripled, then the force is tripled. This is not in agreement with a force that is proportional to the sum of the charges instead of to the product of the charges. Also, a charged object is not attracted to or repelled from a neutral object, which would be the case if the numerator in Coulomb's law were proportional to the sum of the charges.
13. When a charged ruler attracts small pieces of paper, the charge on the ruler causes a separation of charge in the paper. For example, if the ruler is negatively charged, it will force the electrons in the paper to the edge of the paper farthest from the ruler, leaving the near edge positively charged. If the paper touches the ruler, electrons will be transferred from the ruler to the paper, neutralizing the positive charge. This action leaves the paper with a net negative charge, which will cause it to be repelled by the negatively charged ruler.
14. The test charges used to measure electric fields are small in order to minimize their contribution to the field. Large test charges would substantially change the field being investigated.
15. When determining an electric field, it is best, but not required, to use a positive test charge. A negative test charge would be fine for determining the magnitude of the field. But the direction of the electrostatic force on a negative test charge will be opposite to the direction of the electric field. The electrostatic force on a positive test charge will be in the same direction as the electric field. In order to avoid confusion, it is better to use a positive test charge.
16. See Figure 21-34b. A diagram of the electric field lines around two negative charges would be just like this diagram except that the arrows on the field lines would point towards the charges instead of away from them. The distance between the charges is $l$.
17. The electric field will be strongest to the right of the positive charge (between the two charges) and weakest to the left of the positive charge. To the right of the positive charge, the contributions to the field from the two charges point in the same direction, and therefore add. To the left of the positive charge, the contributions to the field from the two charges point in opposite directions, and therefore subtract. Note that this is confirmed by the density of field lines in Figure 21-34a.
18. At point C , the positive test charge would experience zero net force. At points A and B , the direction of the force on the positive test charge would be the same as the direction of the field. This direction is indicated by the arrows on the field lines. The strongest field is at point A, followed (in order of decreasing field strength) by B and then C .
19. Electric field lines can never cross because they give the direction of the electrostatic force on a positive test charge. If they were to cross, then the force on a test charge at a given location would be in more than one direction. This is not possible.
20. The field lines must be directed radially toward or away from the point charge (see rule 1). The spacing of the lines indicates the strength of the field (see rule 2). Since the magnitude of the field due to the point charge depends only on the distance from the point charge, the lines must be distributed symmetrically.
21. The two charges are located along a line as shown in the diagram.
(a) If the signs of the charges are opposite then the point on the line where $E=0$ will lie to the left of Q . In that region
 the electric fields from the two charges will point in opposite directions, and the point will be closer to the smaller charge.
(b) If the two charges have the same sign, then the point on the line where $E=0$ will lie between the two charges, closer to the smaller charge. In this region, the electric fields from the two charges will point in opposite directions.
22. The electric field at point P would point in the negative $x$-direction. The magnitude of the field would be the same as that calculated for a positive distribution of charge on the ring:

$$
E=\frac{1}{4 \pi \varepsilon_{o}} \frac{Q x}{\left(x^{2}+a^{2}\right)^{3 / 2}}
$$

23. The velocity of the test charge will depend on its initial velocity. The field line gives the direction of the change in velocity, not the direction of the velocity. The acceleration of the test charge will be along the electric field line.
24. The value measured will be slightly less than the electric field value at that point before the test charge was introduced. The test charge will repel charges on the surface of the conductor and these charges will move along the surface to increase their distances from the test charge. Since they will then be at greater distances from the point being tested, they will contribute a smaller amount to the field.
25. The motion of the electron in Example 21-16 is projectile motion. In the case of the gravitational force, the acceleration of the projectile is in the same direction as the field and has a value of $g$; in the case of an electron in an electric field, the direction of the acceleration of the electron and the field direction are opposite, and the value of the acceleration varies.
26. Initially, the dipole will spin clockwise. It will "overshoot" the equilibrium position (parallel to the field lines), come momentarily to rest and then spin counterclockwise. The dipole will continue to oscillate back and forth if no damping forces are present. If there are damping forces, the amplitude will decrease with each oscillation until the dipole comes to rest aligned with the field.
27. If an electric dipole is placed in a nonuniform electric field, the charges of the dipole will experience forces of different magnitudes whose directions also may not be exactly opposite. The addition of these forces will leave a net force on the dipole.

## Solutions to Problems

1. Use Coulomb's law to calculate the magnitude of the force.

$$
F=k \frac{Q_{1} Q_{2}}{r^{2}}=\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{\left(1.602 \times 10^{-19} \mathrm{C}\right)\left(26 \times 1.602 \times 10^{-19} \mathrm{C}\right)}{\left(1.5 \times 10^{-12} \mathrm{~m}\right)^{2}}=2.7 \times 10^{-3} \mathrm{~N}
$$

2. Use the charge per electron to find the number of electrons.

$$
\left(-38.0 \times 10^{-6} \mathrm{C}\right)\left(\frac{1 \text { electron }}{-1.602 \times 10^{-19} \mathrm{C}}\right)=2.37 \times 10^{14} \text { electrons }
$$

3. Use Coulomb's law to calculate the magnitude of the force.

$$
F=k \frac{Q_{1} Q_{2}}{r^{2}}=\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{\left(25 \times 10^{-6} \mathrm{C}\right)\left(2.5 \times 10^{-3} \mathrm{C}\right)}{(0.28 \mathrm{~m})^{2}}=7200 \mathrm{~N}
$$

4. Use Coulomb's law to calculate the magnitude of the force.

$$
F=k \frac{Q_{1} Q_{2}}{r^{2}}=\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{\left(1.602 \times 10^{-19} \mathrm{C}\right)^{2}}{\left(4.0 \times 10^{-15} \mathrm{~m}\right)^{2}}=14 \mathrm{~N}
$$

5. The charge on the plastic comb is negative, so the comb has gained electrons.

$$
\frac{\Delta m}{m}=\frac{\left(3.0 \times 10^{-6} \mathrm{C}\right)\left(\frac{1 \mathrm{e}^{-}}{1.602 \times 10^{-19} \mathrm{C}}\right)\left(\frac{9.109 \times 10^{-31} \mathrm{~kg}}{1 \mathrm{e}^{-}}\right)}{0.035 \mathrm{~kg}}=4.9 \times 10^{-16}=4.9 \times 10^{-14 \%} \%
$$

6. Since the magnitude of the force is inversely proportional to the square of the separation distance, $F \propto \frac{1}{r^{2}}$, if the distance is multiplied by a factor of $1 / 8$, the force will be multiplied by a factor of 64 .

$$
F=64 F_{0}=64\left(3.2 \times 10^{-2} \mathrm{~N}\right)=2.0 \mathrm{~N}
$$

7. Since the magnitude of the force is inversely proportional to the square of the separation distance, $F \propto \frac{1}{r^{2}}$, if the force is tripled, the distance has been reduced by a factor of $\sqrt{3}$.

$$
r=\frac{r_{0}}{\sqrt{3}}=\frac{8.45 \mathrm{~cm}}{\sqrt{3}}=4.88 \mathrm{~cm}
$$

8. Use the charge per electron and the mass per electron.

$$
\begin{aligned}
& \left(-46 \times 10^{-6} \mathrm{C}\right)\left(\frac{1 \text { electron }}{-1.602 \times 10^{-19} \mathrm{C}}\right)=2.871 \times 10^{14} \approx 2.9 \times 10^{14} \text { electrons } \\
& \left(2.871 \times 10^{14} \mathrm{e}^{-}\right)\left(\frac{9.109 \times 10^{-31} \mathrm{~kg}}{1 \mathrm{e}^{-}}\right)=2.6 \times 10^{-16} \mathrm{~kg}
\end{aligned}
$$

9. To find the number of electrons, convert the mass to moles, the moles to atoms, and then multiply by the number of electrons in an atom to find the total electrons. Then convert to charge.

$$
\begin{aligned}
15 \mathrm{~kg} \mathrm{Au} & =(15 \mathrm{~kg} \mathrm{Au})\left(\frac{1 \mathrm{~mole} \mathrm{Al}}{0.197 \mathrm{~kg}}\right)\left(\frac{6.022 \times 10^{23} \text { atoms }}{1 \text { mole }}\right)\left(\frac{79 \text { electrons }}{1 \text { molecule }}\right)\left(\frac{-1.602 \times 10^{-19} \mathrm{C}}{\text { electron }}\right) \\
& =-5.8 \times 10^{8} \mathrm{C}
\end{aligned}
$$

The net charge of the bar is 0 C , since there are equal numbers of protons and electrons.
10. Take the ratio of the electric force divided by the gravitational force.

$$
\frac{F_{\mathrm{E}}}{F_{\mathrm{G}}}=\frac{k \frac{Q_{1} Q_{2}}{r^{2}}}{G \frac{m_{1} m_{2}}{r^{2}}}=\frac{k Q_{1} Q_{2}}{G m_{1} m_{2}}=\frac{\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)\left(1.602 \times 10^{-19} \mathrm{C}\right)^{2}}{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(9.11 \times 10^{-31} \mathrm{~kg}\right)\left(1.67 \times 10^{-27} \mathrm{~kg}\right)}=2.3 \times 10^{39}
$$

The electric force is about $2.3 \times 10^{39}$ times stronger than the gravitational force for the given scenario.
11. (a) Let one of the charges be $q$, and then the other charge is $Q_{\mathrm{T}}-q$. The force between the charges is $F_{\mathrm{E}}=k \frac{q\left(Q_{\mathrm{T}}-q\right)}{r^{2}}=\frac{k}{r^{2}}\left(q Q_{\mathrm{T}}-q^{2}\right)$. To find the maximum and minimum force, set the first derivative equal to 0 . Use the second derivative test as well.

$$
\begin{aligned}
& F_{\mathrm{E}}=\frac{k}{r^{2}}\left(q Q_{\mathrm{T}}-q^{2}\right) ; \frac{d F_{\mathrm{E}}}{d q}=\frac{k}{r^{2}}\left(Q_{\mathrm{T}}-2 q\right)=0 \rightarrow q=\frac{1}{2} Q_{\mathrm{T}} \\
& \frac{d^{2} F_{\mathrm{E}}}{d q^{2}}=-\frac{2 k}{r^{2}}<0 \rightarrow q=\frac{1}{2} Q_{\mathrm{T}} \text { gives }\left(F_{\mathrm{E}}\right)_{\max }
\end{aligned}
$$

So $q_{1}=q_{2}=\frac{1}{2} Q_{\mathrm{T}}$ gives the maximum force.
(b) If one of the charges has all of the charge, and the other has no charge, then the force between them will be 0 , which is the minimum possible force. So $q_{1}=0, q_{2}=Q_{\mathrm{T}}$ gives the minimum force.
12. Let the right be the positive direction on the line of charges. Use the fact that like charges repel and unlike charges attract to determine the direction of the forces. In the following expressions, $k=8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}$.

$$
\begin{aligned}
& \overrightarrow{\mathbf{F}}_{+75}=-k \frac{(75 \mu \mathrm{C})(48 \mu \mathrm{C})}{(0.35 \mathrm{~m})^{2}} \hat{\mathbf{i}}+k \frac{(75 \mu \mathrm{C})(85 \mu \mathrm{C})}{(0.70 \mathrm{~m})^{2}} \hat{\mathbf{i}}=-147.2 \mathrm{~N} \hat{\mathbf{i}} \approx-150 \mathrm{~N} \hat{\mathbf{i}} \\
& \overrightarrow{\mathbf{F}}_{+48}=k \frac{(75 \mu \mathrm{C})(48 \mu \mathrm{C})}{(0.35 \mathrm{~m})^{2}} \hat{\mathbf{i}}+k \frac{(48 \mu \mathrm{C})(85 \mu \mathrm{C})}{(0.35 \mathrm{~m})^{2}} \hat{\mathbf{i}}=563.5 \mathrm{~N} \hat{\mathbf{i}} \approx 560 \mathrm{~N} \hat{\mathbf{i}} \\
& \overrightarrow{\mathbf{F}}_{-85}=-k \frac{(85 \mu \mathrm{C})(75 \mu \mathrm{C})}{(0.70 \mathrm{~m})^{2}} \hat{\mathbf{i}}-k \frac{(85 \mu \mathrm{C})(48 \mu \mathrm{C})}{(0.35 \mathrm{~m})^{2}} \hat{\mathbf{i}}=-416.3 \mathrm{~N} \hat{\mathbf{i}} \approx-420 \mathrm{~N} \hat{\mathbf{i}}
\end{aligned}
$$

13. The forces on each charge lie along a line connecting the charges. Let the variable $d$ represent the length of a side of the triangle. Since the triangle is equilateral, each angle is $60^{\circ}$. First calculate the magnitude of each individual force.

$$
\begin{aligned}
F_{12} & =k \frac{\left|Q_{1} Q_{2}\right|}{d^{2}}=\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{\left(7.0 \times 10^{-6} \mathrm{C}\right)\left(8.0 \times 10^{-6} \mathrm{C}\right)}{(1.20 \mathrm{~m})^{2}} \\
& =0.3495 \mathrm{~N} \\
F_{13} & =k \frac{\left|Q_{1} Q_{3}\right|}{d^{2}}=\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{\left(7.0 \times 10^{-6} \mathrm{C}\right)\left(6.0 \times 10^{-6} \mathrm{C}\right)}{(1.20 \mathrm{~m})^{2}} \\
& =0.2622 \mathrm{~N} \\
F_{23} & =k \frac{\left|Q_{2} Q_{3}\right|}{d^{2}}=\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{\left(8.0 \times 10^{-6} \mathrm{C}\right)\left(6.0 \times 10^{-6} \mathrm{C}\right)}{(1.20 \mathrm{~m})^{2}}=0.2996 \mathrm{~N}=F_{32}
\end{aligned}
$$



Now calculate the net force on each charge and the direction of that net force, using components.

$$
\begin{aligned}
& F_{1 x}=F_{12 x}+F_{13 x}=-(0.3495 \mathrm{~N}) \cos 60^{\circ}+(0.2622 \mathrm{~N}) \cos 60^{\circ}=-4.365 \times 10^{-2} \mathrm{~N} \\
& F_{1 y}=F_{12 y}+F_{13 y}=-(0.3495 \mathrm{~N}) \sin 60^{\circ}-(0.2622 \mathrm{~N}) \sin 60^{\circ}=-5.297 \times 10^{-1} \mathrm{~N} \\
& F_{1}=\sqrt{F_{1 x}^{2}+F_{1 y}^{2}}=0.53 \mathrm{~N} \quad \theta_{1}=\tan ^{-1} \frac{F_{1 y}}{F_{1 x}}=\tan ^{-1} \frac{-5.297 \times 10^{-1} \mathrm{~N}}{-4.365 \times 10^{-2} \mathrm{~N}}=265^{\circ} \\
& F_{2 x}=F_{21 x}+F_{23 x}=(0.3495 \mathrm{~N}) \cos 60^{\circ}-(0.2996 \mathrm{~N})=-1.249 \times 10^{-1} \mathrm{~N} \\
& F_{2 y}=F_{21 y}+F_{23 y}=(0.3495 \mathrm{~N}) \sin 60^{\circ}+0=3.027 \times 10^{-1} \mathrm{~N} \\
& F_{2}=\sqrt{F_{2 x}^{2}+F_{2 y}^{2}}=0.33 \mathrm{~N} \quad \theta_{2}=\tan ^{-1} \frac{F_{2 y}}{F_{2 x}}=\tan ^{-1} \frac{3.027 \times 10^{-1} \mathrm{~N}}{-1.249 \times 10^{-1} \mathrm{~N}}=112^{\circ} \\
& F_{3 x}=F_{31 x}+F_{32 x}=-(0.2622 \mathrm{~N}) \cos 60^{\circ}+(0.2996 \mathrm{~N})=1.685 \times 10^{-1} \mathrm{~N} \\
& F_{3 y}=F_{31 y}+F_{32 y}=(0.2622 \mathrm{~N}) \sin 60^{\circ}+0=2.271 \times 10^{-1} \mathrm{~N} \\
& F_{3}=\sqrt{F_{3 x}^{2}+F_{3 y}^{2}}=0.26 \mathrm{~N} \quad \theta_{3}=\tan ^{-1} \frac{F_{3 y}}{F_{3 x}}=\tan ^{-1} \frac{2.271 \times 10^{-1} \mathrm{~N}}{1.685 \times 10^{-1} \mathrm{~N}}=53^{\circ}
\end{aligned}
$$

14. (a) If the force is repulsive, both charges must be positive since the total charge is positive. Call the total charge $Q$.

$$
\begin{aligned}
& Q_{1}+Q_{2}=Q \quad F=\frac{k Q_{1} Q_{2}}{d^{2}}=\frac{k Q_{1}\left(Q-Q_{1}\right)}{d^{2}} \rightarrow Q_{1}^{2}-Q Q_{1}+\frac{F d^{2}}{k}=0 \\
& Q_{1} \\
& =\frac{Q \pm \sqrt{Q^{2}-4 \frac{F d^{2}}{k}}}{2}=\frac{Q \pm \sqrt{Q^{2}-4 \frac{F d^{2}}{k}}}{2} \\
& \\
& =\frac{1}{2}\left[\left(90.0 \times 10^{-6} \mathrm{C}\right) \pm \sqrt{\left(90.0 \times 10^{-6} \mathrm{C}\right)^{2}-4 \frac{(12.0 \mathrm{~N})(1.16 \mathrm{~m})^{2}}{\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)}}\right] \\
& \\
& =60.1 \times 10^{-6} \mathrm{C}, 29.9 \times 10^{-6} \mathrm{C}
\end{aligned}
$$

(b) If the force is attractive, then the charges are of opposite sign. The value used for $F$ must then be negative. Other than that, the solution method is the same as for part $(a)$.

$$
\begin{aligned}
& Q_{1}+Q_{2}=Q \quad F=\frac{k Q_{1} Q_{2}}{d^{2}}=\frac{k Q_{1}\left(Q-Q_{1}\right)}{d^{2}} \rightarrow Q_{1}^{2}-Q Q_{1}+\frac{F d^{2}}{k}=0 \\
& Q_{1}
\end{aligned} \begin{aligned}
& 2 \pm \sqrt{Q^{2}-4 \frac{F d^{2}}{k}}=\frac{Q \pm \sqrt{Q^{2}-4 \frac{F d^{2}}{k}}}{2} \\
& \\
& =\frac{1}{2}\left[\left(90.0 \times 10^{-6} \mathrm{C}\right) \pm \sqrt{\left(90.0 \times 10^{-6} \mathrm{C}\right)^{2}-4 \frac{(-12.0 \mathrm{~N})(1.16 \mathrm{~m})^{2}}{\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)}}\right] \\
& \\
& =106.8 \times 10^{-6} \mathrm{C},-16.8 \times 10^{-6} \mathrm{C}
\end{aligned}
$$

15. Determine the force on the upper right charge, and then use the symmetry of the configuration to determine the force on the other three charges. The force at the upper right corner of the square is the vector sum of the forces due to the other three charges. Let the variable $d$ represent the 0.100 m length of a side of the square, and let the variable $Q$ represent the 4.15 mC charge at each corner.

$$
\begin{aligned}
& F_{41}=k \frac{Q^{2}}{d^{2}} \rightarrow F_{41 x}=k \frac{Q^{2}}{d^{2}}, F_{41 y}=0 \\
& F_{42}=k \frac{Q^{2}}{2 d^{2}} \rightarrow F_{42 x}=k \frac{Q^{2}}{2 d^{2}} \cos 45^{\circ}=k \frac{\sqrt{2} Q^{2}}{4 d^{2}}, F_{42 y}=k \frac{\sqrt{2} Q^{2}}{4 d^{2}} \\
& F_{43}=k \frac{Q^{2}}{d^{2}} \rightarrow F_{43 x}=0, F_{43 y}=k \frac{Q^{2}}{d^{2}}
\end{aligned}
$$



Add the $x$ and $y$ components together to find the total force, noting that $F_{4 x}=F_{4 y}$.

$$
\begin{aligned}
& F_{4 x}=F_{41 x}+F_{42 x}+F_{43 x}=k \frac{Q^{2}}{d^{2}}+k \frac{\sqrt{2} Q^{2}}{4 d^{2}}+0=k \frac{Q^{2}}{d^{2}}\left(1+\frac{\sqrt{2}}{4}\right)=F_{4 y} \\
& F_{4}=\sqrt{F_{4 x}^{2}+F_{4 y}^{2}}=k \frac{Q^{2}}{d^{2}}\left(1+\frac{\sqrt{2}}{4}\right) \sqrt{2}=k \frac{Q^{2}}{d^{2}}\left(\sqrt{2}+\frac{1}{2}\right)
\end{aligned}
$$

$$
\begin{aligned}
& =\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{\left(4.15 \times 10^{-3} \mathrm{C}\right)^{2}}{(0.100 \mathrm{~m})^{2}}\left(\sqrt{2}+\frac{1}{2}\right)=2.96 \times 10^{7} \mathrm{~N} \\
\theta & =\tan ^{-1} \frac{F_{4 y}}{F_{4 x}}=45^{\circ} \text { above the } x \text {-direction. }
\end{aligned}
$$

For each charge, the net force will be the magnitude determined above, and will lie along the line from the center of the square out towards the charge.
16. Determine the force on the upper right charge, and then use the symmetry of the configuration to determine the force on the other charges.

The force at the upper right corner of the square is the vector sum of the forces due to the other three charges. Let the variable $d$ represent the 0.100 m length of a side of the square, and let the variable $Q$ represent the 4.15 mC charge at each corner.

$$
\begin{aligned}
& F_{41}=k \frac{Q^{2}}{d^{2}} \rightarrow F_{41 x}=-k \frac{Q^{2}}{d^{2}}, F_{41 y}=0 \\
& F_{42}=k \frac{Q^{2}}{2 d^{2}} \rightarrow F_{42 x}=k \frac{Q^{2}}{2 d^{2}} \cos 45^{\circ}=k \frac{\sqrt{2} Q^{2}}{4 d^{2}}, F_{42 y}=k \frac{\sqrt{2} Q^{2}}{4 d^{2}} \\
& F_{43}=k \frac{Q^{2}}{d^{2}} \rightarrow F_{43 x}=0, F_{43 y}=-k \frac{Q^{2}}{d^{2}}
\end{aligned}
$$



Add the $x$ and $y$ components together to find the total force, noting that $F_{4 x}=F_{4 y}$.

$$
\begin{aligned}
F_{4 x} & =F_{41 x}+F_{42 x}+F_{43 x}=-k \frac{Q^{2}}{d^{2}}+k \frac{\sqrt{2} Q^{2}}{4 d^{2}}+0=k \frac{Q^{2}}{d^{2}}\left(-1+\frac{\sqrt{2}}{4}\right)=-0.64645 k \frac{Q^{2}}{d^{2}}=F_{4 y} \\
F_{4} & =\sqrt{F_{4 x}^{2}+F_{4 y}^{2}}=k \frac{Q^{2}}{d^{2}}(0.64645) \sqrt{2}=k \frac{Q^{2}}{d^{2}}(0.9142) \\
& =\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{\left(4.15 \times 10^{-3} \mathrm{C}\right)^{2}}{(0.100 \mathrm{~m})^{2}}(0.9142)=1.42 \times 10^{7} \mathrm{~N}
\end{aligned}
$$

$$
\theta=\tan ^{-1} \frac{F_{4 y}}{F_{4 x}}=225^{\circ} \text { from the } x \text {-direction, or exactly towards the center of the square. }
$$

For each charge, there are two forces that point towards the adjacent corners, and one force that points away from the center of the square. Thus for each charge, the net force will be the magnitude of $1.42 \times 10^{7} \mathrm{~N}$ and will lie along the line from the charge inwards towards the center of the square.
17. The spheres can be treated as point charges since they are spherical, and so Coulomb's law may be used to relate the amount of charge to the force of attraction. Each sphere will have a magnitude $Q$ of charge, since that amount was removed from one sphere and added to the other, being initially uncharged.

$$
\begin{aligned}
F=k \frac{Q_{1} Q_{2}}{r^{2}}=k \frac{Q^{2}}{r^{2}} \rightarrow Q & =r \sqrt{\frac{F}{k}}=(0.12 \mathrm{~m}) \sqrt{\frac{1.7 \times 10^{-2} \mathrm{~N}}{8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}}} \\
& =1.650 \times 10^{-7} \mathrm{C}\left(\frac{1 \text { electron }}{1.602 \times 10^{-19} \mathrm{C}}\right)=1.0 \times 10^{12} \text { electrons }
\end{aligned}
$$

18. The negative charges will repel each other, and so the third charge must put an opposite force on each of the original charges.
Consideration of the various possible configurations leads to the conclusion that the third charge must be positive and must be between the other two charges. See the diagram for the definition of variables.


For each negative charge, equate the magnitudes of the two forces on the charge. Also note that $0<x<\ell$.
left: $k \frac{Q_{0} Q}{x^{2}}=k \frac{4 Q_{0}^{2}}{\ell^{2}} \quad$ right: $k \frac{4 Q_{0} Q}{(\ell-x)^{2}}=k \frac{4 Q_{0}^{2}}{\ell^{2}} \quad \rightarrow$
$k \frac{Q_{0} Q}{x^{2}}=k \frac{4 Q_{0} Q}{(\ell-x)^{2}} \rightarrow x=\frac{1}{3} \ell$
$k \frac{Q_{0} Q}{x^{2}}=k \frac{4 Q_{0}^{2}}{\ell^{2}} \rightarrow Q=4 Q_{0} \frac{x^{2}}{\ell^{2}}=Q_{0} \frac{4}{(3)^{2}}=\frac{4}{9} Q_{0}$
Thus the charge should be of magnitude $\frac{4}{9} Q_{0}$, and a distance $\frac{1}{3} \ell$ from $-Q_{0}$ towards $-4 Q_{0}$.
19. (a) The charge will experience a force that is always pointing towards the origin. In the diagram, there is a greater force of
$\frac{Q q}{4 \pi \varepsilon_{0}(d-x)^{2}}$ to the left, and a lesser force of $\frac{Q q}{4 \pi \varepsilon_{0}(d+x)^{2}}$ to

the right. So the net force is towards the origin. The same would be true if the mass were to the left of the origin. Calculate the net force.

$$
\begin{aligned}
F_{\text {net }} & =\frac{Q q}{4 \pi \varepsilon_{0}(d+x)^{2}}-\frac{Q q}{4 \pi \varepsilon_{0}(d-x)^{2}}=\frac{Q q}{4 \pi \varepsilon_{0}(d+x)^{2}(d-x)^{2}}\left[(d-x)^{2}-(d+x)^{2}\right] \\
& =\frac{-4 Q q d}{4 \pi \varepsilon_{0}(d+x)^{2}(d-x)^{2}} x=\frac{-Q q d}{\pi \varepsilon_{0}(d+x)^{2}(d-x)^{2}} x
\end{aligned}
$$

We assume that $x \ll d$.

$$
F_{\mathrm{net}}=\frac{-Q q d}{\pi \varepsilon_{0}(d+x)^{2}(d-x)^{2}} x \approx \frac{-Q q}{\pi \varepsilon_{0} d^{3}} x
$$

This has the form of a simple harmonic oscillator, where the "spring constant" is $k_{\text {elastic }}=\frac{Q q}{\pi \varepsilon_{0} d^{3}}$.
The spring constant can be used to find the period. See Eq. 14-7b.

$$
T=2 \pi \sqrt{\frac{m}{k_{\text {elastic }}}}=2 \pi \sqrt{\frac{m}{\frac{Q q}{\pi \varepsilon_{0} d^{3}}}}=2 \pi \sqrt{\frac{m \pi \varepsilon_{0} d^{3}}{Q q}}
$$

(b) Sodium has an atomic mass of 23.

$$
\begin{aligned}
& T=2 \pi \sqrt{\frac{m \pi \varepsilon_{0} d^{3}}{Q q}}=2 \pi \sqrt{\frac{(29)\left(1.66 \times 10^{-27} \mathrm{~kg}\right) \pi\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)\left(3 \times 10^{-10} \mathrm{~m}\right)^{3}}{\left(1.60 \times 10^{-19} \mathrm{C}\right)^{2}}} \\
& =2.4 \times 10^{-13} \mathrm{~s}\left(\frac{10^{12} \mathrm{ps}}{1 \mathrm{~s}}\right)=0.24 \mathrm{ps} \approx 0.2 \mathrm{ps}
\end{aligned}
$$

20. If all of the angles to the vertical (in both cases) are assumed to be small, then the spheres only have horizontal displacement, and so the electric force of repulsion is always horizontal. Likewise, the small angle condition leads to $\tan \theta \approx \sin \theta \approx \theta$ for all small angles. See the free-body diagram for each sphere, showing the three forces of gravity, tension, and the electrostatic force. Take to the right to be the positive
 horizontal direction, and up to be the positive vertical direction. Since the spheres are in equilibrium, the net force in each direction is zero.
(a) $\sum F_{1 x}=F_{\mathrm{T} 1} \sin \theta_{1}-F_{\mathrm{El} 1}=0 \rightarrow F_{\mathrm{E} 1}=F_{\mathrm{T} 1} \sin \theta_{1}$
$\sum F_{1 y}=F_{\mathrm{T} 1} \cos \theta_{1}-m_{1} g \rightarrow F_{\mathrm{T} 1}=\frac{m_{1} g}{\cos \theta_{1}} \rightarrow F_{\mathrm{E} 1}=\frac{m_{1} g}{\cos \theta_{1}} \sin \theta_{1}=m_{1} g \tan \theta_{1}=m_{1} g \theta_{1}$
A completely parallel analysis would give $F_{\mathrm{E} 2}=m_{2} g \theta_{2}$. Since the electric forces are a
Newton's third law pair, they can be set equal to each other in magnitude.

$$
F_{\mathrm{E} 1}=F_{\mathrm{E} 2} \rightarrow m_{1} g \theta_{1}=m_{2} g \theta_{2} \quad \rightarrow \quad \theta_{1} / \theta_{2}=m_{2} / m_{1}=1
$$

(b) The same analysis can be done for this case.

$$
F_{\mathrm{E} 1}=F_{\mathrm{E} 2} \rightarrow m_{1} g \theta_{1}=m_{2} g \theta_{2} \rightarrow \theta_{1} / \theta_{2}=m_{1} / m_{1}=2
$$

(c) The horizontal distance from one sphere to the other is s by the small angle approximation. See the diagram. Use the relationship derived above that $F_{\mathrm{E}}=m g \theta$ to solve for the distance.
Case 1: $\quad d=\ell\left(\theta_{1}+\theta_{2}\right)=2 \ell \theta_{1} \rightarrow \theta_{1}=\frac{d}{2 \ell}$

$$
m_{1} g \theta_{1}=F_{\mathrm{E} 1}=\frac{k Q(2 Q)}{d^{2}}=m g \frac{d}{2 \ell} \rightarrow d=\left(\frac{4 \ell k Q^{2}}{m g}\right)^{1 / 3}
$$



Case 2: $d=\ell\left(\theta_{1}+\theta_{2}\right)=\frac{3}{2} \ell \theta_{1} \rightarrow \theta_{1}=\frac{2 d}{3 \ell}$

$$
m_{1} g \theta_{1}=F_{\mathrm{E} 1}=\frac{k Q(2 Q)}{d^{2}}=m g \frac{2 d}{3 \ell} \rightarrow d=\left(\frac{3 \ell k Q^{2}}{m g}\right)^{1 / 3}
$$

21. Use Eq. 21-3 to calculate the force. Take east to be the positive $x$ direction.

$$
\overrightarrow{\mathbf{E}}=\frac{\overrightarrow{\mathbf{F}}}{q} \rightarrow \overrightarrow{\mathbf{F}}=q \overrightarrow{\mathbf{E}}=\left(-1.602 \times 10^{-19} \mathrm{C}\right)(1920 \mathrm{~N} / \mathrm{C} \hat{\mathbf{i}})=-3.08 \times 10^{-16} \mathrm{~N} \hat{\mathbf{i}}=3.08 \times 10^{-16} \mathrm{~N} \text { west }
$$

22. Use Eq. 21-3 to calculate the electric field. Take north to be the positive $y$ direction.

$$
\overrightarrow{\mathbf{E}}=\frac{\overrightarrow{\mathbf{F}}}{q}=\frac{-2.18 \times 10^{-14} \mathrm{~N} \hat{\mathbf{j}}}{1.602 \times 10^{-19} \mathrm{C}}=-1.36 \times 10^{5} \mathrm{~N} / \mathrm{C} \hat{\mathbf{j}}=1.36 \times 10^{5} \mathrm{~N} / \mathrm{C} \text { south }
$$

23. Use Eq. 21-4a to calculate the electric field due to a point charge.

$$
E=k \frac{Q}{r^{2}}=\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{33.0 \times 10^{-6} \mathrm{C}}{(0.164 \mathrm{~m})^{2}}=1.10 \times 10^{7} \mathrm{~N} / \mathrm{C} \text { up }
$$

Note that the electric field points away from the positive charge.
24. Use Eq. $21-3$ to calculate the electric field.

$$
\overrightarrow{\mathbf{E}}=\frac{\overrightarrow{\mathbf{F}}}{q}=\frac{8.4 \mathrm{~N} \text { down }}{-8.8 \times 10^{-6} \mathrm{C}}=9.5 \times 10^{5} \mathrm{~N} / \mathrm{C} \mathrm{up}
$$

25. Use the definition of the electric field, Eq. 21-3.

$$
\overrightarrow{\mathbf{E}}=\frac{\overrightarrow{\mathbf{F}}}{q}=\frac{\left(7.22 \times 10^{-4} \mathrm{~N} \hat{\mathbf{j}}\right)}{4.20 \times 10^{-6} \mathrm{C}}=172 \mathrm{~N} / \mathrm{C} \hat{\mathbf{j}}
$$

26. Use the definition of the electric field, Eq. 21-3.

$$
\overrightarrow{\mathbf{E}}=\frac{\overrightarrow{\mathbf{F}}}{q}=\frac{(3.0 \hat{\mathbf{i}}-3.9 \hat{\mathbf{j}}) \times 10^{-3} \mathrm{~N}}{1.25 \times 10^{-6} \mathrm{C}}=(2400 \hat{\mathbf{i}}-3100 \hat{\mathbf{j}}) \mathrm{N} / \mathrm{C}
$$

27. Assuming the electric force is the only force on the electron, then Newton's second law may be used to find the acceleration.

$$
\overrightarrow{\mathbf{F}}_{\mathrm{net}}=m \overrightarrow{\mathbf{a}}=q \overrightarrow{\mathbf{E}} \rightarrow a=\frac{|q|}{m} E=\frac{\left(1.602 \times 10^{-19} \mathrm{C}\right)}{\left(9.109 \times 10^{-31} \mathrm{~kg}\right)}(576 \mathrm{~N} / \mathrm{C})=1.01 \times 10^{14} \mathrm{~m} / \mathrm{s}^{2}
$$

Since the charge is negative, the direction of the acceleration is opposite to the field.
28. The electric field due to the negative charge will point toward the negative charge, and the electric field due to the positive charge will point away from the positive charge. Thus both fields point in the same direction, towards the
 negative charge, and so can be added.

$$
\begin{aligned}
E & =\left|E_{1}\right|+\left|E_{2}\right|=k \frac{\left|Q_{1}\right|}{r_{1}^{2}}+k \frac{\left|Q_{2}\right|}{r_{2}^{2}}=k \frac{\left|Q_{1}\right|}{(\ell / 2)^{2}}+k \frac{\left|Q_{2}\right|}{(\ell / 2)^{2}}=\frac{4 k}{\ell^{2}}\left(\left|Q_{1}\right|+\left|Q_{2}\right|\right) \\
& =\frac{4\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)}{(0.080 \mathrm{~m})^{2}}\left(8.0 \times 10^{-6} \mathrm{C}+5.8 \times 10^{-6} \mathrm{C}\right)=7.8 \times 10^{7} \mathrm{~N} / \mathrm{C}
\end{aligned}
$$

The direction is towards the negative charge.
29.

30. Assuming the electric force is the only force on the electron, then Newton's second law may be used to find the electric field strength.

$$
F_{n e t}=m a=q E \rightarrow E=\frac{m a}{q}=\frac{\left(1.673 \times 10^{-27} \mathrm{~kg}\right)\left(1.8 \times 10^{6}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{\left(1.602 \times 10^{-19} \mathrm{C}\right)}=0.18 \mathrm{~N} / \mathrm{C}
$$

31. The field at the point in question is the vector sum of the two fields shown in Figure 21-56. Use the results of Example 21-11 to find the field of the long line of charge.

$$
\begin{aligned}
& \overrightarrow{\mathbf{E}}_{\text {trread }}=\frac{1}{2 \pi \varepsilon_{0}} \frac{\lambda}{y} \hat{\mathbf{j}} ; \overrightarrow{\mathbf{E}}_{Q}=\frac{1}{4 \pi \varepsilon_{0}} \frac{|Q|}{d^{2}}(-\cos \theta \hat{\mathbf{i}}-\sin \theta \hat{\mathbf{j}}) \rightarrow \\
& \overrightarrow{\mathbf{E}}=\left(-\frac{1}{4 \pi \varepsilon_{0}} \frac{|Q|}{d^{2}} \cos \theta\right) \hat{\mathbf{i}}+\left(\frac{1}{2 \pi \varepsilon_{0}} \frac{\lambda}{y}-\frac{1}{4 \pi \varepsilon_{0}} \frac{|Q|}{d^{2}} \sin \theta\right) \hat{\mathbf{j}} \\
& d^{2}=(0.070 \mathrm{~m})^{2}+(0.120 \mathrm{~m})^{2}=0.0193 \mathrm{~m}^{2} ; y=0.070 \mathrm{~m} ; \theta=\tan ^{-1} \frac{12.0 \mathrm{~cm}}{7.0 \mathrm{~cm}}=59.7^{\circ} \\
& E_{x}=-\frac{1}{4 \pi \varepsilon_{0}} \frac{|Q|}{d^{2}} \cos \theta=-\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{(2.0 \mathrm{C})}{0.0193 \mathrm{~m}^{2}} \cos 59.7^{\circ}=-4.699 \times 10^{11} \mathrm{~N} / \mathrm{C} \\
& E_{y}=\frac{1}{2 \pi \varepsilon_{0}} \frac{\lambda}{y}-\frac{1}{4 \pi \varepsilon_{0}} \frac{|Q|}{d^{2}} \sin \theta=\frac{1}{4 \pi \varepsilon_{0}}\left(\frac{2 \lambda}{y}-\frac{|Q|}{d^{2}} \sin \theta\right) \\
&=\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)\left[\frac{2(2.5 \mathrm{C} / \mathrm{m})}{0.070 \mathrm{~cm}}-\frac{(2.0 \mathrm{C})}{0.0193 \mathrm{~m}^{2}} \sin 59.7^{\circ}\right]=-1.622 \times 10^{11} \mathrm{~N} / \mathrm{C} \\
& \overrightarrow{\mathbf{E}}=\left(-4.7 \times 10^{11} \mathrm{~N} / \mathrm{C}\right) \hat{\mathbf{i}}+\left(-1.6 \times 10^{11} \mathrm{~N} / \mathrm{C}\right) \hat{\mathbf{j}} \\
& E=\sqrt{E_{x}^{2}+E_{y}^{2}}=\sqrt{\left(-4.699 \times 10^{11} \mathrm{~N} / \mathrm{C}\right)^{2}+\left(-1.622 \times 10^{11} \mathrm{~N} / \mathrm{C}\right)^{2}}=5.0 \times 10^{11} \mathrm{~N} / \mathrm{C} \\
& \theta_{E}=\tan ^{-1} \frac{\left(-1.622 \times 10^{11} \mathrm{~N} / \mathrm{C}\right)}{\left(-4.699 \times 10^{11} \mathrm{~N} / \mathrm{C}\right)}=199^{\circ}
\end{aligned}
$$

32. The field due to the negative charge will point towards the negative charge, and the field due to the positive charge will point towards the negative charge. Thus the magnitudes of the two fields can be added together to find
 the charges.

$$
E_{\text {net }}=2 E_{Q}=2 k \frac{Q}{(\ell / 2)^{2}}=\frac{8 k Q}{\ell^{2}} \rightarrow Q=\frac{E \ell^{2}}{8 k}=\frac{(586 \mathrm{~N} / \mathrm{C})(0.160 \mathrm{~m})^{2}}{8\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)}=2.09 \times 10^{-10} \mathrm{C}
$$

33. The field at the upper right corner of the square is the vector sum of the fields due to the other three charges. Let the variable $\ell$ represent the 1.0 m length of a side of the square, and let the variable $Q$ represent the charge at each of the three occupied corners.

$$
\begin{aligned}
& E_{1}=k \frac{Q}{\ell^{2}} \rightarrow E_{1 x}=k \frac{Q}{\ell^{2}}, E_{1 y}=0 \\
& E_{2}=k \frac{Q}{2 \ell^{2}} \rightarrow E_{2 x}=k \frac{Q}{2 \ell^{2}} \cos 45^{\circ}=k \frac{\sqrt{2} Q}{4 \ell^{2}}, E_{2 y}=k \frac{\sqrt{2} Q}{4 \ell^{2}} \\
& E_{3}=k \frac{Q}{\ell^{2}} \rightarrow E_{3 x}=0, E_{1 y}=k \frac{Q}{\ell^{2}}
\end{aligned}
$$



Add the $x$ and $y$ components together to find the total electric field, noting that $E_{x}=E_{y}$.

$$
\begin{aligned}
E_{x} & =E_{1 x}+E_{2 x}+E_{3 x}=k \frac{Q}{\ell^{2}}+k \frac{\sqrt{2} Q}{4 \ell^{2}}+0=k \frac{Q}{\ell^{2}}\left(1+\frac{\sqrt{2}}{4}\right)=E_{y} \\
E & =\sqrt{E_{x}^{2}+E_{y}^{2}}=k \frac{Q}{\ell^{2}}\left(1+\frac{\sqrt{2}}{4}\right) \sqrt{2}=k \frac{Q}{\ell^{2}}\left(\sqrt{2}+\frac{1}{2}\right) \\
& =\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{\left(2.25 \times 10^{-6} \mathrm{C}\right)}{(1.22 \mathrm{~m})^{2}}\left(\sqrt{2}+\frac{1}{2}\right)=2.60 \times 10^{4} \mathrm{~N} / \mathrm{C} \\
\theta & =\tan ^{-1} \frac{E_{y}}{E_{x}}=45.0^{\circ} \text { from the } x \text {-direction. }
\end{aligned}
$$

34. The field at the center due to the two $-27.0 \mu \mathrm{C}$ negative charges on opposite corners (lower right and upper left in the diagram) will cancel each other, and so only the other two charges need to be considered. The field due to each of the other charges will point directly toward the charge. Accordingly, the two fields are in opposite directions and can be combined algebraically.

$$
\begin{aligned}
E & =E_{1}-E_{2}=k \frac{\left|Q_{1}\right|}{\ell^{2} / 2}-k \frac{\left|Q_{2}\right|}{\ell^{2} / 2}=k \frac{\left|Q_{1}\right|-\left|Q_{2}\right|}{\ell^{2} / 2} \\
& =\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{(38.6-27.0) \times 10^{-6} \mathrm{C}}{(0.525 \mathrm{~m})^{2} / 2} \\
& =7.57 \times 10^{6} \mathrm{~N} / \mathrm{C}, \text { towards the }-38.6 \mu \mathrm{C} \text { charge }
\end{aligned}
$$


35. Choose the rightward direction to be positive. Then the field due to $+Q$ will be positive, and the field due to $-Q$ will be negative.

$$
E=k \frac{Q}{(x+a)^{2}}-k \frac{Q}{(x-a)^{2}}=k Q\left(\frac{1}{(x+a)^{2}}-\frac{1}{(x-a)^{2}}\right)=\frac{-4 k Q x a}{\left(x^{2}-a^{2}\right)^{2}}
$$

The negative sign means the field points to the left.
36. For the net field to be zero at point P , the magnitudes of the fields created by $Q_{1}$ and $Q_{2}$ must be equal. Also, the distance $x$ will be taken as positive to the left of $Q_{1}$. That is the only region where the total field due to the two charges can be zero. Let the variable $\ell$ represent the 12 cm distance, and note that $\left|Q_{1}\right|=\frac{1}{2} Q_{2}$.

$$
\begin{aligned}
& \left|\overrightarrow{\mathbf{E}}_{1}\right|=\left|\overrightarrow{\mathbf{E}}_{2}\right| \rightarrow k \frac{\left|Q_{2}\right|}{x^{2}}=k \frac{Q_{2}}{(x+\ell)^{2}} \rightarrow \\
& x=\ell \frac{\sqrt{\left|Q_{1}\right|}}{\left(\sqrt{Q_{2}}-\sqrt{\left|Q_{1}\right|}\right)}=(12 \mathrm{~cm}) \frac{\sqrt{25 \mu \mathrm{C}}}{(\sqrt{45 \mu \mathrm{C}}-\sqrt{25 \mu \mathrm{C}})}=35 \mathrm{~cm}
\end{aligned}
$$

37. Make use of Example 21-11. From that, we see that the electric field due to the line charge along the $y$ axis is $\overrightarrow{\mathbf{E}}_{1}=\frac{1}{2 \pi \varepsilon_{0}} \frac{\lambda}{x} \hat{\mathbf{i}}$. In particular, the field due to that line of charge has no $y$ dependence. In a similar fashion, the electric field due to the line charge along the $x$ axis is $\overrightarrow{\mathbf{E}}_{2}=\frac{1}{2 \pi \varepsilon_{0}} \frac{\lambda}{y} \hat{\mathbf{j}}$. Then the total field at $(x, y)$ is the vector sum of the two fields.

$$
\begin{aligned}
& \overrightarrow{\mathbf{E}}=\overrightarrow{\mathbf{E}}_{1}+\overrightarrow{\mathbf{E}}_{2}=\frac{1}{2 \pi \varepsilon_{0}} \frac{\lambda}{x} \hat{\mathbf{i}}+\frac{1}{2 \pi \varepsilon_{0}} \frac{\lambda}{y} \hat{\mathbf{j}}=\frac{\lambda}{2 \pi \varepsilon_{0}}\left(\frac{1}{x} \hat{\mathbf{i}}+\frac{1}{y} \hat{\mathbf{j}}\right) \\
& E=\frac{\lambda}{2 \pi \varepsilon_{0}} \sqrt{\frac{1}{x^{2}}+\frac{1}{y^{2}}}=\frac{\lambda}{2 \pi \varepsilon_{0} x y} \sqrt{x^{2}+y^{2}} ; \theta=\tan ^{-1} \frac{E_{y}}{E_{x}}=\tan ^{-1} \frac{\frac{1}{\frac{2 \pi \varepsilon_{0}}{} \frac{\lambda}{2 \pi \varepsilon_{0}} \frac{\lambda}{x}}=\tan ^{-1} \frac{x}{y}}{}
\end{aligned}
$$

38. (a) The field due to the charge at A will point straight downward, and the field due to the charge at B will point along the line from A to the origin, $30^{\circ}$ below the negative $x$ axis.

$$
\begin{aligned}
& E_{\mathrm{A}}=k \frac{Q}{\ell^{2}} \rightarrow E_{\mathrm{Ax}}=0, E_{\mathrm{Ax}}=-k \frac{Q}{\ell^{2}} \\
& E_{\mathrm{B}}=k \frac{Q}{\ell^{2}} \rightarrow E_{\mathrm{Bx}}=-k \frac{Q}{\ell^{2}} \cos 30^{\circ}=-k \frac{\sqrt{3} Q}{2 \ell^{2}}, \\
& E_{\mathrm{B} y}=-k \frac{Q}{\ell^{2}} \sin 30^{\circ}=-k \frac{Q}{2 \ell^{2}} \\
& E_{x}=E_{\mathrm{Ax}}+E_{\mathrm{B} x}=-k \frac{\sqrt{3} Q}{2 \ell^{2}} \quad E_{y}=E_{\mathrm{A} y}+E_{\mathrm{B} y}=-k \frac{3 Q}{2 \ell^{2}} \\
& E=\sqrt{E_{x}^{2}+E_{y}^{2}}=\sqrt{\frac{3 k^{2} Q^{2}}{4 \ell^{4}}+\frac{9 k^{2} Q^{2}}{4 \ell^{4}}}=\sqrt{\frac{12 k^{2} Q^{2}}{4 \ell^{4}}}=\frac{\sqrt{3} k Q}{\ell^{2}} \\
& \theta=\tan ^{-1} \frac{E_{y}}{E_{x}}=\tan ^{-1} \frac{-k \frac{3 Q}{2 \ell^{2}}}{-k \frac{\sqrt{3} Q}{2 \ell^{2}}}=\tan ^{-1} \frac{-3}{-\sqrt{3}}=\tan ^{-1} \sqrt{3}=240^{\circ}
\end{aligned}
$$


(b) Now reverse the direction of $\overrightarrow{\mathbf{E}}_{\mathrm{A}}$

$$
\begin{aligned}
& E_{\mathrm{A}}=k \frac{Q}{\ell^{2}} \rightarrow E_{\mathrm{Ax}}=0, E_{\mathrm{Ax}}=-k \frac{Q}{\ell^{2}} \\
& E_{\mathrm{B}}=k \frac{Q}{\ell^{2}} \rightarrow E_{\mathrm{Bx}}=k \frac{Q}{\ell^{2}} \cos 30^{\circ}=k \frac{\sqrt{3 Q}}{2 \ell^{2}}, E_{\mathrm{B} y}=k \frac{Q}{\ell^{2}} \sin 30^{\circ}=k \frac{Q}{2 \ell^{2}} \\
& E_{x}=E_{\mathrm{Ax}}+E_{\mathrm{Bx}}=k \frac{\sqrt{3} Q}{2 \ell^{2}} \quad E_{y}=E_{\mathrm{A} y}+E_{\mathrm{B} y}=-k \frac{Q}{2 \ell^{2}} \\
& E=\sqrt{E_{x}^{2}+E_{y}^{2}}=\sqrt{\frac{3 k^{2} Q^{2}}{4 \ell^{4}}+\frac{k^{2} Q^{2}}{4 \ell^{4}}}=\sqrt{\frac{4 k^{2} Q^{2}}{4 \ell^{4}}}=\frac{k Q}{\ell^{2}}
\end{aligned}
$$

$$
\theta=\tan ^{-1} \frac{E_{y}}{E_{x}}=\tan ^{-1} \frac{k \frac{Q}{2 \ell^{2}}}{-k \frac{\sqrt{3} Q}{2 \ell^{2}}}=\tan ^{-1} \frac{1}{-\sqrt{3}}=330^{\circ}
$$

39. Near the plate, the lines should come from it almost vertically, because it is almost like an infinite line of charge when the observation point is close. When the observation point is far away, it will look like a point charge.

40. Consider Example 21-9. We use the result from this example, but shift the center of the ring to be at $x=\frac{1}{2} \ell$ for the ring on the right, and at $x=-\frac{1}{2} \ell$ for the ring on the left. The fact that the original expression has a factor of $x$ results in the interpretation that the sign of the field expression will give the direction of the field. No special consideration needs to be given to the location of the point at which the field is to be calculated.

$$
\begin{aligned}
\overrightarrow{\mathbf{E}} & =\overrightarrow{\mathbf{E}}_{\text {right }}+\overrightarrow{\mathbf{E}}_{\text {left }} \\
& =\frac{1}{4 \pi \varepsilon_{0}} \frac{Q\left(x-\frac{1}{2} \ell\right)}{\left[\left(x-\frac{1}{2} \ell\right)^{2}+R^{2}\right]^{3 / 2}} \hat{\mathbf{i}}+\frac{1}{4 \pi \varepsilon_{0}} \frac{Q\left(x+\frac{1}{2} \ell\right)}{\left[\left(x+\frac{1}{2} \ell\right)^{2}+R^{2}\right]^{3 / 2}} \hat{\mathbf{i}} \\
& =\hat{\mathbf{i}} \frac{Q}{4 \pi \varepsilon_{0}}\left\{\frac{\left(x-\frac{1}{2} \ell\right)}{\left[\left(x-\frac{1}{2} \ell\right)^{2}+R^{2}\right]^{3 / 2}}+\frac{\left(x+\frac{1}{2} \ell\right)}{\left[\left(x+\frac{1}{2} \ell\right)^{2}+R^{2}\right]^{3 / 2}}\right\}
\end{aligned}
$$

41. Both charges must be of the same sign so that the electric fields created by the two charges oppose each other, and so can add to zero. The magnitudes of the two electric fields must be equal.

$$
E_{1}=E_{2} \rightarrow k \frac{Q_{1}}{(\ell / 3)^{2}}=k \frac{Q_{2}}{(2 \ell / 3)^{2}} \rightarrow 9 Q_{1}=\frac{9 Q_{2}}{4} \rightarrow \frac{Q_{1}}{Q_{2}}=\frac{1}{4}
$$

42. In each case, find the vector sum of the field caused by the charge on the left $\left(\overrightarrow{\mathbf{E}}_{\text {leff }}\right)$ and the field caused by the charge on the right $\left(\overrightarrow{\mathbf{E}}_{\text {right }}\right)$

Point A: From the symmetry of the geometry, in calculating the electric field at point A only the vertical components of the fields need to be considered. The horizontal components will cancel each other.

$$
\begin{aligned}
& \theta=\tan ^{-1} \frac{5.0}{10.0}=26.6^{\circ} \\
& d=\sqrt{(5.0 \mathrm{~cm})^{2}+(10.0 \mathrm{~cm})^{2}}=0.1118 \mathrm{~m}
\end{aligned}
$$

$$
E_{\mathrm{A}}=2 \frac{k Q}{d^{2}} \sin \theta=2\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{5.7 \times 10^{-6} \mathrm{C}}{(0.1118 \mathrm{~m})^{2}} \sin 26.6^{\circ}=3.7 \times 10^{6} \mathrm{~N} / \mathrm{C} \quad \theta_{\mathrm{A}}=90^{\circ}
$$

Point B: Now the point is not symmetrically placed, and so horizontal and vertical components of each individual field need to be calculated to find the resultant electric field.

$$
\begin{aligned}
& \theta_{\text {left }}=\tan ^{-1} \frac{5.0}{5.0}=45^{\circ} \quad \theta_{\text {right }}=\tan ^{-1} \frac{5.0}{15.0}=18.4^{\circ} \\
& d_{\text {left }}=\sqrt{(5.0 \mathrm{~cm})^{2}+(5.0 \mathrm{~cm})^{2}}=0.0707 \mathrm{~m} \\
& d_{\text {right }}=\sqrt{(5.0 \mathrm{~cm})^{2}+(15.0 \mathrm{~cm})^{2}}=0.1581 \mathrm{~m} \\
& E_{x}=\left(\overrightarrow{\mathbf{E}}_{\text {left }}\right)_{x}+\left(\overrightarrow{\mathbf{E}}_{\text {right }}\right)_{x}=k \frac{Q}{d_{\text {left }}^{2}} \cos \theta_{\text {left }}-k \frac{Q}{d_{\text {right }}^{2}} \cos \theta_{\text {right }} \\
& =\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)\left(5.7 \times 10^{-6} \mathrm{C}\right)\left[\frac{\cos 45^{\circ}}{(0.0707 \mathrm{~m})^{2}}-\frac{\cos 18.4^{\circ}}{(0.1581 \mathrm{~m})^{2}}\right]=5.30 \times 10^{6} \mathrm{~N} / \mathrm{C} \\
& E_{y}=\left(\overrightarrow{\mathbf{E}}_{\text {left }}\right)_{y}+\left(\overrightarrow{\mathbf{E}}_{\text {right }}\right)_{y}=k \frac{Q}{d_{\text {left }}^{2}} \sin \theta_{\text {left }}+k \frac{Q}{d_{\text {right }}^{2}} \sin \theta_{\text {right }} \\
& =\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)\left(5.7 \times 10^{-6} \mathrm{C}\right)\left[\frac{\sin 45^{\circ}}{(0.0707 \mathrm{~m})^{2}}+\frac{\sin 18.4^{\circ}}{(0.1581 \mathrm{~m})^{2}}\right]=7.89 \times 10^{6} \mathrm{~N} / \mathrm{C} \\
& E_{\mathrm{B}}=\sqrt{E_{x}^{2}+E_{y}^{2}}=9.5 \times 10^{6} \mathrm{~N} / \mathrm{C} \quad \theta_{B}=\tan ^{-1} \frac{E_{y}}{E_{x}}=56^{\circ}
\end{aligned}
$$



This has to be a maximum, because the magnitude is positive, the field is 0 midway between the charges, and $E \rightarrow 0$ as $y \rightarrow \infty$.
44. From Example 21-9, the electric field along the $x$-axis is $E=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q x}{\left(x^{2}+a^{2}\right)^{\frac{3}{2}}}$. To find the position where the magnitude is a maximum, we differentiate and set the first derivative equal to zero.

$$
\begin{aligned}
\frac{d E}{d x} & =\frac{Q}{4 \pi \varepsilon_{0}} \frac{\left(x^{2}+a^{2}\right)^{\frac{3}{2}}-x \frac{3}{2}\left(x^{2}+a^{2}\right)^{\frac{1}{2}} 2 x}{\left(x^{2}+a^{2}\right)^{3}}=\frac{Q}{4 \pi \varepsilon_{0}\left(x^{2}+a^{2}\right)^{\frac{5}{2}}}\left[\left(x^{2}+a^{2}\right)-3 x^{2}\right] \\
& =\frac{Q}{4 \pi \varepsilon_{0}\left(x^{2}+a^{2}\right)^{\frac{5}{2}}}\left[a^{2}-2 x^{2}\right]=0 \rightarrow x_{M}= \pm \frac{a}{\sqrt{2}}
\end{aligned}
$$

Note that $E=0$ at $x=0$ and $x=\infty$, and that $|E|>0$ for $0<|x|<\infty$. Thus the value of the magnitude of $E$ at $x=x_{M}$ must be a maximum. We could also show that the value is a maximum by using the second derivative test.
45. Because the distance from the wire is much smaller than the length of the wire, we can approximate the electric field by the field of an infinite wire, which is derived in Example 21-11.

$$
E=\frac{1}{2 \pi \varepsilon_{0}} \frac{\lambda}{x}=\frac{1}{4 \pi \varepsilon_{0}} \frac{2 \lambda}{x}=\left(8.988 \times 10^{9} \frac{\mathrm{~N} \cdot \mathrm{~m}^{2}}{\mathrm{C}^{2}}\right)^{2\left(\frac{4.75 \times 10^{-6} \mathrm{C}}{2.0 \mathrm{~m}}\right)} \frac{1.8 \times 10^{6} \mathrm{~N} / \mathrm{C}}{\left(2.4 \times 10^{-2} \mathrm{~m}\right)}=\begin{aligned}
& \text { away from the wire }
\end{aligned}
$$

46. This is essentially Example 21-11 again, but with different limits of integration. From the diagram here, we see that the maximum angle is given by $\sin \theta=\frac{\ell / 2}{\sqrt{x^{2}+(\ell / 2)^{2}}}$. We evaluate the results at that angle.

$$
\begin{aligned}
E & =\left.\frac{\lambda}{4 \pi \varepsilon_{0} x}(\sin \theta)\right|_{\sin \theta=\frac{-\ell / 2}{\sqrt{x^{2}+(\ell / 2)^{2}}}} ^{\sin \theta=\frac{\ell / 2}{\sqrt{x^{2}+(\ell / 2)^{2}}}} \\
& =\frac{\lambda}{4 \pi \varepsilon_{0} x}\left[\frac{\ell / 2}{\sqrt{x^{2}+(\ell / 2)^{2}}}-\left(-\frac{\ell / 2}{\sqrt{x^{2}+(\ell / 2)^{2}}}\right)\right]=\frac{\lambda \rightarrow \overrightarrow{\mathbf{E}}}{4 \pi \varepsilon_{0} x \sqrt{x^{2}+(\ell / 2)^{2}}}=\frac{\lambda}{2 \pi \varepsilon_{0}} \frac{\ell}{x\left(4 x^{2}+\ell^{2}\right)^{1 / 2}}
\end{aligned}
$$

47. If we consider just one wire, then from the answer to problem 46, we would have the following. Note that the distance from the wire to the point in question is $x=\sqrt{z^{2}+(\ell / 2)^{2}}$.

$$
E_{\text {wire }}=\frac{\lambda}{2 \pi \varepsilon_{0}} \frac{\ell}{\sqrt{z^{2}+(\ell / 2)^{2}}\left(4\left[z^{2}+(\ell / 2)^{2}\right]+\ell^{2}\right)^{1 / 2}}
$$

But the total field is not simply four times the above expression, because the fields due to the four wires are not parallel to each other.


Consider a side view of the problem. The two dots represent two parallel wires, on opposite sides of the square. Note that only the vertical component of the field due to each wire will actually contribute to the total field. The horizontal components will cancel.

$$
\begin{aligned}
E_{\text {wire }} & =4\left(E_{\text {wire }}\right) \cos \theta=4\left(E_{\text {wire }}\right) \frac{z}{\sqrt{z^{2}+(\ell / 2)^{2}}} \\
E_{\text {wire }} & =4\left[\frac{\lambda}{2 \pi \varepsilon_{0}} \frac{\ell}{\sqrt{z^{2}+(\ell / 2)^{2}}\left(4\left[z^{2}+(\ell / 2)^{2}\right]+\ell^{2}\right)^{1 / 2}}\right] \frac{z}{\sqrt{z^{2}+(\ell / 2)^{2}}} \\
& =\frac{8 \lambda \ell z}{\pi \varepsilon_{0}\left(4 z^{2}+\ell^{2}\right)\left(4 z^{2}+2 \ell^{2}\right)^{1 / 2}}
\end{aligned}
$$

The direction is vertical, perpendicular to the loop.
48. From the diagram, we see that the $x$ components of the two fields will cancel each other at the point P. Thus the net electric field will be in the negative $y$-direction, and will be twice the $y$-component of either electric field vector.

$$
\begin{aligned}
E_{\text {net }} & =2 E \sin \theta=2 \frac{k Q}{x^{2}+a^{2}} \sin \theta \\
& =\frac{2 k Q}{x^{2}+a^{2}} \frac{a}{\left(x^{2}+a^{2}\right)^{1 / 2}} \\
& =\frac{2 k Q a}{\left(x^{2}+a^{2}\right)^{3 / 2}} \text { in the negative } y \text { direction }
\end{aligned}
$$


49. Select a differential element of the arc which makes an angle of $\theta$ with the $x$ axis. The length of this element is $R d \theta$, and the charge on that element is $d q=\lambda R d \theta$. The magnitude of the field produced by that element is $d E=\frac{1}{4 \pi \varepsilon_{0}} \frac{\lambda R d \theta}{R^{2}}$. From the diagram, considering pieces of the arc that are symmetric with respect to the $x$ axis, we see that the total field will only have an $x$ component. The vertical components of the field due to symmetric portions of the arc will cancel each other.
So we have the following.


$$
\begin{aligned}
& d E_{\text {horizontal }}=\frac{1}{4 \pi \varepsilon_{0}} \frac{\lambda R d \theta}{R^{2}} \cos \theta \\
& E_{\text {horizontal }}=\int_{-\theta_{0}}^{\theta_{0}} \frac{1}{4 \pi \varepsilon_{0}} \cos \theta \frac{\lambda R d \theta}{R^{2}}=\frac{\lambda}{4 \pi \varepsilon_{0} R} \int_{-\theta_{0}}^{\theta_{0}} \cos \theta d \theta=\frac{\lambda}{4 \pi \varepsilon_{0} R}\left[\sin \theta_{0}-\sin \left(-\theta_{0}\right)\right]=\frac{2 \lambda \sin \theta_{0}}{4 \pi \varepsilon_{0} R}
\end{aligned}
$$

The field points in the negative $x$ direction, so $E=-\frac{2 \lambda \sin \theta_{0}}{4 \pi \varepsilon_{0} R} \hat{\mathbf{i}}$
50. (a) Select a differential element of the arc which makes an angle of $\theta$ with the $x$ axis. The length of this element is $R d \theta$, and the charge on that element is $d q=\lambda R d \theta$. The magnitude of the field produced by that element is $d E=\frac{1}{4 \pi \varepsilon_{0}} \frac{\lambda R d \theta}{R^{2}}$. From the diagram, considering pieces of the arc that are symmetric with respect to the $x$ axis, we see that the total field will only have a $y$ component, because the magnitudes of the fields due to those two pieces are the same. From the diagram
 we see that the field will point down. The horizontal components of the field cancel.

$$
\begin{aligned}
d E_{\text {verical }} & =\frac{1}{4 \pi \varepsilon_{0}} \frac{\lambda R d \theta}{R^{2}} \sin \theta=\frac{\lambda_{0}}{4 \pi \varepsilon_{0} R} \sin ^{2} \theta d \theta \\
E_{\text {verical }} & =\int_{-\pi / 2}^{\pi / 2} \frac{\lambda_{0}}{4 \pi \varepsilon_{0} R} \sin ^{2} \theta d \theta=\frac{\lambda_{0}}{4 \pi \varepsilon_{0} R} \int_{-\pi / 2}^{\pi / 2} \sin ^{2} \theta d \theta=\frac{\lambda_{0}}{4 \pi \varepsilon_{0} R}\left(\frac{1}{2} \theta-\frac{1}{4} \sin 2 \theta\right)_{-\pi / 2}^{\pi / 2} \\
& =\frac{\lambda_{0}}{4 \pi \varepsilon_{0} R}\left(\frac{1}{2} \pi\right)=\frac{\lambda_{0}}{8 \varepsilon_{0} R} \rightarrow \overrightarrow{\mathbf{E}}=-\frac{\lambda_{0}}{8 \varepsilon_{0} R} \hat{\mathbf{j}}
\end{aligned}
$$

(b) The force on the electron is given by Eq. 21-3. The acceleration is found from the force.

$$
\begin{aligned}
\overrightarrow{\mathbf{F}} & =m \overrightarrow{\mathbf{a}}=q \overrightarrow{\mathbf{E}}=-\frac{q \lambda_{0}}{8 \varepsilon_{0} R} \hat{\mathbf{j}} \rightarrow \\
\overrightarrow{\mathbf{a}} & =-\frac{q \lambda_{0}}{8 m \varepsilon_{0} R} \hat{\mathbf{j}}=\frac{e \lambda_{0}}{8 m \varepsilon_{0} R} \hat{\mathbf{j}}=\frac{\left(1.60 \times 10^{-19} \mathrm{C}\right)\left(1.0 \times 10^{-6} \mathrm{C} / \mathrm{m}\right)}{8\left(9.11 \times 10^{-31} \mathrm{~kg}\right)\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)(0.010 \mathrm{~m})} \hat{\mathbf{j}} \\
& =2.5 \times 10^{17} \mathrm{~m} / \mathrm{s}^{2} \hat{\mathbf{j}}
\end{aligned}
$$

51. (a) If we follow the first steps of Example 21-11, and refer to Figure 21-29, then the differential electric field due to the segment of wire is still $d E=\frac{1}{4 \pi \varepsilon_{0}} \frac{\lambda d y}{\left(x^{2}+y^{2}\right)}$. But now there is no symmetry, and so we calculate both components of the field.

$$
\begin{aligned}
& d E_{x}=d E \cos \theta=\frac{1}{4 \pi \varepsilon_{0}} \frac{\lambda d y}{\left(x^{2}+y^{2}\right)} \cos \theta=\frac{1}{4 \pi \varepsilon_{0}} \frac{\lambda x d y}{\left(x^{2}+y^{2}\right)^{3 / 2}} \\
& d E_{y}=-d E \sin \theta=-\frac{1}{4 \pi \varepsilon_{0}} \frac{\lambda d y}{\left(x^{2}+y^{2}\right)} \sin \theta=-\frac{1}{4 \pi \varepsilon_{0}} \frac{\lambda y d y}{\left(x^{2}+y^{2}\right)^{3 / 2}}
\end{aligned}
$$

The anti-derivatives needed are in Appendix B4.

$$
\begin{aligned}
E_{x} & =\int_{0}^{\ell} \frac{1}{4 \pi \varepsilon_{0}} \frac{\lambda x d y}{\left(x^{2}+y^{2}\right)^{3 / 2}}=\frac{\lambda x}{4 \pi \varepsilon_{0}} \int_{0}^{\ell} \frac{d y}{\left(x^{2}+y^{2}\right)^{3 / 2}}=\frac{\lambda x}{4 \pi \varepsilon_{0}}\left(\frac{y}{x^{2} \sqrt{x^{2}+y^{2}}}\right)_{0}^{\ell} \\
& =\frac{\lambda \ell}{4 \pi \varepsilon_{0} x \sqrt{x^{2}+\ell^{2}}}
\end{aligned}
$$

$$
\begin{aligned}
E_{y} & =-\int_{0}^{\ell} \frac{1}{4 \pi \varepsilon_{0}} \frac{\lambda y d y}{\left(x^{2}+y^{2}\right)^{3 / 2}}=-\frac{\lambda}{4 \pi \varepsilon_{0}} \int_{0}^{\ell} \frac{y d y}{\left(x^{2}+y^{2}\right)^{3 / 2}}=-\frac{\lambda}{4 \pi \varepsilon_{0}}\left(\frac{-1}{\sqrt{x^{2}+y^{2}}}\right)_{0}^{\ell} \\
& =\frac{\lambda}{4 \pi \varepsilon_{0}}\left(\frac{1}{\sqrt{x^{2}+\ell^{2}}}-\frac{1}{x}\right)=\frac{\lambda}{4 \pi \varepsilon_{0} x \sqrt{x^{2}+\ell^{2}}}\left(x-\sqrt{x^{2}+\ell^{2}}\right)
\end{aligned}
$$

Note that $E_{y}<0$, and so the electric field points to the right and down.
(b) The angle that the electric field makes with the $x$ axis is given as follows.

$$
\tan \theta=\frac{E_{y}}{E_{x}}=\frac{\frac{\lambda}{4 \pi \varepsilon_{0} x \sqrt{x^{2}+\ell^{2}}}\left(x-\sqrt{x^{2}+\ell^{2}}\right)}{\frac{\lambda \ell}{4 \pi \varepsilon_{0} x \sqrt{x^{2}+\ell^{2}}}}=\frac{x-\sqrt{x^{2}+\ell^{2}}}{\ell}=\frac{x}{\ell}-\sqrt{1+\frac{x^{2}}{\ell^{2}}}
$$

As $\ell \rightarrow \infty$, the expression becomes $\tan \theta=-1$, and so the field makes an angle of $45^{\circ}$ below the $x$ axis.
52. Please note: the first printing of the textbook gave the length of the charged wire as 6.00 m , but it should have been 6.50 m . That error has been corrected in later printings, and the following solution uses a length of 6.50 m .
(a) If we follow the first steps of Example 21-11, and refer to Figure 21-29, then the differential electric field due to the segment of wire is still $d E=\frac{1}{4 \pi \varepsilon_{0}} \frac{\lambda d y}{\left(x^{2}+y^{2}\right)}$. But now there is no symmetry, and so we calculate both components of the field.

$$
\begin{aligned}
& d E_{x}=d E \cos \theta=\frac{1}{4 \pi \varepsilon_{0}} \frac{\lambda d y}{\left(x^{2}+y^{2}\right)} \cos \theta=\frac{1}{4 \pi \varepsilon_{0}} \frac{\lambda x d y}{\left(x^{2}+y^{2}\right)^{3 / 2}} \\
& d E_{y}=-d E \cos \theta=-\frac{1}{4 \pi \varepsilon_{0}} \frac{\lambda d y}{\left(x^{2}+y^{2}\right)} \sin \theta=-\frac{1}{4 \pi \varepsilon_{0}} \frac{\lambda y d y}{\left(x^{2}+y^{2}\right)^{3 / 2}}
\end{aligned}
$$

The anti-derivatives needed are in Appendix B4.

$$
\begin{aligned}
E_{x} & =\int_{y_{\min }}^{y_{\max }} \frac{1}{4 \pi \varepsilon_{0}} \frac{\lambda x d y}{\left(x^{2}+y^{2}\right)^{3 / 2}}=\frac{\lambda x}{4 \pi \varepsilon_{0}} \int_{y_{\min }}^{y_{\max }} \frac{d y}{\left(x^{2}+y^{2}\right)^{3 / 2}}=\frac{\lambda x}{4 \pi \varepsilon_{0}}\left(\frac{y}{x^{2} \sqrt{x^{2}+y^{2}}}\right)_{y_{\min }}^{y_{\max }} \\
& =\frac{\lambda}{4 \pi \varepsilon_{0} x}\left(\frac{y_{\max }}{\sqrt{x^{2}+y_{\max }^{2}}}-\frac{y_{\min }}{\sqrt{x^{2}+y_{\min }^{2}}}\right) \\
& =\left(8.99 \times 10^{9} \frac{\mathrm{~N} \cdot \mathrm{~m}^{2}}{\mathrm{C}^{2}}\right) \frac{\left(3.15 \times 10^{-6} \mathrm{C}\right) /(6.50 \mathrm{~m})}{(0.250 \mathrm{~m})} \\
& \left(\frac{2.50 \mathrm{~m}}{\sqrt{(0.250 \mathrm{~m})^{2}+(2.50 \mathrm{~m})^{2}}}-\frac{(-4.00 \mathrm{~m})}{\sqrt{(0.250 \mathrm{~m})^{2}+(-4.00 \mathrm{~m})^{2}}}\right) \\
& =3.473 \times 10^{4} \mathrm{~N} / \mathrm{C} \approx 3.5 \times 10^{4} \mathrm{~N} / \mathrm{C}
\end{aligned}
$$

$$
\begin{aligned}
E_{y} & =-\int_{y_{\min }}^{y_{\max }} \frac{1}{4 \pi \varepsilon_{0}} \frac{\lambda y d y}{\left(x^{2}+y^{2}\right)^{3 / 2}}=-\frac{\lambda}{4 \pi \varepsilon_{0}} \int_{y_{\min }}^{y_{\max }} \frac{y d y}{\left(x^{2}+y^{2}\right)^{3 / 2}}=-\frac{\lambda}{4 \pi \varepsilon_{0}}\left(\frac{-1}{\sqrt{x^{2}+y^{2}}}\right)_{y_{\min }}^{y_{\max }} \\
& =\frac{\lambda}{4 \pi \varepsilon_{0}}\left(\frac{1}{\sqrt{x^{2}+y_{\max }^{2}}}-\frac{1}{\sqrt{x^{2}+y_{\min }^{2}}}\right) \\
& =\left(8.99 \times 10^{9} \frac{\mathrm{~N} \cdot \mathrm{~m}^{2}}{\mathrm{C}^{2}}\right) \frac{\left(3.15 \times 10^{-6} \mathrm{C}\right)}{(6.50 \mathrm{~m})} \\
& =647 \mathrm{~N} / \mathrm{C} \approx 650 \mathrm{~N} / \mathrm{C}
\end{aligned}
$$

(b) We calculate the infinite line of charge result, and calculate the errors.

$$
\begin{aligned}
& E=\frac{\lambda}{2 \pi \varepsilon_{0} x}=\frac{2 \lambda}{4 \pi \varepsilon_{0} x}=2\left(8.99 \times 10^{9} \frac{\mathrm{~N} \cdot \mathrm{~m}^{2}}{\mathrm{C}^{2}}\right) \frac{\left(3.15 \times 10^{-6} \mathrm{C}\right)}{(6.50 \mathrm{~m})(0.250 \mathrm{~m})}=3.485 \times 10^{4} \mathrm{~N} / \mathrm{m} \\
& \frac{E_{x}-E}{E}=\frac{\left(3.473 \times 10^{4} \mathrm{~N} / \mathrm{C}\right)-\left(3.485 \times 10^{4} \mathrm{~N} / \mathrm{m}\right)}{\left(3.485 \times 10^{4} \mathrm{~N} / \mathrm{m}\right)}=-0.0034 \\
& \frac{E_{y}}{E}=\frac{(647 \mathrm{~N} / \mathrm{C})}{\left(3.485 \times 10^{4} \mathrm{~N} / \mathrm{m}\right)}=0.019
\end{aligned}
$$

And so we see that $E_{x}$ is only about $0.3 \%$ away from the value obtained from the infinite line of charge, and $E_{y}$ is only about $2 \%$ of the value obtained from the infinite line of charge. The field of an infinite line of charge result would be a good approximation for the field due to this wire segment.
53. Choose a differential element of the rod $d x^{\prime}$ a distance $x^{\prime}$ from the origin, as shown in the diagram. The charge on that differential element is $d q=\frac{Q}{\ell} d x^{\prime}$. The variable $x^{\prime}$ is treated as positive,

so that the field due to this differential element is $d E=\frac{1}{4 \pi \varepsilon_{0}} \frac{d q}{\left(x+x^{\prime}\right)^{2}}=\frac{Q}{4 \pi \varepsilon_{0} \ell} \frac{d x^{\prime}}{\left(x+x^{\prime}\right)^{2}}$. Integrate along the rod to find the total field.

$$
\begin{aligned}
E & =\int d E=\int_{0}^{\ell} \frac{Q}{4 \pi \varepsilon_{0} \ell} \frac{d x^{\prime}}{\left(x+x^{\prime}\right)^{2}}=\frac{Q}{4 \pi \varepsilon_{0} \ell} \int_{0}^{\ell} \frac{d x^{\prime}}{\left(x+x^{\prime}\right)^{2}}=\frac{Q}{4 \pi \varepsilon_{0} \ell}\left(-\frac{1}{x+x^{\prime}}\right)_{0}^{\ell}=\frac{Q}{4 \pi \varepsilon_{0} \ell}\left(\frac{1}{x}-\frac{1}{x+\ell}\right) \\
& =\frac{Q}{4 \pi \varepsilon_{0} x(x+\ell)}
\end{aligned}
$$

54. As suggested, we divide the plane into long narrow strips of width $d y$ and length $\ell$. The charge on the strip is the area of the strip times the charge per unit area: $d q=\sigma \ell d y$. The charge per unit length on the strip is $\lambda=\frac{d q}{\ell}=\sigma d y$. From Example 21-11, the field due to that narrow strip is $d E=\frac{\lambda}{2 \pi \varepsilon_{0} \sqrt{y^{2}+z^{2}}}=\frac{\sigma d y}{2 \pi \varepsilon_{0} \sqrt{y^{2}+z^{2}}}$. From Figure 21-68 in the textbook, we see that this field does not point vertically. From the symmetry of the plate, there is another long narrow strip a distance $y$ on the other side of the origin, which would create the same magnitude electric field. The horizontal components of those two fields would cancel each other, and so we only need calculate the vertical component of the field. Then we integrate along the $y$ direction to find the total field.

$$
\begin{aligned}
d E & =\frac{\sigma d y}{2 \pi \varepsilon_{0} \sqrt{y^{2}+z^{2}}} ; d E_{z}=d E \cos \theta=\frac{\sigma z d y}{2 \pi \varepsilon_{0}\left(y^{2}+z^{2}\right)} \\
E & =E_{z}=\int_{-\infty}^{\infty} \frac{\sigma z d y}{2 \pi \varepsilon_{0}\left(y^{2}+z^{2}\right)}=\frac{\sigma z}{2 \pi \varepsilon_{0}} \int_{-\infty}^{\infty} \frac{d y}{\left(y^{2}+z^{2}\right)}=\frac{\sigma z}{2 \pi \varepsilon_{0}} \frac{1}{z}\left(\tan ^{-1} \frac{y}{z}\right)_{-\infty}^{\infty} \\
& =\frac{\sigma}{2 \pi \varepsilon_{0}}\left[\tan ^{-1}(\infty)-\tan ^{-1}(-\infty)\right]=\frac{\sigma}{2 \pi \varepsilon_{0}}\left[\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)\right]=\frac{\sigma}{2 \varepsilon_{0}}
\end{aligned}
$$

55. Take Figure 21-28 and add the angle $\phi$, measured from the $-z$ axis, as indicated in the diagram. Consider an infinitesimal length of the ring $a d \phi$. The charge on that infinitesimal length is $d q=\lambda(a d \phi)$ $=\frac{Q}{\pi a}(a d \phi)=\frac{Q}{\pi} d \phi$. The charge creates an infinitesimal electric
field, $d \overrightarrow{\mathbf{E}}$, with magnitude $d E=\frac{1}{4 \pi \varepsilon_{0}} \frac{d q}{r^{2}}=\frac{1}{4 \pi \varepsilon_{0}} \frac{\frac{Q}{\pi} d \phi}{x^{2}+a^{2}}$. From the

symmetry of the figure, we see that the $z$ component of $d \overrightarrow{\mathbf{E}}$ will be cancelled by the $z$ component due to the piece of the ring that is on the opposite side of the $y$ axis. The trigonometric relationships give $d E_{x}=d E \cos \theta$ and $d E_{y}=-d E \sin \theta \sin \phi$. The factor of $\sin \phi$ can be justified by noting that $d E_{y}=0$ when $\phi=0$, and $d E_{y}=-d E \sin \theta$ when $\phi=\pi / 2$.

$$
\begin{aligned}
& d E_{x}=d E \cos \theta=\frac{Q}{4 \pi^{2} \varepsilon_{0}} \frac{d \phi}{x^{2}+a^{2}} \frac{x}{\sqrt{x^{2}+a^{2}}}=\frac{Q x}{4 \pi^{2} \varepsilon_{0}} \frac{d \phi}{\left(x^{2}+a^{2}\right)^{3 / 2}} \\
& E_{x}=\frac{Q x}{4 \pi^{2} \varepsilon_{0}\left(x^{2}+a^{2}\right)^{3 / 2}} \int_{0}^{\pi} d \phi=\frac{Q x}{4 \pi \varepsilon_{0}\left(x^{2}+a^{2}\right)^{3 / 2}} \\
& d E_{y}=-d E \sin \theta \sin \phi=-\frac{Q}{4 \pi^{2} \varepsilon_{0}} \frac{d \phi}{x^{2}+a^{2}} \frac{a}{\sqrt{x^{2}+a^{2}}} \sin \phi=-\frac{Q a}{4 \pi^{2} \varepsilon_{0}\left(x^{2}+a^{2}\right)^{3 / 2}} \sin \phi d \phi \\
& E_{y}=-\frac{Q a}{4 \pi^{2} \varepsilon_{0}\left(x^{2}+a^{2}\right)^{3 / 2}} \int_{0}^{\pi} \sin \phi d \phi=-\frac{Q a}{4 \pi^{2} \varepsilon_{0}\left(x^{2}+a^{2}\right)^{3 / 2}}[(-\cos \pi)-(-\cos 0)]
\end{aligned}
$$

$$
=-\frac{2 Q a}{4 \pi^{2} \varepsilon_{0}\left(x^{2}+a^{2}\right)^{3 / 2}}
$$

We can write the electric field in vector notation.

$$
\overrightarrow{\mathbf{E}}=\frac{Q x}{4 \pi \varepsilon_{0}\left(x^{2}+a^{2}\right)^{3 / 2}} \hat{\mathbf{i}}-\frac{2 Q a}{4 \pi^{2} \varepsilon_{0}\left(x^{2}+a^{2}\right)^{3 / 2}} \hat{\mathbf{j}}=\frac{Q}{4 \pi \varepsilon_{0}\left(x^{2}+a^{2}\right)^{3 / 2}}\left(x \hat{\mathbf{i}}-\frac{2 a}{\pi} \hat{\mathbf{j}}\right)
$$

56. (a) Since the field is uniform, the electron will experience a constant force in the direction opposite to its velocity, so the acceleration is constant and negative. Use constant acceleration relationships with a final velocity of 0 .

$$
\begin{aligned}
& F=m a=q E=-e E \rightarrow a=-\frac{e E}{m} ; v^{2}=v_{0}^{2}+2 a \Delta x=0 \rightarrow \\
& \Delta x=-\frac{v_{0}^{2}}{2 a}=-\frac{v_{0}^{2}}{2\left(-\frac{e E}{m}\right)}=\frac{m v_{0}^{2}}{2 e E}=\frac{\left(9.11 \times 10^{-31} \mathrm{~kg}\right)\left(27.5 \times 10^{6} \mathrm{~m} / \mathrm{s}\right)^{2}}{2\left(1.60 \times 10^{-19} \mathrm{C}\right)\left(11.4 \times 10^{3} \mathrm{~N} / \mathrm{C}\right)}=0.189 \mathrm{~m}
\end{aligned}
$$

(b) Find the elapsed time from constant acceleration relationships. Upon returning to the original position, the final velocity will be the opposite of the initial velocity.

$$
\begin{aligned}
& v=v_{0}+a t \rightarrow \\
& t=\frac{v-v_{0}}{a}=\frac{-2 v_{0}}{\left(-\frac{e E}{m}\right)}=\frac{2 m v_{0}}{e E}=\frac{2\left(9.11 \times 10^{-31} \mathrm{~kg}\right)\left(27.5 \times 10^{6} \mathrm{~m} / \mathrm{s}\right)}{\left(1.60 \times 10^{-19} \mathrm{C}\right)\left(11.4 \times 10^{3} \mathrm{~N} / \mathrm{C}\right)}=2.75 \times 10^{-8} \mathrm{~s}
\end{aligned}
$$

57. (a) The acceleration is produced by the electric force.

$$
\begin{aligned}
& \overrightarrow{\mathbf{F}}_{\text {net }}=m \overrightarrow{\mathbf{a}}=q \overrightarrow{\mathbf{E}}=-e \overrightarrow{\mathbf{E}} \rightarrow \\
& \overrightarrow{\mathbf{a}}=-\frac{e}{m} \overrightarrow{\mathbf{E}}=-\frac{\left(1.60 \times 10^{-19} \mathrm{C}\right)}{\left(9.11 \times 10^{-31} \mathrm{~kg}\right)}\left[(2.0 \hat{\mathbf{i}}+8.0 \hat{\mathbf{j}}) \times 10^{4} \mathrm{~N} / \mathrm{C}\right]=\left(-3.513 \times 10^{15} \hat{\mathbf{i}}-1.405 \times 10^{16} \hat{\mathbf{j}}\right) \mathrm{m} / \mathrm{s}^{2} \\
& \approx-3.5 \times 10^{15} \mathrm{~m} / \mathrm{s}^{2} \hat{\mathbf{i}}-1.4 \times 10^{16} \mathrm{~m} / \mathrm{s}^{2} \hat{\mathbf{j}}
\end{aligned}
$$

(b) The direction is found from the components of the velocity.

$$
\begin{aligned}
& \overrightarrow{\mathbf{v}}=\overrightarrow{\mathbf{v}}+\overrightarrow{\mathbf{a}} t=\left(8.0 \times 10^{4} \mathrm{~m} / \mathrm{s}\right) \hat{\mathbf{j}}+\left[\left(-3.513 \times 10^{15} \hat{\mathbf{i}}-1.405 \times 10^{16} \hat{\mathbf{j}}\right) \mathrm{m} / \mathrm{s}^{2}\right]\left(1.0 \times 10^{-9} \mathrm{~s}\right) \\
&=\left(-3.513 \times 10^{6} \hat{\mathbf{i}}-1.397 \times 10^{\prime} \hat{\mathbf{j}}\right) \mathrm{m} / \mathrm{s} \\
& \tan ^{-1} \frac{v_{y}}{v_{x}}=\tan ^{-1}\left(\frac{-1.397 \times 10^{7} \mathrm{~m} / \mathrm{s}}{-3.513 \times 10^{6} \mathrm{~m} / \mathrm{s}}\right)=256^{\circ} \text { or }-104^{\circ}
\end{aligned}
$$

This is the direction relative to the $x$ axis. The direction of motion relative to the initial direction is measured from the $y$ axis, and so is $\theta=166^{\circ}$ counter-clockwise from the initial direction.
58. (a) The electron will experience a force in the opposite direction to the electric field. Since the electron is to be brought to rest, the electric field must be in the same direction as the initial velocity of the electron, and so is to the right.
(b) Since the field is uniform, the electron will experience a constant force, and therefore have a constant acceleration. Use constant acceleration relationships to find the field strength.

$$
\begin{aligned}
& F=q E=m a \rightarrow a=\frac{q E}{m} \quad v^{2}=v_{0}^{2}+2 a \Delta x=v_{0}^{2}+2 \frac{q E}{m} \Delta x \rightarrow \\
& E=\frac{m\left(v^{2}-v_{0}^{2}\right)}{2 q \Delta x}=\frac{-m v_{0}^{2}}{2 q \Delta x}=-\frac{\left(9.109 \times 10^{-31} \mathrm{~kg}\right)\left(7.5 \times 10^{5} \mathrm{~m} / \mathrm{s}\right)^{2}}{2\left(-1.602 \times 10^{-19} \mathrm{C}\right)(0.040 \mathrm{~m})}=40 \mathrm{~N} / \mathrm{C}(2 \text { sig. fig. })
\end{aligned}
$$

59. The angle is determined by the velocity. The $x$ component of the velocity is constant. The time to pass through the plates can be found from the $x$ motion. Then the $y$ velocity can be found using constant acceleration relationships.

$$
\begin{aligned}
& x=v_{0} t \rightarrow t=\frac{x}{v_{0}} ; v_{y}=v_{y 0}+a_{y} t=-\frac{e E}{m} \frac{x}{v_{0}} \\
& \tan \theta=\frac{v_{y}}{v_{x}}=\frac{-\frac{e E}{m} \frac{x}{v_{0}}}{v_{0}}=-\frac{e E x}{m v_{0}^{2}}=-\frac{\left(1.60 \times 10^{-19} \mathrm{C}\right)\left(5.0 \times 10^{3} \mathrm{~N} / \mathrm{C}\right)(0.049 \mathrm{~m})}{\left(9.11 \times 10^{-31} \mathrm{~kg}\right)\left(1.00 \times 10^{7} \mathrm{~m} / \mathrm{s}\right)^{2}}=-.4303 \rightarrow \\
& \theta=\tan ^{-1}(-0.4303)=-23^{\circ}
\end{aligned}
$$

60. Since the field is constant, the force on the electron is constant, and so the acceleration is constant. Thus constant acceleration relationships can be used. The initial conditions are $x_{0}=0, y_{0}=0$, $v_{x 0}=1.90 \mathrm{~m} / \mathrm{s}$, and $v_{y 0}=0$.

$$
\begin{aligned}
\overrightarrow{\mathbf{F}} & =m \overrightarrow{\mathbf{a}}=q \overrightarrow{\mathbf{E}} \rightarrow \overrightarrow{\mathbf{a}}=\frac{q}{m} \overrightarrow{\mathbf{E}}=-\frac{e}{m} \overrightarrow{\mathbf{E}} ; a_{x}=-\frac{e}{m} E_{x}, a_{y}=-\frac{e}{m} E_{y} \\
x & =x_{0}+v_{x 0} t+\frac{1}{2} a_{x} t^{2}=v_{x 0} t-\frac{e E_{x}}{2 m} t^{2} \\
& =(1.90 \mathrm{~m} / \mathrm{s})(2.0 \mathrm{~s})-\frac{\left(1.60 \times 10^{-19} \mathrm{C}\right)\left(2.00 \times 10^{-11} \mathrm{~N} / \mathrm{C}\right)}{2\left(9.11 \times 10^{-31} \mathrm{~kg}\right)}(2.0 \mathrm{~s})^{2}=-3.2 \mathrm{~m} \\
y & =y_{0}+v_{y 0} t+\frac{1}{2} a_{y} t^{2}=-\frac{e E_{y}}{2 m} t^{2}=-\frac{\left(1.60 \times 10^{-19} \mathrm{C}\right)\left(-1.20 \times 10^{-11} \mathrm{~N} / \mathrm{C}\right)}{2\left(9.11 \times 10^{-31} \mathrm{~kg}\right)}(2.0 \mathrm{~s})^{2}=4.2 \mathrm{~m}
\end{aligned}
$$

61. (a) The field along the axis of the ring is given in Example 21-9, with the opposite sign because this ring is negatively charged. The force on the charge is the field times the charge $q$. Note that if $x$ is positive, the force is to the left, and if $x$ is negative, the force is to the right. Assume that $x \ll R$.

$$
F=q E=\frac{q}{4 \pi \varepsilon_{0}} \frac{(-Q) x}{\left(x^{2}+R^{2}\right)^{3 / 2}}=\frac{-q Q x}{4 \pi \varepsilon_{0}} \frac{1}{\left(x^{2}+R^{2}\right)^{3 / 2}} \approx \frac{-q Q x}{4 \pi \varepsilon_{0} R^{3}}
$$

This has the form of a simple harmonic oscillator, where the "spring constant" is

$$
k_{\text {elastic }}=\frac{Q q}{4 \pi \varepsilon_{0} R^{3}}
$$

(b) The spring constant can be used to find the period. See Eq. 14-7b.

$$
T=2 \pi \sqrt{\frac{m}{k_{\text {elastic }}}}=2 \pi \sqrt{\frac{m}{\frac{Q q}{4 \pi \varepsilon_{0} R^{3}}}}=2 \pi \sqrt{\frac{m 4 \pi \varepsilon_{0} R^{3}}{Q q}}=4 \pi \sqrt{\frac{m \pi \varepsilon_{0} R^{3}}{Q q}}
$$

62. (a) The dipole moment is given by the product of the positive charge and the separation distance.

$$
p=Q \ell=\left(1.60 \times 10^{-19} \mathrm{C}\right)\left(0.68 \times 10^{-9} \mathrm{~m}\right)=1.088 \times 10^{-28} \mathrm{C} \cdot \mathrm{~m} \approx 1.1 \times 10^{-28} \mathrm{C} \cdot \mathrm{~m}
$$

(b) The torque on the dipole is given by Eq. 21-9a.

$$
\tau=p E \sin \theta=\left(1.088 \times 10^{-28} \mathrm{C} \cdot \mathrm{~m}\right)\left(2.2 \times 10^{4} \mathrm{~N} / \mathrm{C}\right)\left(\sin 90^{\circ}\right)=2.4 \times 10^{-24} \mathrm{C} \cdot \mathrm{~m}
$$

(c) $\tau=p E \sin \theta=\left(1.088 \times 10^{-28} \mathrm{C} \cdot \mathrm{m}\right)\left(2.2 \times 10^{4} \mathrm{~N} / \mathrm{C}\right)\left(\sin 45^{\circ}\right)=1.7 \times 10^{-24} \mathrm{~N} \cdot \mathrm{~m}$
(d) The work done by an external force is the change in potential energy. Use Eq. 21-10.

$$
\begin{aligned}
W & =\Delta U=\left(-p E \cos \theta_{\text {final }}\right)-\left(-p E \cos \theta_{\text {initial }}\right)=p E\left(\cos \theta_{\text {initial }}-\cos \theta_{\text {final }}\right) \\
& =\left(1.088 \times 10^{-28} \mathrm{C} \cdot \mathrm{~m}\right)\left(2.2 \times 10^{4} \mathrm{~N} / \mathrm{C}\right)[1-(-1)]=4.8 \times 10^{-24} \mathrm{~J}
\end{aligned}
$$

63. (a) The dipole moment is the effective charge of each atom times the separation distance.

$$
p=Q \ell \rightarrow Q=\frac{p}{\ell}=\frac{3.4 \times 10^{-30} \mathrm{C} \cdot \mathrm{~m}}{1.0 \times 10^{-10} \mathrm{~m}}=3.4 \times 10^{-20} \mathrm{C}
$$

(b) $\frac{Q}{e}=\frac{3.4 \times 10^{-20} \mathrm{C}}{1.60 \times 10^{-19} \mathrm{C}}=0.21 \mathrm{No}$, the net charge on each atom is not an integer multiple of $e$. This is an indication that the H and Cl atoms are not ionized - they haven't fully gained or lost an electron. But rather, the electrons spend more time near the Cl atom than the H atom, giving the molecule a net dipole moment. The electrons are not distributed symmetrically about the two nuclei.
(c) The torque is given by Eq. 21-9a.

$$
\tau=p E \sin \theta \quad \rightarrow \quad \tau_{\max }=p E=\left(3.4 \times 10^{-30} \mathrm{C} \cdot \mathrm{~m}\right)\left(2.5 \times 10^{4} \mathrm{~N} / \mathrm{C}\right)=8.5 \times 10^{-26} \mathrm{~N} \cdot \mathrm{~m}
$$

(d) The energy needed from an external force is the change in potential energy. Use Eq. 21-10.

$$
\begin{aligned}
W & =\Delta U=\left(-p E \cos \theta_{\text {final }}\right)-\left(-p E \cos \theta_{\text {initial }}\right)=p E\left(\cos \theta_{\text {initial }}-\cos \theta_{\text {final }}\right) \\
& =\left(3.4 \times 10^{-30} \mathrm{C} \cdot \mathrm{~m}\right)\left(2.5 \times 10^{4} \mathrm{~N} / \mathrm{C}\right)\left[1-\cos 45^{\circ}\right]=2.5 \times 10^{-26} \mathrm{~J}
\end{aligned}
$$

64. (a) From the symmetry in the diagram, we see that the resultant field will be in the $y$ direction. The vertical components of the two fields add together, while the horizontal components cancel.

$$
\begin{aligned}
E_{\text {net }} & =2 E \sin \phi=2 \frac{Q}{4 \pi \varepsilon_{0}\left(r^{2}+\ell^{2}\right)} \frac{r}{\left(r^{2}+\ell^{2}\right)^{1 / 2}} \\
& =\frac{2 Q r}{4 \pi \varepsilon_{0}\left(r^{2}+\ell^{2}\right)^{3 / 2}} \approx \frac{2 Q r}{4 \pi \varepsilon_{0}\left(r^{3}\right)}=\frac{2 Q}{4 \pi \varepsilon_{0} r^{2}}
\end{aligned}
$$


(b) Both charges are the same sign. A long distance away from the charges, they will look like a single charge of magnitude $2 Q$, and so $E=k \frac{q}{r^{2}}=\frac{2 Q}{4 \pi \varepsilon_{0} r^{2}}$.
65. (a) There will be a torque on the dipole, in a direction to decrease $\theta$. That torque will give the dipole an angular acceleration, in the opposite direction of $\theta$.

$$
\tau=-p E \sin \theta=I \alpha \rightarrow \alpha=\frac{d^{2} \theta}{d t^{2}}=-\frac{p E}{I} \sin \theta
$$

If $\theta$ is small, so that $\sin \theta \approx \theta$, then the equation is in the same form as Eq. 14-3, the equation of motion for the simple harmonic oscillator.

$$
\frac{d^{2} \theta}{d t^{2}}=-\frac{p E}{I} \sin \theta \approx-\frac{p E}{I} \theta \rightarrow \frac{d^{2} \theta}{d t^{2}}+\frac{p E}{I} \theta=0
$$

(b) The frequency can be found from the coefficient of $\theta$ in the equation of motion.

$$
\omega^{2}=\frac{p E}{I} \rightarrow f=\frac{1}{2 \pi} \sqrt{\frac{p E}{I}}
$$

66. If the dipole is of very small extent, then the potential energy is a function of position, and so Eq. 2110 gives $U(x)=-\overrightarrow{\mathbf{p}} \cdot \overrightarrow{\mathbf{E}}(x)$. Since the potential energy is known, we can use Eq. 8-7.

$$
F_{x}=-\frac{d U}{d x}=-\frac{d}{d x}[-\overrightarrow{\mathbf{p}} \cdot \overrightarrow{\mathbf{E}}(x)]=\overrightarrow{\mathbf{p}} \cdot \frac{d \overrightarrow{\mathbf{E}}}{d x}
$$

Since the field does not depend on the $y$ or $z$ coordinates, all other components of the force will be 0 .
Thus $\overrightarrow{\mathbf{F}}=F_{x} \hat{\mathbf{i}}=\left(\overrightarrow{\mathbf{p}} \cdot \frac{d \overrightarrow{\mathbf{E}}}{d x}\right) \hat{\mathbf{i}}$.
67. (a) Along the $x$ axis the fields from the two charges are parallel so the magnitude is found as follows.

$$
\begin{aligned}
E_{\mathrm{net}} & =E_{+Q}+E_{-Q}=\frac{Q}{4 \pi \varepsilon_{0}\left(r-\frac{1}{2} \ell\right)^{2}}+\frac{(-Q)}{4 \pi \varepsilon_{0}\left(r+\frac{1}{2} \ell\right)^{2}} \\
& =\frac{Q\left[\left(r+\frac{1}{2} \ell\right)^{2}-\left(r-\frac{1}{2} \ell\right)^{2}\right]}{4 \pi \varepsilon_{0}\left(r+\frac{1}{2} \ell\right)^{2}\left(r-\frac{1}{2} \ell\right)^{2}} \\
& =\frac{Q(2 r \ell)}{4 \pi \varepsilon_{0}\left(r+\frac{1}{2} \ell\right)^{2}\left(r-\frac{1}{2} \ell\right)^{2}} \approx \frac{Q(2 r \ell)}{4 \pi \varepsilon_{0} r^{4}}=\frac{2 Q \ell}{4 \pi \varepsilon_{0} r^{3}}=\frac{1}{4 \pi \varepsilon_{0}} \frac{2 p}{r^{3}}
\end{aligned}
$$



The same result is obtained if the point is to the left of $-Q$.
(b) The electric field points in the same direction as the dipole moment vector.
68. Set the magnitude of the electric force equal to the magnitude of the force of gravity and solve for the distance.

$$
\begin{aligned}
& F_{\mathrm{E}}=F_{\mathrm{G}} \rightarrow k \frac{e^{2}}{r^{2}}=m g \rightarrow \\
& r=e \sqrt{\frac{k}{m g}}=\left(1.602 \times 10^{-19} \mathrm{C}\right) \sqrt{\frac{\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)}{\left(9.11 \times 10^{-31} \mathrm{~kg}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}}=5.08 \mathrm{~m}
\end{aligned}
$$

69. Water has an atomic mass of 18 , so 1 mole of water molecules has a mass of 18 grams. Each water molecule contains 10 protons.

$$
65 \mathrm{~kg}\left(\frac{6.02 \times 10^{23} \mathrm{H}_{2} \mathrm{O} \text { molecules }}{0.018 \mathrm{~kg}}\right)\left(\frac{10 \text { protons }}{1 \text { molecule }}\right)\left(\frac{1.60 \times 10^{-19} \mathrm{C}}{\text { proton }}\right)=3.5 \times 10^{9} \mathrm{C}
$$

70. Calculate the total charge on all electrons in 3.0 g of copper, and compare $38 \mu \mathrm{C}$ to that value.

$$
\begin{aligned}
& \text { Total electron charge }=3.0 \mathrm{~g}\left(\frac{1 \text { mole }}{63.5 \mathrm{~g}}\right)\left(\frac{6.02 \times 10^{23} \text { atoms }}{\text { mole }}\right)\left(\frac{29 \mathrm{e}}{\text { atoms }}\right)\left(\frac{1.602 \times 10^{-19} \mathrm{C}}{1 \mathrm{e}}\right)=1.32 \times 10^{5} \mathrm{C} \\
& \text { Fraction lost }=\frac{38 \times 10^{-6} \mathrm{C}}{1.32 \times 10^{5} \mathrm{C}}=2.9 \times 10^{-10}
\end{aligned}
$$

71. Use Eq. 21-4a to calculate the magnitude of the electric charge on the Earth.

$$
E=k \frac{Q}{r^{2}} \rightarrow Q=\frac{E r^{2}}{k}=\frac{(150 \mathrm{~N} / \mathrm{C})\left(6.38 \times 10^{6} \mathrm{~m}\right)^{2}}{8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}}=6.8 \times 10^{5} \mathrm{C}
$$

Since the electric field is pointing towards the Earth's center, the charge must be negative.
72. (a) From problem 71, we know that the electric field is pointed towards the Earth's center. Thus an electron in such a field would experience an upwards force of magnitude $F_{\mathrm{E}}=e E$. The force of gravity on the electron will be negligible compared to the electric force.

$$
\begin{aligned}
& F_{\mathrm{E}}=e E=m a \rightarrow \\
& a=\frac{e E}{m}=\frac{\left(1.602 \times 10^{-19} \mathrm{C}\right)(150 \mathrm{~N} / \mathrm{C})}{\left(9.11 \times 10^{-31} \mathrm{~kg}\right)}=2.638 \times 10^{13} \mathrm{~m} / \mathrm{s}^{2} \approx 2.6 \times 10^{13} \mathrm{~m} / \mathrm{s}^{2}, \mathrm{up}
\end{aligned}
$$

(b) A proton in the field would experience a downwards force of magnitude $F_{\mathrm{E}}=e E$. The force of gravity on the proton will be negligible compared to the electric force.

$$
\begin{aligned}
& F_{\mathrm{E}}=e E=m a \rightarrow \\
& a=\frac{e E}{m}=\frac{\left(1.602 \times 10^{-19} \mathrm{C}\right)(150 \mathrm{~N} / \mathrm{C})}{\left(1.67 \times 10^{-27} \mathrm{~kg}\right)}=1.439 \times 10^{10} \mathrm{~m} / \mathrm{s}^{2} \approx 1.4 \times 10^{10} \mathrm{~m} / \mathrm{s}^{2}, \mathrm{down}
\end{aligned}
$$

(c) Electron: $\frac{a}{g}=\frac{2.638 \times 10^{13} \mathrm{~m} / \mathrm{s}^{2}}{9.80 \mathrm{~m} / \mathrm{s}^{2}} \approx 2.7 \times 10^{12} ; \quad$ Proton: $\frac{a}{g}=\frac{1.439 \times 10^{10} \mathrm{~m} / \mathrm{s}^{2}}{9.80 \mathrm{~m} / \mathrm{s}^{2}} \approx 1.5 \times 10^{9}$
73. For the droplet to remain stationary, the magnitude of the electric force on the droplet must be the same as the weight of the droplet. The mass of the droplet is found from its volume times the density of water. Let $n$ be the number of excess electrons on the water droplet.

$$
\begin{aligned}
& F_{\mathrm{E}}=|q| E=m g \rightarrow n e E=\frac{4}{3} \pi r^{3} \rho g \rightarrow \\
& n=\frac{4 \pi r^{3} \rho g}{3 e E}=\frac{4 \pi\left(1.8 \times 10^{-5} \mathrm{~m}\right)^{3}\left(1.00 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{3\left(1.602 \times 10^{-19} \mathrm{C}\right)(150 \mathrm{~N} / \mathrm{C})}=9.96 \times 10^{6} \approx 1.0 \times 10^{7} \text { electrons }
\end{aligned}
$$

74. There are four forces to calculate. Call the rightward direction the positive direction. The value of $k$ is $8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}$ and the value of $e$ is $1.602 \times 10^{-19} \mathrm{C}$.

$$
\begin{aligned}
F_{\mathrm{net}} & =F_{\mathrm{CH}}+F_{\mathrm{CN}}+F_{\mathrm{OH}}+F_{\mathrm{ON}}=\frac{k(0.40 e)(0.20 e)}{\left(1 \times 10^{-9} \mathrm{~m}\right)^{2}}\left[-\frac{1}{(0.30)^{2}}+\frac{1}{(0.40)^{2}}+\frac{1}{(0.18)^{2}}-\frac{1}{(0.28)^{2}}\right] \\
& =2.445 \times 10^{-10} \mathrm{~N} \approx 2.4 \times 10^{-10} \mathrm{~N}
\end{aligned}
$$

75. Set the Coulomb electrical force equal to the Newtonian gravitational force on one of the bodies (the Moon).

$$
\begin{aligned}
& F_{\mathrm{E}}=F_{\mathrm{G}} \rightarrow k \frac{Q^{2}}{r_{\text {orbit }}^{2}}=G \frac{M_{\text {Moon }} M_{\text {Earth }}}{r_{\text {orbit }}^{2}} \rightarrow \\
& Q=\sqrt{\frac{G M_{\text {Moon }} M_{\text {Earth }}}{k}}=\sqrt{\frac{\left(6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(7.35 \times 10^{22} \mathrm{~kg}\right)\left(5.98 \times 10^{24} \mathrm{~kg}\right)}{\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)}}=5.71 \times 10^{13} \mathrm{C}
\end{aligned}
$$

76. The electric force must be a radial force in order for the electron to move in a circular orbit.

$$
\begin{aligned}
& F_{\mathrm{E}}=F_{\text {radial }} \rightarrow k \frac{Q^{2}}{r_{\text {orbit }}^{2}}=\frac{m v^{2}}{r_{\text {orbit }}} \rightarrow \\
& r_{\text {orbit }}=k \frac{Q^{2}}{m v^{2}}=\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{\left(1.602 \times 10^{-19} \mathrm{C}\right)^{2}}{\left(9.109 \times 10^{-31} \mathrm{~kg}\right)\left(2.2 \times 10^{6} \mathrm{~m} / \mathrm{s}\right)^{2}}=5.2 \times 10^{-11} \mathrm{~m}
\end{aligned}
$$

77. Because of the inverse square nature of the electric field, any location where the field is zero must be closer to the weaker charge $\left(Q_{2}\right)$. Also, in between the two charges,
 the fields due to the two charges are parallel to each other and cannot cancel. Thus the only places where the field can be zero are closer to the weaker charge, but not between them. In the diagram, this means that $\ell$ must be positive.

$$
\begin{aligned}
& E=-k \frac{\left|Q_{2}\right|}{\ell^{2}}+k \frac{Q_{1}}{(\ell+d)^{2}}=0 \rightarrow\left|Q_{2}\right|(\ell+d)^{2}=Q_{1} \ell^{2} \rightarrow \\
& \ell=\frac{\sqrt{\left|Q_{2}\right|}}{\sqrt{Q_{1}}-\sqrt{\left|Q_{2}\right|}} d=\frac{\sqrt{5.0 \times 10^{-6} \mathrm{C}}}{\sqrt{2.5 \times 10^{-5} \mathrm{C}}-\sqrt{5.0 \times 10^{-6} \mathrm{C}}}(2.0 \mathrm{~m})=\begin{array}{l}
1.6 \mathrm{~m} \text { from } Q_{2} \\
3.6 \mathrm{~m} \text { from } Q_{1}
\end{array}
\end{aligned}
$$

78. We consider that the sock is only acted on by two forces - the force of gravity, acting downward, and the electrostatic force, acting upwards. If charge $Q$ is on the sweater, then it will create an electric field of $E=\frac{\sigma}{2 \varepsilon_{0}}=\frac{Q / A}{2 \varepsilon_{0}}$, where $A$ is the surface area of one side of the sweater. The same magnitude of charge will be on the sock, and so the attractive force between the sweater and sock is $F_{E}=Q E=\frac{Q^{2}}{2 \varepsilon_{0} A}$. This must be equal to the weight of the sweater. We estimate the sweater area as $0.10 \mathrm{~m}^{2}$, which is roughly a square foot.

$$
\begin{aligned}
& F_{E}=Q E=\frac{Q^{2}}{2 \varepsilon_{0} A}=m g \rightarrow \\
& Q=\sqrt{2 \varepsilon_{0} A m g}=\sqrt{2\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)\left(0.10 \mathrm{~m}^{2}\right)(0.040 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=8 \times 10^{-7} \mathrm{C}
\end{aligned}
$$

79. The sphere will oscillate sinusoidally about the equilibrium point, with an amplitude of 5.0 cm . The angular frequency of the sphere is given by $\omega=\sqrt{k / m}=\sqrt{126 \mathrm{~N} / \mathrm{m} / 0.650 \mathrm{~kg}}=13.92 \mathrm{rad} / \mathrm{s}$. The distance of the sphere from the table is given by $r=[0.150-0.0500 \cos (13.92 t)] \mathrm{m}$. Use this distance
and the charge to give the electric field value at the tabletop. That electric field will point upwards at all times, towards the negative sphere.

$$
\begin{aligned}
E & =k \frac{|Q|}{r^{2}}=\frac{\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)\left(3.00 \times 10^{-6} \mathrm{C}\right)}{[0.150-0.0500 \cos (13.92 t)]^{2} \mathrm{~m}^{2}}=\frac{2.70 \times 10^{4}}{[0.150-0.0500 \cos (13.92 t)]^{2}} \mathrm{~N} / \mathrm{C} \\
& =\frac{1.08 \times 10^{7}}{[3.00-\cos (13.9 t)]^{2}} \mathrm{~N} / \mathrm{C}, \text { upwards }
\end{aligned}
$$

80. The wires form two sides of an isosceles triangle, and so the two charges are separated by a distance $\ell=2(78 \mathrm{~cm}) \sin 26^{\circ}=68.4 \mathrm{~cm}$ and are directly horizontal from each other. Thus the electric force on each charge is horizontal. From the freebody diagram for one of the spheres, write the net force in both the horizontal and vertical directions and solve for the electric force. Then write the electric force by Coulomb's law, and equate the two expressions for the electric force to find the
 charge.

$$
\begin{aligned}
& \sum F_{y}=F_{\mathrm{T}} \cos \theta-m g=0 \rightarrow F_{\mathrm{T}}=\frac{m g}{\cos \theta} \\
& \sum F_{x}=F_{\mathrm{T}} \sin \theta-F_{\mathrm{E}}=0 \rightarrow F_{\mathrm{E}}=F_{\mathrm{T}} \sin \theta=\frac{m g}{\cos \theta} \sin \theta=m g \tan \theta \\
& F_{\mathrm{E}}=k \frac{(Q / 2)^{2}}{\ell^{2}}=m g \tan \theta \rightarrow Q=2 \ell \sqrt{\frac{m g \tan \theta}{k}} \\
& =2(0.684 \mathrm{~m}) \sqrt{\frac{\left(24 \times 10^{-3} \mathrm{~kg}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \tan 26^{\circ}}{\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)}}=4.887 \times 10^{-6} \mathrm{C} \approx 4.9 \times 10^{-6} \mathrm{C}
\end{aligned}
$$

81. The electric field at the surface of the pea is given by Eq. 21-4a. Solve that equation for the charge.

$$
E=k \frac{Q}{r^{2}} \rightarrow Q=\frac{E r^{2}}{k}=\frac{\left(3 \times 10^{6} \mathrm{~N} / \mathrm{C}\right)\left(3.75 \times 10^{-3} \mathrm{~m}\right)^{2}}{8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}}=5 \times 10^{-9} \mathrm{C}
$$

This corresponds to about 3 billion electrons.
82. There will be a rightward force on $Q_{1}$ due to $Q_{2}$, given by Coulomb's law. There will be a leftward force on $Q_{1}$ due to the electric field created by the parallel plates. Let right be the positive direction.

$$
\begin{aligned}
\sum F & =k \frac{\left|Q_{1} Q_{2}\right|}{x^{2}}-\left|Q_{1}\right| E \\
& =\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{\left(6.7 \times 10^{-6} \mathrm{C}\right)\left(1.8 \times 10^{-6} \mathrm{C}\right)}{(0.34 \mathrm{~m})^{2}}-\left(6.7 \times 10^{-6} \mathrm{C}\right)\left(7.3 \times 10^{4} \mathrm{~N} / \mathrm{C}\right) \\
& =0.45 \mathrm{~N}, \text { right }
\end{aligned}
$$

83. The weight of the sphere is the density times the volume. The electric force is given by Eq. 21-1, with both spheres having the same charge, and the separation distance equal to their diameter.

$$
\begin{aligned}
& m g=k \frac{Q^{2}}{(d)^{2}} \rightarrow \rho \frac{4}{3} \pi r^{3} g=\frac{k Q^{2}}{(2 r)^{2}} \rightarrow \\
& Q=\sqrt{\frac{16 \rho \pi g r^{5}}{3 k}}=\sqrt{\frac{16\left(35 \mathrm{~kg} / \mathrm{m}^{3}\right) \pi\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)\left(1.0 \times 10^{-2} \mathrm{~m}\right)^{5}}{3\left(8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)}}=8.0 \times 10^{-9} \mathrm{C}
\end{aligned}
$$

84. From the symmetry, we see that the resultant field will be in the $y$ direction. So we take the vertical component of each field.

$$
\begin{aligned}
E_{\text {net }} & =2 E_{+} \sin \theta-E_{-}=2 \frac{Q}{4 \pi \varepsilon_{0}\left(r^{2}+\ell^{2}\right)} \frac{r}{\left(r^{2}+\ell^{2}\right)^{1 / 2}}-\frac{2 Q}{4 \pi \varepsilon_{0} r^{2}} \\
& =\frac{2 Q}{4 \pi \varepsilon_{0}}\left[\frac{r}{\left(r^{2}+\ell^{2}\right)^{3 / 2}}-\frac{1}{r^{2}}\right] \\
& =\frac{2 Q}{4 \pi \varepsilon_{0}\left(r^{2}+\ell^{2}\right)^{3 / 2} r^{2}}\left[r^{3}-\left(r^{2}+\ell^{2}\right)^{3 / 2}\right] \\
& =\frac{2 Q r^{3}\left[1-\left(1+\frac{\ell^{2}}{r^{2}}\right)^{3 / 2}\right]}{4 \pi \varepsilon_{0} r^{5}\left(1+\frac{\ell^{2}}{r^{2}}\right)^{3 / 2}}
\end{aligned}
$$

Use the binomial expansion, assuming $r \gg \ell$.

$$
E_{\mathrm{net}}=\frac{2 Q r^{3}\left[1-\left(1+\frac{\ell^{2}}{r^{2}}\right)^{3 / 2}\right]}{4 \pi \varepsilon_{0} r^{5}\left(1+\frac{\ell^{2}}{r^{2}}\right)^{3 / 2}} \approx \frac{2 Q r^{3}\left[1-\left(1+\frac{3}{2} \frac{\ell^{2}}{r^{2}}\right)\right]}{4 \pi \varepsilon_{0} r^{5}\left(1+\frac{3}{2} \frac{\ell^{2}}{r^{2}}\right)}=\frac{2 Q r^{3}\left(-\frac{3}{2} \frac{\ell^{2}}{r^{2}}\right)}{4 \pi \varepsilon_{0} r^{5}(1)}=-\frac{3 Q \ell^{2}}{4 \pi \varepsilon_{0} r^{4}}
$$

Notice that the field points toward the negative charges.
85. This is a constant acceleration situation, similar to projectile motion in a uniform gravitational field. Let the width of the plates be $\ell$, the vertical gap between the plates be $h$, and the initial velocity be $v_{0}$. Notice that the vertical motion has a maximum displacement of $h / 2$. Let upwards be the positive vertical direction. We calculate the vertical acceleration produced by the electric field and the time $t$ for the electron to cross the region of the field. We then use constant acceleration equations to solve for the angle.

$$
\begin{aligned}
& F_{y}=m a_{y}=q E=-e E \rightarrow a_{y}=-\frac{e E}{m} ; \ell=v_{0} \cos \theta_{0}(t) \rightarrow t=\frac{\ell}{v_{0} \cos \theta_{0}} \\
& v_{y}=v_{0 y}+a_{y} t_{\text {top }} \rightarrow 0=v_{0} \sin \theta_{0}-\frac{e E}{m}\left(\frac{1}{2} \frac{\ell}{v_{0} \cos \theta_{0}}\right) \rightarrow v_{0}^{2}=\frac{e E}{2 m}\left(\frac{\ell}{\sin \theta_{0} \cos \theta_{0}}\right)
\end{aligned}
$$

$$
\begin{aligned}
& y_{\text {top }}=y_{0}+v_{0 y} t_{\text {top }}+\frac{1}{2} a_{y} t^{2} \rightarrow \frac{1}{2} h=v_{0} \sin \theta_{0}\left(\frac{1}{2} \frac{\ell}{v_{0} \cos \theta_{0}}\right)-\frac{1}{2} \frac{e E}{m}\left(\frac{1}{2} \frac{\ell}{v_{0} \cos \theta_{0}}\right)^{2} \rightarrow \\
& h=\ell \tan \theta_{0}-\frac{e E \ell^{2}}{4 m \cos ^{2} \theta_{0}} \frac{1^{2}}{v_{0}^{2}}=\ell \tan \theta_{0}-\frac{e E \ell^{2}}{4 m \cos ^{2} \theta_{0}} \frac{1}{\frac{e E}{2 m}\left(\frac{\ell}{\sin \theta_{0} \cos \theta_{0}}\right)}=\ell \tan \theta_{0}-\frac{1}{2} \ell \tan \theta_{0} \\
& h=\frac{1}{2} \ell \tan \theta_{0} \rightarrow \theta_{0}=\tan ^{-1} \frac{2 h}{\ell}=\tan ^{-1} \frac{2(1.0 \mathrm{~cm})}{6.0 \mathrm{~cm}}=18^{\circ}
\end{aligned}
$$

86. (a) The electric field from the long wire is derived in Example 21-11.

$$
E=\frac{1}{2 \pi \varepsilon_{0}} \frac{\lambda}{r} \text {, radially away from the wire }
$$

(b) The force on the electron will point radially in, producing a centripetal acceleration.

$$
\begin{aligned}
|F| & =|q E|=\frac{e}{2 \pi \varepsilon_{0}} \frac{\lambda}{r}=\frac{m v^{2}}{r} \rightarrow \\
v & =\sqrt{2 \frac{1}{4 \pi \varepsilon_{0}} \frac{e \lambda}{m}}=\sqrt{2\left(8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{\left(1.60 \times 10^{-19} \mathrm{C}\right)\left(0.14 \times 10^{-6} \mathrm{C} / \mathrm{m}\right)}{9.11 \times 10^{-31} \mathrm{~kg}}} \\
& =2.1 \times 10^{7} \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Note that this speed is independent of $r$.
87. We treat each of the plates as if it were infinite, and then use Eq. 21-7. The fields due to the first and third plates point towards their respective plates, and the fields due to the second plate point away from it. See the diagram. The directions of the fields are given by the arrows, so we calculate the magnitude of the fields from Eq. 21-7. Let the positive direction be to the right.

$$
\begin{aligned}
E_{\mathrm{A}} & =E_{1}-E_{2}+E_{3}=\frac{\left|\sigma_{1}\right|}{2 \varepsilon_{0}}-\frac{\sigma_{2}}{2 \varepsilon_{0}}+\frac{\left|\sigma_{1}\right|}{2 \varepsilon_{0}} \\
& =\frac{(0.50-0.25+0.35) \times 10^{-6} \mathrm{C} / \mathrm{m}^{2}}{2\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)}=3.4 \times 10^{4} \mathrm{~N} / \mathrm{C}, \text { to the right } \\
E_{\mathrm{B}} & =-E_{1}-E_{2}+E_{3}=-\frac{\left|\sigma_{1}\right|}{2 \varepsilon_{0}}-\frac{\sigma_{2}}{2 \varepsilon_{0}}+\frac{\left|\sigma_{1}\right|}{2 \varepsilon_{0}} \\
& =\frac{(-0.50-0.25+0.35) \times 10^{-6} \mathrm{C} / \mathrm{m}^{2}}{2\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)}=-2.3 \times 10^{4} \mathrm{~N} / \mathrm{C}=2.3 \times 10^{4} \mathrm{~N} / \mathrm{C} \text { to the left } \\
E_{\mathrm{C}} & =-E_{1}+E_{2}+E_{3}=-\frac{\left|\sigma_{1}\right|}{2 \varepsilon_{0}}-\frac{\sigma_{2}}{2 \varepsilon_{0}}+\frac{\left|\sigma_{1}\right|}{2 \varepsilon_{0}} \\
& =\frac{(-0.50+0.25+0.35) \times 10^{-6} \mathrm{C} / \mathrm{m}^{2}}{2\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)}=5.6 \times 10^{3} \mathrm{~N} / \mathrm{C} \text { to the right }
\end{aligned}
$$



$$
\begin{aligned}
E_{\mathrm{D}} & =-E_{1}+E_{2}-E_{3}=-\frac{\left|\sigma_{1}\right|}{2 \varepsilon_{0}}-\frac{\sigma_{2}}{2 \varepsilon_{0}}+\frac{\left|\sigma_{1}\right|}{2 \varepsilon_{0}} \\
& =\frac{(-0.50+0.25-0.35) \times 10^{-6} \mathrm{C} / \mathrm{m}^{2}}{2\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)}=-3.4 \times 10^{4} \mathrm{~N} / \mathrm{C}=3.4 \times 10^{3} \mathrm{~N} / \mathrm{C} \text { to the left }
\end{aligned}
$$

88. Since the electric field exerts a force on the charge in the same direction as the electric field, the charge is positive. Use the free-body diagram to write the equilibrium equations for both the horizontal and vertical directions, and use those equations to find the magnitude of the charge.


$$
\begin{aligned}
& \theta=\cos ^{-1} \frac{43 \mathrm{~cm}}{55 \mathrm{~cm}}=38.6^{\circ} \\
& \sum F_{x}=F_{\mathrm{E}}-F_{\mathrm{T}} \sin \theta=0 \rightarrow F_{\mathrm{E}}=F_{\mathrm{T}} \sin \theta=Q E \\
& \sum F_{y}=F_{\mathrm{T}} \cos \theta-m g=0 \rightarrow F_{\mathrm{T}}=\frac{m g}{\cos \theta} \rightarrow Q E=m g \tan \theta \\
& Q=\frac{m g \tan \theta}{E}=\frac{\left(1.0 \times 10^{-3} \mathrm{~kg}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \tan 38.6^{\circ}}{\left(1.5 \times 10^{4} \mathrm{~N} / \mathrm{C}\right)}=5.2 \times 10^{-7} \mathrm{C}
\end{aligned}
$$

89. A negative charge must be placed at the center of the square. Let $Q=8.0 \mu \mathrm{C}$ be the charge at each corner, let $-q$ be the magnitude of negative charge in the center, and let $d=9.2 \mathrm{~cm}$ be the side length of the square. By the symmetry of the problem, if we make the net force on one of the corner charges be zero, the net force on each other corner charge will also be zero.

$$
\begin{aligned}
& F_{41}=k \frac{Q^{2}}{d^{2}} \rightarrow F_{41 x}=k \frac{Q^{2}}{d^{2}}, F_{41 y}=0 \\
& F_{42}=k \frac{Q^{2}}{2 d^{2}} \rightarrow F_{42 x}=k \frac{Q^{2}}{2 d^{2}} \cos 45^{\circ}=k \frac{\sqrt{2} Q^{2}}{4 d^{2}}, F_{42 y}=k \frac{\sqrt{2} Q^{2}}{4 d^{2}} \\
& F_{43}=k \frac{Q^{2}}{d^{2}} \rightarrow F_{43 x}=0, F_{43 y}=k \frac{Q^{2}}{d^{2}} \\
& F_{4 q}=k \frac{q Q}{d^{2} / 2} \rightarrow F_{4 q x}=-k \frac{2 q Q}{d^{2}} \cos 45^{\circ}=-k \frac{\sqrt{2} q Q}{d^{2}}=F_{4 q y}
\end{aligned}
$$

The net force in each direction should be zero.

$$
\begin{aligned}
& \sum F_{x}=k \frac{Q^{2}}{d^{2}}+k \frac{\sqrt{2} Q^{2}}{4 d^{2}}+0-k \frac{\sqrt{2} q Q}{d^{2}}=0 \rightarrow \\
& q=Q\left(\frac{1}{\sqrt{2}}+\frac{1}{4}\right)=\left(8.0 \times 10^{-6} \mathrm{C}\right)\left(\frac{1}{\sqrt{2}}+\frac{1}{4}\right)=7.66 \times 10^{-6} \mathrm{C}
\end{aligned}
$$

So the charge to be placed is $-q=-7.7 \times 10^{-6} \mathrm{C}$.

This is an unstable equilibrium. If the center charge were slightly displaced, say towards the right, then it would be closer to the right charges than the left, and would be attracted more to the right. Likewise the positive charges on the right side of the square would be closer to it and would be attracted more to it, moving from their corner positions. The system would not have a tendency to return to the symmetric shape, but rather would have a tendency to move away from it if disturbed.
90. (a) The force of sphere B on sphere A is given by Coulomb's law.

$$
F_{\mathrm{AB}}=\frac{k Q^{2}}{R^{2}}, \text { away from B }
$$

(b) The result of touching sphere B to uncharged sphere C is that the charge on B is shared between the two spheres, and so the charge on B is reduced to $Q / 2$. Again use Coulomb's law.

$$
F_{\mathrm{AB}}=k \frac{Q Q / 2}{R^{2}}=\frac{k Q^{2}}{2 R^{2}}, \text { away from B }
$$

(c) The result of touching sphere A to sphere C is that the charge on the two spheres is shared, and so the charge on A is reduced to $3 Q / 4$. Again use Coulomb's law.

$$
F_{\mathrm{AB}}=k \frac{(3 Q / 4)(Q / 2)}{R^{2}}=\frac{3 k Q^{2}}{8 R^{2}}, \text { away from } \mathrm{B}
$$

91. (a) The weight of the mass is only about 2 N . Since the tension in the string is more than that, there must be a downward electric force on the positive charge, which means that the electric field must be pointed down. Use the free-body diagram to write an expression for the magnitude of the electric field.

$$
\begin{aligned}
& \sum F=F_{\mathrm{T}}-m g-F_{\mathrm{E}}=0 \rightarrow F_{\mathrm{E}}=Q E=F_{\mathrm{T}}-m g \rightarrow \\
& E=\frac{F_{\mathrm{T}}-m g}{Q}=\frac{5.18 \mathrm{~N}-(0.210 \mathrm{~kg})\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}{3.40 \times 10^{-7} \mathrm{C}}=9.18 \times 10^{6} \mathrm{~N} / \mathrm{C}
\end{aligned}
$$


(b) Use Eq. 21-7.

$$
E=\frac{\sigma}{2 \varepsilon_{0}} \rightarrow \sigma=2 E \varepsilon_{0}=2\left(9.18 \times 10^{6} \mathrm{~N} / \mathrm{C}\right)\left(8.854 \times 10^{-12}\right)=1.63 \times 10^{-4} \mathrm{C} / \mathrm{m}^{2}
$$

92. (a) The force will be attractive. Each successive charge is another distance $d$ farther than the previous charge. The magnitude of the charge on the electron is $e$.

$$
\begin{aligned}
F & =k \frac{e Q}{(d)^{2}}+k \frac{e Q}{(2 d)^{2}}+k \frac{e Q}{(3 d)^{2}}+k \frac{e Q}{(4 d)^{2}}+\cdots=k \frac{e Q}{d^{2}}\left(\frac{1}{1^{2}}+\frac{1}{2^{2}}+\frac{1}{3^{2}}+\frac{1}{4^{2}}+\cdots\right) \\
& =k \frac{e Q}{d^{2}} \sum_{n=1}^{\infty} \frac{1}{n^{2}}=\frac{1}{4 \pi \varepsilon_{0}} \frac{e Q}{d^{2}} \frac{\pi^{2}}{6}=\frac{\pi e Q}{24 \varepsilon_{0} d^{2}}
\end{aligned}
$$

(b) Now the closest $Q$ is a distance of $3 d$ from the electron.

$$
\begin{aligned}
F & =k \frac{e Q}{(3 d)^{2}}+k \frac{e Q}{(4 d)^{2}}+k \frac{e Q}{(5 d)^{2}}+k \frac{e Q}{(6 d)^{2}}+\cdots=k \frac{e Q}{d^{2}}\left(\frac{1}{3^{2}}+\frac{1}{4^{2}}+\frac{1}{5^{2}}+\frac{1}{6^{2}}+\cdots\right) \\
& =k \frac{e Q}{d^{2}} \sum_{n=3}^{\infty} \frac{1}{n^{2}}=k \frac{e Q}{d^{2}}\left[\left(\sum_{n=1}^{\infty} \frac{1}{n^{2}}\right)-\frac{1}{1^{2}}-\frac{1}{2^{2}}\right]=k \frac{e Q}{d^{2}}\left[\frac{\pi^{2}}{6}-\frac{5}{4}\right]=\frac{e Q}{4 \pi \varepsilon_{0} d^{2}}\left[\frac{\pi^{2}}{6}-\frac{5}{4}\right]
\end{aligned}
$$

93. (a) Take $\frac{d E}{d x}$, set it equal to 0 , and solve for the location of the maximum.

$$
\begin{aligned}
& E=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q x}{\left(x^{2}+a^{2}\right)^{3 / 2}} \\
& \frac{d E}{d x}=\frac{Q}{4 \pi \varepsilon_{0}}\left[\frac{\left(x^{2}+a^{2}\right)^{3 / 2}-x \frac{3}{2}}{\left(x^{2}+a\right.}\right. \\
& x=\frac{a}{\sqrt{2}}=\frac{10.0 \mathrm{~cm}}{\sqrt{2}}=7.07 \mathrm{~cm}
\end{aligned}
$$

$$
\frac{d E}{d x}=\frac{Q}{4 \pi \varepsilon_{0}}\left[\frac{\left(x^{2}+a^{2}\right)^{3 / 2}-x \frac{3}{2}\left(x^{2}+a^{2}\right)^{1 / 2} 2 x}{\left(x^{2}+a^{2}\right)^{3}}\right]=\frac{Q}{4 \pi \varepsilon_{0}} \frac{\left(a^{2}-2 x^{2}\right)}{\left(x^{2}+a^{2}\right)^{5 / 2}}=0 \rightarrow a^{2}-2 x^{2}=0 \rightarrow
$$

(b) Yes, the maximum of the graph does coincide with the analytic maximum. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH21.XLS," on tab "Problem 21.93b."
(c) The field due to the ring is $E_{\text {ring }}=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q x}{\left(x^{2}+a^{2}\right)^{3 / 2}}$.
(d) The field due to the point charge is $E_{\text {ring }}=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{x^{2}}$. Both are plotted on the graph. The graph shows that the two fields converge at large distances from the origin. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH21.XLS," on tab

 "Problem 21.93cd."
(e) According to the spreadsheet, $E_{\text {ring }}=0.9 E_{\text {point }}$ at about 37 cm . An analytic calculation gives the same result.

$$
\begin{aligned}
& E_{\text {ring }}=0.9 E_{\text {point }} \rightarrow \frac{1}{4 \pi \varepsilon_{0}} \frac{Q x}{\left(x^{2}+a^{2}\right)^{3 / 2}}=0.9 \frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{x^{2}} \rightarrow \\
& x^{3}=0.9\left(x^{2}+a^{2}\right)^{3 / 2}=0.9 x^{3}\left(1+\frac{a^{2}}{x^{2}}\right)^{3 / 2} \rightarrow x=\frac{a}{\sqrt{\left(\frac{1}{0.9}\right)^{2 / 3}-1}}=\frac{10.0 \mathrm{~cm}}{\sqrt{\left(\frac{1}{0.9}\right)^{2 / 3}-1}}=37.07 \mathrm{~cm}
\end{aligned}
$$

94. (a) Let $q_{1}=8.00 \mu \mathrm{C}, q_{2}=2.00 \mu \mathrm{C}$, and
$d=0.0500 \mathrm{~m}$. The field directions due to the charges are shown in the diagram. We take care with the signs of the $x$ coordinate used to calculate the magnitude of the field.


$$
\begin{aligned}
& E_{x<-d}=E_{2}-E_{1}=\frac{1}{4 \pi \varepsilon_{0}} \frac{q_{2}}{(|x|-d)^{2}}-\frac{1}{4 \pi \varepsilon_{0}} \frac{q_{1}}{(|x|+d)^{2}}=\frac{1}{4 \pi \varepsilon_{0}} \frac{q_{2}}{(-x-d)^{2}}-\frac{1}{4 \pi \varepsilon_{0}} \frac{q_{1}}{(-x+d)^{2}} \\
& E_{-d<x<0}=-E_{2}-E_{1}=-\frac{1}{4 \pi \varepsilon_{0}} \frac{q_{2}}{(d-|x|)^{2}}-\frac{1}{4 \pi \varepsilon_{0}} \frac{q_{1}}{(|x|+d)^{2}}=-\frac{1}{4 \pi \varepsilon_{0}} \frac{q_{2}}{(d+x)^{2}}-\frac{1}{4 \pi \varepsilon_{0}} \frac{q_{1}}{(-x+d)^{2}} \\
& E_{0<x<d}=-E_{2}-E_{1}=-\frac{1}{4 \pi \varepsilon_{0}} \frac{q_{2}}{(d+x)^{2}}-\frac{1}{4 \pi \varepsilon_{0}} \frac{q_{1}}{(d-x)^{2}} \\
& E_{d<x}=E_{1}-E_{2}=\frac{1}{4 \pi \varepsilon_{0}} \frac{q_{1}}{(x-d)^{2}}-\frac{1}{4 \pi \varepsilon_{0}} \frac{q_{2}}{(x+d)^{2}}
\end{aligned}
$$



The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH21.XLS," on tab "Problem 21.94a."
(b) Now for points on the y axis. See the diagram for this case.

$$
E_{x}=-E_{1} \cos \theta-E_{2} \cos \theta
$$

$$
\begin{aligned}
& =-\frac{1}{4 \pi \varepsilon_{0}} \frac{q_{1} \cos \theta}{\left(d^{2}+y^{2}\right)}-\frac{1}{4 \pi \varepsilon_{0}} \frac{q_{2} \cos \theta}{\left(d^{2}+y^{2}\right)} \\
& =-\frac{1}{4 \pi \varepsilon_{0}} \frac{\left(q_{1}+q_{2}\right)}{\left(d^{2}+y^{2}\right)} \cos \theta=-\frac{1}{4 \pi \varepsilon_{0}} \frac{\left(q_{1}+q_{2}\right)}{\left(d^{2}+y^{2}\right)} \frac{d}{\sqrt{d^{2}+y^{2}}} \\
& =-\frac{1}{4 \pi \varepsilon_{0}} \frac{\left(q_{1}+q_{2}\right) d}{\left(d^{2}+y^{2}\right)^{3 / 2}}
\end{aligned}
$$


$E_{y}=E_{1} \sin \theta-E_{2} \sin \theta=\frac{1}{4 \pi \varepsilon_{0}} \frac{q_{1} \sin \theta}{\left(d^{2}+y^{2}\right)}-\frac{1}{4 \pi \varepsilon_{0}} \frac{q_{2} \sin \theta}{\left(d^{2}+y^{2}\right)}$

$$
=\frac{1}{4 \pi \varepsilon_{0}} \frac{\left(q_{1}-q_{2}\right)}{\left(d^{2}+y^{2}\right)} \sin \theta=\frac{1}{4 \pi \varepsilon_{0}} \frac{\left(q_{1}-q_{2}\right)}{\left(d^{2}+y^{2}\right)} \frac{y}{\sqrt{d^{2}+y^{2}}}=\frac{1}{4 \pi \varepsilon_{0}} \frac{\left(q_{1}-q_{2}\right) y}{\left(d^{2}+y^{2}\right)^{3 / 2}}
$$



The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH21.XLS," on tab "Problem 21.94b."

## CHAPTER 22: Gauss's Law

## Responses to Questions

1. No. If the net electric flux through a surface is zero, then the net charge contained in the surface is zero. However, there may be charges both inside and outside the surface that affect the electric field at the surface. The electric field could point outward from the surface at some points and inward at others. Yes. If the electric field is zero for all points on the surface, then the net flux through the surface must be zero and no net charge is contained within the surface.
2. No. The electric field in the expression for Gauss's law refers to the total electric field, not just the electric field due to any enclosed charge. Notice, though, that if the electric field is due to a charge outside the Gaussian surface, then the net flux through the surface due to this charge will be zero.
3. The electric flux will be the same. The flux is equal to the net charge enclosed by the surface divided by $\varepsilon_{0}$. If the same charge is enclosed, then the flux is the same, regardless of the shape of the surface.
4. The net flux will be zero. An electric dipole consists of two charges that are equal in magnitude but opposite in sign, so the net charge of an electric dipole is zero. If the closed surface encloses a zero net charge, than the net flux through it will be zero.
5. Yes. If the electric field is zero for all points on the surface, then the integral of $\overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}$ over the surface will be zero, the flux through the surface will be zero, and no net charge will be contained in the surface. No. If a surface encloses no net charge, then the net electric flux through the surface will be zero, but the electric field is not necessarily zero for all points on the surface. The integral of $\overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}$ over the surface must be zero, but the electric field itself is not required to be zero. There may be charges outside the surface that will affect the values of the electric field at the surface.
6. The electric flux through a surface is the scalar (dot) product of the electric field vector and the area vector of the surface. Thus, in magnitude, $\Phi_{\mathrm{E}}=E A \cos \theta$. By analogy, the gravitational flux through a surface would be the product of the gravitational field (or force per unit mass) and the area, or $\Phi_{\mathrm{g}}=g A \cos \theta$. Any mass, such as a planet, would be a "sink" for gravitational field. Since there is not "anti-gravity" there would be no sources.
7. No. Gauss's law is most useful in cases of high symmetry, where a surface can be defined over which the electric field has a constant value and a constant relationship to the direction of the outward normal to the surface. Such a surface cannot be defined for an electric dipole.
8. When the ball is inflated and charge is distributed uniformly over its surface, the field inside is zero. When the ball is collapsed, there is no symmetry to the charge distribution, and the calculation of the electric field strength and direction inside the ball is difficult (and will most likely give a non-zero result).
9. For an infinitely long wire, the electric field is radially outward from the wire, resulting from contributions from all parts of the wire. This allows us to set up a Gaussian surface that is cylindrical, with the cylinder axis parallel to the wire. This surface will have zero flux through the top and bottom of the cylinder, since the net electric field and the outward surface normal are perpendicular at all points over the top and bottom. In the case of a short wire, the electric field is not radially outward from the wire near the ends; it curves and points directly outward along the axis of
the wire at both ends. We cannot define a useful Gaussian surface for this case, and the electric field must be computed directly.
10. In Example 22-6, there is no flux through the flat ends of the cylindrical Gaussian surface because the field is directed radially outward from the wire. If instead the wire extended only a short distance past the ends of the cylinder, there would be a component of the field through the ends of the cylinder. The result of the example would be altered because the value of the field at a given point would now depend not only on the radial distance from the wire but also on the distance from the ends.
11. The electric flux through the sphere remains the same, since the same charge is enclosed. The electric field at the surface of the sphere is changed, because different parts of the sphere are now at different distances from the charge. The electric field will not have the same magnitude for all parts of the sphere, and the direction of the electric field will not be parallel to the outward normal for all points on the surface of the sphere. The electric field will be stronger on the side closer to the charge and weaker on the side further from the charge.
12. (a) A charge of $(Q-q)$ will be on the outer surface of the conductor. The total charge $Q$ is placed on the conductor but since $+q$ will reside on the inner surface, the leftover, $(Q-q)$, will reside on the outer surface.
(b) A charge of $+q$ will reside on the inner surface of the conductor since that amount is attracted by the charge $-q$ in the cavity. (Note that E must be zero inside the conductor.)
13. Yes. The charge $q$ will induce a charge $-q$ on the inside surface of the thin metal shell, leaving the outside surface with a charge $+q$. The charge $Q$ outside the sphere will feel the same electric force as it would if the metal shell were not present.
14. The total flux through the balloon's surface will not change because the enclosed charge does not change. The flux per unit surface area will decrease, since the surface area increases while the total flux does not change.

## Solutions to Problems

1. The electric flux of a uniform field is given by Eq. 22-1b.
(a)

$$
\begin{aligned}
& \Phi_{\mathrm{E}}=\overrightarrow{\mathbf{E}} \cdot \overrightarrow{\mathbf{A}}=E A \cos \theta=(580 \mathrm{~N} / \mathrm{C}) \pi(0.13 \mathrm{~m})^{2} \cos 0=31 \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C} \\
& \Phi_{\mathrm{E}}=\overrightarrow{\mathbf{E}} \cdot \overrightarrow{\mathbf{A}}=E A \cos \theta=(580 \mathrm{~N} / \mathrm{C}) \pi(0.13 \mathrm{~m})^{2} \cos 45^{\circ}=22 \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C} \\
& \Phi_{\mathrm{E}}=\overrightarrow{\mathbf{E}} \cdot \overrightarrow{\mathbf{A}}=E A \cos \theta=(580 \mathrm{~N} / \mathrm{C}) \pi(0.13 \mathrm{~m})^{2} \cos 90^{\circ}=0
\end{aligned}
$$

(c)
2. Use Eq. 22-1b for the electric flux of a uniform field. Note that the surface area vector points radially outward, and the electric field vector points radially inward. Thus the angle between the two is $180^{\circ}$.

$$
\begin{aligned}
\Phi_{\mathrm{E}} & =\overrightarrow{\mathbf{E}} \cdot \overrightarrow{\mathbf{A}}=E A \cos \theta=(150 \mathrm{~N} / \mathrm{C}) 4 \pi R_{\mathrm{E}}^{2} \cos 180^{\circ}=-4 \pi(150 \mathrm{~N} / \mathrm{C})\left(6.38 \times 10^{6} \mathrm{~m}\right)^{2} \\
& =-7.7 \times 10^{16} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}
\end{aligned}
$$

3. (a) Since the field is uniform, no lines originate or terminate inside the cube, and so the net flux is $\Phi_{\text {net }}=0$.
(b) There are two opposite faces with field lines perpendicular to the faces. The other four faces have field lines parallel to those faces. For the faces parallel to the field lines, no field lines enter or exit the faces. Thus $\Phi_{\text {parallel }}=0$.

Of the two faces that are perpendicular to the field lines, one will have field lines entering the cube, and so the angle between the field lines and the face area vector is $180^{\circ}$. The other will have field lines exiting the cube, and so the angle between the field lines and the face area vector is $0^{\circ}$. Thus we have $\Phi_{\text {entering }}=\vec{E} \cdot \vec{A}=E_{0} A \cos 180^{\circ}=-E_{0} \ell^{2}$ and

$$
\Phi_{\text {leaving }}=\vec{E} \cdot \vec{A}=E_{0} A \cos 0^{\circ}=E_{0} \ell^{2} \text {. }
$$

4. (a) From the diagram in the textbook, we see that the flux outward through the hemispherical surface is the same as the flux inward through the circular surface base of the hemisphere. On that surface all of the flux is perpendicular to the surface. Or, we say that on the circular base,
$\overrightarrow{\mathbf{E}} \| \overrightarrow{\mathbf{A}}$. Thus $\Phi_{\mathrm{E}}=\overrightarrow{\mathbf{E}} \cdot \overrightarrow{\mathbf{A}}=\pi r^{2} E$.
(b) $\overrightarrow{\mathbf{E}}$ is perpendicular to the axis, then every field line would both enter through the hemispherical surface and leave through the hemispherical surface, and so $\Phi_{\mathrm{E}}=0$.
5. Use Gauss's law to determine the enclosed charge.

$$
\Phi_{\mathrm{E}}=\frac{Q_{\text {encl }}}{\varepsilon_{o}} \rightarrow Q_{\text {encl }}=\Phi_{\mathrm{E}} \varepsilon_{o}=\left(1840 \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}\right)\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)=1.63 \times 10^{-8} \mathrm{C}
$$

6. The net flux through each closed surface is determined by the net charge inside. Refer to the picture in the textbook.

$$
\begin{aligned}
& \Phi_{1}=(+Q-3 Q) / \varepsilon_{0}=-2 Q / \varepsilon_{0} ; ~ \Phi_{2}=(+Q+2 Q-3 Q) / \varepsilon_{0}=0 ; \\
& \Phi_{3}=(+2 Q-3 Q) / \varepsilon_{0}=-Q / \varepsilon_{0} ; ~ \Phi_{4}=0 ; \Phi_{5}=+2 Q / \varepsilon_{0}
\end{aligned}
$$

(a) Use Gauss's law to determine the electric flux.

$$
\Phi_{\mathrm{E}}=\frac{Q_{\text {encl }}}{\varepsilon_{o}}=\frac{-1.0 \times 10^{-6} \mathrm{C}}{8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}}=-1.1 \times 10^{5} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}
$$

(b) Since there is no charge enclosed by surface $\mathrm{A}_{2}, \Phi_{\mathrm{E}}=0$.
8. The net flux is only dependent on the charge enclosed by the surface. Since both surfaces enclose the same amount of charge, the flux through both surfaces is the same. Thus the ratio is $1: 1$.
9. The only contributions to the flux are from the faces perpendicular to the electric field. Over each of these two surfaces, the magnitude of the field is constant, so the flux is just $\overrightarrow{\mathbf{E}} \cdot \overrightarrow{\mathbf{A}}$ on each of these two surfaces.

$$
\Phi_{\mathrm{E}}=(\overrightarrow{\mathbf{E}} \cdot \overrightarrow{\mathbf{A}})_{\text {right }}+(\overrightarrow{\mathbf{E}} \cdot \overrightarrow{\mathbf{A}})_{\text {left }}=E_{\text {right }} \ell^{2}-E_{\text {leff }} \ell^{2}=\frac{Q_{\text {encl }}}{\varepsilon_{0}} \rightarrow
$$

$$
Q_{\text {encl }}=\left(E_{\text {right }}-E_{\text {left }}\right) \ell^{2} \varepsilon_{0}=(410 \mathrm{~N} / \mathrm{C}-560 \mathrm{~N} / \mathrm{C})(25 \mathrm{~m})^{2}\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)=-8.3 \times 10^{-7} \mathrm{C}
$$

10. Because of the symmetry of the problem one sixth of the total flux will pass through each face.

$$
\Phi_{\text {face }}=\frac{1}{6} \Phi_{\text {total }}=\frac{1}{6} \frac{Q_{\text {encl }}}{\varepsilon_{0}}=\frac{Q_{\text {encl }}}{6 \varepsilon_{0}}
$$

Notice that the side length of the cube did not enter into the calculation.
11. The charge density can be found from Eq. $22-4$, Gauss's law. The charge is the charge density times the length of the rod.

$$
\Phi=\frac{Q_{\mathrm{encl}}}{\varepsilon_{0}}=\frac{\lambda \ell}{\varepsilon_{0}}=\rightarrow \lambda=\frac{\Phi \varepsilon_{0}}{\ell}=\frac{\left(7.3 \times 10^{5} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}\right)\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)}{0.15 \mathrm{~m}}=4.3 \times 10^{-5} \mathrm{C} / \mathrm{m}
$$

12. 


13. The electric field can be calculated by Eq. 21-4a, and that can be solved for the magnitude of the charge.

$$
E=k \frac{Q}{r^{2}} \rightarrow Q=\frac{E r^{2}}{k}=\frac{\left(6.25 \times 10^{2} \mathrm{~N} / \mathrm{C}\right)\left(3.50 \times 10^{-2} \mathrm{~m}\right)^{2}}{8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}}=8.52 \times 10^{-11} \mathrm{C}
$$

This corresponds to about $5 \times 10^{8}$ electrons. Since the field points toward the ball, the charge must be negative. Thus $Q=-8.52 \times 10^{-11} \mathrm{C}$.
14. The charge on the spherical conductor is uniformly distributed over the surface area of the sphere, so $\sigma=\frac{Q}{4 \pi R^{2}}$. The field at the surface of the sphere is evaluated at $r=R$.

$$
E(r=R)=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{R^{2}}=\frac{1}{4 \pi \varepsilon_{0}} \frac{4 \pi R^{2} \sigma}{R^{2}}=\frac{\sigma}{\varepsilon_{0}}
$$

15. The electric field due to a long thin wire is given in Example 22-6 as $E=\frac{1}{2 \pi \varepsilon_{0}} \frac{\lambda}{R}$.
(a) $E=\frac{1}{2 \pi \varepsilon_{0}} \frac{\lambda}{R}=\frac{1}{4 \pi \varepsilon_{0}} \frac{2 \lambda}{R}=\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{2\left(-7.2 \times 10^{-6} \mathrm{C} / \mathrm{m}\right)}{(5.0 \mathrm{~m})}=-2.6 \times 10^{4} \mathrm{~N} / \mathrm{C}$

The negative sign indicates the electric field is pointed towards the wire.
(b) $E=\frac{1}{2 \pi \varepsilon_{0}} \frac{\lambda}{R}=\frac{1}{4 \pi \varepsilon_{0}} \frac{2 \lambda}{R}=\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{2\left(-7.2 \times 10^{-6} \mathrm{C} / \mathrm{m}\right)}{(1.5 \mathrm{~m})}=-8.6 \times 10^{4} \mathrm{~N} / \mathrm{C}$

The negative sign indicates the electric field is pointed towards the wire.
16. Because the globe is a conductor, the net charge of -1.50 mC will be arranged symmetrically around the sphere.

17. Due to the spherical symmetry of the problem, the electric field can be evaluated using Gauss's law and the charge enclosed by a spherical Gaussian surface of radius $r$.

$$
\oint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=E\left(4 \pi r^{2}\right)=\frac{Q_{\text {encl }}}{\varepsilon_{0}} \rightarrow E=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q_{\text {encl }}}{r^{2}}
$$

Since the charge densities are constant, the charge enclosed is found by multiplying the appropriate charge density times the volume of charge enclosed by the Gaussian sphere. Let $r_{1}=6.0 \mathrm{~cm}$ and $r_{2}=12.0 \mathrm{~cm}$.
(a) Negative charge is enclosed for $r<r_{1}$.

$$
\begin{aligned}
E & =\frac{1}{4 \pi \varepsilon_{0}} \frac{Q_{\text {encl }}}{r^{2}}=\frac{1}{4 \pi \varepsilon_{0}} \frac{\rho_{(-)}\left(\frac{4}{3} \pi r^{3}\right)}{r^{2}}=\frac{\rho_{(-)} r}{3 \varepsilon_{0}}=\frac{\left(-5.0 \mathrm{C} / \mathrm{m}^{3}\right) r}{3\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)} \\
& =\left(-1.9 \times 10^{11} \mathrm{~N} / \mathrm{C} \cdot \mathrm{~m}\right) r
\end{aligned}
$$

(b) In the region $r_{1}<r<r_{2}$, all of the negative charge and part of the positive charge is enclosed.

$$
\begin{aligned}
E & =\frac{1}{4 \pi \varepsilon_{0}} \frac{Q_{\text {encl }}}{r^{2}}=\frac{1}{4 \pi \varepsilon_{0}} \frac{\rho_{(-)}\left(\frac{4}{3} \pi r_{1}^{3}\right)+\rho_{(+)}\left[\frac{4}{3} \pi\left(r^{3}-r_{1}^{3}\right)\right]}{r^{2}}=\frac{\left(\rho_{(-)}-\rho_{(+)}\right)\left(r_{1}^{3}\right)}{3 \varepsilon_{0} r^{2}}+\frac{\rho_{(+)} r}{3 \varepsilon_{0}} \\
& =\frac{\left[\left(-5.0 \mathrm{C} / \mathrm{m}^{3}\right)-\left(8.0 \mathrm{C} / \mathrm{m}^{3}\right)\right](0.060 \mathrm{~m})^{3}}{3\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right) r^{2}}+\frac{\left(8.0 \mathrm{C} / \mathrm{m}^{3}\right) r}{3\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)} \\
& =\frac{\left(-1.1 \times 10^{8} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}\right)}{r^{2}}+\left(3.0 \times 10^{11} \mathrm{~N} / \mathrm{C} \cdot \mathrm{~m}\right) r
\end{aligned}
$$

(c) In the region $r_{2}<r$, all of the charge is enclosed.

$$
\begin{aligned}
E & =\frac{1}{4 \pi \varepsilon_{0}} \frac{Q_{\text {encl }}}{r^{2}}=\frac{1}{4 \pi \varepsilon_{0}} \frac{\rho_{(-)}\left(\frac{4}{3} \pi r_{1}^{3}\right)+\rho_{(+)}\left[\frac{4}{3} \pi\left(r_{2}^{3}-r_{1}^{3}\right)\right]}{r^{2}}=\frac{\left(\rho_{(-)}-\rho_{(+)}\right)\left(r_{1}^{3}\right)+\rho_{(+)}\left(r_{2}^{3}\right)}{3 \varepsilon_{0} r^{2}}= \\
& =\frac{\left[\left(-5.0 \mathrm{C} / \mathrm{m}^{3}\right)-\left(8.0 \mathrm{C} / \mathrm{m}^{3}\right)\right](0.060 \mathrm{~m})^{3}+\left(8.0 \mathrm{C} / \mathrm{m}^{3}\right)(0.120 \mathrm{~m})^{3}}{3\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right) r^{2}}=\frac{\left(4.1 \times 10^{8} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}\right)}{r^{2}}
\end{aligned}
$$

(d) See the adjacent plot. The field is continuous at the edges of the layers. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH22.XLS," on tab "Problem 22.17d."

18. See Example 22-3 for a detailed discussion related to this problem.
(a) Inside a solid metal sphere the electric field is 0 .
(b) Inside a solid metal sphere the electric field is 0 .
(c) Outside a solid metal sphere the electric field is the same as if all the charge were concentrated at the center as a point charge.

$$
|E|=\frac{1}{4 \pi \varepsilon_{0}} \frac{|Q|}{r^{2}}=\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{\left(5.50 \times 10^{-6} \mathrm{C}\right)}{(3.10 \mathrm{~m})^{2}}=5140 \mathrm{~N} / \mathrm{C}
$$

The field would point towards the center of the sphere.
(d) Same reasoning as in part (c).

$$
|E|=\frac{1}{4 \pi \varepsilon_{0}} \frac{|Q|}{r^{2}}=\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{\left(5.50 \times 10^{-6} \mathrm{C}\right)}{(8.00 \mathrm{~m})^{2}}=772 \mathrm{~N} / \mathrm{C}
$$

The field would point towards the center of the sphere.
(e) The answers would be no different for a thin metal shell.
(f) The solid sphere of charge is dealt with in Example 22-4. We see from that Example that the field inside the sphere is given by $|E|=\frac{1}{4 \pi \varepsilon_{0}} \frac{|Q|}{r_{0}^{3}} r$. Outside the sphere the field is no different. So we have these results for the solid sphere.

$$
\begin{aligned}
& |E(r=0.250 \mathrm{~m})|=\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{5.50 \times 10^{-6} \mathrm{C}}{(3.00 \mathrm{~m})^{3}}(0.250 \mathrm{~m})=458 \mathrm{~N} / \mathrm{C} \\
& |E(r=2.90 \mathrm{~m})|=\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{5.50 \times 10^{-6} \mathrm{C}}{(3.00 \mathrm{~m})^{3}}(2.90 \mathrm{~m})=5310 \mathrm{~N} / \mathrm{C} \\
& |E(r=3.10 \mathrm{~m})|=\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{5.50 \times 10^{-6} \mathrm{C}}{(3.10 \mathrm{~m})^{2}}=5140 \mathrm{~N} / \mathrm{C} \\
& |E(r=8.00 \mathrm{~m})|=\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{5.50 \times 10^{-6} \mathrm{C}}{(3.10 \mathrm{~m})^{2}}=772 \mathrm{~N} / \mathrm{C}
\end{aligned}
$$

All point towards the center of the sphere.
19. For points inside the nonconducting spheres, the electric field will be determined by the charge inside the spherical surface of radius $r$.

$$
Q_{\mathrm{encl}}=Q\left(\frac{\frac{4}{3} \pi r^{3}}{\frac{4}{3} \pi r_{0}^{3}}\right)=Q\left(\frac{r}{r_{0}}\right)^{3}
$$

The electric field for $r \leq r_{0}$ can be calculated from Gauss's law.

$$
\begin{aligned}
& E\left(r \leq r_{0}\right)=\frac{Q_{\text {encl }}}{4 \pi \varepsilon_{0} r^{2}} \\
& =Q\left(\frac{r}{r_{0}}\right)^{3} \frac{1}{4 \pi \varepsilon_{0} r^{2}}=\left(\frac{Q}{4 \pi \varepsilon_{0} r_{0}^{3}}\right) r
\end{aligned}
$$



The electric field outside the sphere is calculated from Gauss's law with $Q_{\text {encl }}=Q$.

$$
E\left(r \geq r_{0}\right)=\frac{Q_{\text {encl }}}{4 \pi \varepsilon_{0} r^{2}}=\frac{Q}{4 \pi \varepsilon_{0} r^{2}}
$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH22.XLS," on tab "Problem 22.19."
20. (a) When close to the sheet, we approximate it as an infinite sheet, and use the result of Example 22-7. We assume the charge is over both surfaces of the aluminum.

$$
E=\frac{\sigma}{2 \varepsilon_{o}}=\frac{\frac{275 \times 10^{-9} \mathrm{C}}{(0.25 \mathrm{~m})^{2}}}{2\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)}=2.5 \times 10^{5} \mathrm{~N} / \mathrm{C}, \text { away from the sheet }
$$

(b) When far from the sheet, we approximate it as a point charge.

$$
E=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{r^{2}}=\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{275 \times 10^{-9} \mathrm{C}}{(15 \mathrm{~m})^{2}}=11 \mathrm{~N} / \mathrm{C}, \text { away from the sheet }
$$

21. (a) Consider a spherical gaussian surface at a radius of 3.00 cm . It encloses all of the charge.

$$
\begin{aligned}
& \oint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=E\left(4 \pi r^{2}\right)=\frac{Q}{\varepsilon_{0}} \rightarrow \\
& E=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{r^{2}}=\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{5.50 \times 10^{-6} \mathrm{C}}{\left(3.00 \times 10^{-2} \mathrm{~m}\right)^{2}}=5.49 \times 10^{7} \mathrm{~N} / \mathrm{C}, \text { radially outward }
\end{aligned}
$$

(b) A radius of 6.00 cm is inside the conducting material, and so the field must be 0 . Note that there must be an induced charge of $-5.50 \times 10^{-6} \mathrm{C}$ on the surface at $r=4.50 \mathrm{~cm}$, and then an induced charge of $5.50 \times 10^{-6} \mathrm{C}$ on the outer surface of the sphere.
(c) Consider a spherical gaussian surface at a radius of 3.00 cm . It encloses all of the charge.

$$
\begin{aligned}
& \oint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=E\left(4 \pi r^{2}\right)=\frac{Q}{\varepsilon_{0}} \rightarrow \\
& E=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{r^{2}}=\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{5.50 \times 10^{-6} \mathrm{C}}{\left(30.0 \times 10^{-2} \mathrm{~m}\right)^{2}}=5.49 \times 10^{5} \mathrm{~N} / \mathrm{C}, \text { radially outward }
\end{aligned}
$$

22. (a) Inside the shell, the field is that of the point charge, $E=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{r^{2}}$.
(b) There is no field inside the conducting material: $E=0$.
(c) Outside the shell, the field is that of the point charge, $E=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{r^{2}}$.
(d) The shell does not affect the field due to $Q$ alone, except in the shell material, where the field is 0 . The charge $Q$ does affect the shell - it polarizes it. There will be an induced charge of $-Q$ uniformly distributed over the inside surface of the shell, and an induced charge of $+Q$ uniformly distributed over the outside surface of the shell.
23. (a) There can be no field inside the conductor, and so there must be an induced charge of $-8.00 \mu \mathrm{C}$ on the surface of the spherical cavity.
(b) Any charge on the conducting material must reside on its boundaries. If the net charge of the cube is $-6.10 \mu \mathrm{C}$, and there is a charge of $-8.00 \mu \mathrm{C}$ on its inner surface, there must be a charge of $+1.90 \mu \mathrm{C}$ on the outer surface.
24. Since the charges are of opposite sign, and since the charges are free to move since they are on conductors, the charges will attract each other and move to the inside or facing edges of the plates. There will be no charge on the outside edges of the plates. And there cannot be charge in the plates themselves, since they are conductors. All of the charge must reside on surfaces. Due to the symmetry of the problem, all field lines must be perpendicular to the plates, as discussed in Example 22-7.
(a) To find the field between the plates, we choose a gaussian cylinder, perpendicular to the plates, with area A for the ends of the cylinder. We place one end inside the left plate (where the field must be zero), and the other end between the plates. No flux passes through the curved surface of the cylinder.

$$
\begin{aligned}
& \oint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=\int_{\text {ends }} \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}+\int_{\substack{\text { side }}} \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=\int_{\substack{\text { right } \\
\text { end }}} \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=\frac{Q_{\text {encl }}}{\varepsilon_{0}} \rightarrow \\
& E_{\text {between }} A=\frac{\sigma A}{\varepsilon_{0}} \rightarrow E_{\text {between }}=\frac{\sigma}{\varepsilon_{0}}
\end{aligned}
$$



The field lines between the plates leave the inside surface of the left plate, and terminate on the inside surface of the right plate. A similar derivation could have been done with the right end of the cylinder inside of the right plate, and the left end of the cylinder in the space between the plates.
(b) If we now put the cylinder from above so that the right end is inside the conducting material, and the left end is to the left of the left plate, the only possible location for flux is through the left end of the cylinder. Note that there is NO charge enclosed by the Gaussian cylinder.

$$
\oint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=\int_{\text {ends }} \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}+\int_{\text {side }} \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=\int_{\substack{\text { left } \\ \text { end }}} \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=\frac{Q_{\text {encl }}}{\varepsilon_{0}} \rightarrow
$$



$$
E_{\text {outside }} A=\frac{0}{\varepsilon_{0}} \rightarrow E_{\text {outside }}=\frac{0}{\varepsilon_{0}}
$$

(c) If the two plates were nonconductors, the results would not change. The charge would be distributed over the two plates in a different fashion, and the field inside of the plates would not be zero, but the charge in the empty regions of space would be the same as when the plates are conductors.
25. Example 22-7 gives the electric field from a positively charged plate as $E=\sigma / 2 \varepsilon_{0}$ with the field pointing away from the plate.
The fields from the two plates will add, as shown in the figure.
(a) Between the plates the fields are equal in magnitude, but point in opposite directions.

$$
E_{\text {between }}=\frac{\sigma}{2 \varepsilon_{0}}-\frac{\sigma}{2 \varepsilon_{0}}=0
$$

(b) Outside the two plates the fields are equal in magnitude and
 point in the same direction.

$$
E_{\text {outside }}=\frac{\sigma}{2 \varepsilon_{0}}+\frac{\sigma}{2 \varepsilon_{0}}=\frac{\sigma}{\varepsilon_{0}}
$$

(c) When the plates are conducting the charge lies on the surface of the plates. For nonconducting plates the same charge will be spread across the plate. This will not affect the electric field between or outside the two plates. It will, however, allow for a non-zero field inside each plate.
26. Because $3.0 \mathrm{~cm} \ll 1.0 \mathrm{~m}$, we can consider the plates to be infinite in size, and ignore any edge effects. We use the result from Problem 24(a).

$$
E=\frac{\sigma}{\varepsilon_{0}}=\frac{Q / A}{\varepsilon_{0}} \rightarrow Q=E A \varepsilon_{0}=(160 \mathrm{~N} / \mathrm{C})(1.0 \mathrm{~m})^{2}\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)=1.4 \times 10^{-9} \mathrm{C}
$$

27. (a) In the region $0<r<r_{1}$, a gaussian surface would enclose no charge. Thus, due to the spherical symmetry, we have the following.

$$
\oint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=E\left(4 \pi r^{2}\right)=\frac{Q_{\mathrm{encl}}}{\varepsilon_{0}}=0 \rightarrow E=0
$$

(b) In the region $r_{1}<r<r_{2}$, only the charge on the inner shell will be enclosed.

$$
\oint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=E\left(4 \pi r^{2}\right)=\frac{Q_{\text {encl }}}{\varepsilon_{0}}=\frac{\sigma_{1} 4 \pi r_{1}^{2}}{\varepsilon_{0}} \rightarrow E=\frac{\sigma_{1} r_{1}^{2}}{\varepsilon_{0} r^{2}}
$$

(c) In the region $r_{2}<r$, the charge on both shells will be enclosed.

$$
\oint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=E\left(4 \pi r^{2}\right)=\frac{Q_{\text {encl }}}{\varepsilon_{0}}=\frac{\sigma_{1} 4 \pi r_{1}^{2}+\sigma_{2} 4 \pi r_{2}^{2}}{\varepsilon_{0}} \rightarrow E=\frac{\sigma_{1} r_{1}^{2}+\sigma_{2} r_{2}^{2}}{\varepsilon_{0} r^{2}}
$$

(d) To make $E=0$ for $r_{2}<r$, we must have $\sigma_{1} r_{1}^{2}+\sigma_{2} r_{2}^{2}=0$. This implies that the shells are of opposite charge.
(e) To make $E=0$ for $r_{1}<r<r_{2}$, we must have $\sigma_{1}=0$. Or, if a charge $Q=-4 \pi \sigma_{1} r_{1}^{2}$ were placed at the center of the shells, that would also make $E=0$.
28. If the radius is to increase from $r_{0}$ to $2 r_{0}$ linearly during an elapsed time of $T$, then the rate of increase must be $r_{0} / T$. The radius as a function of time is then $r=r_{0}+\frac{r_{0}}{T} t=r_{0}\left(1+\frac{t}{T}\right)$. Since the balloon is spherical, the field outside the balloon will have the same form as the field due to a point charge.
(a) Here is the field just outside the balloon surface.

$$
E=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{r^{2}}=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{r_{0}^{2}\left(1+\frac{t}{T}\right)^{2}}
$$

(b) Since the balloon radius is always smaller than $3.2 r_{0}$, the total charge enclosed in a gaussian surface at $r=3.2 r_{0}$ does not change in time.

$$
E=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{r^{2}}=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{\left(3.2 r_{0}\right)^{2}}
$$

29. Due to the spherical symmetry of the problem, Gauss's law using a sphere of radius $r$ leads to the following.

$$
\oint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=E\left(4 \pi r^{2}\right)=\frac{Q_{\text {encl }}}{\varepsilon_{0}} \rightarrow E=\frac{Q_{\text {encl }}}{4 \pi \varepsilon_{0} r^{2}}
$$

(a) For the region $0<r<r_{1}$, the enclosed charge is 0 .

$$
E=\frac{Q_{\text {encl }}}{4 \pi \varepsilon_{0} r^{2}}=0
$$

(b) For the region $r_{1}<r<r_{0}$, the enclosed charge is the product of the volume charge density times the volume of charged material enclosed. The charge density is given by $\rho=\frac{Q}{\frac{4}{3} \pi r_{0}^{3}-\frac{4}{3} \pi r_{1}^{3}}$

$$
=\frac{3 Q}{4 \pi\left(r_{0}^{3}-r_{1}^{3}\right)} .
$$

$$
E=\frac{Q_{\text {encl }}}{4 \pi \varepsilon_{0} r^{2}}=\frac{\rho V_{\text {encl }}}{4 \pi \varepsilon_{0} r^{2}}=\frac{\rho\left[\frac{4}{3} \pi r^{3}-\frac{4}{3} \pi r_{1}^{3}\right]}{4 \pi \varepsilon_{0} r^{2}}=\frac{\frac{3 Q}{4 \pi\left(r_{0}^{3}-r_{1}^{3}\right)}\left[\frac{4}{3} \pi r^{3}-\frac{4}{3} \pi r_{1}^{3}\right]}{4 \pi \varepsilon_{0} r^{2}}=\frac{Q}{4 \pi \varepsilon_{0} r^{2}} \frac{\left(r^{3}-r_{1}^{3}\right)}{\left(r_{0}^{3}-r_{1}^{3}\right)}
$$

(c) For the region $r>r_{0}$, the enclosed charge is the total charge, $Q$.

$$
E=\frac{Q}{4 \pi \varepsilon_{0} r^{2}}
$$

30. By the superposition principle for electric fields (Section 21-6), we find the field for this problem by adding the field due to the point charge at the center to the field found in Problem 29. At any location $r>0$, the field due to the point charge is $E=\frac{q}{4 \pi \varepsilon_{0} r^{2}}$.
(a) $E=E_{q}+E_{Q}=\frac{q}{4 \pi \varepsilon_{0} r^{2}}+0=\frac{q}{4 \pi \varepsilon_{0} r^{2}}$
(b) $E=E_{q}+E_{Q}=\frac{q}{4 \pi \varepsilon_{0} r^{2}}+\frac{Q}{4 \pi \varepsilon_{0} r^{2}} \frac{\left(r^{3}-r_{1}^{3}\right)}{\left(r_{0}^{3}-r_{1}^{3}\right)}=\frac{1}{4 \pi \varepsilon_{0} r^{2}}\left[q+Q \frac{\left(r^{3}-r_{1}^{3}\right)}{\left(r_{0}^{3}-r_{1}^{3}\right)}\right]$
(c) $E=E_{q}+E_{Q}=\frac{q}{4 \pi \varepsilon_{0} r^{2}}+\frac{Q}{4 \pi \varepsilon_{0} r^{2}}=\frac{q+Q}{4 \pi \varepsilon_{0} r^{2}}$
31. (a) Create a gaussian surface that just encloses the inner surface of the spherical shell. Since the electric field inside a conductor must be zero, Gauss's law requires that the enclosed charge be zero. The enclosed charge is the sum of the charge at the center and charge on the inner surface of the conductor.

$$
Q_{\mathrm{enc}}=q+Q_{\text {imer }}=0
$$

Therefore $Q_{\text {imer }}=-q$.
(b) The total charge on the conductor is the sum of the charges on the inner and outer surfaces.

$$
Q=Q_{\text {outer }}+Q_{\text {imer }} \rightarrow Q_{\text {outer }}=Q-Q_{\text {imer }}=Q+q
$$

(c) A gaussian surface of radius $r<r_{1}$ only encloses the center charge, $q$. The electric field will therefore be the field of the single charge.

$$
E\left(r<r_{1}\right)=\frac{q}{4 \pi \varepsilon_{0} r^{2}}
$$

(d) A gaussian surface of radius $r_{1}<r<r_{0}$ is inside the conductor so $E=0$.
(e) A gaussian surface of radius $r>r_{0}$ encloses the total charge $q+Q$. The electric field will then be the field from the sum of the two charges.

$$
E\left(r>r_{0}\right)=\frac{q+Q}{4 \pi \varepsilon_{0} r^{2}}
$$

32. (a) For points inside the shell, the field will be due to the point charge only.

$$
E\left(r<r_{0}\right)=\frac{q}{4 \pi \varepsilon_{0} r^{2}}
$$

(b) For points outside the shell, the field will be that of a point charge, equal to the total charge.

$$
E\left(r>r_{0}\right)=\frac{q+Q}{4 \pi \varepsilon_{0} r^{2}}
$$

(c) If $q=Q$, we have $E\left(r<r_{0}\right)=\frac{Q}{4 \pi \varepsilon_{0} r^{2}}$ and $E\left(r>r_{0}\right)=\frac{2 Q}{4 \pi \varepsilon_{0} r^{2}}$.
(d) If $q=-Q$, we have $E\left(r<r_{0}\right)=\frac{-Q}{4 \pi \varepsilon_{0} r^{2}}$ and $E\left(r>r_{0}\right)=0$.
33. We follow the development of Example 22-6. Because of the symmetry, we expect the field to be directed radially outward (no fringing effects near the ends of the cylinder) and to depend only on the perpendicular distance, $R$, from the symmetry axis of the shell. Because of the cylindrical symmetry, the field will be the same at all points on a gaussian surface that is a cylinder whose axis coincides with the axis of the shell. The gaussian surface is of radius $r$ and
 length $\ell$. $\overrightarrow{\mathbf{E}}$ is perpendicular to this surface at all points. In order to apply Gauss's law, we need a closed surface, so we include the flat ends of the cylinder. Since $\overrightarrow{\mathbf{E}}$ is parallel to the flat ends, there is no flux through the ends. There is only flux through the curved wall of the gaussian cylinder.

$$
\oint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=E(2 \pi R \ell)=\frac{Q_{\text {encl }}}{\varepsilon_{0}}=\frac{\sigma A_{\text {encl }}}{\varepsilon_{0}} \rightarrow E=\frac{\sigma A_{\text {encl }}}{2 \pi \varepsilon_{0} R \ell}
$$

(a) For $R>R_{0}$, the enclosed surface area of the shell is $A_{\text {encl }}=2 \pi R_{0} \ell$.

$$
E=\frac{\sigma A_{\text {encl }}}{2 \pi \varepsilon_{0} R \ell}=\frac{\sigma 2 \pi R_{0} \ell}{2 \pi \varepsilon_{0} R \ell}=\frac{\sigma R_{0}}{\varepsilon_{0} R} \text {, radially outward }
$$

(b) For $R<R_{0}$, the enclosed surface area of the shell is $A_{\text {encl }}=0$, and so $E=0$.
(c) The field for $R>R_{0}$ due to the shell is the same as the field due to the long line of charge, if we substitute $\lambda=2 \pi R_{0} \sigma$.
34. The geometry of this problem is similar to Problem 33, and so we use the same development, following Example 22-6. See the solution of Problem 33 for details.

$$
\oint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=E(2 \pi R \ell)=\frac{Q_{\text {encl }}}{\varepsilon_{0}}=\frac{\rho_{\mathrm{E}} V_{\text {encl }}}{\varepsilon_{0}} \rightarrow E=\frac{\rho_{\mathrm{E}} V_{\text {encl }}}{2 \pi \varepsilon_{0} R \ell}
$$

(a) For $R>R_{0}$, the enclosed volume of the shell is


$$
\begin{aligned}
& V_{\text {encl }}=\pi R_{0}^{2} \ell . \\
& \qquad E=\frac{\rho_{\mathrm{E}} V_{\text {encl }}}{2 \pi \varepsilon_{0} R \ell}=\frac{\rho_{\mathrm{E}} R_{0}^{2}}{2 \varepsilon_{0} R}, \text { radially outward }
\end{aligned}
$$

(b) For $R<R_{0}$, the enclosed volume of the shell is $V_{\text {encl }}=\pi R^{2} \ell$.

$$
E=\frac{\rho_{\mathrm{E}} V_{\text {encl }}}{2 \pi \varepsilon_{0} R \ell}=\frac{\rho_{\mathrm{E}} R}{2 \varepsilon_{0}}, \text { radially outward }
$$

35. The geometry of this problem is similar to Problem 33, and so we use the same development, following Example 22-6. See the solution of Problem 33 for details. We choose the gaussian cylinder to be the same length as the cylindrical shells.

$$
\oint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=E(2 \pi R \ell)=\frac{Q_{\text {encl }}}{\varepsilon_{0}} \rightarrow E=\frac{Q_{\text {encl }}}{2 \pi \varepsilon_{0} R \ell}
$$

(a) For $0<R<R_{1}$, no charge is enclosed, and so $E=\frac{Q_{\text {encl }}}{2 \pi \varepsilon_{0} R \ell}=0$.
(b) For $R_{1}<R<R_{2}$, charge $+Q$ is enclosed, and so $E=\frac{Q}{2 \pi \varepsilon_{0} R \ell}$, radially outward.
(c) For $R>R_{2}$, both charges of $+Q$ and $-Q$ are enclosed, and so $E=\frac{Q_{\text {encl }}}{2 \pi \varepsilon_{0} R \ell}=0$.
(d) The force on an electron between the cylinders points in the direction opposite to the electric field, and so the force is inward. The electric force produces the centripetal acceleration for the electron to move in the circular orbit.

$$
F_{\text {centrip }}=e E=\frac{e Q}{2 \pi \varepsilon_{0} R \ell}=m \frac{v^{2}}{R} \rightarrow K=\frac{1}{2} m v^{2}=\frac{e Q}{4 \pi \varepsilon_{0} \ell}
$$

Note that this is independent of the actual value of the radius, as long as $R_{1}<R<R_{2}$.
36. The geometry of this problem is similar to Problem 33, and so we use the same development, following Example 22-6. See the solution of Problem 33 for details. We choose the gaussian cylinder to be the same length as the cylindrical shells.

$$
\oint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=E(2 \pi R \ell)=\frac{Q_{\mathrm{encl}}}{\varepsilon_{0}} \rightarrow E=\frac{Q_{\mathrm{encl}}}{2 \pi \varepsilon_{0} R \ell}
$$

(a) At a distance of $R=3.0 \mathrm{~cm}$, no charge is enclosed, and so $E=\frac{Q_{\mathrm{encl}}}{2 \pi \varepsilon_{0} R \ell}=0$.
(b) At a distance of $R=7.0 \mathrm{~cm}$, the charge on the inner cylinder is enclosed.

$$
E=\frac{Q_{\text {encl }}}{2 \pi \varepsilon_{0} R \ell}=\frac{2}{4 \pi \varepsilon_{0}} \frac{Q_{\text {encl }}}{R \ell}=2\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{\left(-0.88 \times 10^{-6} \mathrm{C}\right)}{(0.070 \mathrm{~m})(5.0 \mathrm{~m})}=-4.5 \times 10^{4} \mathrm{~N} / \mathrm{C}
$$

The negative sign indicates that the field points radially inward.
(c) At a distance of $R=12.0 \mathrm{~cm}$, the charge on both cylinders is enclosed.

$$
E=\frac{Q_{\text {encl }}}{2 \pi \varepsilon_{0} R \ell}=\frac{2}{4 \pi \varepsilon_{0}} \frac{Q_{\text {encl }}}{R \ell}=2\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{(1.56-0.88) \times 10^{-6} \mathrm{C}}{(0.120 \mathrm{~m})(5.0 \mathrm{~m})}=2.0 \times 10^{4} \mathrm{~N} / \mathrm{C}
$$

The field points radially outward.
37. (a) The final speed can be calculated from the work-energy theorem, where the work is the integral of the force on the electron between the two shells.

$$
W=\int \vec{F} \cdot d \vec{r}=\frac{1}{2} m v^{2}-\frac{1}{2} m v_{0}^{2}
$$

Setting the force equal to the electric field times the charge on the electron, and inserting the electric field from Problem 36 gives the work done on the electron.

$$
\begin{aligned}
W & =\int_{R_{1}}^{R_{2}} \frac{q Q}{2 \pi \varepsilon_{0} \ell R} d R=\frac{q Q}{2 \pi \varepsilon_{0} \ell} \ln \left(\frac{R_{2}}{R_{1}}\right) \\
& =\frac{\left(-1.60 \times 10^{-19} \mathrm{C}\right)(-0.88 \mu \mathrm{C})}{2 \pi\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{Nm}^{2}\right)(5.0 \mathrm{~m})} \ln \left(\frac{9.0 \mathrm{~cm}}{6.5 \mathrm{~cm}}\right)=1.65 \times 10^{-16} \mathrm{~J}
\end{aligned}
$$

Solve for the velocity from the work-energy theorem.

$$
v=\sqrt{\frac{2 W}{m}}=\sqrt{\frac{2\left(1.65 \times 10^{-16} \mathrm{~J}\right)}{9.1 \times 10^{-31} \mathrm{~kg}}}=1.9 \times 10^{7} \mathrm{~m} / \mathrm{s}
$$

(b) The electric force on the proton provides its centripetal acceleration.

$$
F_{c}=\frac{m v^{2}}{R}=q E=\frac{|q Q|}{2 \pi \varepsilon_{0} \ell R}
$$

The velocity can be solved for from the centripetal acceleration.

$$
v=\sqrt{\frac{\left(1.60 \times 10^{-19} \mathrm{C}\right)(0.88 \mu \mathrm{C})}{2 \pi\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{Nm}^{2}\right)\left(1.67 \times 10^{-27} \mathrm{~kg}\right)(5.0 \mathrm{~m})}}=5.5 \times 10^{5} \mathrm{~m} / \mathrm{s}
$$

Note that as long as the proton is between the two cylinders, the velocity is independent of the radius.
38. The geometry of this problem is similar to Problem 33, and so we use the same development, following Example 22-6. See the solution of Problem 33 for details.

$$
\oint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=E(2 \pi R \ell)=\frac{Q_{\text {encl }}}{\varepsilon_{0}} \rightarrow E=\frac{Q_{\text {encl }}}{2 \pi \varepsilon_{0} R \ell}
$$

(a) For $0<R<R_{1}$, the enclosed charge is the volume of charge enclosed, times the charge density.


$$
E=\frac{Q_{\text {encl }}}{2 \pi \varepsilon_{0} R \ell}=\frac{\rho_{\mathrm{E}} \pi R^{2} \ell}{2 \pi \varepsilon_{0} R \ell}=\frac{\rho_{\mathrm{E}} R}{2 \varepsilon_{0}}
$$

(b) For $R_{1}<R<R_{2}$, the enclosed charge is all of the charge on the inner cylinder.

$$
E=\frac{Q_{\text {encl }}}{2 \pi \varepsilon_{0} R \ell}=\frac{\rho_{\mathrm{E}} \pi R_{1}^{2} \ell}{2 \pi \varepsilon_{0} R \ell}=\frac{\rho_{\mathrm{E}} R_{1}^{2}}{2 \varepsilon_{0} R}
$$

(c) For $R_{2}<R<R_{3}$, the enclosed charge is all of the charge on the inner cylinder, and the part of the charge on the shell that is enclosed by the gaussian cylinder.

$$
E=\frac{Q_{\text {encl }}}{2 \pi \varepsilon_{0} R \ell}=\frac{\rho_{\mathrm{E}} \pi R_{1}^{2} \ell+\rho_{\mathrm{E}}\left(\pi R^{2} \ell-\pi R_{2}^{2} \ell\right)}{2 \pi \varepsilon_{0} R \ell}=\frac{\rho_{\mathrm{E}}\left(R_{1}^{2}+R^{2}-R_{2}^{2}\right)}{2 \varepsilon_{0} R}
$$

(d) For $R>R_{3}$, the enclosed charge is all of the charge on both the inner cylinder and the shell.

$$
E=\frac{Q_{\text {encl }}}{2 \pi \varepsilon_{0} R \ell}=\frac{\rho_{\mathrm{E}} \pi R_{1}^{2} \ell+\rho_{\mathrm{E}}\left(\pi R_{3}^{2} \ell-\pi R_{2}^{2} \ell\right)}{2 \pi \varepsilon_{0} R \ell}=\frac{\rho_{\mathrm{E}}\left(R_{1}^{2}+R_{3}^{2}-R_{2}^{2}\right)}{2 \varepsilon_{0} R}
$$

(e) See the graph. The spreadsheet used for this problem can be found on the Media Manager, with filename
"PSE4_ISM_CH22.XLS," on tab "Problem 22.38e."

39. Due to the spherical symmetry of the geometry, we have the following to find the electric field at any radius $r$. The field will point either radially out or radially in.

$$
\oint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=E\left(4 \pi r^{2}\right)=\frac{Q_{\mathrm{encl}}}{\varepsilon_{0}} \rightarrow E=\frac{Q_{\mathrm{encl}}}{4 \pi \varepsilon_{0} r^{2}}
$$

(a) For $0<r<r_{0}$, the enclosed charge is due to the part of the charged sphere that has a radius smaller than $r$.

$$
E=\frac{Q_{\mathrm{encl}}}{4 \pi \varepsilon_{0} r^{2}}=\frac{\rho_{\mathrm{E}}\left(\frac{4}{3} \pi r^{3}\right)}{4 \pi \varepsilon_{0} r^{2}}=\frac{\rho_{\mathrm{E}} r}{3 \varepsilon_{0}}
$$

(b) For $r_{0}<r<r_{1}$, the enclosed charge is due to the entire charged sphere of radius $r_{0}$.

$$
E=\frac{Q_{\mathrm{encl}}}{4 \pi \varepsilon_{0} r^{2}}=\frac{\rho_{\mathrm{E}}\left(\frac{4}{3} \pi r_{0}^{3}\right)}{4 \pi \varepsilon_{0} r^{2}}=\frac{\rho_{\mathrm{E}} r_{0}^{3}}{3 \varepsilon_{0} r^{2}}
$$

(c) For $r_{1}<r<r_{2}, r$ is in the interior of the conducting spherical shell, and so $E=0$. This implies that $Q_{\text {encl }}=0$, and so there must be an induced charge of magnitude $-\frac{4}{3} \rho_{\mathrm{E}} \pi r_{0}^{3}$ on the inner surface of the conducting shell, at $r_{1}$.
(d) For $r>r_{2}$, the enclosed charge is the total charge of both the sphere and the shell.

$$
E=\frac{Q_{\mathrm{encl}}}{4 \pi \varepsilon_{0} r^{2}}=\frac{Q+\rho_{\mathrm{E}}\left(\frac{4}{3} \pi r_{0}^{3}\right)}{4 \pi \varepsilon_{0} r^{2}}=\left(\frac{Q}{4 \pi \varepsilon_{0}}+\frac{\rho_{\mathrm{E}} r_{0}^{3}}{3 \varepsilon_{0}}\right) \frac{1}{r^{2}}
$$

40. The conducting outer tube is uncharged, and the electric field is 0 everywhere within the conducting material. Because there will be no electric field inside the conducting material of the outer cylinder tube, the charge on the inner nonconducting cylinder will induce an oppositely signed, equal magnitude charge on the inner surface of the conducting tube. This charge will NOT be uniformly distributed, because the inner cylinder is not in the center of the tube. Since the conducting tube has no net charge, there will be an induced charge on the OUTER surface of the conducting tube, equal in magnitude to the charge on the inner cylinder, and of the same sign. This charge will be uniformly distributed. Since there is no electric field in the conducting material of the tube, there is no way for the charges in the region interior to the tube to influence the charge distribution on the outer surface. Thus the field outside the tube is due to a cylindrically symmetric distribution of charge. Application of Gauss's law as in Example 22-6, for a Gaussian cylinder with a radius larger than the conducting tube, and a length $\ell$ leads to $E(2 \pi R \ell)=\frac{Q_{\text {encl }}}{\varepsilon_{0}}$. The enclosed charge is the amount of charge on the inner cylinder.

$$
Q_{\mathrm{encl}}=\rho_{\mathrm{E}} \pi R_{1}^{2} \ell \rightarrow E=\frac{Q_{\mathrm{encl}}}{\varepsilon_{0}(2 \pi R \ell)}=\frac{\rho_{\mathrm{E}} R_{1}^{2}}{2 \varepsilon_{0} R}
$$

41. We treat the source charge as a disk of positive charge of radius concentric with a disk of negative charge of radius $R_{0}$. In order for the net charge of the inner space to be 0 , the charge per unit area of the source disks must both have the same magnitude but opposite sign. The field due to the annulus is then the sum of the fields due to both the positive and negative rings.
(a) At a distance of $0.25 R_{0}$ from the center of the ring, we can approximate both of the disks as infinite planes, each producing a uniform field. Since those two uniform fields will be of the same magnitude and opposite sign, the net field is 0 .
(b) At a distance of $75 R_{0}$ from the center of the ring, it appears to be approximately a point charge, and so the field will approximate that of a point charge, $E=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{\left(75 R_{0}\right)^{2}}$
42. The conducting sphere is uncharged, and the electric field is 0 everywhere within its interior, except for in the cavities. When charge $Q_{1}$ is placed in the first cavity, a charge $-Q_{1}$ will be drawn from the conducting material to the inner surface of the cavity, and the electric field will remain 0 in the conductor. That charge $-Q_{1}$ will NOT be distributed symmetrically on the cavity surface. Since the conductor is neutral, a compensating charge $Q_{1}$ will appear on the outer surface of the conductor (charge can only be on the surfaces of conductors in electrostatics). Likewise, when charge $Q_{2}$ is placed in the second cavity, a charge $-Q_{2}$ will be drawn from the conducting material, and a compensating charge $Q_{2}$ will appear on the outer surface. Since there is no electric field in the conducting material, there is no way for the charges in the cavities to influence the charge distribution on the outer surface. So the distribution of charge on the outer surface is uniform, just as it would be if there were no inner charges, and instead a charge $Q_{1}+Q_{2}$ were simply placed on the conductor. Thus the field outside the conductor is due to a spherically symmetric distribution of $Q_{1}+Q_{2}$. Application of Gauss's law leads to $E=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q_{1}+Q_{2}}{r^{2}}$. If $Q_{1}+Q_{2}>0$, the field will point radially outward. If $Q_{1}+Q_{2}<0$, the field will point radially inward.
43. (a) Choose a cylindrical gaussian surface with the flat ends parallel to and equidistant from the slab. By symmetry the electric field must point perpendicularly away from the slab, resulting in no flux passing through the curved part of the gaussian cylinder. By symmetry the flux through each end of the cylinder must be equal with the electric field constant across the surface.

$$
\oint \vec{E} \cdot d \vec{A}=2 E A
$$

The charge enclosed by the surface is the charge density of the slab multiplied by the volume of the slab enclosed by the surface.

$$
q_{e n c}=\rho_{E}(A d)
$$

Gauss's law can then be solved for the electric field.

$$
\oint \vec{E} \cdot d \vec{A}=2 E A=\frac{\rho_{E} A d}{\varepsilon_{0}} \rightarrow E=\frac{\rho_{E} d}{2 \varepsilon_{0}}
$$

Note that this electric field is independent of the distance from the slab.
(b) When the coordinate system of this problem is changed to axes parallel $(\hat{\mathbf{z}})$ and perpendicular $(\hat{\mathbf{r}})$ to the slab, it can easily be seen that the particle will hit the slab if the initial perpendicular velocity is sufficient for the particle to reach the slab before the acceleration decreases its velocity to zero. In the new coordinate system the axes are rotated by $45^{\circ}$.

$$
\vec{r}_{0}=y_{0} \cos 45^{\circ} \hat{\mathbf{r}}+y_{0} \sin 45^{\circ} \hat{\mathbf{z}}=\frac{y_{0}}{\sqrt{2}} \hat{\mathbf{r}}+\frac{y_{0}}{\sqrt{2}} \hat{\mathbf{z}}
$$

$$
\begin{aligned}
& \vec{v}_{0}=-v_{0} \sin 45^{\circ} \hat{\mathbf{r}}+v_{0} \cos 45^{\circ} \hat{\mathbf{z}}=-\frac{v_{0}}{\sqrt{2}} \hat{\mathbf{r}}+\frac{v_{0}}{\sqrt{2}} \hat{\mathbf{z}} \\
& \vec{a}=q E / m \hat{\mathbf{r}}
\end{aligned}
$$

The perpendicular components are then inserted into Eq. 2-12c, with the final velocity equal to zero.

$$
0=v_{r 0}^{2}-2 a\left(r-r_{0}\right)=\frac{v_{0}^{2}}{2}-2 \frac{q}{m}\left(\frac{\rho_{E} d}{2 \varepsilon_{0}}\right)\left(\frac{y_{0}}{\sqrt{2}}-0\right)
$$

Solving for the velocity gives the minimum speed that the particle can have to reach the slab.

$$
v_{0} \geq \sqrt{\frac{\sqrt{2} q \rho_{E} d y_{0}}{m \varepsilon_{0}}}
$$

44. Due to the spherical symmetry of the problem, Gauss's law using a sphere of radius $r$ leads to the following.

$$
\oint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=E\left(4 \pi r^{2}\right)=\frac{Q_{\text {encl }}}{\varepsilon_{0}} \rightarrow E=\frac{Q_{\text {encl }}}{4 \pi \varepsilon_{0} r^{2}}
$$

(a) For the region $0<r<r_{1}$, the enclosed charge is 0 .

$$
E=\frac{Q_{\text {encl }}}{4 \pi \varepsilon_{0} r^{2}}=0
$$

(b) For the region $r_{1}<r<r_{0}$, the enclosed charge is the product of the volume charge density times the volume of charged material enclosed. The charge density is given by $\rho=\rho_{0} \frac{r_{1}}{r}$. We must integrate to find the total charge. We follow the procedure given in Example 22-5. We divide the sphere up into concentric thin shells of thickness $d r$, as shown in Fig. 22-14. We then integrate to find the charge.

$$
\begin{aligned}
& Q_{\text {encl }}=\int \rho_{\mathrm{E}} d V=\int_{r_{1}}^{r} \rho_{0} \frac{r_{1}}{r^{\prime}} 4 \pi\left(r^{\prime}\right)^{2} d r^{\prime}=4 \pi r_{1} \rho_{0} \int_{r_{1}}^{r} r^{\prime} d r^{\prime}=2 \pi r_{1} \rho_{0}\left(r^{2}-r_{1}^{2}\right) \\
& E=\frac{Q_{\text {encl }}}{4 \pi \varepsilon_{0} r^{2}}=\frac{2 \pi r_{1} \rho_{0}\left(r^{2}-r_{1}^{2}\right)}{4 \pi \varepsilon_{0} r^{2}}=\frac{\rho_{0} r_{1}\left(r^{2}-r_{1}^{2}\right)}{2 \varepsilon_{0} r^{2}}
\end{aligned}
$$

(c) For the region $r>r_{0}$, the enclosed charge is the total charge, found by integration in a similar fashion to part (b).

$$
\begin{aligned}
& Q_{\text {encl }}=\int \rho_{\mathrm{E}} d V=\int_{r_{1}}^{r_{0}} \rho_{0} \frac{r_{1}}{r^{\prime}} 4 \pi\left(r^{\prime}\right)^{2} d r^{\prime}=4 \pi r_{1} \rho_{0} \int_{r_{i}}^{r_{0}} r^{\prime} d r^{\prime}=2 \pi r_{1} \rho_{0}\left(r_{0}^{2}-r_{1}^{2}\right) \\
& E=\frac{Q_{\text {encl }}}{4 \pi \varepsilon_{0} r^{2}}=\frac{2 \pi r_{1} \rho_{0}\left(r_{0}^{2}-r_{1}^{2}\right)}{4 \pi \varepsilon_{0} r^{2}}=\frac{\rho_{0} r_{1}\left(r_{0}^{2}-r_{1}^{2}\right)}{2 \varepsilon_{0} r^{2}}
\end{aligned}
$$

(d) See the attached graph. We have chosen $r_{1}=\frac{1}{2} r_{0}$. Let

$$
E_{0}=E\left(r=r_{0}\right)=\frac{\rho_{0} r_{1}\left(r_{0}^{2}-r_{1}^{2}\right)}{2 \varepsilon_{0} r_{0}^{2}} .
$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH22.XLS," on tab "Problem 22.44d."

45. (a) The force felt by one plate will be the charge on that plate multiplied by the electric field caused by the other plate. The field due to one plate is found in Example 22-7. Let the positive plate be on the left, and the negative plate on the right. We find the force on the negative plate due to the positive plate.

$$
\begin{aligned}
& =\frac{\left(-15 \times 10^{-6} \mathrm{C} / \mathrm{m}^{2}\right)\left(1.0 \mathrm{~m}^{2}\right)\left(-15 \times 10^{-6} \mathrm{C} / \mathrm{m}^{2}\right)}{2\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{Nm}^{2}\right)}=-12.71 \mathrm{~N} \\
& \approx 13 \mathrm{~N}, \text { towards the other plate }
\end{aligned}
$$

(b) Since the field due to either plate is constant, the force on the other plate is constant, and then the work is just the force times the distance. Since the plates are oppositely charged, they will attract, and so a force equal to and opposite the force above will be required to separate them. The force will be in the same direction as the displacement of the plates.

$$
W=\overrightarrow{\mathbf{F}} \cdot \Delta \overrightarrow{\mathbf{x}}=(12.71 \mathrm{~N})\left(\cos 0^{\circ}\right)\left(5.0 \times 10^{-3} \mathrm{~m}\right)=0.064 \mathrm{~J}
$$

46. Because the slab is very large, and we are considering only distances from the slab much less than its height or breadth, the symmetry of the slab results in the field being perpendicular to the slab, with a constant magnitude for a constant distance from the center. We assume that $\rho_{\mathrm{E}}>0$ and so the electric field points away from the center of the slab.
(a) To determine the field inside the slab, choose a cylindrical gaussian surface, of length $2 x<d$ and cross-sectional area $A$. Place it so that it is centered in the slab. There will be no flux through the curved wall of the cylinder. The electric field is parallel to the surface area vector on both ends, and is the same magnitude on both ends. Apply Gauss's law to find the electric field at a distance $x<\frac{1}{2} d$ from the center of the slab.
 See the first diagram.

$$
\begin{aligned}
& \oint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=\int_{\text {ends }} \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}+\int_{\text {side }} \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=\int_{\text {ends }} \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}+0=\frac{Q_{\text {encl }}}{\varepsilon_{0}} \rightarrow 2 E A=\frac{\rho(2 x A)}{\varepsilon_{0}} \rightarrow \\
& E_{\text {inside }}=\frac{\rho x}{\varepsilon_{0}} ;|x|<\frac{1}{2} d
\end{aligned}
$$

(b) Use a similar arrangement to determine the field outside the slab. Now let $2 x>d$. See the second diagram.

$$
\begin{aligned}
& \oint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=\int_{\text {ends }} \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=\frac{Q_{\text {encl }}}{\varepsilon_{0}} \rightarrow \\
& 2 E A=\frac{\rho(d A)}{\varepsilon_{0}} \rightarrow E_{\text {outside }}=\frac{\rho d}{2 \varepsilon_{0}} ;|x|>\frac{1}{2} d
\end{aligned}
$$

Notice that electric field is continuous at the boundary of the slab.
47. (a) In Problem 46, it is shown that the field outside a flat slab of nonconducting material with a uniform charge density is given by $E=\frac{\rho d}{2 \varepsilon_{0}}$. If the charge density is positive, the field points away from the slab, and if the charge density is negative, the field points towards the slab. So for this problem's configuration, the field outside of both half-slabs is the vector sum of the fields from each half-slab. Since those fields are equal in magnitude and opposite in direction, the field outside the slab is 0 .
(b) To find the field in the positively charged half-slab, we use a cylindrical gaussian surface of cross sectional area $A$. Place it so that its left end is in the positively charged half-slab, a distance $x>0$ from the center of the slab. Its right end is external to the slab. Due to the symmetry of the configuration, there will be no flux through the curved wall of the cylinder. The electric field is parallel to the surface area vector on the left end, and is 0 on the right end. We assume that the electric field is pointing to the left. Apply Gauss's law to find the
 electric field a distance $0<x<d$ from the center of the slab. See the diagram.

$$
\begin{aligned}
& \oint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=\int_{\text {ends }} \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}+\int_{\text {side }} \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=\int_{\substack{\text { left } \\
\text { end }}} \overrightarrow{\mathbf{E}} d \overrightarrow{\mathbf{A}}+0=\frac{Q_{\text {encl }}}{\varepsilon_{0}} \rightarrow \\
& E A=\frac{\rho_{0}(d-x) A}{\varepsilon_{0}} \rightarrow E_{x>0}=\frac{\rho_{0}(d-x)}{\varepsilon_{0}}
\end{aligned}
$$

Since the field is pointing to the left, we can express this as $E_{x>0}=-\frac{\rho_{0}(d-x)}{\varepsilon_{0}} \hat{\mathbf{i}}$.
(c) To find the field in the negatively charged half-slab, we use a cylindrical gaussian surface of cross sectional area $A$. Place it so that its right end is in the negatively charged half-slab, a distance $x<0$ from the center of the slab. Its left end is external to the slab. Due to the symmetry of the configuration, there will be no flux through the curved wall of the cylinder. The electric field is parallel to the surface area vector on the left end, and is 0 on the right end. We assume that the electric field is pointing to the right. Apply Gauss's law to find the electric field at a distance $-d<x<0$ from the center of the slab. See the diagram.


$$
\oint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=\int_{\text {ends }} \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}+\int_{\text {side }} \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=\int_{\substack{\text { right } \\ \text { end }}} \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}+0=\frac{Q_{\text {encl }}}{\varepsilon_{0}} \rightarrow
$$

$$
E A=\frac{-\rho_{0}(d+x) A}{\varepsilon_{0}} \rightarrow E_{x<0}=\frac{-\rho_{0}(d+x)}{\varepsilon_{0}}
$$

Since the field is pointing to the left, we can express this as $E_{x<0}=-\frac{\rho_{0}(d+x)}{\varepsilon_{0}} \hat{\mathbf{i}}$.
Notice that the field is continuous at all boundaries. At the left edge $(x=-d), E_{x<0}=E_{\text {ousside }}$. At the center $(x=0), E_{x<0}=E_{>0}$. And at the right edge $(x=d), E_{x>0}=E_{\text {ousidide }}$.
48. We follow the development of Example 22-6. Because of the symmetry, we expect the field to be directed radially outward (no fringing effects near the ends of the cylinder) and to depend only on the perpendicular distance, $R$, from the symmetry axis of the cylinder. Because of the cylindrical symmetry, the field will be the same at all points on a gaussian surface that is a cylinder whose axis coincides with the axis of the cylinder.


The gaussian surface is of radius $r$ and length $\ell . \overrightarrow{\mathbf{E}}$ is perpendicular to this surface at all points. In order to apply Gauss's law, we need a closed surface, so we include the flat ends of the cylinder.
Since $\overrightarrow{\mathbf{E}}$ is parallel to the flat ends, there is no flux through the ends. There is only flux through the curved wall of the gaussian cylinder.

$$
\oint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=E(2 \pi R \ell)=\frac{Q_{\text {encl }}}{\varepsilon_{0}} \rightarrow E=\frac{Q_{\text {encl }}}{2 \pi \varepsilon_{0} R \ell}
$$

To find the field inside the cylinder, we must find the charge enclosed in the gaussian cylinder. We divide the gaussian cylinder up into coaxial thin cylindrical shells of length $\ell$ and thickness $d R$. That shell has volume $d V=2 \pi R \ell d R$. The total charge in the gaussian cylinder is found by integration.

$$
\begin{gathered}
R<R_{0}: Q_{\text {encl }}=\int_{0}^{R} \rho_{\mathrm{E}} d V=\int_{0}^{R} \rho_{0}\left(\frac{R}{R_{0}}\right)^{2} 2 \pi R \ell d R=\frac{2 \pi \rho_{0} \ell}{R_{0}^{2}} \int_{0}^{R} R^{3} d R=\frac{\pi \rho_{0} \ell R^{4}}{2 R_{0}^{2}} \\
E=\frac{Q_{\text {encl }}}{2 \pi \varepsilon_{0} R \ell}=\frac{\frac{\pi \rho_{0} \ell R^{4}}{2 R_{0}^{2}}}{2 \pi \varepsilon_{0} R \ell}=\frac{\rho_{0} R^{3}}{4 \varepsilon_{0} R_{0}^{2}}, \text { radially out } \\
R<R_{0}: \quad Q_{\text {encl }}=\int_{0}^{R_{0}} \rho_{\mathrm{E}} d V=\frac{2 \pi \rho_{0} \ell}{R_{0}^{2}} \int_{0}^{R_{R}} R^{3} d R=\frac{\pi \rho_{0} \ell R_{0}^{2}}{2} \\
E=\frac{Q_{\text {encl }}}{2 \pi \varepsilon_{0} R \ell}=\frac{\frac{\pi \rho_{0} \ell R_{0}^{2}}{2}}{2 \pi \varepsilon_{0} R \ell}=\frac{\rho_{0} R_{0}^{2}}{4 \varepsilon_{0} R}, \text { radially out }
\end{gathered}
$$

49. The symmetry of the charge distribution allows the electric field inside the sphere to be calculated using Gauss's law with a concentric gaussian sphere of radius $r \leq r_{0}$. The enclosed charge will be found by integrating the charge density over the enclosed volume.

$$
Q_{\mathrm{encl}}=\int \rho_{\mathrm{E}} d V=\int_{0}^{r} \rho_{0}\left(\frac{r^{\prime}}{r_{0}}\right) 4 \pi r^{\prime 2} d r^{\prime}=\frac{\rho_{0} \pi r^{4}}{r_{0}}
$$

The enclosed charge can be written in terms of the total charge by setting

$r=r_{0}$ and solving for the charge density in terms of the total charge.

$$
Q=\frac{\rho_{0} \pi r_{0}^{4}}{r_{0}}=\rho_{0} \pi r_{0}^{3} \rightarrow \rho_{0}=\frac{Q}{\pi r_{0}^{3}} \rightarrow Q_{\text {encl }}(r)=\frac{\rho_{0} \pi r^{4}}{r_{0}}=Q\left(\frac{r}{r_{0}}\right)^{4}
$$

The electric field is then found from Gauss's law

$$
\oint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=\frac{Q_{\mathrm{encl}}}{\varepsilon_{0}} \rightarrow E\left(4 \pi r^{2}\right)=\frac{Q}{\varepsilon_{0}}\left(\frac{r}{r_{0}}\right)^{4} \rightarrow E=\frac{Q}{4 \pi \varepsilon_{0}} \frac{r^{2}}{r_{0}^{4}}
$$

The electric field points radially outward since the charge distribution is positive.
50. By Gauss's law, the total flux through the cylinder is $Q / \varepsilon_{0}$. We find the flux through the ends of the cylinder, and then subtract that from the total flux to find the flux through the curved sides. The electric field is that of a point charge. On the ends of the cylinder, that field will vary in both magnitude and direction. Thus we must do a detailed integration to find the flux through the ends of the cylinder. Divide the ends into a series of concentric circular rings, of radius $R$ and thickness $d R$. Each ring will have an area of $2 \pi R d R$. The angle between $\overrightarrow{\mathbf{E}}$ and $d \overrightarrow{\mathbf{A}}$ is $\theta$, where $\tan \theta=R / R_{0}$. See the diagram of the left half of the cylinder.


$$
\Phi_{\substack{\text { left } \\ \text { end }}}=\int \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=\int_{0}^{R_{0}} \frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{r^{2}} \cos \theta(2 \pi R) d R
$$

The flux integral has three variables: $r, R$, and $\theta$. We express $r$ and $\theta$ in terms of $R$ in order to integrate. The anti-derivative is found in Appendix B-4.

$$
\begin{aligned}
& r=\sqrt{R^{2}+R_{0}^{2}} ; \cos \theta=\frac{R_{0}}{r}=\frac{R_{0}}{\sqrt{R^{2}+R_{0}^{2}}} \\
& \begin{array}{c}
\Phi_{\text {left }} \text { end } \\
\text { end }
\end{array} \int_{0}^{R_{0}} \frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{\left(R^{2}+R_{0}^{2}\right)} \frac{R_{0}}{\sqrt{R^{2}+R_{0}^{2}}}(2 \pi R) d R=\frac{2 \pi Q R_{0}}{4 \pi \varepsilon_{0}} \int_{0}^{R_{0}} \frac{R d R}{\left(R^{2}+R_{0}^{2}\right)^{3 / 2}}=\frac{Q R_{0}}{2 \varepsilon_{0}}\left[-\frac{1}{\sqrt{R^{2}+R_{0}^{2}}}\right]_{0}^{R_{0}} \\
& \quad=\frac{Q}{2 \varepsilon_{0}}\left[1-\frac{1}{\sqrt{2}}\right] ; \Phi_{\substack{\text { both } \\
\text { ends }}}=2 \Phi_{\substack{\text { left } \\
\text { end }}}=\frac{Q}{\varepsilon_{0}}\left[1-\frac{1}{\sqrt{2}}\right] \\
& \Phi_{\text {total }}=\frac{Q}{\varepsilon_{0}}=\Phi_{\text {sides }}+\Phi_{\substack{\text { both } \\
\text { ends }}} \rightarrow \Phi_{\text {sides }}=\frac{Q}{\varepsilon_{0}}-\Phi_{\substack{\text { both } \\
\text { ends }}}=\frac{Q}{\varepsilon_{0}}-\frac{Q}{\varepsilon_{0}}\left[1-\frac{1}{\sqrt{2}}\right]=\frac{Q}{\sqrt{2} \varepsilon_{0}}
\end{aligned}
$$

51. The gravitational field a distance $r$ from a point mass $M$ is given by Eq. $6-8, \overrightarrow{\mathbf{g}}=-\frac{G M}{r^{2}} \hat{\mathbf{r}}$, where $\hat{\mathbf{r}}$ is a unit vector pointing radially outward from mass $M$. Compare this to the electric field of a point charge, $\overrightarrow{\mathbf{E}}=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{r^{2}} \hat{\mathbf{r}}$. To change the electric field to the gravitational field, we would make these changes: $\overrightarrow{\mathbf{E}} \rightarrow \overrightarrow{\mathbf{g}} ; Q / \varepsilon_{0} \rightarrow-4 \pi G M$. Make these substitutions in Gauss's law.

$$
\oint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=\frac{Q_{\text {encl }}}{\varepsilon_{0}} \rightarrow \oint \overrightarrow{\mathbf{g}} \cdot d \overrightarrow{\mathbf{A}}=-4 \pi G M_{\text {encl }}
$$

52. (a) We use Gauss's law for a spherically symmetric charge distribution, and assume that all the charge is on the surface of the Earth. Note that the field is pointing radially inward, and so the dot product introduces a negative sign.

$$
\begin{aligned}
& \oint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=-E\left(4 \pi r^{2}\right)=Q_{\text {encl }} / \varepsilon_{0} \rightarrow \\
& Q_{\text {encl }}=-4 \pi \varepsilon_{0} E R_{\text {Earth }}^{2}=\frac{-(150 \mathrm{~N} / \mathrm{C})\left(6.38 \times 10^{6} \mathrm{~m}\right)^{2}}{\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)}=-6.793 \times 10^{5} \mathrm{C} \approx-6.8 \times 10^{5} \mathrm{C}
\end{aligned}
$$

(b) Find the surface density of electrons. Let $n$ be the total number of electrons.

$$
\begin{aligned}
\sigma & =\frac{Q}{A}=-\frac{n e}{A} \rightarrow \\
\frac{n}{A} & =-\frac{Q}{e A}=-\frac{-4 \pi \varepsilon_{0} E R_{\text {Earth }}^{2}}{e\left(4 \pi R_{\text {Earth }}^{2}\right)}=\frac{\varepsilon_{0} E}{e}=\frac{\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)(150 \mathrm{~N} / \mathrm{C})}{\left(1.60 \times 10^{-19} \mathrm{C}\right)} \\
& =8.3 \times 10^{9} \text { electrons } / \mathrm{m}^{2}
\end{aligned}
$$

53. The electric field is strictly in the $y$ direction. So, referencing the diagram, there is no flux through the top, bottom, front, or back faces of the cube. Only the "left" and "right" faces will have flux through them. And since the flux is only dependent on the $y$ coordinate, the flux through each of those two faces is particularly simple. Calculate the flux and use Gauss's law to find the enclosed charge.

$$
\begin{aligned}
\Phi & =\oint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=\int_{\substack{\text { left } \\
\text { face }}} \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}+\int_{\substack{\text { right } \\
\text { fface }}} \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}} \\
& =\int_{\substack{\text { left } \\
\text { face }}} b \hat{\mathbf{j}} \cdot(-\hat{\mathbf{j}} d A)+\int_{\substack{\text { right } \\
\text { face }}}(a \ell+b) \hat{\mathbf{j}} \cdot(\hat{\mathbf{j}} d A)=-b \ell^{2}+a \ell^{3}+b \ell^{2} \\
& =a \ell^{3}=Q_{\text {encl }} / \varepsilon_{0} \rightarrow Q_{\text {encl }}=\varepsilon_{0} a \ell^{3}
\end{aligned}
$$


54. (a) Find the value of $b$ by integrating the charge density over the entire sphere. Follow the development given in Example 22-5.

$$
Q=\int \rho_{\mathrm{E}} d V=\int_{0}^{r_{0}} b r\left(4 \pi r^{2} d r\right)=4 \pi b\left(\frac{1}{4} r_{0}^{4}\right) \rightarrow b=\frac{Q}{\pi r_{0}^{4}}
$$

(b) To find the electric field inside the sphere, we apply Gauss's law to an imaginary sphere of radius $r$, calculating the charge enclosed by that sphere. The spherical symmetry allows us to evaluate the flux integral simply.

$$
\begin{aligned}
& \Phi=\oint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=\frac{Q_{\text {encl }}}{\varepsilon_{0}} ; Q=\int \rho_{\mathrm{E}} d V=\int_{0}^{r} \frac{Q}{\pi r_{0}^{4}} r\left(4 \pi r^{2} d r\right)=\frac{Q r^{4}}{r_{0}^{4}} \rightarrow \\
& E=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q r^{2}}{r_{0}^{4}}, r<r_{0}
\end{aligned}
$$

(c) As discussed in Example 22-4, the field outside a spherically symmetric distribution of charge is the same as that for a point charge of the same magnitude located at the center of the sphere.

$$
E=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{r^{2}}, r>r_{0}
$$

55. The flux through a gaussian surface depends only on the charge enclosed by the surface. For both of these spheres the two point charges are enclosed within the sphere. Therefore the flux is the same for both spheres.

$$
\Phi=\frac{Q_{\text {encl }}}{\varepsilon_{0}}=\frac{\left(9.20 \times 10^{-9} \mathrm{C}\right)+\left(-5.00 \times 10^{-9} \mathrm{C}\right)}{8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}}=475 \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}
$$

56. (a) The flux through any closed surface containing the total charge must be the same, so the flux through the larger sphere is the same as the flux through the smaller sphere, $+235 \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}$.
(b) Use Gauss's law to determine the enclosed charge.

$$
\Phi=\frac{Q_{\text {encl }}}{\varepsilon_{0}} \rightarrow Q_{\text {encl }}=\varepsilon_{0} \Phi=\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)\left(+235 \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}\right)=2.08 \times 10^{-9} \mathrm{C}
$$

57. (a) There is no charge enclosed within the sphere, and so no flux lines can originate or terminate inside the sphere. All field lines enter and leave the sphere. Thus the net flux is 0 .
(b) The maximum electric field will be at the point on the sphere closest to $Q$, which is the top of the sphere. The minimum electric field will be at the point on the sphere farthest from Q , which is the bottom of the sphere.

$$
\begin{aligned}
& E_{\max }=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{r_{\text {closest }}^{2}}=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{\left(\frac{1}{2} r_{0}\right)^{2}}=\frac{1}{\pi \varepsilon_{0}} \frac{Q}{r_{0}^{2}} \\
& E_{\min }=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{r_{\text {farthest }}^{2}}=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{\left(\frac{5}{2} r_{0}\right)^{2}}=\frac{1}{25 \pi \varepsilon_{0}} \frac{Q}{r_{0}^{2}}
\end{aligned}
$$



Thus the range of values is $\frac{1}{\pi \varepsilon_{0}} \frac{Q}{r_{0}^{2}} \leq E_{\substack{\text { sphere } \\ \text { surface }}} \leq \frac{1}{25 \pi \varepsilon_{0}} \frac{Q}{r_{0}^{2}}$.
(c) $\overrightarrow{\mathbf{E}}$ is not perpendicular at all points. It is only perpendicular at the two points already discussed: the point on the sphere closest to the point charge, and the point on the sphere farthest from the point charge.
(d) The electric field is not perpendicular or constant over the surface of the sphere. Therefore Gauss's law is not useful for obtaining E at the surface of the sphere because a gaussian surface cannot be chosen that simplifies the flux integral.
58. The force on a sheet is the charge on the sheet times the average electric field due to the other sheets: But the fields due to the "other" sheets is uniform, so the field is the same over the entire sheet. The force per unit area is then the charge per unit area, times the field due to the other sheets.

$$
\begin{aligned}
& F_{\substack{\text { on } \\
\text { shect }}}=q_{\substack{\text { on } \\
\text { sheet }}}^{\bar{E}_{\text {oherer }}}=q_{\text {onects }} E_{\substack{\text { onheet } \\
\text { shere }}}^{\text {shets }} \rightarrow \\
& \left(\frac{F}{A}\right)_{\substack{\text { on } \\
\text { sheet }}}=\left(\frac{q}{A}\right)_{\substack{\text { on } \\
\text { sheet }}} E_{\text {other }}^{\text {sheets }}=\sigma_{\substack{\text { on } \\
\text { sheet }}} E_{\text {other }}^{\text {sheets }}
\end{aligned}
$$



The uniform fields from each of the three sheets are indicated on the diagram. Take the positive direction as upwards. We take the direction from the diagram, and so use the absolute value of each charge density. The electric field magnitude due to each sheet is given by $E=\sigma / 2 \varepsilon_{0}$.

$$
\begin{aligned}
\left(\frac{F}{A}\right)_{\mathrm{I}} & =\sigma_{\mathrm{I}}\left(E_{\mathrm{III}}-E_{\mathrm{II}}\right)=\frac{\sigma_{\mathrm{I}}}{2 \varepsilon_{0}}\left(\left|\sigma_{\mathrm{III}}\right|-\left|\sigma_{\mathrm{II}}\right|\right)=\frac{6.5 \times 10^{-9} \mathrm{C} / \mathrm{m}^{2}}{2\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)}\left[(5.0-2.0) \times 10^{-9} \mathrm{C} / \mathrm{m}^{2}\right] \\
& =1.1 \times 10^{-6} \mathrm{~N} / \mathrm{m}^{2}(\text { up }) \\
\left(\frac{F}{A}\right)_{\text {II }} & =\sigma_{\mathrm{II}}\left(E_{\mathrm{III}}-E_{\mathrm{I}}\right)=\frac{\sigma_{\mathrm{II}}}{2 \varepsilon_{0}}\left(\left|\sigma_{\mathrm{III}}\right|-\left|\sigma_{\mathrm{I}}\right|\right)=\frac{-2.0 \times 10^{-9} \mathrm{C} / \mathrm{m}^{2}}{2\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)}\left[(5.0-6.5) \times 10^{-9} \mathrm{C} / \mathrm{m}^{2}\right] \\
& =1.7 \times 10^{-7} \mathrm{~N} / \mathrm{m}^{2}(\text { up }) \\
\left(\frac{F}{A}\right)_{\text {III }} & =\sigma_{\mathrm{III}}\left(E_{\mathrm{III}}-E_{\mathrm{I}}\right)=\frac{\sigma_{\text {III }}}{2 \varepsilon_{0}}\left(\left|\sigma_{\mathrm{II}}\right|-\left|\sigma_{\mathrm{I}}\right|\right)=\frac{5.0 \times 10^{-9} \mathrm{C} / \mathrm{m}^{2}}{2\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)}\left[(2.0-6.5) \times 10^{-9} \mathrm{C} / \mathrm{m}^{2}\right] \\
& =-1.3 \times 10^{-6} \mathrm{~N} / \mathrm{m}^{2}(\text { down })
\end{aligned}
$$

59. (a) The net charge inside a sphere of radius $a_{0}$ will be made of two parts - the positive point charge at the center of the sphere, and some fraction of the total negative charge, since the negative charge is distributed over all space, as described by the charge density. To evaluate the portion of the negative charge inside the sphere, we must determine the coefficient $A$. We do that by integrating the charge density over all space, in the manner of Example 22-5. Use an integral from Appendix B-5.

$$
\begin{aligned}
& -e=\int \rho_{E} d V=\int_{0}^{\infty}\left(-A e^{-2 r / a_{0}}\right)\left(4 \pi r^{2} d r\right)=-4 \pi A \int_{0}^{\infty} e^{-2 r / a_{0}} r^{2} d r=-4 \pi A \frac{2!}{\left(2 / a_{0}\right)^{3}}=-\pi A a_{0}^{3} \rightarrow \\
& A=\frac{e}{\pi a_{0}^{3}}
\end{aligned}
$$

Now we find the negative charge inside the sphere of radius $a_{0}$, using an integral from
Appendix B-4. We are indicating the elementary charge by $(e)$, so as to not confuse it with the base of the natural logarithms.

$$
\begin{aligned}
Q_{\text {neg }} & =\int_{0}^{a_{0}}\left(-A e^{-2 r / a_{0}}\right)\left(4 \pi r^{2} d r\right)=-\frac{4 \pi(e)}{\pi a_{0}^{3}} \int_{0}^{a_{0}} e^{-2 r / a_{0}} r^{2} d r \\
& =-\frac{4 \pi(e)}{\pi a_{0}^{3}}\left\{\left[-\frac{e^{-2 r / a_{0}}}{\left(2 / a_{0}\right)^{3}}\right]\left[\left(2 / a_{0}\right)^{2} r^{2}+2\left(2 / a_{0}\right) r+2\right]\right\}_{0}^{a_{0}}=(e)\left[5 e^{-2}-1\right] \\
Q_{\text {net }} & =Q_{\text {neg }}+Q_{\text {pos }}=(e)\left[5 e^{-2}-1\right]+(e)=(e) 5 e^{-2}=\left(1.6 \times 10^{-19} \mathrm{C}\right) 5 e^{-2}=1.083 \times 10^{-19} \mathrm{C} \\
& \approx 1.1 \times 10^{-19} \mathrm{C}
\end{aligned}
$$

(b) The field at a distance $r=a_{0}$ is that of a point charge of magnitude $Q_{\text {net }}$ at the origin, because of the spherical symmetry and Gauss's law.

$$
E=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q_{\text {encl }}}{r^{2}}=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q_{\text {net }}}{a_{0}^{2}}=\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{\left(1.083 \times 10^{-19} \mathrm{C}\right)}{\left(0.53 \times 10^{-10} \mathrm{~m}\right)^{2}}=3.5 \times 10^{11} \mathrm{~N} / \mathrm{C}
$$

60. The field due to the plane is $E_{\text {plane }}=\frac{\sigma}{2 \varepsilon_{0}}$, as discussed in Example 22-7. Because the slab is very large, and we assume that we are considering only distances from the slab much less than its height or breadth, the symmetry of the slab results in its field being perpendicular to the slab, with a constant magnitude for a constant distance from its center. We also assume that $\rho_{\mathrm{E}}>0$ and so the electric field of the slab points away from the center of the slab.
(a) To determine the field to the left of the plane, we choose a cylindrical gaussian surface, of length $x>d$ and cross-sectional area $A$. Place it so that the plane is centered inside the cylinder. See the diagram. There will be no flux through the curved wall of the cylinder. From the symmetry, the electric field is parallel to the surface area vector on both ends. We already know that the
 field due to the plane is the same on both ends, and by the symmetry of the problem, the field due to the slab must also be the same on both ends. Thus the total field is the same magnitude on both ends.

$$
\begin{aligned}
& \oint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=\int_{\text {ends }} \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}+\int_{\text {side }} \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=\int_{\text {ends }} \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}+0=\frac{Q_{\text {encl }}}{\varepsilon_{0}} \rightarrow 2 E_{\text {outside }} A=\frac{\sigma A+\rho_{\mathrm{E}} d A}{\varepsilon_{0}} \rightarrow \\
& E_{\text {outside }}=E_{\text {left }}=\frac{\sigma+\rho_{\mathrm{E}} d}{\text { of phane }} \\
& 2 \varepsilon_{0}
\end{aligned}
$$

(b) As argued above, the field is symmetric on the outside of the charged matter.

$$
E_{\text {right }}=\frac{\sigma+\rho_{\mathrm{E}} d}{2 \varepsilon_{0}}
$$

(c) To determine the field inside the slab, we choose a cylindrical gaussian surface of cross-sectional area $A$ with one face to the left of the plane, and the other face inside the slab, a distance $x$ from the plane. Due to symmetry, the field again is parallel to the surface area vector on both ends, has a constant value on each end, and no flux pierces the curved walls. Apply Gauss's law.


$$
\begin{aligned}
& \oint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=\int_{\substack{\text { left } \\
\text { end }}} \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}+\int_{\substack{\text { right } \\
\text { end }}} \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}+\int_{\text {side }} \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=E_{\text {outside }} A+E_{\text {inside }} A+0=\frac{Q_{\text {encl }}}{\varepsilon_{0}} \\
& Q_{\text {encl }}=\sigma A+\rho_{\mathrm{E}} x A \rightarrow\left(\frac{\sigma+\rho_{\mathrm{E}} d}{2 \varepsilon_{0}}\right) A+E_{\text {inside }} A=\frac{\sigma A+\rho_{\mathrm{E}} x A}{\varepsilon_{0}} \rightarrow \\
& E_{\text {inside }}=\frac{\sigma+\rho_{\mathrm{E}}(2 x-d)}{2 \varepsilon_{0}}, 0<x<d
\end{aligned}
$$

Notice that the field is continuous from "inside" to "outside" at the right edge of the slab, but not at the left edge of the slab. That discontinuity is due to the surface charge density.
61. Consider this sphere as a combination of two spheres. Sphere 1 is a solid sphere of radius $r_{0}$ and charge density $\rho_{\mathrm{E}}$ centered at A and sphere 2 is a second sphere of radius $r_{0} / 2$ and density $-\rho_{\mathrm{E}}$ centered at C .
(a) The electric field at A will have zero contribution from sphere 1 due to its symmetry about point A. The electric field is then calculated by creating a gaussian surface centered at point C with radius $r_{0} / 2$.

$$
\oint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=\frac{q_{\mathrm{enc}}}{\varepsilon_{0}} \rightarrow E \cdot 4 \pi\left(\frac{1}{2} r_{0}\right)^{2}=\frac{\left(-\rho_{\mathrm{E}}\right) \frac{4}{3} \pi\left(\frac{1}{2} r_{0}\right)^{3}}{\varepsilon_{0}} \rightarrow E=-\frac{\rho_{\mathrm{E}} r_{0}}{6 \varepsilon_{0}}
$$

Since the electric field points into the gaussian surface (negative) the electric field at point A points to the right.
(b) At point B the electric field will be the sum of the electric fields from each sphere. The electric field from sphere 1 is calculated using a gaussian surface of radius $r_{0}$ centered at A.

$$
\oint \overrightarrow{\mathbf{E}}_{1} \cdot d \overrightarrow{\mathbf{A}}=\frac{q_{\mathrm{enc}}}{\varepsilon_{0}} \rightarrow E_{1} \cdot 4 \pi r_{0}^{2}=\frac{\frac{4}{3} \pi r_{0}^{3}\left(\rho_{\mathrm{E}}\right)}{\varepsilon_{0}} \rightarrow E_{1}=\frac{\rho_{\mathrm{E}} r_{0}}{3 \varepsilon_{0}}
$$

At point B the field from sphere 1 points toward the left. The electric field from sphere 2 is calculated using a gaussian surface centered at C of radius $3 r_{0} / 2$.

$$
\oint \overrightarrow{\mathbf{E}}_{2} \cdot d \overrightarrow{\mathbf{A}}=\frac{q_{\mathrm{enc}}}{\varepsilon_{0}} \rightarrow E_{2} \cdot 4 \pi\left(\frac{3}{2} r_{0}\right)^{2}=\frac{\left(-\rho_{\mathrm{E}}\right) \frac{4}{3} \pi\left(\frac{1}{2} r_{0}\right)^{3}}{\varepsilon_{0}} \rightarrow E_{2}=-\frac{\rho_{\mathrm{E}} r_{0}}{54 \varepsilon_{0}}
$$

At point B, the electric field from sphere 2 points toward the right. The net electric field is the sum of these two fields. The net field points to the left.

$$
E=E_{1}+E_{2}=\frac{\rho_{\mathrm{E}} r_{0}}{3 \varepsilon_{0}}+\frac{-\rho_{\mathrm{E}} r_{0}}{54 \varepsilon_{0}}=\frac{17 \rho_{\mathrm{E}} r_{0}}{54 \varepsilon_{0}} .
$$

62. We assume the charge is uniformly distributed, and so the field of the pea is that of a point charge.

$$
\begin{aligned}
& E(r=R)=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{R^{2}} \rightarrow \\
& Q=E 4 \pi \varepsilon_{0} R^{2}=\left(3 \times 10^{6} \mathrm{~N} / \mathrm{C}\right) 4 \pi\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)(0.00375 \mathrm{~m})^{2}=5 \times 10^{-9} \mathrm{C}
\end{aligned}
$$

63. (a) In an electrostatic situation, there is no electric field inside a conductor. Thus $E=0$ inside the conductor.
(b) The positive sheet produces an electric field, external to itself, directed away from the plate with a magnitude as given in Example 22-7, of $E_{1}=\frac{\left|\sigma_{1}\right|}{2 \varepsilon_{0}}$. The negative sheet produces an electric field, external to itself, directed towards the plate with a magnitude of $E_{2}=\frac{\left|\sigma_{2}\right|}{2 \varepsilon_{0}}$. Between the left
 and middle sheets, those two fields are parallel and so add to each other.

$$
E_{\substack{\text { left } \\ \text { middle }}}=E_{1}+E_{2}=\frac{\left|\sigma_{1}\right|+\left|\sigma_{2}\right|}{2 \varepsilon_{0}}=\frac{2\left(5.00 \times 10^{-6} \mathrm{C} / \mathrm{m}^{2}\right)}{2\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)}=5.65 \times 10^{5} \mathrm{~N} / \mathrm{C} \text {, to the right }
$$

(c) The same field is between the middle and right sheets. See the diagram.

$$
E_{\substack{\text { middle } \\ \text { right }}}=5.65 \times 10^{5} \mathrm{~N} / \mathrm{C} \text {, to the right }
$$

(d) To find the charge density on the surface of the left side of the middle sheet, choose a gaussian cylinder with ends of area $A$. Let one end be inside the conducting sheet, where there is no electric field, and the other end be in the area between the left and middle sheets. Apply Gauss's law in the manner of Example 22-16. Note that there is no flux through the curved sides of the cylinder, and there is no flux through the right end since it is in conducting material. Also note that the field through the left end is in the opposite direction as the area vector of the left end.

$$
\begin{aligned}
& \oint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=\int_{\substack{\text { left } \\
\text { end }}} \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}+\int_{\substack{\text { right } \\
\text { end }}} \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}+\int_{\substack{\text { side }}} \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=-E_{\substack{\text { left } \\
\text { middle }}} A+0+0=\frac{Q_{\text {encl }}}{\varepsilon_{0}}=\frac{\sigma_{\text {lef }} A}{\varepsilon_{0}} \rightarrow \\
& \sigma_{\text {left }}=-\varepsilon_{0} E_{\substack{\text { left } \\
\text { middle }}}=-\varepsilon_{0}\left(\frac{\left|\sigma_{1}\right|+\left|\sigma_{2}\right|}{2 \varepsilon_{0}}\right)=-5.00 \times 10^{-6} \mathrm{C} / \mathrm{m}^{2}
\end{aligned}
$$

(e) Because the middle conducting sheet has no net charge, the charge density on the right side must be the opposite of the charge density on the left side.

$$
\sigma_{\text {right }}=-\sigma_{\text {left }}=5.00 \times 10^{-6} \mathrm{C} / \mathrm{m}^{2}
$$

Alternatively, we could have applied Gauss's law on the right side in the same manner that we did on the left side. The same answer would result.
64. Because the electric field has only $x$ and $y$ components, there will be no flux through the top or bottom surfaces. For the other faces, we choose a horizontal strip of height $d z$ and width $a$ for a differential element and integrate to find the flux. The total flux is used to determine the enclosed charge.

$$
\begin{aligned}
\Phi_{\substack{\text { font } \\
(x=a)}} & =\int_{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=\int_{0}^{a}\left[E_{0}\left(1+\frac{z}{a}\right) \hat{\mathbf{i}}+E_{0}\left(\frac{z}{a}\right) \hat{\mathbf{j}}\right] \cdot(a d z \hat{\mathbf{i}}) \\
& =E_{0} a \int_{0}^{a}\left(1+\frac{z}{a}\right) d z=E_{0} a\left(z+\frac{z^{2}}{2 a}\right)_{0}^{a}=\frac{3}{2} E_{0} a^{2} \\
\Phi_{\substack{\text { back } \\
(x=0)}} & =\int_{0}^{a}\left[E_{0}\left(1+\frac{z}{a}\right) \hat{\mathbf{i}}+E_{0}\left(\frac{z}{a}\right) \hat{\mathbf{j}}\right] \cdot(-a d \hat{\mathbf{i}})=-\frac{3}{2} E_{0} a^{2} \\
\Phi_{\substack{\text { right } \\
(y=a)}} & =\int_{0}^{a}\left[E_{0}\left(1+\frac{z}{a}\right) \hat{\mathbf{i}}+E_{0}\left(\frac{z}{a}\right) \hat{\mathbf{j}}\right] \cdot(a d \hat{z} \hat{\mathbf{j}})=E_{0} a \int_{0}^{a}\left(\frac{z}{a}\right) d z=E_{0} a\left(\frac{z^{2}}{2 a}\right)_{0}^{a}=\frac{1}{2} E_{0} a^{2} \\
\Phi_{\substack{\text { leti } \\
(y=a)}} & =\int_{0}^{a}\left[E_{0}\left(1+\frac{z}{a}\right) \hat{\mathbf{i}}+E_{0}\left(\frac{z}{a}\right) \hat{\mathbf{j}}\right] \cdot(-a d z \hat{\mathbf{j}})=-\frac{1}{2} E_{0} a^{2} \\
\Phi_{\text {total }} & =\Phi_{\text {froont }}+\Phi_{\text {back }}+\Phi_{\text {right }}+\Phi_{\text {left }}+\Phi_{\text {top }}+\Phi_{\text {botom }}=\frac{3}{2} E_{0} a^{2}-\frac{3}{2} E_{0} a^{2}+\frac{1}{2} E_{0} a^{2}-\frac{1}{2} E_{0} a^{2}=0+0 \\
& =0=\frac{Q_{\text {encl }}}{\varepsilon_{0}} \rightarrow Q_{\text {encl }}=0
\end{aligned}
$$

65. (a) Because the shell is a conductor, there is no electric field in the conducting material, and all charge must reside on its surfaces. All of the field lines that originate from the point charge at the center must terminate on the inner surface of the shell. Therefore the inner surface must have an equal but opposite charge to the point charge at the center. Since the conductor has the same magnitude of charge as the point charge at the center, all of the charge on the conductor is on the inner surface of the shell, in a spherically symmetric distribution.
(b) By Gauss's law and the spherical symmetry of the problem, the electric field can be calculated by $E=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q_{\text {encl }}}{r^{2}}$.

$$
r<0.10 \mathrm{~m}: E=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q_{\text {encl }}}{r^{2}}=\frac{\left(8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)\left(3.0 \times 10^{-6} \mathrm{C}\right)}{r^{2}}=\frac{2.7 \times 10^{4} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}}{r^{2}}
$$

$$
r>0.15 \mathrm{~m}: E=0
$$

And since there is no electric field in the shell, we could express the second answer as $r>0.10 \mathrm{~m}: E=0$.
66. (a) At a strip such as is marked in the textbook diagram, $d \overrightarrow{\mathbf{A}}$ is perpendicular to the surface, and $\overrightarrow{\mathbf{E}}$ is inclined at an angle $\theta$ relative to $d \overrightarrow{\mathbf{A}}$.

$$
\begin{aligned}
\Phi_{\text {hemisphere }} & =\int \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=\int_{0}^{\pi / 2} E \cos \theta\left(2 \pi R^{2} \sin \theta d \theta\right) \\
& =2 \pi R^{2} E \int_{0}^{\pi / 2} \cos \theta \sin \theta d \theta=2 \pi R^{2} E\left(\frac{1}{2} \sin ^{2} \theta\right)_{0}^{\pi / 2}=\pi R^{2} E
\end{aligned}
$$

(b) Choose a closed gaussian surface consisting of the hemisphere and the circle of radius $R$ at the base of the hemisphere. There is no charge inside that closed gaussian surface, and so the total flux through the two surfaces (hemisphere and base) must be zero. The field lines are all perpendicular to the circle, and all of the same magnitude, and so that flux is very easy to calculate.

$$
\begin{aligned}
& \Phi_{\text {circle }}=\int \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=\int E\left(\cos 180^{\circ}\right) d \overrightarrow{\mathbf{A}}=-E A=-E \pi R^{2} \\
& \Phi_{\text {total }}=0=\Phi_{\text {circle }}+\Phi_{\text {hemisphere }}=-E \pi R^{2}+\Phi_{\text {hemisphere }} \rightarrow \Phi_{\text {hemisphere }}=\pi R^{2} E
\end{aligned}
$$

67. The flux is the sum of six integrals, each of the form $\iint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}$. Because the electric field has only $x$ and $y$ components, there will be no flux through the top or bottom surfaces. For the other faces, we choose a vertical strip of height $a$ and width $d y$ (for the front and back faces) or $d x$ (for the left and right faces). See the diagram for an illustration of a strip on the front face. The total flux is then calculated, and used to determine
 the enclosed charge.

$$
\Phi_{\substack{\text { ropm } \\(x=a)}}=\int_{0}^{a}\left(E_{x 0} e^{-\left(-\frac{x+y}{a}\right)^{2}} \hat{\mathbf{i}}+E_{y 0} e^{-\left(\frac{x+y}{a}\right)} \hat{\mathbf{j}}\right) \cdot a d y \hat{\mathbf{i}}=a E_{x 0} \int_{0}^{a-\left(\frac{a+y}{a}\right)^{2}} d y
$$

This integral does not have an analytic anti-derivative, and so must be integrated numerically. We ar imate the integral by a sum: $\int_{0}^{a} e^{-\left(\frac{a+y}{a}\right)^{2}} d y \approx \sum_{i=1}^{n} e^{-\left(\frac{a+y_{i}}{a}\right)^{2}} \Delta y$. The region of integration is divided
into $n$ elements, and so $\Delta y=\frac{a-0}{n}$ and $y_{i}=i \Delta y$. We initially evaluate the sum for $n=10$. Then we evaluate it for $n=20$. If the two sums differ by no more than $2 \%$, we take that as the value of the integral. If they differ by more than $2 \%$, we choose a larger $n$, compute the sum, and compare that to the result for $n=20$. We continue until a difference of $2 \%$ or less is reached. This integral, for $n=$ 100 and $a=1.0 \mathrm{~m}$, is 0.1335 m . So we have this intermediate result.

$$
\Phi_{\substack{\text { front } \\(x=a)}}=a E_{x 0} \sum_{i=1}^{n} e^{-\left(\frac{a+y_{i}}{a}\right)^{2}} \Delta y=(1.0 \mathrm{~m})(50 \mathrm{~N} / \mathrm{C})(0.1335 \mathrm{~m})=6.675 \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}
$$

Now do the integral over the back face.

$$
\Phi_{\substack{\text { back } \\(x=0)}}=\int_{0}^{a}\left(E_{x 0} e^{-\left(\frac{x+y}{a}\right)^{2}} \hat{\mathbf{i}}+E_{y 0} e^{-\left(\frac{x+y}{a}\right)^{2}} \hat{\mathbf{j}}\right) \cdot(-a d y \hat{\mathbf{i}})=-a E_{x 0} \int_{0}^{a} e^{-\left(\frac{y}{a}\right)^{2}} d y
$$

We again get an integral that cannot be evaluated analytically. A similar process to that used for the front face is applied again, and so we make this approximation: $-a E_{x 0} \int_{0}^{a} e^{-\left(\frac{y}{a}\right)^{2}} d y \approx-a E_{x 0} \sum_{i=1}^{n} e^{-\left(\frac{v_{y}}{a}\right)^{2}} \Delta y$. The numeric integration gives a value of 0.7405 m .

$$
\Phi_{\substack{\text { back } \\(x=0)}}=-a E_{x 0} \sum_{i=1}^{n} e^{-\left(\frac{v_{i}}{a}\right)^{2}} \Delta y=-(1.0 \mathrm{~m})(50 \mathrm{~N} / \mathrm{C})(0.7405 \mathrm{~m})=-37.025 \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C} .
$$

Now consider the right side.

$$
\Phi_{\substack{\text { right } \\(y=a)}}=\int_{0}^{a}\left(E_{x 0} e^{-\left(\frac{x+y}{a}\right)^{2}} \hat{\mathbf{i}}+E_{y 0} e^{-\left(\frac{x+y}{a}\right)^{2}} \hat{\mathbf{j}}\right) \cdot a d x \hat{\mathbf{j}}=a E_{y 0} \int_{0}^{a} e^{-\left(\frac{x+a}{a}\right)^{2}} d x
$$

Notice that the same integral needs to be evaluated as for the front side. All that has changed is the variable name. Thus we have the following.

$$
\Phi_{\substack{\text { right } \\(y=a)}}=a E_{y 0} \int_{0}^{a} e^{-\left(\frac{x+a}{a}\right)^{2}} d x \approx(1.0 \mathrm{~m})(25 \mathrm{~N} / \mathrm{C})(0.1335 \mathrm{~m})=3.3375 \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}
$$

Finally, do the left side, following the same process. The same integral arises as for the back face.

$$
\left.\left.\begin{array}{rl}
\Phi_{\substack{\text { left } \\
(y=0)}} & =\int_{0}^{a}\left(E_{x 0} e^{-\left(\frac{x+y}{a}\right)^{2}}\right. \\
\mathbf{i}
\end{array}+E_{y 0} e^{-\left(\frac{x+y}{a}\right)^{2}} \hat{\mathbf{j}}\right) \cdot(-a d x \hat{\mathbf{j}})=-a E_{y 0} \int_{0}^{a} e^{-\left(\frac{x}{a}\right)^{2}} d x\right] \text { } \quad \approx-(1.0 \mathrm{~m})(25 \mathrm{~N} / \mathrm{C})(0.7405 \mathrm{~m})=-18.5125 \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}
$$

Sum to find the total flux, and multiply by $\varepsilon_{0}$ to find the enclosed charge.

$$
\begin{aligned}
\Phi_{\text {total }} & =\Phi_{\text {front }}+\Phi_{\text {back }}+\Phi_{\text {right }}+\Phi_{\text {left }}+\Phi_{\text {top }}+\Phi_{\text {botom }} \\
& =(6.675-37.025+3.3375-18.5125) \mathrm{N} \cdot \mathrm{~m}^{2} / \mathrm{C}=-45.525 \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C} \approx-46 \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C} \\
Q_{\text {encl }} & =\varepsilon_{0} \Phi_{\text {total }}=\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)\left(-45.525 \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}\right)=-4.0 \times 10^{-10} \mathrm{C}
\end{aligned}
$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH22.XLS," on tab "Problem 22.67."

## CHAPTER 23: Electric Potential

## Responses to Questions

Not necessarily. If two points are at the same potential, then no net work is done in moving a charge from one point to the other, but work (both positive and negative) could be done at different parts of the path. No. It is possible that positive work was done over one part of the path, and negative work done over another part of the path, so that these two contributions to the net work sum to zero. In this case, a non-zero force would have to be exerted over both parts of the path.
2. The negative charge will move toward a region of higher potential and the positive charge will move toward a region of lower potential. In both cases, the potential energy of the charge will decrease.
3. (a) The electric potential is the electric potential energy per unit charge. The electric potential is a scalar. The electric field is the electric force per unit charge, and is a vector.
(b) Electric potential is the electric potential energy per unit charge.
4. Assuming the electron starts from rest in both cases, the final speed will be twice as great. If the electron is accelerated through a potential difference that is four times as great, then its increase in kinetic energy will also be greater by a factor of four. Kinetic energy is proportional to the square of the speed, so the final speed will be greater by a factor of two.
5. Yes. If the charge on the particle is negative and it moves from a region of low electric potential to a region of high electric potential, its electric potential energy will decrease.
6. No. Electric potential is the potential energy per unit charge at a point in space and electric field is the electric force per unit charge at a point in space. If one of these quantities is zero, the other is not necessarily zero. For example, the point exactly between two charges with equal magnitudes and opposite signs will have a zero electric potential because the contributions from the two charges will be equal in magnitude and opposite in sign. (Net electric potential is a scalar sum.) This point will not have a zero electric field, however, because the electric field contributions will be in the same direction (towards the negative and away from the positive) and so will add. (Net electric field is a vector sum.) As another example, consider the point exactly between two equal positive point charges. The electric potential will be positive since it is the sum of two positive numbers, but the electric field will be zero since the field contributions from the two charges will be equal in magnitude but opposite in direction.
(a) $V$ at other points would be lower by 10 V . $E$ would be unaffected, since $E$ is the negative gradient of $V$, and a change in $V$ by a constant value will not change the value of the gradient.
(b) If $V$ represents an absolute potential, then yes, the fact that the Earth carries a net charge would affect the value of $V$ at the surface. If $V$ represents a potential difference, then no, the net charge on the Earth would not affect the choice of $V$.
8. No. An equipotential line is a line connecting points of equal electric potential. If two equipotential lines crossed, it would indicate that their intersection point has two different values of electric potential simultaneously, which is impossible. As an analogy, imagine contour lines on a topographic map. They also never cross because one point on the surface of the Earth cannot have two different values for elevation above sea level.
9. The equipotential lines (in black) are perpendicular to the electric field lines (in red).

(b)

10. The electric field is zero in a region of space where the electric potential is constant. The electric field is the gradient of the potential; if the potential is constant, the gradient is zero.
11. The Earth's gravitational equipotential lines are roughly circular, so the orbit of the satellite would have to be roughly circular.
12. The potential at point P would be unchanged. Each bit of positive charge will contribute an amount to the potential based on its charge and its distance from point P . Moving charges to different locations on the ring does not change their distance from P , and hence does not change their contributions to the potential at P .

The value of the electric field will change. The electric field is the vector sum of all the contributions to the field from the individual charges. When the charge $Q$ is distributed uniformly about the ring, the $y$-components of the field contributions cancel, leaving a net field in the $x$-direction. When the charge is not distributed uniformly, the $y$-components will not cancel, and the net field will have both $x$ - and $y$-components, and will be larger than for the case of the uniform charge distribution. There is no discrepancy here, because electric potential is a scalar and electric field is a vector.
13. The charge density and the electric field strength will be greatest at the pointed ends of the football because the surface there has a smaller radius of curvature than the middle.
14. No. You cannot calculate electric potential knowing only electric field at a point and you cannot calculate electric field knowing only electric potential at a point. As an example, consider the uniform field between two charged, conducting plates. If the potential difference between the plates is known, then the distance between the plates must also be known in order to calculate the field. If the field between the plates is known, then the distance to a point of interest between the plates must also be known in order to calculate the potential there. In general, to find $V$, you must know E and be able to integrate it. To find E, you must know $V$ and be able to take its derivative. Thus you need E or $V$ in the region around the point, not just at the point, in order to be able to find the other variable.
15. (a) Once the two spheres are placed in contact with each other, they effectively become one larger conductor. They will have the same potential because the potential everywhere on a conducting surface is constant.
(b) Because the spheres are identical in size, an amount of charge $Q / 2$ will flow from the initially charged sphere to the initially neutral sphere so that they will have equal charges.
(c) Even if the spheres do not have the same radius, they will still be at the same potential once they are brought into contact because they still create one larger conductor. However, the amount of charge that flows will not be exactly equal to half the total charge. The larger sphere will end up with the larger charge.
16. If the electric field points due north, the change in the potential will be $(a)$ greatest in the direction opposite the field, south; (b) least in the direction of the field, north; and (c) zero in a direction perpendicular to the field, east and west.
17. Yes. In regions of space where the equipotential lines are closely spaced, the electric field is stronger than in regions of space where the equipotential lines are farther apart.
18. If the electric field in a region of space is uniform, then you can infer that the electric potential is increasing or decreasing uniformly in that region. For example, if the electric field is $10 \mathrm{~V} / \mathrm{m}$ in a region of space then you can infer that the potential difference between two points 1 meter apart (measured parallel to the direction of the field) is 10 V . If the electric potential in a region of space is uniform, then you can infer that the electric field there is zero.
19. The electric potential energy of two unlike charges is negative. The electric potential energy of two like charges is positive. In the case of unlike charges, work must be done to separate the charges. In the case of like charges, work must be done to move the charges together.

## Solutions to Problems

1. Energy is conserved, so the change in potential energy is the opposite of the change in kinetic energy. The change in potential energy is related to the change in potential.

$$
\begin{aligned}
& \Delta U=q \Delta V=-\Delta K \rightarrow \\
& \Delta V=\frac{-\Delta K}{q}=\frac{K_{\text {intial }}-K_{\text {final }}}{q}=\frac{m v^{2}}{2 q}=\frac{\left(9.11 \times 10^{-31} \mathrm{~kg}\right)\left(5.0 \times 10^{5} \mathrm{~m} / \mathrm{s}\right)^{2}}{2\left(-1.60 \times 10^{-19} \mathrm{C}\right)}=-0.71 \mathrm{~V}
\end{aligned}
$$

The final potential should be lower than the initial potential in order to stop the electron.
2. The work done by the electric field can be found from Eq. 23-2b.

$$
V_{\mathrm{ba}}=-\frac{W_{\mathrm{ba}}}{q} \rightarrow W_{\mathrm{ba}}=-q V_{\mathrm{ba}}=-\left(1.60 \times 10^{-19} \mathrm{C}\right)[-55 \mathrm{~V}-185 \mathrm{~V}]=3.84 \times 10^{-17} \mathrm{~J}
$$

3. The kinetic energy gained by the electron is the work done by the electric force. Use Eq. 23-2b to calculate the potential difference.

$$
V_{\mathrm{ba}}=-\frac{W_{\mathrm{ba}}}{q}=-\frac{5.25 \times 10^{-16} \mathrm{~J}}{\left(-1.60 \times 10^{-19} \mathrm{C}\right)}=3280 \mathrm{~V}
$$

The electron moves from low potential to high potential, so plate $B$ is at the higher potential.
4. By the work energy theorem, the total work done, by the external force and the electric field together, is the change in kinetic energy. The work done by the electric field is given by Eq. 23-2b.

$$
\begin{aligned}
& W_{\text {extemal }}+W_{\text {electric }}=\mathrm{KE}_{\text {final }}-\mathrm{KE}_{\text {initial }} \rightarrow W_{\text {extemal }}-q\left(V_{\mathrm{b}}-V_{\mathrm{a}}\right)=\mathrm{KE}_{\text {final }} \rightarrow \\
& \left(V_{\mathrm{b}}-V_{\mathrm{a}}\right)=\frac{W_{\text {extemal }}-\mathrm{KE}_{\text {final }}}{q}=\frac{7.00 \times 10^{-4} \mathrm{~J}-2.10 \times 10^{-4} \mathrm{~J}}{-9.10 \times 10^{-6} \mathrm{C}}=-53.8 \mathrm{~V}
\end{aligned}
$$

Since the potential difference is negative, we see that $V_{\mathrm{a}}>V_{\mathrm{b}}$.
5. As an estimate, the length of the bolt would be the voltage difference of the bolt divided by the breakdown electric field of air.

$$
\frac{1 \times 10^{8} \mathrm{~V}}{3 \times 10^{6} \mathrm{~V} / \mathrm{m}}=33 \mathrm{~m} \approx 30 \mathrm{~m}
$$

6. The distance between the plates is found from Eq. 23-4b, using the magnitude of the electric field.

$$
|E|=\frac{V_{\mathrm{ba}}}{d} \rightarrow d=\frac{V_{\mathrm{ba}}}{|E|}=\frac{45 \mathrm{~V}}{1300 \mathrm{~V} / \mathrm{m}}=3.5 \times 10^{-2} \mathrm{~m}
$$

7. The maximum charge will produce an electric field that causes breakdown in the air. We use the same approach as in Examples 23-4 and 23-5.

$$
\begin{aligned}
& V_{\text {surface }}=r_{0} E_{\text {breakkown }} \text { and } V_{\text {surface }}=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{r_{0}} \rightarrow \\
& Q=4 \pi \varepsilon_{0} r_{0}^{2} E_{\text {breakdown }}=\left(\frac{1}{8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}}\right)(0.065 \mathrm{~m})^{2}\left(3 \times 10^{6} \mathrm{~V} / \mathrm{m}\right)=1.4 \times 10^{-6} \mathrm{C}
\end{aligned}
$$

8. We assume that the electric field is uniform, and so use Eq. 23-4b, using the magnitude of the electric field.

$$
E=\frac{V_{\mathrm{ba}}}{d}=\frac{110 \mathrm{~V}}{4.0 \times 10^{-3} \mathrm{~m}}=2.8 \times 10^{4} \mathrm{~V} / \mathrm{m}
$$

9. To find the limiting value, we assume that the E-field at the radius of the sphere is the minimum value that will produce breakdown in air. We use the same approach as in Examples 23-4 and 23-5.

$$
\begin{aligned}
V_{\text {surface }}=r_{0} E_{\text {breakdown }} \rightarrow r_{0} & =\frac{V_{\text {surface }}}{E_{\text {breakdown }}}=\frac{35,000 \mathrm{~V}}{3 \times 10^{6} \mathrm{~V} / \mathrm{m}}=0.0117 \mathrm{~m} \approx 0.012 \mathrm{~m} \\
V_{\text {surface }}=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{r_{0}} \rightarrow Q & =4 \pi \varepsilon_{0} V_{\text {surface }} r_{0}=\left(\frac{1}{8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}}\right)(35,000 \mathrm{~V})(0.0117 \mathrm{~m}) \\
& =4.6 \times 10^{-8} \mathrm{C}
\end{aligned}
$$

10. If we assume the electric field is uniform, then we can use Eq. $23-4 \mathrm{~b}$ to estimate the magnitude of the electric field. From Problem 22-24 we have an expression for the electric field due to a pair of oppositely charged planes. We approximate the area of a shoe as $30 \mathrm{~cm} \times 8 \mathrm{~cm}$.

$$
\begin{aligned}
& E=\frac{V}{d}=\frac{\sigma}{\varepsilon_{0}}=\frac{Q}{\varepsilon_{0} A} \rightarrow \\
& Q=\frac{\varepsilon_{0} A V}{d}=\frac{\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{Nm}^{2}\right)\left(0.024 \mathrm{~m}^{2}\right)\left(5.0 \times 10^{3} \mathrm{~V}\right)}{1.0 \times 10^{-3} \mathrm{~m}}=1.1 \times 10^{-6} \mathrm{C}
\end{aligned}
$$

11. Since the field is uniform, we may apply Eq. 23-4b. Note that the electric field always points from high potential to low potential.
(a) $V_{\mathrm{BA}}=0$. The distance between the two points is exactly perpendicular to the field lines.
(b) $V_{\mathrm{CB}}=V_{\mathrm{C}}-V_{\mathrm{B}}=(-4.20 \mathrm{~N} / \mathrm{C})(7.00 \mathrm{~m})=-29.4 \mathrm{~V}$
(c) $V_{\mathrm{CA}}=V_{\mathrm{C}}-V_{\mathrm{A}}=V_{\mathrm{C}}-V_{\mathrm{B}}+V_{\mathrm{B}}-V_{\mathrm{A}}=V_{\mathrm{CB}}+V_{\mathrm{BA}}=-29.4 \mathrm{~V}+0=-29.4 \mathrm{~V}$
12. From Example 22-7, the electric field produced by a large plate is uniform with magnitude $E=\frac{\sigma}{2 \varepsilon_{0}}$. The field points away from the plate, assuming that the charge is positive. Apply Eq. 23-41.

$$
V(x)-V(0)=V(x)-V_{0}=-\int_{0}^{x} \overrightarrow{\mathbf{E}}^{\bullet} \cdot(d \overrightarrow{\boldsymbol{\ell}})=-\int_{0}^{x}\left(\frac{\sigma}{2 \varepsilon_{0}} \hat{\mathbf{i}}\right) \cdot(d x \hat{\mathbf{i}})=-\frac{\sigma x}{2 \varepsilon_{0}} \rightarrow V(x)=V_{0}-\frac{\sigma x}{2 \varepsilon_{0}}
$$

13. (a) The electric field at the surface of the Earth is the same as that of a point charge, $E=\frac{Q}{4 \pi \varepsilon_{0} r_{0}^{2}}$. The electric potential at the surface, relative to $V(\infty)=0$ is given by Eq. 23-5. Writing this in terms of the electric field and radius of the earth gives the electric potential.

$$
V=\frac{Q}{4 \pi \varepsilon_{0} r_{0}}=E r_{0}=(-150 \mathrm{~V} / \mathrm{m})\left(6.38 \times 10^{6} \mathrm{~m}\right)=-0.96 \mathrm{GV}
$$

(b) Part (a) demonstrated that the potential at the surface of the earth is 0.96 GV lower than the potential at infinity. Therefore if the potential at the surface of the Earth is taken to be zero, the potential at infinity must be $V(\infty)=0.96 \mathrm{GV}$. If the charge of the ionosphere is included in the calculation, the electric field outside the ionosphere is basically zero. The electric field between the earth and the ionosphere would remain the same. The electric potential, which would be the integral of the electric field from infinity to the surface of the earth, would reduce to the integral of the electric field from the ionosphere to the earth. This would result in a negative potential, but of a smaller magnitude.
14. (a) The potential at the surface of a charged sphere is derived in Example 23-4.

$$
\begin{aligned}
V_{0} & =\frac{Q}{4 \pi \varepsilon_{0} r_{0}} \rightarrow Q=4 \pi \varepsilon_{0} r_{0} V_{0} \rightarrow \\
\sigma & =\frac{Q}{\text { Area }}=\frac{Q}{4 \pi r_{0}^{2}}=\frac{4 \pi \varepsilon_{0} r_{0} V_{0}}{4 \pi r_{0}^{2}}=\frac{V_{0} \varepsilon_{0}}{r_{0}}=\frac{(680 \mathrm{~V})\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{Nm}^{2}\right)}{(0.16 \mathrm{~m})}=3.761 \times 10^{-8} \mathrm{C} / \mathrm{m}^{2} \\
& \approx 3.8 \times 10^{-8} \mathrm{C} / \mathrm{m}^{2}
\end{aligned}
$$

(b) The potential away from the surface of a charged sphere is also derived in Example 23-4.

$$
V=\frac{Q}{4 \pi \varepsilon_{0} r}=\frac{4 \pi \varepsilon_{0} r_{0} V_{0}}{4 \pi \varepsilon_{0} r}=\frac{r_{0} V_{0}}{r} \rightarrow r=\frac{r_{0} V_{0}}{V}=\frac{(0.16 \mathrm{~m})(680 \mathrm{~V})}{(25 \mathrm{~V})}=4.352 \mathrm{~m} \approx 4.4 \mathrm{~m}
$$

15. (a) After the connection, the two spheres are at the same potential. If they were at different potentials, then there would be a flow of charge in the wire until the potentials were equalized.
(b) We assume the spheres are so far apart that the charge on one sphere does not influence the charge on the other sphere. Another way to express this would be to say that the potential due to either of the spheres is zero at the location of the other sphere. The charge splits between the spheres so that their potentials (due to the charge on them only) are equal. The initial charge on sphere 1 is $Q$, and the final charge on sphere 1 is $Q_{1}$.

$$
\begin{aligned}
& V_{1}=\frac{Q_{1}}{4 \pi \varepsilon_{0} r_{1}} ; V_{2}=\frac{Q-Q_{1}}{4 \pi \varepsilon_{0} r_{2}} ; V_{1}=V_{2} \rightarrow \frac{Q_{1}}{4 \pi \varepsilon_{0} r_{1}}=\frac{Q-Q_{1}}{4 \pi \varepsilon_{0} r_{2}} \rightarrow Q_{1}=Q \frac{r_{1}}{\left(r_{1}+r_{2}\right)} \\
& \text { Charge transferred } Q-Q_{1}=Q-Q \frac{r_{1}}{\left(r_{1}+r_{2}\right)}=Q \frac{r_{2}}{\left(r_{1}+r_{2}\right)}
\end{aligned}
$$

16. From Example 22-6, the electric field due to a long wire is radial relative to the wire, and is of magnitude $E=\frac{1}{2 \pi \varepsilon_{0}} \frac{\lambda}{R}$. If the charge density is positive, the field lines point radially away from the wire. Use Eq. 23-41 to find the potential difference, integrating along a line that is radially outward from the wire.

$$
V_{\mathrm{b}}-V_{\mathrm{a}}=-\int_{R_{\mathrm{a}}}^{R_{\mathrm{a}}} \overrightarrow{\mathbf{E}} \cdot(d \overrightarrow{\boldsymbol{\ell}})=-\int_{R_{\mathrm{a}}}^{R_{\mathrm{a}}} \frac{1}{2 \pi \varepsilon_{0}} \frac{\lambda}{R} d R=-\frac{\lambda}{2 \pi \varepsilon_{0}} \ln \left(R_{\mathrm{b}}-R_{\mathrm{a}}\right)=\frac{\lambda}{2 \pi \varepsilon_{0}} \ln \frac{R_{\mathrm{a}}}{\mathrm{R}_{\mathrm{b}}}
$$

17. (a) The width of the end of a finger is about 1 cm , and so consider the fingertip to be a part of a sphere of diameter 1 cm . We assume that the electric field at the radius of the sphere is the minimum value that will produce breakdown in air. We use the same approach as in Examples 23-4 and 23-5.

$$
V_{\text {surface }}=r_{0} E_{\text {breakdown }}=(0.005 \mathrm{~m})\left(3 \times 10^{6} \mathrm{~V} / \mathrm{m}\right)=15,000 \mathrm{~V}
$$

Since this is just an estimate, we might expect anywhere from $10,000 \mathrm{~V}$ to $20,000 \mathrm{~V}$.
(b)

$$
\begin{aligned}
& V_{\text {surface }}=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{r_{0}}=\frac{1}{4 \pi \varepsilon_{0}} \frac{4 \pi r_{0}^{2} \sigma}{r_{0}} \rightarrow \\
& \sigma=V_{\text {surface }} \frac{\varepsilon_{0}}{r_{0}}=(15,000 \mathrm{~V}) \frac{\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)}{0.005 \mathrm{~m}}=2.7 \times 10^{-5} \mathrm{C} / \mathrm{m}^{2}
\end{aligned}
$$

Since this is an estimate, we might say the charge density is on the order of $30 \mu \mathrm{C} / \mathrm{m}^{2}$.
18. We assume the field is uniform, and so Eq. 23-4b applies.

$$
E=\frac{V}{d}=\frac{0.10 \mathrm{~V}}{10 \times 10^{-9} \mathrm{~m}}=1 \times 10^{7} \mathrm{~V} / \mathrm{m}
$$

19. (a) The electric field outside a charged, spherically symmetric volume is the same as that for a point charge of the same magnitude of charge. Integrating the electric field from infinity to the radius of interest will give the potential at that radius.

$$
E\left(r \geq r_{0}\right)=\frac{Q}{4 \pi \varepsilon_{0} r^{2}} ; V\left(r \geq r_{0}\right)=-\int_{\infty}^{r} \frac{Q}{4 \pi \varepsilon_{0} r^{2}} d r=\left.\frac{Q}{4 \pi \varepsilon_{0} r}\right|_{\infty} ^{r}=\frac{Q}{4 \pi \varepsilon_{0} r}
$$

(b) Inside the sphere the electric field is obtained from Gauss's Law using the charge enclosed by a sphere of radius $r$.

$$
4 \pi r^{2} E=\frac{Q}{\varepsilon_{0}} \frac{\frac{4}{3} \pi r^{3}}{\frac{4}{3} \pi r_{0}^{3}} \rightarrow E\left(r<r_{0}\right)=\frac{Q r}{4 \pi \varepsilon_{0} r_{0}^{3}}
$$

Integrating the electric field from the surface to $r<r_{0}$ gives the electric potential inside the sphere.

$$
V\left(r<r_{0}\right)=V\left(r_{0}\right)-\int_{r_{0}}^{r} \frac{Q r}{4 \pi \varepsilon_{0} r_{0}^{3}} d r=\frac{Q}{4 \pi \varepsilon_{0} r_{0}}-\left.\frac{Q r^{2}}{8 \pi \varepsilon_{0} r_{0}^{3}}\right|_{r_{0}} ^{r}=\frac{Q}{8 \pi \varepsilon_{0} r_{0}}\left(3-\frac{r^{2}}{r_{0}^{2}}\right)
$$

(c) To plot, we first calculate $V_{0}=V\left(r=r_{0}\right)=\frac{Q}{4 \pi \varepsilon_{0} r_{0}}$ and $E_{0}=E\left(r=r_{0}\right)=\frac{Q}{4 \pi \varepsilon_{0} r_{0}^{2}}$. Then we plot $V / V_{0}$ and $E / E_{0}$ as functions of $r / r_{0}$.

For $r<r_{0}: \quad V / V_{0}=\frac{\frac{Q}{8 \pi \varepsilon_{0} r_{0}}\left(3-\frac{r^{2}}{r_{0}^{2}}\right)}{\frac{Q}{4 \pi \varepsilon_{0} r_{0}}}=\frac{1}{2}\left(3-\frac{r^{2}}{r_{0}^{2}}\right) ; E / E_{0}=\frac{\frac{Q r}{4 \pi \varepsilon_{0} r_{0}^{3}}}{\frac{Q}{4 \pi \varepsilon_{0} r_{0}^{2}}}=\frac{r}{r_{0}}$
For $r>r_{0}: \quad V / V_{0}=\frac{\frac{Q}{4 \pi \varepsilon_{0} r}}{\frac{Q}{4 \pi \varepsilon_{0} r_{0}}}=\frac{r_{0}}{r}=\left(r / r_{0}\right)^{-1} \quad ; \quad E / E_{0}=\frac{\frac{Q}{4 \pi \varepsilon_{0} r^{2}}}{\frac{Q}{4 \pi \varepsilon_{0} r_{0}^{2}}}=\frac{r_{0}^{2}}{r^{2}}=\left(r / r_{0}\right)^{-2}$
The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH23.XLS," on tab "Problem 23.19c."


20. We assume the total charge is still $Q$, and let $\rho_{\mathrm{E}}=k r^{2}$. We evaluate the constant $k$ by calculating the total charge, in the manner of Example 22-5.

$$
Q=\int \rho_{\mathrm{E}} d V=\int_{0}^{r_{0}} k r^{2}\left(4 \pi r^{2} d r\right)=\frac{4}{5} k \pi r_{0}^{5} \rightarrow k=\frac{5 Q}{4 \pi r_{0}^{5}}
$$

(a) The electric field outside a charged, spherically symmetric volume is the same as that for a point charge of the same magnitude of charge. Integrating the electric field from infinity to the radius of interest gives the potential at that radius.

$$
E\left(r \geq r_{0}\right)=\frac{Q}{4 \pi \varepsilon_{0} r^{2}} ; V\left(r \geq r_{0}\right)=-\int_{\infty}^{r} \frac{Q}{4 \pi \varepsilon_{0} r^{2}} d r=\left.\frac{Q}{4 \pi \varepsilon_{0} r}\right|_{\infty} ^{r}=\frac{Q}{4 \pi \varepsilon_{0} r}
$$

(b) Inside the sphere the electric field is obtained from Gauss's Law using the charge enclosed by a sphere of radius $r$.

$$
4 \pi r^{2} E=\frac{Q_{\text {encl }}}{\varepsilon_{0}} ; Q_{\text {encl }}=\int \rho_{\mathrm{E}} d V=\frac{5 Q}{4 \pi r_{0}^{5}} \int_{0}^{r} r^{2}\left(4 \pi r^{2} d r\right)=\frac{5 Q}{4 \pi r_{0}^{5}} \frac{4}{5} \pi r^{5}=\frac{Q r^{5}}{r_{0}^{5}} \rightarrow
$$

$$
E\left(r<r_{0}\right)=\frac{Q_{\text {encl }}}{4 \pi \varepsilon_{0} r^{2}}=\frac{Q r^{3}}{4 \pi \varepsilon_{0} r_{0}^{5}}
$$

Integrating the electric field from the surface to $r<r_{0}$ gives the electric potential inside the sphere.

$$
V\left(r<r_{0}\right)=V\left(r_{0}\right)-\int_{r_{0}}^{r} \frac{Q r^{3}}{4 \pi \varepsilon_{0} r_{0}^{5}} d r=\frac{Q}{4 \pi \varepsilon_{0} r_{0}}-\left.\frac{Q r^{4}}{16 \pi \varepsilon_{0} r_{0}^{5}}\right|_{r_{0}} ^{r}=\frac{Q}{16 \pi \varepsilon_{0} r_{0}}\left(5-\frac{r^{4}}{r_{0}^{4}}\right)
$$

(c) To plot, we first calculate $V_{0}=V\left(r=r_{0}\right)=\frac{Q}{4 \pi \varepsilon_{0} r_{0}}$ and $E_{0}=E\left(r=r_{0}\right)=\frac{Q}{4 \pi \varepsilon_{0} r_{0}^{2}}$. Then we plot $V / V_{0}$ and $E / E_{0}$ as functions of $r / r_{0}$.
For $r<r_{0}: \quad V / V_{0}=\frac{\frac{Q}{16 \pi \varepsilon_{0} r_{0}}\left(5-\frac{r^{4}}{r_{0}^{4}}\right)}{\frac{Q}{4 \pi \varepsilon_{0} r_{0}}}=\frac{1}{4}\left(5-\frac{r^{4}}{r_{0}^{4}}\right) ; E / E_{0}=\frac{\frac{Q r^{3}}{4 \pi \varepsilon_{0} r_{0}^{5}}}{\frac{Q}{4 \pi \varepsilon_{0} r_{0}^{2}}}=\frac{r^{3}}{r_{0}^{3}}$
For $r>r_{0}: \quad V / V_{0}=\frac{\frac{Q}{4 \pi \varepsilon_{0} r}}{\frac{Q}{4 \pi \varepsilon_{0} r_{0}}}=\frac{r_{0}}{r}=\left(r / r_{0}\right)^{-1} \quad ; \quad E / E_{0}=\frac{\frac{Q}{4 \pi \varepsilon_{0} r^{2}}}{\frac{Q}{4 \pi \varepsilon_{0} r_{0}^{2}}}=\frac{r_{0}^{2}}{r^{2}}=\left(r / r_{0}\right)^{-2}$
The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH23.XLS," on tab "Problem 23.20c."


21. We first need to find the electric field. Since the charge distribution is spherically symmetric, Gauss's law tells us the electric field everywhere.

$$
\oint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=E\left(4 \pi r^{2}\right)=\frac{Q_{\text {encl }}}{\varepsilon_{0}} \rightarrow E=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q_{\text {encl }}}{r^{2}}
$$

If $r<r_{0}$, calculate the charge enclosed in the manner of Example 22-5.

$$
Q_{\text {encl }}=\int \rho_{\mathrm{E}} d V=\int_{0}^{r} \rho_{0}\left[1-\frac{r^{2}}{r_{0}^{2}}\right] 4 \pi r^{2} d r=4 \pi \rho_{0} \int_{0}^{r}\left[r^{2}-\frac{r^{4}}{r_{0}^{2}}\right] d r=4 \pi \rho_{0}\left[\frac{r^{3}}{3}-\frac{r^{5}}{5 r_{0}^{2}}\right]
$$

The total charge in the sphere is the above expression evaluated at $r=r_{0}$.

$$
Q_{\text {total }}=4 \pi \rho_{0}\left[\frac{r_{0}^{3}}{3}-\frac{r_{0}^{5}}{5 r_{0}^{2}}\right]=\frac{8 \pi \rho_{0} r_{0}^{3}}{15}
$$

Outside the sphere, we may treat it as a point charge, and so the potential at the surface of the sphere is given by Eq. 23-5, evaluated at the surface of the sphere.

$$
V\left(r=r_{0}\right)=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q_{\text {total }}}{r_{0}}=\frac{1}{4 \pi \varepsilon_{0}} \frac{\frac{8 \pi \rho_{0} r_{0}^{3}}{15}}{r_{0}}=\frac{2 \rho_{0} r_{0}^{2}}{15 \varepsilon_{0}}
$$

The potential inside is found from Eq. 23-4a. We need the field inside the sphere to use Eq. 23-4a.
The field is radial, so we integrate along a radial line so that $\overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\boldsymbol{\ell}}=E d r$.

$$
\begin{aligned}
E\left(r<r_{0}\right) & =\frac{1}{4 \pi \varepsilon_{0}} \frac{Q_{\text {encl }}}{r^{2}}=\frac{1}{4 \pi \varepsilon_{0}} \frac{4 \pi \rho_{0}\left[\frac{r^{3}}{3}-\frac{r^{5}}{5 r_{0}^{2}}\right]}{r^{2}}=\frac{\rho_{0}}{\varepsilon_{0}}\left[\frac{r}{3}-\frac{r^{3}}{5 r_{0}^{2}}\right] \\
V_{r}-V_{r_{0}} & =-\int_{r_{0}}^{r} \overrightarrow{\mathbf{E}} \cdot d \vec{\ell}=-\int_{r_{0}}^{r} E d r=-\int_{r_{0}}^{r} \frac{\rho_{0}}{\varepsilon_{0}}\left[\frac{r}{3}-\frac{r^{3}}{5 r_{0}^{2}}\right] d r=-\frac{\rho_{0}}{\varepsilon_{0}}\left[\frac{r^{2}}{6}-\frac{r^{4}}{20 r_{0}^{2}}\right]_{r_{0}}^{r} \\
V_{r} & =V_{r_{0}}+\left(-\frac{\rho_{0}}{\varepsilon_{0}}\left[\frac{r^{2}}{6}-\frac{r^{4}}{20 r_{0}^{2}}\right]_{r_{0}}^{r}\right)=\frac{2 \rho_{0} r_{0}^{2}}{15 \varepsilon_{0}}-\frac{\rho_{0}}{\varepsilon_{0}}\left[\left(\frac{r^{2}}{6}-\frac{r^{4}}{20 r_{0}^{2}}\right)-\left(\frac{r_{0}^{2}}{6}-\frac{r_{0}^{4}}{20 r_{0}^{2}}\right)\right] \\
& \left.=\frac{\rho_{0}}{\varepsilon_{0}}\left(\frac{r_{0}^{2}}{4}-\frac{r^{2}}{6}+\frac{r^{4}}{20 r_{0}^{2}}\right)\right]
\end{aligned}
$$

22. Because of the spherical symmetry of the problem, the electric field in each region is the same as that of a point charge equal to the net enclosed charge.
(a) For $r>r_{2}: E=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q_{\text {encl }}}{r^{2}}=\frac{1}{4 \pi \varepsilon_{0}} \frac{\frac{3}{2} Q}{r^{2}}=\frac{3}{8 \pi \varepsilon_{0}} \frac{Q}{r^{2}}$

For $r_{1}<r<r_{2}: E=0$, because the electric field is 0 inside of conducting material.
For $0<r<r_{1}: E=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q_{\text {encl }}}{r^{2}}=\frac{1}{4 \pi \varepsilon_{0}} \frac{\frac{1}{2} Q}{r^{2}}=\frac{1}{8 \pi \varepsilon_{0}} \frac{Q}{r^{2}}$
(b) For $r>r_{2}$, the potential is that of a point charge at the center of the sphere.

$$
V=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{r}=\frac{1}{4 \pi \varepsilon_{0}} \frac{\frac{3}{2} Q}{r}=\frac{3}{8 \pi \varepsilon_{0}} \frac{Q}{r}, r>r_{2}
$$

(c) For $r_{1}<r<r_{2}$, the potential is constant and equal to its value on the outer shell, because there is no electric field inside the conducting material.

$$
V=V\left(r=r_{2}\right)=\frac{3}{8 \pi \varepsilon_{0}} \frac{Q}{r_{2}}, r_{1}<r<r_{2}
$$

(d) For $0<r<r_{1}$, we use Eq. 23-4a. The field is radial, so we integrate along a radial line so that $\overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\boldsymbol{\ell}}=E d r$.

$$
\begin{aligned}
& V_{r}-V_{r_{\mathrm{i}}}=-\int_{r_{\mathrm{i}}}^{r} \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\boldsymbol{\ell}}=-\int_{r_{\mathrm{i}}}^{r} E d r=-\int_{r_{\mathrm{i}}}^{r} \frac{1}{8 \pi \varepsilon_{0}} \frac{Q}{r^{2}} d r=\frac{Q}{8 \pi \varepsilon_{0}}\left(\frac{1}{r}-\frac{1}{r_{1}}\right) \\
& V_{r}=V_{r_{\mathrm{i}}}+\frac{Q}{8 \pi \varepsilon_{0}}\left(\frac{1}{r}-\frac{1}{r_{1}}\right)=\frac{Q}{8 \pi \varepsilon_{0}}\left(\frac{1}{2 r_{1}}+\frac{1}{r}\right)=\frac{Q}{8 \pi \varepsilon_{0}}\left(\frac{1}{r_{2}}+\frac{1}{r}\right), 0<r<r_{1}
\end{aligned}
$$

(e) To plot, we first calculate $V_{0}=V\left(r=r_{2}\right)=\frac{3 Q}{8 \pi \varepsilon_{0} r_{2}}$ and $E_{0}=E\left(r=r_{2}\right)=\frac{3 Q}{8 \pi \varepsilon_{0} r_{2}^{2}}$. Then we plot $V / V_{0}$ and $E / E_{0}$ as functions of $r / r_{2}$.
For $0<r<r_{1}: \quad \frac{V}{V_{0}}=\frac{\frac{Q}{8 \pi \varepsilon_{0}}\left(\frac{1}{r_{2}}+\frac{1}{r}\right)}{\frac{3 Q}{8 \pi \varepsilon_{0} r_{2}}}=\frac{1}{3}\left[1+\left(r / r_{2}\right)^{-1}\right] ; \frac{E}{E_{0}}=\frac{\frac{1}{8 \pi \varepsilon_{0}} \frac{Q}{r^{2}}}{\frac{3 Q}{8 \pi \varepsilon_{0} r_{2}^{2}}}=\frac{1}{3} \frac{r_{2}^{2}}{r^{2}}=\frac{1}{3}\left(r / r_{2}\right)^{-2}$
For $r_{1}<r<r_{2}: \quad \frac{V}{V_{0}}=\frac{\frac{3}{8 \pi \varepsilon_{0}} \frac{Q}{r_{2}}}{\frac{3 Q}{8 \pi \varepsilon_{0} r_{2}}}=1 ; \frac{E}{E_{0}}=\frac{0}{\frac{3 Q}{8 \pi \varepsilon_{0} r_{2}^{2}}}=0$
For $r>r_{2}: \quad \frac{V}{V_{0}}=\frac{\frac{3}{8 \pi \varepsilon_{0}} \frac{Q}{r}}{\frac{3 Q}{8 \pi \varepsilon_{0} r_{2}}}=\frac{r_{2}}{r}=\left(r / r_{2}\right)^{-1} ; \frac{E}{E_{0}}=\frac{\frac{3}{8 \pi \varepsilon_{0}} \frac{Q}{r^{2}}}{\frac{3 Q}{8 \pi \varepsilon_{0} r_{2}^{2}}}=\frac{r_{2}^{2}}{r^{2}}=\left(r / r_{2}\right)^{-2}$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH23.XLS," on tab "Problem 23.22e."


23. The field is found in Problem 22-33. The field inside the cylinder is 0 , and the field outside the cylinder is $\frac{\sigma R_{0}}{\varepsilon_{0} R}$.
(a) Use Eq. 23-4a to find the potential. Integrate along a radial line, so that $\overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\boldsymbol{\ell}}=E d R$.

$$
\begin{aligned}
V_{R}-V_{R_{0}} & =-\int_{R_{0}}^{R} \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\boldsymbol{\ell}}=-\int_{R_{0}}^{R} E d R=-\int_{R_{0}}^{R} \frac{\sigma R_{0}}{\varepsilon_{0} R} d R=-\frac{\sigma R_{0}}{\varepsilon_{0}} \ln \frac{R}{R_{0}} \rightarrow \\
V_{R} & =V_{0}-\frac{\sigma R_{0}}{\varepsilon_{0}} \ln \frac{R}{R_{0}}, R>R_{0}
\end{aligned}
$$

(b) The electric field inside the cylinder is 0 , so the potential inside is constant and equal to the potential on the surface, $V_{0}$.
(c) No, we are not able to assume that $V=0$ at $R=\infty . \quad V \neq 0$ because there would be charge at infinity for an infinite cylinder. And from the formula derived in (a), if $R=\infty, V_{R}=-\infty$.
24. Use Eq. 23-5 to find the charge.

$$
V=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{r} \rightarrow Q=\left(4 \pi \varepsilon_{0}\right) r V=\left(\frac{1}{8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}}\right)(0.15 \mathrm{~m})(185 \mathrm{~V})=3.1 \times 10^{-9} \mathrm{C}
$$

25. (a) The electric potential is given by Eq. 23-5.

$$
V=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{r}=\left(8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{1.60 \times 10^{-19} \mathrm{C}}{0.50 \times 10^{-10} \mathrm{~m}}=28.77 \mathrm{~V} \approx 29 \mathrm{~V}
$$

(b) The potential energy of the electron is the charge of the electron times the electric potential due to the proton.

$$
U=Q V=\left(-1.60 \times 10^{-19} \mathrm{C}\right)(28.77 \mathrm{~V})=-4.6 \times 10^{-18} \mathrm{~J}
$$

26. (a) Because of the inverse square nature of the electric field, any location where the field is zero must be closer to the weaker charge $\left(q_{2}\right)$. Also, in between the
 two charges, the fields due to the two charges are parallel to each other (both to the left) and cannot cancel. Thus the only places where the field can be zero are closer to the weaker charge,
but not between them. In the diagram, this is the point to the left of $q_{2}$. Take rightward as the positive direction.

$$
\begin{aligned}
& E=\frac{1}{4 \pi \varepsilon_{0}} \frac{\left|q_{2}\right|}{x^{2}}-\frac{1}{4 \pi \varepsilon_{0}} \frac{q_{1}}{(d+x)^{2}}=0 \rightarrow\left|q_{2}\right|(d+x)^{2}=q_{1} x^{2} \rightarrow \\
& x=\frac{\sqrt{\left|q_{2}\right|}}{\sqrt{q_{1}}-\sqrt{\left|q_{2}\right|}} d=\frac{\sqrt{2.0 \times 10^{-6} \mathrm{C}}}{\sqrt{3.4 \times 10^{-6} \mathrm{C}}-\sqrt{2.0 \times 10^{-6} \mathrm{C}}}(5.0 \mathrm{~cm})=16 \mathrm{~cm} \mathrm{left} \text { of } q_{2}
\end{aligned}
$$

(b) The potential due to the positive charge is positive everywhere, and the potential due to the negative charge is negative everywhere. Since the negative charge is smaller in magnitude than the positive charge,
 any point where the potential is zero must be closer to the negative charge. So consider locations between the charges (position $x_{1}$ ) and to the left of the negative charge (position $x_{2}$ ) as shown in the diagram.

$$
\begin{aligned}
V_{\text {location } 1} & =\frac{1}{4 \pi \varepsilon_{0}}\left[\frac{q_{1}}{\left(d-x_{1}\right)}+\frac{q_{2}}{x_{1}}\right]=0 \rightarrow x_{1}=\frac{q_{2} d}{\left(q_{2}-q_{1}\right)}=\frac{\left(-2.0 \times 10^{-6} \mathrm{C}\right)(5.0 \mathrm{~cm})}{\left(-5.4 \times 10^{-6} \mathrm{C}\right)}=1.852 \mathrm{~cm} \\
V_{\text {loation } 2} & =\frac{1}{4 \pi \varepsilon_{0}}\left[\frac{q_{1}}{\left(d+x_{2}\right)}+\frac{q_{2}}{x_{2}}\right]=0 \rightarrow \\
x_{2} & =-\frac{q_{2} d}{\left(q_{1}+q_{2}\right)}=-\frac{\left(-2.0 \times 10^{-6} \mathrm{C}\right)(5.0 \mathrm{~cm})}{\left(1.4 \times 10^{-6} \mathrm{C}\right)}=7.143 \mathrm{~cm}
\end{aligned}
$$

So the two locations where the potential is zero are 1.9 cm from the negative charge towards the positive charge, and 7.1 cm from the negative charge away from the positive charge.
27. The work required is the difference in the potential energy of the charges, calculated with the test charge at the two different locations. The potential energy of a pair of charges is given in Eq. 23-10 as $U=\frac{1}{4 \pi \varepsilon_{0}} \frac{q Q}{r}$. So to find the work, calculate the difference in potential energy with the test charge at the two locations. Let $Q$ represent the $25 \mu \mathrm{C}$ charge, let $q$ represent the $0.18 \mu \mathrm{C}$ test charge, $D$ represent the 6.0 cm distance, and let $d$ represent the 1.0 cm distance. Since the potential energy of the two $25 \mu \mathrm{C}$ charges doesn't change, we don't include it in the calculation.

$$
\begin{aligned}
& U_{\text {intial }}=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q q}{D / 2}+\frac{1}{4 \pi \varepsilon_{0}} \frac{Q q}{D / 2} \quad U_{\text {final }}=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q q}{[D / 2-d]}+\frac{1}{4 \pi \varepsilon_{0}} \frac{Q q}{[D / 2+d]} \\
& \begin{aligned}
\begin{array}{c}
\text { Work } \\
\text { exteremal } \\
\text { force }
\end{array} & =U_{\text {frial }}-U_{\text {initial }}=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q q}{[D / 2-d]}+\frac{1}{4 \pi \varepsilon_{0}} \frac{Q q}{[D / 2+d]}-2\left(\frac{1}{4 \pi \varepsilon_{0}} \frac{Q q}{D / 2}\right) \\
& =\frac{2 Q q}{4 \pi \varepsilon_{0}}\left[\frac{1}{[D-2 d]}+\frac{1}{[D+2 d]}-\frac{1}{D / 2}\right] \\
& =2\left(8.99 \times 10^{9} \mathrm{~N}^{2} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)\left(25 \times 10^{-6} \mathrm{C}\right)\left(0.18 \times 10^{-6} \mathrm{C}\right)\left[\frac{1}{0.040 \mathrm{~m}}+\frac{1}{0.080 \mathrm{~m}}-\frac{1}{0.030 \mathrm{~m}}\right] \\
& =0.34 \mathrm{~J}
\end{aligned}
\end{aligned}
$$

An external force needs to do positive work to move the charge.
28. (a) The potential due to a point charge is given by Eq. 23-5.

$$
\begin{aligned}
V_{\mathrm{ba}} & =V_{\mathrm{b}}-V_{\mathrm{a}}=\frac{1}{4 \pi \varepsilon_{0}} \frac{q}{r_{\mathrm{b}}}-\frac{1}{4 \pi \varepsilon_{0}} \frac{q}{r_{\mathrm{a}}}=\frac{q}{4 \pi \varepsilon_{0}}\left(\frac{1}{r_{\mathrm{b}}}-\frac{1}{r_{\mathrm{a}}}\right) \\
& =\left(8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)\left(-3.8 \times 10^{-6} \mathrm{C}\right)\left(\frac{1}{0.36 \mathrm{~m}}-\frac{1}{0.26 \mathrm{~m}}\right)=3.6 \times 10^{4} \mathrm{~V}
\end{aligned}
$$

(b) The magnitude of the electric field due to a point charge is given by Eq. 21-4a. The direction of the electric field due to a negative charge is towards the charge, so the field at point a will point downward, and the field at point $b$ will point to the right. See the vector diagram.


$$
\begin{aligned}
& \overrightarrow{\mathbf{E}}_{\mathrm{b}}=\frac{1}{4 \pi \varepsilon_{0}} \frac{|q|}{r_{\mathrm{b}}^{2}} \hat{\mathbf{i}}=\frac{\left(8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)\left(3.8 \times 10^{-6} \mathrm{C}\right)}{(0.36 \mathrm{~m})^{2}} \hat{\mathbf{i}}=2.636 \times 10^{5} \mathrm{~V} / \mathrm{m} \hat{\mathbf{i}} \\
& \overrightarrow{\mathbf{E}}_{\mathrm{a}}=-\frac{1}{4 \pi \varepsilon_{0}} \frac{|q|}{r_{\mathrm{a}}^{2}} \hat{\mathbf{j}}=-\frac{\left(8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)\left(3.8 \times 10^{-6} \mathrm{C}\right)}{(0.26 \mathrm{~m})^{2}} \hat{\mathbf{j}}=-5.054 \times 10^{5} \mathrm{~V} / \mathrm{m} \hat{\mathbf{j}} \\
& \overrightarrow{\mathbf{E}}_{\mathrm{b}}-\overrightarrow{\mathbf{E}}_{\mathrm{a}}=2.636 \times 10^{5} \mathrm{~V} / \mathrm{m} \hat{\mathbf{i}}+5.054 \times 10^{5} \mathrm{~V} / \mathrm{m} \hat{\mathbf{j}} \\
& \left|\overrightarrow{\mathbf{E}}_{\mathrm{b}}-\overrightarrow{\mathbf{E}}_{\mathrm{a}}\right|=\sqrt{\left(2.636 \times 10^{5} \mathrm{~V} / \mathrm{m}\right)^{2}+\left(5.054 \times 10^{5} \mathrm{~V} / \mathrm{m}\right)^{2}}=5.7 \times 10^{5} \mathrm{~V} / \mathrm{m} \\
& \theta=\tan ^{-1} \frac{-E_{\mathrm{a}}}{E_{\mathrm{b}}}=\tan ^{-1} \frac{5.054 \times 10^{5}}{2.636 \times 10^{5}}=62^{\circ}
\end{aligned}
$$

29. We assume that all of the energy the proton gains in being accelerated by the voltage is changed to potential energy just as the proton's outer edge reaches the outer radius of the silicon nucleus.

$$
\begin{aligned}
& U_{\text {initial }}=U_{\text {final }} \rightarrow e V_{\text {initial }}=\frac{1}{4 \pi \varepsilon_{0}} \frac{e(14 e)}{r} \rightarrow \\
& V_{\text {initial }}=\frac{1}{4 \pi \varepsilon_{0}} \frac{14 e}{r}=\left(8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{(14)\left(1.60 \times 10^{-19} \mathrm{C}\right)}{(1.2+3.6) \times 10^{-15} \mathrm{~m}}=4.2 \times 10^{6} \mathrm{~V}
\end{aligned}
$$


30. By energy conservation, all of the initial potential energy of the charges will change to kinetic energy when the charges are very far away from each other. By momentum conservation, since the initial momentum is zero and the charges have identical masses, the charges will have equal speeds in opposite directions from each other as they move. Thus each charge will have the same kinetic energy.

$$
\begin{aligned}
& E_{\text {initial }}=E_{\text {final }} \rightarrow U_{\text {initial }}=K_{\text {final }} \rightarrow \frac{1}{4 \pi \varepsilon_{0}} \frac{Q^{2}}{r}=2\left(\frac{1}{2} m v^{2}\right) \rightarrow \\
& v=\sqrt{\frac{1}{4 \pi \varepsilon_{0}} \frac{Q^{2}}{m r}}=\sqrt{\frac{\left(8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)\left(5.5 \times 10^{-6} \mathrm{C}\right)^{2}}{\left(1.0 \times 10^{-6} \mathrm{~kg}\right)(0.065 \mathrm{~m})}}=2.0 \times 10^{3} \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

31. By energy conservation, all of the initial potential energy will change to kinetic energy of the electron when the electron is far away. The other charge is fixed, and so has no kinetic energy. When the electron is far away, there is no potential energy.

$$
\begin{aligned}
& E_{\text {initial }}=E_{\text {final }} \rightarrow U_{\text {initial }}=K_{\text {final }} \rightarrow \frac{(-e)(Q)}{4 \pi \varepsilon_{0} r}=\frac{1}{2} m v^{2} \rightarrow \\
& v \\
& =\sqrt{\frac{2(-e)(Q)}{\left(4 \pi \varepsilon_{0}\right) m r}}=\sqrt{\frac{2\left(8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)\left(-1.60 \times 10^{-19} \mathrm{C}\right)\left(-1.25 \times 10^{-10} \mathrm{C}\right)}{\left(9.11 \times 10^{-31} \mathrm{~kg}\right)(0.425 \mathrm{~m})}} \\
& \\
& =9.64 \times 10^{5} \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

32. Use Eq. 23-2b and Eq. 23-5.

$$
\begin{aligned}
V_{\mathrm{BA}} & =V_{\mathrm{B}}-V_{\mathrm{A}}=\left(\frac{1}{4 \pi \varepsilon_{0}} \frac{q}{d-b}+\frac{1}{4 \pi \varepsilon_{0}} \frac{(-q)}{b}\right)-\left(\frac{1}{4 \pi \varepsilon_{0}} \frac{q}{b}+\frac{1}{4 \pi \varepsilon_{0}} \frac{(-q)}{d-b}\right) \\
& =\frac{q}{4 \pi \varepsilon_{0}}\left(\frac{1}{d-b}-\frac{1}{b}-\frac{1}{b}+\frac{1}{d-b}\right)=2 \frac{1}{4 \pi \varepsilon_{0}} q\left(\frac{1}{d-b}-\frac{1}{b}\right)=\frac{2 q(2 b-d)}{4 \pi \varepsilon_{0} b(d-b)}
\end{aligned}
$$

33. (a) For every element $d q$ as labeled in Figure 23-14 on the top half of the ring, there will be a diametrically opposite element of charge $-d q$. The potential due to those two infinitesimal elements will cancel each other, and so the potential due to the entire ring is 0 .
(b) We follow Example 21-9 from the textbook. But because the upper and lower halves of the ring are oppositely
 charged, the parallel components of the fields from diametrically opposite infinitesimal segments of the ring will cancel each other, and the perpendicular components add, in the negative $y$ direction. We know then that $E_{x}=0$.

$$
\begin{aligned}
d E_{y} & =-d E \sin \theta=-\frac{1}{4 \pi \varepsilon_{0}} \frac{d q}{r^{2}} \sin \theta=-\frac{1}{4 \pi \varepsilon_{0}} \frac{\frac{Q}{2 \pi R} d \ell}{\left(x^{2}+R^{2}\right)} \frac{R}{\left(x^{2}+R^{2}\right)^{1 / 2}}=-\frac{Q}{8 \pi^{2} \varepsilon_{0}} \frac{d \ell}{\left(x^{2}+R^{2}\right)^{3 / 2}} \\
E_{y} & =\int_{0}^{2 \pi R} d E_{y}=-\frac{Q}{8 \pi^{2} \varepsilon_{0}} \frac{1}{\left(x^{2}+R^{2}\right)^{3 / 2}} \int_{0}^{2 \pi R} d \ell=-\frac{Q}{4 \pi \varepsilon_{0}} \frac{R}{\left(x^{2}+R^{2}\right)^{3 / 2}} \rightarrow \\
\overrightarrow{\mathbf{E}} & =-\frac{Q}{4 \pi \varepsilon_{0}} \frac{R}{\left(x^{2}+R^{2}\right)^{3 / 2}} \hat{\mathbf{j}}
\end{aligned}
$$

Note that for $x \gg R$, this reduces to $\overrightarrow{\mathbf{E}}=-\frac{Q}{4 \pi \varepsilon_{0}} \frac{R}{x^{3}} \hat{\mathbf{j}}$, which has the typical distance dependence for the field of a dipole, along the axis of the dipole.
34. The potential at the corner is the sum of the potentials due to each of the charges, using Eq. 23-5.

$$
V=\frac{1}{4 \pi \varepsilon_{0}} \frac{(3 Q)}{\ell}+\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{\sqrt{2} \ell}+\frac{1}{4 \pi \varepsilon_{0}} \frac{(-2 Q)}{\ell}=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{\ell}\left(1+\frac{1}{\sqrt{2}}\right)=\frac{1}{4 \pi \varepsilon_{0}} \frac{\sqrt{2} Q}{2 \ell}(\sqrt{2}+1)
$$

35. We follow the development of Example 23-9, with Figure 23-15. The charge on a thin ring of radius $R$ and thickness $d R$ is $d q=\sigma d A=\sigma(2 \pi R d R)$. Use Eq. 23-6b to find the potential of a continuous charge distribution.

$$
\begin{aligned}
V & =\frac{1}{4 \pi \varepsilon_{0}} \int \frac{d q}{r}=\frac{1}{4 \pi \varepsilon_{0}} \int_{R_{1}}^{R_{2}} \frac{\sigma(2 \pi R d R)}{\sqrt{x^{2}+R^{2}}}=\frac{\sigma}{2 \varepsilon_{0}} \int_{R_{1}}^{R_{2}} \frac{R}{\sqrt{x^{2}+R^{2}}} d R=\left.\frac{\sigma}{2 \varepsilon_{0}}\left(x^{2}+R^{2}\right)^{1 / 2}\right|_{R_{1}} ^{R_{2}} \\
& =\frac{\sigma}{2 \varepsilon_{0}}\left(\sqrt{x^{2}+R_{2}^{2}}-\sqrt{x^{2}+R_{1}^{2}}\right)
\end{aligned}
$$

36. All of the charge is the same distance from the center of the semicircle - the radius of the semicircle. Use Eq 23-6b to calculate the potential.

$$
\ell=\pi r_{0} \rightarrow r_{0}=\frac{\ell}{\pi} ; V=\frac{1}{4 \pi \varepsilon_{0}} \int \frac{d q}{r}=\frac{1}{4 \pi \varepsilon_{0} r_{0}} \int d q=\frac{Q}{4 \pi \varepsilon_{0} \frac{\ell}{\pi}}=\frac{Q}{4 \varepsilon_{0} \ell}
$$

37. The electric potential energy is the product of the point charge and the electric potential at the location of the charge. Since all points on the ring are equidistant from any point on the axis, the electric potential integral is simple.

$$
U=q V=q \int \frac{d q}{4 \pi \varepsilon_{0} \sqrt{r^{2}+x^{2}}}=\frac{q}{4 \pi \varepsilon_{0} \sqrt{r^{2}+x^{2}}} \int d q=\frac{q Q}{4 \pi \varepsilon_{0} \sqrt{r^{2}+x^{2}}}
$$

Energy conservation is used to obtain a relationship between the potential and kinetic energies at the center of the loop and at a point 2.0 m along the axis from the center.

$$
\begin{aligned}
K_{0}+U_{0} & =K+U \\
0+\frac{q Q}{4 \pi \varepsilon_{0} \sqrt{r^{2}}} & =\frac{1}{2} m v^{2}+\frac{q Q}{4 \pi \varepsilon_{0} \sqrt{r^{2}+x^{2}}}
\end{aligned}
$$

This is equation is solved to obtain the velocity at $x=2.0 \mathrm{~m}$.

$$
\begin{aligned}
v & =\sqrt{\frac{q Q}{2 \pi \varepsilon_{0} m}\left(\frac{1}{r}-\frac{1}{\sqrt{r^{2}+x^{2}}}\right)} \\
& =\sqrt{\frac{(3.0 \mu \mathrm{C})(15.0 \mu \mathrm{C})}{2 \pi\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{Nm}^{2}\right)\left(7.5 \times 10^{-3} \mathrm{~kg}\right)}\left(\frac{1}{0.12 \mathrm{~m}}-\frac{1}{\sqrt{(0.12 \mathrm{~m})^{2}+(2.0 \mathrm{~m})^{2}}}\right)} \\
& =29 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

38. Use Eq. 23-6b to find the potential of a continuous charge distribution. Choose a differential element of length $d x^{\prime}$ at position $x^{\prime}$ along the rod. The charge on the element is $d q=\frac{Q}{2 \ell} d x^{\prime}$, and the element is a distance $r=\sqrt{x^{\prime 2}+y^{2}}$ from a point on the $y$ axis. Use an indefinite integral from Appendix B-4, page A-7.

$$
\begin{aligned}
V_{y \text { axis }} & =\frac{1}{4 \pi \varepsilon_{0}} \int \frac{d q}{r}=\frac{1}{4 \pi \varepsilon_{0}} \int_{-\ell}^{\ell} \frac{\frac{Q}{2 \ell} d x^{\prime}}{\sqrt{x^{\prime 2}+y^{2}}} \\
& =\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{2 \ell}\left[\ln \left(\sqrt{x^{\prime 2}+y^{2}}+x^{\prime}\right)\right]_{-\ell}^{\ell}=\frac{Q}{8 \pi \varepsilon_{0} \ell}\left[\ln \left(\frac{\sqrt{\ell^{2}+y^{2}}+\ell}{\sqrt{\ell^{2}+y^{2}}-\ell}\right)\right]
\end{aligned}
$$

39. Use Eq. 23-6b to find the potential of a continuous charge distribution. Choose a differential element of length $d x^{\prime}$ at position $x^{\prime}$ along the rod. The charge on the element is
 $d q=\frac{Q}{2 \ell} d x^{\prime}$, and the element is a distance $x-x^{\prime}$ from a point outside the rod on the $x$ axis.

$$
V=\frac{1}{4 \pi \varepsilon_{0}} \int_{r} \frac{d q}{r}=\frac{1}{4 \pi \varepsilon_{0}} \int_{-\ell}^{\ell} \frac{Q}{2 \ell} d x^{\prime} x^{\prime}=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{2 \ell}\left[-\ln \left(x-x^{\prime}\right)\right]_{-\ell}^{\ell}=\frac{Q}{8 \pi \varepsilon_{0} \ell}\left[\ln \left(\frac{x+\ell}{x-\ell}\right)\right], x>\ell
$$

40. For both parts of the problem, use Eq. 23-6b to find the potential of a continuous charge distribution. Choose a differential element of length $d x^{\prime}$ at position $x^{\prime}$ along the rod. The charge on the element is $d q=\lambda d x^{\prime}=a x^{\prime} d x^{\prime}$.
(a) The element is a distance $r=\sqrt{x^{\prime 2}+y^{2}}$ from a point on the $y$ axis.

$$
V=\frac{1}{4 \pi \varepsilon_{0}} \int \frac{d q}{r}=\frac{1}{4 \pi \varepsilon_{0}} \int_{-\ell}^{\ell} \frac{a x^{\prime} d x^{\prime}}{\sqrt{x^{\prime 2}+y^{2}}}=0
$$

The integral is equal to 0 because the region of integration is "even" with respect to the origin, while the integrand is "odd." Alternatively, the antiderivative can be found, and the integral can be shown to be 0 . This is to be expected since the potential from points symmetric about the origin would cancel on the $y$ axis.
(b) The element is a distance $x-x^{\prime}$ from a point outside the rod on the $x$ axis.

$$
V=\frac{1}{4 \pi \varepsilon_{0}} \int^{d q} \frac{d}{4 \pi \varepsilon_{0}} \int_{-\ell}^{\ell} \frac{a x^{\prime} d x^{\prime}}{x-x^{\prime}}=\frac{a}{4 \pi \varepsilon_{0}} \int_{-\ell}^{\ell} \frac{x^{\prime} d x^{\prime}}{x-x^{\prime}}
$$



A substitution of $z=x-x^{\prime}$ can be used to do the integration.

$$
\begin{aligned}
V & =\frac{a}{4 \pi \varepsilon_{0}} \int_{-\ell}^{\ell} \frac{x^{\prime} d x^{\prime}}{x-x^{\prime}}=\frac{a}{4 \pi \varepsilon_{0}} \int_{x+\ell}^{x-\ell} \frac{(x-z)(-d z)}{z}=\frac{a}{4 \pi \varepsilon_{0}} \int_{x-\ell}^{x+\ell}\left(\frac{x}{z}-1\right) d z \\
& =\frac{a}{4 \pi \varepsilon_{0}}(x \ln z-z)_{x-\ell}^{x+\ell}=\frac{a}{4 \pi \varepsilon_{0}}\left[x \ln \left(\frac{x+\ell}{x-\ell}\right)-2 \ell\right], x>\ell
\end{aligned}
$$

41. We follow the development of Example 23-9, with Figure 23-15. The charge on a thin ring of radius $R$ and thickness $d R$ will now be $d q=\sigma d A=\left(a R^{2}\right)(2 \pi R d R)$. Use Eq. 23-6b to find the potential of a continuous charge distribution.

$$
V=\frac{1}{4 \pi \varepsilon_{0}} \int \frac{d q}{r}=\frac{1}{4 \pi \varepsilon_{0}} \int_{0}^{R_{0}} \frac{\left(a R^{2}\right)(2 \pi R d R)}{\sqrt{x^{2}+R^{2}}}=\frac{a}{2 \varepsilon_{0}} \int_{0}^{R_{0}} \frac{R^{3} d R}{\sqrt{x^{2}+R^{2}}}
$$

A substitution of $x^{2}+R^{2}=u^{2}$ can be used to do the integration.

$$
\begin{aligned}
x^{2} & +R^{2}=u^{2} \rightarrow R^{2}=u^{2}-x^{2} ; 2 R d R=2 u d u \\
V & =\frac{a}{2 \varepsilon_{0}} \int_{0}^{R_{0}} \frac{R^{3} d R}{\sqrt{x^{2}+R^{2}}}=\frac{a}{2 \varepsilon_{0}} \int_{R=0}^{R=R_{0}} \frac{\left(u^{2}-x^{2}\right) u d u}{u}=\frac{a}{2 \varepsilon_{0}}\left[\frac{1}{3} u^{3}-u x^{2}\right]_{R=0}^{R=R_{0}} \\
& =\frac{a}{2 \varepsilon_{0}}\left[\frac{1}{3}\left(x^{2}+R^{2}\right)^{3 / 2}-x^{2}\left(x^{2}+R^{2}\right)^{1 / 2}\right]_{R=0}^{R=R_{0}} \\
& =\frac{a}{2 \varepsilon_{0}}\left[\left\{\frac{1}{3}\left(x^{2}+R_{0}^{2}\right)^{3 / 2}-x^{2}\left(x^{2}+R_{0}^{2}\right)^{1 / 2}\right\}+\frac{2}{3} x^{3}\right] \\
& =\frac{a}{6 \varepsilon_{0}}\left[\left(R_{0}^{2}-2 x^{2}\right)\left(x^{2}+R_{0}^{2}\right)^{1 / 2}+2 x^{3}\right], x>0
\end{aligned}
$$

42. 


43. The electric field from a large plate is uniform with magnitude $E=\sigma / 2 \varepsilon_{0}$, with the field pointing away from the plate on both sides. Equation 23-4(a) can be integrated between two arbitrary points to calculate the potential difference between those points.

$$
\Delta V=-\int_{x_{0}}^{x_{1}} \frac{\sigma}{2 \varepsilon_{0}} d x=\frac{\sigma\left(x_{0}-x_{1}\right)}{2 \varepsilon_{0}}
$$

Setting the change in voltage equal to 100 V and solving for $x_{0}-x_{1}$ gives the distance between field lines.

$$
x_{0}-x_{1}=\frac{2 \varepsilon_{0} \Delta V}{\sigma}=\frac{2\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{Nm}^{2}\right)(100 \mathrm{~V})}{0.75 \times 10^{-6} \mathrm{C} / \mathrm{m}^{2}}=2.36 \times 10^{-3} \mathrm{~m} \approx 2 \mathrm{~mm}
$$

44. The potential at the surface of the sphere is $V_{0}=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{r_{0}}$. The potential outside the sphere is $V=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{r}=V_{0} \frac{r_{0}}{r}$, and decreases as you move away from the surface. The difference in potential between a given location and the surface is to be a multiple of 100 V .

$$
\begin{aligned}
& V_{0}=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{r_{0}}=\left(8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)\left(\frac{0.50 \times 10^{-6} \mathrm{C}}{0.44 \mathrm{~m}}\right)=10,216 \mathrm{~V} \\
& V_{0}-V=V_{0}-V_{0} \frac{r_{0}}{r}=(100 \mathrm{~V}) n \rightarrow r=\frac{V_{0}}{\left[V_{0}-(100 \mathrm{~V}) n\right]^{0}} r_{0} \\
& \text { (a) } \quad r_{1}=\frac{V_{0}}{\left[V_{0}-(100 \mathrm{~V}) 1\right]} r_{0}=\frac{10,216 \mathrm{~V}}{10,116 \mathrm{~V}}(0.44 \mathrm{~m})=0.444 \mathrm{~m}
\end{aligned}
$$

Note that to within the appropriate number of significant figures, this location is at the surface of the sphere. That can be interpreted that we don't know the voltage well enough to be working with a $100-\mathrm{V}$ difference.
(b) $r_{10}=\frac{V_{0}}{\left[V_{0}-(100 \mathrm{~V}) 10\right]} r_{0}=\frac{10,216 \mathrm{~V}}{9,216 \mathrm{~V}}(0.44 \mathrm{~m})=0.49 \mathrm{~m}$
(c) $r_{100}=\frac{V_{0}}{\left[V_{0}-(100 \mathrm{~V}) 100\right]} r_{0}=\frac{10,216 \mathrm{~V}}{216 \mathrm{~V}}(0.44 \mathrm{~m})=21 \mathrm{~m}$
45. The potential due to the dipole is given by Eq. 23-7.
(a) $V=\frac{1}{4 \pi \varepsilon_{0}} \frac{p \cos \theta}{r^{2}}=\frac{\left(8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)\left(4.8 \times 10^{-30} \mathrm{C} \cdot \mathrm{m}\right) \cos 0}{\left(4.1 \times 10^{-9} \mathrm{~m}\right)^{2}}$


$$
=2.6 \times 10^{-3} \mathrm{~V}
$$

(b) $V=\frac{1}{4 \pi \varepsilon_{0}} \frac{p \cos \theta}{r^{2}}=\frac{\left(8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)\left(4.8 \times 10^{-30} \mathrm{C} \cdot \mathrm{m}\right) \cos 45^{\circ}}{\left(4.1 \times 10^{-9} \mathrm{~m}\right)^{2}}$

$$
=1.8 \times 10^{-3} \mathrm{~V}
$$

(c) $\begin{aligned} V & =\frac{1}{4 \pi \varepsilon_{0}} \frac{p \cos \theta}{r^{2}}=\frac{\left(8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)\left(4.8 \times 10^{-30} \mathrm{C} \cdot \mathrm{m}\right) \cos 135^{\circ}}{\left(1.1 \times 10^{-9} \mathrm{~m}\right)^{2}} \\ & =-1.8 \times 10^{-3} \mathrm{~V}\end{aligned}$

46. (a) We assume that $\overrightarrow{\mathbf{p}}_{1}$ and $\overrightarrow{\mathbf{p}}_{2}$ are equal in magnitude, and that each makes a $52^{\circ}$ angle with $\overrightarrow{\mathbf{p}}$. The magnitude of $\overrightarrow{\mathbf{p}}_{1}$ is also given by $p_{1}=q d$, where $q$ is the net charge on the hydrogen atom, and $d$ is the distance between the H and the O .

$$
\begin{aligned}
& p=2 p_{1} \cos 52^{\circ} \rightarrow p_{1}=\frac{p}{2 \cos 52^{\circ}}=q d \rightarrow \\
& q=\frac{p}{2 d \cos 52^{\circ}}=\frac{6.1 \times 10^{-30} \mathrm{C} \cdot \mathrm{~m}}{2\left(0.96 \times 10^{-10} \mathrm{~m}\right) \cos 52^{\circ}}=5.2 \times 10^{-20} \mathrm{C}
\end{aligned}
$$

This is about 0.32 times the charge on an electron.
(b) Since we are considering the potential far from the dipoles, we will take the potential of each dipole to be given by Eq. 23-7. See the diagram for the angles involved. From part (a), $p_{1}=p_{2}=\frac{p}{2 \cos 52^{\circ}}$.

$$
\begin{aligned}
V & =V_{p_{1}}+V_{p_{2}} \\
& =\frac{1}{4 \pi \varepsilon_{0}} \frac{p_{1} \cos \left(52^{\circ}-\theta\right)}{r}+\frac{1}{4 \pi \varepsilon_{0}} \frac{p_{2} \cos \left(52^{\circ}+\theta\right)}{r} \\
& =\frac{1}{4 \pi \varepsilon_{0} r} \frac{p}{2 \cos 52^{\circ}}\left[\cos \left(52^{\circ}-\theta\right)+\cos \left(52^{\circ}+\theta\right)\right]
\end{aligned}
$$

$$
=\frac{1}{4 \pi \varepsilon_{0} r} \frac{p}{2 \cos 52^{\circ}}\left(\cos 52^{\circ} \cos \theta+\sin 52^{\circ} \cos \theta+\cos 52^{\circ} \cos \theta-\sin 52^{\circ} \cos \theta\right)
$$

$$
=\frac{1}{4 \pi \varepsilon_{0} r} \frac{p}{2 \cos 52^{\circ}}\left(2 \cos 52^{\circ} \cos \theta\right)=\frac{1}{4 \pi \varepsilon_{0}} \frac{p \cos \theta}{r}
$$

47. $E=-\frac{d V}{d r}=-\frac{d}{d r}\left(\frac{1}{4 \pi \varepsilon_{0}} \frac{q}{r}\right)=-\frac{q}{4 \pi \varepsilon_{0}} \frac{d}{d r}\left(\frac{1}{r}\right)=-\frac{q}{4 \pi \varepsilon_{0}}\left(-\frac{1}{r^{2}}\right)=\frac{1}{4 \pi \varepsilon_{0}} \frac{q}{r^{2}}$
48. The potential gradient is the negative of the electric field. Outside of a spherically symmetric charge distribution, the field is that of a point charge at the center of the distribution.

$$
\frac{d V}{d r}=-E=-\frac{1}{4 \pi \varepsilon_{0}} \frac{q}{r^{2}}=-\left(8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{(92)\left(1.60 \times 10^{-19} \mathrm{C}\right)}{\left(7.5 \times 10^{-15} \mathrm{~m}\right)^{2}}=-2.4 \times 10^{21} \mathrm{~V} / \mathrm{m}
$$

49. The electric field between the plates is obtained from the negative derivative of the potential.

$$
E=-\frac{d V}{d x}=-\frac{d}{d x}[(8.0 \mathrm{~V} / \mathrm{m}) x+5.0 \mathrm{~V}]=-8.0 \mathrm{~V} / \mathrm{m}
$$

The charge density on the plates (assumed to be conductors) is then calculated from the electric field between two large plates, $E=\sigma / \varepsilon_{0}$.

$$
\sigma=E \varepsilon_{0}=(8.0 \mathrm{~V} / \mathrm{m})\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{Nm}^{2}\right)=7.1 \times 10^{-11} \mathrm{C} / \mathrm{m}^{2}
$$

The plate at the origin has the charge $-7.1 \times 10^{-11} \mathrm{C} / \mathrm{m}^{2}$ and the other plate, at a positive $x$, has charge $+7.1 \times 10^{-11} \mathrm{C} / \mathrm{m}^{2}$ so that the electric field points in the negative direction.
50. We use Eq. 23-9 to find the components of the electric field.

$$
\begin{aligned}
& E_{x}=-\frac{\partial V}{\partial x}=0 ; E_{z}=-\frac{\partial V}{\partial z}=0 \\
& E_{y}=-\frac{\partial V}{\partial y}=-\frac{\partial}{\partial y}\left[\frac{b y}{\left(a^{2}+y^{2}\right)}\right]=-\frac{\left(a^{2}+y^{2}\right) b-b y(2 y)}{\left(a^{2}+y^{2}\right)^{2}}=\frac{\left(y^{2}-a^{2}\right) b}{\left(a^{2}+y^{2}\right)^{2}} \\
& \overrightarrow{\mathbf{E}}=\frac{\left(y^{2}-a^{2}\right) b}{\left(a^{2}+y^{2}\right)^{2}} \hat{\mathbf{j}}
\end{aligned}
$$

51. We use Eq. 23-9 to find the components of the electric field.

$$
\begin{aligned}
& E_{x}=-\frac{\partial V}{\partial x}=-2.5 y+3.5 y z ; E_{y}=-\frac{\partial V}{\partial y}=-2 y-2.5 x+3.5 x z ; E_{z}=-\frac{\partial V}{\partial z}=3.5 x y \\
& \overrightarrow{\mathbf{E}}=(-2.5 y+3.5 y z) \hat{\mathbf{i}}+(-2 y-2.5 x+3.5 x z) \hat{\mathbf{j}}+(3.5 x y) \hat{\mathbf{k}}
\end{aligned}
$$

52. We use the potential to find the electric field, the electric field to find the force, and the force to find the acceleration.

$$
\begin{aligned}
& E_{x}=-\frac{\partial V}{\partial x} ; F_{x}=q E_{x} ; a_{x}=\frac{F_{x}}{m}=\frac{q E_{x}}{m}=-\frac{q}{m} \frac{\partial V}{\partial x}=-\frac{q}{m} \frac{\partial V}{\partial x} \\
& a_{x}(x=2.0 \mathrm{~m})=-\frac{2.0 \times 10^{-6} \mathrm{C}}{5.0 \times 10^{-5} \mathrm{~kg}}\left[2\left(2.0 \mathrm{~V} / \mathrm{m}^{2}\right)(2.0 \mathrm{~m})-3\left(3.0 \mathrm{~V} / \mathrm{m}^{3}\right)(2.0 \mathrm{~m})^{2}\right]=1.1 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

53. (a) The potential along the $y$ axis was derived in Problem 38.

$$
\begin{aligned}
& V_{y a x i s}=\frac{Q}{8 \pi \varepsilon_{0} \ell}\left[\ln \left(\frac{\sqrt{\ell^{2}+y^{2}}+\ell}{\sqrt{\ell^{2}+y^{2}}-\ell}\right)\right]=\frac{Q}{8 \pi \varepsilon_{0} \ell}\left[\ln \left(\sqrt{\ell^{2}+y^{2}}+\ell\right)-\ln \left(\sqrt{\ell^{2}+y^{2}}-\ell\right)\right] \\
& E_{y}=-\frac{\partial V}{\partial y}=-\frac{Q}{8 \pi \varepsilon_{0} \ell}\left[\frac{\frac{1}{2}\left(\ell^{2}+y^{2}\right)^{-1 / 2} 2 y}{\sqrt{\ell^{2}+y^{2}}+\ell}-\frac{\frac{1}{2}\left(\ell^{2}+y^{2}\right)^{-1 / 2} 2 y}{\sqrt{\ell^{2}+y^{2}}-\ell}\right]=\frac{Q}{4 \pi \varepsilon_{0} y \sqrt{\ell^{2}+y^{2}}}
\end{aligned}
$$

From the symmetry of the problem, this field will point along the $y$ axis.

$$
\overrightarrow{\mathbf{E}}=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{y \sqrt{\ell^{2}+y^{2}}} \hat{\mathbf{j}}
$$

Note that for $y \gg \ell$, this reduces to the field of a point charge at the origin.
(b) The potential along the $x$ axis was derived in Problem 39.

$$
\begin{aligned}
& V_{x \text { axis }}=\frac{Q}{8 \pi \varepsilon_{0} \ell}\left[\ln \left(\frac{x+\ell}{x-\ell}\right)\right]=\frac{Q}{8 \pi \varepsilon_{0} \ell}[\ln (x+\ell)-\ln (x-\ell)] \\
& E_{x}=-\frac{\partial V}{\partial x}=-\frac{Q}{8 \pi \varepsilon_{0} \ell}\left[\frac{1}{x+\ell}-\frac{1}{x-\ell}\right]=\frac{1}{4 \pi \varepsilon_{0}}\left(\frac{Q}{x^{2}-\ell^{2}}\right)
\end{aligned}
$$

From the symmetry of the problem, this field will point along the $x$ axis.

$$
\overrightarrow{\mathbf{E}}=\frac{1}{4 \pi \varepsilon_{0}}\left(\frac{Q}{x^{2}-\ell^{2}}\right) \hat{\mathbf{i}}
$$

Note that for $x \gg \ell$, this reduces to the field of a point charge at the origin.
54. Let the side length of the equilateral triangle be $\ell$. Imagine bringing the electrons in from infinity one at a time. It takes no work to bring the first electron to its final location, because there are no other charges present. Thus $W_{1}=0$. The work done in bringing in the second electron to its final location is equal to the charge on the electron times the potential
 (due to the first electron) at the final location of the second electron.
Thus $W_{2}=(-e)\left(-\frac{1}{4 \pi \varepsilon_{0}} \frac{e}{\ell}\right)=\frac{1}{4 \pi \varepsilon_{0}} \frac{e^{2}}{L}$. The work done in bringing the third electron to its final
location is equal to the charge on the electron times the potential (due to the first two electrons).
Thus $W_{3}=(-e)\left(-\frac{1}{4 \pi \varepsilon_{0}} \frac{e}{\ell}-\frac{1}{4 \pi \varepsilon_{0}} \frac{e}{\ell}\right)=\frac{1}{4 \pi \varepsilon_{0}} \frac{2 e^{2}}{\ell}$. The total work done is the sum $W_{1}+W_{2}+W_{3}$.

$$
\begin{aligned}
W & =W_{1}+W_{2}+W_{3}=0+\frac{1}{4 \pi \varepsilon_{0}} \frac{e^{2}}{\ell}+\frac{1}{4 \pi \varepsilon_{0}} \frac{2 e^{2}}{\ell}=\frac{1}{4 \pi \varepsilon_{0}} \frac{3 e^{2}}{\ell}=\frac{3\left(8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)\left(1.60 \times 10^{-19} \mathrm{C}\right)^{2}}{\left(1.0 \times 10^{-10} \mathrm{~m}\right)} \\
& =6.9 \times 10^{-18} \mathrm{~J}=6.9 \times 10^{-18} \mathrm{~J}\left(\frac{1 \mathrm{eV}}{1.60 \times 10^{-19} \mathrm{~J}}\right)=43 \mathrm{eV}
\end{aligned}
$$

55. The gain of kinetic energy comes from a loss of potential energy due to conservation of energy, and the magnitude of the potential difference is the energy per unit charge. The helium nucleus has a charge of $2 e$.

$$
\Delta V=\frac{\Delta U}{q}=-\frac{\Delta K}{q}=-\frac{125 \times 10^{3} \mathrm{eV}}{2 e}=-62.5 \mathrm{kV}
$$

The negative sign indicates that the helium nucleus had to go from a higher potential to a lower potential.
56. The kinetic energy of the particle is given in each case. Use the kinetic energy to find the speed.
(a) $\frac{1}{2} m v^{2}=K \rightarrow v=\sqrt{\frac{2 K}{m}}=\sqrt{\frac{2(1500 \mathrm{eV})\left(1.60 \times 10^{-19} \mathrm{~J} / \mathrm{eV}\right)}{9.11 \times 10^{-31} \mathrm{~kg}}}=2.3 \times 10^{7} \mathrm{~m} / \mathrm{s}$
(b) $\frac{1}{2} m v^{2}=K \rightarrow v=\sqrt{\frac{2 K}{m}}=\sqrt{\frac{2(1500 \mathrm{eV})\left(1.60 \times 10^{-19} \mathrm{~J} / \mathrm{eV}\right)}{1.67 \times 10^{-27} \mathrm{~kg}}}=5.4 \times 10^{5} \mathrm{~m} / \mathrm{s}$
57. The potential energy of the two-charge configuration (assuming they are both point charges) is given by Eq. 23-10.

$$
\begin{aligned}
U & =\frac{1}{4 \pi \varepsilon_{0}} \frac{Q_{1} Q_{2}}{r}=-\frac{1}{4 \pi \varepsilon_{0}} \frac{e^{2}}{r} \\
\Delta U & =U_{\text {final }}-U_{\text {initial }}=\frac{e^{2}}{4 \pi \varepsilon_{0}}\left(\frac{1}{r_{\text {initial }}}-\frac{1}{r_{\text {final }}}\right) \\
& =\left(8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)\left(1.60 \times 10^{-19} \mathrm{C}\right)^{2}\left(\frac{1}{0.110 \times 10^{-9} \mathrm{~m}}-\frac{1}{0.100 \times 10^{-9} \mathrm{~m}}\right)\left(\frac{1 \mathrm{eV}}{1.60 \times 10^{-19} \mathrm{~J}}\right) \\
& =-1.31 \mathrm{eV}
\end{aligned}
$$

Thus 1.3 eV of potential energy was lost.
58. The kinetic energy of the alpha particle is given. Use the kinetic energy to find the speed.

$$
\frac{1}{2} m v^{2}=K \rightarrow v=\sqrt{\frac{2 K}{m}}=\sqrt{\frac{2\left(5.53 \times 10^{6} \mathrm{eV}\right)\left(1.60 \times 10^{-19} \mathrm{~J} / \mathrm{eV}\right)}{6.64 \times 10^{-27} \mathrm{~kg}}}=1.63 \times 10^{7} \mathrm{~m} / \mathrm{s}
$$

59. Following the same method as presented in Section 23-8, we get the following results.
(a) 1 charge: No work is required to move a single charge into a position, so $U_{1}=0$.

2 charges: This represents the interaction between $Q_{1}$ and $Q_{2}$.

$$
U_{2}=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q_{1} Q_{2}}{r_{12}}
$$

3 charges: $\quad$ This now adds the interactions between $Q_{1} \& Q_{3}$ and $Q_{2} \& Q_{3}$.

$$
U_{3}=\frac{1}{4 \pi \varepsilon_{0}}\left(\frac{Q_{1} Q_{2}}{r_{12}}+\frac{Q_{1} Q_{3}}{r_{13}}+\frac{Q_{2} Q_{3}}{r_{23}}\right)
$$

4 charges: $\quad$ This now adds the interaction between $Q_{1} \& Q_{4}, Q_{2} \& Q_{4}$, and $Q_{3} \& Q_{4}$.

$$
U_{4}=\frac{1}{4 \pi \varepsilon_{0}}\left(\frac{Q_{1} Q_{2}}{r_{12}}+\frac{Q_{1} Q_{3}}{r_{13}}+\frac{Q_{1} Q_{4}}{r_{14}}+\frac{Q_{2} Q_{3}}{r_{23}}+\frac{Q_{2} Q_{4}}{r_{24}}+\frac{Q_{3} Q_{4}}{r_{34}}\right)
$$


(b) 5 charges: This now adds the interaction between $Q_{1} \& Q_{5}, Q_{2} \& Q_{5}, Q_{3} \& Q_{5}$, and $Q_{4} \& Q_{5}$.

$$
U_{5}=\frac{1}{4 \pi \varepsilon_{0}}\left(\frac{Q_{1} Q_{2}}{r_{12}}+\frac{Q_{1} Q_{3}}{r_{13}}+\frac{Q_{1} Q_{4}}{r_{14}}+\frac{Q_{1} Q_{5}}{r_{15}}+\frac{Q_{2} Q_{3}}{r_{23}}+\frac{Q_{2} Q_{4}}{r_{24}}+\frac{Q_{2} Q_{5}}{r_{25}}+\frac{Q_{3} Q_{4}}{r_{34}}+\frac{Q_{3} Q_{5}}{r_{35}}+\frac{Q_{4} Q_{5}}{r_{45}}\right)
$$


60. (a) The potential energy of the four-charge configuration was derived in Problem 59. Number the charges clockwise, starting in the upper right hand corner of the square.

$$
\begin{aligned}
U_{4} & =\frac{1}{4 \pi \varepsilon_{0}}\left(\frac{Q_{1} Q_{2}}{r_{12}}+\frac{Q_{1} Q_{3}}{r_{13}}+\frac{Q_{1} Q_{4}}{r_{14}}+\frac{Q_{2} Q_{3}}{r_{23}}+\frac{Q_{2} Q_{4}}{r_{24}}+\frac{Q_{3} Q_{4}}{r_{34}}\right) \\
& =\frac{Q^{2}}{4 \pi \varepsilon_{0}}\left(\frac{1}{b}+\frac{1}{\sqrt{2} b}+\frac{1}{b}+\frac{1}{b}+\frac{1}{\sqrt{2} b}+\frac{1}{b}\right)=\frac{Q^{2}}{4 \pi \varepsilon_{0} b}(4+\sqrt{2})
\end{aligned}
$$

(b) The potential energy of the fifth charge is due to the interaction between the fifth charge and each of the other four charges. Each of those interaction terms is of the same magnitude since the fifth charge is the same distance from each of the other four charges.

$$
U_{\substack{5 \text { th } \\ \text { charge }}}=\frac{Q^{2}}{4 \pi \varepsilon_{0} b}(4 \sqrt{2})
$$

(c) If the center charge were moved away from the center, it would be moving closer to 1 or 2 of the other charges. Since the charges are all of the same sign, by moving closer, the center charge would be repelled back towards its original position. Thus it is in a place of stable equilibrium.
(d) If the center charge were moved away from the center, it would be moving closer to 1 or 2 of the other charges. Since the corner charges are of the opposite sign as the center charge, the center charge would be attracted towards those closer charges, making the center charge move even farther from the center. So it is in a place of unstable equilibrium.
61. (a) The electron was accelerated through a potential difference of 1.33 kV (moving from low potential to high potential) in gaining 1.33 keV of kinetic energy. The proton is accelerated through the opposite potential difference as the electron, and has the exact opposite charge. Thus the proton gains the same kinetic energy, 1.33 keV .
(b) Both the proton and the electron have the same KE. Use that to find the ratio of the speeds.

$$
\frac{1}{2} m_{\mathrm{p}} v_{\mathrm{p}}^{2}=\frac{1}{2} m_{\mathrm{e}} v_{\mathrm{e}}^{2} \rightarrow \frac{v_{\mathrm{e}}}{v_{\mathrm{p}}}=\sqrt{\frac{m_{\mathrm{p}}}{m_{e}}}=\sqrt{\frac{1.67 \times 10^{-27} \mathrm{~kg}}{9.11 \times 10^{-31} \mathrm{~kg}}}=42.8
$$

The lighter electron is moving about 43 times faster than the heavier proton.
62. We find the energy by bringing in a small amount of charge at a time, similar to the method given in Section 23-8. Consider the sphere partially charged, with charge $q<Q$. The potential at the surface of the sphere is $V=\frac{1}{4 \pi \varepsilon_{0}} \frac{q}{r_{0}}$, and the work to add a charge $d q$ to that sphere will increase the potential energy by $d U=V d q$. Integrate over the entire charge to find the total potential energy.

$$
U=\int d U=\int_{0}^{Q} \frac{1}{4 \pi \varepsilon_{0}} \frac{q}{r_{0}} d q=\frac{1}{8 \pi \varepsilon_{0}} \frac{Q^{2}}{r_{0}}
$$

63. The two fragments can be treated as point charges for purposes of calculating their potential energy. Use Eq. 23-10 to calculate the potential energy. Using energy conservation, the potential energy is all converted to kinetic energy as the two fragments separate to a large distance.

$$
\begin{aligned}
& E_{\text {initial }}=E_{\text {final }} \rightarrow U_{\text {initial }}=K_{\text {final }}=\frac{1}{4 \pi \varepsilon_{0}} \frac{q_{1} q_{2}}{r} V \\
& \quad=\left(8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{(38)(54)\left(1.60 \times 10^{-19} \mathrm{C}\right)^{2}}{\left(5.5 \times 10^{-15} \mathrm{~m}\right)+\left(6.2 \times 10^{-15} \mathrm{~m}\right)}\left(\frac{1 \mathrm{eV}}{1.60 \times 10^{-19} \mathrm{~J}}\right)=250 \times 10^{6} \mathrm{eV} \\
& \quad=250 \mathrm{MeV}
\end{aligned}
$$

This is about $25 \%$ greater than the observed kinetic energy of 200 MeV .
64. We find the energy by bringing in a small amount of spherically symmetric charge at a time, similar to the method given in Section 23-8. Consider that the sphere has been partially constructed, and so has a charge $q<Q$, contained in a radius $r<r_{0}$. Since the sphere is made of uniformly charged material, the charge density of the sphere must be $\rho_{\mathrm{E}}=\frac{Q}{\frac{4}{3} \pi r_{0}^{3}}$. Thus the partially constructed sphere also satisfies $\rho_{\mathrm{E}}=\frac{q}{\frac{4}{3} \pi r^{3}}$, and so $\frac{q}{\frac{4}{3} \pi r^{3}}=\frac{Q}{\frac{4}{3} \pi r_{0}^{3}} \rightarrow q=\frac{Q r^{3}}{r_{0}^{3}}$. The potential at the surface of that sphere can now found.

$$
V=\frac{1}{4 \pi \varepsilon_{0}} \frac{q}{r}=\frac{1}{4 \pi \varepsilon_{0}} \frac{\frac{Q r^{3}}{r_{0}^{3}}}{r}=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q r^{2}}{r_{0}^{3}}
$$

We now add another infinitesimally thin shell to the partially constructed sphere. The charge of that shell is $d q=\rho_{\mathrm{E}} 4 \pi r^{2} d r$. The work to add charge $d q$ to the sphere will increase the potential energy by $d U=V d q$. Integrate over the entire sphere to find the total potential energy.

$$
U=\int d U=\int V d q=\int_{0}^{r_{0}} \frac{1}{4 \pi \varepsilon_{0}} \frac{Q r^{2}}{r_{0}^{3}} \rho_{\mathrm{E}} 4 \pi r^{2} d r=\frac{\rho_{\mathrm{E}}}{\varepsilon_{0}} \frac{Q}{r_{0}^{r_{0}^{3}}} r_{0}^{r_{0}} r^{4} d r=\frac{3 Q^{2}}{20 \pi \varepsilon_{0} r_{0}}
$$

65. The ideal gas model, from Eq. $18-4$, says that $K=\frac{1}{2} m v_{\mathrm{ms}}^{2}=\frac{3}{2} k T$.

$$
\begin{aligned}
& K=\frac{1}{2} m v_{\mathrm{rms}}^{2}=\frac{3}{2} k T \rightarrow v_{\substack{\mathrm{rms} \\
273 \mathrm{~K}}}=\sqrt{\frac{3 k T}{m}}=\sqrt{\frac{3\left(1.38 \times 10^{-23} \mathrm{~J} / \mathrm{K}\right)(273 \mathrm{~K})}{9.11 \times 10^{-31} \mathrm{~kg}}}=1.11 \times 10^{5} \mathrm{~m} / \mathrm{s} \\
& v_{\mathrm{rms}}=\sqrt{\frac{3 k T}{m}}=\sqrt{\frac{3\left(1.38 \times 10^{-23} \mathrm{~J} / \mathrm{K}\right)(2700 \mathrm{~K})}{9.11 \times 10^{-31} \mathrm{~kg}}}=3.5 \times 10^{5} \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

66. If there were no deflecting field, the electrons would hit the center of the screen. If an electric field of a certain direction moves the electrons towards one extreme of the screen, then the opposite field will move the electrons to the opposite extreme of the screen. So we solve for the field to move the electrons to one extreme of the screen. Consider three parts to the
electron's motion, and see the diagram, which is a top view. First, during the horizontal acceleration phase, energy will be
 conserved and so the horizontal speed of the electron $v_{x}$ can be found from the accelerating potential $V$. Secondly, during the deflection phase, a vertical force will be applied by the uniform electric field which gives the electron a leftward velocity, $v_{y}$. We assume that there is very little leftward displacement during this time. Finally, after the electron leaves the region of electric field, it travels in a straight line to the left edge of the screen.

Acceleration:

$$
U_{\text {initial }}=K_{\text {final }} \rightarrow e V=\frac{1}{2} m v_{x}^{2} \rightarrow v_{x}=\sqrt{\frac{2 e V}{m}}
$$

## Deflection:

$$
\begin{aligned}
& \text { time in field: } \Delta x_{\text {field }}=v_{x} t_{\text {field }} \rightarrow t_{\text {field }}=\frac{\Delta x_{\text {field }}}{v_{x}} \\
& F_{y}=e E=m a_{y} \rightarrow a_{y}=\frac{e E}{m} \quad v_{y}=v_{0}+a_{y} t_{\text {field }}=0+\frac{e E \Delta x_{\text {field }}}{m v_{x}}
\end{aligned}
$$

Screen:

$$
\begin{aligned}
& \Delta x_{\text {screen }}=v_{x} t_{\text {screen }} \rightarrow t_{\text {screen }}=\frac{\Delta x_{\text {screen }}}{v_{x}} \quad \Delta y_{\text {screen }}=v_{y} t_{\text {screen }}=v_{y} \frac{\Delta x_{\text {screen }}}{v_{x}} \\
& \frac{\Delta y_{\text {screen }}}{\Delta x_{\text {screen }}}=\frac{v_{y}}{v_{x}}=\frac{\frac{e E \Delta x_{\text {field }}}{m v_{x}}}{v_{x}}=\frac{e E \Delta x_{\text {field }}}{m v_{x}^{2}} \rightarrow \\
& E=\frac{\Delta y_{\text {screen }} m v_{x}^{2}}{\Delta x_{\text {screen }} e \Delta x_{\text {field }}}=\frac{\Delta y_{\text {screen }} m \frac{2 e V}{m}}{\Delta x_{\text {screen }} e \Delta x_{\text {field }}}=\frac{2 V \Delta y_{\text {screen }}}{\Delta x_{\text {screen }} \Delta x_{\text {field }}}=\frac{2\left(6.0 \times 10^{3} \mathrm{~V}\right)(0.14 \mathrm{~m})}{(0.34 \mathrm{~m})(0.026 \mathrm{~m})} \\
& \quad=1.90 \times 10^{5} \mathrm{~V} / \mathrm{m} \approx 1.9 \times 10^{5} \mathrm{~V} / \mathrm{m}
\end{aligned}
$$

As a check on our assumptions, we calculate the upward distance that the electron would move while in the electric field.

$$
\begin{aligned}
\Delta y & =v_{0} t_{\text {field }}+\frac{1}{2} a_{y} t_{\text {field }}^{2}=0+\frac{1}{2}\left(\frac{e E}{m}\right)\left(\frac{\Delta x_{\text {field }}}{v_{x}}\right)^{2}=\frac{e E\left(\Delta x_{\text {field }}\right)^{2}}{2 m\left(\frac{2 e V}{m}\right)}=\frac{E\left(\Delta x_{\text {field }}\right)^{2}}{4 V} \\
& =\frac{\left(1.90 \times 10^{5} \mathrm{~V} / \mathrm{m}\right)(0.026 \mathrm{~m})^{2}}{4(6000 \mathrm{~V})}=5.4 \times 10^{-3} \mathrm{~m}
\end{aligned}
$$

This is about $4 \%$ of the total 15 cm vertical deflection, and so for an estimation, our approximation is acceptable. And so the field must vary from $+1.9 \times 10^{5} \mathrm{~V} / \mathrm{m}$ to $-1.9 \times 10^{5} \mathrm{~V} / \mathrm{m}$
67. Consider three parts to the electron's motion. First, during the horizontal acceleration phase, energy will be conserved and so the horizontal speed of the electron $v_{x}$ can be found from the accelerating potential, $V$. Secondly, during the deflection phase, a vertical force will be applied by the uniform

electric field which gives the electron an upward velocity, $v_{y} . \Delta x_{\text {field }}$
We assume that there is very little upward displacement during this time. Finally, after the electron leaves the region of electric field, it travels in a straight line to the top of the screen.

Acceleration:

$$
U_{\mathrm{initial}}=K_{\mathrm{final}} \rightarrow e V=\frac{1}{2} m v_{x}^{2} \rightarrow v_{x}=\sqrt{\frac{2 e V}{m}}
$$

## Deflection:

$$
\begin{aligned}
& \text { time in field: } \Delta x_{\text {field }}=v_{x} t_{\text {field }} \rightarrow t_{\text {field }}=\frac{\Delta x_{\text {field }}}{v_{x}} \\
& F_{y}=e E=m a_{y} \rightarrow a_{y}=\frac{e E}{m} \quad v_{y}=v_{0}+a_{y} t_{\text {field }}=0+\frac{e E \Delta x_{\text {field }}}{m v_{x}}
\end{aligned}
$$

Screen:

$$
\begin{aligned}
& \Delta x_{\text {screen }}=v_{x} t_{\text {screen }} \rightarrow t_{\text {screen }}=\frac{\Delta x_{\text {screen }}}{v_{x}} \quad \Delta y_{\text {screen }}=v_{y} t_{\text {screen }}=v_{y} \frac{\Delta x_{\text {screen }}}{v_{x}} \\
& \frac{\Delta y_{\text {screen }}}{\Delta x_{\text {screen }}}=\frac{v_{y}}{v_{x}}=\frac{\frac{e E \Delta x_{\text {field }}}{m v_{x}}}{v_{x}}=\frac{e E \Delta x_{\text {field }}}{m v_{x}^{2}} \rightarrow \\
& E=\frac{\Delta y_{\text {screen }} m v_{x}^{2}}{\Delta x_{\text {screen }} \Delta \Delta x_{\text {field }}}=\frac{\Delta y_{\text {screen }} m \frac{2 e V}{m}}{\Delta x_{\text {screen }} e \Delta x_{\text {field }}}=\frac{2 V \Delta y_{\text {screen }}}{\Delta x_{\text {screen }} \Delta x_{\text {field }}}=\frac{2(7200 \mathrm{~V})(0.11 \mathrm{~m})}{(0.22 \mathrm{~m})(0.028 \mathrm{~m})} \\
& \quad=2.57 \times 10^{5} \mathrm{~V} / \mathrm{m} \approx 2.6 \times 10^{5} \mathrm{~V} / \mathrm{m}
\end{aligned}
$$

As a check on our assumptions, we calculate the upward distance that the electron would move while in the electric field.

$$
\begin{aligned}
\Delta y & =v_{0} t_{\text {field }}+\frac{1}{2} a_{y} t_{\text {field }}^{2}=0+\frac{1}{2}\left(\frac{e E}{m}\right)\left(\frac{\Delta x_{\text {field }}}{v_{x}}\right)^{2}=\frac{e E\left(\Delta x_{\text {field }}\right)^{2}}{2 m\left(\frac{2 e V}{m}\right)}=\frac{E\left(\Delta x_{\text {field }}\right)^{2}}{4 V} \\
& =\frac{\left(2.97 \times 10^{5} \mathrm{~V} / \mathrm{m}\right)(0.028 \mathrm{~m})^{2}}{4(7200 \mathrm{~V})}=8.1 \times 10^{-3} \mathrm{~m}
\end{aligned}
$$

This is about $7 \%$ of the total 11 cm vertical deflection, and so for an estimation, our approximation is acceptable.
68. The potential of the earth will increase because the "neutral" Earth will now be charged by the removing of the electrons. The excess charge will be the elementary charge times the number of electrons removed. We approximate this change in potential by using a spherical Earth with all the excess charge at the surface.

$$
\begin{aligned}
Q & =\left(\frac{1.602 \times 10^{-19} \mathrm{C}}{e^{-}}\right)\left(\frac{10 e^{-}}{\mathrm{H}_{2} \mathrm{O} \text { molecule }}\right)\left(\frac{6.02 \times 10^{23} \text { molecules }}{0.018 \mathrm{~kg}}\right)\left(\frac{1000 \mathrm{~kg}}{\mathrm{~m}^{3}}\right) \frac{4}{3} \pi(0.00175 \mathrm{~m})^{3} \\
& =1203 \mathrm{C} \\
V & =\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{R_{\text {Earth }}}=\left(8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{1203 \mathrm{C}}{6.38 \times 10^{6} \mathrm{~m}}=1.7 \times 10^{6} \mathrm{~V}
\end{aligned}
$$

69. The potential at the surface of a charged sphere is that of a point charge of the same magnitude, located at the center of the sphere.

$$
V=\frac{1}{4 \pi \varepsilon_{0}} \frac{q}{r}=\left(8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{\left(1 \times 10^{-8} \mathrm{C}\right)}{(0.15 \mathrm{~m})}=599.3 \mathrm{~V} \approx 600 \mathrm{~V}
$$

70. 


71. Let $d_{1}$ represent the distance from the left charge to point b , and let $d_{2}$ represent the distance from the right charge to point b . Let $Q$ represent the positive charges, and let $q$ represent the negative charge that moves. The change in potential energy is given by Eq. 23-2b.

$$
\begin{aligned}
d_{1}=\sqrt{12^{2}+14^{2}} \mathrm{~cm}=18.44 \mathrm{~cm} \quad d_{2}=\sqrt{14^{2}+24^{2}} \mathrm{~cm}=27.78 \mathrm{~cm} \\
\begin{aligned}
U_{\mathrm{b}}-U_{\mathrm{a}} & =q\left(V_{\mathrm{b}}-V_{\mathrm{a}}\right)=q \frac{1}{4 \pi \varepsilon_{0}}\left[\left(\frac{Q}{0.1844 \mathrm{~m}}+\frac{Q}{0.2778 \mathrm{~m}}\right)-\left(\frac{Q}{0.12 \mathrm{~m}}+\frac{Q}{0.24 \mathrm{~m}}\right)\right] \\
& =\frac{1}{4 \pi \varepsilon_{0}} Q q\left[\left(\frac{1}{0.1844 \mathrm{~m}}+\frac{1}{0.2778 \mathrm{~m}}\right)-\left(\frac{1}{0.12 \mathrm{~m}}+\frac{1}{0.24 \mathrm{~m}}\right)\right] \\
& =\left(8.99 \times 10^{9} \mathrm{~N}^{2} \mathrm{~m}^{2} / \mathrm{C}^{2}\right)\left(-1.5 \times 10^{-6} \mathrm{C}\right)\left(33 \times 10^{-6} \mathrm{C}\right)\left(-3.477 \mathrm{~m}^{-1}\right)=1.547 \mathrm{~J} \approx 1.5 \mathrm{~J}
\end{aligned}
\end{aligned}
$$

72. (a) All eight charges are the same distance from the center of the cube. Use Eq. 23-5 for the potential of a point charge.

$$
V_{\text {center }}=8 \frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{\frac{\sqrt{3}}{2} \ell}=\frac{16}{\sqrt{3}} \frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{\ell} \approx 9.24 \frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{\ell}
$$

(b) For the seven charges that produce the potential at a corner, three are a distance $\ell$ away from that corner, three are a distance $\sqrt{2} \ell$ away from that corner, and one is a distance $\sqrt{3} \ell$ away from that corner.

$$
V_{\text {corner }}=3 \frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{\ell}+3 \frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{\sqrt{2 \ell}}+\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{\sqrt{3 \ell}}=\left(3+\frac{3}{\sqrt{2}}+\frac{1}{\sqrt{3}}\right) \frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{\ell} \approx 5.70 \frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{\ell}
$$

(c) The total potential energy of the system is half the energy found by multiplying each charge times the potential at a corner. The factor of half comes from the fact that if you took each charge times the potential at a corner, you would be counting each pair of charges twice.

$$
U=\frac{1}{2} 8\left(Q V_{\text {corner }}\right)=4\left(3+\frac{3}{\sqrt{2}}+\frac{1}{\sqrt{3}}\right) \frac{1}{4 \pi \varepsilon_{0}} \frac{Q^{2}}{\ell} \approx 22.8 \frac{1}{4 \pi \varepsilon_{0}} \frac{Q^{2}}{\ell}
$$

73. The electric force on the electron must be the same magnitude as the weight of the electron. The magnitude of the electric force is the charge on the electron times the magnitude of the electric field. The electric field is the potential difference per meter: $E=V / d$.

$$
\begin{aligned}
& F_{\mathrm{E}}=m g ; F_{\mathrm{E}}=|q| E=\mathrm{eV} / \mathrm{d} \rightarrow \mathrm{eV} / d=m g \rightarrow \\
& V=\frac{m g d}{e}=\frac{\left(9.11 \times 10^{-31} \mathrm{~kg}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.035 \mathrm{~m})}{1.60 \times 10^{-19} \mathrm{C}}=2.0 \times 10^{-12} \mathrm{~V}
\end{aligned}
$$

Since it takes such a tiny voltage to balance gravity, the thousands of volts in a television set are more than enough (by many orders of magnitude) to move electrons upward against the force of gravity.
74. From Problem 59, the potential energy of a configuration of four charges is $U=\frac{1}{4 \pi \varepsilon_{0}}\left(\frac{Q_{1} Q_{2}}{r_{12}}+\frac{Q_{1} Q_{3}}{r_{13}}+\frac{Q_{1} Q_{4}}{r_{14}}+\frac{Q_{2} Q_{3}}{r_{23}}+\frac{Q_{2} Q_{4}}{r_{24}}+\frac{Q_{3} Q_{4}}{r_{34}}\right)$.
Let a side of the square be $\ell$, and number the charges clockwise starting with the upper left corner.

$$
\begin{aligned}
U & =\frac{1}{4 \pi \varepsilon_{0}}\left(\frac{Q_{1} Q_{2}}{r_{12}}+\frac{Q_{1} Q_{3}}{r_{13}}+\frac{Q_{1} Q_{4}}{r_{14}}+\frac{Q_{2} Q_{3}}{r_{23}}+\frac{Q_{2} Q_{4}}{r_{24}}+\frac{Q_{3} Q_{4}}{r_{34}}\right) \quad 2 Q--\ell- \\
& =\frac{1}{4 \pi \varepsilon_{0}}\left(\frac{Q(2 Q)}{\ell}+\frac{Q(-3 Q)}{\sqrt{2} \ell}+\frac{Q(2 Q)}{\ell}+\frac{(2 Q)(-3 Q)}{\ell}+\frac{(2 Q)(2 Q)}{\sqrt{2} \ell}+\frac{(-3 Q)(2 Q)}{\ell}\right) \\
& =\frac{Q^{2}}{4 \pi \varepsilon_{0} \ell}\left(\frac{1}{\sqrt{2}}-8\right)=\left(8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{\left(3.1 \times 10^{-6} \mathrm{C}\right)^{2}}{0.080 \mathrm{~m}}\left(\frac{1}{\sqrt{2}}-8\right)=-7.9 \mathrm{~J}
\end{aligned}
$$

75. The kinetic energy of the electrons (provided by the UV light) is converted completely to potential energy at the plate since they are stopped. Use energy conservation to find the emitted speed, taking the 0 of PE to be at the surface of the barium.

$$
\begin{aligned}
& \mathrm{KE}_{\text {initial }}=\mathrm{PE}_{\text {final }} \rightarrow \frac{1}{2} m v^{2}=q V \rightarrow \\
& v=\sqrt{\frac{2 q V}{m}}=\sqrt{\frac{2\left(-1.60 \times 10^{-19} \mathrm{C}\right)(-3.02 \mathrm{~V})}{9.11 \times 10^{-31} \mathrm{~kg}}}=1.03 \times 10^{6} \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

76. To find the angle, the horizontal and vertical components of the velocity are needed. The horizontal component can be found using conservation of energy for the initial acceleration of the electron. That component is not changed as the electron passes through the plates. The vertical component can be found using the vertical acceleration due to the potential difference of the plates, and the time the electron spends between the plates.

## Horizontal:

$$
\mathrm{PE}_{\text {inital }}=\mathrm{KE}_{\text {final }} \rightarrow q V=\frac{1}{2} m v_{x}^{2} \quad t=\frac{\Delta x}{v_{x}}
$$

## Vertical:

$$
F_{\mathrm{E}}=q E_{y}=m a=m \frac{\left(v_{y}-v_{y 0}\right)}{t} \rightarrow v_{y}=\frac{q E_{y} t}{m}=\frac{q E_{y} \Delta x}{m v_{x}}
$$

Combined:

$$
\begin{aligned}
& \tan \theta=\frac{v_{y}}{v_{x}}=\frac{\frac{q E_{y} \Delta x}{m v_{x}}}{v_{x}}=\frac{q E_{y} \Delta x}{m v_{x}^{2}}=\frac{q E_{y} \Delta x}{2 q V}=\frac{E_{y} \Delta x}{2 V}=\frac{\left(\frac{250 \mathrm{~V}}{0.013 \mathrm{~m}}\right)(0.065 \mathrm{~m})}{2(5500 \mathrm{~V})}=0.1136 \\
& \theta=\tan ^{-1} 0.1136=6.5^{\circ}
\end{aligned}
$$

77. Use Eq. 23-5 to find the potential due to each charge. Since the triangle is equilateral, the 30-60-90 triangle relationship says that the distance from a corner to the midpoint of the opposite side is $\sqrt{3} \ell / 2$.

$$
\begin{aligned}
V_{\mathrm{A}} & =\frac{1}{4 \pi \varepsilon_{0}} \frac{(-Q)}{\ell / 2}+\frac{1}{4 \pi \varepsilon_{0}} \frac{(-3 Q)}{\ell / 2}+\frac{1}{4 \pi \varepsilon_{0}} \frac{(Q)}{\sqrt{3} \ell / 2}=\frac{1}{4 \pi \varepsilon_{0}} \frac{2 Q}{\ell}\left(-4+\frac{1}{\sqrt{3}}\right) \\
& =\frac{Q}{\pi \varepsilon_{0} \ell}\left(\frac{\sqrt{3}}{6}-2\right) \\
V_{\mathrm{B}} & =\frac{1}{4 \pi \varepsilon_{0}} \frac{(-Q)}{\ell / 2}+\frac{1}{4 \pi \varepsilon_{0}} \frac{(Q)}{\ell / 2}+\frac{1}{4 \pi \varepsilon_{0}} \frac{(-3 Q)}{\sqrt{3} \ell / 2}=-\frac{1}{4 \pi \varepsilon_{0}} \frac{6 Q}{\sqrt{3} \ell}=-\frac{\sqrt{3} Q}{2 \pi \varepsilon_{0} \ell} \\
V_{\mathrm{C}} & =\frac{1}{4 \pi \varepsilon_{0}} \frac{(Q)}{\ell / 2}+\frac{1}{4 \pi \varepsilon_{0}} \frac{(-3 Q)}{\ell / 2}+\frac{1}{4 \pi \varepsilon_{0}} \frac{(-Q)}{\sqrt{3} \ell / 2}=-\frac{1}{4 \pi \varepsilon_{0}} \frac{2 Q}{\ell}\left(2+\frac{1}{\sqrt{3}}\right)=-\frac{Q}{\pi \varepsilon_{0} \ell}\left(1+\frac{\sqrt{3}}{6}\right)
\end{aligned}
$$

78. Since the E-field points downward, the surface of the Earth is a lower potential than points above the surface. Call the surface of the Earth 0 volts. Then a height of 2.00 m has a potential of 300 V . We also call the surface of the Earth the 0 location for gravitational PE. Write conservation of energy relating the charged spheres at 2.00 m (where their speed is 0 ) and at ground level (where their electrical and gravitational potential energies are 0 ).

$$
\begin{aligned}
& E_{\text {initial }}=E_{\text {final }} \rightarrow m g h+q V=\frac{1}{2} m v^{2} \rightarrow v=\sqrt{2\left(g h+\frac{q V}{m}\right)} \\
& v_{+}=\sqrt{2\left[\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(2.00 \mathrm{~m})+\frac{\left(4.5 \times 10^{-4} \mathrm{C}\right)(300 \mathrm{~V})}{(0.340 \mathrm{~kg})}\right]}=6.3241 \mathrm{~m} / \mathrm{s} \\
& v_{-}=\sqrt{2\left[\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(2.00 \mathrm{~m})+\frac{\left(-4.5 \times 10^{-4} \mathrm{C}\right)(300 \mathrm{~V})}{(0.340 \mathrm{~kg})}\right]}=6.1972 \mathrm{~m} / \mathrm{s} \\
& v_{+}-v_{-}=6.3241 \mathrm{~m} / \mathrm{s}-6.1972 \mathrm{~m} / \mathrm{s}=0.13 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

79. (a) The energy is related to the charge and the potential difference by Eq. 23-3.

$$
\Delta U=q \Delta V \rightarrow \Delta V=\frac{\Delta U}{q}=\frac{4.8 \times 10^{6} \mathrm{~J}}{4.0 \mathrm{C}}=1.2 \times 10^{6} \mathrm{~V}
$$

(b) The energy (as heat energy) is used to raise the temperature of the water and boil it. Assume that room temperature is $20^{\circ} \mathrm{C}$.

$$
Q=m c \Delta T+m L_{\mathrm{f}} \quad \rightarrow
$$

$$
m=\frac{Q}{c \Delta T+L_{\mathrm{f}}}=\frac{4.8 \times 10^{6} \mathrm{~J}}{\left(4186 \frac{\mathrm{~J}}{\mathrm{~kg} \cdot \mathrm{C}^{\circ}}\right)\left(80 \mathrm{C}^{\circ}\right)+\left(22.6 \times 10^{5} \frac{\mathrm{~J}}{\mathrm{~kg}}\right)}=1.8 \mathrm{~kg}
$$

80. Use Eq. 23-7 for the dipole potential, and use Eq. 23-9 to determine the electric field.

$$
\begin{aligned}
V & =\frac{1}{4 \pi \varepsilon_{0}} \frac{p \cos \theta}{r^{2}}=\frac{p}{4 \pi \varepsilon_{0}} \frac{\frac{x}{\left(x^{2}+y^{2}\right)^{1 / 2}}}{x^{2}+y^{2}}=\frac{p}{4 \pi \varepsilon_{0}} \frac{x}{\left(x^{2}+y^{2}\right)^{3 / 2}} \\
E_{x} & =-\frac{\partial V}{\partial x}=-\frac{p}{4 \pi \varepsilon_{0}}\left[\frac{\left(x^{2}+y^{2}\right)^{3 / 2}-x \frac{3}{2}\left(x^{2}+y^{2}\right)^{1 / 2} 2 x}{\left(x^{2}+y^{2}\right)^{3}}\right]=\frac{p}{4 \pi \varepsilon_{0}}\left[\frac{2 x^{2}-y^{2}}{\left(x^{2}+y^{2}\right)^{5 / 2}}\right] \\
& =\frac{p}{4 \pi \varepsilon_{0}}\left[\frac{2 \cos ^{2} \theta-\sin ^{2} \theta}{r^{3}}\right] \\
E_{y} & =-\frac{\partial V}{\partial y}=-\frac{p x}{4 \pi \varepsilon_{0}}\left[-\frac{3}{2}\left(x^{2}+y^{2}\right)^{-5 / 2} 2 y\right]=\frac{p}{4 \pi \varepsilon_{0}}\left[\frac{3 x y}{\left.\left(x^{2}+y^{2}\right)^{5 / 2}\right]}\right] \frac{p}{4 \pi \varepsilon_{0}}\left[\frac{3 \cos \theta \sin \theta}{r^{3}}\right]
\end{aligned}
$$

Notice the $\frac{1}{r^{3}}$ dependence in both components, which is indicative of a dipole field.
81. (a) Since the reference level is given as $V=0$ at $r=\infty$, the potential outside the shell is that of a point charge with the same total charge.

$$
V=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{r}=\frac{1}{4 \pi \varepsilon_{0}} \frac{\rho_{\mathrm{E}}\left(\frac{4}{3} \pi r_{2}^{3}-\frac{4}{3} \pi r_{1}^{3}\right)}{r}=\frac{\rho_{\mathrm{E}}}{3 \varepsilon_{0}} \frac{\left(r_{2}^{3}-r_{1}^{3}\right)}{r}, r>r_{2}
$$

Note that the potential at the surface of the shell is $V_{r_{2}}=\frac{\rho_{\mathrm{E}}}{3 \varepsilon_{0}}\left(r_{2}^{2}-\frac{r_{1}^{3}}{r_{2}}\right)$.
(b) To find the potential in the region $r_{1}<r<r_{2}$, we need the electric field in that region. Since the charge distribution is spherically symmetric, Gauss's law may be used to find the electric field.

$$
\oint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=E\left(4 \pi r^{2}\right)=\frac{Q_{\text {encl }}}{\varepsilon_{0}} \rightarrow E=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q_{\text {encl }}}{r^{2}}=\frac{1}{4 \pi \varepsilon_{0}} \frac{\rho_{\mathrm{E}}\left(\frac{4}{3} \pi r^{3}-\frac{4}{3} \pi r_{1}^{3}\right)}{r^{2}}=\frac{\rho_{\mathrm{E}}}{3 \varepsilon_{0}} \frac{\left(r^{3}-r_{1}^{3}\right)}{r^{2}}
$$

The potential in that region is found from Eq. 23-4a. The electric field is radial, so we integrate along a radial line so that $\overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\boldsymbol{\ell}}=E d r$.

$$
\begin{aligned}
& V_{r}-V_{r_{2}}=-\int_{r_{2}}^{r} \overrightarrow{\mathbf{E}} \cdot d \vec{\ell}=-\int_{r_{2}}^{r} E d r=-\int_{r_{2}}^{r} \frac{\rho_{\mathrm{E}}}{3 \varepsilon_{0}} \frac{\left(r^{3}-r_{1}^{3}\right)}{r^{2}} d r=-\frac{\rho_{\mathrm{E}}}{3 \varepsilon_{0}} \int_{r_{2}}^{r}\left(r-\frac{r_{1}^{3}}{r^{2}}\right) d r=-\frac{\rho_{\mathrm{E}}}{3 \varepsilon_{0}}\left(\frac{1}{2} r^{2}+\frac{r_{1}^{3}}{r}\right)_{r_{2}}^{r} \\
& V_{r}=V_{r_{2}}+\left[-\frac{\rho_{\mathrm{E}}}{3 \varepsilon_{0}}\left(\frac{1}{2} r^{2}+\frac{r_{1}^{3}}{r}\right)_{r_{2}}^{r}\right]=\frac{\rho_{\mathrm{E}}}{3 \varepsilon_{0}}\left(\frac{3}{2} r_{2}^{2}-\frac{1}{2} r^{2}-\frac{r_{1}^{3}}{r}\right)=\frac{\rho_{\mathrm{E}}}{\varepsilon_{0}}\left(\frac{1}{2} r_{2}^{2}-\frac{1}{6} r^{2}-\frac{1}{3} \frac{r_{1}^{3}}{r}\right), r_{1}<r<r_{2}
\end{aligned}
$$

(c) Inside the cavity there is no electric field, so the potential is constant and has the value that it has on the cavity boundary.

$$
V_{r_{1}}=\frac{\rho_{\mathrm{E}}}{\varepsilon_{0}}\left(\frac{1}{2} r_{2}^{2}-\frac{1}{6} r_{1}^{2}-\frac{1}{3} \frac{r_{1}^{3}}{r_{1}}\right)=\frac{\rho_{\mathrm{E}}}{2 \varepsilon_{0}}\left(r_{2}^{2}-r_{1}^{2}\right), r<r_{1}
$$

The potential is continuous at both boundaries.
82. We follow the development of Example 23-9, with Figure 23-15. The charge density of the ring is $\sigma=\left(\frac{Q}{\pi R_{0}^{2}-\pi\left(\frac{1}{2} R_{0}\right)^{2}}\right)=\frac{4 Q}{3 \pi R_{0}^{2}}$. The charge on a thin ring of radius $R$ and thickness $d R$ is $d q=\sigma d A=\frac{4 Q}{3 \pi R_{0}^{2}}(2 \pi R d R)$. Use Eq. $23-6 \mathrm{~b}$ to find the potential of a continuous charge distribution.

$$
\begin{aligned}
V & =\frac{1}{4 \pi \varepsilon_{0}} \int \frac{d q}{r}=\frac{1}{4 \pi \varepsilon_{0}} \int_{\frac{⿺}{2} R_{0}}^{R_{0}} \frac{\frac{4 Q}{3 \pi R_{0}^{2}}(2 \pi R d R)}{\sqrt{x^{2}+R^{2}}}=\frac{2 Q}{3 \varepsilon_{0} \pi R_{0}^{2}} \int_{\frac{1}{2} R_{0}}^{R_{0}} \frac{R}{\sqrt{x^{2}+R^{2}}} d R=\left.\frac{2 Q}{3 \varepsilon_{0} \pi R_{0}^{2}}\left(x^{2}+R^{2}\right)^{1 / 2}\right|_{\frac{1}{2} R_{0}} ^{R_{0}} \\
& =\frac{2 Q}{3 \varepsilon_{0} \pi R_{0}^{2}}\left(\sqrt{x^{2}+R_{0}^{2}}-\sqrt{x^{2}+\frac{1}{4} R_{0}^{2}}\right)
\end{aligned}
$$

83. From Example 22-6, the electric field due to a long wire is radial relative to the wire, and is of magnitude $E=\frac{1}{2 \pi \varepsilon_{0}} \frac{\lambda}{R}$. If the charge density is positive, the field lines point radially away from the wire. Use Eq. 23-41 to find the potential difference, integrating along a line that is radially outward from the wire.

$$
V_{\mathrm{a}}-V_{\mathrm{b}}=-\int_{R_{\mathrm{b}}}^{R_{\mathrm{a}}} \overrightarrow{\mathbf{E}}_{\mathrm{b}} \cdot(d \overrightarrow{\boldsymbol{\ell}})=-\int_{R_{\mathrm{b}}}^{R_{\mathrm{a}}} \frac{1}{2 \pi \varepsilon_{0}} \frac{\lambda}{R} d R=-\frac{\lambda}{2 \pi \varepsilon_{0}} \ln \left(R_{\mathrm{a}}-R_{\mathrm{b}}\right)=\frac{\lambda}{2 \pi \varepsilon_{0}} \ln \frac{R_{\mathrm{b}}}{\mathrm{R}_{\mathrm{a}}}
$$

84. (a) We may treat the sphere as a point charge located at the center of the field. Then the electric field at the surface is $E_{\text {surface }}=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{r_{0}^{2}}$, and the potential at the surface is $V_{\text {surface }}=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{r_{0}}$.

$$
V_{\text {surface }}=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{r_{0}}=E_{\text {surface }} r_{0}=E_{\text {breakdown }} r_{0}=\left(3 \times 10^{6} \mathrm{~V} / \mathrm{m}\right)(0.20 \mathrm{~m})=6 \times 10^{5} \mathrm{~V}
$$

(b)

$$
V_{\text {surface }}=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{r_{0}} \rightarrow Q=\left(4 \pi \varepsilon_{0}\right) r_{0} V_{\text {surface }}=\frac{(0.20 \mathrm{~m})\left(6 \times 10^{5} \mathrm{~V}\right)}{\left(8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)}=1.33 \times 10^{-5} \mathrm{C} \approx 1 \times 10^{-5} \mathrm{C}
$$

85. (a) The voltage at $x=0.20 \mathrm{~m}$ is obtained by inserting the given data directly into the voltage equation.

$$
V(0.20 \mathrm{~m})=\frac{B}{\left(x^{2}+R^{2}\right)^{2}}=\frac{150 \mathrm{~V} \cdot \mathrm{~m}^{4}}{\left[(0.20 \mathrm{~m})^{2}+(0.20 \mathrm{~m})^{2}\right]^{2}}=23 \mathrm{kV}
$$

(b) The electric field is the negative derivative of the potential.

$$
\overrightarrow{\mathbf{E}}(x)=-\frac{d}{d x}\left[\frac{B}{\left(x^{2}+R^{2}\right)^{2}}\right] \hat{\mathbf{i}}=\frac{4 B x \hat{\mathbf{i}}}{\left(x^{2}+R^{2}\right)^{3}}
$$

Since the voltage only depends on $x$ the electric field points in the positive $x$ direction.
(c) Inserting the given values in the equation of part (b) gives the electric field at $x=0.20 \mathrm{~m}$

$$
\overrightarrow{\mathbf{E}}(0.20 \mathrm{~m})=\frac{4\left(150 \mathrm{~V} \cdot \mathrm{~m}^{4}\right)(0.20 \mathrm{~m}) \hat{\mathbf{i}}}{\left[(0.20 \mathrm{~m})^{2}+(0.20 \mathrm{~m})^{2}\right]^{3}}=2.3 \times 10^{5} \mathrm{~V} / \mathrm{m} \hat{\mathbf{i}}
$$

86. Use energy conservation, equating the energy of charge $-q_{1}$ at its initial position to its final position at infinity. Take the speed at infinity to be 0 , and take the potential of the point charges to be 0 at infinity.

$$
\begin{aligned}
& E_{\text {initial }}=E_{\text {final }} \rightarrow K_{\text {initial }}+U_{\text {initial }}=K_{\text {final }}+U_{\text {final }} \rightarrow \frac{1}{2} m v_{0}^{2}+\left(-q_{1}\right) V_{\text {intial }}=\frac{1}{2} m v_{\text {final }}^{2}+\left(-q_{1}\right) V_{\text {final }}^{\text {point }} \text { point }
\end{aligned}, \begin{aligned}
& \frac{1}{2} m v_{0}^{2}+\left(-q_{1}\right) \frac{1}{4 \pi \varepsilon_{0}} \frac{2 q_{2}}{\sqrt{\frac{1}{a^{2}+b^{2}}}=0+0 \rightarrow v_{0}=\sqrt{\frac{q_{1} q_{2}}{m \pi \varepsilon_{0}}} \sqrt{a^{2}+b^{2}}}
\end{aligned}
$$

87. (a) From the diagram, the potential at $x$ is the potential of two point charges.

$$
\begin{aligned}
V_{\text {exact }} & =\frac{1}{4 \pi \varepsilon_{0}}\left(\frac{q}{x-d}\right)+\frac{1}{4 \pi \varepsilon_{0}}\left(\frac{-q}{x+d}\right) \\
& =\frac{1}{4 \pi \varepsilon_{0}}\left[\frac{2 q d}{\left(x^{2}-d^{2}\right)}\right], q=1.0 \times 10^{-9} \mathrm{C}, d=0.010 \mathrm{~m}
\end{aligned}
$$


(b) The approximate potential is given by Eq. 23-7, with $\theta=0, p=2 q d$, and $r=x$.

$$
V_{\text {approx }}=\frac{1}{4 \pi \varepsilon_{0}} \frac{2 q d}{x^{2}}
$$

To make the difference at small distances more apparent, we have plotted from 2.0 cm to 8.0 cm . The spreadsheet used for this problem can be found on the Media Manager, with filename
"PSE4_ISM_CH23.XLS," on tab "Problem 23.87."

88. The electric field can be determined from the potential by using Eq. 23-8, differentiating with respect to $x$.

$$
\begin{aligned}
E(x) & =-\frac{d V(x)}{d x}=-\frac{d}{d x}\left[\frac{Q}{2 \pi \varepsilon_{0} R_{0}^{2}}\left[\left(x^{2}+R_{0}^{2}\right)^{1 / 2}-x\right]\right]=-\frac{Q}{2 \pi \varepsilon_{0} R_{0}^{2}}\left[\frac{1}{2}\left(x^{2}+R_{0}^{2}\right)^{-1 / 2}(2 x)-1\right] \\
& =\frac{Q}{2 \pi \varepsilon_{0} R_{0}^{2}}\left[1-\frac{x}{\left(x^{2}+R_{0}^{2}\right)^{1 / 2}}\right]
\end{aligned}
$$

Express $V$ and $E$ in terms of $x / R_{0}$. Let $X=x / R_{0}$.

$$
\begin{aligned}
V(x) & =\frac{Q}{2 \pi \varepsilon_{0} R_{0}^{2}}\left[\left(x^{2}+R_{0}^{2}\right)^{1 / 2}-x\right]=\frac{2 Q}{4 \pi \varepsilon_{0} R_{0}}\left(\sqrt{X^{2}+1}-X\right) \\
& =\left(8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{2\left(5.0 \times 10^{-6} \mathrm{C}\right)}{0.10 \mathrm{~m}}\left(\sqrt{X^{2}+1}-X\right)=\left(8.99 \times 10^{5} \mathrm{~V}\right)\left(\sqrt{X^{2}+1}-X\right) \\
E(x) & =\frac{Q}{2 \pi \varepsilon_{0} R_{0}^{2}}\left[1-\frac{x}{\left(x^{2}+R_{0}^{2}\right)^{1 / 2}}\right]=\frac{2 Q}{4 \pi \varepsilon_{0} R_{0}^{2}}\left[1-\frac{X}{\sqrt{X^{2}+1}}\right] \\
& =\left(8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{2\left(5.0 \times 10^{-6} \mathrm{C}\right)}{(0.10 \mathrm{~m})^{2}}\left[1-\frac{X}{\sqrt{X^{2}+1}}\right] \\
& =\left(8.99 \times 10^{6} \mathrm{~V} / \mathrm{m}\right)\left[1-\frac{X}{\sqrt{X^{2}+1}}\right]
\end{aligned}
$$

The spreadsheet used for this problem can be found on the Media Manager, with filename
"PSE4_ISM_CH23.XLS," on tab "Problem 23.88."


89. (a) If the field is caused by a point charge, the potential will have a graph that has the appearance of $1 / r$ behavior, which means that the potential change per unit of distance will decrease as potential is measured farther from the charge. If the field is caused by a sheet of charge, the potential will have a linear decrease with distance. The graph indicates
 that the field is caused by a point charge. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH23.XLS," on tab "Problem 23.89a."
(b) Assuming the field is caused by a point charge, we assume the charge is at $x=d$, and then the potential is given by $V=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{x-d}$. This can be rearranged to the following.

$$
\begin{aligned}
V & =\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{x-d} \rightarrow \\
x & =\frac{1}{V} \frac{Q}{4 \pi \varepsilon_{0}}+d
\end{aligned}
$$

If we plot $x$ vs. $\frac{1}{V}$, the slope is $\frac{Q}{4 \pi \varepsilon_{0}}$, which can be used to determine the charge.


$$
\begin{aligned}
& \text { slope }=0.1392 \mathrm{~m} \cdot \mathrm{~V}=\frac{Q}{4 \pi \varepsilon_{0}} \rightarrow \\
& Q=4 \pi \varepsilon_{0}(0.1392 \mathrm{~m} \cdot \mathrm{~V})=\frac{(0.1392 \mathrm{~m} \cdot \mathrm{~V})}{\left(8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)}=1.5 \times 10^{-11} \mathrm{C}
\end{aligned}
$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH23.XLS," on tab "Problem 23.89b."
(c) From the above equation, the $y$ intercept of the graph is the location of the charge, $d$. So the charge is located at $x=d=-0.0373 \mathrm{~m} \approx 3.7 \mathrm{~cm}$ from the first measured position.

## CHAPTER 24: Capacitance, Dielectrics, Electric Energy Storage

## Responses to Questions

1. Yes. If the conductors have different shapes, then even if they have the same charge, they will have different charge densities and therefore different electric fields near the surface. There can be a potential difference between them. The definition of capacitance $C=Q / V$ cannot be used here because it is defined for the case where the charges on the two conductors of the capacitor are equal and opposite.
2. Underestimate. If the separation between the plates is not very small compared to the plate size, then fringing cannot be ignored and the electric field (for a given charge) will actually be smaller. The capacitance is inversely proportional to potential and, for parallel plates, also inversely proportional to the field, so the capacitance will actually be larger than that given by the formula.
3. Ignoring fringing field effects, the capacitance would decrease by a factor of 2, since the area of overlap decreases by a factor of 2. (Fringing effects might actually be noticeable in this configuration.)
4. When a capacitor is first connected to a battery, charge flows to one plate. Because the plates are separated by an insulating material, charge cannot cross the gap. An equal amount of charge is therefore repelled from the opposite plate, leaving it with a charge that is equal and opposite to the charge on the first plate. The two conductors of a capacitor will have equal and opposite charges even if they have different sizes or shapes.
5. Charge a parallel-plate capacitor using a battery with a known voltage $V$. Let the capacitor discharge through a resistor with a known resistance $R$ and measure the time constant. This will allow calculation of the capacitance $C$. Then use $C=\varepsilon_{0} A / d$ and solve for $\varepsilon_{0}$.
6. Parallel. The equivalent capacitance of the three capacitors in parallel will be greater than that of the same three capacitors in series, and therefore they will store more energy when connected to a given potential difference if they are in parallel.
7. If a large copper sheet of thickness $\ell$ is inserted between the plates of a parallel-plate capacitor, the charge on the capacitor will appear on the large flat surfaces of the copper sheet, with the negative side of the copper facing the positive side of the capacitor. This arrangement can be considered to be two capacitors in series, each with a thickness of $\frac{1}{2}(d-\ell)$. The new net capacitance will be $C^{\prime}=\varepsilon_{0} A /(d-\ell)$, so the capacitance of the capacitor will be reduced.
8. A force is required to increase the separation of the plates of an isolated capacitor because you are pulling a positive plate away from a negative plate. The work done in increasing the separation goes into increasing the electric potential energy stored between the plates. The capacitance decreases, and the potential between the plates increases since the charge has to remain the same.
9. (a) The energy stored quadruples since the potential difference across the plates doubles and the capacitance doesn't change: $U=\frac{1}{2} C V^{2}$.
(b) The energy stored quadruples since the charge doubles and the capacitance doesn't change:

$$
U=\frac{1}{2} \frac{Q^{2}}{C} .
$$

(c) If the separation between the plates doubles, the capacitance is halved. The potential difference across the plates doesn't change if the capacitor remains connected to the battery, so the energy stored is also halved: $U=\frac{1}{2} C V^{2}$.
10. (c) If the voltage across a capacitor is doubled, the amount of energy it can store is quadrupled: $U=\frac{1}{2} C V^{2}$.
11. The dielectric will be pulled into the capacitor by the electrostatic attractive forces between the charges on the capacitor plates and the polarized charges on the dielectric's surface. (Note that the addition of the dielectric decreases the energy of the system.)
12. If the battery remains connected to the capacitor, the energy stored in the electric field of the capacitor will increase as the dielectric is inserted. Since the energy of the system increases, work must be done and the dielectric will have to be pushed into the area between the plates. If it is released, it will be ejected.
13. (a) If the capacitor is isolated, $Q$ remains constant, and $U=\frac{1}{2} \frac{Q^{2}}{C}$ becomes $U^{\prime}=\frac{1}{2} \frac{Q^{2}}{K C}$ and the stored energy decreases.
(b) If the capacitor remains connected to a battery so $V$ does not change, $U=\frac{1}{2} C V^{2}$ becomes $U^{\prime}=\frac{1}{2} K C V^{2}$, and the stored energy increases.
14. For dielectrics consisting of polar molecules, one would expect the dielectric constant to decrease with temperature. As the thermal energy increases, the molecular vibrations will increase in amplitude, and the polar molecules will be less likely to line up with the electric field.
15. When the dielectric is removed, the capacitance decreases. The potential difference across the plates remains the same because the capacitor is still connected to the battery. If the potential difference remains the same and the capacitance decreases, the charge on the plates and the energy stored in the capacitor must also decrease. (Charges return to the battery.) The electric field between the plates will stay the same because the potential difference across the plates and the distance between the plates remain constant.
16. For a given configuration of conductors and dielectrics, $C$ is the proportionality constant between the voltage between the plates and the charge on the plates.
17. The dielectric constant is the ratio of the capacitance of a capacitor with the dielectric between the plates to the capacitance without the dielectric. If a conductor were inserted between the plates of a capacitor such that it filled the gap and touched both plates, the capacitance would drop to zero since charge would flow from one plate to the other. So, the dielectric constant of a good conductor would be zero.

## Solutions to Problems

1. The capacitance is found from Eq. 24-1.

$$
P_{1}=C V \rightarrow C=\frac{Q}{V}=\frac{2.8 \times 10^{-3} \mathrm{C}}{930 \mathrm{~V}}=3.0 \times 10^{-6} \mathrm{~F}=3.0 \mu \mathrm{~F}
$$

2. We assume the capacitor is fully charged, according to Eq. 24-1.

$$
Q=C V=\left(12.6 \times 10^{-6} \mathrm{~F}\right)(12.0 \mathrm{~V})=1.51 \times 10^{-4} \mathrm{C}
$$

3. The capacitance is found from Eq. 24-1.

$$
Q=C V \rightarrow C=\frac{Q}{V}=\frac{75 \times 10^{-12} \mathrm{C}}{24.0 \mathrm{~V}}=3.1 \times 10^{-12} \mathrm{~F}=3.1 \mathrm{pF}
$$

4. Let $Q_{1}$ and $V_{1}$ be the initial charge and voltage on the capacitor, and let $Q_{2}$ and $V_{2}$ be the final charge and voltage on the capacitor. Use Eq. 24-1 to relate the charges and voltages to the capacitance.

$$
\begin{aligned}
& Q_{1}=C V_{1} \quad Q_{2}=C V_{2} \quad Q_{2}-Q_{1}=C V_{2}-C V_{1}=C\left(V_{2}-V_{1}\right) \rightarrow \\
& C=\frac{Q_{2}-Q_{1}}{V_{2}-V_{1}}=\frac{26 \times 10^{-6} \mathrm{C}}{50 \mathrm{~V}}=5.2 \times 10^{-7} \mathrm{~F}=0.52 \mu \mathrm{~F}
\end{aligned}
$$

5. After the first capacitor is disconnected from the battery, the total charge must remain constant. The voltage across each capacitor must be the same when they are connected together, since each capacitor plate is connected to a corresponding plate on the other capacitor by a constant-potential connecting wire. Use the total charge and the final potential difference to find the value of the second capacitor.

$$
\begin{aligned}
& Q_{\text {Total }}=C_{1} V_{1} \quad Q_{\text {intial }}=C_{1} V_{\text {final }} \quad Q_{\text {final }}=C_{2} V_{\text {final }} \\
& Q_{\text {Total }}=\underset{\text { final }}{Q_{1}}+\underset{\text { final }}{Q_{2}}=\left(C_{1}+C_{2}\right) V_{\text {final }} \rightarrow \underset{\substack{ \\
C_{1} V_{1} \\
\text { initial }}}{ }=\left(C_{1}+C_{2}\right) V_{\text {final }} \rightarrow \\
& C_{2}=C_{1}\left(\begin{array}{l}
V_{1} \\
\text { initial } \\
V_{\text {final }}
\end{array}\right)=\left(7.7 \times 10^{-6} \mathrm{~F}\right)\left(\frac{125 \mathrm{~V}}{15 \mathrm{~V}}-1\right)=5.6 \times 10^{-5} \mathrm{~F}=56 \mu \mathrm{~F}
\end{aligned}
$$

6. The total charge will be conserved, and the final potential difference across the capacitors will be the same.

$$
\begin{aligned}
& Q_{0}=Q_{1}+Q_{2} ; V_{1}=V_{2} \rightarrow \frac{Q_{1}}{C_{1}}=\frac{Q_{2}}{C_{2}}=\frac{Q_{0}-Q_{1}}{C_{2}} \rightarrow Q_{1}=Q_{0} \frac{C_{1}}{C_{1}+C_{2}} \\
& Q_{2}=Q_{0}-Q_{1}=Q_{0}-Q_{0} \frac{C_{1}}{C_{1}+C_{2}}=Q_{2}=Q_{0}\left(\frac{C_{2}}{C_{1}+C_{2}}\right) \\
& V_{1}=V_{2}=\frac{Q_{1}}{C_{1}}=\frac{Q_{0} \frac{C_{1}}{C_{1}+C_{2}}}{C_{1}}=V=\frac{Q_{0}}{C_{1}+C_{2}}
\end{aligned}
$$

7. The work to move the charge between the capacitor plates is $W=q V$, where $V$ is the voltage difference between the plates, assuming that $q \ll Q$ so that the charge on the capacitor does not change appreciably. The charge is then found from Eq. $24-1$. The assumption that $q \ll Q$ is justified.

$$
W=q V=q\left(\frac{Q}{C}\right) \rightarrow Q=\frac{C W}{q}=\frac{(15 \mu \mathrm{~F})(15 \mathrm{~J})}{0.20 \mathrm{mC}}=1.1 \mathrm{C}
$$

8. (a) The total charge on the combination of capacitors is the sum of the charges on the two individual capacitors, since there is no battery connected to them to supply additional charge, and there is no neutralization of charge by combining positive and negative charges. The voltage across each capacitor must be the same after they are connected, since each capacitor plate is connected to a corresponding plate on the other capacitor by a constant-potential connecting wire. Use the total charge and the fact of equal potentials to find the charge on each capacitor and the common potential difference.

$$
\begin{aligned}
& \underset{\text { initial }}{Q_{1}}=C_{1} V_{1} \quad \underset{\text { initial }}{Q_{2}}=C_{\text {initial }} V_{2} \quad \underset{\text { initial }}{Q_{1}}=C_{1} V_{\text {final }} \quad \underset{\text { final }}{Q_{2}}=C_{2} V_{\text {final }} \\
& Q_{\text {Total }}=\underset{\text { initial }}{Q_{1}}+\underset{\text { initial }}{Q_{2}}=\underset{\text { final }}{Q_{1}}+\underset{\text { final }}{Q_{2}}=C_{1} V_{1}+\underset{\text { initial }}{C_{2} V_{2}}=C_{\text {initial }} V_{\text {final }}+C_{2} V_{\text {final }} \rightarrow \\
& V_{\text {final }}=\frac{C_{1} V_{1}+C_{2} V_{2}}{\text { initial }} \underset{\text { initial }}{ }=\frac{\left(2.70 \times 10^{-6} \mathrm{~F}\right)(475 \mathrm{~V})+\left(4.00 \times 10^{-6} \mathrm{~F}\right)(525 \mathrm{~V})}{\left(6.70 \times 10^{-6} \mathrm{~F}\right)} \\
& =504.85 \mathrm{~V} \approx 505 \mathrm{~V}=V_{1}=V_{2} \\
& \underset{\text { final }}{Q_{1}}=C_{1} V_{\text {final }}=\left(2.70 \times 10^{-6} \mathrm{~F}\right)(504.85 \mathrm{~V})=1.36 \times 10^{-3} \mathrm{C} \\
& \underset{\text { final }}{Q_{2}}=C_{2} V_{\text {final }}=\left(4.00 \times 10^{-6} \mathrm{~F}\right)(504.85 \mathrm{~V})=2.02 \times 10^{-3} \mathrm{C}
\end{aligned}
$$

(b) By connecting plates of opposite charge, the total charge will be the difference of the charges on the two individual capacitors. Once the charges have equalized, the two capacitors will again be at the same potential.

$$
\begin{aligned}
& \underset{\text { initial }}{Q_{1}}=C_{1} V_{1} \quad \underset{\text { initial }}{Q_{2}}=C_{2} V_{2} \quad \underset{\text { initial }}{Q_{1}}=C_{1} V_{\text {final }} \quad \underset{\text { final }}{Q_{2}}=C_{2} V_{\text {final }} \\
& Q_{\text {Total }}=\left|\underset{\text { initial }}{Q_{1}}-\underset{\text { initial }}{Q_{2}}\right|=\underset{\text { final }}{Q_{1}}+\underset{\text { final }}{Q_{2}} \rightarrow\left|C_{1} V_{1} \quad-C_{2} V_{\text {initial }} V_{\text {initial }}\right|=C_{1} V_{\text {final }}+C_{2} V_{\text {final }} \rightarrow \\
& V_{\text {final }}=\frac{\mid C_{1} V_{1}-C_{2} V_{2}}{\text { initial }} \begin{array}{l}
\text { initial }
\end{array} \left\lvert\,=\frac{\left|\left(2.70 \times 10^{-6} \mathrm{~F}\right)(475 \mathrm{~V})-\left(4.00 \times 10^{-6} \mathrm{~F}\right)(525 \mathrm{~V})\right|}{C_{1}+C_{2}}\right. \\
& =122.01 \mathrm{~V} \approx 120 \mathrm{~V}=V_{1}=V_{2} \\
& Q_{\text {final }}=C_{1} V_{\text {final }}=\left(2.70 \times 10^{-6} \mathrm{~F}\right)(122.01 \mathrm{~V})=3.3 \times 10^{-4} \mathrm{C} \\
& Q_{2}=C_{2} V_{\text {final }}=\left(4.00 \times 10^{-6} \mathrm{~F}\right)(122.01 \mathrm{~V})=4.9 \times 10^{-4} \mathrm{C}
\end{aligned}
$$

9. Use Eq. 24-1.

$$
\Delta Q=C \Delta V ; t=\frac{\Delta Q}{\Delta Q / \Delta t}=\frac{C \Delta V}{\Delta Q / \Delta t}=\frac{(1200 \mathrm{~F})(6.0 \mathrm{~V})}{1.0 \times 10^{-3} \mathrm{C} / \mathrm{s}}=7.2 \times 10^{6} \mathrm{~s}\left(\frac{1 \mathrm{~d}}{86,400 \mathrm{~s}}\right)=83 \mathrm{~d}
$$

10. (a) The absolute value of the charge on each plate is given by Eq. 24-1. The plate with electrons has a net negative charge.

$$
Q=C V \rightarrow N(-e)=-C V \rightarrow
$$

$$
N=\frac{C V}{e}=\frac{\left(35 \times 10^{-15} \mathrm{~F}\right)(1.5 \mathrm{~V})}{1.60 \times 10^{-19} \mathrm{C}}=3.281 \times 10^{5} \approx 3.3 \times 10^{5} \text { electrons }
$$

(b) Since the charge is directly proportional to the potential difference, a $1.0 \%$ decrease in potential difference corresponds to a $1.0 \%$ decrease in charge.

$$
\Delta Q=0.01 Q ;
$$

$$
\Delta t=\frac{\Delta Q}{\Delta Q / \Delta t}=\frac{0.01 Q}{\Delta Q / \Delta t}=\frac{0.01 C V}{\Delta Q / \Delta t}=\frac{0.01\left(35 \times 10^{-15} \mathrm{~F}\right)(1.5 \mathrm{~V})}{0.30 \times 10^{-15} \mathrm{C} / \mathrm{s}}=1.75 \mathrm{~s} \approx 1.8 \mathrm{~s}
$$

11. Use Eq. 24-2.

$$
C=\varepsilon_{0} \frac{A}{d} \rightarrow A=\frac{C d}{\varepsilon_{0}}=\frac{\left(0.40 \times 10^{-6} \mathrm{~F}\right)\left(2.8 \times 10^{-3} \mathrm{~m}\right)}{\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)}=126.6 \mathrm{~m}^{2} \approx 130 \mathrm{~m}^{2}
$$

If the capacitor plates were square, they would be about 11.2 m on a side.
12. The capacitance per unit length of a coaxial cable is derived in Example 24-2

$$
\frac{C}{\ell}=\frac{2 \pi \varepsilon_{0}}{\ln \left(R_{\text {outside }} / R_{\text {inside }}\right)}=\frac{2 \pi\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)}{\ln (5.0 \mathrm{~mm} / 1.0 \mathrm{~mm})}=3.5 \times 10^{-11} \mathrm{~F} / \mathrm{m}
$$

13. Inserting the potential at the surface of a spherical conductor into Eq. 24.1 gives the capacitance of a conducting sphere. Then inserting the radius of the Earth yields the Earth's capacitance.

$$
C=\frac{Q}{V}=\frac{Q}{\left(Q / 4 \pi \varepsilon_{0} r\right)}=4 \pi \varepsilon_{0} r=4 \pi\left(8.85 \times 10^{-12} \mathrm{~F} / \mathrm{m}\right)\left(6.38 \times 10^{6} \mathrm{~m}\right)=7.10 \times 10^{-4} \mathrm{~F}
$$

14. From the symmetry of the charge distribution, any electric field must be radial, away from the cylinder axis, and its magnitude must be independent of the location around the axis (for a given radial location). We assume the cylinders have charge of magnitude $Q$ in a length $\ell$. Choose a Gaussian cylinder of length $d$ and radius $R$, centered on the capacitor's axis, with $d \ll \ell$ and the Gaussian cylinder far away from both ends of the capacitor. On the ends of this cylinder, $\overrightarrow{\mathbf{E}} \perp d \overrightarrow{\mathbf{A}}$ and so there is no flux through the ends. On the curved side of the


Gaussian cylinder of radius $R$ cylinder, the field has a constant magnitude and $\overrightarrow{\mathbf{E}} \| d \overrightarrow{\mathbf{A}}$. Thus $\overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=E d A$. Write Gauss's law.

$$
\oiint \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{A}}=E_{\substack{\text { curved } \\ \text { walls }}} A_{\text {curved }}^{\text {walls }}<=E(2 \pi R d)=\frac{Q_{\text {encl }}}{\varepsilon_{0}}
$$

For $R<R_{\mathrm{b}}, Q_{\text {encl }}=0 \rightarrow E(2 \pi R d) \varepsilon_{0}=0 \rightarrow E=0$.
For $R>R_{\mathrm{a}}, Q_{\text {encl }}=\frac{Q}{\ell} d+\frac{-Q}{\ell} d=0$, and so $Q_{\text {encl }}=0 \rightarrow E(2 \pi R d) \varepsilon_{0}=0 \rightarrow E=0$.
15. We assume there is a uniform electric field between the capacitor plates, so that $V=E d$, and then use Eqs. 24-1 and 24-2.

$$
\begin{aligned}
Q_{\max } & =C V_{\max }=\varepsilon_{0} \frac{A}{d}\left(E_{\max } d\right)=\varepsilon_{0} A E_{\max }=\left(8.85 \times 10^{-12} \mathrm{~F} / \mathrm{m}\right)\left(6.8 \times 10^{-4} \mathrm{~m}^{2}\right)\left(3.0 \times 10^{6} \mathrm{~V} / \mathrm{m}\right) \\
& =1.8 \times 10^{-8} \mathrm{C}
\end{aligned}
$$

16. We assume there is a uniform electric field between the capacitor plates, so that $V=E d$, and then use Eqs. 24-1 and 24-2.

$$
\begin{aligned}
Q & =C V=\varepsilon_{0} \frac{A}{d}(E d)=\varepsilon_{0} A E=\left(8.85 \times 10^{-12} \mathrm{~F} / \mathrm{m}\right)\left(21.0 \times 10^{-4} \mathrm{~m}^{2}\right)\left(4.80 \times 10^{5} \mathrm{~V} / \mathrm{m}\right) \\
& =8.92 \times 10^{-9} \mathrm{C}
\end{aligned}
$$

17. We assume there is a uniform electric field between the capacitor plates, so that $V=E d$, and then use Eqs. 24-1 and 24-2.

$$
Q=C V=C E d \rightarrow E=\frac{Q}{C d}=\frac{92 \times 10^{-6} \mathrm{C}}{\left(0.80 \times 10^{-6} \mathrm{~F}\right)\left(2.0 \times 10^{-3} \mathrm{~m}\right)}=5.8 \times 10^{4} \mathrm{~V} / \mathrm{m}
$$

18. (a) The uncharged plate will polarize so that negative charge will be drawn towards the positive capacitor plate, and positive charge will be drawn towards the negative capacitor plate. The same charge will be on each face of the plate as on the original capacitor plates. The same electric field will be in the gaps as before the plate was inserted. Use that electric field
 to determine the potential difference between the two original plates, and the new capacitance. Let $x$ be the distance from one original plate to the nearest face of the sheet, and so $d-\ell-x$ is the distance from the other original plate to the other face of the sheet.

$$
\begin{gathered}
E=\frac{\sigma}{\varepsilon_{0}}=\frac{Q}{A \varepsilon_{0}} ; V_{1}=E x=\frac{Q x}{A \varepsilon_{0}} ; V_{2}=E(d-\ell-x)=\frac{Q(d-\ell-x)}{A \varepsilon_{0}} \\
\Delta V=V_{1}+V_{2}=\frac{Q x}{A \varepsilon_{0}}+\frac{Q(d-\ell-x)}{A \varepsilon_{0}}=\frac{Q(d-\ell)}{A \varepsilon_{0}}=\frac{Q}{C} \rightarrow C=\varepsilon_{0} \frac{A}{(d-\ell)} \\
\text { (b) } C_{\text {initial }}=\varepsilon_{0} \frac{A}{d} ; C_{\text {final }}=\varepsilon_{0} \frac{A}{(d-\ell)} ; \frac{C_{\text {final }}}{C_{\text {initial }}}=\frac{\varepsilon_{0} \frac{A}{(d-\ell)}}{\varepsilon_{0} \frac{A}{d}}=\frac{d}{d-\ell}=\frac{d}{d-0.40 d}=\frac{1}{0.60}=1.7
\end{gathered}
$$

19. (a) The distance between plates is obtained from Eq. 24-2.

$$
C=\frac{\varepsilon_{0} A}{x} \rightarrow x=\frac{\varepsilon_{0} A}{C}
$$

Inserting the maximum capacitance gives the minimum plate separation and the minimum capacitance gives the maximum plate separation.

$$
\begin{aligned}
& \quad x_{\min }=\frac{\varepsilon_{0} A}{C_{\max }}=\frac{(8.85 \mathrm{pF} / \mathrm{m})\left(25 \times 10^{-6} \mathrm{~m}^{2}\right)}{1000.0 \times 10^{-12} \mathrm{~F}}=0.22 \mu \mathrm{~m} \\
& x_{\max }=\frac{\varepsilon_{0} A}{C_{\min }}=\frac{(8.85 \mathrm{pF} / \mathrm{m})\left(25 \times 10^{-6} \mathrm{~m}^{2}\right)}{1.0 \mathrm{pF}}=0.22 \mathrm{~mm}=220 \mu \mathrm{~m} \\
& \text { So } 0.22 \mu \mathrm{~m} \leq x \leq 220 \mu \mathrm{~m} .
\end{aligned}
$$

(b) Differentiating the distance equation gives the approximate uncertainty in distance.

$$
\Delta x \approx \frac{d x}{d C} \Delta C=\frac{d}{d C}\left[\frac{\varepsilon_{0} A}{C}\right] \Delta C=-\frac{\varepsilon_{0} A}{C^{2}} \Delta C .
$$

The minus sign indicates that the capacitance increases as the plate separation decreases. Since only the magnitude is desired, the minus sign can be dropped. The uncertainty is finally written in terms of the plate separation using Eq. 24-2.

$$
\Delta x \approx \frac{\varepsilon_{0} A}{\left(\frac{\varepsilon_{0} A}{x}\right)^{2}} \Delta C=\frac{x^{2} \Delta C}{\varepsilon_{0} A}
$$

(c) The percent uncertainty in distance is obtained by dividing the uncertainty by the separation distance.

$$
\begin{aligned}
& \frac{\Delta x_{\min }}{x_{\min }} \times 100 \%=\frac{x_{\min } \Delta C}{\varepsilon_{0} A} \times 100 \%=\frac{(0.22 \mu \mathrm{~m})(0.1 \mathrm{pF})(100 \%)}{(8.85 \mathrm{pF} / \mathrm{m})\left(25 \mathrm{~mm}^{2}\right)}=0.01 \% \\
& \frac{\Delta x_{\max }}{x_{\max }} \times 100 \%=\frac{x_{\max } \Delta C}{\varepsilon_{0} A} \times 100 \%=\frac{(0.22 \mathrm{~mm})(0.1 \mathrm{pF})(100 \%)}{(8.85 \mathrm{pF} / \mathrm{m})\left(25 \mathrm{~mm}^{2}\right)}=10 \%
\end{aligned}
$$

20. The goal is to have an electric field of strength $E_{\mathrm{s}}$ at a radial distance of $5.0 R_{\mathrm{b}}$ from the center of the cylinder. Knowing the electric field at a specific distance allows us to calculate the linear charge density on the inner cylinder. From the linear charge density and the capacitance we can find the potential difference needed to create the field. From the cylindrically symmetric geometry and Gauss's law, the field in between the cylinders is given by $E=\frac{1}{2 \pi \varepsilon_{0}} \frac{\lambda}{R}$. The capacitance of a cylindrical capacitor is given in Example 24-2.

$$
\begin{aligned}
& E\left(R=5.0 R_{\mathrm{b}}\right)=\frac{1}{2 \pi \varepsilon_{0}} \frac{\lambda}{\left(5.0 R_{\mathrm{b}}\right)}=E_{\mathrm{s}} \rightarrow \lambda=2 \pi \varepsilon_{0}\left(5.0 R_{\mathrm{b}}\right) E_{\mathrm{s}}=\frac{Q}{\ell} \\
& \begin{array}{l}
Q=C V \rightarrow V=\frac{Q}{C}=\frac{Q}{\frac{2 \pi \varepsilon_{0} \ell}{\ln \left(R_{\mathrm{a}} / R_{\mathrm{b}}\right)}}=\frac{Q}{\ell} \frac{\ln \left(R_{\mathrm{a}} / R_{\mathrm{b}}\right)}{2 \pi \varepsilon_{0}}=\left[2 \pi \varepsilon_{0}\left(5.0 R_{\mathrm{b}}\right) E_{\mathrm{s}}\right] \frac{\ln \left(R_{\mathrm{a}} / R_{\mathrm{b}}\right)}{2 \pi \varepsilon_{0}} \\
\\
\quad=\left(5.0 R_{\mathrm{b}}\right) E_{\mathrm{s}} \ln \left(R_{\mathrm{a}} / R_{\mathrm{b}}\right)=\left[5.0\left(1.0 \times 10^{-4} \mathrm{~m}\right)\right]\left(2.7 \times 10^{6} \mathrm{~N} / \mathrm{C}\right) \ln \left(\frac{0.100 \mathrm{~m}}{1.0 \times 10^{-4} \mathrm{~m}}\right)=9300 \mathrm{~V}
\end{array}
\end{aligned}
$$

21. To reduce the net capacitance, another capacitor must be added in series.

$$
\begin{aligned}
& \frac{1}{C_{\mathrm{eq}}}=\frac{1}{C_{1}}+\frac{1}{C_{2}} \rightarrow \frac{1}{C_{2}}=\frac{1}{C_{\mathrm{eq}}}-\frac{1}{C_{1}}=\frac{C_{1}-C_{\mathrm{eq}}}{C_{1} C_{\mathrm{eq}}} \rightarrow \\
& C_{2}=\frac{C_{1} C_{\mathrm{eq}}}{C_{1}-C_{\mathrm{eq}}}=\frac{\left(2.9 \times 10^{-9} \mathrm{~F}\right)\left(1.6 \times 10^{-9} \mathrm{~F}\right)}{\left(2.9 \times 10^{-9} \mathrm{~F}\right)-\left(1.6 \times 10^{-9} \mathrm{~F}\right)}=3.57 \times 10^{-9} \mathrm{~F} \approx 3600 \mathrm{pF}
\end{aligned}
$$

Yes, an existing connection needs to be broken in the process. One of the connections of the original capacitor to the circuit must be disconnected in order to connect the additional capacitor in series.
22. (a) Capacitors in parallel add according to Eq. 24-3.

$$
C_{\mathrm{eq}}=C_{1}+C_{2}+C_{3}+C_{4}+C_{5}+C_{6}=6\left(3.8 \times 10^{-6} \mathrm{~F}\right)=2.28 \times 10^{-5} \mathrm{~F}=22.8 \mu \mathrm{~F}
$$

(b) Capacitors in series add according to Eq. 24-4.

$$
\begin{aligned}
C_{\mathrm{eq}} & =\left(\frac{1}{C_{1}}+\frac{1}{C_{2}}+\frac{1}{C_{3}}+\frac{1}{C_{4}}+\frac{1}{C_{5}}+\frac{1}{C_{6}}\right)^{-1}=\left(\frac{6}{3.8 \times 10^{-6} \mathrm{~F}}\right)^{-1}=\frac{3.8 \times 10^{-6} \mathrm{~F}}{6}=6.3 \times 10^{-7} \mathrm{~F} \\
& =0.63 \mu \mathrm{~F}
\end{aligned}
$$

23. We want a small voltage drop across $C_{1}$. Since $V=Q / C$, if we put the smallest capacitor in series with the battery, there will be a large voltage drop across it. Then put the two larger capacitors in parallel, so that their equivalent capacitance is large and therefore will have a small voltage drop across them. So put $C_{1}$ and $C_{3}$ in parallel with each other, and then put that combination in series with $C_{2}$. See the diagram. To calculate the voltage across $C_{1}$, find the equivalent
 capacitance and the net charge. That charge is used to find the voltage drop across $C_{2}$, and then that voltage is subtracted from the battery voltage to find the voltage across the parallel combination.

$$
\begin{aligned}
\frac{1}{C_{\mathrm{eq}}} & =\frac{1}{C_{2}}+\frac{1}{C_{1}+C_{3}}=\frac{C_{1}+C_{2}+C_{3}}{C_{2}\left(C_{1}+C_{3}\right)} \rightarrow C_{\mathrm{eq}}=\frac{C_{2}\left(C_{1}+C_{3}\right)}{C_{1}+C_{2}+C_{3}} ; Q_{\mathrm{eq}}=C_{\mathrm{eq}} V_{0} ; V_{2}=\frac{Q_{2}}{C_{2}}=\frac{Q_{\mathrm{eq}}}{C_{2}} ; \\
V_{1} & =V_{0}-V_{2}=V_{0}-\frac{Q_{\mathrm{eq}}}{C_{2}}=V_{0}-\frac{C_{\mathrm{eq}} V_{0}}{C_{2}}=V_{0}-\frac{\frac{C_{2}\left(C_{1}+C_{3}\right)}{C_{1}+C_{2}+C_{3}} V_{0}}{C_{2}}=\frac{C_{2}}{C_{1}+C_{2}+C_{3}} V_{0}=\frac{1.5 \mu \mathrm{~F}}{6.5 \mu \mathrm{~F}}(12 \mathrm{~V}) \\
& =2.8 \mathrm{~V}
\end{aligned}
$$

24. The capacitors are in parallel, and so the potential is the same for each capacitor, and the total charge on the capacitors is the sum of the individual charges. We use Eqs. 24-1 and 24-2.

$$
\begin{aligned}
& Q_{1}=C_{1} V=\varepsilon_{0} \frac{A_{1}}{d_{1}} V ; Q_{2}=C_{2} V=\varepsilon_{0} \frac{A_{2}}{d_{2}} V ; Q_{3}=C_{3} V=\varepsilon_{0} \frac{A_{3}}{d_{3}} V \\
& Q_{\text {total }}=Q_{1}+Q_{2}+Q_{3}=\varepsilon_{0} \frac{A_{1}}{d_{1}} V+\varepsilon_{0} \frac{A_{2}}{d_{2}} V+\varepsilon_{0} \frac{A_{3}}{d_{3}} V=\left(\varepsilon_{0} \frac{A_{1}}{d_{1}}+\varepsilon_{0} \frac{A_{2}}{d_{2}}+\varepsilon_{0} \frac{A_{3}}{d_{3}}\right) V \\
& C_{\text {net }}=\frac{Q_{\text {toal }}}{V}=\frac{\left(\varepsilon_{0} \frac{A_{1}}{d_{1}}+\varepsilon_{0} \frac{A_{2}}{d_{2}}+\varepsilon_{0} \frac{A_{3}}{d_{3}}\right) V}{V}=\left(\varepsilon_{0} \frac{A_{1}}{d_{1}}+\varepsilon_{0} \frac{A_{2}}{d_{2}}+\varepsilon_{0} \frac{A_{3}}{d_{3}}\right)=C_{1}+C_{2}+C_{3}
\end{aligned}
$$

Capacitors in parallel add linearly, and so adding a capacitor in parallel will increase the net capacitance without removing the $5.0 \mu \mathrm{~F}$ capacitor.

$$
5.0 \mu \mathrm{~F}+C=16 \mu \mathrm{~F} \rightarrow C=11 \mu \mathrm{~F} \text { connected in parallel }
$$

26. (a) The two capacitors are in parallel. Both capacitors have their high voltage plates at the same potential (the middle plate), and both capacitors have their low voltage plates at the same potential (the outer plates, which are connected).
(b) The capacitance of two capacitors in parallel is the sum of the individual capacitances.

$$
C=C_{1}+C_{2}=\frac{\varepsilon_{0} A}{d_{1}}+\frac{\varepsilon_{0} A}{d_{2}}=\varepsilon_{0} A\left(\frac{1}{d_{1}}+\frac{1}{d_{2}}\right)=\varepsilon_{0} A\left(\frac{d_{1}+d_{2}}{d_{1} d_{2}}\right)
$$

(c) Let $\ell=d_{1}+d_{2}=$ constant. Then $C=\frac{\varepsilon_{0} A \ell}{d_{1} d_{2}}=\frac{\varepsilon_{0} A \ell}{d_{1}\left(\ell-d_{1}\right)}$. We see that $C \rightarrow \infty$ as $d_{1} \rightarrow 0$ or $d_{1} \rightarrow \ell$ (which is $d_{2} \rightarrow 0$ ). Of course, a real capacitor would break down as the plates got too close to each other. To find the minimum capacitance, set $\frac{d C}{d\left(d_{1}\right)}=0$ and solve for $d_{1}$.

$$
\begin{aligned}
& \frac{d C}{d\left(d_{1}\right)}=\frac{d}{d\left(d_{1}\right)}\left[\frac{\varepsilon_{0} A \ell}{d_{1} \ell-d_{1}^{2}}\right]=\varepsilon_{0} A \ell \frac{\left(\ell-2 d_{1}\right)}{\left(d_{1} \ell-d_{1}^{2}\right)^{2}}=0 \rightarrow d_{1}=\frac{1}{2} \ell=d_{2} \\
& C_{\min }=\varepsilon_{0} A\left(\frac{d_{1}+d_{2}}{d_{1} d_{2}}\right)_{d_{1}-\frac{1}{2} \ell}=\varepsilon_{0} A\left(\frac{\ell}{\left(\frac{1}{2} \ell\right)\left(\frac{1}{2} \ell\right)}\right)=\varepsilon_{0} A\left(\frac{4}{\ell}\right)=\varepsilon_{0} A\left(\frac{4}{d_{1}+d_{2}}\right) \\
& C_{\min }=\frac{4 \varepsilon_{0} A}{d_{1}+d_{2}} ; C_{\max }=\infty
\end{aligned}
$$

27. The maximum capacitance is found by connecting the capacitors in parallel.

$$
C_{\max }=C_{1}+C_{2}+C_{3}=3.6 \times 10^{-9} \mathrm{~F}+5.8 \times 10^{-9} \mathrm{~F}+1.00 \times 10^{-8} \mathrm{~F}=1.94 \times 10^{-8} \mathrm{~F} \text { in parallel }
$$

The minimum capacitance is found by connecting the capacitors in series.

$$
C_{\min }=\left(\frac{1}{C_{1}}+\frac{1}{C_{2}}+\frac{1}{C_{3}}\right)^{-1}=\left(\frac{1}{3.6 \times 10^{-9} \mathrm{~F}}+\frac{1}{5.8 \times 10^{-9} \mathrm{~F}}+\frac{1}{1.00 \times 10^{-8} \mathrm{~F}}\right)^{-1}=1.82 \times 10^{-9} \mathrm{~F} \text { in series }
$$

28. When the capacitors are connected in series, they each have the same charge as the net capacitance.
(a)

$$
\begin{aligned}
Q_{1} & =Q_{2}=Q_{\mathrm{eq}}=C_{\mathrm{eq}} V=\left(\frac{1}{C_{1}}+\frac{1}{C_{2}}\right)^{-1} V=\left(\frac{1}{0.50 \times 10^{-6} \mathrm{~F}}+\frac{1}{0.80 \times 10^{-6} \mathrm{~F}}\right)^{-1}(9.0 \mathrm{~V}) \\
& =2.769 \times 10^{-6} \mathrm{C} \\
V_{1} & =\frac{Q_{1}}{C_{1}}=\frac{2.769 \times 10^{-6} \mathrm{C}}{0.50 \times 10^{-6} \mathrm{~F}}=5.538 \mathrm{~V} \approx 5.5 \mathrm{~V} \quad V_{2}=\frac{Q_{2}}{C_{2}}=\frac{2.769 \times 10^{-6} \mathrm{C}}{0.80 \times 10^{-6} \mathrm{~F}}=3.461 \mathrm{~V} \approx 3.5 \mathrm{~V} \\
Q_{1} & =Q_{2}=Q_{\mathrm{eq}}=2.769 \times 10^{-6} \mathrm{C} \approx 2.8 \times 10^{-6} \mathrm{C}
\end{aligned}
$$

(b)

When the capacitors are connected in parallel, they each have the full potential difference.
(c) $V_{1}=9.0 \mathrm{~V} \quad V_{2}=9.0 \mathrm{~V} \quad Q_{1}=C_{1} V_{1}=\left(0.50 \times 10^{-6} \mathrm{~F}\right)(9.0 \mathrm{~V})=4.5 \times 10^{-6} \mathrm{C}$

$$
Q_{2}=C_{2} V_{2}=\left(0.80 \times 10^{-6} \mathrm{~F}\right)(9.0 \mathrm{~V})=7.2 \times 10^{-6} \mathrm{C}
$$

29. (a) From the diagram, we see that $C_{1}$ and $C_{2}$ are in series. That combination is in parallel with $C_{3}$, and then that combination is in series with $C_{4}$. Use those combinations to find the equivalent capacitance. We use subscripts to indicate which capacitors have been combined.

$$
\begin{aligned}
& \frac{1}{C_{12}}=\frac{1}{C}+\frac{1}{C} \rightarrow C_{12}=\frac{1}{2} C ; C_{123}=C_{12}+C_{3}=\frac{1}{2} C+C=\frac{3}{2} C ; \\
& \frac{1}{C_{1234}}=\frac{1}{C_{123}}+\frac{1}{C_{4}}=\frac{2}{3 C}+\frac{1}{C}=\frac{5}{3 C} \rightarrow C_{1234}=\frac{3}{5} C
\end{aligned}
$$

(b) The charge on the equivalent capacitor $C_{1234}$ is given by $Q_{1234}=C_{1234} V=\frac{3}{5} C V$. This is the charge on both of the series components of $C_{1234}$.

$$
\begin{aligned}
& Q_{123}=\frac{3}{5} C V=C_{123} V_{123}=\frac{3}{2} C V_{123} \rightarrow V_{123}=\frac{2}{5} V \\
& Q_{4}=\frac{3}{5} C V=C_{4} V_{4} \rightarrow V_{4}=\frac{3}{5} V
\end{aligned}
$$

The voltage across the equivalent capacitor $C_{123}$ is the voltage across both of its parallel components. Note that the sum of the charges across the two parallel components of $C_{123}$ is the same as the total charge on the two components, $\frac{3}{5} C V$.

$$
\begin{aligned}
& V_{123}=\frac{2}{5} V=V_{12} ; Q_{12}=C_{12} V_{12}=\left(\frac{1}{2} C\right)\left(\frac{2}{5} V\right)=\frac{1}{5} C V \\
& V_{123}=\frac{2}{5} V=V_{3} ; Q_{3}=C_{3} V_{3}=C\left(\frac{2}{5} V\right)=\frac{2}{5} C V
\end{aligned}
$$

Finally, the charge on the equivalent capacitor $C_{12}$ is the charge on both of the series components of $C_{12}$.

$$
Q_{12}=\frac{1}{5} C V=Q_{1}=C_{1} V_{1} \quad \rightarrow \quad V_{1}=\frac{1}{5} V ; Q_{12}=\frac{1}{5} C V=Q_{2}=C_{1} V_{2} \quad \rightarrow \quad V_{2}=\frac{1}{5} V
$$

Here are all the results, gathered together.

$$
\begin{aligned}
& Q_{1}=Q_{2}=\frac{1}{5} C V ; Q_{3}=\frac{2}{5} C V ; Q_{4}=\frac{3}{5} C V \\
& V_{1}=V_{2}=\frac{1}{5} V ; V_{3}=\frac{2}{5} V ; V_{4}=\frac{3}{5} V
\end{aligned}
$$

30. $C_{1}$ and $C_{2}$ are in series, so they both have the same charge. We then use that charge to find the voltage across each of $C_{1}$ and $C_{2}$. Then their combined voltage is the voltage across $C_{3}$. The voltage across $C_{3}$ is used to find the charge on $C_{3}$.

$$
\begin{aligned}
& Q_{1}=Q_{2}=12.4 \mu \mathrm{C} ; V_{1}=\frac{Q_{1}}{C_{1}}=\frac{12.4 \mu \mathrm{C}}{16.0 \mu \mathrm{~F}}=0.775 \mathrm{~V} ; V_{2}=\frac{Q_{2}}{C_{2}}=\frac{12.4 \mu \mathrm{C}}{16.0 \mu \mathrm{~F}}=0.775 \mathrm{~V} \\
& V_{3}=V_{1}+V_{2}=1.55 \mathrm{~V} ; Q_{3}=C_{3} V_{3}=(16.0 \mu \mathrm{~F})(1.55 \mathrm{~V})=24.8 \mu \mathrm{C}
\end{aligned}
$$

From the diagram, $C_{4}$ must have the same charge as the sum of the charges on $C_{1}$ and $C_{3}$. Then the voltage across the entire combination is the sum of the voltages across $C_{4}$ and $C_{3}$.

$$
\begin{aligned}
& Q_{4}=Q_{1}+Q_{3}=12.4 \mu \mathrm{C}+24.8 \mu \mathrm{C}=37.2 \mu \mathrm{C} ; V_{4}=\frac{Q_{4}}{C_{4}}=\frac{37.2 \mu \mathrm{C}}{28.5 \mu \mathrm{~F}}=1.31 \mathrm{~V} \\
& V_{\mathrm{ab}}=V_{4}+V_{3}=1.31 \mathrm{~V}+1.55 \mathrm{~V}=2.86 \mathrm{~V}
\end{aligned}
$$

Here is a summary of all results.

$$
\begin{aligned}
& Q_{1}=Q_{2}=12.4 \mu \mathrm{C} ; Q_{3}=24.8 \mu \mathrm{C} ; Q_{4}=37.2 \mu \mathrm{C} \\
& V_{1}=V_{2}=0.775 \mathrm{~V} ; V_{3}=1.55 \mathrm{~V} ; V_{4}=1.31 \mathrm{~V} ; V_{\mathrm{ab}}=2.86 \mathrm{~V}
\end{aligned}
$$

31. When the switch is down the initial charge on $C_{2}$ is calculated from Eq. 24-1.

$$
Q_{2}=C_{2} V_{0}
$$

When the switch is moved up, charge will flow from $C_{2}$ to $C_{1}$ until the voltage across the two capacitors is equal.

$$
V=\frac{Q_{2}^{\prime}}{C_{2}}=\frac{Q_{1}^{\prime}}{C_{1}} \quad \rightarrow \quad Q_{2}^{\prime}=Q_{1}^{\prime} \frac{C_{2}}{C_{1}}
$$



The sum of the charges on the two capacitors is equal to the initial charge on $C_{2}$.

$$
Q_{2}=Q_{2}^{\prime}+Q_{1}^{\prime}=Q_{1}^{\prime} \frac{C_{2}}{C_{1}}+Q_{1}^{\prime}=Q_{1}^{\prime}\left(\frac{C_{2}+C_{1}}{C_{1}}\right)
$$

Inserting the initial charge in terms of the initial voltage gives the final charges.

$$
Q_{1}^{\prime}\left(\frac{C_{2}+C_{1}}{C_{1}}\right)=C_{2} V_{0} \rightarrow Q_{1}^{\prime}=\frac{C_{1} C_{2}}{C_{2}+C_{1}} V_{0} ; Q_{2}^{\prime}=Q_{1}^{\prime} \frac{C_{2}}{C_{1}}=\frac{C_{2}^{2}}{C_{2}+C} V_{0}
$$

32. (a) From the diagram, we see that $C_{1}$ and $C_{2}$ are in parallel, and $C_{3}$ and $C_{4}$ are in parallel. Those two combinations are then in series with each other. Use those combinations to find the equivalent capacitance. We use subscripts to indicate which capacitors have been combined.


$$
\begin{aligned}
& C_{12}=C_{1}+C_{2} ; C_{34}=C_{3}+C_{4} ; \\
& \frac{1}{C_{1234}}=\frac{1}{C_{12}}+\frac{1}{C_{34}}=\frac{1}{C_{1}+C_{2}}+\frac{1}{C_{3}+C_{4}} \rightarrow \\
& C_{1234}=\frac{\left(C_{1}+C_{2}\right)\left(C_{3}+C_{4}\right)}{\left(C_{1}+C_{2}+C_{3}+C_{4}\right)}
\end{aligned}
$$


(b) The charge on the equivalent capacitor $C_{1234}$ is given by $Q_{1234}=C_{1234} V$. This is the charge on both of the series components of $C_{1234}$. Note that $V_{12}+V_{34}=V$.

$$
\begin{aligned}
& Q_{12}=C_{1234} V=C_{12} V_{12} \rightarrow V_{12}=\frac{C_{1234}}{C_{12}} V=\frac{\frac{\left(C_{1}+C_{2}\right)\left(C_{3}+C_{4}\right)}{\left(C_{1}+C_{2}+C_{3}+C_{4}\right)}}{C_{1}+C_{2}} V=\frac{\left(C_{3}+C_{4}\right)}{\left(C_{1}+C_{2}+C_{3}+C_{4}\right)} V \\
& Q_{34}=C_{1234} V=C_{34} V_{34} \rightarrow V_{34}=\frac{C_{1234}}{C_{34}} V=\frac{\frac{\left(C_{1}+C_{2}\right)\left(C_{3}+C_{4}\right)}{\left(C_{1}+C_{2}+C_{3}+C_{4}\right)}}{C_{3}+C_{4}} V=\frac{\left(C_{1}+C_{2}\right)}{\left(C_{1}+C_{2}+C_{3}+C_{4}\right)} V
\end{aligned}
$$

The voltage across the equivalent capacitor $C_{12}$ is the voltage across both of its parallel components, and the voltage across the equivalent $C_{34}$ is the voltage across both its parallel components.

$$
\begin{aligned}
& V_{12}=V_{1}=V_{2}=\frac{\left(C_{3}+C_{4}\right)}{\left(C_{1}+C_{2}+C_{3}+C_{4}\right)} V ; \\
& C_{1} V_{1}=Q_{1}=\frac{C_{1}\left(C_{3}+C_{4}\right)}{\left(C_{1}+C_{2}+C_{3}+C_{4}\right)} V ; C_{2} V_{2}=Q_{2}=\frac{C_{2}\left(C_{3}+C_{4}\right)}{\left(C_{1}+C_{2}+C_{3}+C_{4}\right)} V \\
& V_{34}=V_{3}=V_{2}=\frac{\left(C_{1}+C_{2}\right)}{\left(C_{1}+C_{2}+C_{3}+C_{4}\right)} V ; \\
& C_{3} V_{3}=Q_{3}=\frac{C_{3}\left(C_{1}+C_{2}\right)}{\left(C_{1}+C_{2}+C_{3}+C_{4}\right)} V ; C_{4} V_{4}=Q_{4}=\frac{C_{4}\left(C_{1}+C_{2}\right)}{\left(C_{1}+C_{2}+C_{3}+C_{4}\right)} V
\end{aligned}
$$

33. (a) The voltage across $C_{3}$ and $C_{4}$ must be the same, since they are in parallel.

$$
V_{3}=V_{4} \rightarrow \frac{Q_{3}}{C_{3}}=\frac{Q_{4}}{C_{4}} \rightarrow Q_{4}=Q_{3} \frac{C_{4}}{C_{3}}=(23 \mu \mathrm{C}) \frac{16 \mu \mathrm{~F}}{8 \mu \mathrm{~F}}=46 \mu \mathrm{C}
$$

The parallel combination of $C_{3}$ and $C_{4}$ is in series with the parallel combination of $C_{1}$ and $C_{2}$, and so $Q_{3}+Q_{4}=Q_{1}+Q_{2}$. That total charge then divides between $C_{1}$ and $C_{2}$ in such a way that $V_{1}=V_{2}$.

$$
\begin{aligned}
& Q_{1}+Q_{2}=Q_{3}+Q_{4}=69 \mu \mathrm{C} ; V_{1}=V_{2} \rightarrow \frac{Q_{1}}{C_{1}}=\frac{Q_{4}}{C_{4}}=\frac{69 \mu \mathrm{C}-Q_{1}}{C_{4}} \rightarrow \\
& Q_{1}=\frac{C_{1}}{C_{4}+C_{1}}(69 \mu \mathrm{C})=\frac{8.0 \mu \mathrm{~F}}{24.0 \mu \mathrm{~F}}(69 \mu \mathrm{C})=23 \mu \mathrm{C} ; Q_{2}=69 \mu \mathrm{C}-23 \mu \mathrm{C}=46 \mu \mathrm{C}
\end{aligned}
$$

Notice the symmetry in the capacitances and the charges.
(b) Use Eq. 24-1.

$$
\begin{aligned}
& V_{1}=\frac{Q_{1}}{C_{1}}=\frac{23 \mu \mathrm{C}}{8.0 \mu \mathrm{~F}}=2.875 \mathrm{~V} \approx 2.9 \mathrm{~V} ; V_{2}=V_{1}=2.9 \mathrm{~V} \\
& V_{3}=\frac{Q_{3}}{C_{3}}=\frac{23 \mu \mathrm{C}}{8.0 \mu \mathrm{~F}}=2.875 \mathrm{~V} \approx 2.9 \mathrm{~V} ; V_{4}=V_{3}=2.9 \mathrm{~V}
\end{aligned}
$$

(c) $V_{\text {ba }}=V_{1}+V_{3}=2.875 \mathrm{~V}+2.875 \mathrm{~V}=5.75 \mathrm{~V} \approx 5.8 \mathrm{~V}$
34. We have $C_{\mathrm{P}}=C_{1}+C_{2}$ and $\frac{1}{C_{\mathrm{s}}}=\frac{1}{C_{1}}+\frac{1}{C_{2}}$. Solve for $C_{1}$ and $C_{2}$ in terms of $C_{\mathrm{P}}$ and $C_{\mathrm{S}}$.

$$
\begin{aligned}
\frac{1}{C_{\mathrm{s}}} & =\frac{1}{C_{1}}+\frac{1}{C_{2}}=\frac{1}{C_{1}}+\frac{1}{C_{\mathrm{P}}-C_{1}}=\frac{\left(C_{\mathrm{P}}-C_{1}\right)+C_{1}}{C_{1}\left(C_{\mathrm{P}}-C_{1}\right)}=\frac{C_{\mathrm{p}}}{C_{1}\left(C_{\mathrm{P}}-C_{1}\right)} \rightarrow \\
\frac{1}{C_{\mathrm{s}}} & =\frac{C_{\mathrm{P}}}{C_{1}\left(C_{\mathrm{P}}-C_{1}\right)} \rightarrow C_{1}^{2}-C_{\mathrm{P}} C_{1}+C_{\mathrm{P}} C_{\mathrm{s}}=0 \rightarrow \\
C_{1} & =\frac{C_{\mathrm{P}} \pm \sqrt{C_{\mathrm{P}}^{2}-4 C_{\mathrm{P}} C_{\mathrm{S}}}}{2}=\frac{35.0 \mu \mathrm{~F} \pm \sqrt{(35.0 \mu \mathrm{~F})^{2}-4(35.0 \mu \mathrm{~F})(5.5 \mu \mathrm{~F})}}{2} \\
& =28.2 \mu \mathrm{~F}, 6.8 \mu \mathrm{~F} \\
C_{2} & =C_{\mathrm{P}}-C_{1}=35.0 \mu \mathrm{~F}-28.2 \mu \mathrm{~F}=6.8 \mu \mathrm{~F} \text { or } 35.0 \mu \mathrm{~F}-6.8 \mu \mathrm{~F}=28.2 \mu \mathrm{~F}
\end{aligned}
$$

So the two values are $28.2 \mu \mathrm{~F}$ and $6.8 \mu \mathrm{~F}$.
35. Since there is no voltage between points $a$ and $b$, we can imagine there being a connecting wire between points a and b . Then capacitors $C_{1}$ and $C_{2}$ are in parallel, and so have the same voltage. Also capacitors $C_{3}$ and $C_{x}$ are in parallel, and so have the same voltage.

$$
V_{1}=V_{2} \rightarrow \frac{Q_{1}}{C_{1}}=\frac{Q_{2}}{C_{2}} ; V_{3}=V_{x} \rightarrow \frac{Q_{3}}{C_{3}}=\frac{Q_{x}}{C_{x}}
$$

Since no charge flows through the voltmeter, we could also remove it from the circuit and have no change in the circuit. In that case, capacitors $C_{1}$ and $C_{x}$ are in series and so have the same charge. Likewise capacitors $C_{2}$ and $C_{3}$ are in series, and so have the same
 charge.

$$
Q_{1}=Q_{x} ; Q_{2}=Q_{3}
$$

Solve this system of equations for $C_{x}$.

$$
\frac{Q_{3}}{C_{3}}=\frac{Q_{x}}{C_{x}} \rightarrow C_{x}=C_{3} \frac{Q_{x}}{Q_{3}}=C_{3} \frac{Q_{1}}{Q_{2}}=C_{3} \frac{C_{1}}{C_{2}}=(4.8 \mu \mathrm{~F})\left(\frac{8.9 \mu \mathrm{~F}}{18.0 \mu \mathrm{~F}}\right)=2.4 \mu \mathrm{~F}
$$

36. The initial equivalent capacitance is the series combination of the two individual capacitances. Each individual capacitor will have the same charge as the equivalent capacitance. The sum of the two initial charges will be the sum of the two final charges, because charge is conserved. The final potential of both capacitors will be equal.

$$
\begin{aligned}
& \frac{1}{C_{\mathrm{eq}}}=\frac{1}{C_{1}}+\frac{1}{C_{2}} \rightarrow \\
& C_{\mathrm{eq}}=\frac{C_{1} C_{2}}{C_{1}+C_{2}} ; Q_{\mathrm{eq}}=C_{\mathrm{eq}} V_{0}=\frac{C_{1} C_{2}}{C_{1}+C_{2}} V_{0}=\frac{(3200 \mathrm{pF})(1800 \mathrm{pF})}{5000 \mathrm{pF}}(12.0 \mathrm{~V})=13,824 \mathrm{pC} \\
& Q_{\text {final }}+Q_{\text {final }}=2 Q_{\mathrm{eq}} ; V_{1}=V_{\text {final }} \rightarrow \underset{\text { final }}{Q_{1}} \frac{Q_{2}}{C_{1}}=\frac{Q_{\text {final }}}{C_{2}}=\frac{2 Q_{\mathrm{eq}}-Q_{1}}{C_{2}} \rightarrow \\
& Q_{\text {final }}^{1}=2 \frac{C_{1}}{C_{1}+C_{2}} Q_{\mathrm{eq}}=2 \frac{3200 \mathrm{pF}}{5000 \mathrm{pF}}(13,824 \mathrm{pC})=17,695 \mathrm{pC} \approx 1.8 \times 10^{-8} \mathrm{C} \\
& Q_{\text {final }}=2 Q_{\mathrm{eq}}-\underbrace{}_{\text {final }}=2(13,824 \mathrm{pC})-17,695 \mathrm{pC}=9953 \mathrm{pC} \approx 1.0 \times 10^{-8} \mathrm{C}
\end{aligned}
$$

37. (a) The series capacitors add reciprocally, and then the parallel combination is found by adding linearly.

$$
C_{\mathrm{eq}}=C_{1}+\left(\frac{1}{C_{2}}+\frac{1}{C_{3}}\right)^{-1}=C_{1}+\left(\frac{C_{3}}{C_{2} C_{3}}+\frac{C_{2}}{C_{2} C_{3}}\right)^{-1}=C_{1}+\left(\frac{C_{2}+C_{3}}{C_{2} C_{3}}\right)^{-1}=C_{1}+\frac{C_{2} C_{3}}{C_{2}+C_{3}}
$$

(b) For each capacitor, the charge is found by multiplying the capacitance times the voltage. For $C_{1}$, the full 35.0 V is across the capacitance, so $Q_{1}=C_{1} V=\left(24.0 \times 10^{-6} \mathrm{~F}\right)(35.0 \mathrm{~V})=$
$8.40 \times 10^{-4} \mathrm{C}$. The equivalent capacitance of the series combination of $C_{2}$ and $C_{3}$ has the full 35.0 V across it, and the charge on the series combination is the same as the charge on each of the individual capacitors.

$$
C_{\mathrm{eq}}=\left(\frac{1}{C}+\frac{1}{C / 2}\right)^{-1}=\frac{C}{3} \quad Q_{\mathrm{eq}}=C_{\mathrm{eq}} V=\frac{1}{3}\left(24.0 \times 10^{-6} \mathrm{~F}\right)(35.0 \mathrm{~V})=2.80 \times 10^{-4} \mathrm{C}=Q_{2}=Q_{3}
$$

38. From the circuit diagram, we see that $C_{1}$ is in parallel with the voltage, and so $V_{1}=24 \mathrm{~V}$.

Capacitors $C_{2}$ and $C_{3}$ both have the same charge, so their voltages are inversely proportional to their capacitance, and their voltages must total to 24.0 V .

$$
\begin{aligned}
& Q_{2}=Q_{3} \rightarrow C_{2} V_{2}=C_{3} V_{3} ; V_{2}+V_{3}=V \\
& V_{2}+\frac{C_{2}}{C_{3}} V_{2}=V \rightarrow V_{2}=\frac{C_{3}}{C_{2}+C_{3}} V=\frac{4.00 \mu \mathrm{~F}}{7.00 \mu \mathrm{~F}}(24.0 \mathrm{~V})=13.7 \mathrm{~V} \\
& V_{3}=V-V_{2}=24.0 \mathrm{~V}-13.7 \mathrm{~V}=10.3 \mathrm{~V}
\end{aligned}
$$

39. For an infinitesimal area element of the capacitance a distance $y$ up from the small end, the distance between the plates is $d+x=d+y \tan \theta \approx d+y \theta$. Since the capacitor plates are square, they are of dimension $\sqrt{A} \times \sqrt{A}$, and the area of the infinitesimal strip is $d A=\sqrt{A} d y$. The infinitesimal capacitance $d C$ of the strip is calculated, and then the total capacitance is found by adding together all of the infinitesimal capacitances, in parallel with each other.

$$
\begin{aligned}
C & =\varepsilon_{0} \frac{A}{d} \rightarrow d C=\varepsilon_{0} \frac{d A}{d+y \theta}=\varepsilon_{0} \frac{\sqrt{A} d y}{d+y \theta} \\
C & =\int d C=\int_{0}^{\sqrt{A}} \varepsilon_{0} \frac{\sqrt{A} d y}{d+y \theta}=\left.\frac{\varepsilon_{0} \sqrt{A}}{\theta} \ln (d+y \theta)\right|_{0} ^{\sqrt{A}} \\
& =\frac{\varepsilon_{0} \sqrt{A}}{\theta}[\ln (d+\theta \sqrt{A})-\ln d]=\frac{\varepsilon_{0} \sqrt{A}}{\theta} \ln \left(\frac{d+\theta \sqrt{A}}{d}\right)=\frac{\varepsilon_{0} \sqrt{A}}{\theta} \ln \left(1+\frac{\theta \sqrt{A}}{d}\right)
\end{aligned}
$$



We use the approximation from page A-1 that $\ln (1+x) \approx x-\frac{1}{2} x^{2}$.

$$
C=\frac{\varepsilon_{0} \sqrt{A}}{\theta} \ln \left(1+\frac{\theta \sqrt{A}}{d}\right)=\frac{\varepsilon_{0} \sqrt{A}}{\theta}\left[\frac{\theta \sqrt{A}}{d}-\frac{1}{2}\left(\frac{\theta \sqrt{A}}{d}\right)^{2}\right]=\frac{\varepsilon_{0} A}{d}\left(1-\frac{\theta \sqrt{A}}{2 d}\right)
$$

40. No two capacitors are in series or in parallel in the diagram, and so we may not simplify by that method. Instead use the hint as given in the problem. We consider point a as the higher voltage. The equivalent capacitance must satisfy $Q_{\text {tot }}=C_{\text {eq }} V$.
(a) The potential between a and b can be written in three ways. Alternate but equivalent expressions are shown in parentheses.

$$
V=V_{2}+V_{1} ; V=V_{2}+V_{3}+V_{4} ; V=V_{5}+V_{4} \quad\left(V_{2}+V_{3}=V_{5} ; V_{3}+V_{4}=V_{1}\right)
$$

There are also three independent charge relationships. Alternate but equivalent expressions are shown in parentheses. Convert the charge expressions to voltage - capacitance expression.

$$
\begin{array}{ll}
Q_{\text {tot }}=Q_{2}+Q_{5} \quad ; Q_{\text {tot }}=Q_{4}+Q_{1} \quad ; Q_{2}=Q_{1}+Q_{3}\left(Q_{4}=Q_{3}+Q_{5}\right) \\
C_{\text {eq }} V=C_{2} V_{2}+C_{5} V_{5} ; C_{\mathrm{eq}} V=C_{4} V_{4}+C_{1} V_{1} ; C_{2} V_{2}=C_{1} V_{1}+C_{3} V_{3}
\end{array}
$$

We have a set of six equations: $\quad V=V_{2}+V_{1}(1) ; V=V_{2}+V_{3}+V_{4}(2) ; V=V_{5}+V_{4}$ (3)

$$
\begin{equation*}
C_{\mathrm{eq}} V=C_{2} V_{2}+C_{5} V_{5}(4) ; C_{\mathrm{eq}} V=C_{4} V_{4}+C_{1} V_{1}(5) ; C_{2} V_{2}=C_{1} V_{1}+C_{3} V_{3} \tag{6}
\end{equation*}
$$

Solve for $C_{\text {eq }}$ as follows.
(i) From Eq. (1), $V_{1}=V-V_{2}$. Rewrite equations (5) and (6). $V_{1}$ has been eliminated.

$$
\begin{equation*}
C_{\mathrm{eq}} V=C_{4} V_{4}+C_{1} V-C_{1} V_{2}(5) ; C_{2} V_{2}=C_{1} V-C_{1} V_{2}+C_{3} V_{3} \tag{6}
\end{equation*}
$$

(ii) From Eq. (3), $V_{5}=V-V_{4}$. Rewrite equation (4). $V_{5}$ has been eliminated.

$$
C_{\mathrm{eq}} V=C_{2} V_{2}+C_{5} V-C_{5} V_{4}
$$

(iii) From Eq. (2), $V_{3}=V-V_{2}-V_{4}$. Rewrite equation (6). $V_{3}$ has been eliminated.

$$
\begin{aligned}
& C_{2} V_{2}=C_{1} V-C_{1} V_{2}+C_{3} V-C_{3} V_{2}-C_{3} V_{4}(6) \rightarrow \\
& \left(C_{1}+C_{2}+C_{3}\right) V_{2}+C_{3} V_{4}=\left(C_{1}+C_{3}\right) V(6)
\end{aligned}
$$

Here is the current set of equations.

$$
\begin{aligned}
& C_{\mathrm{eq}} V=C_{2} V_{2}+C_{5} V-C_{5} V_{4} \\
& C_{\mathrm{eq}} V=C_{4} V_{4}+C_{1} V-C_{1} V_{2} \\
& \left(C_{1}+C_{2}+C_{3}\right) V_{2}+C_{3} V_{4}=\left(C_{1}+C_{3}\right) V
\end{aligned}
$$

(iv) From Eq. (4), $V_{4}=\frac{1}{C_{5}}\left(C_{2} V_{2}+C_{5} V-C_{\mathrm{eq}} V\right)$. Rewrite equations (5) and (6).

$$
\begin{aligned}
& C_{5} C_{\mathrm{eq}} V=C_{4}\left[\left(C_{2} V_{2}+C_{5} V-C_{\mathrm{eq}} V\right)\right]+C_{5} C_{1} V-C_{5} C_{1} V_{2} \\
& C_{5}\left(C_{1}+C_{2}+C_{3}\right) V_{2}+C_{3}\left[\left(C_{2} V_{2}+C_{5} V-C_{\mathrm{eq}} V\right)\right]=C_{5}\left(C_{1}+C_{3}\right) V
\end{aligned}
$$

(v) Group all terms by common voltage.

$$
\begin{align*}
& \left(C_{5} C_{\mathrm{eq}}+C_{4} C_{\mathrm{eq}}-C_{4} C_{5}-C_{5} C_{1}\right) V=\left(C_{4} C_{2}-C_{5} C_{1}\right) V_{2}(5)  \tag{5}\\
& {\left[C_{5}\left(C_{1}+C_{3}\right)+C_{3} C_{\mathrm{eq}}-C_{3} C_{5}\right] V=\left[C_{5}\left(C_{1}+C_{2}+C_{3}\right)+C_{3} C_{2}\right] V_{2}}
\end{align*}
$$

(vi) Divide the two equations to eliminate the voltages, and solve for the equivalent capacitance.

$$
\begin{aligned}
& \frac{\left(C_{5} C_{\mathrm{eq}}+C_{4} C_{\mathrm{eq}}-C_{4} C_{5}-C_{5} C_{1}\right)}{\left[C_{5}\left(C_{1}+C_{3}\right)+C_{3} C_{\mathrm{eq}}-C_{3} C_{5}\right]}=\frac{\left(C_{4} C_{2}-C_{5} C_{1}\right)}{\left[C_{5}\left(C_{1}+C_{2}+C_{3}\right)+C_{3} C_{2}\right]} \rightarrow \\
& C_{\mathrm{eq}}=\frac{C_{1} C_{2} C_{3}+C_{1} C_{2} C_{4}+C_{1} C_{2} C_{5}+C_{1} C_{3} C_{5}+C_{1} C_{4} C_{5}+C_{2} C_{3} C_{4}+C_{2} C_{4} C_{5}+C_{3} C_{4} C_{5}}{C_{1} C_{3}+C_{1} C_{4}+C_{1} C_{5}+C_{2} C_{3}+C_{2} C_{4}+C_{2} C_{5}+C_{3} C_{4}+C_{3} C_{5}}
\end{aligned}
$$

(b) Evaluate with the given data. Since all capacitances are in $\mu \mathrm{F}$, and the expression involves capacitance cubed terms divided by capacitance squared terms, the result will be in $\mu \mathrm{F}$.

$$
\begin{aligned}
C_{\mathrm{eq}} & =\frac{C_{1} C_{2} C_{3}+C_{1} C_{2} C_{4}+C_{1} C_{2} C_{5}+C_{1} C_{3} C_{5}+C_{1} C_{4} C_{5}+C_{2} C_{3} C_{4}+C_{2} C_{4} C_{5}+C_{3} C_{4} C_{5}}{C_{1} C_{3}+C_{1} C_{4}+C_{1} C_{5}+C_{2} C_{3}+C_{2} C_{4}+C_{2} C_{5}+C_{3} C_{4}+C_{3} C_{5}} \\
& =\frac{C_{1}\left[C_{2}\left(C_{3}+C_{4}+C_{5}\right)+C_{5}\left(C_{3}+C_{4}\right)\right]+C_{4}\left(C_{2} C_{3}+C_{2} C_{5}+C_{3} C_{5}\right)}{C_{1}\left(C_{3}+C_{4}+C_{5}\right)+C_{2}\left(C_{3}+C_{4}+C_{5}\right)+C_{3}\left(C_{4}+C_{5}\right)} \mu \mathrm{F} \\
& =\frac{(4.5)\{(8.0)(17.0)+(4.5)(12.5)\}+(8.0)[(8.0)(4.5)+(8.0)(4.5)+(4.5)(4.5)]}{(4.5)(17.0)+(8.0)(17.0)+(4.5)(12.5)} \\
& =6.0 \mu \mathrm{~F}
\end{aligned}
$$

41. The stored energy is given by Eq. 24-5.

$$
U=\frac{1}{2} C V^{2}=\frac{1}{2}\left(2.8 \times 10^{-9} \mathrm{~F}\right)(2200 \mathrm{~V})^{2}=6.8 \times 10^{-3} \mathrm{~J}
$$

42. The energy density is given by Eq. 24-6.

$$
u=\frac{1}{2} \varepsilon_{0} E^{2}=\frac{1}{2}\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)(150 \mathrm{~V} / \mathrm{m})^{2}=1.0 \times 10^{-7} \mathrm{~J} / \mathrm{m}^{3}
$$

43. The energy stored is obtained from Eq. 24-5, with the capacitance of Eq. 24-2.

$$
U=\frac{Q^{2}}{2 C}=\frac{Q^{2} d}{2 \varepsilon_{0} A}=\frac{\left(4.2 \times 10^{-4} \mathrm{C}\right)^{2}(0.0013 \mathrm{~m})}{2\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)(0.080 \mathrm{~m})^{2}}=2.0 \times 10^{3} \mathrm{~J}
$$

44. (a) The charge is constant, and the tripling of separation reduces the capacitance by a factor of 3 .

$$
\frac{U_{2}}{U_{1}}=\frac{\frac{Q^{2}}{2 C_{2}}}{\frac{Q^{2}}{2 C_{1}}}=\frac{C_{1}}{C_{2}}=\frac{\varepsilon_{0} \frac{A}{d}}{\varepsilon_{0} \frac{A}{3 d}}=3
$$

(b) The work done is the change in energy stored in the capacitor.

$$
U_{2}-U_{1}=3 U_{1}-U_{1}=2 U_{1}=2 \frac{Q^{2}}{2 C_{1}}=\frac{Q^{2}}{\varepsilon_{0} \frac{A}{d}}=\frac{Q^{2} d}{\varepsilon_{0} A}
$$

45. The equivalent capacitance is formed by $C_{1}$ in parallel with the series combination of $C_{2}$ and $C_{3}$.

Then use Eq. 24-5 to find the energy stored.

$$
\begin{aligned}
& C_{\text {net }}=C_{1}+\frac{C_{2} C_{3}}{C_{2}+C_{3}}=C+\frac{C^{2}}{2 C}=\frac{3}{2} C \\
& U=\frac{1}{2} C_{\text {net }} V^{2}=\frac{3}{4} C V^{2}=\frac{3}{4}\left(22.6 \times 10^{-6} \mathrm{~F}\right)(10.0 \mathrm{~V})^{2}=1.70 \times 10^{-3} \mathrm{~J}
\end{aligned}
$$

46. (a) Use Eqs. 24-3 and 24-5.

$$
U_{\text {parallel }}=\frac{1}{2} C_{\text {eq }} V^{2}=\frac{1}{2}\left(C_{1}+C_{2}\right) V^{2}=\frac{1}{2}\left(0.65 \times 10^{-6} \mathrm{~F}\right)(28 \mathrm{~V})^{2}=2.548 \times 10^{-4} \mathrm{~J} \approx 2.5 \times 10^{-4} \mathrm{~J}
$$

(b) Use Eqs. 24-4 and 24-5.

$$
\begin{aligned}
U_{\text {series }} & =\frac{1}{2} C_{\text {eq }} V^{2}=\frac{1}{2}\left(\frac{C_{1} C_{2}}{C_{1}+C_{2}}\right) V^{2}=\frac{1}{2}\left(\frac{\left(0.45 \times 10^{-6} \mathrm{~F}\right)\left(0.20 \times 10^{-6} \mathrm{~F}\right)}{0.65 \times 10^{-6} \mathrm{~F}}\right)(28 \mathrm{~V})^{2} \\
& =5.428 \times 10^{-5} \mathrm{~J} \approx 5.4 \times 10^{-5} \mathrm{~J}
\end{aligned}
$$

(c) The charge can be found from Eq. 24-5.

$$
\begin{aligned}
& U=\frac{1}{2} Q V \rightarrow Q=\frac{2 U}{V} \rightarrow Q_{\text {parallel }}=\frac{2\left(2.548 \times 10^{-4} \mathrm{~J}\right)}{28 \mathrm{~V}}=1.8 \times 10^{-5} \mathrm{C} \\
& Q_{\text {series }}=\frac{2\left(5.428 \times 10^{-5} \mathrm{~J}\right)}{28 \mathrm{~V}}=3.9 \times 10^{-6} \mathrm{C}
\end{aligned}
$$

47. The capacitance of a cylindrical capacitor is given in Example 24-2 as $C=\frac{2 \pi \varepsilon_{0} \ell}{\ln \left(R_{\mathrm{a}} / R_{\mathrm{b}}\right)}$.
(a) If the charge is constant, the energy can be calculated by $U=\frac{1}{2} \frac{Q^{2}}{C}$.

$$
\frac{U_{2}}{U_{1}}=\frac{\frac{1}{2} \frac{Q^{2}}{C_{2}}}{\frac{1}{2} \frac{Q^{2}}{C_{1}}}=\frac{C_{1}}{C_{2}}=\frac{\frac{2 \pi \varepsilon_{0} \ell}{\ln \left(R_{\mathrm{a}} / R_{\mathrm{b}}\right)}}{\frac{2 \pi \varepsilon_{0} \ell}{\ln \left(3 R_{\mathrm{a}} / R_{\mathrm{b}}\right)}}=\frac{\ln \left(3 R_{\mathrm{a}} / R_{\mathrm{b}}\right)}{\ln \left(R_{\mathrm{a}} / R_{\mathrm{b}}\right)}>1
$$

The energy comes from the work required to separate the capacitor components.
(b) If the voltage is constant, the energy can be calculated by $U=\frac{1}{2} C V^{2}$.

$$
\frac{U_{2}}{U_{1}}=\frac{\frac{1}{2} C_{2} V^{2}}{\frac{1}{2} C_{1} V^{2}}=\frac{C_{2}}{C_{1}}=\frac{\frac{2 \pi \varepsilon_{0} \ell}{\ln \left(3 R_{\mathrm{a}} / R_{\mathrm{b}}\right)}}{\frac{2 \pi \varepsilon_{0} \ell}{\ln \left(R_{\mathrm{a}} / R_{\mathrm{b}}\right)}}=\frac{\ln \left(R_{\mathrm{a}} / R_{\mathrm{b}}\right)}{\ln \left(3 R_{\mathrm{a}} / R_{\mathrm{b}}\right)}<1
$$

Since the voltage remained constant, and the capacitance decreased, the amount of charge on the capacitor components decreased. Charge flowed back into the battery that was maintaining the constant voltage.
48. (a) Before the capacitors are connected, the only stored energy is in the initially-charged capacitor. Use Eq. 24-5.

$$
U_{1}=\frac{1}{2} C_{1} V_{0}^{2}=\frac{1}{2}\left(2.20 \times 10^{-6} \mathrm{~F}\right)(12.0 \mathrm{~V})^{2}=1.584 \times 10^{-4} \mathrm{~J} \approx 1.58 \times 10^{-4} \mathrm{~J}
$$

(b) The total charge available is the charge on the initial capacitor. The capacitance changes to the equivalent capacitance of the two capacitors in parallel.

$$
\begin{aligned}
Q=Q_{1}=C_{1} V_{0} ; C_{\mathrm{eq}}=C_{1}+C_{2} ; U_{2} & =\frac{1}{2} \frac{Q^{2}}{C_{\mathrm{eq}}}=\frac{1}{2} \frac{C_{1}^{2} V_{0}^{2}}{C_{1}+C_{2}}=\frac{1}{2} \frac{\left(2.20 \times 10^{-6} \mathrm{~F}\right)^{2}(12.0 \mathrm{~V})^{2}}{\left(5.70 \times 10^{-6} \mathrm{~F}\right)} \\
& =6.114 \times 10^{-5} \mathrm{~J} \approx 6.11 \times 10^{-5} \mathrm{~J}
\end{aligned}
$$

(c) $\Delta U=U_{2}-U_{1}=6.114 \times 10^{-5} \mathrm{~J}-1.584 \times 10^{-4} \mathrm{~J}=-9.73 \times 10^{-5} \mathrm{~J}$
49. (a) With the plate inserted, the capacitance is that of two series capacitors of plate separations $d_{1}=x$ and $d_{2}=d-\ell-x$.

$$
C_{i}=\left[\frac{x}{\varepsilon_{0} A}+\frac{d-x-\ell}{\varepsilon_{0} A}\right]^{-1}=\frac{\varepsilon_{0} A}{d-\ell}
$$

With the plate removed the capacitance is obtained directly from Eq. 24-2.

$$
C_{f}=\frac{\varepsilon_{0} A}{d}
$$

Since the voltage remains constant the energy of the capacitor will be given by Eq. $24-5$ written in terms of voltage and capacitance. The work will be the change in energy as the plate is removed.

$$
\begin{aligned}
& W=U_{f}-U_{i}=\frac{1}{2}\left(C_{f}-C_{i}\right) V^{2} \\
& =\frac{1}{2}\left(\frac{\varepsilon_{0} A}{d}-\frac{\varepsilon_{0} A}{d-\ell}\right) V^{2}=-\frac{\varepsilon_{0} A \ell V^{2}}{2 d(d-\ell)}
\end{aligned}
$$

The net work done is negative. Although the person pulling the plate out must do work, charge is returned to the battery, resulting in a net negative work done.
(b) Since the charge now remains constant, the energy of the capacitor will be given by Eq. 24-5 written in terms of capacitance and charge.

$$
W=\frac{Q^{2}}{2}\left(\frac{1}{C_{f}}-\frac{1}{C_{i}}\right)=\frac{Q^{2}}{2}\left(\frac{d}{\varepsilon_{0} A}-\frac{d-\ell}{\varepsilon_{0} A}\right)=\frac{Q^{2} \ell}{2 \varepsilon_{0} A}
$$

The original charge is $Q=C V_{0}=\frac{\varepsilon_{0} A}{d-\ell} V_{0}$ and so $W=\frac{\left(\frac{\varepsilon_{0} A}{d-\ell} V_{0}\right)^{2} \ell}{2 \varepsilon_{0} A}=\frac{\varepsilon_{0} A V_{0}^{2} \ell}{2(d-\ell)^{2}}$.
50. (a) The charge remains constant, so we express the stored energy as $U=\frac{1}{2} \frac{Q^{2}}{C}=\frac{1}{2} \frac{Q^{2} x}{\varepsilon_{0} A}$, where $x$ is the separation of the plates. The work required to increase the separation by $d x$ is $d W=F d x$, where $F$ is the force on one plate exerted by the other plate. That work results in an increase in potential energy, $d U$.

$$
d W=F d x=d U=\frac{1}{2} \frac{Q^{2} d x}{\varepsilon_{0} A} \rightarrow F=\frac{1}{2} \frac{Q^{2}}{\varepsilon_{0} A}
$$

(b) We cannot use $F=Q E=Q \frac{\sigma}{\varepsilon_{0}}=Q \frac{Q}{\varepsilon_{0} A}=\frac{Q^{2}}{\varepsilon_{0} A}$ because the electric field is due to both plates, and charge cannot put a force on itself by the field it creates. By the symmetry of the geometry, the electric field at one plate, due to just the other plate, is $\frac{1}{2} E$. See Example 24-10.
51. (a) The electric field outside the spherical conductor is that of an equivalent point charge at the center of the sphere, so $E=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{r^{2}}, r>R$. Consider a differential volume of radius $d r$, and volume $d V=4 \pi r^{2} d r$, as used in Example 22-5. The energy in that volume is $d U=u d V$. Integrate over the region outside the conductor.

$$
\begin{aligned}
U & =\int d U=\int u d V=\frac{1}{2} \varepsilon_{0} \int E^{2} d V=\frac{1}{2} \varepsilon_{0} \int_{R}^{\infty}\left(\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{r^{2}}\right)^{2} 4 \pi r^{2} d r=\frac{Q^{2}}{8 \pi \varepsilon_{0}} \int_{R}^{\infty} \frac{1}{r^{2}} d r=-\left.\frac{Q^{2}}{8 \pi \varepsilon_{0}} \frac{1}{r}\right|_{R} ^{\infty} \\
& =\frac{Q^{2}}{8 \pi \varepsilon_{0} R}
\end{aligned}
$$

(b) Use Eq. 24-5 with the capacitance of an isolated sphere, from the text immediately after Example 24-3.

$$
U=\frac{1}{2} \frac{Q^{2}}{C}=\frac{1}{2} \frac{Q^{2}}{4 \pi \varepsilon_{0} R}=\frac{Q^{2}}{8 \pi \varepsilon_{0} R}
$$

(c) When there is a charge $q<Q$ on the sphere, the potential of the sphere is $V=\frac{1}{4 \pi \varepsilon_{0}} \frac{q}{R}$. The work required to add a charge $d q$ to the sphere is then $d W=V d q=\frac{1}{4 \pi \varepsilon_{0}} \frac{q}{R} d q$. That work increase the potential energy by the same amount, so $d U=d W=V d q=\frac{1}{4 \pi \varepsilon_{0}} \frac{q}{R} d q$. Build up the entire charge from 0 to $Q$, calculating the energy as the charge increases.

$$
U=\int d U=\int d W=\int V d q=\int_{0}^{Q} \frac{1}{4 \pi \varepsilon_{0}} \frac{q}{R} d q=\frac{1}{4 \pi \varepsilon_{0} R} \int_{0}^{Q} q d q=\frac{Q^{2}}{8 \pi \varepsilon_{0} R}
$$

52. In both configurations, the voltage across the combination of capacitors is the same. So use $U=\frac{1}{2} C V^{2}$.

$$
\begin{aligned}
& U_{\mathrm{P}}=\frac{1}{2} C_{\mathrm{P}} V^{2}=\frac{1}{2}\left(C_{1}+C_{2}\right) V^{2} ; U_{\mathrm{s}}=\frac{1}{2} C_{\mathrm{S}} V^{2}=\frac{1}{2} \frac{C_{1} C_{2}}{\left(C_{1}+C_{2}\right)} V^{2} \\
& U_{\mathrm{P}}=5 U_{\mathrm{s}} \rightarrow \frac{1}{2}\left(C_{1}+C_{2}\right) V^{2}=5\left(\frac{1}{2}\right) \frac{C_{1} C_{2}}{\left(C_{1}+C_{2}\right)} V^{2} \rightarrow\left(C_{1}+C_{2}\right)^{2}=5 C_{1} C_{2} \rightarrow \\
& C_{1}^{2}-3 C_{1} C_{2}+C_{2}^{2}=0 \rightarrow C_{1}=\frac{3 C_{2} \pm \sqrt{9 C_{2}^{2}-4 C_{2}^{2}}}{2}=C_{2} \frac{3 \pm \sqrt{5}}{2} \rightarrow \\
& \frac{C_{1}}{C_{2}}=\frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}=2.62,0.382
\end{aligned}
$$

53. First find the ratio of energy requirements for a logical operation in the past to the current energy requirements for a logical operation.

$$
\frac{E_{\text {past }}}{E_{\text {present }}}=\frac{N\left(\frac{1}{2} C V^{2}\right)_{\text {past }}}{N\left(\frac{1}{2} C V^{2}\right)_{\text {present }}}=\left(\frac{C_{\text {past }}}{C_{\text {present }}}\right)\left(\frac{V_{\text {past }}}{V_{\text {present }}}\right)^{2}=\left(\frac{20}{1}\right)\left(\frac{5.0}{1.5}\right)^{2}=220
$$

So past operations would have required 220 times more energy. Since 5 batteries in the past were required to hold the same energy as a present battery, it would have taken 1100 times as many batteries in the past. And if it takes 2 batteries for a modern PDA, it would take 2200 batteries to power the PDA in the past. It would not fit in a pocket or purse. The volume of a present-day battery is $V=\pi r^{2} \ell=\pi(0.5 \mathrm{~cm})^{2}(4 \mathrm{~cm})=3 \mathrm{~cm}^{3}$. The volume of 2200 of them would be $6600 \mathrm{~cm}^{3}$, which would require a cube about 20 cm in side length.
54. Use Eq. 24-8 to calculate the capacitance with a dielectric.

$$
C=K \varepsilon_{0} \frac{A}{d}=(2.2)\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right) \frac{\left(4.2 \times 10^{-2} \mathrm{~m}\right)^{2}}{\left(1.8 \times 10^{-3} \mathrm{~m}\right)}=1.9 \times 10^{-11} \mathrm{~F}
$$

55. The change in energy of the capacitor is obtained from Eq. $24-5$ in terms of the constant voltage and the capacitance.

$$
\Delta U=U_{\mathrm{f}}-U_{\mathrm{i}}=\frac{1}{2} C_{0} V^{2}-\frac{1}{2} K C_{0} V^{2}=-\frac{1}{2}(K-1) C_{0} V^{2}
$$

The work done by the battery in maintaining a constant voltage is equal to the voltage multiplied by the change in charge, with the charge given by Eq. 24-1.

$$
W_{\text {battery }}=V\left(Q_{\mathrm{f}}-Q_{\mathrm{i}}\right)=V\left(C_{0} V-K C_{0} V\right)=-(K-1) C_{0} V^{2}
$$

The work done in pulling the dielectric out of the capacitor is equal to the difference between the change in energy of the capacitor and the energy done by the battery.

$$
\begin{aligned}
W & =\Delta U-W_{\text {battery }}=-\frac{1}{2}(K-1) C_{0} V^{2}+(K-1) C_{0} V^{2} \\
& =\frac{1}{2}(K-1) C_{0} V^{2}=(3.4-1)\left(8.8 \times 10^{-9} \mathrm{~F}\right)(100 \mathrm{~V})^{2}=1.1 \times 10^{-4} \mathrm{~J}
\end{aligned}
$$

56. We assume the charge and dimensions are the same as in Problem 43. Use Eq. 24-5 with charge and capacitance.

$$
U=\frac{1}{2} \frac{Q^{2}}{C}=\frac{1}{2} \frac{Q^{2}}{K C_{0}}=\frac{1}{2} \frac{Q^{2} d}{K \varepsilon_{0} A}=\frac{1}{2} \frac{\left(420 \times 10^{-6} \mathrm{C}\right)^{2}(0.0013 \mathrm{~m})}{(7)\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)\left(64 \times 10^{-4} \mathrm{~m}^{2}\right)}=289.2 \mathrm{~J} \approx 290 \mathrm{~J}
$$

57. From Problem 10, we have $C=35 \times 10^{-15} \mathrm{~F}$. Use Eq. 24-8 to calculate the area.

$$
\begin{aligned}
C=K \varepsilon_{0} \frac{A}{d} \rightarrow A & =\frac{C d}{K \varepsilon_{0}}=\frac{\left(35 \times 10^{-15} \mathrm{~F}\right)\left(2.0 \times 10^{-9} \mathrm{~m}\right)}{(25)\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)}=3.164 \times 10^{-13} \mathrm{~m}^{2}\left(\frac{10^{6} \mu \mathrm{~m}}{1 \mathrm{~m}}\right)^{2} \\
& =0.3164 \mu \mathrm{~m}^{2} \approx 0.32 \mu \mathrm{~m}^{2}
\end{aligned}
$$

Half of the area of the cell is used for capacitance, so $1.5 \mathrm{~cm}^{2}$ is available for capacitance. Each capacitor is one "bit."

$$
1.5 \mathrm{~cm}^{2}\left(\frac{10^{6} \mu \mathrm{~m}}{10^{2} \mathrm{~cm}}\right)^{2}\left(\frac{1 \text { bit }}{0.32 \mu \mathrm{~m}^{2}}\right)\left(\frac{1 \text { byte }}{8 \text { bits }}\right)=5.86 \times 10^{7} \text { bytes } \approx 59 \mathrm{Mbytes}
$$

58. The initial charge on the capacitor is $Q_{\text {initial }}=C_{\text {initial }} V$. When the mica is inserted, the capacitance changes to $C_{\text {final }}=K C_{\text {initial }}$, and the voltage is unchanged since the capacitor is connected to the same battery. The final charge on the capacitor is $Q_{\text {final }}=C_{\text {final }} V$.

$$
\begin{aligned}
\Delta Q & =Q_{\text {final }}-Q_{\text {intitial }}=C_{\text {final }} V-C_{\text {initial }} V=(K-1) C_{\text {initial }} V=(7-1)\left(3.5 \times 10^{-9} \mathrm{~F}\right)(32 \mathrm{~V}) \\
& =6.7 \times 10^{-7} \mathrm{C}
\end{aligned}
$$

59. The potential difference is the same on each half of the capacitor, so it can be treated as two capacitors in parallel. Each parallel capacitor has half of the total area of the original capacitor.

$$
C=C_{1}+C_{2}=K_{1} \varepsilon_{0} \frac{\frac{1}{2} A}{d}+K_{2} \varepsilon_{0} \frac{\frac{1}{2} A}{d}=\frac{1}{2}\left(K_{1}+K_{2}\right) \varepsilon_{0} \frac{A}{d}
$$


60. The intermediate potential at the boundary of the two dielectrics can be treated as the "low" potential plate of one half and the "high" potential plate of the other half, so we treat it as two capacitors in series. Each series capacitor has half of the inter-plate distance of the original capacitor.


$$
\frac{1}{C}=\frac{1}{C_{1}}+\frac{1}{C_{2}}=\frac{\frac{1}{2} d}{K_{1} \varepsilon_{0} A}+\frac{\frac{1}{2} d}{K_{2} \varepsilon_{0} A}=\frac{d}{2 \varepsilon_{0} A} \frac{K_{1}+K_{2}}{K_{1} K_{2}} \rightarrow C=\frac{2 \varepsilon_{0} A}{d} \frac{K_{1} K_{2}}{K_{1}+K_{2}}
$$

61. The capacitor can be treated as two series capacitors with the same areas, but different plate separations and dielectrics. Substituting Eq. 24-8 into Eq. 24-4 gives the effective capacitance.

$$
C=\left(\frac{1}{C_{1}}+\frac{1}{C_{2}}\right)^{-1}=\left(\frac{d_{1}}{K_{1} A \varepsilon_{0}}+\frac{d_{2}}{K_{2} A \varepsilon_{0}}\right)^{-1}=\frac{A \varepsilon_{0} K_{1} K_{2}}{d_{1} K_{2}+d_{2} K_{1}}
$$


62. (a) Since the capacitors each have the same charge and the same voltage in the initial situation, each has the same capacitance of $C=\frac{Q_{0}}{V_{0}}$. When the dielectric is inserted, the total charge of $2 Q_{0}$ will not change, but the charge will no longer be divided equally between the two capacitors. Some charge will move from the capacitor without the dielectric $\left(C_{1}\right)$ to the capacitor with the dielectric $\left(C_{2}\right)$. Since the capacitors are in parallel, their voltages will be the same.

$$
\begin{aligned}
V_{1} & =V_{2} \rightarrow \frac{Q_{1}}{C_{1}}=\frac{Q_{2}}{C_{2}} \rightarrow \frac{Q_{1}}{C}=\frac{2 Q_{0}-Q_{1}}{K C} \rightarrow \\
Q_{1} & =\frac{2}{(K+1)} Q_{0}=\frac{2}{4.2} Q_{0}=0.48 Q_{0} ; Q_{2}=1.52 Q_{0} \\
\text { (b) } V_{1}=V_{2} & =\frac{Q_{1}}{C_{1}}=\frac{0.48 Q_{0}}{Q_{0} / V_{0}}=0.48 V_{0}=\frac{Q_{2}}{C_{2}}=\frac{1.52 Q_{0}}{3.2 Q_{0} / V_{0}}
\end{aligned}
$$

63. (a) We treat this system as two capacitors, one with a dielectric, and one without a dielectric. Both capacitors have their high voltage plates in contact and their low voltage plates in contact, so they are in parallel. Use Eq. 24-2 and 24-8 for the capacitance. Note that $x$ is measured from the right edge of
 the capacitor, and is positive to the left in the diagram.

$$
C=C_{1}+C_{2}=\varepsilon_{0} \frac{\ell(\ell-x)}{d}+K \varepsilon_{0} \frac{\ell x}{d}=\varepsilon_{0} \frac{\ell^{2}}{d}\left[1+(K-1) \frac{x}{\ell}\right]
$$

(b) Both "capacitors" have the same potential difference, so use $U=\frac{1}{2} C V^{2}$.

$$
U=\frac{1}{2}\left(C_{1}+C_{2}\right) V_{0}^{2}=\varepsilon_{0} \frac{\ell^{2}}{2 d}\left[1+(K-1) \frac{x}{\ell}\right] V_{0}^{2}
$$

(c) We must be careful here. When the voltage across a capacitor is constant and a dielectric is inserted, charge flows from the battery to the capacitor. So the battery will lose energy and the capacitor gain energy as the dielectric is inserted. As in Example 24-10, we assume that work is done by an external agent $\left(W_{\mathrm{nc}}\right)$ in such a way that the dielectric has no kinetic energy. Then the work-energy principle (Chapter 8) can be expressed as $W_{\mathrm{nc}}=\Delta U$ or $d W_{\mathrm{nc}}=d U$. This is analogous to moving an object vertically at constant speed. To increase (decrease) the gravitational potential energy, positive (negative) work must be done by an outside, nongravitational source.

In this problem, the potential energy of the voltage source and the potential energy of the capacitor both change as $x$ changes. Also note that the change in charge stored on the capacitor is the opposite of the change in charge stored in the voltage supply.

$$
\begin{aligned}
d W_{\text {nc }} & =d U=d U_{\text {cap }}+d U_{\text {battery }} \rightarrow F_{\text {nc }} d x=d\left(\frac{1}{2} C V_{0}^{2}\right)+d\left(Q_{\text {batery }} V_{0}\right) \rightarrow \\
F_{\text {nc }} & =\frac{1}{2} V_{0}^{2} \frac{d C}{d x}+V_{0} \frac{d Q_{\text {batery }}}{d x}=\frac{1}{2} V_{0}^{2} \frac{d C}{d x}-V_{0} \frac{d Q_{\text {cap }}}{d x}=\frac{1}{2} V_{0}^{2} \frac{d C}{d x}-V_{0}^{2} \frac{d C}{d x}=-\frac{1}{2} V_{0}^{2} \frac{d C}{d x} \\
& =-\frac{1}{2} V_{0}^{2} \varepsilon_{0} \frac{\ell^{2}}{d}\left[\frac{(K-1)}{\ell}\right]=-\frac{V_{0}^{2} \varepsilon_{0}^{\ell}}{2 d}(K-1)
\end{aligned}
$$

Note that this force is in the opposite direction of $d x$, and so is to the right. Since this force is being applied to keep the dielectric from accelerating, there must be a force of equal magnitude to the left pulling on the dielectric. This force is due to the attraction of the charged plates and the induced charge on the dielectric. The magnitude and direction of this attractive force are
$\frac{V_{0}^{2} \varepsilon_{0} \ell}{2 d}(K-1)$, left .
64. (a) We consider the cylinder as two cylindrical capacitors in parallel. The two "negative plates" are the (connected) halves of the inner cylinder (half of which is in contact with liquid, and half of which is in contact with vapor). The two "positive plates" are the (connected) halves of the outer cylinder (half of which is in contact with liquid, and half of which is in contact with vapor). Schematically, it is like Figure 24-30 in Problem 59. The capacitance of a cylindrical capacitor is given in Example 24-2.

$$
\begin{aligned}
& C=C_{\text {liq }}+C_{\mathrm{v}}=\frac{2 \pi \varepsilon_{0} K_{\text {liq }} h}{\ln \left(R_{\mathrm{a}} / R_{\mathrm{b}}\right)}+\frac{2 \pi \varepsilon_{0} K_{\mathrm{v}}(\ell-h)}{\ln \left(R_{\mathrm{a}} / R_{\mathrm{b}}\right)}=\frac{2 \pi \varepsilon_{0} \ell}{\ln \left(R_{\mathrm{a}} / R_{\mathrm{b}}\right)}\left[\left(K_{\text {liq }}-K_{\mathrm{v}}\right) \frac{h}{\ell}+K_{\mathrm{v}}\right]=C \rightarrow \\
& \frac{h}{\ell}=\frac{1}{\left(K_{\text {liq }}-K_{\mathrm{v}}\right)}\left[\frac{C \ln \left(R_{\mathrm{a}} / R_{\mathrm{b}}\right)}{2 \pi \varepsilon_{0} \ell}-K_{\mathrm{v}}\right]
\end{aligned}
$$

(b) For the full tank, $\frac{h}{\ell}=1$, and for the empty tank, $\frac{h}{\ell}=0$.

Full:

$$
\text { Full: } \quad \begin{aligned}
C & =\frac{2 \pi \varepsilon_{0} \ell}{\ln \left(R_{\mathrm{a}} / R_{\mathrm{b}}\right)}\left[\left(K_{\text {liq }}-K_{\mathrm{v}}\right) \frac{h}{\ell}+K_{\mathrm{v}}\right]=\frac{2 \pi \varepsilon_{0} \ell K_{\text {liq }}}{\ln \left(R_{\mathrm{a}} / R_{\mathrm{b}}\right)} \\
& =\frac{2 \pi\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)(2.0 \mathrm{~m})(1.4)}{\ln (5.0 \mathrm{~mm} / 4.5 \mathrm{~mm})}=1.5 \times 10^{-9} \mathrm{~F} \\
\text { Empty: } \quad C & =\frac{2 \pi \varepsilon_{0} \ell}{\ln \left(R_{\mathrm{a}} / R_{\mathrm{b}}\right)}\left[\left(K_{\text {liq }}-K_{\mathrm{v}}\right) \frac{h}{\ell}+K_{\mathrm{v}}\right]=\frac{2 \pi \varepsilon_{0} \ell K_{\mathrm{v}}}{\ln \left(R_{\mathrm{a}} / R_{\mathrm{b}}\right)} \\
& =\frac{2 \pi\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)(2.0 \mathrm{~m})(1.0)}{\ln (5.0 \mathrm{~mm} / 4.5 \mathrm{~mm})}=1.1 \times 10^{-9} \mathrm{~F}
\end{aligned}
$$

65. Consider the dielectric as having a layer of equal and opposite charges at each side of the dielectric. Then the geometry is like three capacitors in series. One air gap is taken to be $d_{1}$, and then the other air gap is $d-d_{1}-\ell$.

$$
\begin{aligned}
& \frac{1}{C}=\frac{1}{C_{1}}+\frac{1}{C_{2}}+\frac{1}{C_{3}}=\frac{d_{1}}{\varepsilon_{0} A}+\frac{\ell}{K \varepsilon_{0} A}+\frac{d-d_{1}-\ell}{\varepsilon_{0} A}=\frac{1}{\varepsilon_{0} A}\left(\left[\frac{\ell}{K}+(d-\ell)\right]\right) \rightarrow \\
& C=\frac{\varepsilon_{0} A}{\left[\frac{\ell}{K}+(d-\ell)\right]}=\frac{\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)\left(2.50 \times 10^{-2} \mathrm{~m}^{2}\right)}{\left[\frac{1.00 \times 10^{-3} \mathrm{~m}}{3.50}+\left(1.00 \times 10^{-3} \mathrm{~m}\right)\right]}=1.72 \times 10^{-10} \mathrm{~F}
\end{aligned}
$$

66. By leaving the battery connected, the voltage will not change when the dielectric is inserted, but the amount of charge will change. That will also change the electric field.
(a) Use Eq. 24-2 to find the capacitance.

$$
C_{0}=\varepsilon_{0} \frac{A}{d}=\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)\left(\frac{2.50 \times 10^{-2} \mathrm{~m}^{2}}{2.00 \times 10^{-3} \mathrm{~m}}\right)=1.106 \times 10^{-10} \mathrm{~F} \approx 1.11 \times 10^{-10} \mathrm{~F}
$$

(b) Use Eq. 24-1 to find the initial charge on each plate.

$$
Q_{0}=C_{0} V=\left(1.106 \times 10^{-10} \mathrm{~F}\right)(150 \mathrm{~V})=1.659 \times 10^{-8} \mathrm{C} \approx 1.66 \times 10^{-8} \mathrm{C}
$$

In Example 24-12, the charge was constant, so it was simple to calculate the induced charge and then the electric fields from those charges. But now the voltage is constant, and so we calculate the fields first, and then calculate the charges. So we are solving the problem parts in a different order.
(d) We follow the same process as in part ( $f$ ) of Example 24-12.

$$
\begin{align*}
V & =E_{0}(d-\ell)+E_{\mathrm{D}} \ell=E_{0}(d-\ell)+\frac{E_{0}}{K} \ell \rightarrow \\
E_{0} & =\frac{V}{d-\ell+\frac{\ell}{K}}=\frac{(150 \mathrm{~V})}{\left(2.00 \times 10^{-3} \mathrm{~m}\right)-\left(1.00 \times 10^{-3} \mathrm{~m}\right)+\frac{\left(1.00 \times 10^{-3} \mathrm{~m}\right)}{(3.50)}}=1.167 \times 10^{5} \mathrm{~V} / \mathrm{m}  \tag{3.50}\\
& \approx 1.17 \times 10^{5} \mathrm{~V} / \mathrm{m}
\end{align*}
$$

(e) $E_{\mathrm{D}}=\frac{E_{0}}{K}=\frac{1.167 \times 10^{5} \mathrm{~V} / \mathrm{m}}{3.50}=3.333 \times 10^{4} \mathrm{~V} / \mathrm{m} \approx 3.33 \times 10^{4} \mathrm{~V} / \mathrm{m}$
(h) $E_{0}=\frac{\sigma}{\varepsilon_{0}}=\frac{Q}{A \varepsilon_{0}} \rightarrow$
$Q=E A \varepsilon_{0}=\left(1.167 \times 10^{5} \mathrm{~V} / \mathrm{m}\right)\left(0.0250 \mathrm{~m}^{2}\right)\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{m}^{2}\right)=2.582 \times 10^{-8} \mathrm{C}$

$$
\approx 2.58 \times 10^{-8} \mathrm{C}
$$

(c) $Q_{\text {ind }}=Q\left(1-\frac{1}{K}\right)=\left(2.582 \times 10^{-8} \mathrm{C}\right)\left(1-\frac{1}{3.50}\right)=1.84 \times 10^{-8} \mathrm{C}$
(f) Because the battery voltage does not change, the potential difference between the plates is unchanged when the dielectric is inserted, and so is $V=150 \mathrm{~V}$.
(g) $C=\frac{Q}{V}=\frac{2.582 \times 10^{-8} \mathrm{C}}{150 \mathrm{~V}}=1.72 \times 10^{-10} \mathrm{pF}$

Notice that the capacitance is the same as in Example 24-12. Since the capacitance is a constant (function of geometry and material, not charge and voltage), it should be the same value.
67. The capacitance will be given by $C=Q / V$. When a charge $Q$ is placed on one plate and a charge $-Q$ is placed on the other plate, an electric field will be set up between the two plates. The electric field in the air-filled region is just the electric field between two charged plates, $E_{0}=\frac{\sigma}{\varepsilon_{0}}=\frac{Q}{A \varepsilon_{0}}$. The electric field in the dielectric is equal to the electric field in the air,
divided by the dielectric constant: $E_{D}=\frac{E_{0}}{K}=\frac{Q}{K A \varepsilon_{0}}$.
The voltage drop between the two plates is obtained by integrating the electric field between the two plates. One plate is set at the origin with the dielectric touching this plate. The dielectric ends at $x=\ell$. The rest of the distance to $x=d$ is then air filled.

$$
V=-\int_{0}^{d} \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{x}}=\int_{0}^{\ell} \frac{Q d x}{K A \varepsilon_{0}}+\int_{\ell}^{d} \frac{Q d x}{A \varepsilon_{0}}=\frac{Q}{A \varepsilon_{0}}\left(\frac{\ell}{K}+(d-\ell)\right)
$$

The capacitance is the ratio of the voltage to the charge.

$$
C=\frac{Q}{V}=\frac{Q}{\frac{Q}{A \varepsilon_{0}}\left(\frac{\ell}{K}+(d-\ell)\right)}=\frac{\varepsilon_{0} A}{d-\ell+\frac{\ell}{K}}
$$

68. Find the energy in each region from the energy density and the volume. The energy density in the "gap" is given by $u_{\text {gap }}=\frac{1}{2} \varepsilon_{0} E_{\text {gap }}^{2}$, and the energy density in the dielectric is given by $u_{\mathrm{D}}=\frac{1}{2} \varepsilon_{D} E_{\mathrm{D}}^{2}$ $=\frac{1}{2} K \varepsilon_{0}\left(\frac{E_{\text {gap }}}{K}\right)^{2}=\frac{1}{2} \varepsilon_{0} \frac{E_{\text {gap }}^{2}}{K}$, where Eq. 24-10 is used.

$$
\begin{aligned}
\frac{U_{\mathrm{D}}}{U_{\text {toal }}} & =\frac{U_{\mathrm{D}}}{U_{\text {gap }}+U_{\mathrm{D}}}=\frac{u_{\mathrm{D}} \mathrm{Vol}_{\mathrm{D}}}{u_{\text {gap }} \mathrm{Vol}_{\text {gap }}+u_{\mathrm{D}} \mathrm{Vol}_{\mathrm{D}}}=\frac{\frac{1}{2} \varepsilon_{0} \frac{E_{\text {gap }}^{2}}{K} A \ell}{\frac{1}{2} \varepsilon_{0} E_{\text {gap }}^{2} A(d-\ell)+\frac{1}{2} \varepsilon_{0} \frac{E_{\text {gap }}^{2}}{K} A \ell} \\
& =\frac{\frac{\ell}{K}}{(d-\ell)+\frac{\ell}{K}}=\frac{\ell}{(d-\ell) K+\ell}=\frac{(1.00 \mathrm{~mm})}{(1.00 \mathrm{~mm})(3.50)+(1.00 \mathrm{~mm})}=0.222
\end{aligned}
$$

69. There are two uniform electric fields - one in the air, and one in the gap. They are related by Eq. 2410. In each region, the potential difference is the field times the distance in the direction of the field over which the field exists.

$$
\begin{aligned}
V & =E_{\text {air }} d_{\text {air }}+E_{\text {glass }} d_{\text {glass }}=E_{\text {air }} d_{\text {air }}+\frac{E_{\text {air }}}{K_{\text {glass }}} d_{\text {glass }} \rightarrow \\
E_{\text {air }} & =V \frac{K_{\text {glass }}}{d_{\text {air }} K_{\text {glass }}+d_{\text {glass }}} \\
& =(90.0 \mathrm{~V}) \frac{5.80}{\left(3.00 \times 10^{-3} \mathrm{~m}\right)(5.80)+\left(2.00 \times 10^{-3} \mathrm{~m}\right)} \\
& =2.69 \times 10^{4} \mathrm{~V} / \mathrm{m} \\
E_{\text {glass }} & =\frac{E_{\text {air }}}{K_{\text {glass }}}=\frac{2.69 \times 10^{4} \mathrm{~V} / \mathrm{m}}{5.80}=4.64 \times 10^{3} \mathrm{~V} / \mathrm{m}
\end{aligned}
$$



The charge on the plates can be calculated from the field at the plate, using Eq. 22-5. Use Eq. 24$11^{11}$ calculate the charge on the dielectric.

$$
\begin{aligned}
E_{\text {air }} & =\frac{\sigma_{\text {plate }}}{\varepsilon_{0}}=\frac{Q_{\text {plate }}}{\varepsilon_{0} A} \rightarrow \\
Q_{\text {plate }} & =E_{\text {air }} \varepsilon_{0} A=\left(2.69 \times 10^{4} \mathrm{~V} / \mathrm{m}\right)\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)\left(1.45 \mathrm{~m}^{2}\right)=3.45 \times 10^{-7} \mathrm{C} \\
Q_{\text {ind }} & =Q\left(1-\frac{1}{K}\right)=\left(3.45 \times 10^{-7} \mathrm{C}\right)\left(1-\frac{1}{5.80}\right)=2.86 \times 10^{-7} \mathrm{C}
\end{aligned}
$$

70. (a) The capacitance of a single isolated conducting sphere is given after example 24-3.

$$
\begin{aligned}
& C=4 \pi \varepsilon_{0} r \rightarrow \\
& \frac{C}{r}=4 \pi\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)=\left(1.11 \times 10^{-10} \frac{\mathrm{~F}}{\mathrm{~m}}\right)\left(\frac{1 \mathrm{~m}}{100 \mathrm{~cm}}\right)\left(\frac{10^{12} \mathrm{pF}}{1 \mathrm{~F}}\right)=1.11 \mathrm{pF} / \mathrm{cm}
\end{aligned}
$$

And so $C=(1.11 \mathrm{pF} / \mathrm{cm}) r \rightarrow C(\mathrm{pF}) \approx r(\mathrm{~cm})$.
(b) We assume that the human body is a sphere of radius 100 cm . Thus the rule $C(\mathrm{pF}) \approx r(\mathrm{~cm})$ says that the capacitance of the human body is about 100 pF .
(c) A $0.5-\mathrm{cm}$ spark would require a potential difference of about $15,000 \mathrm{~V}$. Use Eq. 24-1.

$$
Q=C V=(100 \mathrm{pF})(15,000 \mathrm{~V})=1.5 \mu \mathrm{C}
$$

71. Use Eq. 24-5 to find the capacitance.
$U=\frac{1}{2} C V^{2} \rightarrow C=\frac{2 U}{V^{2}}=\frac{2(1200 \mathrm{~J})}{(7500 \mathrm{~V})^{2}}=4.3 \times 10^{-5} \mathrm{~F}$
72. (a) We approximate the configuration as a parallel-plate capacitor, and so use Eq. 24-2 to calculate the capacitance.

$$
\begin{aligned}
C & =\varepsilon_{0} \frac{A}{d}=\varepsilon_{0} \frac{\pi r^{2}}{d}=\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right) \frac{\pi[(4.5 \mathrm{in})(0.0254 \mathrm{~m} / \mathrm{in})]^{2}}{0.050 \mathrm{~m}}=7.265 \times 10^{-12} \mathrm{~F} \\
& \approx 7 \times 10^{-12} \mathrm{~F}
\end{aligned}
$$

(b) Use Eq. 24-1.

$$
Q=C V=\left(7.265 \times 10^{-12} \mathrm{~F}\right)(9 \mathrm{~V})=6.539 \times 10^{-11} \mathrm{C} \approx 7 \times 10^{-11} \mathrm{C}
$$

(c) The electric field is uniform, and is the voltage divided by the plate separation.

$$
E=\frac{V}{d}=\frac{9 \mathrm{~V}}{0.050 \mathrm{~m}}=180 \mathrm{~V} / \mathrm{m} \approx 200 \mathrm{~V} / \mathrm{m}
$$

(d) The work done by the battery to charge the plates is equal to the energy stored by the capacitor. Use Eq. 24-5.

$$
U=\frac{1}{2} C V^{2}=\frac{1}{2}\left(7.265 \times 10^{-12} \mathrm{~F}\right)(9 \mathrm{~V})^{2}=2.942 \times 10^{-10} \mathrm{~J} \approx 3 \times 10^{-10} \mathrm{~J}
$$

(e) The electric field will stay the same, because the voltage will stay the same (since the capacitor is still connected to the battery) and the plate separation will stay the same. The capacitance changes, and so the charge changes (by Eq. 24-1), and so the work done by the battery changes (by Eq. 24-5).
73. Since the capacitor is disconnected from the battery, the charge on it cannot change. The capacitance of the capacitor is increased by a factor of $K$, the dielectric constant.

$$
Q=C_{\text {initial }} V_{\text {initial }}=C_{\text {final }} V_{\text {final }} \rightarrow V_{\text {final }}=V_{\text {initial }} \frac{C_{\text {initial }}}{C_{\text {final }}}=V_{\text {initial }} \frac{C_{\text {initial }}}{K C_{\text {initial }}}=(34.0 \mathrm{~V}) \frac{1}{2.2}=15 \mathrm{~V}
$$

74. The energy is given by Eq. 24-5. Calculate the energy difference for the two different amounts of charge, and then solve for the difference.

$$
\begin{aligned}
& U=\frac{1}{2} \frac{Q^{2}}{C} \rightarrow \Delta U=\frac{1}{2} \frac{(Q+\Delta Q)^{2}}{C}-\frac{1}{2} \frac{Q^{2}}{C}=\frac{1}{2 C}\left[(Q+\Delta Q)^{2}-Q^{2}\right]=\frac{\Delta Q}{2 C}[2 Q+\Delta Q] \rightarrow \\
& Q=\frac{C \Delta U}{\Delta Q}-\frac{1}{2} \Delta Q=\frac{\left(17.0 \times 10^{-6} \mathrm{~F}\right)(18.5 \mathrm{~J})}{\left(13.0 \times 10^{-3} \mathrm{C}\right)}-\frac{1}{2}\left(13.0 \times 10^{-3} \mathrm{C}\right)=17.7 \times 10^{-3} \mathrm{C}=17.7 \mathrm{mC}
\end{aligned}
$$

75. The energy in the capacitor, given by Eq. $24-5$, is the heat energy absorbed by the water, given by Eq. 19-2.

$$
\begin{aligned}
& U=Q_{\text {heat }} \rightarrow \frac{1}{2} C V^{2}=m c \Delta T \rightarrow \\
& V=\sqrt{\frac{2 m c \Delta T}{C}}=\sqrt{\frac{2(3.5 \mathrm{~kg})\left(4186 \frac{\mathrm{~J}}{\mathrm{~kg} \cdot{ }^{\circ} \mathrm{C}}\right)\left(95^{\circ} \mathrm{C}-22^{\circ} \mathrm{C}\right)}{3.0 \mathrm{~F}}}=844 \mathrm{~V} \approx 840 \mathrm{~V}
\end{aligned}
$$

76. (a) The capacitance per unit length of a cylindrical capacitor with no dielectric is derived in Example 24-2, as $\frac{C}{\ell}=\frac{2 \pi \varepsilon_{0}}{\ln \left(R_{\text {outside }} / R_{\text {inside }}\right)}$. The addition of a dielectric increases the capacitance by a factor of $K$.

$$
\frac{C}{\ell}=\frac{2 \pi \varepsilon_{0} K}{\ln \left(R_{\text {outside }} / R_{\text {inside }}\right)}
$$

(b) $\frac{C}{\ell}=\frac{2 \pi \varepsilon_{0} K}{\ln \left(R_{\text {outside }} / R_{\text {inside }}\right)}=\frac{2 \pi\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{m}^{2}\right) 2.6}{\ln (9.0 \mathrm{~mm} / 2.5 \mathrm{~mm})}=1.1 \times 10^{-10} \mathrm{~F} / \mathrm{m}$
77. The potential can be found from the field and the plate separation. Then the capacitance is found from Eq. 24-1, and the area from Eq. 24-8.

$$
\begin{aligned}
& E=\frac{V}{d} ; Q=C V=C E d \rightarrow \\
& C=\frac{Q}{E d}=\frac{\left(0.675 \times 10^{-6} \mathrm{C}\right)}{\left(9.21 \times 10^{4} \mathrm{~V} / \mathrm{m}\right)\left(1.95 \times 10^{-3} \mathrm{~m}\right)}=3.758 \times 10^{-9} \mathrm{~F} \approx 3.76 \times 10^{-9} \mathrm{~F} \\
& C=K \varepsilon_{0} \frac{A}{d} \rightarrow A=\frac{C d}{K \varepsilon_{0}}=\frac{\left(3.758 \times 10^{-9} \mathrm{~F}\right)\left(1.95 \times 10^{-3} \mathrm{~m}\right)}{(3.75)\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)}=0.221 \mathrm{~m}^{2}
\end{aligned}
$$

78. (a) If $N$ electrons flow onto the plate, the charge on the top plate is $-N e$, and the positive charge associated with the capacitor is $Q=N e$. Since $Q=C V$, we have $N e=C V \rightarrow V=N e / C$, showing that $V$ is proportional to $N$.
(b) Given $\Delta V=1 \mathrm{mV}$ and we want $\Delta N=1$, solve for the capacitance.

$$
\begin{aligned}
& V=\frac{N e}{C} \rightarrow \Delta V=\frac{e \Delta N}{C} \rightarrow \\
& C=e \frac{\Delta N}{\Delta V}=\left(1.60 \times 10^{-19} \mathrm{C}\right) \frac{1}{1 \times 10^{-3} \mathrm{~V}}=1.60 \times 10^{-16} \mathrm{~F} \approx 2 \times 10^{-16} \mathrm{~F}
\end{aligned}
$$

(c) Use Eq. 24-8.

$$
\begin{aligned}
& C=\varepsilon_{0} K \frac{A}{d}=\varepsilon_{0} K \frac{\ell^{2}}{d} \rightarrow \\
& \ell=\sqrt{\frac{C d}{\varepsilon_{0} K}}=\sqrt{\frac{\left(1.60 \times 10^{-16} \mathrm{~F}\right)\left(100 \times 10^{-9} \mathrm{~m}\right)}{\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)(3)}}=7.76 \times 10^{-7} \mathrm{~m}\left(\frac{10^{6} \mu \mathrm{~m}}{1 \mathrm{~m}}\right)=0.8 \mu \mathrm{~m}
\end{aligned}
$$

79. The relative change in energy can be obtained by inserting Eq. 24-8 into Eq. 24-5.

$$
\frac{U}{U_{0}}=\frac{\frac{Q^{2}}{2 C}}{\frac{Q^{2}}{2 C}}=\frac{C_{0}}{C}=\frac{\frac{A \varepsilon_{0}}{d}}{\frac{K A \varepsilon_{0}}{\left(\frac{1}{2} d\right)}}=\frac{1}{2 K}
$$

The dielectric is attracted to the capacitor. As such, the dielectric will gain kinetic energy as it enters the capacitor. An external force is necessary to stop the dielectric. The negative work done by this force results in the decrease in energy within the capacitor.

Since the charge remains constant, and the magnitude of the electric field depends on the charge, and not the separation distance, the electric field will not be affected by the change in distance between the plates. The electric field between the plates will be reduced by the dielectric constant, as given in Eq. 24-10.

$$
\frac{E}{E_{0}}=\frac{E_{0} / K}{E_{0}}=\frac{1}{K}
$$

80. (a) Use Eq. 24-2.

$$
C=\frac{\varepsilon_{0} A}{d}=\frac{\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)\left(120 \times 10^{6} \mathrm{~m}^{2}\right)}{(1500 \mathrm{~m})}=7.08 \times 10^{-7} \mathrm{~F} \approx 7.1 \times 10^{-7} \mathrm{~F}
$$

(b) Use Eq. 24-1.

$$
Q=C V=\left(7.08 \times 10^{-7} \mathrm{~F}\right)\left(3.5 \times 10^{7} \mathrm{~V}\right)=24.78 \mathrm{C} \approx 25 \mathrm{C}
$$

(c) Use Eq. 24-5.

$$
U=\frac{1}{2} Q V=\frac{1}{2}(24.78 \mathrm{C})\left(3.5 \times 10^{7} \mathrm{~V}\right)=4.337 \times 10^{8} \mathrm{~J} \approx 4.3 \times 10^{8} \mathrm{~J}
$$

81. We treat this as $N$ capacitors in parallel, so that the total capacitance is $N$ times the capacitance of a single capacitor. The maximum voltage and dielectric strength are used to find the plate separation of a single capacitor.

$$
\begin{aligned}
& d=\frac{V}{E_{\mathrm{s}}}=\frac{100 \mathrm{~V}}{30 \times 10^{6} \mathrm{~V} / \mathrm{m}}=3.33 \times 10^{-6} \mathrm{~m} ; N=\frac{\ell}{d}=\frac{6.0 \times 10^{-3} \mathrm{~m}}{3.33 \times 10^{-6} \mathrm{~m}}=1800 \\
& C_{\mathrm{eq}}=N C=N \varepsilon_{0} K \frac{A}{d} \rightarrow
\end{aligned}
$$

$$
K=\frac{C_{\text {eq }} d}{N \varepsilon_{0} A}=\frac{\left(1.0 \times 10^{-6} \mathrm{~F}\right)\left(3.33 \times 10^{-6} \mathrm{~m}\right)}{1800\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)\left(12.0 \times 10^{-3} \mathrm{~m}\right)\left(14.0 \times 10^{-3} \mathrm{~m}\right)}=1.244 \approx 1.2
$$

82. The total charge doesn't change when the second capacitor is connected, since the two-capacitor combination is not connected to a source of charge. The final voltage across the two capacitors must be the same. Use Eq. 24-1.

$$
\begin{aligned}
& Q_{0}=C_{1} V_{0}=Q_{1}+Q_{2}=C_{1} V_{1}+C_{2} V_{2}=C_{1} V_{1}+C_{2} V_{1} \\
& C_{2}=C_{1} \frac{\left(V_{0}-V_{1}\right)}{V_{1}}=(3.5 \mu \mathrm{~F})\left(\frac{12.4 \mathrm{~V}-5.9 \mathrm{~V}}{5.9 \mathrm{~V}}\right)=3.856 \mu \mathrm{~F} \approx 3.9 \mu \mathrm{~F}
\end{aligned}
$$

83. (a) Use Eq. 24-5 to calculate the stored energy.

$$
U=\frac{1}{2} C V^{2}=\frac{1}{2}\left(8.0 \times 10^{-8} \mathrm{~F}\right)\left(2.5 \times 10^{4} \mathrm{~V}\right)^{2}=25 \mathrm{~J}
$$

(b) The power is the energy converted per unit time.

$$
P=\frac{\text { Energy }}{\text { time }}=\frac{0.15(25 \mathrm{~J})}{4.0 \times 10^{-6} \mathrm{~s}}=9.38 \times 10^{5} \mathrm{~W} \approx 940 \mathrm{~kW}
$$

84. The pressure is the force per unit area on a face of the dielectric. The force is related to the potential energy stored in the capacitor by Eq. $8-7, F=-\frac{d U}{d x}$, where $x$ is the separation of the capacitor plates.

$$
\begin{aligned}
& U=\frac{1}{2} C V^{2}=\frac{1}{2}\left(K \varepsilon_{0} \frac{A}{x}\right) V^{2} \rightarrow F=-\frac{d U}{d x}=\frac{K \varepsilon_{0} A V^{2}}{2 x^{2}} ; P=\frac{F}{A}=\frac{K \varepsilon_{0} V^{2}}{2 x^{2}} \rightarrow \\
& V=\sqrt{\frac{2 x^{2} P}{K \varepsilon_{0}}}=\sqrt{\frac{2\left(1.0 \times 10^{-4} \mathrm{~m}\right)^{2}(40.0 \mathrm{~Pa})}{(3.1)\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)}}=170 \mathrm{~V}
\end{aligned}
$$

85. (a) From the diagram, we see that one group of 4 plates is connected together, and the other group of 4 plates is connected together. This common grouping shows that the capacitors are connected in parallel.
(b) Since they are connected in parallel, the equivalent capacitance is the sum of the individual capacitances. The variable area will change the equivalent capacitance.

$$
\begin{aligned}
& C_{\text {eq }}=7 C=7 \varepsilon_{0} \frac{A}{d} \\
& C_{\min }=7 \varepsilon_{0} \frac{A_{\min }}{d}=7\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right) \frac{\left(2.0 \times 10^{-4} \mathrm{~m}^{2}\right)}{\left(1.6 \times 10^{-3} \mathrm{~m}\right)}=7.7 \times 10^{-12} \mathrm{~F} \\
& C_{\max }=7 \varepsilon_{0} \frac{A_{\max }}{d}=7\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right) \frac{\left(9.0 \times 10^{-4} \mathrm{~m}^{2}\right)}{\left(1.6 \times 10^{-3} \mathrm{~m}\right)}=3.5 \times 10^{-11} \mathrm{~F}
\end{aligned}
$$

And so the range is from 7.7 pF to 35 pF .
86. (a) Since the capacitor is charged and then disconnected from the power supply, the charge is constant. Use Eq. 24-1 to find the new voltage.

$$
Q=C V=\text { constant } \rightarrow C_{1} V_{1}=C_{2} V_{2} \rightarrow V_{2}=V_{1} \frac{C_{1}}{C_{2}}=(7500 \mathrm{~V}) \frac{8.0 \mathrm{pF}}{1.0 \mathrm{pF}}=6.0 \times 10^{4} \mathrm{~V}
$$

(b) In using this as a high voltage power supply, once it discharges, the voltage drops, and it needs to be recharged. So it is not a constant source of high voltage. You would also have to be sure it was designed to not have breakdown of the capacitor material when the voltage gets so high. Another disadvantage is that it has only a small amount of energy stored: $U=\frac{1}{2} C V^{2}$ $=\frac{1}{2}\left(1.0 \times 10^{-12} \mathrm{C}\right)\left(6.0 \times 10^{4} \mathrm{~V}\right)^{2}=1.8 \times 10^{-3} \mathrm{~J}$, and so could actually only supply a small amount of power unless the discharge time was extremely short.
87. Since the two capacitors are in series, they will both have the same charge on them.

$$
\begin{aligned}
& Q_{1}=Q_{2}=Q_{\text {series }} ; \frac{1}{C_{\text {series }}}=\frac{V}{Q_{\text {series }}}=\frac{1}{C_{1}}+\frac{1}{C_{2}} \rightarrow \\
& C_{2}=\frac{Q_{\text {series }} C_{1}}{C_{1} V-Q_{\text {series }}}=\frac{\left(125 \times 10^{-12} \mathrm{C}\right)\left(175 \times 10^{-12} \mathrm{~F}\right)}{\left(175 \times 10^{-12} \mathrm{~F}\right)(25.0 \mathrm{~V})-\left(125 \times 10^{-12} \mathrm{C}\right)}=5.15 \times 10^{-12} \mathrm{~F}
\end{aligned}
$$

88. (a) The charge can be determined from Eqs. 24-1 and 24-2.

$$
\begin{aligned}
Q & =C V=\varepsilon_{0} \frac{A}{d} V=\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right) \frac{\left(2.0 \times 10^{-4} \mathrm{~m}^{2}\right)}{\left(5.0 \times 10^{-4} \mathrm{~m}\right)}(12 \mathrm{~V})=4.248 \times 10^{-11} \mathrm{C} \\
& \approx 4.2 \times 10^{-11} \mathrm{C}
\end{aligned}
$$

(b) Since the battery is disconnected, no charge can flow to or from the plates. Thus the charge is constant.

$$
Q=4.2 \times 10^{-11} \mathrm{C}
$$

(c) The capacitance has changed and the charge has stayed constant, and so the voltage has changed.

$$
\begin{aligned}
& Q=C V=\text { constant } \rightarrow C_{1} V_{1}=C_{0} V_{0} \rightarrow \varepsilon_{0} \frac{A}{d_{1}} V_{1}=\varepsilon_{0} \frac{A}{d_{0}} V_{0} \rightarrow \\
& V_{1}=\frac{d_{1}}{d_{0}} V_{0}=\frac{0.75 \mathrm{~mm}}{0.50 \mathrm{~mm}}(12 \mathrm{~V})=18 \mathrm{~V}
\end{aligned}
$$

(d) The work is the change in stored energy.

$$
W=\Delta U=\frac{1}{2} Q V_{1}-\frac{1}{2} Q V_{0}=\frac{1}{2} Q\left(V_{1}-V_{0}\right)=\frac{1}{2}\left(4.248 \times 10^{-11} \mathrm{C}\right)(6.0 \mathrm{~V})=1.3 \times 10^{-10} \mathrm{~J}
$$

89. The first capacitor is charged, and so has a certain amount of charge on its plates. Then, when the switch is moved, the capacitors are not connected to a source of charge, and so the final charge is equal to the initial charge. Initially treat capacitors $C_{2}$ and $C_{3}$ as their equivalent capacitance, $C_{23}=\frac{C_{2} C_{3}}{C_{2}+C_{3}}=\frac{(2.0 \mu \mathrm{~F})(2.4 \mu \mathrm{~F})}{4.4 \mu \mathrm{~F}}=1.091 \mu \mathrm{~F}$. The final voltage across $C_{1}$ and $C_{23}$ must be the same. The charge on $C_{2}$ and $C_{3}$ must be the same. Use Eq. 24-1.

$$
\begin{aligned}
& Q_{0}=C_{1} V_{0}=Q_{1}+Q_{23}=C_{1} V_{1}+C_{23} V_{23}=C_{1} V_{1}+C_{23} V_{1} \rightarrow \\
& V_{1}=\frac{C_{1}}{C_{1}+C_{23}} V_{0}=\frac{1.0 \mu \mathrm{~F}}{1.0 \mu \mathrm{~F}+1.091 \mu \mathrm{~F}}(24 \mathrm{~V})=11.48 \mathrm{~V}=V_{1}=V_{23} \\
& Q_{1}=C_{1} V_{1}=(1.0 \mu \mathrm{~F})(11.48 \mathrm{~V})=11.48 \mu \mathrm{C} \\
& Q_{23}=C_{23} V_{23}=(1.091 \mu \mathrm{~F})(11.48 \mathrm{~V})=12.52 \mu \mathrm{C}=Q_{2}=Q_{3} \\
& V_{2}=\frac{Q_{2}}{C_{2}}=\frac{12.52 \mu \mathrm{C}}{2.0 \mu \mathrm{~F}}=6.26 \mathrm{~V} ; V_{3}=\frac{Q_{3}}{C_{3}}=\frac{12.52 \mu \mathrm{C}}{2.4 \mu \mathrm{~F}}=5.22 \mathrm{~V}
\end{aligned}
$$

To summarize: $Q_{1}=11 \mu \mathrm{C}, V_{1}=11 \mathrm{~V} ; Q_{2}=13 \mu \mathrm{C}, V_{2}=6.3 \mathrm{~V} ; Q_{3}=13 \mu \mathrm{C}, V_{3}=5.2 \mathrm{~V}$
90. The metal conducting strips connecting cylinders b and c mean that b and c are at the same potential. Due to the positive charge on the inner cylinder and the negative charge on the outer cylinder, cylinders b and c will polarize according to the first diagram, with negative charge on cylinder c , and positive charge on cylinder $b$. This is then two capacitors in series, as illustrated in the second diagram. The capacitance per unit length of a cylindrical capacitor is derived in Example 24-2.

$$
\begin{aligned}
C_{1} & =\frac{2 \pi \varepsilon_{0} \ell}{\ln \left(R_{\mathrm{a}} / R_{\mathrm{b}}\right)} ; C_{2}=\frac{2 \pi \varepsilon_{0} \ell}{\ln \left(R_{\mathrm{c}} / R_{\mathrm{d}}\right)} ; \frac{1}{C_{\text {net }}}=\frac{1}{C_{1}}+\frac{1}{C_{2}} \rightarrow \\
C_{\text {net }} & =\frac{C_{1} C_{2}}{C_{1}+C_{2}}=\frac{\left[\frac{2 \pi \varepsilon_{0} \ell}{\ln \left(R_{\mathrm{a}} / R_{\mathrm{b}}\right)}\right]\left[\frac{2 \pi \varepsilon_{0} \ell}{\ln \left(R_{\mathrm{c}} / R_{\mathrm{d}}\right)}\right]}{\frac{2 \pi \varepsilon_{0} \ell}{\ln \left(R_{\mathrm{a}} / R_{\mathrm{b}}\right)}+\frac{2 \pi \varepsilon_{0} \ell}{\ln \left(R_{\mathrm{c}} / R_{\mathrm{d}}\right)}} \\
& =\frac{2 \pi \varepsilon_{0} \ell}{\ln \left(R_{\mathrm{c}} / R_{\mathrm{d}}\right)+\ln \left(R_{\mathrm{a}} / R_{\mathrm{b}}\right)}=\frac{2 \pi \varepsilon_{0} \ell}{\ln \left(R_{\mathrm{a}} R_{\mathrm{c}} / R_{\mathrm{b}} R_{\mathrm{d}}\right)} \rightarrow \\
\frac{C}{\ell} & =\frac{2 \pi \varepsilon_{0}}{\ln \left(R_{\mathrm{a}} R_{\mathrm{c}} / R_{\mathrm{b}} R_{\mathrm{d}}\right)}
\end{aligned}
$$


91. The force acting on one plate by the other plate is equal to the electric field produced by one charged plate multiplied by the charge on the second plate.

$$
F=E Q=\left(\frac{Q}{2 A \varepsilon_{0}}\right) Q=\frac{Q^{2}}{2 A \varepsilon_{0}}
$$

The force is attractive since the plates are oppositely charged. Since the force is constant, the work done in pulling the two plates apart by a distance $x$ is just the force times distance.

$$
W=F x=\frac{Q^{2} x}{2 A \varepsilon_{0}}
$$

The change in energy stored between the plates is obtained using Eq. 24-5.

$$
W=\Delta U=\frac{Q^{2}}{2}\left(\frac{1}{C_{2}}-\frac{1}{C_{1}}\right)=\frac{Q^{2}}{2}\left(\frac{2 x}{\varepsilon_{0} A}-\frac{x}{\varepsilon_{0} A}\right)=\frac{Q^{2} x}{2 \varepsilon_{0} A}
$$

Tl () rrk done in pulling the plates apart is equal to the increase in energy between the plates.
92. Since the other values in this problem manifestly have 2 significant figures, we assume that the capacitance also has 2 significant figures.
(a) The number of electrons is found from the charge on the capacitor.

$$
Q=C V=N e \rightarrow N=\frac{C V}{e}=\frac{\left(30 \times 10^{-15} \mathrm{~F}\right)(1.5 \mathrm{~V})}{1.60 \times 10^{-19} \mathrm{C}}=2.8 \times 10^{5} e^{\prime} \mathrm{s}
$$

(b) The thickness is determined from the dielectric strength.

$$
E_{\max }=\frac{V}{d_{\min }} \rightarrow d_{\min }=\frac{V}{E_{\max }}=\frac{1.5 \mathrm{~V}}{1.0 \times 10^{9} \mathrm{~V} / \mathrm{m}}=1.5 \times 10^{-9} \mathrm{~m}
$$

(c) The area is found from Eq. 24-8.

$$
C=K \varepsilon_{0} \frac{A}{d} \rightarrow A=\frac{C d}{K \varepsilon_{0}}=\frac{\left(30 \times 10^{-15} \mathrm{~F}\right)\left(1.5 \times 10^{-9} \mathrm{~m}\right)}{25\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)}=2.0 \times 10^{-13} \mathrm{~m}
$$

93. Use Eq. 24-2 for the capacitance.

$$
C=\frac{\varepsilon_{0} A}{d} \rightarrow d=\frac{\varepsilon_{0} A}{C}=\frac{\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)\left(1.0 \times 10^{-4} \mathrm{~m}^{2}\right)}{(1 \mathrm{~F})}=9 \times 10^{-16} \mathrm{~m}
$$

No, this is not practically achievable. The gap would have to be smaller than the radius of a proton.
94. See the schematic diagram for the arrangement. The two "capacitors" are in series, and so have the same charge. Thus their voltages, which must total 25 kV , will be inversely proportional to their capacitances. Let $C_{1}$ be the glass-filled capacitor, and $C_{2}$ be the vinyl capacitor. The area of the foot is approximately twice the area of the hand, and since there are two feet on the floor and only one hand on the screen, the area ratio is $\frac{A_{\text {foot }}}{A_{\text {hand }}}=\frac{4}{1}$.


$$
\begin{aligned}
& Q=C_{1} V_{1}=C_{2} V_{2} \rightarrow V_{1}=V_{2} \frac{C_{2}}{C_{1}} \\
& C_{1}=\frac{\varepsilon_{0} K_{\text {glass }} A_{\text {hand }}}{d_{\text {glass }}} ; C_{2}=\frac{\varepsilon_{0} K_{\text {viny1 }} A_{\text {foot }}}{d_{\text {vinyl }}} \\
& \frac{C_{2}}{C_{1}}=\frac{\frac{\varepsilon_{0} K_{\text {viny }} A_{\text {foot }}}{d_{\text {viny }}}}{\frac{\varepsilon_{0} K_{\text {glass }}}{d_{\text {hand }}}}=\frac{K_{\text {vinyl }} A_{\text {foot }} d_{\text {glass }}}{K_{\text {glass }} A_{\text {hand }} d_{\text {vinyl }}}=\frac{(3)(4)(0.63)}{(5)(1)(1.0)}=1.5
\end{aligned}
$$

$$
V=V_{1}+V_{2}=V_{2} \frac{C_{2}}{C_{1}}+V_{2}=2.5 V_{2}=25,000 \mathrm{~V} \rightarrow V_{2}=10,000 \mathrm{~V}
$$

95. (a) Use Eq. 24-2 to calculate the capacitance.

$$
C_{0}=\frac{\varepsilon_{0} A}{d}=\frac{\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)\left(2.0 \mathrm{~m}^{2}\right)}{\left(3.0 \times 10^{-3} \mathrm{~m}\right)}=5.9 \times 10^{-9} \mathrm{~F}
$$

Use Eq. 24-1 to calculate the charge.

$$
Q_{0}=C_{0} V_{0}=\left(5.9 \times 10^{-9} \mathrm{~F}\right)(45 \mathrm{~V})=2.655 \times 10^{-7} \mathrm{C} \approx 2.7 \times 10^{-7} \mathrm{C}
$$

The electric field is the potential difference divided by the plate separation.

$$
E_{0}=\frac{V_{0}}{d}=\frac{45 \mathrm{~V}}{3.0 \times 10^{-3} \mathrm{~m}}=15000 \mathrm{~V} / \mathrm{m}
$$

Use Eq. 24-5 to calculate the energy stored.

$$
U_{0}=\frac{1}{2} C_{0} V_{0}^{2}=\frac{1}{2}\left(5.9 \times 10^{-9} \mathrm{~F}\right)(45 \mathrm{~V})^{2}=6.0 \times 10^{-6} \mathrm{~J}
$$

(b) Now include the dielectric. The capacitance is multiplied by the dielectric constant.

$$
C=K C_{0}=3.2\left(5.9 \times 10^{-9} \mathrm{~F}\right)=1.888 \times 10^{-8} \mathrm{~F} \approx 1.9 \times 10^{-8} \mathrm{~F}
$$

The voltage doesn't change. Use Eq. 24-1 to calculate the charge.

$$
Q=C V=K C_{0} V=3.2\left(5.9 \times 10^{-9} \mathrm{~F}\right)(45 \mathrm{~V})=8.496 \times 10^{-7} \mathrm{C} \approx 8.5 \times 10^{-7} \mathrm{C}
$$

Since the battery is still connected, the voltage is the same as before, and so the electric field doesn't change.

$$
E=E_{0}=15000 \mathrm{~V} / \mathrm{m}
$$

Use Eq. 24-5 to calculate the energy stored.

$$
U=\frac{1}{2} C V^{2}=\frac{1}{2} K C_{0} V^{2}=\frac{1}{2}(3.2)\left(5.9 \times 10^{-9} \mathrm{~F}\right)(45 \mathrm{~V})^{2}=1.9 \times 10^{-5} \mathrm{~J}
$$

96. (a) For a plane conducting surface, the electric field is given by Eq. 22-5.

$$
\begin{aligned}
E=\frac{\sigma}{\varepsilon_{0}}=\frac{Q}{\varepsilon_{0} A} \rightarrow Q_{\max } & =E_{\mathrm{S}} \varepsilon_{0} A=\left(3 \times 10^{6} \mathrm{~N} / \mathrm{C}\right)\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)\left(150 \times 10^{-4} \mathrm{~m}^{2}\right) \\
& =3.98 \times 10^{-7} \mathrm{C} \approx 4 \times 10^{-7} \mathrm{C}
\end{aligned}
$$

(b) The capacitance of an isolated sphere is derived in the text, right after Example 24-3.

$$
C=4 \pi \varepsilon_{0} r=4 \pi\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)(1 \mathrm{~m})=1.11 \times 10^{-10} \mathrm{~F} \approx 1 \times 10^{-10} \mathrm{~F}
$$

(c) Use Eq. 24-1, with the maximum charge from part (a) and the capacitance from part (b).

$$
Q=C V \quad \rightarrow \quad V=\frac{Q}{C}=\frac{3.98 \times 10^{-7} \mathrm{C}}{1.11 \times 10^{-10} \mathrm{~F}}=3586 \mathrm{~V} \approx 4000 \mathrm{~V}
$$

97. (a) The initial capacitance is obtained directly from Eq. 24-8.

$$
C_{0}=\frac{K \varepsilon_{0} A}{d}=\frac{3.7(8.85 \mathrm{pF} / \mathrm{m})(0.21 \mathrm{~m})(0.14 \mathrm{~m})}{0.030 \times 10^{-3} \mathrm{~m}}=32 \mathrm{nF}
$$

(b) Maximum charge will occur when the electric field between the plates is equal to the dielectric strength. The charge will be equal to the capacitance multiplied by the maximum voltage, where the maximum voltage is the electric field times the separation distance of the plates.

$$
\begin{aligned}
Q_{\max } & =C_{0} V=C_{0} E d=(32 \mathrm{nF})\left(15 \times 10^{6} \mathrm{~V} / \mathrm{m}\right)\left(0.030 \times 10^{-3} \mathrm{~m}\right) \\
& =14 \mu \mathrm{C}
\end{aligned}
$$

(c) The sheets of foil would be separated by sheets of paper with alternating sheets connected together on each side. This capacitor would consist of 100 sheets of paper with 101 sheets of foil.

$$
\begin{aligned}
t & =101 d_{\mathrm{Al}}+100 d_{\text {paper }}=101(0.040 \mathrm{~mm})+100(0.030 \mathrm{~mm}) \\
& =7.0 \mathrm{~mm}
\end{aligned}
$$


(d) Since the capacitors are in parallel, each capacitor has the same voltage which is equal to the total voltage. Therefore breakdown will occur when the voltage across a single capacitor provides an electric field across that capacitor equal to the dielectric strength.

$$
V_{\text {max }}=E_{\text {max }} d=\left(15 \times 10^{6} \mathrm{~V} / \mathrm{m}\right)\left(0.030 \times 10^{-3} \mathrm{~m}\right)=450 \mathrm{~V}
$$

98. From Eq. 24-2, $C=\frac{\varepsilon_{0}}{d} A$. So if we plot $C$ vs. $A$, we should get a straight line with a slope of $\frac{\varepsilon_{0}}{d}$.

$$
\begin{aligned}
\frac{\varepsilon_{0}}{d} & =\text { slope } \rightarrow \\
d & =\frac{\varepsilon_{0}}{\text { slope }} \\
& =\frac{8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}}{8606 \times 10^{-12} \mathrm{~F} / \mathrm{m}^{2}} \\
& =1.03 \times 10^{-3} \mathrm{~m} \approx 1.0 \mathrm{~mm}
\end{aligned}
$$



The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH24.XLS," on tab "Problem 24.98."

## CHAPTER 25: Electric Currents and Resistance

## Responses to Questions

1. A battery rating in ampere-hours gives the total amount of charge available in the battery.
2. The chemical reactions within the cell cause electrons to pile up on the negative electrode. If the terminals of the battery are connected in a circuit, then electrons flow from the negative terminal because it has an excess of electrons. Once the electrons return to the cell, the electrolyte again causes them to move to the negative terminal.
3. When a flashlight is operated, the battery energy is being used up.
4. The terminal of the car battery connected to "ground" is actually connected to the metal frame of the car. This provides a large "sink" or "source" for charge. The metal frame serves as the common ground for all electrical devices in the car, and all voltages are measured with respect to the car's frame.
5. Generally, water is already in the faucet spout, but it will not come out until the faucet valve is opened. Opening the valve provides the pressure difference needed to force water out of the spout. The same thing is essentially true when you connect a wire to the terminals of a battery. Electrons already exist in the wires. The battery provides the potential that causes them to move, producing a current.
6. Yes. They might have the same resistance if the aluminum wire is thicker. If the lengths of the wires are the same, then the ratios of resistivity to cross-sectional area must also be the same for the resistances to be the same. Aluminum has a higher resistivity than copper, so if the cross-sectional area of the aluminum is also larger by the same proportion, the two wires will have the same resistance.
7. If the emf in a circuit remains constant and the resistance in the circuit is increased, less current will flow, and the power dissipated in the circuit will decrease. Both power equations support this result. If the current in a circuit remains constant and the resistance is increased, then the emf must increase and the power dissipated in the circuit will increase. Both equations also support this result. There is no contradiction, because the voltage, current, and resistance are related to each other by $V=I R$.
8. When a lightbulb burns out, the filament breaks, creating a gap in the circuit so that no current flows.
9. If the resistance of a small immersion heater were increased, it would slow down the heating process. The emf in the circuit made up of the heater and the wires that connect it to the wall socket is maintained at a constant rms value. If the resistance in the circuit is increased, less current will flow, and the power dissipated in the circuit will decrease, slowing the heating process.
10. Resistance is proportional to length and inversely proportional to cross-sectional area.
(a) For the least resistance, you want to connect the wires to maximize area and minimize length. Therefore, connect them opposite to each other on the faces that are $2 a$ by $3 a$.
(b) For the greatest resistance, you want to minimize area and maximize length. Therefore, connect the wires to the faces that are $1 a$ by $2 a$.
11. When a light is turned on, the filament is cool, and has a lower resistance than when it is hot. The current through the filament will be larger, due to the lower resistance. This momentary high current will heat the wire rapidly, possibly causing the filament to break due to thermal stress or vaporize. After the light has been on for some time, the filament is at a constant high temperature, with a higher resistance and a lower current. Since the temperature is constant, there is less thermal stress on the filament than when the light is first turned on.
12. When connected to the same potential difference, the $100-\mathrm{W}$ bulb will draw more current $(P=I V)$. The 75-W bulb has the higher resistance ( $V=I R$ or $P=V^{2} / R$ ).
13. The electric power transferred by the lines is $P=I V$. If the voltage across the transmission lines is large, then the current in the lines will be small. The power lost in the transmission lines is $P=I^{2} R$. The power dissipated in the lines will be small, because $I$ is small.
14. If the circuit has a $15-\mathrm{A}$ fuse, then it is rated to carry current of no more than 15 A . Replacing the $15-$ A fuse with a $25-$-A fuse will allow the current to increase to a level that is dangerously high for the wiring, which might result in overheating and possibly a fire.
15. The human eye and brain cannot distinguish the on-off cycle of lights when they are operated at the normal 60 Hz frequency. At much lower frequencies, such as 5 Hz , the eye and brain are able to process the on-off cycle of the lights, and they will appear to flicker.
16. The electrons are not "used up" as they pass through the lamp. Their energy is dissipated as light and heat, but with each cycle of the alternating voltage, their potential energy is raised again. As long as the electrons keep moving (converting potential energy into kinetic energy, light, and heat) the lamp will stay lit.
17. Immediately after the toaster is turned on, the Nichrome wire heats up and its resistance increases. Since the (rms) potential across the element remains constant, the current in the heating element must decrease.
18. No. Energy is dissipated in a resistor but current, the rate of flow of charge, is not "used up."
19. In the two wires described, the drift velocities of the electrons will be about the same, but the current density, and therefore the current, in the wire with twice as many free electrons per atom will be twice as large as in the other wire.
20. (a) If the length of the wire doubles, its resistance also doubles, and so the current in the wire will be reduced by a factor of two. Drift velocity is proportional to current, so the drift velocity will be halved.
(b) If the wire's radius is doubled, the drift velocity remains the same. (Although, since there are more charge carriers, the current will quadruple.)
(c) If the potential difference doubles while the resistance remains constant, the drift velocity and current will also double.
21. If you turn on an electric appliance when you are outside with bare feet, and the appliance shorts out through you, the current has a direct path to ground through your feet, and you will receive a severe shock. If you are inside wearing socks and shoes with thick soles, and the appliance shorts out, the current will not have an easy path to ground through you, and will most likely find an alternate path. $Y$ Yight receive a mild shock, but not a severe one.

## Solutions to Problems

1. Use the definition of current, Eq. 25-1 a.

$$
I=\frac{\Delta Q}{\Delta t} \rightarrow 1.30 \mathrm{~A}=\frac{1.30 \mathrm{C}}{\mathrm{~s}} \times \frac{1 \text { electron }}{1.60 \times 10^{-19} \mathrm{C}}=8.13 \times 10^{18} \text { electrons } / \mathrm{s}
$$

2. Use the definition of current, Eq. 25-1a.

$$
I=\frac{\Delta Q}{\Delta t} \rightarrow \Delta Q=I \Delta t=(6.7 \mathrm{~A})(5.0 \mathrm{~h})(3600 \mathrm{~s} / \mathrm{h})=1.2 \times 10^{5} \mathrm{C}
$$

3. Use the definition of current, Eq. 25-1a.

$$
I=\frac{\Delta Q}{\Delta t}=\frac{(1200 \text { ions })\left(1.60 \times 10^{-19} \mathrm{C} / \text { ion }\right)}{3.5 \times 10^{-6} \mathrm{~s}}=5.5 \times 10^{-11} \mathrm{~A}
$$

4. Solve Eq. 25-2a for resistance.

$$
R=\frac{V}{I}=\frac{120 \mathrm{~V}}{4.2 \mathrm{~A}}=29 \Omega
$$

5. (a) Use Eq. 25-2b to find the current.

$$
V=I R \rightarrow I=\frac{V}{R}=\frac{240 \mathrm{~V}}{8.6 \Omega}=27.91 \mathrm{~A} \approx 28 \mathrm{~A}
$$

(b) Use the definition of current, Eq. 25-1a.

$$
I=\frac{\Delta Q}{\Delta t} \rightarrow \Delta Q=I \Delta t=(27.91 \mathrm{~A})(50 \mathrm{~min})(60 \mathrm{~s} / \mathrm{min})=8.4 \times 10^{4} \mathrm{C}
$$

6. (a) Solve Eq. 25-2a for resistance.

$$
R=\frac{V}{I}=\frac{120 \mathrm{~V}}{9.5 \mathrm{~A}}=12.63 \Omega \approx 13 \Omega
$$

(b) Use the definition of average current, Eq. 25-1a.

$$
I=\frac{\Delta Q}{\Delta t} \rightarrow \Delta Q=I \Delta t=(9.5 \mathrm{~A})(15 \mathrm{~min})(60 \mathrm{~s} / \mathrm{min})=8600 \mathrm{C}
$$

7. Use Ohm's Law, Eq. 25-2a, to find the current. Then use the definition of current, Eq. 25-1a, to calculate the number of electrons per minute.

$$
I=\frac{V}{R}=\frac{\Delta Q}{\Delta t}=\frac{4.5 \mathrm{~V}}{1.6 \Omega}=\frac{2.8 \mathrm{C}}{\mathrm{~s}} \times \frac{1 \text { electron }}{1.60 \times 10^{-19} \mathrm{C}} \times \frac{60 \mathrm{~s}}{1 \mathrm{~min}}=1.1 \times 10^{21} \frac{\text { electrons }}{\text { minute }}
$$

8. Find the potential difference from the resistance and the current.

$$
\begin{aligned}
& R=\left(2.5 \times 10^{-5} \Omega / \mathrm{m}\right)\left(4.0 \times 10^{-2} \mathrm{~m}\right)=1.0 \times 10^{-6} \Omega \\
& V=I R=(3100 \mathrm{~A})\left(1.0 \times 10^{-6} \Omega\right)=3.1 \times 10^{-3} \mathrm{~V}
\end{aligned}
$$

9. (a) Use Eq. $25-2 \mathrm{~b}$ to find the resistance.

$$
R=\frac{V}{I}=\frac{12 \mathrm{~V}}{0.60 \mathrm{~A}}=20 \Omega \text { (2 sig. fig.) }
$$

(b) An amount of charge $\Delta Q$ loses a potential energy of $(\Delta Q) V$ as it passes through the resistor. The amount of charge is found from Eq. 25-1a.

$$
\Delta U=(\Delta Q) V=(I \Delta t) V=(0.60 \mathrm{~A})(60 \mathrm{~s})(12 \mathrm{~V})=430 \mathrm{~J}
$$

10. (a) If the voltage drops by $15 \%$, and the resistance stays the same, then by Eq. $25-2 \mathrm{~b}, V=I R$, the current will also drop by $15 \%$.

$$
I_{\text {final }}=0.85 I_{\text {initial }}=0.85(6.50 \mathrm{~A})=5.525 \mathrm{~A} \approx 5.5 \mathrm{~A}
$$

(b) If the resistance drops by $15 \%$ (the same as being multiplied by 0.85 ), and the voltage stays the same, then by Eq. $25-2$ b, the current must be divided by 0.85 .

$$
I_{\text {final }}=\frac{I_{\text {initial }}}{0.85}=\frac{6.50 \mathrm{~A}}{0.85}=7.647 \mathrm{~A} \approx 7.6 \mathrm{~A}
$$

11. Use Eq. 25-3 to find the diameter, with the area as $A=\pi r^{2}=\pi d^{2} / 4$.

$$
R=\rho \frac{\ell}{A}=\rho \frac{4 \ell}{\pi d^{2}} \rightarrow d=\sqrt{\frac{4 \ell \rho}{\pi R}}=\sqrt{\frac{4(1.00 \mathrm{~m})\left(5.6 \times 10^{-8} \Omega \cdot \mathrm{~m}\right)}{\pi(0.32 \Omega)}}=4.7 \times 10^{-4} \mathrm{~m}
$$

12. Use Eq. 25-3 to calculate the resistance, with the area as $A=\pi r^{2}=\pi d^{2} / 4$.

$$
R=\rho \frac{\ell}{A}=\rho \frac{4 \ell}{\pi d^{2}}=\left(1.68 \times 10^{-8} \Omega \cdot \mathrm{~m}\right) \frac{4(4.5 \mathrm{~m})}{\pi\left(1.5 \times 10^{-3} \mathrm{~m}\right)^{2}}=4.3 \times 10^{-2} \Omega
$$

13. Use Eq. 25-3 to calculate the resistances, with the area as $A=\pi r^{2}=\pi d^{2} / 4$.

$$
\begin{aligned}
& R=\rho \frac{\ell}{A}=\rho \frac{4 \ell}{\pi d^{2}} . \\
& \frac{R_{\mathrm{Al}}}{R_{\mathrm{Cu}}}=\frac{\rho_{\mathrm{Al}} \frac{4 \ell_{\mathrm{Al}}}{\pi d_{\mathrm{Al}}^{2}}}{\rho_{\mathrm{Cu}} \frac{4 \ell_{\mathrm{Cu}}}{\pi d_{\mathrm{Cu}}^{2}}}=\frac{\rho_{\mathrm{Al}} \ell_{\mathrm{Al}} d_{\mathrm{Cu}}^{2}}{\rho_{\mathrm{Cu}} \ell_{\mathrm{Cu}}^{2} d_{\mathrm{Al}}^{2}}=\frac{\left(2.65 \times 10^{-8} \Omega \cdot \mathrm{~m}\right)(10.0 \mathrm{~m})(1.8 \mathrm{~mm})^{2}}{\left(1.68 \times 10^{-8} \Omega \cdot \mathrm{~m}\right)(20.0 \mathrm{~m})(2.0 \mathrm{~mm})^{2}}=0.64
\end{aligned}
$$

14. Use Eq. 25-3 to express the resistances, with the area as $A=\pi r^{2}=\pi d^{2} / 4$, and so $R=\rho \frac{\ell}{A}=\rho \frac{4 \ell}{\pi d^{2}}$.

$$
\begin{aligned}
& R_{\mathrm{w}}=R_{\mathrm{Cu}} \rightarrow \rho_{\mathrm{w}} \frac{4 \ell}{\pi d_{\mathrm{w}}^{2}}=\rho_{\mathrm{Cu}} \frac{4 \ell}{\pi d_{\mathrm{Cu}}^{2}} \rightarrow \\
& d_{\mathrm{w}}=d_{\mathrm{Cu}} \sqrt{\frac{\rho_{\mathrm{w}}}{\rho_{\mathrm{Cu}}}}=(2.2 \mathrm{~mm}) \sqrt{\frac{5.6 \times 10^{-8} \Omega \cdot \mathrm{~m}}{1.68 \times 10^{-8} \Omega \cdot \mathrm{~m}}}=4.0 \mathrm{~mm}
\end{aligned}
$$

The diameter of the tungsten should be 4.0 mm .
15. (a) If the wire obeys Ohm's law, then $V=I R$ or $I=\frac{1}{R} V$, showing a linear relationship between $I$ and $V$. A graph of $I$ vs. $V$ should give a straight line with a slope of $\frac{1}{R}$ and a $y$-intercept of 0 .
(b) From the graph and the calculated linear fit, we see that the wire obeys Ohm's law.

$$
\begin{gathered}
\text { slope }=\frac{1}{R} \rightarrow \\
R=\frac{1}{0.720} \mathrm{~A} / \mathrm{V} \\
=1.39 \Omega
\end{gathered}
$$

The spreadsheet used for this
 problem can be found on the
Media Manager, with filename "PSE4_ISM_CH25.XLS," on tab "Problem 25.15b."
(c) Use Eq. 25-3 to find the resistivity.

$$
R=\rho \frac{\ell}{A} \rightarrow \rho=\frac{A R}{\ell}=\frac{\pi d^{2} R}{4 \ell}=\frac{\pi\left(3.2 \times 10^{-4} \mathrm{~m}\right)^{2}(1.39 \Omega)}{4(0.11 \mathrm{~m})}=1.0 \times 10^{-6} \Omega \cdot \mathrm{~m}
$$

From Table 25-1, the material is nichrome.
16. Use Eq. 25-5 multiplied by $\ell / A$ so that it expresses resistance instead of resistivity.

$$
\begin{aligned}
& R=R_{0}\left[1+\alpha\left(T-T_{0}\right)\right]=1.15 R_{0} \rightarrow 1+\alpha\left(T-T_{0}\right)=1.15 \rightarrow \\
& T-T_{0}=\frac{0.15}{\alpha}=\frac{0.15}{.0068\left(\mathrm{C}^{\circ}\right)^{-1}}=22 \mathrm{C}^{\circ}
\end{aligned}
$$

So raise the temperature by $22 \mathrm{C}^{\circ}$ to a final temperature of $42^{\circ} \mathrm{C}$.
17. Since the resistance is directly proportional to the length, the length of the long piece must be 4.0 times the length of the short piece.

$$
\ell=\ell_{\text {short }}+\ell_{\text {long }}=\ell_{\text {short }}+4.0 \ell_{\text {short }}=5.0 \ell_{\text {short }} \rightarrow \ell_{\text {short }}=0.20 \ell, \ell_{\text {long }}=0.80 \ell
$$

Make the cut at $20 \%$ of the length of the wire.

$$
\ell_{\text {short }}=0.20 \ell, \ell_{\text {long }}=0.80 \ell \rightarrow R_{\text {short }}=0.2 R=2.0 \Omega, R_{\text {long }}=0.8 R=8.0 \Omega
$$

18. Use Eq. 25-5 for the resistivity.

$$
\begin{aligned}
& \rho_{\mathrm{TAI}}=\rho_{0 \mathrm{Al}}\left[1+\alpha_{\mathrm{Al}}\left(T-T_{0}\right)\right]=\rho_{0 \mathrm{w}} \rightarrow \\
& T=T_{0}+\frac{1}{\alpha_{\mathrm{Al}}}\left(\frac{\rho_{0 \mathrm{~W}}}{\rho_{0 \mathrm{Al}}}-1\right)=20^{\circ} \mathrm{C}+\frac{1}{0.00429\left(\mathrm{C}^{\circ}\right)^{-1}}\left(\frac{5.6 \times 10^{-8} \Omega \cdot \mathrm{~m}}{2.65 \times 10^{-8} \Omega \cdot \mathrm{~m}}-1\right)=279.49^{\circ} \mathrm{C} \approx 280^{\circ} \mathrm{C}
\end{aligned}
$$

19. Use Eq. 25-5 multiplied by $\ell / A$ so that it expresses resistances instead of resistivity.

$$
\begin{aligned}
& R=R_{0}\left[1+\alpha\left(T-T_{0}\right)\right] \rightarrow \\
& T=T_{0}+\frac{1}{\alpha}\left(\frac{R}{R_{0}}-1\right)=20^{\circ} \mathrm{C}+\frac{1}{0.0045\left(\mathrm{C}^{\circ}\right)^{-1}}\left(\frac{140 \Omega}{12 \Omega}-1\right)=2390^{\circ} \mathrm{C} \approx 2400^{\circ} \mathrm{C}
\end{aligned}
$$

20. Calculate the voltage drop by combining Ohm's Law (Eq. 25-2b) with the expression for resistance, Eq. 25-3.

$$
V=I R=I \frac{\rho \ell}{A}=I \frac{4 \rho \ell}{\pi d^{2}}=(12 \mathrm{~A}) \frac{4\left(1.68 \times 10^{-8} \Omega \cdot \mathrm{~m}\right)(26 \mathrm{~m})}{\pi\left(1.628 \times 10^{-3} \mathrm{~m}\right)^{2}}=2.5 \mathrm{~V}
$$

21. The wires have the same resistance and the same resistivity.

$$
R_{\text {long }}=R_{\text {stort }} \rightarrow \frac{\rho \ell_{\text {long }}}{A_{1}}=\frac{\rho \ell_{\text {slort }}}{A_{2}} \rightarrow \frac{(4) 2 \ell_{\text {short }}}{\pi d_{\text {long }}^{2}}=\frac{4 \ell_{\text {short }}}{\pi d_{\text {short }}^{2}} \rightarrow \frac{d_{\text {long }}}{d_{\text {short }}}=\sqrt{2}
$$

22. In each case calculate the resistance by using Eq. 25-3 for resistance.
(a) $R_{x}=\frac{\rho \ell_{x}}{A_{y z}}=\frac{\left(3.0 \times 10^{-5} \Omega \cdot \mathrm{~m}\right)\left(1.0 \times 10^{-2} \mathrm{~m}\right)}{\left(2.0 \times 10^{-2} \mathrm{~m}\right)\left(4.0 \times 10^{-2} \mathrm{~m}\right)}=3.75 \times 10^{-4} \Omega \approx 3.8 \times 10^{-4} \Omega$
(b) $\quad R_{y}=\frac{\rho \ell_{y}}{A_{x z}}=\frac{\left(3.0 \times 10^{-5} \Omega \cdot \mathrm{~m}\right)\left(2.0 \times 10^{-2} \mathrm{~m}\right)}{\left(1.0 \times 10^{-2} \mathrm{~m}\right)\left(4.0 \times 10^{-2} \mathrm{~m}\right)}=1.5 \times 10^{-3} \Omega$
(c) $R_{z}=\frac{\rho \ell_{z}}{A_{x y}}=\frac{\left(3.0 \times 10^{-5} \Omega \cdot \mathrm{~m}\right)\left(4.0 \times 10^{-2} \mathrm{~m}\right)}{\left(1.0 \times 10^{-2} \mathrm{~m}\right)\left(2.0 \times 10^{-2} \mathrm{~m}\right)}=6.0 \times 10^{-3} \Omega$
23. The original resistance is $R_{0}=V / I_{0}$, and the high temperature resistance is $R=V / I$, where the two voltages are the same. The two resistances are related by Eq. $25-5$, multiplied by $\ell / A$ so that it expresses resistance instead of resistivity.

$$
\begin{aligned}
R=R_{0}\left[1+\alpha\left(T-T_{0}\right)\right] \rightarrow T & =T_{0}+\frac{1}{\alpha}\left(\frac{R}{R_{0}}-1\right)=T_{0}+\frac{1}{\alpha}\left(\frac{V / I}{V / I_{0}}-1\right)=T_{0}+\frac{1}{\alpha}\left(\frac{I_{0}}{I}-1\right) \\
& =20.0^{\circ} \mathrm{C}+\frac{1}{0.00429\left(\mathrm{C}^{\circ}\right)^{-1}}\left(\frac{0.4212 \mathrm{~A}}{0.3818 \mathrm{~A}}-1\right)=44.1^{\circ} \mathrm{C}
\end{aligned}
$$

24. For the cylindrical wire, its (constant) volume is given by $V=\ell_{0} A_{0}=\ell A$, and so $A=\frac{V}{\ell}$. Combine this relationship with Eq. 25-3. We assume that $\Delta \ell \ll \ell_{0}$.

$$
\begin{aligned}
& R_{0}=\rho \frac{\ell_{0}}{A_{0}}=\rho \frac{\ell_{0}^{2}}{V_{0}} ; R=\rho \frac{\ell}{A}=\rho \frac{\ell^{2}}{V_{0}} ; \frac{d R}{d \ell}=2 \rho \frac{\ell}{V_{0}} \\
& \Delta R \approx \frac{d R}{d \ell} \Delta \ell=2 \rho \frac{\ell}{V_{0}} \Delta \ell \rightarrow \Delta \ell=\frac{V_{0} \Delta R}{2 \rho \ell} \rightarrow \frac{\Delta \ell}{\ell}=\frac{V_{0} \Delta R}{2 \rho \ell^{2}}=\frac{\Delta R}{2 \frac{\rho \ell^{2}}{V_{0}}}=\frac{1}{2} \frac{\Delta R}{R}
\end{aligned}
$$

This is true for any initial conditions, and so $\frac{\Delta \ell}{\ell_{0}}=\frac{1}{2} \frac{\Delta R}{R_{0}}$
25. The resistance depends on the length and area as $R=\rho \ell / A$. Cutting the wire and running the wires side by side will halve the length and double the area.

$$
R_{2}=\frac{\rho\left(\frac{1}{2} \ell\right)}{2 A}=\frac{1}{4} \frac{\rho \ell}{A}=\frac{1}{4} R_{1}
$$

26. The total resistance is to be 3700 ohms $\left(R_{\text {total }}\right)$ at all temperatures. Write each resistance in terms of Eq.25-5 (with $T_{0}=0^{\circ} \mathrm{C}$ ), multiplied by $\ell / A$ to express resistance instead of resistivity.

$$
\begin{aligned}
R_{\text {total }} & =R_{\mathrm{oC}}\left[1+\alpha_{\mathrm{C}} T\right]+R_{\mathrm{oN}}\left[1+\alpha_{\mathrm{N}} T\right]=R_{\mathrm{oC}}+R_{\mathrm{oC}} \alpha_{\mathrm{C}} T+R_{\mathrm{oN}}+R_{\mathrm{oN}} \alpha_{\mathrm{N}} T \\
& =R_{\mathrm{oC}}+R_{\mathrm{oN}}+\left(R_{\mathrm{oC}} \alpha_{\mathrm{C}}+R_{\mathrm{oN}} \alpha_{\mathrm{N}}\right) T
\end{aligned}
$$

For the above to be true, the terms with a temperature dependence must cancel, and the terms without a temperature dependence must add to $R_{\text {total }}$. Thus we have two equations in two unknowns.

$$
\begin{aligned}
& 0=\left(R_{\mathrm{OC}} \alpha_{\mathrm{C}}+R_{\mathrm{oN}} \alpha_{\mathrm{N}}\right) T \rightarrow R_{\mathrm{oN}}=-\frac{R_{\mathrm{oc}} \alpha_{\mathrm{C}}}{\alpha_{\mathrm{N}}} \\
& R_{\text {total }}=R_{\mathrm{OC}}+R_{\mathrm{ON}}=R_{\mathrm{oC}}-\frac{R_{\mathrm{oc}} \alpha_{\mathrm{C}}}{\alpha_{\mathrm{N}}}=\frac{R_{\mathrm{OC}}\left(\alpha_{\mathrm{N}}-\alpha_{\mathrm{C}}\right)}{\alpha_{\mathrm{N}}} \rightarrow \\
& R_{\mathrm{oC}}=R_{\text {tooal }} \frac{\alpha_{\mathrm{N}}}{\left(\alpha_{\mathrm{N}}-\alpha_{\mathrm{C}}\right)}=(3700 \Omega) \frac{0.0004\left(\mathrm{C}^{\mathrm{o}}\right)^{-1}}{0.0004\left(\mathrm{C}^{\mathrm{o}}\right)^{-1}+0.0005\left(\mathrm{C}^{\mathrm{o}}\right)^{-1}}=1644 \Omega \approx 1600 \Omega \\
& R_{0 \mathrm{~N}}=R_{\text {toala }}-R_{\mathrm{OC}}=3700 \Omega-1644 \Omega=2056 \Omega \approx 2100 \Omega
\end{aligned}
$$

27. We choose a spherical shell of radius $r$ and thickness $d r$ as a differential element. The area of this element is $4 \pi r^{2}$. Use Eq. 25-3, but for an infinitesimal resistance. Then integrate over the radius of the sphere.

$$
R=\rho \frac{\ell}{A} \rightarrow d R=\rho \frac{d \ell}{A}=\frac{d r}{4 \pi \sigma r^{2}} \rightarrow R=\int d R=\int_{r_{i}}^{r_{2}} \frac{d r}{4 \pi \sigma r^{2}}=\frac{1}{4 \pi \sigma}\left(-\frac{1}{r}\right)_{r_{i}}^{r_{1}}=\frac{1}{4 \pi \sigma}\left(\frac{1}{r_{1}}-\frac{1}{r_{2}}\right)
$$

28. (a) Let the values at the lower temperature be indicated by a subscript " 0 ". Thus $R_{0}=\rho_{0} \frac{\ell_{0}}{A_{0}}$ $=\rho_{0} \frac{4 \ell_{0}}{\pi d_{0}^{2}}$. The change in temperature results in new values for the resistivity, the length, and the diameter. Let $\alpha$ represent the temperature coefficient for the resistivity, and $\alpha_{\mathrm{T}}$ represent the thermal coefficient of expansion, which will affect the length and diameter.

$$
\begin{aligned}
R & =\rho \frac{\ell}{A}=\rho \frac{4 \ell}{\pi d^{2}}=\rho_{0}\left[1+\alpha\left(T-T_{0}\right)\right] \frac{4 \ell_{0}\left[1+\alpha_{\mathrm{T}}\left(T-T_{0}\right)\right]}{\pi\left\{d_{0}\left[1+\alpha_{\mathrm{T}}\left(T-T_{0}\right)\right]\right\}^{2}}=\rho_{0} \frac{4 \ell_{0}}{\pi d_{0}^{2}} \frac{\left[1+\alpha\left(T-T_{0}\right)\right]}{\left[1+\alpha_{\mathrm{T}}\left(T-T_{0}\right)\right]} \\
& =R_{0} \frac{\left[1+\alpha\left(T-T_{0}\right)\right]}{\left[1+\alpha_{\mathrm{T}}\left(T-T_{0}\right)\right]} \rightarrow R\left[1+\alpha_{\mathrm{T}}\left(T-T_{0}\right)\right]=R_{0}\left[1+\alpha\left(T-T_{0}\right)\right] \rightarrow
\end{aligned}
$$

$$
\begin{aligned}
T & =T_{0}+\frac{\left(R-R_{0}\right)}{\left(R_{0} \alpha-R \alpha_{\mathrm{T}}\right)}=20^{\circ} \mathrm{C}+\frac{(140 \Omega-12 \Omega)}{\left[(12 \Omega)\left(0.0045 \mathrm{C}^{\circ-1}\right)-(140 \Omega)\left(5.5 \times 10^{-6} \mathrm{C}^{\circ-1}\right)\right]} \\
& =20^{\circ} \mathrm{C}+2405^{\circ} \mathrm{C}=2425^{\circ} \mathrm{C} \approx 2400^{\circ} \mathrm{C}
\end{aligned}
$$

(b) The net effect of thermal expansion is that both the length and diameter increase, which lowers the resistance.

$$
\begin{aligned}
& \frac{R}{R_{0}}=\frac{\rho_{0} \frac{4 \ell}{\pi d^{2}}}{\rho_{0} \frac{4 \ell_{0}}{\pi d_{0}^{2}}}=\frac{\ell d_{0}^{2}}{\ell_{0} d^{2}}=\frac{\ell_{0}\left[1+\alpha_{\mathrm{T}}\left(T-T_{0}\right)\right]}{\ell_{0}} \frac{d_{0}^{2}}{\left\{d_{0}\left[1+\alpha_{\mathrm{T}}\left(T-T_{0}\right)\right]\right\}^{2}}=\frac{1}{\left[1+\alpha_{\mathrm{T}}\left(T-T_{0}\right)\right]} \\
& \quad=\frac{1}{\left[1+\left(5.5 \times 10^{-6} \mathrm{C}^{\circ-1}\right)\left(2405^{\circ} \mathrm{C}\right)\right]}=0.9869 \\
& \% \text { change }=\left(\frac{R-R_{0}}{R_{0}}\right) 100=\left(\frac{R}{R_{0}}-1\right) 100=-1.31 \approx-1.3 \%
\end{aligned}
$$

The net effect of resistivity change is that the resistance increases.

$$
\begin{aligned}
& \begin{aligned}
& \frac{R}{R_{0}}=\frac{\rho \frac{4 \ell_{0}}{\pi d_{0}^{2}}}{\rho_{0} \frac{4 \ell_{0}}{\pi d_{0}^{2}}}=\frac{\rho}{\rho_{0}}=\frac{\rho_{0}\left[1+\alpha\left(T-T_{0}\right)\right]}{\rho_{0}}=\left[1+\alpha\left(T-T_{0}\right)\right]=\left[1+\left(0.0045 \mathrm{C}^{0-1}\right)\left(2405^{\circ} \mathrm{C}\right)\right] \\
&=11.82 \\
& \% \text { change }=\left(\frac{R-R_{0}}{R_{0}}\right) 100=\left(\frac{R}{R_{0}}-1\right) 100=1082 \approx 1100 \%
\end{aligned}
\end{aligned}
$$

29. (a) Calculate each resistance separately using Eq. 25-3, and then add the resistances together to find the total resistance.

$$
\begin{aligned}
& R_{\mathrm{Cu}}=\frac{\rho_{\mathrm{Cu}} \ell}{A}=\frac{4 \rho_{\mathrm{Cu}} \ell}{\pi d^{2}}=\frac{4\left(1.68 \times 10^{-8} \Omega \cdot \mathrm{~m}\right)(5.0 \mathrm{~m})}{\pi\left(1.4 \times 10^{-3} \mathrm{~m}\right)^{2}}=0.054567 \Omega \\
& R_{\mathrm{Al}}=\frac{\rho_{\mathrm{Al}} \ell}{A}=\frac{4 \rho_{\mathrm{A} 1} \ell}{\pi d^{2}}=\frac{4\left(2.65 \times 10^{-8} \Omega \cdot \mathrm{~m}\right)(5.0 \mathrm{~m})}{\pi\left(1.4 \times 10^{-3} \mathrm{~m}\right)^{2}}=0.086074 \Omega \\
& R_{\mathrm{tooal}}=R_{\mathrm{Cu}}+R_{\mathrm{Al}}=0.054567 \Omega+0.086074 \Omega=0.140641 \Omega \approx 0.14 \Omega
\end{aligned}
$$

(b) The current through the wire is the voltage divided by the total resistance.

$$
I=\frac{V}{R_{\text {total }}}=\frac{85 \times 10^{-3} \mathrm{~V}}{0.140641 \Omega}=0.60438 \mathrm{~A} \approx 0.60 \mathrm{~A}
$$

(c) For each segment of wire, Ohm's law is true. Both wires have the current found in $(b)$ above.

$$
\begin{aligned}
& V_{\mathrm{Cu}}=I R_{\mathrm{Cu}}=(0.60438 \mathrm{~A})(0.054567 \Omega) \approx 0.033 \mathrm{~V} \\
& V_{\mathrm{Al}}=I R_{\mathrm{Al}}=(0.60438 \mathrm{~A})(0.086074 \Omega) \approx 0.052 \mathrm{~V}
\end{aligned}
$$

Notice that the total voltage is 85 mV .
30. (a) Divide the cylinder up into concentric cylindrical shells of radius $r$, thickness $d r$, and length $\ell$. See the diagram. The resistance of one of those shells, from Eq. 25-3, is found. Note that the "length" in Eq. 25-3 is in the direction of the current flow, so we must substitute in $d r$ for the "length" in Eq. 25-3. The area is the surface area of the thin cylindrical shell. Then integrate over the range of radii to find the total resistance.

$$
\begin{aligned}
& R=\rho \frac{\text { " } \ell "}{A} \rightarrow d R=\rho \frac{d r}{2 \pi r \ell} \\
& R=\int d R=\int_{r_{i}}^{r_{2}} \rho \frac{d r}{2 \pi r \ell}=\frac{\rho}{2 \pi \ell} \ln \frac{r_{2}}{r_{1}}
\end{aligned}
$$

(b) Use the data given to calculate the resistance from the above
 formula.

$$
R=\frac{\rho}{2 \pi \ell} \ln \frac{r_{2}}{r_{1}}=\frac{15 \times 10^{-5} \Omega \cdot \mathrm{~m}}{2 \pi(0.024 \mathrm{~m})} \ln \left(\frac{1.8 \mathrm{~mm}}{1.0 \mathrm{~mm}}\right)=5.8 \times 10^{-4} \Omega
$$

(c) For resistance along the axis, we again use Eq. 25-3, but the current is flowing in the direction of length $\ell$. The area is the cross-sectional area of the face of the hollow cylinder.

$$
R=\frac{\rho \ell}{A}=\frac{\rho \ell}{\pi\left(r_{2}^{2}-r_{1}^{2}\right)}=\frac{\left(15 \times 10^{-5} \Omega \cdot \mathrm{~m}\right)(0.024 \mathrm{~m})}{\pi\left[\left(1.8 \times 10^{-3} \mathrm{~m}\right)^{2}-\left(1.0 \times 10^{-3} \mathrm{~m}\right)^{2}\right]}=0.51 \Omega
$$

31. Use Eq. 25-6 to find the power from the voltage and the current.

$$
P=I V=(0.27 \mathrm{~A})(3.0 \mathrm{~V})=0.81 \mathrm{~W}
$$

32. Use Eq. $25-7 \mathrm{~b}$ to find the resistance from the voltage and the power.

$$
P=\frac{V^{2}}{R} \rightarrow R=\frac{V^{2}}{P}=\frac{(240 \mathrm{~V})^{2}}{3300 \mathrm{~W}}=17 \Omega
$$

33. Use Eq. $25-7 \mathrm{~b}$ to find the voltage from the power and the resistance.

$$
P=\frac{V^{2}}{R} \rightarrow V=\sqrt{R P}=\sqrt{(3300 \Omega)(0.25 \mathrm{~W})}=29 \mathrm{~V}
$$

34. Use Eq. $25-7$ b to find the resistance, and Eq. 25-6 to find the current.
(a) $P=\frac{V^{2}}{R} \rightarrow R=\frac{V^{2}}{P}=\frac{(110 \mathrm{~V})^{2}}{75 \mathrm{~W}}=161.3 \Omega \approx 160 \Omega$

$$
P=I V \rightarrow I=\frac{P}{V}=\frac{75 \mathrm{~W}}{110 \mathrm{~V}}=0.6818 \mathrm{~A} \approx 0.68 \mathrm{~A}
$$

(b)

$$
\begin{aligned}
& P=\frac{V^{2}}{R} \rightarrow R=\frac{V^{2}}{P}=\frac{(110 \mathrm{~V})^{2}}{440 \mathrm{~W}}=27.5 \Omega \approx 28 \Omega \\
& P=I V \rightarrow I=\frac{P}{V}=\frac{440 \mathrm{~W}}{110 \mathrm{~V}}=4.0 \mathrm{~A}
\end{aligned}
$$

35. (a) From Eq. 25-6, if power $P$ is delivered to the transmission line at voltage $V$, there must be a current $I=P / V$. As this current is carried by the transmission line, there will be power losses of $I^{2} R$ due to the resistance of the wire. This power loss can be expressed as $\Delta P=I^{2} R$ $=P^{2} R / V^{2}$. Equivalently, there is a voltage drop across the transmission lines of $V^{\prime}=I R$. Thus the voltage available to the users is $V-V^{\prime}$, and so the power available to the users is $P^{\prime}=\left(V-V^{\prime}\right) I=V I-V^{\prime} I=V I-I^{2} R=P-I^{2} R$. The power loss is $\Delta P=P-P^{\prime}=P-\left(P-I^{2} R\right)$ $=I^{2} R=P^{2} R / V^{2}$.
(b) Since $\Delta P \propto \frac{1}{V^{2}}, V$ should be as large as possible to minimize $\Delta P$.
36. (a) Since $P=\frac{V^{2}}{R} \rightarrow R=\frac{V^{2}}{P}$ says that the resistance is inversely proportional to the power for a constant voltage, we predict that the 850 W setting has the higher resistance.
(b) $R=\frac{V^{2}}{P}=\frac{(120 \mathrm{~V})^{2}}{850 \mathrm{~W}}=17 \Omega$
(c) $R=\frac{V^{2}}{P}=\frac{(120 \mathrm{~V})^{2}}{1250 \mathrm{~W}}=12 \Omega$
37. (a) Use Eq. 25-6 to find the current.

$$
P=I V \rightarrow I=\frac{P}{V}=\frac{95 \mathrm{~W}}{115 \mathrm{~V}}=0.83 \mathrm{~A}
$$

(b) Use Eq. $25-7 \mathrm{~b}$ to find the resistance.

$$
P=\frac{V^{2}}{R} \rightarrow R=\frac{V^{2}}{P}=\frac{(115 \mathrm{~V})^{2}}{95 \mathrm{~W}} \approx 140 \Omega
$$

38. The power (and thus the brightness) of the bulb is proportional to the square of the voltage, according to Eq. $25-7 \mathrm{~b}, P=\frac{V^{2}}{R}$. Since the resistance is assumed to be constant, if the voltage is cut in half from 240 V to 120 V , the power will be reduced by a factor of 4 . Thus the bulb will appear only about $1 / 4$ as bright in the United States as in Europe.
39. To find the kWh of energy, multiply the kilowatts of power consumption by the number of hours in operation.

$$
\text { Energy }=P(\text { in } \mathrm{kW}) t(\text { in } \mathrm{h})=(550 \mathrm{~W})\left(\frac{1 \mathrm{~kW}}{1000 \mathrm{~W}}\right)(6.0 \mathrm{~min})\left(\frac{1 \mathrm{~h}}{60 \mathrm{~min}}\right)=0.055 \mathrm{kWh}
$$

To find the cost of the energy used in a month, multiply times 4 days per week of usage, times 4 weeks per month, times the cost per kWh .

$$
\text { Cost }=\left(0.055 \frac{\mathrm{kWh}}{\mathrm{~d}}\right)\left(\frac{4 \mathrm{~d}}{1 \text { week }}\right)\left(\frac{4 \text { week }}{1 \text { month }}\right)\left(\frac{9.0 \text { cents }}{\mathrm{kWh}}\right)=7.9 \text { cents } / \text { month }
$$

40. To find the cost of the energy, multiply the kilowatts of power consumption by the number of hours in operation times the cost per kWh .

$$
\text { Cost }=(25 \mathrm{~W})\left(\frac{1 \mathrm{~kW}}{1000 \mathrm{~W}}\right)(365 \text { day })\left(\frac{24 \mathrm{~h}}{1 \text { day }}\right)\left(\frac{\$ 0.095}{\mathrm{kWh}}\right) \approx \$ 21
$$

41. The $\mathrm{A} \cdot \mathrm{h}$ rating is the amount of charge that the battery can deliver. The potential energy of the charge is the charge times the voltage.

$$
U=Q V=(75 \mathrm{~A} \cdot \mathrm{~h})\left(\frac{3600 \mathrm{~s}}{1 \mathrm{~h}}\right)(12 \mathrm{~V})=3.2 \times 10^{6} \mathrm{~J}=0.90 \mathrm{kWh}
$$

42. (a) Calculate the resistance from Eq. 25-2b and the power from Eq. 25-6.

$$
R=\frac{V}{I}=\frac{3.0 \mathrm{~V}}{0.38 \mathrm{~A}}=7.895 \Omega \approx 7.9 \Omega \quad P=I V=(0.38 \mathrm{~A})(3.0 \mathrm{~V})=1.14 \mathrm{~W} \approx 1.1 \mathrm{~W}
$$

(b) If four D-cells are used, the voltage will be doubled to 6.0 V . Assuming that the resistance of the bulb stays the same (by ignoring heating effects in the filament), the power that the bulb would need to dissipate is given by Eq. 25-7b, $P=\frac{V^{2}}{R}$. A doubling of the voltage means the power is increased by a factor of 4 . This should not be tried because the bulb is probably not rated for such a high wattage. The filament in the bulb would probably burn out, and the glass bulb might even explode if the filament burns violently.
43. Each bulb will draw an amount of current found from Eq. 25-6.

$$
P=I V \quad \rightarrow \quad I_{\text {bulb }}=\frac{P}{V}
$$

The number of bulbs to draw 15 A is the total current divided by the current per bulb.

$$
I_{\text {total }}=n I_{\text {bulb }}=n \frac{P}{V} \rightarrow n=\frac{V I_{\text {total }}}{P}=\frac{(120 \mathrm{~V})(15 \mathrm{~A})}{75 \mathrm{~W}}=24 \mathrm{bulbs}
$$

44. Find the power dissipated in the cord by Eq. 25-7a, using Eq. 25-3 for the resistance.

$$
\begin{aligned}
P & =I^{2} R=I^{2} \rho \frac{\ell}{A}=I^{2} \rho \frac{\ell}{\pi d^{2} / 4}=I^{2} \rho \frac{4 \ell}{\pi d^{2}}=(15.0 \mathrm{~A})^{2}\left(1.68 \times 10^{-8} \Omega \cdot \mathrm{~m}\right) \frac{4(5.4 \mathrm{~m})}{\pi\left(0.129 \times 10^{-2} \mathrm{~m}\right)^{2}} \\
& =15.62 \mathrm{~W} \approx 16 \mathrm{~W}
\end{aligned}
$$

45. Find the current used to deliver the power in each case, and then find the power dissipated in the resistance at the given current.

$$
\begin{aligned}
& P=I V \rightarrow I=\frac{P}{V} \quad P_{\text {disisipated }}=I^{2} R=\frac{P^{2}}{V^{2}} R \\
& P_{\substack{\text { disispated } \\
\text { 12,000 }}}=\frac{\left(7.5 \times 10^{5} \mathrm{~W}\right)^{2}}{\left(1.2 \times 10^{4} \mathrm{~V}\right)^{2}}(3.0 \Omega)=11719 \mathrm{~W} \\
& \underbrace{}_{\substack{\text { Hisipipated } \\
50,00 \mathrm{~V}}}=\frac{\left(7.5 \times 10^{5} \mathrm{~W}\right)^{2}}{\left(5 \times 10^{4} \mathrm{~V}\right)^{2}}(3.0 \Omega)=675 \mathrm{~W} \quad \text { difference }=11719 \mathrm{~W}-675 \mathrm{~W}=1.1 \times 10^{4} \mathrm{~W}
\end{aligned}
$$

46. (a) By conservation of energy and the efficiency claim, $75 \%$ of the electrical power dissipated by the heater must be the rate at which energy is absorbed by the water.

$$
\begin{aligned}
& 0.75_{\substack{\text { emitted by } \\
\text { electromagnet }}}=P_{\substack{\text { absorbed } \\
\text { by water }}} \rightarrow 0.75(I V)=\frac{Q_{\text {heat water }}}{t}=\frac{m c \Delta T}{t} \rightarrow \\
& I=\frac{m c \Delta T}{0.75 \mathrm{~V} t}=\frac{(0.120 \mathrm{~kg})(4186 \mathrm{~J} / \mathrm{kg})\left(95^{\circ} \mathrm{C}-25^{\circ} \mathrm{C}\right)}{(0.75)(12 \mathrm{~V})(480 \mathrm{~s})}=8.139 \mathrm{~A} \approx 8.1 \mathrm{~A}
\end{aligned}
$$

(b) Use Ohm's law to find the resistance of the heater.

$$
V=I R \quad \rightarrow \quad R=\frac{V}{I}=\frac{12 \mathrm{~V}}{8.139 \mathrm{~A}}=1.5 \Omega
$$

47. The water temperature rises by absorbing the heat energy that the electromagnet dissipates. Express both energies in terms of power, which is energy per unit time.

$$
\begin{aligned}
& P_{\text {electric }}=P_{\substack{\text { to heat } \\
\text { water }}} \rightarrow I V=\frac{Q_{\text {heat water }}}{t}=\frac{m c \Delta T}{t} \rightarrow \\
& \frac{m}{t}=\frac{I V}{c \Delta T}=\frac{(17.5 \mathrm{~A})(240 \mathrm{~V})}{\left(4186 \mathrm{~J} / \mathrm{kg} \cdot{ }^{\circ} \mathrm{C}\right)\left(6.50 \mathrm{C}^{\circ}\right)}=0.154 \mathrm{~kg} / \mathrm{s} \approx 0.15 \mathrm{~kg} / \mathrm{s}
\end{aligned}
$$

This is $154 \mathrm{~mL} / \mathrm{s}$.
48. For the wire to stay a constant temperature, the power generated in the resistor is to be dissipated by radiation. Use Eq. $25-7$ a and $19-18$, both expressions of power (energy per unit time). We assume that the dimensions requested and dimensions given are those at the higher temperature, and do not take any thermal expansion effects into account. We also use Eq. 25-3 for resistance.

$$
\begin{aligned}
I^{2} R & =\varepsilon \sigma A\left(T_{\text {high }}^{4}-T_{\text {low }}^{4}\right) \rightarrow I^{2} \frac{4 \rho \ell}{\pi d^{2}}=\varepsilon \sigma \pi d \ell\left(T_{\text {high }}^{4}-T_{\text {low }}^{4}\right) \rightarrow \\
d & =\left(\frac{4 I^{2} \rho}{\pi^{2} \varepsilon \sigma\left(T_{\text {high }}^{4}-T_{\text {low }}^{4}\right)}\right)^{1 / 3}=\left(\frac{4(15.0 \mathrm{~A})^{2}\left(5.6 \times 10^{-8} \Omega \cdot \mathrm{~m}\right)}{\pi^{2}(1.0)\left(5.67 \times 10^{-8} \mathrm{~W} / \mathrm{m}^{2} \cdot \mathrm{~K}^{4}\right)\left[(3100 \mathrm{~K})^{4}-(293 \mathrm{~K})^{4}\right]}\right)^{1 / 3} \\
& =9.92 \times 10^{-5} \mathrm{~m} \approx 0.099 \mathrm{~mm}
\end{aligned}
$$

49. Use Ohm's law and the relationship between peak and rms values.

$$
I_{\text {peak }}=\sqrt{2} I_{\mathrm{rms}}=\sqrt{2} \frac{V_{\mathrm{rms}}}{R}=\sqrt{2} \frac{220 \mathrm{~V}}{2700 \Omega}=0.12 \mathrm{~A}
$$

50. Find the peak current from Ohm's law, and then find the rms current from Eq. 25-9a.

$$
I_{\text {peak }}=\frac{V_{\text {peak }}}{R}=\frac{180 \mathrm{~V}}{380 \Omega}=0.47368 \mathrm{~A} \approx 0.47 \mathrm{~A} \quad I_{\text {rms }}=I_{\text {peak }} / \sqrt{2}=(0.47368 \mathrm{~A}) / \sqrt{2}=0.33 \mathrm{~A}
$$

51. (a) When everything electrical is turned off, no current will be flowing into the house, even though a voltage is being supplied. Since for a given voltage, the more resistance, the lower the current, a zero current corresponds to an infinite resistance.
(b) Use Eq. 25-7a to calculate the resistance.

$$
P=\frac{V^{2}}{R} \rightarrow R=\frac{V^{2}}{P}=\frac{(120 \mathrm{~V})^{2}}{2(75 \mathrm{~W})}=96 \Omega
$$

52. The power and current can be used to find the peak voltage, and then the rms voltage can be found from the peak voltage.

$$
\bar{P}=I_{\mathrm{rms}} V_{\mathrm{rms}}=\frac{I_{\mathrm{peak}}}{\sqrt{2}} V_{\mathrm{rms}} \rightarrow V_{\mathrm{rms}}=\frac{\sqrt{2} \bar{P}}{I_{\text {peak }}}=\frac{\sqrt{2}(1500 \mathrm{~W})}{5.4 \mathrm{~A}}=390 \mathrm{~V}
$$

53. Use the average power and rms voltage to calculate the peak voltage and peak current.
(a) $V_{\text {paak }}=\sqrt{2} V_{\text {rms }}=\sqrt{2}(660 \mathrm{~V})=933.4 \mathrm{~V} \approx 930 \mathrm{~V}$
(b) $\bar{P}=I_{\mathrm{rms}} V_{\mathrm{rms}}=\frac{I_{\text {peak }}}{\sqrt{2}} V_{\mathrm{rms}} \rightarrow I_{\text {peak }}=\frac{\sqrt{2} \bar{P}}{V_{\mathrm{rms}}}=\frac{\sqrt{2}(1800 \mathrm{~W})}{660 \mathrm{~V}}=3.9 \mathrm{~A}$
54. (a) We assume that the 2.5 hp is the average power, so the maximum power is twice that, or 5.0 hp , as seen in Figure 25-22.

$$
5.0 \mathrm{hp}\left(\frac{746 \mathrm{~W}}{1 \mathrm{hp}}\right)=3730 \mathrm{~W} \approx 3700 \mathrm{~W}
$$

(b) Use the average power and the rms voltage to find the peak current.

$$
\bar{P}=I_{\mathrm{rms}} V_{\mathrm{rms}}=\frac{I_{\text {peak }}}{\sqrt{2}} V_{\mathrm{rms}} \rightarrow I_{\mathrm{peak}}=\frac{\sqrt{2} \bar{P}}{V_{\mathrm{rms}}}=\frac{\sqrt{2}\left[\frac{1}{2}(3730 \mathrm{~W})\right]}{240 \mathrm{~V}}=11 \mathrm{~A}
$$

55. (a) The average power used can be found from the resistance and the rms voltage by Eq. 25-10c.

$$
\bar{P}=\frac{V_{\mathrm{ms}}^{2}}{R}=\frac{(240 \mathrm{~V})^{2}}{44 \Omega}=1309 \mathrm{~W} \approx 1300 \mathrm{~W}
$$

(b) The maximum power is twice the average power, and the minimum power is 0 .

$$
P_{\max }=2 \bar{P}=2(1309 \mathrm{~W}) \approx 2600 \mathrm{~W} \quad P_{\text {min }}=0 \mathrm{~W}
$$

56. (a) Find $V_{\mathrm{rms}}$. Use an integral from Appendix B-4, page A-7.

$$
V_{\mathrm{rms}}=\left[\frac{1}{T} \int_{0}^{T}\left(V_{0} \sin \frac{2 \pi t}{T}\right)^{2} d t\right]^{1 / 2}=\left[\frac{V_{0}^{2}}{T}\left(\frac{t}{2}-\frac{\sin \left(\frac{4 \pi t}{T}\right)}{\frac{8 \pi}{T}}\right)^{T}\right]^{1 / 2}=\left(\frac{V_{0}^{2}}{2}\right)^{1 / 2}=\frac{V_{0}}{\sqrt{2}}
$$

(b) Find $V_{\mathrm{rms}}$.

$$
V_{\mathrm{rms}}=\left[\frac{1}{T} \int_{0}^{T} V^{2} d t\right]^{1 / 2}=\left[\frac{1}{T} \int_{0}^{T / 2} V_{0}^{2} d t+\frac{1}{T} \int_{T / 2}^{T}(0)^{2} d t\right]^{1 / 2}=\left[\frac{V_{0}^{2}}{T} \frac{T}{2}+0\right]^{1 / 2}=\frac{V_{0}}{\sqrt{2}}
$$

57. (a) We follow the derivation in Example 25-14. Start with Eq. 25-14, in absolute value.

$$
\begin{aligned}
& j=n e v_{\mathrm{d}} \rightarrow v_{\mathrm{d}}=\frac{j}{n e}=\frac{I}{n e A}=\frac{I}{\left(\frac{N(1 \text { mole })}{m(1 \text { mole })} \rho_{\mathrm{D}}\right) e\left[\pi\left(\frac{1}{2} d\right)^{2}\right]}=\frac{4 I \mathrm{~m}}{N \rho_{\mathrm{D}} e \pi d^{2}} \\
& v_{\mathrm{d}}=\frac{4\left(2.3 \times 10^{-6} \mathrm{~A}\right)\left(63.5 \times 10^{-3} \mathrm{~kg}\right)}{\left(6.02 \times 10^{23}\right)\left(8.9 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)\left(1.60 \times 10^{-19} \mathrm{C}\right) \pi\left(0.65 \times 10^{-3} \mathrm{~m}\right)^{2}}=5.1 \times 10^{-10} \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(b) Calculate the current density from Eq. 25-11.

$$
j=\frac{I}{A}=\frac{I}{\pi r^{2}}=\frac{4 I}{\pi d^{2}}=\frac{4\left(2.3 \times 10^{-6} \mathrm{~A}\right)}{\pi\left(6.5 \times 10^{-4} \mathrm{~m}\right)^{2}}=6.931 \mathrm{~A} / \mathrm{m}^{2} \approx 6.9 \mathrm{~A} / \mathrm{m}^{2}
$$

(c) The electric field is calculated from Eq. 25-17.

$$
j=\frac{1}{\rho} E \rightarrow E=\rho j=\left(1.68 \times 10^{-8} \Omega \cdot \mathrm{~m}\right)\left(6.931 \mathrm{~A} / \mathrm{m}^{2}\right)=1.2 \times 10^{-7} \mathrm{~V} / \mathrm{m}
$$

58. (a) Use Ohm's law to find the resistance.

$$
V=I R \rightarrow R=\frac{V}{I}=\frac{0.0220 \mathrm{~V}}{0.75 \mathrm{~A}}=0.02933 \Omega \approx 0.029 \Omega
$$

(b) Find the resistivity from Eq. 25-3.

$$
\begin{aligned}
& R=\frac{\rho \ell}{A} \rightarrow \\
& \rho=\frac{R A}{\ell}=\frac{R \pi r^{2}}{\ell}=\frac{(0.02933 \Omega) \pi\left(1.0 \times 10^{-3} \mathrm{~m}\right)^{2}}{(5.80 \mathrm{~m})}=1.589 \times 10^{-8} \Omega \cdot \mathrm{~m} \approx 1.6 \times 10^{-8} \Omega \cdot \mathrm{~m}
\end{aligned}
$$

(c) Use Eq. 25-11 to find the current density.

$$
j=\frac{I}{A}=\frac{I}{\pi r^{2}}=\frac{0.75}{\pi(0.0010 \mathrm{~m})^{2}}=2.387 \times 10^{5} \mathrm{~A} / \mathrm{m}^{2} \approx 2.4 \times 10^{5} \mathrm{~A} / \mathrm{m}^{2}
$$

(d) Use Eq. 25-17 to find the electric field.

$$
\begin{aligned}
& j=\frac{1}{\rho} E \rightarrow \\
& E=\rho j=\left(1.589 \times 10^{-8} \Omega \cdot \mathrm{~m}\right)\left(2.387 \times 10^{5} \mathrm{~A} / \mathrm{m}^{2}\right)=3.793 \times 10^{-3} \mathrm{~V} / \mathrm{m} \approx 3.8 \times 10^{-3} \mathrm{~V} / \mathrm{m}
\end{aligned}
$$

(e) Find the number of electrons per unit volume from the absolute value of Eq. 25-14.

$$
j=n e v_{\mathrm{d}} \rightarrow n=\frac{j}{v_{\mathrm{d}} e}=\frac{2.387 \times 10^{5} \mathrm{~A} / \mathrm{m}^{2}}{\left(1.7 \times 10^{-5} \mathrm{~m} / \mathrm{s}\right)\left(1.60 \times 10^{-19} \mathrm{C}\right)}=8.8 \times 10^{28} \mathrm{e}^{-} / \mathrm{m}^{3}
$$

59. We are given a charge density and a speed (like the drift speed) for both types of ions. From that we can use Eq. 25-13 (without the negative sign) to determine the current per unit area. Both currents are in the same direction in terms of conventional current - positive charge moving north has the same effect as negative charge moving south - and so they can be added.

$$
\begin{aligned}
& I=n e A v_{\mathrm{d}} \rightarrow \\
& \frac{I}{A}=\left(\text { nev }_{\mathrm{d}}\right)_{\mathrm{He}}+\left(\text { nev }_{\mathrm{d}}\right)_{\mathrm{o}}= {\left[\left(2.8 \times 10^{12} \text { ions } / \mathrm{m}^{3}\right) 2\left(1.60 \times 10^{-19} \mathrm{C} / \text { ion }\right)\left(2.0 \times 10^{6} \mathrm{~m} / \mathrm{s}\right)\right]+} \\
& {\left[\left(7.0 \times 10^{11} \text { ions } / \mathrm{m}^{3}\right)\left(1.60 \times 10^{-19} \mathrm{C} / \text { ion }\right)\left(6.2 \times 10^{6} \mathrm{~m} / \mathrm{s}\right)\right] }
\end{aligned}
$$

$$
=2.486 \mathrm{~A} / \mathrm{m}^{2} \approx 2.5 \mathrm{~A} / \mathrm{m}^{2}, \text { North }
$$

60. The magnitude of the electric field is the voltage change per unit meter.

$$
|E|=\frac{\Delta V}{\Delta x}=\frac{70 \times 10^{-3} \mathrm{~V}}{1.0 \times 10^{-8} \mathrm{~m}}=7.0 \times 10^{6} \mathrm{~V} / \mathrm{m}
$$

61. The speed is the change in position per unit time.

$$
v=\frac{\Delta x}{\Delta t}=\frac{7.20 \times 10^{-2} \mathrm{~m}-3.40 \times 10^{-2} \mathrm{~m}}{0.0063 \mathrm{~s}-0.0052 \mathrm{~s}}=35 \mathrm{~m} / \mathrm{s}
$$

Two measurements are needed because there may be a time delay from the stimulation of the nerve to the generation of the action potential.
62. The power is the work done per unit time. The work done to move a charge through a potential difference is the charge times the potential difference. The charge density must be multiplied by the surface area of the cell (the surface area of an open tube, length times circumference) to find the actual charge moved.

$$
\begin{aligned}
P & =\frac{W}{t}=\frac{Q V}{t}=\frac{Q}{t} V \\
& =\left(3 \times 10^{-7} \frac{\mathrm{~mol}}{\mathrm{~m}^{2} \cdot \mathrm{~s}}\right)\left(6.02 \times 10^{23} \frac{\mathrm{ions}}{\mathrm{~mol}}\right)\left(1.6 \times 10^{-19} \frac{\mathrm{C}}{\mathrm{ion}}\right)(0.10 \mathrm{~m}) \pi\left(20 \times 10^{-6} \mathrm{~m}\right)(0.030 \mathrm{~V}) \\
& =5.4 \times 10^{-9} \mathrm{~W}
\end{aligned}
$$

63. The energy supplied by the battery is the energy consumed by the lights.

$$
\begin{aligned}
& E_{\text {supplied }}=E_{\text {consumed }} \rightarrow Q \Delta V=P t \rightarrow \\
& t=\frac{Q \Delta V}{P}=\frac{(85 \mathrm{~A} \cdot \mathrm{~h})(3600 \mathrm{~s} / \mathrm{h})(12 \mathrm{~V})}{92 \mathrm{~W}}=39913 \mathrm{~s}\left(\frac{1 \mathrm{~h}}{3600 \mathrm{~s}}\right)=11.09 \mathrm{~h} \approx 11 \mathrm{~h}
\end{aligned}
$$

64. The ampere-hour is a unit of charge.

$$
(1.00 \mathrm{~A} \cdot \mathrm{~h})\left(\frac{1 \mathrm{C} / \mathrm{s}}{1 \mathrm{~A}}\right)\left(\frac{3600 \mathrm{~s}}{1 \mathrm{~h}}\right)=3600 \mathrm{C}
$$

65. Use Eqs. 25-3 and 25-7b.

$$
\begin{aligned}
& R=\rho \frac{\ell}{A}=\rho \frac{\ell}{\pi r^{2}}=\frac{4 \rho \ell}{\pi d^{2}} ; P=\frac{V^{2}}{R}=\frac{V^{2}}{\frac{4 \rho \ell}{\pi d^{2}}} \rightarrow \\
& \ell=\frac{V^{2} \pi d^{2}}{4 \rho P}=\frac{(1.5 \mathrm{~V})^{2} \pi\left(5.0 \times 10^{-4} \mathrm{~m}\right)^{2}}{4\left(1.68 \times 10^{-8} \Omega \cdot \mathrm{~m}\right)(15 \mathrm{~W})}=1.753 \mathrm{~m} \approx 1.8 \mathrm{~m}
\end{aligned}
$$

If the voltage increases by a factor of 6 without the resistance changing, the power will increase by a factor of 36 . The blanket would theoretically be able to deliver 540 W of power, which might make the material catch on fire or burn the occupant.
66. Use Eq. 25-6 to calculate the current.

$$
P=I V \rightarrow I=\frac{P}{V}=\frac{746 \mathrm{~W}}{120 \mathrm{~V}}=6.22 \mathrm{~A}
$$

67. 

From Eq. 25-2b, if $R=V / I$, then $G=I / V$

$$
G=\frac{I}{V}=\frac{0.48 \mathrm{~A}}{3.0 \mathrm{~V}}=0.16 \mathrm{~S}
$$

68. Use Eq. $25-7$ b to express the resistance in terms of the power, and Eq. $25-3$ to express the resistance in terms of the wire geometry.

$$
\begin{aligned}
& P=\frac{V^{2}}{R} \rightarrow R=\frac{V^{2}}{P} \quad R=\rho \frac{\ell}{A}=\rho \frac{\ell}{\pi r^{2}}=4 \rho \frac{\ell}{\pi d^{2}} \\
& 4 \rho \frac{\ell}{\pi d^{2}}=\frac{V^{2}}{P} \rightarrow d=\sqrt{\frac{4 \rho \ell P}{\pi V^{2}}}=\sqrt{\frac{4\left(9.71 \times 10^{-8} \Omega \cdot \mathrm{~m}\right)(3.5 \mathrm{~m})(1500 \mathrm{~W})}{\pi(110 \mathrm{~V})^{2}}}=2.3 \times 10^{-4} \mathrm{~m}
\end{aligned}
$$

69. (a) Calculate the total kWh used per day, and then multiply by the number of days and the cost per kWh .

$$
\begin{aligned}
(1.8 \mathrm{~kW}) & (2.0 \mathrm{~h} / \mathrm{d})+4(0.1 \mathrm{~kW})(6.0 \mathrm{~h} / \mathrm{d})+(3.0 \mathrm{~kW})(1.0 \mathrm{~h} / \mathrm{d})+(2.0 \mathrm{kWh} / \mathrm{d}) \\
& =11.0 \mathrm{kWh} / \mathrm{d} \\
\text { Cost }= & (11.0 \mathrm{kWh} / \mathrm{d})(30 \mathrm{~d})\left(\frac{\$ 0.105}{\mathrm{kWh}}\right)=\$ 34.65 \approx \$ 35 \text { per month }
\end{aligned}
$$

(b) The energy required by the household is $35 \%$ of the energy that needs to be supplied by the power plant.

$$
\text { Household Energy }=0.35(\text { coal mass })(\text { coal energy per mass }) \rightarrow
$$

$$
\begin{aligned}
\text { coal mass } & =\frac{\text { Household Energy }}{(0.35)(\text { coal energy per mass })}=\frac{(11.0 \mathrm{kWh} / \mathrm{d})(365 \mathrm{~d})\left(\frac{1000 \mathrm{~W}}{\mathrm{~kW}}\right)\left(\frac{3600 \mathrm{~s}}{1 \mathrm{~h}}\right)}{(0.35)\left(7500 \frac{\mathrm{kcal}}{\mathrm{~kg}}\right)\left(\frac{4186 \mathrm{~J}}{1 \mathrm{kcal}}\right)} \\
& =1315 \mathrm{~kg} \approx 1300 \mathrm{~kg} \text { of coal }
\end{aligned}
$$

70. To deliver 15 MW of power at 120 V requires a current of $I=\frac{P}{V}=\frac{15 \times 10^{6} \mathrm{~W}}{120 \mathrm{~V}}=1.25 \times 10^{5} \mathrm{~A}$.

Calculate the power dissipated in the resistors using the current and the resistance.

$$
\begin{aligned}
P & =I^{2} R=I^{2} \rho \frac{L}{A}=I^{2} \rho \frac{L}{\pi r^{2}}=4 I^{2} \rho \frac{L}{\pi d^{2}}=4\left(1.25 \times 10^{5} \mathrm{~A}\right)^{2}\left(1.68 \times 10^{-8} \Omega \cdot \mathrm{~m}\right) \frac{2(1.0 \mathrm{~m})}{\pi\left(5.0 \times 10^{-3} \mathrm{~m}\right)^{2}} \\
& =2.674 \times 10^{7} \mathrm{~W} \\
\text { Cost } & =(\text { Power })(\text { time })(\text { rate per } \mathrm{kWh})=\left(2.674 \times 10^{7} \mathrm{~W}\right)\left(\frac{1 \mathrm{~kW}}{1000 \mathrm{~W}}\right)(1 \mathrm{~h})\left(\frac{\$ 0.090}{\mathrm{kWh}}\right) \\
& =\$ 2407 \approx \$ 2,400 \text { per hour per meter }
\end{aligned}
$$

71. (a) Use Eq. 25-7b to relate the power to the voltage for a constant resistance.

$$
P=\frac{V^{2}}{R} \rightarrow \frac{P_{105}}{P_{117}}=\frac{(105 \mathrm{~V})^{2} / R}{(117 \mathrm{~V})^{2} / R}=\frac{(105 \mathrm{~V})^{2}}{(117 \mathrm{~V})^{2}}=0.805 \text { or a } 19.5 \% \text { decrease }
$$

(b) The lower power output means that the resistor is generating less heat, and so the resistor's temperature would be lower. The lower temperature results in a lower value of the resistance, which would increase the power output at the lower voltages. Thus the decrease would be smaller than the value given in the first part of the problem.
72. Assume that we have a meter of wire, carrying 35 A of current, and dissipating 1.5 W of heat. The power dissipated is $P_{R}=I^{2} R$, and the resistance is $R=\frac{\rho \ell}{A}$.

$$
\begin{aligned}
& P_{R}=I^{2} R=I^{2} \frac{\rho \ell}{A}=I^{2} \frac{\rho \ell}{\pi r^{2}}=I^{2} \frac{4 \rho \ell}{\pi d^{2}} \rightarrow \\
& d=\sqrt{I^{2} \frac{4 \rho \ell}{P_{R} \pi}}=2 I \sqrt{\frac{\rho \ell}{P_{R} \pi}}=2(35 \mathrm{~A}) \sqrt{\frac{\left(1.68 \times 10^{-8} \Omega \cdot \mathrm{~m}\right)(1.0 \mathrm{~m})}{(1.5 \mathrm{~W}) \pi}}=4.2 \times 10^{-3} \mathrm{~m}
\end{aligned}
$$

73. (a) The resistance at the operating temperature can be calculated directly from Eq. 25-7.

$$
P=\frac{V^{2}}{R} \rightarrow R=\frac{V^{2}}{P}=\frac{(120 \mathrm{~V})^{2}}{75 \mathrm{~W}}=190 \Omega
$$

(b) The resistance at room temperature is found by converting Eq. $25-5$ into an equation for resistances and solving for $R_{0}$.

$$
\begin{aligned}
& R=R_{0}\left[1+\alpha\left(T-T_{0}\right)\right] \\
& R_{0}=\frac{R}{\left[1+\alpha\left(T-T_{0}\right)\right]}=\frac{192 \Omega}{\left[1+\left(0.0045 \mathrm{~K}^{-1}\right)(3000 \mathrm{~K}-293 \mathrm{~K})\right]}=15 \Omega
\end{aligned}
$$

74. (a) The angular frequency is $\omega=210 \mathrm{rad} / \mathrm{s}$.

$$
f=\frac{\omega}{2 \pi}=\frac{210 \mathrm{rad} / \mathrm{s}}{2 \pi}=33.42 \mathrm{~Hz} \approx 33 \mathrm{~Hz}
$$

(b) The maximum current is 1.80 A .

$$
I_{\mathrm{rms}}=\frac{I_{\max }}{\sqrt{2}}=\frac{1.80 \mathrm{~A}}{\sqrt{2}}=1.27 \mathrm{~A}
$$

(c) For a resistor, $V=I R$.

$$
V=I R=(1.80 \mathrm{~A})(\sin 210 t)(24.0 \Omega)=(43.2 \sin 210 t) \mathrm{V}
$$

75. (a) The power delivered to the interior is $65 \%$ of the power drawn from the source.

$$
P_{\text {interior }}=0.65 P_{\text {source }} \rightarrow P_{\text {source }}=\frac{P_{\text {interior }}}{0.65}=\frac{950 \mathrm{~W}}{0.65}=1462 \mathrm{~W} \approx 1500 \mathrm{~W}
$$

(b) The current drawn is current from the source, and so the source power is used to calculate the current.

$$
P_{\text {source }}=I V_{\text {source }} \rightarrow I=\frac{P_{\text {source }}}{V_{\text {source }}}=\frac{1462 \mathrm{~W}}{120 \mathrm{~V}}=12.18 \mathrm{~A} \approx 12 \mathrm{~A}
$$

76. The volume of wire is unchanged by the stretching. The volume is equal to the length of the wire times its cross-sectional area, and since the length was increased by a factor of 1.20 , the area was decreased by a factor of 1.20. Use Eq. 25-3.

$$
R_{0}=\rho \frac{\ell_{0}}{A_{0}} \quad \ell=1.20 \ell_{0} \quad A=\frac{A_{0}}{1.20} \quad R=\rho \frac{\ell}{A}=\rho \frac{1.20 \ell_{0}}{\frac{A_{0}}{1.20}}=(1.20)^{2} \rho \frac{\ell_{0}}{A_{0}}=1.44 R_{0}=1.44 \Omega
$$

77. The long, thick conductor is labeled as conductor number 1 , and the short, thin conductor is labeled as number 2 . The power transformed by a resistor is given by Eq. $25-7 \mathrm{~b}, P=V^{2} / R$, and both have the same voltage applied.

$$
\begin{aligned}
& R_{1}=\rho \frac{\ell_{1}}{A_{1}} \quad R_{2}=\rho \frac{\ell_{2}}{A_{2}} \quad \ell_{1}=2 \ell_{2} \quad A_{1}=4 A_{2}\left(\text { diameter }_{1}=2 \text { diameter }_{2}\right) \\
& \frac{P_{1}}{P_{2}}=\frac{V_{1}^{2} / R_{1}}{V_{2}^{2} / R_{2}}=\frac{R_{2}}{R_{1}}=\frac{\rho \ell_{2} / A_{2}}{\rho \ell_{1} / A_{1}}=\frac{\ell_{2}}{\ell_{1}} \frac{A_{1}}{A_{2}}=\frac{1}{2} \times 4=2 \quad P_{1}: P_{2}=2: 1
\end{aligned}
$$

78. The heater must heat $108 \mathrm{~m}^{3}$ of air per hour from $5^{\circ} \mathrm{C}$ to $20^{\circ} \mathrm{C}$, and also replace the heat being lost at a rate of $850 \mathrm{kcal} / \mathrm{h}$. Use Eq. 19-2 to calculate the energy needed to heat the air. The density of air is found in Table 13-1.

$$
\begin{aligned}
& Q=m c \Delta T \rightarrow \frac{Q}{t}=\frac{m}{t} c \Delta T=\left(108 \frac{\mathrm{~m}^{3}}{\mathrm{~h}}\right)\left(1.29 \frac{\mathrm{~kg}}{\mathrm{~m}^{3}}\right)\left(0.17 \frac{\mathrm{kcal}}{\mathrm{~kg} \cdot \mathrm{C}^{\circ}}\right)\left(15 \mathrm{C}^{\circ}\right)=355 \frac{\mathrm{kcal}}{\mathrm{~h}} \\
& \text { Power required }=355 \frac{\mathrm{kcal}}{\mathrm{~h}}+850 \frac{\mathrm{kcal}}{\mathrm{~h}}=1205 \frac{\mathrm{kcal}}{\mathrm{~h}}\left(\frac{4186 \mathrm{~J}}{\mathrm{kcal}}\right)\left(\frac{1 \mathrm{~h}}{3600 \mathrm{~s}}\right)=1401 \mathrm{~W} \approx 1400 \mathrm{~W}
\end{aligned}
$$

79. (a) Use Eq. 25-7b.

$$
P=\frac{V^{2}}{R} \rightarrow R=\frac{V^{2}}{P}=\frac{(240 \mathrm{~V})^{2}}{2800 \mathrm{~W}}=20.57 \Omega \approx 21 \Omega
$$

(b) Only $75 \%$ of the heat from the oven is used to heat the water. Use Eq. 19-2.

$$
0.75\left(P_{\text {oven }}\right) t=\text { Heat absorbed by water }=m c \Delta T \rightarrow
$$

$$
t=\frac{m c \Delta T}{0.75\left(P_{\text {oven }}\right)}=\frac{(0.120 \mathrm{~L})\left(\frac{1 \mathrm{~kg}}{1 \mathrm{~L}}\right)\left(4186 \mathrm{~J} / \mathrm{kg} \cdot \mathrm{C}^{\circ}\right)\left(85 \mathrm{C}^{\circ}\right)}{0.75(2800 \mathrm{~W})}=20.33 \mathrm{~s} \approx 20 \mathrm{~s}(2 \mathrm{sig} . \text { fig. })
$$

(c) $\frac{11 \text { cents }}{\mathrm{kWh}}(2.8 \mathrm{~kW})(20.33 \mathrm{~s}) \frac{1 \mathrm{~h}}{3600 \mathrm{~s}}=0.17$ cents
80. (a) The horsepower required is the power dissipated by the frictional force, since we are neglecting the energy used for acceleration.

$$
P=F v=(240 \mathrm{~N})(45 \mathrm{~km} / \mathrm{hr})\left(\frac{1 \mathrm{~m} / \mathrm{s}}{3.6 \mathrm{~km} / \mathrm{hr}}\right)=3000 \mathrm{~W}\left(\frac{1 \mathrm{hp}}{746 \mathrm{~W}}\right)=4.0 \mathrm{hp}
$$

(b) The charge available by each battery is $Q=95 \mathrm{~A} \cdot \mathrm{~h}=95 \mathrm{C} / \mathrm{s} \cdot 3600 \mathrm{~s}=3.42 \times 10^{5} \mathrm{C}$, and so the total charge available is 24 times that. The potential energy of that charge is the charge times the voltage. That energy must be delivered (batteries discharged) in a certain amount of time to produce the 3000 W necessary. The speed of the car times the discharge time is the range of the car between recharges.

$$
\begin{aligned}
& P=\frac{U}{t}=\frac{Q V}{t} \rightarrow t=\frac{Q V}{P}=\frac{d}{v} \rightarrow \\
& d=v t=v \frac{Q V}{P}=v \frac{Q V}{F v}=\frac{Q V}{F}=\frac{24\left(3.42 \times 10^{5} \mathrm{C}\right)(12 \mathrm{~V})}{240 \mathrm{~N}}=410 \mathrm{~km}
\end{aligned}
$$

81. The mass of the wire is the density of copper times the volume of the wire, and the resistance of the wire is given by Eq. 25-3. We represent the mass density by $\rho_{\mathrm{m}}$ and the resistivity by $\rho$.

$$
\begin{aligned}
& R=\rho \frac{\ell}{A} \rightarrow A=\frac{\rho \ell}{R} \quad m=\rho_{\mathrm{m}} \ell A=\rho_{\mathrm{m}} \ell \frac{\rho \ell}{R} \rightarrow \\
& \ell=\sqrt{\frac{m R}{\rho_{\mathrm{m}} \rho}}=\sqrt{\frac{(0.0155 \mathrm{~kg})(12.5 \Omega)}{\left(8.9 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)\left(1.68 \times 10^{-8} \Omega \cdot \mathrm{~m}\right)}}=35.997 \mathrm{~m} \approx 36.0 \mathrm{~m} \\
& A=\frac{\rho \ell}{R}=\pi\left(\frac{1}{2} d\right)^{2} \rightarrow d=\sqrt{\frac{4 \rho \ell}{\pi R}}=\sqrt{\frac{4\left(1.68 \times 10^{-8} \Omega \cdot \mathrm{~m}\right)(35.997 \mathrm{~m})}{\pi(12.5 \Omega)}}=2.48 \times 10^{-4} \mathrm{~m}
\end{aligned}
$$

82. The resistance can be calculated from the power and voltage, and then the diameter of the wire can be calculated from the resistance.

$$
\begin{aligned}
& P=\frac{V^{2}}{R} \rightarrow R=\frac{V^{2}}{P} \quad R=\frac{\rho L}{A}=\frac{\rho L}{\pi\left(\frac{1}{2} d\right)^{2}} \rightarrow \frac{V^{2}}{P}=\frac{\rho L}{\pi\left(\frac{1}{2} d\right)^{2}} \rightarrow \\
& d=\sqrt{\frac{4 \rho L P}{\pi V^{2}}}=\sqrt{\frac{4\left(100 \times 10^{-8} \Omega \cdot \mathrm{~m}\right)(3.8 \mathrm{~m})(95 \mathrm{~W})}{\pi(120 \mathrm{~V})^{2}}}=1.787 \times 10^{-4} \mathrm{~m} \approx 1.8 \times 10^{-4} \mathrm{~m}
\end{aligned}
$$

83. Use Eq. 25-7b.
(a) $P=\frac{V^{2}}{R}=\frac{(120 \mathrm{~V})^{2}}{12 \Omega}=1200 \mathrm{~W}$
(b) $P=\frac{V^{2}}{R}=\frac{(120 \mathrm{~V})^{2}}{140 \Omega}=103 \mathrm{~W} \approx 100 \mathrm{~W}$ (2 sig. fig.)
84. Use Eq. $25-7 \mathrm{~b}$ for the power in each case, assuming the resistance is constant.

$$
\frac{P_{13.8 \mathrm{~V}}}{P_{12.0 \mathrm{~V}}}=\frac{\left(V^{2} / R\right)_{13.8 \mathrm{~V}}}{\left(V^{2} / R\right)_{12.0 \mathrm{~V}}}=\frac{13.8^{2}}{12.0^{2}}=1.3225=32 \% \text { increase }
$$

85. Model the protons as moving in a continuous beam of cross-sectional area $A$. Then by Eq. 25-13, $I=n e A v_{\mathrm{d}}$, where we only consider the absolute value of the current. The variable $n$ is the number of protons per unit volume, so $n=\frac{N}{A \ell}$, where $N$ is the number of protons in the beam and $\ell$ is the circumference of the ring. The "drift" velocity in this case is the speed of light.

$$
\begin{aligned}
& I=n e A v_{\mathrm{d}}=\frac{N}{A \ell} e A v_{\mathrm{d}}=\frac{N}{\ell} e v_{\mathrm{d}} \rightarrow \\
& N=\frac{I \ell}{e v_{\mathrm{d}}}=\frac{\left(11 \times 10^{-3}\right)(6300 \mathrm{~m})}{\left(1.60 \times 10^{-19} \mathrm{C}\right)\left(3.00 \times 10^{8} \mathrm{~m} / \mathrm{s}\right)}=1.4 \times 10^{12} \mathrm{protons}
\end{aligned}
$$

86. (a he current can be found from Eq. 25-6.

$$
I=P / V \quad I_{A}=P_{A} / V_{A}=40 \mathrm{~W} / 120 \mathrm{~V}=0.33 \mathrm{~A} \quad I_{B}=P_{B} / V_{B}=40 \mathrm{~W} / 12 \mathrm{~V}=3.3 \mathrm{~A}
$$

(b) The resistance can be found from Eq. 25-7b.

$$
R=\frac{V^{2}}{P} \quad R_{A}=\frac{V_{A}^{2}}{P_{A}}=\frac{(120 \mathrm{~V})^{2}}{40 \mathrm{~W}}=360 \Omega \quad R_{B}=\frac{V_{B}^{2}}{P_{B}}=\frac{(12 \mathrm{~V})^{2}}{40 \mathrm{~W}}=3.6 \Omega
$$

(c) The charge is the current times the time.

$$
\begin{aligned}
Q=I t & Q_{A}=I_{A} t=(0.33 \mathrm{~A})(3600 \mathrm{~s})=1200 \mathrm{C} \\
& Q_{B}=I_{B} t=(3.3 \mathrm{~A})(3600 \mathrm{~s})=12,000 \mathrm{C}
\end{aligned}
$$

(d) The energy is the power times the time, and the power is the same for both bulbs.

$$
E=P t \quad E_{A}=E_{B}=(40 \mathrm{~W})(3600 \mathrm{~s})=1.4 \times 10^{5} \mathrm{~J}
$$

(e) Bulb B requires a larger current, and so should have larger diameter connecting wires to avoid overheating the connecting wires.
87. (a) The power is given by $P=I V$.

$$
P=I V=(14 \mathrm{~A})(220 \mathrm{~V})=3080 \mathrm{~W} \approx 3100 \mathrm{~W}
$$

(b) The power dissipated is given by $P_{R}=I^{2} R$, and the resistance is $R=\frac{\rho \ell}{A}$.

$$
\begin{aligned}
P_{R} & =I^{2} R=I^{2} \frac{\rho \ell}{A}=I^{2} \frac{\rho \ell}{\pi r^{2}}=I^{2} \frac{4 \rho \ell}{\pi d^{2}}=(14 \mathrm{~A})^{2} \frac{4\left(1.68 \times 10^{-8} \Omega \cdot \mathrm{~m}\right)(15 \mathrm{~m})}{\pi\left(1.628 \times 10^{-3} \mathrm{~m}\right)^{2}}=23.73 \mathrm{~W} \\
& \approx 24 \mathrm{~W}
\end{aligned}
$$

(c) $P_{R}=I^{2} \frac{4 \rho L}{\pi d^{2}}=(14 \mathrm{~A})^{2} \frac{4\left(1.68 \times 10^{-8} \Omega \cdot \mathrm{~m}\right)(15 \mathrm{~m})}{\pi\left(2.053 \times 10^{-3} \mathrm{~m}\right)^{2}}=14.92 \mathrm{~W} \approx 15 \mathrm{~W}$
(d) The savings is due to the power difference.

$$
\begin{aligned}
\text { Savings } & =(23.73 \mathrm{~W}-14.92 \mathrm{~W})\left(\frac{1 \mathrm{~kW}}{1000 \mathrm{~W}}\right)(30 \mathrm{~d})\left(\frac{12 \mathrm{~h}}{1 \mathrm{~d}}\right)\left(\frac{\$ 0.12}{1 \mathrm{kWh}}\right) \\
& =\$ 0.3806 / \text { month } \approx 38 \text { cents per month }
\end{aligned}
$$

88. The wasted power is due to losses in the wire. The current in the wire can be found by $I=P / V$.
(a) $P_{R}=I^{2} R=\frac{P^{2}}{V^{2}} R=\frac{P^{2}}{V^{2}} \frac{\rho L}{A}=\frac{P^{2}}{V^{2}} \frac{\rho L}{\pi r^{2}}=\frac{P^{2}}{V^{2}} \frac{4 \rho L}{\pi d^{2}}$

$$
=\frac{(1750 \mathrm{~W})^{2}}{(120 \mathrm{~V})^{2}} \frac{4\left(1.68 \times 10^{-8} \Omega \cdot \mathrm{~m}\right)(25.0 \mathrm{~m})}{\pi\left(2.59 \times 10^{-3} \mathrm{~m}\right)^{2}}=16.954 \mathrm{~W} \approx 17.0 \mathrm{~W}
$$

(b) $P_{R}=\frac{P^{2}}{V^{2}} \frac{4 \rho L}{\pi d^{2}}=\frac{(1750 \mathrm{~W})^{2}}{(120 \mathrm{~V})^{2}} \frac{4\left(1.68 \times 10^{-8} \Omega \cdot \mathrm{~m}\right)(25.0 \mathrm{~m})}{\pi\left(4.12 \times 10^{-3} \mathrm{~m}\right)^{2}}=6.70 \mathrm{~W}$
89. (a) The D-cell provides 25 mA at 1.5 V for 820 h , at a cost of $\$ 1.70$.

$$
\text { Energy }=P t=V I t=(1.5 \mathrm{~V})(0.025 \mathrm{~A})(820 \mathrm{~h})\left(\frac{1 \mathrm{~kW}}{1000 \mathrm{~W}}\right)=0.03075 \mathrm{kWh}
$$

$$
\operatorname{Cost} / \mathrm{kWh}=\frac{\$ 1.70}{0.03075 \mathrm{kWh}}=\$ 55.28 / \mathrm{kWh} \approx \$ 55 / \mathrm{kWh}
$$

(b) The AA-cell provides 25 mA at 1.5 V for 120 h , at a cost of $\$ 1.25$.

$$
\begin{aligned}
& \text { Energy }=P t=V I t=(1.5 \mathrm{~V})(0.025 \mathrm{~A})(120 \mathrm{~h})\left(\frac{1 \mathrm{~kW}}{1000 \mathrm{~W}}\right)=0.0045 \mathrm{kWh} \\
& \text { Cost } / \mathrm{kWh}=\frac{\$ 1.25}{0.0045 \mathrm{kWh}}=\$ 277.78 / \mathrm{kWh} \approx \$ 280 / \mathrm{kWh}
\end{aligned}
$$

The D-cell is $\frac{\$ 55.28 / \mathrm{kWh}}{\$ 0.10 / \mathrm{kWh}} \approx 550 \times$ as costly. The AA-cell is $\frac{\$ 277.78 / \mathrm{kWh}}{\$ 0.10 / \mathrm{kWh}} \approx 2800 \times$ as costly .
90. The electrons are assumed to be moving with simple harmonic motion. During one cycle, an object in simple harmonic motion will move a distance equal to the amplitude from its equilibrium point. From Eq. 14-9a, we know that $v_{\max }=A \omega$, where $\omega$ is the angular frequency of oscillation. From Eq. 25-13 in absolute value, we see that $I_{\max }=n e A v_{\max }$. Finally, the maximum current can be related to the power by Eqs. $25-9$ and 25-10. The charge carrier density, $n$, is calculated in Example 25-14.

$$
\begin{aligned}
\bar{P} & =I_{\mathrm{rms}} V_{\mathrm{rms}}=\frac{1}{\sqrt{2}} I_{\mathrm{max}} V_{\mathrm{rms}} \\
A & =\frac{v_{\max }}{\omega}=\frac{I_{\max }}{\omega n e A}=\frac{\sqrt{2} \bar{P}}{\omega n e \frac{\pi d^{2}}{4} V_{\mathrm{rms}}} \\
& =\frac{4 \sqrt{2}(550 \mathrm{~W})}{2 \pi(60 \mathrm{~Hz})\left(8.4 \times 10^{28} \mathrm{~m}^{-3}\right)\left(1.60 \times 10^{-19} \mathrm{C}\right) \pi\left(1.7 \times 10^{-3} \mathrm{~m}\right)^{2}(120 \mathrm{~V})}=5.6 \times 10^{-7} \mathrm{~m}
\end{aligned}
$$

The electron will move this distance in both directions from its equilibrium point.
91. Eq. 25-3 can be used. The area to be used is the cross-sectional area of the pipe.

$$
R=\frac{\rho \ell}{A}=\frac{\rho \ell}{\pi\left(r_{\text {ouside }}^{2}-r_{\text {mide }}^{2}\right)}=\frac{\left(1.68 \times 10^{-8} \Omega \cdot \mathrm{~m}\right)(10.0 \mathrm{~m})}{\pi\left[\left(2.50 \times 10^{-2} \mathrm{~m}\right)^{2}-\left(1.50 \times 10^{-2} \mathrm{~m}\right)^{2}\right]}=1.34 \times 10^{-4} \Omega
$$

92. We assume that all of the current that enters at $a$ leaves at $b$, so that the current is the same at each end. The current density is given by Eq. 25-11.

$$
\begin{aligned}
& j_{\mathrm{a}}=\frac{I}{A_{\mathrm{a}}}=\frac{I}{\pi\left(\frac{1}{2} a\right)^{2}}=\frac{4 I}{\pi a^{2}}=\frac{4(2.0 \mathrm{~A})}{\pi\left(2.5 \times 10^{-3} \mathrm{~m}\right)^{2}}=4.1 \times 10^{5} \mathrm{~A} / \mathrm{m}^{2} \\
& j_{\mathrm{b}}=\frac{I}{A_{\mathrm{b}}}=\frac{I}{\pi\left(\frac{1}{2} b\right)^{2}}=\frac{4 I}{\pi b^{2}}=\frac{4(2.0 \mathrm{~A})}{\pi\left(4.0 \times 10^{-3} \mathrm{~m}\right)^{2}}=1.6 \times 10^{5} \mathrm{~A} / \mathrm{m}^{2}
\end{aligned}
$$

93. Using Eq. 25-3, we find the infinitesimal resistance first of a thin vertical slice at a horizontal distance $x$ from the center of the left side towards the center of the right side. Let the thickness of that slice be $d x$. That thickness corresponds to the variable $\ell$ in Eq. 25-3. The diameter of this slice is
$a+\frac{x}{\ell}(b-a)$. Then integrate over all the slices to
 find the total resistance.

$$
\begin{aligned}
& R=\rho \frac{\ell}{A} \rightarrow d R=\rho \frac{d x}{\pi \frac{1}{4}\left(a+\frac{x}{\ell}(b-a)\right)^{2}} \rightarrow \\
& R=\int d R=\int_{0}^{\ell} \rho \frac{d x}{\pi \frac{1}{4}\left(a+\frac{x}{\ell}(b-a)\right)^{2}}=-\frac{4 \rho}{\pi} \frac{\ell}{b-a} \frac{1}{\left(a+\frac{x}{\ell}(b-a)\right)}=\frac{4 \rho}{\pi} \frac{\ell}{a b}
\end{aligned}
$$

94. The resistance of the filament when the flashlight is on is $R=\frac{V}{I}=\frac{3.2 \mathrm{~V}}{0.20 \mathrm{~A}}=16 \Omega$. That can be used with a combination of Eqs. 25-3 and 25-5 to find the temperature.

$$
\begin{aligned}
& R=R_{0}\left[1+\alpha\left(T-T_{0}\right)\right] \rightarrow \\
& T=T_{0}+\frac{1}{\alpha}\left(\frac{R}{R_{0}}-1\right)=20^{\circ} \mathrm{C}+\frac{1}{0.0045\left(\mathrm{C}^{\circ}\right)^{-1}}\left(\frac{16 \Omega}{1.5 \Omega}-1\right)=2168^{\circ} \mathrm{C} \approx 2200^{\circ} \mathrm{C}
\end{aligned}
$$

95. When the tank is empty, the entire length of the wire is in a non-superconducting state, and so has a non-zero resistivity, which we call $\rho$. Then the resistance of the wire when the tank is empty is given by $R_{0}=\rho \frac{\ell}{A}=\frac{V_{0}}{I}$. When a length $x$ of the wire is superconducting, that portion of the wire has 0 resistance. Then the resistance of the wire is only due to the length $\ell-x$, and so $R=\rho \frac{\ell-x}{A}=\rho \frac{\ell}{A} \frac{\ell-x}{\ell}=R_{0} \frac{\ell-x}{\ell}$. This resistance, combined with the constant current, gives $V=I R$.

$$
V=I R=\left(\frac{V_{0}}{R_{0}}\right) R_{0} \frac{\ell-x}{\ell}=V_{0}\left(1-\frac{x}{\ell}\right)=V_{0}(1-f) \rightarrow f=1-\frac{V}{V_{0}}
$$

Thus a measurement of the voltage can give the fraction of the tank that is filled with liquid helium.
96. We plot resistance vs. temperature. The graph is shown as follows, with no curve fitted to it. It is apparent that a linear fit will not be a good fit to this data. Both quadratic and exponential equations fit the data well, according to the R -squared coefficient as given by Excel. The equations and the predictions are given below.

$$
\begin{aligned}
& R_{\text {exp }}=\left(30.1 \times 10^{4} e^{-0.0422 T}\right) \Omega \\
& R_{\text {quad }}=\left[\left(7.39 \times 10^{4}\right) T^{2}-8200 T+25.9 \times 10^{4}\right] \Omega
\end{aligned}
$$



Solving these expressions for $R=57,641 \Omega$ (using the spreadsheet) gives $T_{\text {exp }}=37.402^{\circ} \mathrm{C}$ and $T_{\text {quad }}=37.021^{\circ} \mathrm{C}$. So the temperature is probably in the range between those two values:
$37.021^{\circ} \mathrm{C}<T<37.402^{\circ} \mathrm{C}$. The average of those two values is $T=37.21^{\circ} \mathrm{C}$. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH25.XLS," on tab "Problem 25.96."

As an extra comment, how might you choose between the exponential and quadratic fits? While they both give almost identical predictions for this intermediate temperature, they differ significantly at temperatures near $0^{\circ} \mathrm{C}$. The exponential fit would give a resistance of about $301,000 \Omega$ at $0^{\circ} \mathrm{C}$, while the quadratic fit would give a resistance of about $259,000 \Omega$ at $0^{\circ} \mathrm{C}$. So a measurement of resistance near $0^{\circ} \mathrm{C}$ might be very useful.

## CHAPTER 26: DC Circuits

## Responses to Questions

1. Even though the bird's feet are at high potential with respect to the ground, there is very little potential difference between them, because they are close together on the wire. The resistance of the bird is much greater than the resistance of the wire between the bird's feet. These two resistances are in parallel, so very little current will pass through the bird as it perches on the wire. When you put a metal ladder up against a power line, you provide a direct connection between the high potential line and ground. The ladder will have a large potential difference between its top and bottom. A person standing on the ladder will also have a large potential difference between his or her hands and feet. Even if the person's resistance is large, the potential difference will be great enough to produce a current through the person's body large enough to cause substantial damage or death.
2. Series: The main disadvantage of Christmas tree lights connected in series is that when one bulb burns out, a gap is created in the circuit and none of the bulbs remains lit. Finding the burned-out bulb requires replacing each individual bulb one at a time until the string of bulbs comes back on. As an advantage, the bulbs are slightly easier to wire in series.

Parallel: The main advantage of connecting the bulbs in parallel is that one burned-out bulb does not affect the rest of the strand, and is easy to identify and replace. As a disadvantage, wiring the bulbs in parallel is slightly more difficult.
3. Yes. You can put 20 of the $6-\mathrm{V}$ lights in series, or you can put several of the $6-\mathrm{V}$ lights in series with a large resistance.
4. When the bulbs are connected in series, they have the same current through them. $R_{2}$, the bulb with the greater resistance, will be brighter in this case, since $P=I R$. When the bulbs are connected in parallel, they will have the same voltage across them. In this case, $R_{1}$, the bulb with the lower resistance, will have a larger current flowing through it and will be brighter: $P=V^{2} / R$.
5. Double outlets are connected in parallel, since each has 120 V across its terminals and they can be used independently.
6. Arrange the two batteries in series with each other and the two bulbs in parallel across the combined voltage of the batteries. This configuration maximizes the voltage gain and minimizes the equivalent resistance, yielding the maximum power.
7. The battery has to supply less power when the two resistors are connected in series than it has to supply when only one resistor is connected. $P=I V=\frac{V^{2}}{R}$, so if $V$ is constant and $R$ increases, the power decreases.
8. The overall resistance decreases and more current is drawn from the source. A bulb rated at $60-\mathrm{W}$ and $120-\mathrm{V}$ has a resistance of $240 \Omega$. A bulb rated at $100-\mathrm{W}$ and $120-\mathrm{V}$ has a resistance of $144 \Omega$. When only the $60-\mathrm{W}$ bulb is on, the total resistance is $240 \Omega$. When both bulbs are lit, the total resistance is the combination of the two resistances in parallel, which is only $90 \Omega$.
9. No. The sign of the battery's emf does not depend on the direction of the current through the battery. Yes, the terminal voltage of the battery does depend on the direction of the current through the
battery. Note that the sign of the battery's emf in the loop equation does depend on the direction the loop is traversed (+ in the direction of the battery's potential, - in the opposite direction), and the terminal voltage sign and magnitude depend on whether the loop is traversed with or against the current.
10. When resistors are connected in series, the equivalent resistance is the sum of the individual resistances, $R_{\text {eq,series }}=R_{1}+R_{2}+\ldots$. The current has to go through each additional resistance if the resistors are in series and therefore the equivalent resistance is greater than any individual resistance. In contrast, when capacitors are in parallel the equivalent capacitance is equal to the sum of the individual capacitors, $C_{\text {eq,parallel }}=C_{1}+C_{2}+\ldots$. Charge drawn from the battery can go down any one of the different branches and land on any one of the capacitors, so the overall capacitance is greater than that of each individual capacitor.

When resistors are connected in parallel, the current from the battery or other source divides into the different branches and so the equivalent resistance is less than any individual resistor in the circuit. The corresponding expression is $1 / R_{\text {eq,parallel }}=1 / R_{1}+1 / R_{2}+\ldots$. The formula for the equivalent capacitance of capacitors in series follows this same form, $1 / C_{\text {eq,series }}=1 / C_{1}+1 / C_{2}+\ldots$. When capacitors are in series, the overall capacitance is less than the capacitance of any individual capacitor. Charge leaving the first capacitor lands on the second rather than going straight to the battery.
Compare the expressions defining resistance ( $R=V / I$ ) and capacitance ( $C=Q / V$ ). Resistance is proportional to voltage, whereas capacitance is inversely proportional to voltage.
11. When batteries are connected in series, their emfs add together, producing a larger potential. The batteries do not need to be identical in this case. When batteries are connected in parallel, the currents they can generate add together, producing a larger current over a longer time period. Batteries in this case need to be nearly identical, or the battery with the larger emf will end up charging the battery with the smaller emf.
12. Yes. When a battery is being charged, current is forced through it "backwards" and then $V_{\text {terminal }}=$ emf $+I r$, so $V_{\text {terminal }}>\mathrm{emf}$.
13. Put the battery in a circuit in series with a very large resistor and measure the terminal voltage. With a large resistance, the current in the circuit will be small, and the potential across the battery will be mainly due to the emf. Next put the battery in parallel with the large resistor (or in series with a small resistor) and measure the terminal voltage and the current in the circuit. You will have enough information to use the equation $V_{\text {terminal }}=\mathrm{emf}-I r$ to determine the internal resistance $r$.
14. No. As current passes through the resistor in the $R C$ circuit, energy is dissipated in the resistor. Therefore, the total energy supplied by the battery during the charging is the combination of the energy dissipated in the resistor and the energy stored in the capacitor.
15. (a) Stays the same;
(b) Increases;
(c) Decreases;
(d) Increases;
(e) Increases;
(f) Decreases;
(g) Decreases;
(h) Increases;
(i) Remains the same.
16. The capacitance of a parallel plate capacitor is inversely proportional to the distance between the plates: $\quad\left(C=\varepsilon_{0} A / d\right)$. As the diaphragm moves in and out, the distance between the plates changes and therefore the capacitance changes with the same frequency. This changes the amount of charge that can be stored on the capacitor, creating a current as the capacitor charges or discharges. The current oscillates with the same frequency as the diaphragm, which is the same frequency as the incident sound wave, and produces an oscillating $V_{\text {output }}$.
17. See the adjacent figure. If both switches are connected to the same wire, the circuit is complete and the light is on. If they are connected to opposite wires, the light will remain off.

18. In an analog ammeter, the internal resistor, or shunt resistor, has a small value and is in parallel with the galvanometer, so that the overall resistance of the ammeter is very small. In an analog voltmeter, the internal resistor has a large value and is in series with the galvanometer, and the overall resistance of the voltmeter is very large.
19. If you use an ammeter where you need to use a voltmeter, you will short the branch of the circuit. Too much current will pass through the ammeter and you will either blow the fuse on the ammeter or burn out its coil.
20. An ammeter is placed in series with a given circuit element in order to measure the current through that element. If the ammeter did not have very low (ideally, zero) resistance, its presence in the circuit would change the current it is attempting to measure by adding more resistance in series. An ideal ammeter has zero resistance and thus does not change the current it is measuring.

A voltmeter is placed in parallel with a circuit element in order to measure the voltage difference across that element. If the voltmeter does not have a very high resistance, than its presence in parallel will lower the overall resistance and affect the circuit. An ideal voltmeter has infinite resistance so that when placed in parallel with circuit elements it will not change the value of the voltage it is reading.
21. When a voltmeter is connected across a resistor, the voltmeter is in parallel with the resistor. Even if the resistance of the voltmeter is large, the parallel combination of the resistor and the voltmeter will be slightly smaller than the resistor alone. If $R_{\text {eq }}$ decreases, then the overall current will increase, so that the potential drop across the rest of the circuit will increase. Thus the potential drop across the parallel combination will be less than the original voltage drop across the resistor.
22. A voltmeter has a very high resistance. When it is connected to the battery very little current will flow. A small current results in a small voltage drop due to the internal resistance of the battery, and the emf and terminal voltage (measured by the voltmeter) will be very close to the same value. However, when the battery is connected to the lower-resistance flashlight bulb, the current will be higher and the voltage drop due to the internal resistance of the battery will also be higher. As a battery is used, its internal resistance increases. Therefore, the terminal voltage will be significantly lower than the emf: $V_{\text {terminal }}=\mathrm{emf}-I r$. A lower terminal voltage will result in a dimmer bulb, and usually indicates a "used-up" battery.
23. (a) With the batteries in series, a greater voltage is delivered to the lamp, and the lamp will burn brighter.
(b) With the batteries in parallel, the voltage across the lamp is the same as for either battery alone. Each battery supplies only half of the current going through the lamp, so the batteries will last twice as long.

## Solutions to Problems

1. See Figure 26-2 for a circuit diagram for this problem. Using the same analysis as in Example 26-1, the current in the circuit is $I=\frac{\mathscr{E}}{R+r}$. Use Eq. 26-1 to calculate the terminal voltage.
(a) $V_{\mathrm{ab}}=\mathscr{E}-I r=\mathscr{E}-\left(\frac{\mathscr{E}}{R+r}\right) r=\frac{\mathscr{E}(R+r)-\mathscr{E} r}{R+r}=\mathscr{E} \frac{R}{R+r}=(6.00 \mathrm{~V}) \frac{81.0 \Omega}{(81.0+0.900) \Omega}=5.93 \mathrm{~V}$
(b) $V_{\text {ab }}=\mathscr{E} \frac{R}{R+r}=(6.00 \mathrm{~V}) \frac{810 \Omega}{(810+0.900) \Omega}=5.99 \mathrm{~V}$
2. See the circuit diagram below. The current in the circuit is $I$. The voltage $V_{\text {ab }}$ is given by Ohm's law to be $V_{\mathrm{ab}}=I R$. That same voltage is the terminal voltage of the series EMF.


$$
\begin{aligned}
& V_{\mathrm{ab}}=(\mathscr{E}-I r)+(\mathscr{E}-I r)+(\mathscr{E}-I r)+(\mathscr{E}-I r)=4(\mathscr{E}-I r) \text { and } V_{\mathrm{ab}}=I R \\
& 4(\mathscr{E}-I r)=I R \rightarrow r=\frac{\mathscr{E}-\frac{1}{4} I R}{I}=\frac{(1.5 \mathrm{~V})-\frac{1}{4}(0.45 \mathrm{~A})(12 \Omega)}{0.45 \mathrm{~A}}=0.333 \Omega \approx 0.3 \Omega
\end{aligned}
$$

3. We take the low-resistance ammeter to have no resistance. The circuit is shown. The terminal voltage will be 0 volts.

$$
V_{\mathrm{ab}}=\mathscr{E}-I r=0 \rightarrow r=\frac{\mathscr{E}}{I}=\frac{1.5 \mathrm{~V}}{25 \mathrm{~A}}=0.060 \Omega
$$


4. See Figure 26-2 for a circuit diagram for this problem. Use Eq. 26-1.

$$
\begin{aligned}
& V_{\mathrm{ab}}=\mathscr{E}-I r \rightarrow r=\frac{\mathscr{E}-V_{\mathrm{ab}}}{I}=\frac{12.0 \mathrm{~V}-8.4 \mathrm{~V}}{95 \mathrm{~A}}=0.038 \Omega \\
& V_{\mathrm{ab}}=I R \rightarrow R=\frac{V_{\mathrm{ab}}}{I}=\frac{8.4 \mathrm{~V}}{95 \mathrm{~A}}=0.088 \Omega
\end{aligned}
$$

5. The equivalent resistance is the sum of the two resistances: $R_{\text {eq }}=R_{1}+R_{2}$. The current in the circuit is then the voltage divided by the equivalent resistance: $I=\frac{\mathscr{E}}{R_{\mathrm{eq}}}=\frac{\mathscr{E}}{R_{1}+R_{2}}$. The voltage across the $2200-\Omega$ resistor is given by Ohm's law.

$$
V_{2200}=I R_{2}=\frac{\mathscr{E}}{R_{1}+R_{2}} R_{2}=\mathscr{E} \frac{R_{2}}{R_{1}+R_{2}}=(12.0 \mathrm{~V}) \frac{2200 \Omega}{650 \Omega+2200 \Omega}=9.3 \mathrm{~V}
$$

6. (a) For the resistors in series, use Eq. 26-3, which says the resistances add linearly.

$$
R_{\mathrm{eq}}=3(45 \Omega)+3(65 \Omega)=330 \Omega
$$

(b) For the resistors in parallel, use Eq. 26-4, which says the resistances add reciprocally.

$$
\begin{aligned}
& \frac{1}{R_{\text {eq }}}=\frac{1}{45 \Omega}+\frac{1}{45 \Omega}+\frac{1}{45 \Omega}+\frac{1}{65 \Omega}+\frac{1}{65 \Omega}+\frac{1}{65 \Omega}=\frac{3}{45 \Omega}+\frac{3}{65 \Omega}=\frac{3(65 \Omega)+3(45 \Omega)}{(65 \Omega)(45 \Omega)} \rightarrow \\
& R_{\text {eq }}=\frac{(65 \Omega)(45 \Omega)}{3(65 \Omega)+3(45 \Omega)}=8.9 \Omega
\end{aligned}
$$

7. (a) The maximum resistance is made by combining the resistors in series.

$$
R_{\mathrm{eq}}=R_{1}+R_{2}+R_{3}=680 \Omega+720 \Omega+1200 \Omega=2.60 \mathrm{k} \Omega
$$

(b) The minimum resistance is made by combining the resistors in parallel.

$$
\begin{aligned}
& \frac{1}{R_{\mathrm{eq}}}=\frac{1}{R_{1}}+\frac{1}{R_{2}}+\frac{1}{R_{3}} \rightarrow \\
& R_{\mathrm{eq}}=\left(\frac{1}{R_{1}}+\frac{1}{R_{2}}+\frac{1}{R_{3}}\right)^{-1}=\left(\frac{1}{680 \Omega}+\frac{1}{720 \Omega}+\frac{1}{1200 \Omega}\right)^{-1}=270 \Omega
\end{aligned}
$$

8. The equivalent resistance of five $100-\Omega$ resistors in parallel is found, and then that resistance is divided by $10 \Omega$ to find the number of $10-\Omega$ resistors needed.

$$
R_{\mathrm{eq}}=\left(\frac{1}{R_{1}}+\frac{1}{R_{2}}+\frac{1}{R_{3}}+\frac{1}{R_{4}}+\frac{1}{R_{5}}\right)^{-1}=\left(\frac{5}{100 \Omega}\right)^{-1}=20 \Omega=n(10 \Omega) \rightarrow n=\frac{20 \Omega}{10 \Omega}=2
$$

9. Connecting nine of the resistors in series will enable you to make a voltage divider with a 4.0 V output. To get the desired output, measure the voltage across four consecutive series resistors.

$$
\begin{aligned}
& R_{\mathrm{eq}}=9(1.0 \Omega) \quad I=\frac{\mathscr{E}}{R_{\mathrm{eq}}}=\frac{\mathscr{E}}{9.0 \Omega} \\
& V_{\mathrm{ab}}=(4.0 \Omega) I=(4.0 \Omega) \frac{\mathscr{E}}{9.0 \Omega}=(4.0 \Omega) \frac{9.0 \mathrm{~V}}{9.0 \Omega}=4.0 \mathrm{~V}
\end{aligned}
$$


10. The resistors can all be connected in series.

$$
R_{\mathrm{eq}}=R+R+R=3(1.70 \mathrm{k} \Omega)=5.10 \mathrm{k} \Omega
$$

The resistors can all be connected in parallel.

$$
\frac{1}{R_{\mathrm{eq}}}=\frac{1}{R}+\frac{1}{R}+\frac{1}{R} \rightarrow R_{\mathrm{eq}}=\left(\frac{3}{R}\right)^{-1}=\frac{R}{3}=\frac{1.70 \mathrm{k} \Omega}{3}=567 \Omega
$$

Two resistors in series can be placed in parallel with the third.

$$
\frac{1}{R_{\mathrm{eq}}}=\frac{1}{R}+\frac{1}{R+R}=\frac{1}{R}+\frac{1}{2 R}=\frac{3}{2 R} \rightarrow R_{\mathrm{eq}}=\frac{2 R}{3}=\frac{2(1.70 \mathrm{k} \Omega)}{3}=1.13 \mathrm{k} \Omega
$$

Two resistors in parallel can be placed in series with the third.

$$
R_{\mathrm{eq}}=R+\left(\frac{1}{R}+\frac{1}{R}\right)^{-1}=R+\frac{R}{2}=\frac{3}{2}(1.70 \mathrm{k} \Omega)=2.55 \mathrm{k} \Omega
$$

11. The resistance of each bulb can be found from its power rating.

$$
P=\frac{V^{2}}{R} \rightarrow R=\frac{V^{2}}{P}=\frac{(12.0 \mathrm{~V})^{2}}{4.0 \mathrm{~W}}=36 \Omega
$$

Find the equivalent resistance of the two bulbs in parallel.

$$
\frac{1}{R_{\mathrm{eq}}}=\frac{1}{R}+\frac{1}{R}=\frac{2}{R} \rightarrow R_{\mathrm{eq}}=\frac{R}{2}=\frac{36 \Omega}{2}=18 \Omega
$$



The terminal voltage is the voltage across this equivalent resistance.
Use that to find the current drawn from the battery.

$$
V_{\mathrm{ab}}=I R_{\mathrm{eq}} \rightarrow I=\frac{V_{\mathrm{ab}}}{R_{\mathrm{eq}}}=\frac{V_{\mathrm{ab}}}{R / 2}=\frac{2 V_{\mathrm{ab}}}{R}
$$

Finally, use the terminal voltage and the current to find the internal resistance, as in Eq. 26-1.

$$
V_{\mathrm{ab}}=\mathscr{E}-I r \rightarrow r=\frac{\mathscr{E}-V_{\mathrm{ab}}}{I}=\frac{\mathcal{E}-V_{\mathrm{ab}}}{\left(\frac{2 V_{\mathrm{ab}}}{R}\right)}=R \frac{\mathscr{E}-V_{\mathrm{ab}}}{2 V_{\mathrm{ab}}}=(36 \Omega) \frac{12.0 \mathrm{~V}-11.8 \mathrm{~V}}{2(11.8 \mathrm{~V})}=0.305 \Omega \approx 0.3 \Omega
$$

12. (a) Each bulb should get one-eighth of the total voltage, but let us prove that instead of assuming it. Since the bulbs are identical, the net resistance is $R_{\text {eq }}=8 R$. The current flowing through the bulbs is then $V_{\text {tot }}=I R_{\text {eq }} \rightarrow I=\frac{V_{\text {tot }}}{R_{\text {eq }}}=\frac{V_{\text {tot }}}{8 R}$. The voltage across one bulb is found from Ohm's law.

$$
V=I R=\frac{V_{\text {tot }}}{8 R} R=\frac{V_{\text {tot }}}{8}=\frac{110 \mathrm{~V}}{8}=13.75 \mathrm{~V} \approx 14 \mathrm{~V}
$$

(b)

$$
\begin{aligned}
& I=\frac{V_{\text {tot }}}{8 R} \rightarrow R=\frac{V_{\text {tot }}}{8 I}=\frac{110 \mathrm{~V}}{8(0.42 \mathrm{~A})}=32.74 \Omega \approx 33 \Omega \\
& P=I^{2} R=(0.42 \mathrm{~A})^{2}(32.74 \Omega)=5.775 \mathrm{~W} \approx 5.8 \mathrm{~W}
\end{aligned}
$$

13. We model the resistance of the long leads as a single resistor $r$. Since the bulbs are in parallel, the total current is the sum of the current in each bulb, and so $I=8 I_{R}$. The voltage drop across the long leads is $V_{\text {leads }}=\operatorname{Ir}=8 I_{R} r=8(0.24 \mathrm{~A})(1.4 \Omega)=2.688 \mathrm{~V}$. Thus the voltage across each of the parallel resistors is $V_{R}=V_{\text {tot }}-V_{\text {leads }}=110 \mathrm{~V}-2.688 \mathrm{~V}=107.3 \mathrm{~V}$. Since we have the current through each resistor, and the voltage across each resistor, we calculate the resistance using Ohm's law.

$$
V_{R}=I_{R} R \rightarrow R=\frac{V_{R}}{I_{R}}=\frac{107.3 \mathrm{~V}}{0.24 \mathrm{~A}}=447.1 \Omega=450 \Omega
$$

The total power delivered is $P=V_{\text {tot }} I$, and the "wasted" power is $I^{2} r$. The fraction wasted is the ratio of those powers.

$$
\text { fraction wasted }=\frac{I^{2} r}{I V_{\text {tot }}}=\frac{I r}{V_{\text {tot }}}=\frac{8(0.24 \mathrm{~A})(1.4 \Omega)}{110 \mathrm{~V}}=0.024
$$

So about $2.5 \%$ of the power is wasted.
14. The power delivered to the starter is equal to the square of the current in the circuit multiplied by the resistance of the starter. Since the resistors in each circuit are in series we calculate the currents as the battery emf divided by the sum of the resistances.

$$
\begin{aligned}
\frac{P}{P_{0}} & =\frac{I^{2} R_{\mathrm{S}}}{I_{0}^{2} R_{\mathrm{S}}}=\left(\frac{I}{I_{0}}\right)^{2}=\left(\frac{\mathscr{E} / R_{\mathrm{eq}}}{\mathscr{E} / R_{0 \mathrm{eq}}}\right)^{2}=\left(\frac{R_{0 \mathrm{eq}}}{R_{\mathrm{eq}}}\right)^{2}=\left(\frac{r+R_{\mathrm{S}}}{r+R_{\mathrm{S}}+R_{\mathrm{C}}}\right)^{2} \\
& =\left(\frac{0.02 \Omega+0.15 \Omega}{0.02 \Omega+0.15 \Omega+0.10 \Omega}\right)^{2}=0.40
\end{aligned}
$$

15. To fix this circuit, connect another resistor in parallel with the $480-\Omega$ resistor so that the equivalent resistance is the desired $370 \Omega$.

$$
\frac{1}{R_{\mathrm{eq}}}=\frac{1}{R_{1}}+\frac{1}{R_{2}} \rightarrow R_{2}=\left(\frac{1}{R_{\mathrm{eq}}}-\frac{1}{R_{1}}\right)^{-1}=\left(\frac{1}{370 \Omega}-\frac{1}{480 \Omega}\right)^{-1}=1615 \Omega \approx 1600 \Omega
$$

So solder a $1600-\Omega$ resistor in parallel with the $480-\Omega$ resistor.
16. (a) The equivalent resistance is found by combining the $820 \Omega$ and $680 \Omega$ resistors in parallel, and then adding the $960 \Omega$ resistor in series with that parallel combination.

$$
R_{\mathrm{eq}}=\left(\frac{1}{820 \Omega}+\frac{1}{680 \Omega}\right)^{-1}+960 \Omega=372 \Omega+960 \Omega=1332 \Omega \approx 1330 \Omega
$$

(b) The current delivered by the battery is $I=\frac{V}{R_{\mathrm{eq}}}=\frac{12.0 \mathrm{~V}}{1332 \Omega}=9.009 \times 10^{-3} \mathrm{~A}$. This is the current in the $960 \Omega$ resistor. The voltage across that resistor can be found by Ohm's law.

$$
V_{470}=I R=\left(9.009 \times 10^{-3} \mathrm{~A}\right)(960 \Omega)=8.649 \mathrm{~V} \approx 8.6 \mathrm{~V}
$$

Thus the voltage across the parallel combination must be $12.0 \mathrm{~V}-8.6 \mathrm{~V}=3.4 \mathrm{~V}$. This is the voltage across both the $820 \Omega$ and $680 \Omega$ resistors, since parallel resistors have the same voltage across them. Note that this voltage value could also be found as follows.

$$
V_{\text {parallel }}=I R_{\text {parallel }}=\left(9.009 \times 10^{-3} \mathrm{~A}\right)(372 \Omega)=3.351 \mathrm{~V} \approx 3.4 \mathrm{~V}
$$

17. The resistance of each bulb can be found by using Eq. $25-7 \mathrm{~b}, P=V^{2} / R$. The two individual resistances are combined in parallel. We label the bulbs by their wattage.

$$
\begin{aligned}
& P=V^{2} / R \rightarrow \frac{1}{R}=\frac{P}{V^{2}} \\
& R_{\mathrm{eq}}=\left(\frac{1}{R_{75}}+\frac{1}{R_{40}}\right)^{-1}=\left(\frac{75 \mathrm{~W}}{(110 \mathrm{~V})^{2}}+\frac{25 \mathrm{~W}}{(110 \mathrm{~V})^{2}}\right)^{-1}=121 \Omega \approx 120 \Omega
\end{aligned}
$$

18. (a) The three resistors on the far right are in series, so their equivalent resistance is $3 R$. That combination is in parallel with the next resistor to the left, as shown in the dashed box in the second figure. The equivalent resistance of the dashed box is found as follows.

(a)

$$
R_{\mathrm{eq} 1}=\left(\frac{1}{R}+\frac{1}{3 R}\right)^{-1}=\frac{3}{4} R
$$

This equivalent resistance of $\frac{3}{4} R$ is in series with the next two resistors, as shown in the dashed box in the third figure (on the next
 page). The equivalent resistance of that dashed box is $R_{\text {eq } 2}=2 R+\frac{3}{4} R=\frac{11}{4} R$. This $\frac{11}{4} R$ is in
parallel with the next resistor to the left, as shown in the fourth figure. The equivalent resistance of that dashed box is found as follows.

$$
R_{\text {eq } 2}=\left(\frac{1}{R}+\frac{4}{11 R}\right)^{-1}=\frac{11}{15} R .
$$

This is in series with the last two resistors, the ones connected directly to A and B . The final equivalent resistance is given below.

$$
R_{\mathrm{eq}}=2 R+\frac{11}{15} R=\frac{41}{15} R=\frac{41}{15}(125 \Omega)=341.67 \Omega \approx 342 \Omega
$$


(c)

(b) The current flowing from the battery is found from Ohm's law.

$$
I_{\text {toal }}=\frac{V}{R_{\text {eq }}}=\frac{50.0 \mathrm{~V}}{341.67 \Omega}=0.1463 \mathrm{~A} \approx 0.146 \mathrm{~A}
$$

This is the current in the top and bottom resistors. There will be less current in the next resistor because the current splits, with some current passing through the resistor in question, and the rest of the current passing through the equivalent resistance of $\frac{11}{4} R$, as shown in the last figure. The voltage across $R$ and across $\frac{11}{4} R$ must be the same, since they are in parallel. Use this to find the desired current.

$$
\begin{aligned}
& V_{R}=V_{\frac{11}{4} R} \rightarrow I_{R} R=I_{\frac{11}{4} R}\left(\frac{11}{4} R\right)=\left(I_{\text {toal }}-I_{R}\right)\left(\frac{11}{4} R\right) \rightarrow \\
& I_{R}=\frac{11}{15} I_{\text {toaal }}=\frac{11}{15}(0.1463 \mathrm{~A}) I_{\text {total }}=0.107 \mathrm{~A}
\end{aligned}
$$

19. The resistors have been numbered in the accompanying diagram to help in the analysis. $R_{1}$ and $R_{2}$ are in series with an equivalent resistance of $R_{12}=R+R=2 R$. This combination is in parallel with $R_{3}$, with an equivalent resistance of $R_{123}=\left(\frac{1}{R}+\frac{1}{2 R}\right)^{-1}=\frac{2}{3} R$. This combination is in series with $R_{4}$, with an equivalent resistance of $R_{1234}=\frac{2}{3} R+R=\frac{5}{3} R$. This combination is in parallel with $R_{5}$, with an equivalent resistance of $R_{12345}=\left(\frac{1}{R}+\frac{3}{5 R}\right)^{-1}=\frac{5}{8} R$. Finally, this combination is in series with $R_{6}$, and we calculate the final equivalent resistance.


$$
R_{\mathrm{eq}}=\frac{5}{8} R+R=\frac{13}{8} R
$$

20. We reduce the circuit to a single loop by combining series and parallel combinations. We label a combined resistance with the subscripts of the resistors used in the combination. See the successive diagrams. $R_{1}$ and $R_{2}$ are in series.

$$
R_{12}=R_{1}+R_{2}=R+R=2 R
$$

$R_{12}$ and $R_{3}$ are in parallel.

$$
R_{123}=\left(\frac{1}{R_{12}}+\frac{1}{R_{3}}\right)^{-1}=\left(\frac{1}{2 R}+\frac{1}{R}\right)^{-1}=\frac{2}{3} R
$$


$R_{123}$ and $R_{4}$ are in series.

$$
R_{1234}=R_{123}+R_{4}=\frac{2}{3} R+R=\frac{5}{3} R
$$

$R_{1234}$ and $R_{5}$ are in parallel.

$$
R_{12345}=\left(\frac{1}{R_{1234}}+\frac{1}{R_{5}}\right)^{-1}=\left(\frac{1}{\frac{5}{3} R}+\frac{1}{R}\right)^{-1}=\frac{5}{8} R
$$

$R_{12345}$ and $R_{6}$ are in series, producing the equivalent resistance.

$$
R_{\mathrm{eq}}=R_{12345}+R_{6}=\frac{5}{8} R+R=\frac{13}{8} R
$$

Now work "backwards" from the simplified circuit. Resistors in series have the same current as their equivalent resistance, and resistors in parallel have the

same voltage as their equivalent resistance. To avoid rounding errors, we do not use numeric values until the end of the problem.

$$
\begin{aligned}
& I_{\text {eq }}=\frac{\mathscr{E}}{R_{\text {eq }}}=\frac{\mathscr{E}}{\frac{13}{8} R}=\frac{8 \mathscr{E}}{13 R}=I_{6}=I_{12345} \\
& V_{5}=V_{1234}=V_{12345}=I_{12345} R_{12345}=\left(\frac{8 \mathscr{E}}{13 R}\right)\left(\frac{5}{8} R\right)=\frac{5}{13} \mathscr{E} ; I_{5}=\frac{V_{5}}{R_{5}}=\frac{\frac{5}{11} \mathscr{E}}{R}=\frac{5 \mathscr{E}}{13 R}=I_{5} \\
& I_{1234}=\frac{V_{1234}}{R_{1234}}=\frac{\frac{5}{11} \mathscr{E}}{\frac{5}{3} R}=\frac{3 \mathscr{E}}{13 R}=I_{4}=I_{123} ; V_{123}=I_{123} R_{123}=\left(\frac{3 \mathscr{E}}{13 R}\right)\left(\frac{2}{3} R\right)=\frac{2}{13} \mathscr{E}=V_{12}=V_{3} \\
& I_{3}=\frac{V_{3}}{R_{3}}=\frac{2 \mathscr{E}}{13 R}=I_{3} ; I_{12}=\frac{V_{12}}{R_{12}}=\frac{\frac{2}{13} \mathscr{E}}{2 R}=\frac{\mathscr{E}}{13 R}=I_{1}=I_{2}
\end{aligned}
$$

Now substitute in numeric values.

$$
\begin{aligned}
& I_{1}=I_{2}=\frac{\mathscr{E}}{13 R}=\frac{12.0 \mathrm{~V}}{13(1.20 \mathrm{k} \Omega)}=0.77 \mathrm{~mA} ; I_{3}=\frac{2 \mathscr{E}}{13 R}=1.54 \mathrm{~mA} ; I_{4}=\frac{3 \mathscr{E}}{13 R}=2.31 \mathrm{~mA} ; \\
& I_{5}=\frac{5 \mathscr{E}}{13 R}=3.85 \mathrm{~mA} ; I_{6}=\frac{8 \mathscr{E}}{13 R}=6.15 \mathrm{~mA} ; V_{\mathrm{AB}}=V_{3}=\frac{2}{13} \mathscr{E}=1.85 \mathrm{~V}
\end{aligned}
$$

21. The resistors $r$ and $R$ are in series, so the equivalent resistance of the circuit is $R+r$ and the current in the resistors is $I=\frac{\mathscr{E}}{R+r}$. The power delivered to load resistor is found from Eq. 25-7a. To find the value of $R$ that maximizes this delivered power, set $\frac{d P}{d R}=0$ and solve for $R$.

$$
\begin{aligned}
& P=I^{2} R=\left(\frac{\mathscr{E}}{R+r}\right) R=\frac{\mathscr{E}^{2} R}{(R+r)^{2}} ; \frac{d P}{d R}=\mathscr{E}^{2}\left[\frac{(R+r)^{2}-R(2)(R+r)}{(R+r)^{4}}\right]=0 \rightarrow \\
& (R+r)^{2}-R(2)(R+r)=0 \rightarrow R^{2}+2 R r+r^{2}-2 R^{2}-2 R r=0 \rightarrow R=r
\end{aligned}
$$

22. It is given that the power used when the resistors are in series is one-fourth the power used when the resistors are in parallel. The voltage is the same in both cases. Use Eq. $25-7 \mathrm{~b}$, along with the definitions of series and parallel equivalent resistance.

$$
\begin{aligned}
& P_{\text {series }}=\frac{1}{4} P_{\text {parallel }} \rightarrow \frac{V^{2}}{R_{\text {series }}}=\frac{1}{4} \frac{V^{2}}{R_{\text {parallel }}} \rightarrow R_{\text {series }}=4 R_{\text {parallel }} \rightarrow\left(R_{1}+R_{2}\right)=4 \frac{R_{1} R_{2}}{\left(R_{1}+R_{2}\right)} \rightarrow \\
& \left(R_{1}+R_{2}\right)^{2}=4 R_{1} R_{2} \rightarrow R_{1}^{2}+2 R_{1} R_{2}+R_{2}^{2}-4 R_{1} R_{2}=0=\left(R_{1}-R_{2}\right)^{2} \rightarrow R_{1}=R_{2}
\end{aligned}
$$

Thus the two resistors must be the same, and so the "other" resistor is $3.8 \mathrm{k} \Omega$.
23. We label identical resistors from left to right as $R_{\text {left }}, R_{\text {middle }}$, and $R_{\text {right }}$. When the switch is opened, the equivalent resistance of the circuit increases from $\frac{3}{2} R+r$ to $2 R+r$. Thus the current delivered by the battery decreases, from $\frac{\mathscr{E}}{\frac{3}{2} R+r}$ to $\frac{\mathscr{E}}{2 R+r}$. Note that this is LESS than a $50 \%$ decrease.
(a) Because the current from the battery has decreased, the voltage drop across $R_{\text {leff }}$ will decrease, since it will have less current than before. The voltage drop across $R_{\text {right }}$ decreases to 0 , since no current is flowing in it. The voltage drop across $R_{\text {midde }}$ will increase, because even though the total current has decreased, the current flowing through $R_{\text {middle }}$ has increased since before the switch was opened, only half the total current was flowing through $R_{\text {midde }}$.

$$
V_{\text {left }} \text { decreases ; } V_{\text {middle }} \text { increases ; } V_{\text {right }} \text { goes to } 0 \text {. }
$$

(b) By Ohm's law, the current is proportional to the voltage for a fixed resistance.
$I_{\text {leff }}$ decreases ; $I_{\text {middle }}$ increases ; $I_{\text {right }}$ goes to 0
(c) Since the current from the battery has decreased, the voltage drop across $r$ will decrease, and thus the terminal voltage increases.
(d) With the switch closed, the equivalent resistance is $\frac{3}{2} R+r$. Thus the current in the circuit is $I_{\text {closed }}=\frac{\mathscr{E}}{\frac{3}{2} R+r}$, and the terminal voltage is given by Eq. 26-1.

$$
\begin{aligned}
V_{\text {terminal }}^{\text {closed }}
\end{aligned}=\mathscr{E}-I_{\text {closed }} r=\mathscr{E}-\frac{\mathscr{E}}{\frac{3}{2} R+r} r=\mathscr{E}\left(1-\frac{r}{\frac{3}{2} R+r}\right)=(9.0 \mathrm{~V})\left(1-\frac{0.50 \Omega}{\frac{3}{2}(5.50 \Omega)+0.50 \Omega}\right)
$$

(e) With the switch open, the equivalent resistance is $2 R+r$. Thus the current in the circuit is $I_{\text {closed }}=\frac{\mathscr{E}}{2 R+r}$, and again the terminal voltage is given by Eq. 26-1.

$$
\begin{aligned}
V_{\text {terminal }}^{\text {closed }} & =\mathscr{E}-I_{\text {closed }} r=\mathscr{E}-\frac{\mathscr{E}}{2 R+r} r=\mathscr{E}\left(1-\frac{r}{2 R+r}\right)=(9.0 \mathrm{~V})\left(1-\frac{0.50 \Omega}{2(5.50 \Omega)+0.50 \Omega}\right) \\
& =8.609 \mathrm{~V} \approx 8.6 \mathrm{~V}
\end{aligned}
$$

24. Find the maximum current and resulting voltage for each resistor under the power restriction.

$$
\begin{aligned}
& P=I^{2} R=\frac{V^{2}}{R} \rightarrow I=\sqrt{\frac{P}{R}}, V=\sqrt{R P} \\
& I_{1800}=\sqrt{\frac{0.5 \mathrm{~W}}{1.8 \times 10^{3} \Omega}}=0.0167 \mathrm{~A} \quad V_{1800}=\sqrt{(0.5 \mathrm{~W})\left(1.8 \times 10^{3} \Omega\right)}=30.0 \mathrm{~V}
\end{aligned}
$$

$$
\begin{aligned}
& I_{2800}=\sqrt{\frac{0.5 \mathrm{~W}}{2.8 \times 10^{3} \Omega}}=0.0134 \mathrm{~A} \quad V_{2800}=\sqrt{(0.5 \mathrm{~W})\left(2.8 \times 10^{3} \Omega\right)}=37.4 \mathrm{~V} \\
& I_{3700}=\sqrt{\frac{0.5 \mathrm{~W}}{3.7 \times 10^{3} \Omega}}=0.0116 \mathrm{~A} \quad V_{3700}=\sqrt{(0.5 \mathrm{~W})\left(3.7 \times 10^{3} \Omega\right)}=43.0 \mathrm{~V}
\end{aligned}
$$

The parallel resistors have to have the same voltage, and so the voltage across that combination is limited to 37.4 V . That would require a current given by Ohm's law and the parallel combination of the two resistors.

$$
I_{\text {parallel }}=\frac{V_{\text {parallel }}}{R_{\text {parallel }}}=V_{\text {parallel }}\left(\frac{1}{R_{2800}}+\frac{1}{R_{2100}}\right)=(37.4 \mathrm{~V})\left(\frac{1}{2800 \Omega}+\frac{1}{3700 \Omega}\right)=0.0235 \mathrm{~A}
$$

This is more than the maximum current that can be in $R_{1800}$. Thus the maximum current that $R_{1800}$ can carry, 0.0167 A , is the maximum current for the circuit. The maximum voltage that can be applied across the combination is the maximum current times the equivalent resistance. The equivalent resistance is the parallel combination of $R_{2800}$ and $R_{3700}$ added to $R_{1800}$.

$$
\begin{aligned}
V_{\max } & =I_{\max } R_{\mathrm{eq}}=I_{\max }\left[R_{1800}+\left(\frac{1}{R_{2800}}+\frac{1}{R_{3700}}\right)^{-1}\right]=(0.0167 \mathrm{~A})\left[1800 \Omega+\left(\frac{1}{2800 \Omega}+\frac{1}{3700 \Omega}\right)^{-1}\right] \\
& =56.68 \mathrm{~V} \approx 57 \mathrm{~V}
\end{aligned}
$$

25. (a) Note that adding resistors in series always results in a larger resistance, and adding resistors in parallel always results in a smaller resistance. Closing the switch adds another resistor in parallel with $R_{3}$ and $R_{4}$, which lowers the net resistance of the parallel portion of the circuit, and thus lowers the equivalent resistance of the circuit. That means that more current will be delivered by the battery. Since $R_{1}$ is in series with the battery, its voltage will increase.

Because of that increase, the voltage across $R_{3}$ and $R_{4}$ must decrease so that the total voltage drops around the loop are equal to the battery voltage. Since there was no voltage across $R_{2}$ until the switch was closed, its voltage will increase. To summarize:

```
V
```

(b) By Ohm's law, the current is proportional to the voltage for a fixed resistance. Thus

$$
I_{1} \text { and } I_{2} \text { increase } ; I_{3} \text { and } I_{4} \text { decrease }
$$

(c) Since the battery voltage does not change and the current delivered by the battery increases, the power delivered by the battery, found by multiplying the voltage of the battery by the current delivered, increases.
(d) Before the switch is closed, the equivalent resistance is $R_{3}$ and $R_{4}$ in parallel, combined with $R_{1}$ in series.

$$
R_{\mathrm{eq}}=R_{1}+\left(\frac{1}{R_{3}}+\frac{1}{R_{4}}\right)^{-1}=125 \Omega+\left(\frac{2}{125 \Omega}\right)^{-1}=187.5 \Omega
$$

The current delivered by the battery is the same as the current through $R_{1}$.

$$
I_{\text {total }}=\frac{V_{\text {batery }}}{R_{\text {eq }}}=\frac{22.0 \mathrm{~V}}{187.5 \Omega}=0.1173 \mathrm{~A}=I_{1}
$$

The voltage across $R_{1}$ is found by Ohm's law.

$$
V_{1}=I R_{1}=(0.1173 \mathrm{~A})(125 \Omega)=14.66 \mathrm{~V}
$$

The voltage across the parallel resistors is the battery voltage less the voltage across $R_{1}$.

$$
V_{\mathrm{p}}=V_{\text {battery }}-V_{1}=22.0 \mathrm{~V}-14.66 \mathrm{~V}=7.34 \mathrm{~V}
$$

The current through each of the parallel resistors is found from Ohm's law.

$$
I_{3}=\frac{V_{\mathrm{p}}}{R_{2}}=\frac{7.34 \mathrm{~V}}{125 \Omega}=0.0587 \mathrm{~A}=I_{4}
$$

Notice that the current through each of the parallel resistors is half of the total current, within the limits of significant figures. The currents before closing the switch are as follows.

$$
I_{1}=0.117 \mathrm{~A} \quad I_{3}=I_{4}=0.059 \mathrm{~A}
$$

After the switch is closed, the equivalent resistance is $R_{2}, R_{3}$, and $R_{4}$ in parallel, combined with $R_{1}$ in series. Do a similar analysis.

$$
\begin{aligned}
& R_{\text {eq }}=R_{1}+\left(\frac{1}{R_{2}}+\frac{1}{R_{3}}+\frac{1}{R_{4}}\right)^{-1}=125 \Omega+\left(\frac{3}{125 \Omega}\right)^{-1}=166.7 \Omega \\
& I_{\text {total }}=\frac{V_{\text {batery }}}{R_{\text {eq }}}=\frac{22.0 \mathrm{~V}}{166.7 \Omega}=0.1320 \mathrm{~A}=I_{1} \quad V_{1}=I R_{1}=(0.1320 \mathrm{~A})(125 \Omega)=16.5 \mathrm{~V} \\
& V_{\mathrm{p}}=V_{\text {batery }}-V_{1}=22.0 \mathrm{~V}-16.5 \mathrm{~V}=5.5 \mathrm{~V} \quad I_{2}=\frac{V_{\mathrm{p}}}{R_{2}}=\frac{5.5 \mathrm{~V}}{125 \Omega}=0.044 \mathrm{~A}=I_{3}=I_{4}
\end{aligned}
$$

Notice that the current through each of the parallel resistors is one third of the total current, within the limits of significant figures. The currents after closing the switch are as follows.

$$
I_{1}=0.132 \mathrm{~A} \quad I_{2}=I_{3}=I_{4}=0.044 \mathrm{~A}
$$

Yes, the predictions made in part $(b)$ are all confirmed.
26. The goal is to determine $r$ so that $\left.\frac{d P_{R}}{d R}\right|_{R=R_{0}}=0$. This ensures that $R$ produce very little change in $P_{R}$, since $\Delta P_{R} \approx \frac{d P_{R}}{d R} \Delta R$. The power delivered to the heater can be found by $P_{\text {heater }}=V_{\text {heater }}^{2} / R$, and so we need to determine the voltage across the heater. We do this by calculating the current drawn from the voltage source, and then subtracting the voltage drop across $r$ from the source voltage.

$$
\begin{aligned}
& R_{\mathrm{eq}}=r+\frac{R r}{R+r}=\frac{2 R r+r^{2}}{R+r}=\frac{r(2 R+r)}{R+r} ; I_{\text {total }}=\frac{\mathscr{E}}{R_{\mathrm{eq}}}=\frac{\mathscr{E}}{\frac{r(2 R+r)}{R+r}}=\frac{\mathscr{E}(R+r)}{r(2 R+r)} \\
& V_{\text {heater }}=\mathscr{E}-I_{\text {total }} r=\mathscr{E}-\frac{\mathscr{E}(R+r)}{r(2 R+r)} r=\mathscr{E}-\frac{\mathscr{E}(R+r)}{(2 R+r)}=\frac{\mathscr{E} R}{(2 R+r)} ; P_{\text {heater }}=\frac{V_{\text {heater }}^{2}}{R}=\frac{\mathscr{E}^{2} R}{(2 R+r)^{2}} \\
& \left.\frac{d P_{\text {heater }}}{d R}\right|_{R=R_{0}}=\mathscr{E}^{2} \frac{\left(2 R_{0}+r\right)^{2}-R_{0}(2)\left(2 R_{0}+r\right)(2)}{\left(2 R_{0}+r\right)^{4}}=0 \rightarrow\left(2 R_{0}+r\right)^{2}-R_{0}(2)\left(2 R_{0}+r\right)(2)=0 \rightarrow \\
& 4 R_{0}^{2}+4 R_{0} r+r^{2}-8 R_{0}^{2}-4 R_{0} r=0 \rightarrow r^{2}=4 R_{0}^{2} \rightarrow r=2 R_{0}
\end{aligned}
$$

27. All of the resistors are in series, so the equivalent resistance is just the sum of the resistors. Use Ohm's law then to find the current, and show all voltage changes starting at the negative pole of the battery and going counterclockwise.

$$
\begin{aligned}
& I=\frac{\mathcal{E}}{R_{\text {eq }}}=\frac{9.0 \mathrm{~V}}{(9.5+12.0+2.0) \Omega}=0.383 \mathrm{~A} \approx 0.38 \mathrm{~A} \\
& \begin{aligned}
\sum \text { voltages } & =9.0 \mathrm{~V}-(9.5 \Omega)(0.383 \mathrm{~A})-(12.0 \Omega)(0.383 \mathrm{~A})-(2.0 \Omega)(0.383 \mathrm{~A}) \\
& =9.0 \mathrm{~V}-3.638 \mathrm{~V}-4.596 \mathrm{~V}-0.766 \mathrm{~V}=0.00 \mathrm{~V}
\end{aligned}
\end{aligned}
$$

28. Apply Kirchhoff's loop rule to the circuit starting at the upper left corner of the circuit diagram, in order to calculate the current. Assume that the current is flowing clockwise.

$$
-I(2.0 \Omega)+18 \mathrm{~V}-I(6.6 \Omega)-12 \mathrm{~V}-I(1.0 \Omega)=0 \rightarrow I=\frac{6 \mathrm{~V}}{9.6 \Omega}=0.625 \mathrm{~A}
$$

The terminal voltage for each battery is found by summing the potential differences across the internal resistance and EMF from left to right. Note that for the 12 V battery, there is a voltage gain going across the internal resistance from left to right.

$$
\begin{aligned}
& 18 \mathrm{~V} \text { battery: } V_{\text {teminal }}=-I(2.0 \Omega)+18 \mathrm{~V}=-(0.625 \mathrm{~A})(2.0 \Omega)+18 \mathrm{~V}=16.75 \mathrm{~V} \approx 17 \mathrm{~V} \\
& 12 \mathrm{~V} \text { battery: } V_{\text {teminal }}=I(1.0 \Omega)+12 \mathrm{~V}=(0.625 \mathrm{~A})(1.0 \Omega)+12 \mathrm{~V}=12.625 \mathrm{~V} \approx 13 \mathrm{~V}
\end{aligned}
$$

29. To find the potential difference between points $a$ and $b$, the current must be found from Kirchhoff's loop law. Start at point a and go counterclockwise around the entire circuit, taking the current to be counterclockwise.

$$
\begin{aligned}
& -I R+\mathscr{E}-I R-I R+\mathscr{E}-I R=0 \rightarrow I=\frac{\mathscr{E}}{2 R} \\
& V_{\mathrm{ab}}=V_{\mathrm{a}}-V_{\mathrm{b}}=-I R+\mathscr{E}-I R=\mathscr{E}-2 I R=\mathscr{E}-2 \frac{\mathscr{E}}{2 R} R=0 \mathrm{~V}
\end{aligned}
$$

30. (a) We label each of the currents as shown in the accompanying figure. Using Kirchhoff's junction rule and the first three junctions (a-c) we write equations relating the entering and exiting currents.

$$
\begin{align*}
& I=I_{1}+I_{2}  \tag{1}\\
& I_{2}=I_{3}+I_{4}  \tag{2}\\
& I_{1}+I_{4}=I_{5} \tag{3}
\end{align*}
$$

We use Kirchhoff's loop rule to write equations for loops
 abca, abcda, and bdcb.

$$
\begin{align*}
& 0=-I_{2} R-I_{4} R+I_{1} R  \tag{4}\\
& 0=-I_{2} R-I_{3} R+\mathscr{E}  \tag{5}\\
& 0=-I_{3} R+I_{5} R+I_{4} R \tag{6}
\end{align*}
$$

We have six unknown currents and six equations. We solve these equations by substitution. First, insert Eq. [3] into [6] to eliminate current $I_{5}$. Next insert Eq. [2] into Eqs. [1], [4], and [5] to eliminate $I_{2}$.

$$
\begin{align*}
& 0=-I_{3} R+\left(I_{1}+I_{4}\right) R+I_{4} R \rightarrow 0=-I_{3} R+I_{1} R+2 I_{4} R  \tag{*}\\
& I=I_{1}+I_{3}+I_{4}  \tag{1*}\\
& 0=-\left(I_{3}+I_{4}\right) R-I_{4} R+I_{1} R \rightarrow 0=-I_{3} R-2 I_{4} \mathrm{R}+I_{1} R  \tag{*}\\
& 0=-\left(I_{3}+I_{4}\right) R-I_{3} R+\mathscr{E} \rightarrow 0=-I_{4} R-2 I_{3} R+\mathscr{E} \tag{*}
\end{align*}
$$

Next we solve Eq. [4*] for $I_{4}$ and insert the result into Eqs. [1*], [5*], and [6*].

$$
\begin{array}{ll}
0=-I_{3} R-2 I_{4} R+I_{1} R \rightarrow I_{4}=\frac{1}{2} I_{1}-\frac{1}{2} I_{3} & \\
I=I_{1}+I_{3}+\frac{1}{2} I_{1}-\frac{1}{2} I_{3} \rightarrow I=\frac{3}{2} I_{1}+\frac{1}{2} I_{3} & {\left[1^{* *}\right]} \\
0=-I_{3} R+I_{1} R+2\left(\frac{1}{2} I_{1}-\frac{1}{2} I_{3}\right) R=-2 I_{3} R+2 I_{1} R \rightarrow I_{1}=I_{3} & {\left[6^{* *}\right]} \\
0=-\left(\frac{1}{2} I_{1}-\frac{1}{2} I_{3}\right) R-2 I_{3} R+\mathscr{E} \rightarrow 0=-\frac{1}{2} I_{1} R-\frac{3}{2} I_{3} R+\mathscr{E} & {\left[5^{* *}\right]}
\end{array}
$$

Finally we substitute Eq. $\left[6^{* *}\right]$ into $\mathrm{Eq}\left[5^{* *}\right]$ and solve for $I_{1}$. We insert this result into Eq. [1**] to write an equation for the current through the battery in terms of the battery emf and resistance.

$$
0=-\frac{1}{2} I_{1} R-\frac{3}{2} I_{1} R+\mathscr{E} \rightarrow I_{1}=\frac{\mathscr{E}}{2 R} ; I=\frac{3}{2} I_{1}+\frac{1}{2} I_{1}=2 I_{1} \rightarrow I=\frac{\mathscr{E}}{R}
$$

(b) We divide the battery emf by the current to determine the effective resistance.

$$
R_{e q}=\frac{\mathscr{E}}{I}=\frac{\mathscr{E}}{\mathscr{E} / R}=R
$$

31. This circuit is identical to Example 26-9 and Figure 26-13 except for the numeric values. So we may copy the same equations as developed in that Example, but using the current values.

Eq. (a): $\quad I_{3}=I_{1}+I_{2} \quad ;$
Eq. (b): $\quad-34 I_{1}+45-48 I_{3}=0$
Eq. (c): $\quad-34 I_{1}+19 I_{2}-75=0$
Eq. $(d): \quad I_{2}=\frac{75+34 I_{1}}{19}=3.95+1.79 I_{1}$
Eq. (e): $\quad I_{3}=\frac{45-34 I_{1}}{48}=0.938-0.708 I_{1}$

$$
\begin{aligned}
& I_{3}=I_{1}+I_{2} \rightarrow 0.938-0.708 I_{1}=I_{1}+3.95+1.79 I_{1} \rightarrow I_{1}=-0.861 \mathrm{~A} \\
& I_{2}=3.95+1.79 I_{1}=2.41 \mathrm{~A} ; I_{3}=0.938-0.708 I_{1}=1.55 \mathrm{~A}
\end{aligned}
$$

(a) To find the potential difference between points a and d, start at point a and add each individual potential difference until reaching point d. The simplest way to do this is along the top branch.

$$
V_{\mathrm{ad}}=V_{\mathrm{d}}-V_{\mathrm{a}}=-I_{1}(34 \Omega)=-(-0.861 \mathrm{~A})(34 \Omega)=29.27 \mathrm{~V} \approx 29 \mathrm{~V}
$$

Slight differences will be obtained in the final answer depending on the branch used, due to rounding. For example, using the bottom branch, we get the following.

$$
V_{\mathrm{ad}}=V_{\mathrm{d}}-V_{\mathrm{a}}=\mathscr{G}-I_{2}(19 \Omega)=75 \mathrm{~V}-(2.41 \mathrm{~A})(19 \Omega)=29.21 \mathrm{~V} \approx 29 \mathrm{~V}
$$

(b) For the $75-\mathrm{V}$ battery, the terminal voltage is the potential difference from point g to point e . For the $45-\mathrm{V}$ battery, the terminal voltage is the potential difference from point d to point b .

75 V battery: $V_{\text {terminal }}=\mathscr{G}-I_{2} r=75 \mathrm{~V}-(2.41 \mathrm{~A})(1.0 \Omega)=73 \mathrm{~V}$
45 V battery: $V_{\text {terminal }}=\mathscr{E}_{2}-I_{3} r=45 \mathrm{~V}-(1.55 \mathrm{~A})(1.0 \Omega)=43 \mathrm{~V}$
32. There are three currents involved, and so there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction of the three branches at the top center of the circuit.

$$
I_{1}=I_{2}+I_{3}
$$

Another equation comes from Kirchhoff's loop rule applied to the left loop, starting at the negative terminal of the battery and
 progressing counterclockwise.

$$
58 \mathrm{~V}-I_{1}(120 \Omega)-I_{1}(82 \Omega)-I_{2}(64 \Omega)=0 \rightarrow 58=202 I_{1}+64 I_{2}
$$

The final equation comes from Kirchhoff's loop rule applied to the right loop, starting at the negative terminal of the battery and progressing counterclockwise.

$$
3.0 \mathrm{~V}-I_{3}(25 \Omega)+I_{2}(64 \Omega)-I_{3}(110 \Omega)=0 \rightarrow 3=-64 I_{2}+135 I_{3}
$$

Substitute $I_{1}=I_{2}+I_{3}$ into the left loop equation, so that there are two equations with two unknowns.

$$
58=202\left(I_{2}+I_{3}\right)+64 I_{2}=266 I_{2}+202 I_{3}
$$

Solve the right loop equation for $I_{2}$ and substitute into the left loop equation, resulting in an equation with only one unknown, which can be solved.

$$
\begin{aligned}
& 3=-64 I_{2}+135 I_{3} \rightarrow I_{2}=\frac{135 I_{3}-3}{64} ; 58=266 I_{2}+202 I_{3}=266\left(\frac{135 I_{3}-3}{64}\right)+202 I_{3} \rightarrow \\
& I_{3}=0.09235 \mathrm{~A} ; I_{2}=\frac{135 I_{3}-3}{64}=0.1479 \mathrm{~A} ; I_{1}=I_{2}+I_{3}=0.24025 \mathrm{~A}
\end{aligned}
$$

The current in each resistor is as follows:

$$
\begin{array}{|lllll|}
\hline 120 \Omega: 0.24 \mathrm{~A} & 82 \Omega: 0.24 \mathrm{~A} & 64 \Omega: 0.15 \mathrm{~A} & 25 \Omega: 0.092 \mathrm{~A} & 110 \Omega: 0.092 \mathrm{~A} \\
\hline
\end{array}
$$

33. Because there are no resistors in the bottom branch, it is possible to write Kirchhoff loop equations that only have one current term, making them easier to solve. To find the current through $R_{1}$, go around the outer loop counterclockwise, starting at the lower left corner.

$$
V_{3}-I_{1} R_{1}+V_{1}=0 \rightarrow I_{1}=\frac{V_{3}+V_{1}}{R_{1}}=\frac{6.0 \mathrm{~V}+9.0 \mathrm{~V}}{22 \Omega}=0.68 \mathrm{~A}, \text { left }
$$

To find the current through $R_{2}$, go around the lower loop counterclockwise, starting at the lower left corner.

$$
V_{3}-I_{2} R_{2}=0 \rightarrow I_{2}=\frac{V_{3}}{R_{2}}=\frac{6.0 \mathrm{~V}}{18 \Omega}=0.33 \mathrm{~A}, \text { left }
$$

34. (a) There are three currents involved, and so there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction of the three branches on the right of the circuit.

$$
I_{2}=I_{1}+I_{3} \rightarrow I_{1}=I_{2}-I_{3}
$$

Another equation comes from Kirchhoff's loop rule applied to the top loop, starting at the negative terminal of the battery and progressing clockwise.

$$
\mathscr{E}_{1}-I_{1} R_{1}-I_{2} R_{2}=0 \rightarrow 9=25 I_{1}+48 I_{2}
$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, starting at the negative terminal of the battery and

progressing counterclockwise.

$$
\mathscr{E}_{2}-I_{3} R_{3}-I_{2} R_{2}=0 \rightarrow 12=35 I_{3}+48 I_{2}
$$

Substitute $I_{1}=I_{2}-I_{3}$ into the top loop equation, so that there are two equations with two unknowns.

$$
9=25 I_{1}+48 I_{2}=25\left(I_{2}-I_{3}\right)+48 I_{2}=73 I_{2}-25 I_{3} ; 12=35 I_{3}+48 I_{2}
$$

Solve the bottom loop equation for $I_{2}$ and substitute into the top loop equation, resulting in an equation with only one unknown, which can be solved.

$$
\begin{aligned}
& 12=35 I_{3}+48 I_{2} \rightarrow I_{2}=\frac{12-35 I_{3}}{48} \\
& 9=73 I_{2}-25 I_{3}=73\left(\frac{12-35 I_{3}}{48}\right)-25 I_{3} \rightarrow 432=876-2555 I_{3}-1200 I_{3} \rightarrow \\
& I_{3}=\frac{444}{3755}=0.1182 \mathrm{~A} \approx 0.12 \mathrm{~A}, \text { up } ; I_{2}=\frac{12-35 I_{3}}{48}=0.1638 \mathrm{~A} \approx 0.16 \mathrm{~A}, \text { left } \\
& I_{1}=I_{2}-I_{3}=0.0456 \mathrm{~A} \approx 0.046 \mathrm{~A}, \text { right }
\end{aligned}
$$

(b) We can include the internal resistances simply by adding $1.0 \Omega$ to $R_{1}$ and $R_{3}$. So let $R_{1}=26 \Omega$ and let $R_{3}=36 \Omega$. Now re-work the problem exactly as in part (a).

$$
\begin{aligned}
& I_{2}=I_{1}+I_{3} \rightarrow I_{1}=I_{2}-I_{3} \\
& \mathscr{C}_{1}-I_{1} R_{1}-I_{2} R_{2}=0 \rightarrow 9=26 I_{1}+48 I_{2} \\
& \mathscr{E}_{2}-I_{3} R_{3}-I_{2} R_{2}=0 \rightarrow 12=36 I_{3}+48 I_{2} \\
& 9=26 I_{1}+48 I_{2}=26\left(I_{2}-I_{3}\right)+48 I_{2}=74 I_{2}-26 I_{3} ; 12=36 I_{3}+48 I_{2} \\
& 12=36 I_{3}+48 I_{2} \rightarrow I_{2}=\frac{12-36 I_{3}}{48}=\frac{1-3 I_{3}}{4} \\
& 9=74 I_{2}-26 I_{3}=74\left(\frac{1-3 I_{3}}{4}\right)-26 I_{3} \rightarrow 36=74-222 I_{3}-104 I_{3} \rightarrow \\
& I_{3}=\frac{38}{326}=0.1166 \mathrm{~A} \approx 0.12 \mathrm{~A}, \text { up } ; I_{2}=\frac{1-3 I_{3}}{4}=0.1626 \mathrm{~A} \approx 0.16 \mathrm{~A}, \text { left } \\
& I_{1}=I_{2}-I_{3}=0.046 \mathrm{~A}, \text { right }
\end{aligned}
$$

The currents are unchanged to 2 significant figures by the inclusion of the internal resistances.
35. We are to find the ratio of the power used when the resistors are in series, to the power used when the resistors are in parallel. The voltage is the same in both cases. Use Eq. 25-7b, along with the definitions of series and parallel equivalent resistance.

$$
\begin{aligned}
& R_{\text {series }}=R_{1}+R_{2}+\cdots R_{n}=n R ; R_{\text {parallel }}=\left(\frac{1}{R_{1}}+\frac{1}{R_{2}}+\cdots \frac{1}{R_{\mathrm{n}}}\right)^{-1}=\left(\frac{n}{R}\right)^{-1}=\frac{R}{n} \\
& \frac{P_{\text {series }}}{P_{\text {parallel }}}=\frac{V^{2} / R_{\text {serics }}}{V^{2} / R_{\text {pearallel }}}=\frac{R_{\text {parallel }}}{R_{\text {series }}}=\frac{R / n}{n R}=\frac{1}{n^{2}}
\end{aligned}
$$

36. (a) Since there are three currents to determine, there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction near the negative terminal of the middle battery.

$$
I_{1}=I_{2}+I_{3}
$$

Another equation comes from Kirchhoff's loop rule applied to the top loop, starting at the negative terminal of the middle battery, and progressing counterclockwise. We add series resistances.

$$
12.0 \mathrm{~V}-I_{2}(12 \Omega)+12.0 \mathrm{~V}-I_{1}(35 \Omega)=0 \rightarrow 24=35 I_{1}+12 I_{2}
$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, starting at the negative terminal of the middle battery, and progressing clockwise.

$$
12.0 \mathrm{~V}-I_{2}(12 \Omega)-6.0 \mathrm{~V}+I_{3}(34 \Omega)=0 \rightarrow 6=12 I_{2}-34 I_{3}
$$

Substitute $I_{1}=I_{2}+I_{3}$ into the top loop equation, so that there are two equations with two unknowns.

$$
24=35 I_{1}+12 I_{2}=35\left(I_{2}+I_{3}\right)+12 I_{2}=47 I_{2}+35 I_{3}
$$

Solve the bottom loop equation for $I_{2}$ and substitute into the top loop equation, resulting in an equation with only one unknown, which can be solved for $I_{3}$.

$$
\begin{aligned}
& 6=12 I_{2}-34 I_{3} \rightarrow I_{2}=\frac{6+34 I_{3}}{12} ; 24=47 I_{2}+35 I_{3}=47\left(\frac{6+34 I_{3}}{12}\right)+35 I_{3} \rightarrow \\
& I_{3}=2.97 \mathrm{~mA} ; I_{2}=\frac{6+34 I_{3}}{12}=0.508 \mathrm{~A} ; I_{1}=I_{2}+I_{3}=0.511 \mathrm{~A}
\end{aligned}
$$

(b) The terminal voltage of the $6.0-\mathrm{V}$ battery is $6.0 \mathrm{~V}-I_{3} r=6.0 \mathrm{~V}-\left(2.97 \times 10^{-3} \mathrm{~A}\right)(1.0 \Omega)$

$$
=5.997 \mathrm{~V} \approx 6.0 \mathrm{~V}
$$

37. This problem is the same as Problem 36, except the total resistance in the top branch is now $23 \Omega$ instead of $35 \Omega$. We simply reproduce the adjusted equations here without the prose.

$$
\begin{aligned}
& I_{1}=I_{2}+I_{3} \\
& 12.0 \mathrm{~V}-I_{2}(12 \Omega)+12.0 \mathrm{~V}-I_{1}(23 \Omega)=0 \rightarrow 24=23 I_{1}+12 I_{2} \\
& 12.0 \mathrm{~V}-I_{2}(12 \Omega)-6.0 \mathrm{~V}+I_{3}(34 \Omega)=0 \rightarrow 6=12 I_{2}-34 I_{3} \\
& 24=23 I_{1}+12 I_{2}=23\left(I_{2}+I_{3}\right)+12 I_{2}=35 I_{2}+23 I_{3} \\
& 6=12 I_{2}-34 I_{3} \rightarrow I_{2}=\frac{6+34 I_{3}}{12} ; 24=35 I_{2}+23 I_{3}=35\left(\frac{6+34 I_{3}}{12}\right)+23 I_{3} \rightarrow \\
& I_{3}=0.0532 \mathrm{~A} ; I_{2}=\frac{6+34 I_{3}}{12}=0.6508 \mathrm{~A} ; I_{1}=I_{2}+I_{3}=0.704 \mathrm{~A} \approx 0.70 \mathrm{~A}
\end{aligned}
$$

38. The circuit diagram has been labeled with six different currents. We apply the junction rule to junctions $a$, $b$, and $c$. We apply the loop rule to the three loops labeled in the diagram.
1) $\left.I=I_{1}+I_{2} ; 2\right) I_{1}=I_{3}+I_{5}$; 3) $I_{3}+I_{4}=I$
2) $\left.-I_{1} R_{1}-I_{5} R_{5}+I_{2} R_{2}=0 ; 5\right)-I_{3} R_{3}+I_{4} R_{4}+I_{5} R_{5}=0$
3) $\mathscr{E}-I_{2} R_{2}-I_{4} R_{4}=0$

Eliminate $I$ using equations 1) and 3).

1) $I_{3}+I_{4}=I_{1}+I_{2}$; 2) $I_{1}=I_{3}+I_{5}$
2) $\left.-I_{1} R_{1}-I_{5} R_{5}+I_{2} R_{2}=0 ; 5\right)-I_{3} R_{3}+I_{4} R_{4}+I_{5} R_{5}=0$
3) $\mathscr{E}-I_{2} R_{2}-I_{4} R_{4}=0$


Eliminate $I_{1}$ using equation 2.

1) $I_{3}+I_{4}=I_{3}+I_{5}+I_{2} \rightarrow I_{4}=I_{5}+I_{2}$
2) $-\left(I_{3}+I_{5}\right) R_{1}-I_{5} R_{5}+I_{2} R_{2}=0 \rightarrow-I_{3} R_{1}-I_{5}\left(R_{1}+R_{5}\right)+I_{2} R_{2}=0$
3) $-I_{3} R_{3}+I_{4} R_{4}+I_{5} R_{5}=0$
4) $\mathscr{E}-I_{2} R_{2}-I_{4} R_{4}=0$

Eliminate $I_{4}$ using equation 1 .
4) $-I_{3} R_{1}-I_{5}\left(R_{1}+R_{5}\right)+I_{2} R_{2}=0$
5) $-I_{3} R_{3}+\left(I_{5}+I_{2}\right) R_{4}+I_{5} R_{5}=0 \rightarrow-I_{3} R_{3}+I_{5}\left(R_{4}+R_{5}\right)+I_{2} R_{4}=0$
6) $\mathscr{E}-I_{2} R_{2}-\left(I_{5}+I_{2}\right) R_{4}=0 \rightarrow \mathscr{E}-I_{2}\left(R_{2}+R_{4}\right)-I_{5} R_{4}=0$

Eliminate $I_{2}$ using equation 4: $I_{2}=\frac{1}{R_{2}}\left[I_{3} R_{1}+I_{5}\left(R_{1}+R_{5}\right)\right]$.
5) $-I_{3} R_{3}+I_{5}\left(R_{4}+R_{5}\right)+\frac{1}{R_{2}}\left[I_{3} R_{1}+I_{5}\left(R_{1}+R_{5}\right)\right] R_{4}=0 \rightarrow$

$$
I_{3}\left(R_{1} R_{4}-R_{2} R_{3}\right)+I_{5}\left(R_{2} R_{4}+R_{2} R_{5}+R_{1} R_{4}+R_{5} R_{4}\right)=0
$$

6) $\mathscr{E}-\frac{1}{R_{2}}\left[I_{3} R_{1}+I_{5}\left(R_{1}+R_{5}\right)\right]\left(R_{2}+R_{4}\right)-I_{5} R_{4}=0 \rightarrow$

$$
\mathscr{E} R_{2}-I_{3} R_{1}\left(R_{2}+R_{4}\right)-I_{5}\left(R_{1} R_{2}+R_{1} R_{4}+R_{5} R_{2}+R_{5} R_{4}+R_{2} R_{4}\right)=0
$$

Eliminate $I_{3}$ using equation 5: $I_{3}=-I_{5} \frac{\left(R_{2} R_{4}+R_{2} R_{5}+R_{1} R_{4}+R_{5} R_{4}\right)}{\left(R_{1} R_{4}-R_{2} R_{3}\right)}$

$$
\begin{aligned}
& \mathscr{E} R_{2}+\left[I_{5} \frac{\left(R_{2} R_{4}+R_{2} R_{5}+R_{1} R_{4}+R_{5} R_{4}\right)}{\left(R_{1} R_{4}-R_{2} R_{3}\right)}\right] R_{1}\left(R_{2}+R_{4}\right)-I_{5}\left(R_{1} R_{2}+R_{1} R_{4}+R_{5} R_{2}+R_{5} R_{4}+R_{2} R_{4}\right)=0 \\
& \mathscr{E}=-\frac{I_{5}}{R_{2}}\left\{\left[\frac{\left(R_{2} R_{4}+R_{2} R_{5}+R_{1} R_{4}+R_{5} R_{4}\right)}{\left(R_{1} R_{4}-R_{2} R_{3}\right)}\right] R_{1}\left(R_{2}+R_{4}\right)-\left(R_{1} R_{2}+R_{1} R_{4}+R_{5} R_{2}+R_{5} R_{4}+R_{2} R_{4}\right)\right\} \\
& =-\frac{I_{5}}{25 \Omega}\left\{\left[\begin{array}{l}
{\left[\frac{(25 \Omega)(14 \Omega)+(25 \Omega)(15 \Omega)+(22 \Omega)(14 \Omega)+(15 \Omega)(14 \Omega)}{(22 \Omega)(14 \Omega)-(25 \Omega)(12 \Omega)}\right](22 \Omega)(25 \Omega+14 \Omega)} \\
-[(22 \Omega)(25 \Omega)+(22)(14)+(15 \Omega)(25 \Omega)+(15 \Omega)(14 \Omega)+(25 \Omega)(14 \Omega)]
\end{array}\right\}\right.
\end{aligned}
$$

$$
\begin{aligned}
& =-I_{5}(5261 \Omega) \rightarrow I_{5}=-\frac{6.0 \mathrm{~V}}{5261 \Omega}=-1.140 \mathrm{~mA} \text { (upwards) } \\
I_{3} & =-I_{5} \frac{\left(R_{2} R_{4}+R_{2} R_{5}+R_{1} R_{4}+R_{5} R_{4}\right)}{\left(R_{1} R_{4}-R_{2} R_{3}\right)} \\
& =-(-1.140 \mathrm{~mA}) \frac{(25 \Omega)(14 \Omega)+(25 \Omega)(15 \Omega)+(22 \Omega)(14 \Omega)+(15 \Omega)(14 \Omega)}{(22 \Omega)(14 \Omega)-(25 \Omega)(12 \Omega)}=0.1771 \mathrm{~A} \\
I_{2} & =\frac{1}{R_{2}}\left[I_{3} R_{1}+I_{5}\left(R_{1}+R_{5}\right)\right]=\frac{1}{25 \Omega}[(0.1771 \mathrm{~A})(22 \Omega)+(-0.00114 \mathrm{~A})(37 \Omega)]=0.1542 \mathrm{~A} \\
I_{4} & =I_{5}+I_{2}=-0.00114 \mathrm{~A}+0.1542 \mathrm{~A}=0.1531 \mathrm{~A} \\
I_{1} & =I_{3}+I_{5}=0.1771 \mathrm{~A}-0.00114 \mathrm{~A}=0.1760 \mathrm{~A}
\end{aligned}
$$

We keep an extra significant figure to show the slight difference in the currents.

$$
\begin{array}{|lllll|}
\hline I_{22 \Omega}=0.176 \mathrm{~A} & I_{25 \Omega}=0.154 \mathrm{~A} & I_{12 \Omega}=0.177 \mathrm{~A} & I_{14 \Omega}=0.153 \mathrm{~A} & I_{15 \Omega}=0.001 \mathrm{~A}, \text { upwards } \\
\hline
\end{array}
$$

39. The circuit diagram from Problem 38 is reproduced, with $R_{2}=0$. This circuit can now be simplified significantly. Resistors $R_{1}$ and $R_{5}$ are in parallel. Call that combination $R_{15}$. That combination is in series with $R_{3}$. Call that combination $R_{153}$. That combination is in parallel with $R_{4}$. See the second diagram. We calculate the equivalent resistance $R_{153}$, use that to find the current through the top branch in the second diagram, and then use that current to find the current through $R_{5}$.


$$
R_{153}=\left(\frac{1}{R_{1}}+\frac{1}{R_{5}}\right)^{-1}+R_{3}=\left(\frac{1}{22 \Omega}+\frac{1}{15 \Omega}\right)^{-1}+12 \Omega=20.92 \Omega
$$

Use the loop rule for the outside loop to find the current in the top branch.

$$
\mathscr{E}-I_{153} R_{153}=0 \rightarrow I_{153}=\frac{\mathscr{E}}{R_{153}}=\frac{6.0 \mathrm{~V}}{20.92 \Omega}=0.2868 \mathrm{~A}
$$

This current is the sum of the currents in $R_{1}$ and $R_{5}$. Since those two resistors are in parallel, the voltage across them must be the same.


$$
\begin{aligned}
& V_{1}=V_{5} \rightarrow I_{1} R_{1}=I_{5} R_{5} \rightarrow\left(I_{153}-I_{5}\right) R_{1}=I_{5} R_{5} \rightarrow \\
& I_{5}=I_{153} \frac{R_{1}}{\left(R_{5}+R_{1}\right)}=(0.2868 \mathrm{~A}) \frac{22 \Omega}{37 \Omega}=0.17 \mathrm{~A}
\end{aligned}
$$

40. (a) As shown in the diagram, we use symmetry to reduce the number of independent currents to six. Using Kirchhoff's junction rule, we write equations for junctions $\mathrm{a}, \mathrm{c}$, and d . We then use Kirchhoff's loop rule to write the loop equations for loops afgba, hedch, and aba (through the voltage source).

$$
\begin{aligned}
& I=2 I_{1}+I_{2}[1] ; I_{3}+I_{4}=I_{1}[2] ; I_{5}=2 I_{4} \\
& 0=-2 I_{1} R-I_{3} R+I_{2} R[4] ; 0=-2 I_{4} R-I_{5} R+I_{3} R \\
& 0=\mathscr{E}-I_{2} R[6]
\end{aligned}
$$



We have six equations with six unknown currents. We use the method of substitution to reduce the equations to a single equation relating the emf from the power source to the current through the power source. This resulting ratio is the effective resistance between points $a$ and $b$. We insert Eqs. [2], [3], and [6] into the other three equations to eliminate $I_{1}, I_{2}$, and $I_{5}$.

$$
\begin{align*}
I & =2\left(I_{3}+I_{4}\right)+\frac{\mathscr{E}}{R}=2 I_{3}+2 I_{4}+\frac{\mathscr{E}}{R}  \tag{*}\\
0 & =-2\left(I_{3}+I_{4}\right) R-I_{3} R+\frac{\mathscr{C}}{R} R=-2 I_{4} R-3 I_{3} R+\mathscr{E}  \tag{4*}\\
0 & =-2 I_{4} R-2 I_{4} R+I_{3} R=-4 I_{4} R+I_{3} R \tag{5*}
\end{align*}
$$

We solve Eq. [5*] for $I_{3}$ and insert that into Eq. [4*]. We then insert the two results into Eq. [1*] and solve for the effective resistance.

$$
\begin{aligned}
& I_{3}=4 I_{4} ; 0=-2 I_{4} R-3\left(4 I_{4}\right) R+\mathscr{E} \rightarrow I_{4}=\frac{\mathscr{E}}{14 R} \\
& I=2\left(4 I_{4}\right)+2 I_{4}+\frac{\mathscr{E}}{R}=10 I_{4}+\frac{\mathscr{E}}{R}=\frac{10 \mathscr{E}}{14 R}+\frac{\mathscr{E}}{R}=\frac{24 \mathscr{E}}{14 R}=\frac{12 \mathscr{E}}{7 R} \rightarrow R_{\mathrm{eq}}=\frac{\mathscr{E}}{I}=\frac{7}{12} R
\end{aligned}
$$

(b) As shown in the diagram, we use symmetry to reduce the number of currents to four. We use Kirchhoff's junction rule at junctions a and $d$ and the loop rule around loops abca (through the voltage source) and afgdcha. This results in four equations with four unknowns. We solve these equations for the ratio of the voltage source to current $I$, to obtain the effective resistance.

$$
\begin{align*}
& I=2 I_{1}+I_{2}[1] \quad ; 2 I_{3}=I_{2} \\
& 0=-2 I_{2} R+\mathscr{E}[3] ; 0=-2 I_{2} R-2 I_{3} R+2 I_{1} R \tag{4}
\end{align*}
$$

We solve Eq. [3] for $I_{2}$ and Eq. [2] for $I_{3}$. These results are
 inserted into Eq. [4] to determine $I_{1}$. Using these results and Eq. [1] we solve for the effective resistance.

$$
\begin{aligned}
& I_{2}=\frac{\mathscr{E}}{2 R} ; I_{3}=\frac{I_{2}}{2}=\frac{\mathscr{E}}{4 R} ; I_{1}=I_{2}+I_{3}=\frac{\mathscr{E}}{2 R}+\frac{\mathscr{E}}{4 R}=\frac{3 \mathscr{C}}{4 R} \\
& I=2 I_{1}+I_{2}=2\left(\frac{3 \mathscr{E}}{4 R}\right)+\frac{\mathscr{E}}{2 R}=\frac{2 \mathscr{E}}{R} ; R_{\text {eq }}=\frac{\mathscr{E}}{I}=\frac{1}{2} R
\end{aligned}
$$

(c) As shown in the diagram, we again use symmetry to reduce the number of currents to three. We use Kirchhoff's junction rule at points a and b and the loop rule around the loop abgda (through the power source) to write three equations for the three unknown currents. We solve these equations for the ratio of the emf to the current through the emf (I) to calculate the effective resistance.

$$
\begin{aligned}
& I=3 I_{1}[1] ; I_{1}=2 I_{2} \\
& 0=-2 I_{1} R-I_{2} R+\mathscr{E}[3]
\end{aligned}
$$

We insert Eq. [2] into Eq. [3] and solve for $I_{1}$. Inserting $I_{1}$
 into Eq. [1] enables us to solve for the effective resistance.

$$
0=-2 I_{1} R-\frac{1}{2} I_{1} R+\mathscr{E} \rightarrow I_{1}=\frac{2 \mathscr{E}}{5 R} ; I=3 I_{1}=\frac{6 \mathscr{E}}{5 R} \rightarrow R_{\text {eq }}=\frac{\mathscr{E}}{I}=\frac{5}{6} R
$$

41. (a) To find the equivalent resistance between points a and c , apply a voltage between points a and c , find the current that flows from the voltage source, and then calculate $R_{\mathrm{eq}}=\mathscr{E} / I$. There is no symmetry to exploit.
(Bottom Loop) 1) $\mathscr{E}-R I_{3}=0$

| $(\mathrm{a}-\mathrm{d}-\mathrm{b})$ | 2) | $-R I_{1}-R I_{5}+R I_{2}=0$ |
| :--- | :--- | :--- |
| $(\mathrm{a}-\mathrm{b}-\mathrm{c})$ | 3) | $-R I_{2}-R I_{6}+R I_{3}=0$ |
| $(\mathrm{~d}-\mathrm{b}-\mathrm{c})$ | $4)$ | $-R I_{5}-R I_{6}+R^{\prime} I_{4}=0$ |
| (junction a) | 5) | $I=I_{1}+I_{2}+I_{3}$ |
| (junction d) | 6) | $I_{1}=I_{4}+I_{5}$ |
| (junction b) | $7)$ | $I_{2}+I_{5}=I_{6}$ |


(junction b)
7) $I_{2}+I_{5}=I_{6}$

From Eq. 1, substitute $I_{3}=\mathscr{E} / R$.
2) $-R I_{1}-R I_{5}+R I_{2}=0 \rightarrow I_{1}+I_{5}=I_{2}$
3) $-R I_{2}-R I_{6}+R \frac{\mathscr{E}}{R}=0 \rightarrow I_{2}+I_{6}=\frac{\mathscr{E}}{R}$
4) $-R I_{5}-R I_{6}+R^{\prime} I_{4}=0 \rightarrow R\left(I_{5}+I_{6}\right)=R^{\prime} I_{4}$
5) $I=I_{1}+I_{2}+\frac{\mathscr{C}}{R}$; 6) $\left.I_{1}=I_{4}+I_{5} ; 7\right) \quad I_{2}+I_{5}=I_{6}$

From Eq. 7, substitute $I_{6}=I_{2}+I_{5}$
2) $I_{1}+I_{5}=I_{2}$; 3) $I_{2}+I_{2}+I_{5}=\frac{\mathscr{C}}{R} \rightarrow 2 I_{2}+I_{5}=\frac{\mathscr{E}}{R}$
4) $R\left(2 I_{5}+I_{2}\right)=R^{\prime} I_{4} \quad$; 5) $I=I_{1}+I_{2}+\frac{\mathscr{E}}{R} \quad$; 6) $\quad I_{1}=I_{4}+I_{5}$

From Eq. 6, substitute $I_{1}=I_{4}+I_{5} \rightarrow I_{5}=I_{1}-I_{4}$
2) $2 I_{1}-I_{4}=I_{2}$;
3) $2 I_{2}+I_{1}-I_{4}=\frac{\mathscr{E}}{R}$
4) $R\left(2 I_{1}-2 I_{4}+I_{2}\right)=R^{\prime} I_{4} \quad$; 5) $I=I_{1}+I_{2}+\frac{\mathscr{E}}{R}$

From Eq. 2, substitute $2 I_{1}-I_{4}=I_{2} \rightarrow I_{4}=2 I_{1}-I_{2}$
3) $2 I_{2}+I_{1}-\left(2 I_{1}-I_{2}\right)=\frac{\mathscr{E}}{R} \rightarrow 3 I_{2}-I_{1}=\frac{\mathscr{E}}{R}$
4) $R\left(2 I_{1}-2\left(2 I_{1}-I_{2}\right)+I_{2}\right)=R^{\prime}\left(2 I_{1}-I_{2}\right) \rightarrow R\left(3 I_{2}-2 I_{1}\right)=R^{\prime}\left(2 I_{1}-I_{2}\right)$
5) $I=I_{1}+I_{2}+\frac{\mathscr{E}}{R}$

From Eq. 3, substitute $3 I_{2}-I_{1}=\frac{\mathscr{E}}{R} \rightarrow I_{1}=3 I_{2}-\frac{\mathscr{E}}{R}$
4) $R\left(3 I_{2}-2\left(3 I_{2}-\frac{\mathscr{C}}{R}\right)\right)=R^{\prime}\left(2\left(3 I_{2}-\frac{\mathscr{E}}{R}\right)-I_{2}\right) \rightarrow R\left(-3 I_{2}+2 \frac{\mathscr{E}}{R}\right)=R^{\prime}\left(5 I_{2}-2 \frac{\mathscr{E}}{R}\right)$

$$
\text { 5) } \quad I=3 I_{2}-\frac{\mathscr{E}}{R}+I_{2}+\frac{\mathscr{E}}{R} \rightarrow I=4 I_{2}
$$

From Eq. 5, substitute $I_{2}=\frac{1}{4} I$

$$
\text { 4) } R\left(-3\left(\frac{1}{4} I\right)+2 \frac{\mathscr{E}}{R}\right)=R^{\prime}\left(5\left(\frac{1}{4} I\right)-2 \frac{\mathscr{E}}{R}\right) \rightarrow \frac{\mathscr{E}}{I}=R_{\mathrm{eq}}=\frac{R\left(5 R^{\prime}+3 R\right)}{8\left(R+R^{\prime}\right)}
$$

(b) In this case, apply a voltage between points a and b. Now there is symmetry. In this case no current would flow through resistor $R^{\prime}$, and so that branch can be eliminated from the circuit. See the adjusted diagram. Now the upper left two resistors (from a to $d$ to $b$ ) are in series, and the lower right two resistors (from a to c to b ) are in series. These two combinations are in parallel with each other, and with the resistor between $a$ and $b$. The equivalent resistance is now relatively simple to calculate.

$$
R_{\mathrm{eq}}=\left(\frac{1}{2 R}+\frac{1}{R}+\frac{1}{2 R}\right)^{-1}=\left(\frac{4}{2 R}\right)^{-1}=\frac{1}{2} R
$$


42. Define $I_{1}$ to be the current to the right through the 2.00 V battery $\left(\mathscr{C}_{1}\right)$, and $I_{2}$ to be the current to the right through the 3.00 V battery $\left(\mathscr{E}_{2}\right)$. At the junction, they combine to give current $I=I_{1}+I_{2}$ to the left through the top branch. Apply Kirchhoff's loop rule first to the upper loop, and then to the outer loop, and solve for the currents.


$$
\begin{aligned}
& \mathscr{G}-I_{1} r-\left(I_{1}+I_{2}\right) R=0 \rightarrow \mathscr{G}_{1}-(R+r) I_{1}-R I_{2}=0 \\
& \mathscr{E}_{2}-I_{2} r-\left(I_{1}+I_{2}\right) R=0 \rightarrow \mathscr{E}_{2}-R I_{1}-(R+r) I_{2}=0
\end{aligned}
$$

Solve the first equation for $I_{2}$ and substitute into the second equation to solve for $I_{1}$.

$$
\begin{aligned}
& \mathscr{C}_{9}-(R+r) I_{1}-R I_{2}=0 \rightarrow I_{2}=\frac{\mathscr{G}_{1}-(R+r) I_{1}}{R}=\frac{2.00-4.450 I_{1}}{4.00}=0.500-1.1125 I_{1} \\
& \mathscr{E}_{2}-R I_{1}-(R+r) I_{2}=3.00 \mathrm{~V}-(4.00 \Omega) I_{1}-(4.45 \Omega)\left(0.500-1.1125 I_{1}\right)=0 \rightarrow \\
& I_{1}=-0.815 \mathrm{~A} ; I_{2}=0.500-1.1125 I_{1}=1.407 \mathrm{~A}
\end{aligned}
$$

The voltage across $R$ is its resistance times $I=I_{1}+I_{2}$.

$$
V_{R}=R\left(I_{1}+I_{2}\right)=(4.00 \Omega)(-0.815 \mathrm{~A}+1.407 \mathrm{~A})=2.368 \mathrm{~V} \approx 2.37 \mathrm{~V}
$$

Note that the top battery is being charged - the current is flowing through it from positive to negative.
43. We estimate the time between cycles of the wipers to be from 1 second to 15 seconds. We take these times as the time constant of the $R C$ combination.

$$
\tau=R C \rightarrow R_{1 \mathrm{~s}}=\frac{\tau}{C}=\frac{1 \mathrm{~s}}{1 \times 10^{-6} \mathrm{~F}}=10^{6} \Omega ; R_{1 \mathrm{~s}}=\frac{\tau}{C}=\frac{15 \mathrm{~s}}{1 \times 10^{-6} \mathrm{~F}}=15 \times 10^{6} \Omega
$$

So we estimate the range of resistance to be $1 \mathrm{M} \Omega-15 \mathrm{M} \Omega$.
44. (a) From Eq. 26-7 the product $R C$ is equal to the time constant.

$$
\tau=R C \rightarrow C=\frac{\tau}{R}=\frac{24.0 \times 10^{-6} \mathrm{~s}}{15.0 \times 10^{3} \Omega}=1.60 \times 10^{-9} \mathrm{~F}
$$

(b) Since the battery has an EMF of 24.0 V , if the voltage across the resistor is 16.0 V , the voltage across the capacitor will be 8.0 V as it charges. Use the expression for the voltage across a charging capacitor.

$$
\begin{aligned}
& V_{C}=\mathscr{E}\left(1-e^{-t / \tau}\right) \rightarrow e^{-t / \tau}=\left(1-\frac{V_{C}}{\mathscr{E}}\right) \rightarrow-\frac{t}{\tau}=\ln \left(1-\frac{V_{C}}{\mathscr{E}}\right) \rightarrow \\
& t=-\tau \ln \left(1-\frac{V_{C}}{\mathscr{E}}\right)=-\left(24.0 \times 10^{-6} \mathrm{~s}\right) \ln \left(1-\frac{8.0 \mathrm{~V}}{24.0 \mathrm{~V}}\right)=9.73 \times 10^{-6} \mathrm{~s}
\end{aligned}
$$

45. The current for a capacitor-charging circuit is given by Eq. 26-8, with $R$ the equivalent series resistance and $C$ the equivalent series capacitance.

$$
\begin{aligned}
I & =\frac{\mathscr{E}}{R_{\mathrm{eq}}} e^{-\left(\frac{t}{R_{\mathrm{eq}} C_{\mathrm{eq}}}\right)} \rightarrow \\
t & =-R_{\mathrm{eq}} C_{\mathrm{eq}} \ln \left(\frac{I R_{\mathrm{eq}}}{\mathscr{E}}\right)=-\left(R_{1}+R_{2}\right)\left(\frac{C_{1} C_{2}}{C_{1}+C_{2}}\right) \ln \left[\frac{I\left(R_{1}+R_{2}\right)}{\mathscr{E}}\right] \\
& =-(4400 \Omega)\left[\frac{\left(3.8 \times 10^{-6} \mathrm{~F}\right)^{2}}{7.6 \times 10^{-6} \mathrm{~F}}\right] \ln \left[\frac{\left(1.50 \times 10^{-3} \mathrm{~A}\right)(4400 \Omega)}{(12.0 \mathrm{~V})}\right]=5.0 \times 10^{-3} \mathrm{~s}
\end{aligned}
$$


46. Express the stored energy in terms of the charge on the capacitor, using Eq. 24-5. The charge on the capacitor is given by Eq. 26-6a.

$$
\begin{aligned}
& U=\frac{1}{2} \frac{Q^{2}}{C}=\frac{1}{2} \frac{\left[C \mathscr{C}\left(1-e^{-t / \tau}\right)\right]^{2}}{C}=\frac{1}{2} C^{C \mathscr{C}^{2}}\left(1-e^{-t / \tau}\right)^{2}=U_{\max }\left(1-e^{-t / \tau}\right)^{2} ; \\
& U=0.75 U_{\max } \rightarrow U_{\max }\left(1-e^{-t / \tau}\right)^{2}=0.75 U_{\max } \rightarrow\left(1-e^{-t / \tau}\right)^{2}=0.75 \rightarrow \\
& t=-\tau \ln (1-\sqrt{0.75})=2.01 \tau
\end{aligned}
$$

47. The capacitance is given by Eq. 24-8 and the resistance by Eq. 25-3. The capacitor plate separation $d$ is the same as the resistor length $\ell$. Calculate the time constant.

$$
\tau=R C=\left(\frac{\rho d}{A}\right)\left(K \varepsilon_{0} \frac{A}{d}\right)=\rho K \varepsilon_{0}=\left(1.0 \times 10^{12} \Omega \cdot \mathrm{~m}\right)(5.0)\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)=44 \mathrm{~s}
$$

48. The voltage of the discharging capacitor is given by $V_{\mathrm{C}}=V_{0} e^{-t / R C}$. The capacitor voltage is to be $0.0010 V_{0}$.

$$
\begin{aligned}
& V_{\mathrm{C}}=V_{0} e^{-t / R C} \rightarrow 0.0010 V_{0}=V_{0} e^{-t / R C} \rightarrow 0.0010=e^{-t / R C} \rightarrow \ln (0.010)=-\frac{t}{R C} \rightarrow \\
& t=-R C \ln (0.010)=-\left(8.7 \times 10^{3} \Omega\right)\left(3.0 \times 10^{-6} \mathrm{~F}\right) \ln (0.0010)=0.18 \mathrm{~s}
\end{aligned}
$$

49. (a) At $t=0$, the capacitor is uncharged and so there is no voltage difference across it. The capacitor is a "short," and so a simpler circuit can be drawn just by eliminating the capacitor. In this simpler circuit, the two resistors on the right are in parallel with each other, and then in series with the resistor by the switch. The current through the resistor by the switch splits equally when it reaches the junction of the parallel resistors.

$$
R_{\mathrm{eq}}=R+\left(\frac{1}{R}+\frac{1}{R}\right)^{-1}=\frac{3}{2} R \rightarrow I_{1}=\frac{\mathscr{E}}{R_{\mathrm{eq}}}=\frac{\mathscr{E}}{\frac{3}{2} R}=\frac{2 \mathscr{E}}{3 R} ; I_{2}=I_{3}=\frac{1}{2} I_{1}=\frac{\mathscr{E}}{3 R}
$$

(b) At $t=\infty$, the capacitor will be fully charged and there will be no current in the branch containing the capacitor, and so a simpler circuit can be drawn by eliminating that branch. In this simpler circuit, the two resistors are in series, and they both have the same current.

$$
R_{\text {eq }}=R+R=2 R \rightarrow I_{1}=I_{2}=\frac{\mathscr{E}}{R_{\text {eq }}}=\frac{\mathscr{E}}{2 R} ; I_{3}=0
$$

(c) At $t=\infty$, since there is no current through the branch containing the capacitor, there is no potential drop across that resistor. Therefore the voltage difference across the capacitor equals the voltage difference across the resistor through which $I_{2}$ flows.

$$
V_{C}=V_{R_{2}}=I_{2} R=\left(\frac{\mathscr{E}}{2 R}\right) R=\frac{1}{2} \mathscr{E}
$$

50. (a) With the currents and junctions labeled as in the diagram, we use point a for the junction rule and the right and left loops for the loop rule. We set current $I_{3}$ equal to the derivative of the charge on the capacitor and combine the equations to obtain a single differential equation in terms of the capacitor charge. Solving this equation yields the charging time constant.


$$
\begin{equation*}
I_{1}=I_{2}+I_{3}[1] ; \mathscr{E}-I_{1} R_{1}-I_{2} R_{2}=0 \quad[2] ;-\frac{Q}{C}+I_{2} R_{2}=0 \tag{3}
\end{equation*}
$$

We use Eq. [1] to eliminate $I_{1}$ in Eq. [2]. Then we use Eq. [3] to eliminate $I_{2}$ from Eq. [2].

$$
0=\mathscr{E}-\left(I_{2}+I_{3}\right) R_{1}-I_{2} R_{2} ; 0=\mathscr{E}-I_{2}\left(R_{1}+R_{2}\right)-I_{3} R_{1} ; 0=\mathscr{E}-\left(\frac{Q}{R_{2} C}\right)\left(R_{1}+R_{2}\right)-I_{3} R_{1}
$$

We set $I_{3}$ as the derivative of the charge on the capacitor and solve the differential equation by separation of variables.

$$
\begin{aligned}
& 0=\mathscr{E}-\left(\frac{Q}{R_{2} C}\right)\left(R_{1}+R_{2}\right)-\frac{d Q}{d t} R_{1} \rightarrow \int_{0}^{Q} \frac{d Q^{\prime}}{Q^{\prime}-\left(\frac{R_{2} C^{\mathscr{C}}}{R_{1}+R_{2}}\right)}=\int_{0}^{t} \frac{-\left(R_{1}+R_{2}\right)}{R_{1} R_{2} C} d t^{\prime} \rightarrow \\
& \ln \left[Q^{\prime}-\left(\frac{R_{2} C^{\mathscr{E}}}{R_{1}+R_{2}}\right)\right]_{0}^{Q}=-\left.\frac{\left(R_{1}+R_{2}\right)}{R_{1} R_{2} C} t^{\prime}\right|_{0} ^{t} \rightarrow \ln \left[\frac{Q-\left(\frac{R_{2} C^{\mathscr{C}}}{R_{1}+R_{2}}\right)}{\left(\frac{R_{2} C^{\mathscr{E}}}{R_{1}+R_{2}}\right)}\right]=-\frac{\left(R_{1}+R_{2}\right)}{R_{1} R_{2} C} t \rightarrow \\
& Q=\frac{R_{2} C^{\mathscr{E}}}{R_{1}+R_{2}}\left(1-e^{-\frac{\left(R_{1}+R_{2}\right)^{2}}{R_{1} R_{2} C} t}\right)
\end{aligned}
$$

From the exponential term we obtain the time constant, $\tau=\frac{R_{1} R_{2} C}{R_{1}+R_{2}}$.
(b) We obtain the maximum charge on the capacitor by taking the limit as time goes to infinity.

$$
Q_{\max }=\lim _{t \rightarrow \infty} \frac{R_{2} C^{\mathscr{E}}}{R_{1}+R_{2}}\left(1-e^{-\frac{\left(R_{1}+R_{2}\right)}{R_{1} R_{2} C} t}\right)=\frac{R_{2} C^{\mathscr{C}}}{R_{1}+R_{2}}
$$

51. (a) With the switch open, the resistors are in series with each other, and so have the same current. Apply the loop rule clockwise around the left loop, starting at the negative terminal of the source, to find the current.

$$
V-I R_{1}-I R_{2}=0 \rightarrow I=\frac{V}{R_{1}+R_{2}}=\frac{24 \mathrm{~V}}{8.8 \Omega+4.4 \Omega}=1.818 \mathrm{~A}
$$

The voltage at point a is the voltage across the $4.4 \Omega$-resistor.

$$
V_{a}=I R_{2}=(1.818 \mathrm{~A})(4.4 \Omega)=8.0 \mathrm{~V}
$$

(b) With the switch open, the capacitors are in series with each other. Find the equivalent capacitance. The charge stored on the equivalent capacitance is the same value as the charge stored on each capacitor in series.

$$
\begin{aligned}
& \frac{1}{C_{\text {eq }}}=\frac{1}{C_{1}}+\frac{1}{C_{2}} \rightarrow C_{\text {eq }}=\frac{C_{1} C_{2}}{C_{1}+C_{2}}=\frac{(0.48 \mu \mathrm{~F})(0.36 \mu \mathrm{~F})}{(0.48 \mu \mathrm{~F}+0.36 \mu \mathrm{~F})}=0.2057 \mu \mathrm{~F} \\
& Q_{\text {eq }}=V C_{\mathrm{eq}}=(24.0 \mathrm{~V})(0.2057 \mu \mathrm{~F})=4.937 \mu \mathrm{C}=Q_{1}=Q_{2}
\end{aligned}
$$

The voltage at point b is the voltage across the $0.24 \mu \mathrm{~F}$-capacitor.

$$
V_{b}=\frac{Q_{2}}{C_{2}}=\frac{4.937 \mu \mathrm{C}}{0.36 \mu \mathrm{~F}}=13.7 \mathrm{~V} \approx 14 \mathrm{~V}
$$

(c) The switch is now closed. After equilibrium has been reached a long time, there is no current flowing in the capacitors, and so the resistors are again in series, and the voltage of point a must be 8.0 V . Point b is connected by a conductor to point a , and so point b must be at the same potential as point a, 8.0 V . This also means that the voltage across $C_{2}$ is 8.0 V , and the voltage across $C_{1}$ is 16 V .
(d) Find the charge on each of the capacitors, which are no longer in series.

$$
\begin{aligned}
& Q_{1}=V_{1} C_{1}=(16 \mathrm{~V})(0.48 \mu \mathrm{~F})=7.68 \mu \mathrm{C} \\
& Q_{2}=V_{2} C_{2}=(8.0 \mathrm{~V})(0.36 \mu \mathrm{~F})=2.88 \mu \mathrm{C}
\end{aligned}
$$

When the switch was open, point $b$ had a net charge of 0 , because the charge on the negative plate of $C_{1}$ had the same magnitude as the charge on the positive plate of $C_{2}$. With the switch closed, these charges are not equal. The net charge at point $b$ is the sum of the charge on the negative plate of $C_{1}$ and the charge on the positive plate of $C_{2}$.

$$
Q_{\mathrm{b}}=-Q_{1}+Q_{2}=-7.68 \mu \mathrm{C}+2.88 \mu \mathrm{C}=-4.80 \mu \mathrm{C} \approx-4.8 \mu \mathrm{C}
$$

Thus $4.8 \mu \mathrm{C}$ of charge has passed through the switch, from right to left.
52. Because there are no simple series or parallel connections in this circuit, we use Kirchhoff's rules to write equations for the currents, as labeled in our diagram. We write junction equations for the junctions c and d . We then write loop equations for each of the three loops. We set the current through the capacitor equal to the derivative of the charge on the capacitor.

$$
\begin{align*}
& I=I_{1}+I_{3}[1] ; I=I_{2}+I_{4}[2] ; \mathscr{E}-\frac{Q_{1}}{C_{1}}-\frac{Q_{2}}{C_{2}}=0 \\
& \frac{Q_{1}}{C_{1}}-I_{3} R_{3}=0[4] ; \frac{Q_{2}}{C_{2}}-I_{4} R_{4}=0 \tag{5}
\end{align*}
$$



We differentiate Eq. [3] with respect to time and set the derivative of the charge equal to the current.

$$
0=\frac{d \mathscr{E}}{d t}-\frac{d Q_{1}}{d t} \frac{1}{C_{1}}-\frac{d Q_{2}}{d t} \frac{1}{C_{2}}=0-\frac{I_{1}}{C_{1}}-\frac{I_{2}}{C_{2}} \rightarrow I_{2}=-I_{1} \frac{C_{2}}{C_{1}}
$$

We then substitute Eq. [1] into Eq. [2] to eliminate $I$. Then using Eqs. [4] and [5] we eliminate $I_{3}$ and $I_{4}$ from the resulting equation. We eliminate $I_{2}$ using the derivative equation above.

$$
I_{1}+I_{3}=I_{2}+I_{4} ; I_{1}+\frac{Q_{1}}{R_{3} C_{1}}=-I_{1} \frac{C_{2}}{C_{1}}+\frac{Q_{2}}{R_{4} C_{2}}
$$

Finally, we eliminate $Q_{2}$ using Eq.[3].

$$
\begin{aligned}
& I_{1}+\frac{Q_{1}}{R_{3} C_{1}}=-I_{1} \frac{C_{2}}{C_{1}}+\frac{1}{R_{4}}\left(\mathscr{E}-\frac{Q_{1}}{C_{1}}\right) \rightarrow \mathscr{E}=I_{1} R_{4}\left(\frac{C_{1}+C_{2}}{C_{1}}\right)+Q_{1}\left(\frac{R_{4}+R_{3}}{R_{3} C_{1}}\right) \rightarrow \\
& \mathscr{E}=I_{1} R+\frac{Q_{1}}{C} \quad \text { where } \quad R=R_{4}\left(\frac{C_{1}+C_{2}}{C_{1}}\right) \text { and } C=C_{1}\left(\frac{R_{3}}{R_{4}+R_{3}}\right)
\end{aligned}
$$

This final equation represents a simple $R C$ circuit, with time constant $\tau=R C$.

$$
\begin{aligned}
\tau & =R C=R_{4}\left(\frac{C_{1}+C_{2}}{C_{1}}\right) C_{1}\left(\frac{R_{3}}{R_{4}+R_{3}}\right)=\frac{R_{4} R_{3}\left(C_{1}+C_{2}\right)}{R_{4}+R_{3}} \\
& =\frac{(8.8 \Omega)(4.4 \Omega)(0.48 \mu \mathrm{~F}+0.36 \mu \mathrm{~F})}{8.8 \Omega+4.4 \Omega}=2.5 \mu \mathrm{~s}
\end{aligned}
$$

53. The full-scale current is the reciprocal of the sensitivity.

$$
I_{\substack{\text { full- } \\ \text { scale }}}=\frac{1}{35,000 \Omega / \mathrm{V}}=2.9 \times 10^{-5} \mathrm{~A} \text { or } 29 \mu \mathrm{~A}
$$

54. The resistance is the full-scale voltage multiplied by the sensitivity.

$$
R=V_{\substack{\text { full } \\ \text { scale }}}(\text { sensitivity })=(250 \mathrm{~V})(35,000 \Omega / \mathrm{V})=8.75 \times 10^{6} \Omega \approx 8.8 \times 10^{6} \Omega
$$

55. (a) The current for full-scale deflection of the galvanometer is

$$
I_{\mathrm{G}}=\frac{1}{\text { sensitivity }}=\frac{1}{45,000 \Omega / \mathrm{V}}=2.222 \times 10^{-5} \mathrm{~A}
$$

To make an ammeter, a shunt resistor must be placed in parallel with the galvanometer. The voltage across the shunt resistor must be the voltage across the galvanometer. The total current is to be 2.0 A . See Figure 26-28 for a circuit diagram.

$$
\begin{aligned}
I_{\mathrm{G}} r_{\mathrm{G}}=I_{\mathrm{s}} R_{\mathrm{s}} \rightarrow R_{\mathrm{s}} & =\frac{I_{\mathrm{G}}}{I_{\mathrm{s}}} r_{\mathrm{G}}=\frac{I_{\mathrm{G}}}{I_{\text {full }}-I_{\mathrm{G}}} r_{\mathrm{G}}=\frac{2.222 \times 10^{-5} \mathrm{~A}}{2.0 \mathrm{~A}-2.222 \times 10^{-5} \mathrm{~A}}(20.0 \Omega) \\
& =2.222 \times 10^{-4} \Omega \approx 2.2 \times 10^{-4} \Omega \text { in parallel }
\end{aligned}
$$

(b) To make a voltmeter, a resistor must be placed in series with the galvanometer, so that the desired full scale voltage corresponds to the full scale current of the galvanometer. See Figure 26-29 for a circuit diagram. The total current must be the full-scale deflection current.

$$
\begin{aligned}
& V_{\text {full }}=I_{\mathrm{G}}\left(r_{\mathrm{G}}+R\right) \rightarrow \\
& R=\frac{V_{\text {full }}}{I_{\mathrm{G}}}-r_{\mathrm{G}}=\frac{1.00 \mathrm{~V}}{2.222 \times 10^{-5} \mathrm{~A}}-20.0 \Omega=44985 \Omega \approx 45 \mathrm{k} \Omega \text { in series }
\end{aligned}
$$

56. (a) To make an ammeter, a shunt resistor must be placed in parallel with the galvanometer. The voltage across the shunt resistor must be the voltage across the galvanometer. See Figure 26-28 for a circuit diagram.

$$
\begin{aligned}
& V_{\text {shunt }}=V_{\mathrm{G}} \rightarrow\left(I_{\text {full }}-I_{\mathrm{G}}\right) R_{\text {shunt }}=I_{\mathrm{G}} R_{\mathrm{G}} \rightarrow \\
& R_{\text {shunt }}=\frac{I_{\mathrm{G}} R_{\mathrm{G}}}{\left(I_{\text {full }}-I_{\mathrm{G}}\right)}=\frac{\left(55 \times 10^{-6} \mathrm{~A}\right)(32 \Omega)}{\left(25 \mathrm{~A}-55 \times 10^{-6} \mathrm{~A}\right)}=7.0 \times 10^{-5} \Omega
\end{aligned}
$$

(b) To make a voltmeter, a resistor must be placed in series with the galvanometer, so that the desired full-scale voltage corresponds to the full scale current of the galvanometer. See Figure 26-29 for a circuit diagram.

$$
V_{\text {full scale }}=I_{\mathrm{G}}\left(R_{\text {ser }}+R_{\mathrm{G}}\right) \rightarrow R_{\text {ser }}=\frac{V_{\text {full scale }}}{I_{\mathrm{G}}}-R_{\mathrm{G}}=\frac{250 \mathrm{~V}}{55 \times 10^{-6} \mathrm{~A}}-30 \Omega=4.5 \times 10^{6} \Omega
$$

57. We divide the full-scale voltage of the electronic module by the module's internal resistance to determine the current through the module that will give full-scale deflection. Since the module and $R_{2}$ are in parallel they will have the same voltage drop across them $(400 \mathrm{mV})$ and their currents will add to equal the current through $R_{1}$. We set the voltage drop across $R_{1}$ and $R_{2}$ equal to the 40 volts and solve the resulting equation for $R_{2}$.

$$
\begin{aligned}
& I_{\text {meter }}=\frac{V_{\text {meter }}}{r}=\frac{400 \mathrm{mV}}{100 \mathrm{M} \Omega}=4.00 \mathrm{nA} ; I_{2}=\frac{V_{\text {meter }}}{R_{2}} ; I_{1}=I_{2}+I_{\text {meter }}=\frac{V_{\text {meter }}}{R_{2}}+I_{\text {meter }} \\
& V=I_{1} R_{1}+V_{\text {meter }} \rightarrow\left(V-V_{\text {meter }}\right)=\left(\frac{V_{\text {meter }}}{R_{2}}+I_{\text {meter }}\right) R_{1} \rightarrow \\
& R_{2}=\frac{R_{1} V_{\text {meter }}}{\left(V-V_{\text {meter }}\right)-I_{\text {meter }} R_{1}}=\frac{\left(10 \times 10^{6} \Omega\right)(0.400 \mathrm{~V})}{(40 \mathrm{~V}-0.400 \mathrm{~V})-\left(4.00 \times 10^{-9} \mathrm{~A}\right)\left(10 \times 10^{6} \Omega\right)}=100 \mathrm{k} \Omega
\end{aligned}
$$

58. To make a voltmeter, a resistor $R_{\text {ser }}$ must be placed in series with the existing meter so that the desired full scale voltage corresponds to the full scale current of the galvanometer. We know that 25 mA produces full scale deflection of the galvanometer, so the voltage drop across the total meter must be 25 V when the current through the meter is 25 mA .

$$
\begin{aligned}
& V_{\substack{\text { full } \\
\text { scale }}}=I_{\substack{\text { full } \\
\text { scale }}} R_{\text {eq }}=I_{\substack{\text { full } \\
\text { scale }}}\left[R_{\text {ser }}+\left(\frac{1}{R_{\mathrm{G}}}+\frac{1}{R_{\text {shuut }}}\right)^{-1}\right] \rightarrow \\
& R_{\text {ser }}=\frac{V_{\text {full }}^{\text {scale }}}{I_{\substack{\text { full } \\
\text { scale }}}-\left(\frac{1}{R_{\mathrm{G}}}+\frac{1}{R_{\text {shunt }}}\right)^{-1}=\frac{25 \mathrm{~V}}{25 \times 10^{-3} \mathrm{~A}}-\left(\frac{1}{33 \Omega}+\frac{1}{0.20 \Omega}\right)^{-1}=999.8 \Omega \approx 1000 \Omega}
\end{aligned}
$$

The sensitivity is $\frac{1000 \Omega}{25 \mathrm{~V}}=40 \Omega / \mathrm{V}$
59. If the voltmeter were ideal, then the only resistance in the circuit would be the series combination of the two resistors. The current can be found from the battery and the equivalent resistance, and then the voltage across each resistor can be found.

$$
\begin{aligned}
& R_{\text {tot }}=R_{1}+R_{2}=44 \mathrm{k} \Omega+27 \mathrm{k} \Omega=71 \mathrm{k} \Omega ; I=\frac{V}{R_{\text {tot }}}=\frac{45 \mathrm{~V}}{71 \times 10^{3} \Omega}=6.338 \times 10^{-4} \mathrm{~A} \\
& V_{44}=I R_{1}=\left(6.338 \times 10^{-4} \mathrm{~A}\right)\left(44 \times 10^{3} \Omega\right)=27.89 \mathrm{~V} \\
& V_{27}=I R_{2}=\left(6.338 \times 10^{-4} \mathrm{~A}\right)\left(27 \times 10^{3} \Omega\right)=17.11 \mathrm{~V}
\end{aligned}
$$

Now put the voltmeter in parallel with the $44 \mathrm{k} \Omega$ resistor. Find its equivalent resistance, and then follow the same analysis as above.

$$
\begin{aligned}
& R_{\mathrm{eq}}=\left(\frac{1}{44 \mathrm{k} \Omega}+\frac{1}{95 \mathrm{k} \Omega}\right)^{-1}=30.07 \mathrm{k} \Omega \\
& R_{\mathrm{tot}}=R_{\mathrm{eq}}+R_{2}=30.07 \mathrm{k} \Omega+27 \mathrm{k} \Omega=57.07 \mathrm{k} \Omega \quad I=\frac{V}{R_{\mathrm{tot}}}=\frac{45 \mathrm{~V}}{57.07 \times 10^{3} \Omega}=7.885 \times 10^{-4} \mathrm{~A} \\
& V_{44}=V_{\mathrm{eq}}=I R_{\mathrm{eq}}=\left(7.885 \times 10^{-4} \mathrm{~A}\right)\left(30.07 \times 10^{3} \Omega\right)=23.71 \mathrm{~V} \approx 24 \mathrm{~V} \\
& \left.\% \text { error }=\frac{23.71 \mathrm{~V}-27.89 \mathrm{~V}}{27.89 \mathrm{~V}} \times 100=-15 \% \text { (reading too low }\right)
\end{aligned}
$$

And now put the voltmeter in parallel with the $27 \mathrm{k} \Omega$ resistor, and repeat the process.

$$
\begin{aligned}
& R_{\text {eq }}=\left(\frac{1}{27 \mathrm{k} \Omega}+\frac{1}{95 \mathrm{k} \Omega}\right)^{-1}=21.02 \mathrm{k} \Omega \\
& R_{\text {tot }}=R_{\text {eq }}+R_{\mathrm{l}}=21.02 \mathrm{k} \Omega+44 \mathrm{k} \Omega=65.02 \mathrm{k} \Omega \quad I=\frac{V}{R_{\text {to }}}=\frac{45 \mathrm{~V}}{65.02 \times 10^{3} \Omega}=6.921 \times 10^{-4} \mathrm{~A} \\
& V_{27}=V_{\text {eq }}=I R_{\text {eq }}=\left(6.921 \times 10^{-4} \mathrm{~A}\right)\left(21.02 \times 10^{3} \Omega\right)=14.55 \mathrm{~V} \approx 15 \mathrm{~V} \\
& \% \text { error }=\frac{14.55 \mathrm{~V}-17.11 \mathrm{~V}}{17.11 \mathrm{~V}} \times 100=-15 \% \text { (reading too low) }
\end{aligned}
$$

60. The total resistance with the ammeter present is $R_{\mathrm{eq}}=650 \Omega+480 \Omega+53 \Omega=1183 \Omega$. The voltage supplied by the battery is found from Ohm's law to be $V_{\text {batery }}=I R_{\text {eq }}=\left(5.25 \times 10^{-3} \mathrm{~A}\right)(1183 \Omega)$ $=6.211 \mathrm{~V}$. When the ammeter is removed, we assume that the battery voltage does not change. The equivalent resistance changes to $R_{\text {eq }}^{\prime}=1130 \Omega$, and the new current is again found from Ohm's law.

$$
I=\frac{V_{\text {battery }}}{R_{\mathrm{eq}}^{\prime}}=\frac{6.211 \mathrm{~V}}{1130 \Omega}=5.50 \times 10^{-3} \mathrm{~A}
$$

61. Find the equivalent resistance for the entire circuit, and then find the current drawn from the source. That current will be the ammeter reading. The ammeter and voltmeter symbols in the diagram below are each assumed to have resistance.

$$
\begin{aligned}
R_{\mathrm{eq}} & =1.0 \Omega+0.50 \Omega+7500 \Omega+\frac{(7500 \Omega)(15000 \Omega)}{(7500 \Omega+15000 \Omega)} \\
& =12501.5 \Omega \approx 12500 \Omega ; I_{\text {source }}=\frac{\mathscr{E}}{R_{\text {eq }}}=\frac{12.0 \mathrm{~V}}{12500 \Omega}=9.60 \times 10^{-4} \mathrm{~A}
\end{aligned}
$$

The voltmeter reading will be the source current times the equivalent resistance of the resistorvoltmeter combination.

$$
V_{\text {meter }}=I_{\text {source }} R_{\mathrm{eq}}=\left(9.60 \times 10^{-4} \mathrm{~A}\right) \frac{(7500 \Omega)(15000 \Omega)}{(7500 \Omega+15000 \Omega)}=4.8 \mathrm{~V}
$$

62. From the first diagram, write the sum of the currents at junction a, and then substitute in for those currents as shown.

$$
\begin{aligned}
& I_{1}=I_{1 \mathrm{~A}}+I_{1 \mathrm{~V}} \\
& \mathscr{E}-V_{R_{1}}-I_{1} R_{2}=0 \rightarrow I_{1}=\frac{\mathscr{E}-V_{R_{1}}}{R_{2}} ; I_{1 A}=\frac{V_{R_{1}}}{R_{1}} ; I_{1 V}=\frac{V_{1 \mathrm{~V}}}{R_{\mathrm{v}}} \\
& \frac{\mathscr{E}-V_{R_{1}}}{R_{2}}=\frac{V_{R_{1}}}{R_{1}}+\frac{V_{1 \mathrm{v}}}{R_{\mathrm{V}}}
\end{aligned}
$$



Then do a similar procedure for the second diagram.

$$
\begin{aligned}
& I_{2}=I_{2 \mathrm{~A}}+I_{2 \mathrm{~V}} \\
& \mathscr{E}-I_{2} R_{1}-V_{R_{2}}=0 \rightarrow I_{2}=\frac{\mathscr{E}-V_{R_{2}}}{R_{1}} ; I_{2 A}=\frac{V_{R_{2}}}{R_{2}} ; I_{2 V}=\frac{V_{2 \mathrm{~V}}}{R_{\mathrm{V}}} \\
& \frac{\mathscr{E}-V_{R_{2}}}{R_{1}}=\frac{V_{R_{2}}}{R_{2}}+\frac{V_{2 \mathrm{~V}}}{R_{\mathrm{V}}}
\end{aligned}
$$



Now there are two equations in the two unknowns of $R_{1}$ and $R_{2}$. Solve for the reciprocal values and then find the resistances. Assume that all resistances are measured in kilohms.

$$
\begin{aligned}
& \frac{\mathscr{E}-V_{R_{1}}}{R_{2}}=\frac{V_{R_{1}}}{R_{1}}+\frac{V_{1 \mathrm{~V}}}{R_{\mathrm{V}}} \rightarrow \frac{12.0-5.5}{R_{2}}=\frac{5.5}{R_{1}}+\frac{5.5}{18.0} \rightarrow \frac{6.5}{R_{2}}=\frac{5.5}{R_{1}}+0.30556 \\
& \frac{\mathscr{E}-V_{R_{2}}}{R_{1}}=\frac{V_{R_{2}}}{R_{2}}+\frac{V_{2 \mathrm{~V}}}{R_{\mathrm{V}}} \rightarrow \frac{12.0-4.0}{R_{1}}=\frac{4.0}{R_{2}}+\frac{4.0}{18.0} \rightarrow \frac{8.0}{R_{1}}=\frac{4.0}{R_{2}}+0.22222 \\
& \frac{8.0}{R_{1}}=\frac{4.0}{R_{2}}+0.22222 \rightarrow \frac{1}{R_{2}}=\frac{2}{R_{1}}-0.05556 \\
& \frac{6.5}{R_{2}}=\frac{5.5}{R_{1}}+0.30556 \rightarrow 6.5\left(\frac{2}{R_{1}}-0.05556\right)=\frac{5.5}{R_{1}}+0.30556 \rightarrow \frac{1}{R_{1}}=\frac{0.66667}{7.5} \rightarrow
\end{aligned}
$$

$$
R_{1}=11.25 \mathrm{k} \Omega ; \frac{1}{R_{2}}=\frac{2}{R_{1}}-0.05556 \rightarrow R_{2}=8.18 \mathrm{k} \Omega
$$

So the final results are $R_{1}=11 \mathrm{k} \Omega ; R_{2}=8.2 \mathrm{k} \Omega$
63. The sensitivity of the voltmeter is 1000 ohms per volt on the 3.0 volt scale, so it has a resistance of 3000 ohms. The circuit is shown in the diagram. Find the equivalent resistance of the meter-resistor parallel combination and the entire circuit.

$$
\begin{aligned}
& R_{\mathrm{p}}=\left(\frac{1}{R}+\frac{1}{R_{\mathrm{v}}}\right)^{-1}=\frac{R_{\mathrm{v}} R}{R_{\mathrm{v}}+R}=\frac{(3000 \Omega)(9400 \Omega)}{3000 \Omega+9400 \Omega}=2274 \Omega \\
& R_{\mathrm{eq}}=R+R_{\mathrm{p}}=2274 \Omega+9400 \Omega=11674 \Omega
\end{aligned}
$$



Using the meter reading of 2.3 volts, calculate the current into the parallel combination, which is the current delivered by the battery. Use that current to find the EMF of the battery.

$$
\begin{aligned}
& I=\frac{V}{R_{\mathrm{p}}}=\frac{2.3 \mathrm{~V}}{2274 \Omega}=1.011 \times 10^{-3} \mathrm{~A} \\
& \mathscr{E}=I R_{\mathrm{eq}}=\left(1.011 \times 10^{-3} \mathrm{~A}\right)(11674 \Omega)=11.80 \mathrm{~V} \approx 12 \mathrm{~V}
\end{aligned}
$$

64. By calling the voltmeter "high resistance," we can assume it has no current passing through it. Write Kirchhoff's loop rule for the circuit for both cases, starting with the negative pole of the battery and proceeding counterclockwise.

$$
\begin{array}{ll}
\text { Case 1: } V_{\text {meter }}=V_{1}=I_{1} R_{1} & \mathscr{E}-I_{1} r-I_{1} R_{1}=0 \rightarrow \mathscr{E}=I_{1}\left(r+R_{1}\right)=\frac{V_{1}}{R_{1}}\left(r+R_{1}\right) \\
\text { Case 2: } V_{\text {meter }}=V_{2}=I_{2} R_{2} & \mathscr{E}-I_{2} r-I_{2} R_{2}=0 \rightarrow \mathscr{E}=I_{2}\left(r+R_{2}\right)=\frac{V_{2}}{R_{2}}\left(r+R_{2}\right)
\end{array}
$$

Solve these two equations for the two unknowns of $\mathscr{E}$ and $r$.

$$
\begin{aligned}
& \mathscr{E}=\frac{V_{1}}{R_{1}}\left(r+R_{1}\right)=\frac{V_{2}}{R_{2}}\left(r+R_{2}\right) \rightarrow \\
& r=R_{1} R_{2}\left(\frac{V_{2}-V_{1}}{V_{1} R_{2}-V_{2} R_{1}}\right)=(35 \Omega)(14.0 \Omega)\left(\frac{8.1 \mathrm{~V}-9.7 \mathrm{~V}}{(9.7 \mathrm{~V})(14.0 \Omega)-(8.1 \mathrm{~V})(35 \Omega)}\right)=5.308 \Omega \approx 5.3 \Omega \\
& \mathscr{E}=\frac{V_{1}}{R_{1}}\left(r+R_{1}\right)=\frac{9.7 \mathrm{~V}}{35 \Omega}(5.308 \Omega+35 \Omega)=11.17 \mathrm{~V} \approx 11 \mathrm{~V}
\end{aligned}
$$

65. We connect the battery in series with the body and a resistor. The current through this series circuit is the voltage supplied by the battery divided by the sum of the resistances. The voltage drop across the body is equal to the current multiplied by the body's resistance. We set the voltage drop across the body equal to 0.25 V and solve for the necessary resistance.

$$
\begin{aligned}
& I=\frac{\mathscr{E}}{R+R_{B}} \\
& V=I R_{B}=\frac{\mathscr{E} R_{B}}{R+R_{B}} \rightarrow R=\left(\frac{\mathscr{E}}{V}-1\right) R_{B}=\left(\frac{1.5 \mathrm{~V}}{0.25 \mathrm{~V}}-1\right)(1800 \Omega)=9000 \Omega=9.0 \mathrm{k} \Omega
\end{aligned}
$$

66. (a) Since $P=V^{2} / R$ and the voltage is the same for each combination, the power and resistance are inversely related to each other. So for the 50 W output, use the higher-resistance filament. For the 100 W output, use the lower-resistance filament. For the 150 W output, use the filaments in parallel.
(b) $P=V^{2} / R \rightarrow$

$$
R=\frac{V^{2}}{P} \quad R_{50 \mathrm{~W}}=\frac{(120 \mathrm{~V})^{2}}{50 \mathrm{~W}}=288 \Omega \approx 290 \Omega \quad R_{\mathrm{100W}}=\frac{(120 \mathrm{~V})^{2}}{100 \mathrm{~W}}=144 \Omega \approx 140 \Omega
$$

As a check, the parallel combination of the resistors gives the following.

$$
R_{\mathrm{p}}=\frac{R_{1} R_{2}}{R_{1}+R_{2}}=\frac{(288 \Omega)(144 \Omega)}{288 \Omega+144 \Omega}=96 \Omega \quad P=\frac{V^{2}}{R}=\frac{(120 \mathrm{~V})^{2}}{96 \Omega}=150 \mathrm{~W}
$$

67. The voltage drop across the two wires is the 3.0 A current times their total resistance.

$$
V_{\text {wires }}=I R_{\text {wires }}=(3.0 \mathrm{~A})(0.0065 \Omega / \mathrm{m})(130 \mathrm{~m}) R_{\mathrm{p}}=2.535 \mathrm{~V} \approx 2.5 \mathrm{~V}
$$

Thus the voltage applied to the apparatus is $V=V_{\text {source }}-V_{\text {wires }}=120 \mathrm{~V}-2.535 \mathrm{~V}=117.465 \mathrm{~V} \approx 117 \mathrm{~V}$.
68. The charge on the capacitor and the current in the resistor both decrease exponentially, with a time constant of $\tau=R C$. The energy stored in the capacitor is given by $U=\frac{1}{2} \frac{Q^{2}}{C}$, and the power dissipated in the resistor is given by $P=I^{2} R$.

$$
\begin{aligned}
& Q=Q_{0} e^{-t / R C} ; I=I_{0} e^{-t / R C}=\frac{V_{0}}{R} e^{-t / R C}=\frac{Q_{0}}{R C} e^{-t / R C} \\
& \begin{aligned}
U_{\text {decrease }}=-\Delta U=U_{t=0}-U_{t=\tau}=\frac{1}{2}\left(\frac{Q^{2}}{C}\right)_{t=0}-\frac{1}{2}\left(\frac{Q^{2}}{C}\right)_{t=\tau}=\frac{1}{2} \frac{Q_{0}^{2}}{C}-\frac{1}{2}\left(\frac{Q_{0} e^{-1}}{C}\right)^{2}=\frac{1}{2} \frac{Q_{0}^{2}}{C}\left(1-e^{-2}\right) \\
U_{\text {dissipated }}=\int P d t=\int_{0}^{\tau} I^{2} R d t=\int_{0}^{\tau}\left(\frac{Q_{0}}{R C} e^{-t / R C}\right)^{2} R d t=\frac{Q_{0}^{2}}{R C^{2}} \int_{0}^{\tau} e^{-2 t / R C} d t=\frac{Q_{0}^{2}}{R C^{2}}\left(-\frac{R C}{2}\right)\left(e^{-2 t / R C}\right)_{0}^{\tau} \\
\quad=-\frac{1}{2} \frac{Q_{0}^{2}}{C}\left(e^{-2}-1\right)=\frac{1}{2} \frac{Q_{0}^{2}}{C}\left(1-e^{-2}\right)
\end{aligned}
\end{aligned}
$$

And so we see that $U_{\text {decrease }}=U_{\text {dissipated }}$.
69. The capacitor will charge up to $75 \%$ of its maximum value, and then discharge. The charging time is the time for one heartbeat.

$$
\begin{aligned}
& t_{\text {beat }}=\frac{1 \mathrm{~min}}{72 \text { beats }} \times \frac{60 \mathrm{~s}}{1 \mathrm{~min}}=0.8333 \mathrm{~s} \\
& V=V_{0}\left(1-e^{-\frac{t}{R C}}\right) \rightarrow 0.75 V_{0}=V_{0}\left(1-e^{-\frac{t_{\text {cot }}}{R C}}\right) \rightarrow e^{\frac{t_{\text {bat }}}{R C}}=0.25 \rightarrow\left(-\frac{t_{\text {beat }}}{R C}\right)=\ln (0.25) \rightarrow \\
& R=-\frac{t_{\text {beat }}}{C \ln (0.25)}=-\frac{0.8333 \mathrm{~s}}{\left(6.5 \times 10^{-6} \mathrm{~F}\right)(-1.3863)}=9.2 \times 10^{4} \Omega
\end{aligned}
$$

70. (a) Apply Ohm's law to find the current.

$$
I=\frac{V_{\text {body }}}{R_{\text {body }}}=\frac{110 \mathrm{~V}}{950 \Omega}=0.116 \mathrm{~A} \approx 0.12 \mathrm{~A}
$$

(b) The description of "alternative path to ground" is a statement that the $35 \Omega$ path in in parallel with the body. Thus the full 110 V is still applied across the body, and so the current is the same: 0.12 A .
(c) If the current is limited to a total of 1.5 A , then that current will get divided between the person and the parallel path. The voltage across the body and the parallel path will be the same, since they are in parallel.

$$
\begin{aligned}
& V_{\text {body }}=V_{\text {alternate }} \rightarrow I_{\text {body }} R_{\text {body }}=I_{\text {alternate }} R_{\text {alternate }}=\left(I_{\text {total }}-I_{\text {body }}\right) R_{\text {alternate }} \rightarrow \\
& I_{\text {body }}=I_{\text {total }} \frac{R_{\text {alternate }}}{\left(R_{\text {body }}+R_{\text {alternate }}\right)}=(1.5 \mathrm{~A}) \frac{35 \Omega}{950 \Omega+35 \Omega}=0.0533 \mathrm{~A} \approx 53 \mathrm{~mA}
\end{aligned}
$$

This is still a very dangerous current.
71. (a) If the ammeter shows no current with the closing of the switch, then points $B$ and $D$ must be at the same potential, because the ammeter has some small resistance. Any potential difference between points B and D would cause current to flow through the ammeter. Thus the potential drop from $A$ to $B$ must be the same as the drop from $A$ to $D$. Since points $B$ and $D$ are at the same potential, the potential drop from B to C must be the same as the drop from D to C . Use these two potential relationships to find the unknown resistance.

$$
\begin{gathered}
V_{\mathrm{BA}}=V_{\mathrm{DA}} \rightarrow I_{3} R_{3}=I_{1} R_{1} \rightarrow \frac{R_{3}}{R_{1}}=\frac{I_{1}}{I_{3}} \\
V_{\mathrm{CB}}=V_{\mathrm{CD}} \rightarrow I_{3} R_{x}=I_{1} R_{2} \rightarrow R_{x}=R_{2} \frac{I_{1}}{I_{3}}=R_{2} R_{3} / R_{1} \\
R_{x}=R_{2} \frac{R_{3}}{R_{1}}=(972 \Omega)\left(\frac{78.6 \Omega}{630 \Omega}\right)=121 \Omega
\end{gathered}
$$

(b)
72. From the solution to problem 71, the unknown resistance is given by $R_{x}=R_{2} R_{3} / R_{1}$. We use that with Eq. 25-3 to find the length of the wire.

$$
\begin{aligned}
& R_{x}=R_{2} \frac{R_{3}}{R_{1}}=\frac{\rho L}{A}=\frac{\rho L}{\pi(d / 2)^{2}}=\frac{4 \rho L}{\pi d^{2}} \rightarrow \\
& L=\frac{R_{2} R_{3} \pi d^{2}}{4 R_{1} \rho}=\frac{(29.2 \Omega)(3.48 \Omega) \pi\left(1.22 \times 10^{-3} \mathrm{~m}\right)^{2}}{4(38.0 \Omega)\left(10.6 \times 10^{-8} \Omega \cdot \mathrm{~m}\right)}=29.5 \mathrm{~m}
\end{aligned}
$$

73. Divide the power by the required voltage to determine the current drawn by the hearing aid.

$$
I=\frac{P}{V}=\frac{2.5 \mathrm{~W}}{4.0 \mathrm{~V}}=0.625 \mathrm{~A}
$$

Use Eq. 26-1 to calculate the terminal voltage across the three batteries for mercury and dry cells.

$$
\begin{aligned}
& V_{\mathrm{Hg}}=3(\mathscr{E}-I r)=3[1.35 \mathrm{~V}-(0.625 \mathrm{~A})(0.030 \Omega)]=3.99 \mathrm{~V} \\
& V_{\mathrm{D}}=3(\mathscr{E}-I r)=3[1.50 \mathrm{~V}-(0.625 \mathrm{~A})(0.35 \Omega)]=3.84 \mathrm{~V}
\end{aligned}
$$

The terminal voltage of the mercury cell batteries is closer to the required 4.0 V than the voltage from the dry cell.
74. One way is to connect $N$ resistors in series. If each resistor can dissipate 0.5 W , then it will take 7 resistors in series to dissipate 3.5 W . Since the resistors are in series, each resistor will be $1 / 7$ of the total resistance.

$$
R=\frac{R_{\mathrm{eq}}}{7}=\frac{3200 \Omega}{7}=457 \Omega \approx 460 \Omega
$$

So connect 7 resistors of $460 \Omega$ each, rated at $1 / 2 \mathrm{~W}$, in series.
Or, the resistors could be connected in parallel. Again, if each resistor watt can dissipate 0.5 W , then it will take 7 resistors in parallel to dissipate 3.5 W . Since the resistors are in parallel, the equivalent resistance will be $1 / 7$ of each individual resistance.

$$
\frac{1}{R_{\mathrm{eq}}}=7\left(\frac{1}{R}\right) \rightarrow R=7 R_{\mathrm{eq}}=7(3200 \Omega)=22.4 \mathrm{k} \Omega
$$

So connect 7 resistors of $22.4 \mathrm{k} \Omega$ each, rated at $1 / 2 \mathrm{~W}$, in parallel.
75. To build up a high voltage, the cells will have to be put in series. 120 V is needed from a series of 0.80 V cells. Thus $\frac{120 \mathrm{~V}}{0.80 \mathrm{~V} / \text { cell }}=150$ cells are needed to provide the desired voltage. Since these cells are all in series, their current will all be the same at 350 mA . To achieve the higher current desired, banks made of 150 cells each can be connected in parallel. Then their voltage will still be at 120 V , but the currents would add making a total of $\frac{1.3 \mathrm{~A}}{350 \times 10^{-3} \mathrm{~A} / \mathrm{bank}}=3.71$ banks $\approx 4$ banks. So the total number of cells is 600 cells. The panel area is 600 cells $\left(9.0 \times 10^{-4} \mathrm{~m}^{2} /\right.$ cell $)=0.54 \mathrm{~m}^{2}$. The cells should be wired in 4 banks of 150 cells in series per bank, with the banks in parallel. This will produce 1.4 A at 120 V . To optimize the output, always have the panel pointed directly at the sun.
76. (a) If the terminal voltage is to be 3.0 V , then the voltage across $R_{1}$ will be 9.0 V . This can be used to find the current, which then can be used to find the value of $R_{2}$.

$$
\begin{aligned}
& V_{1}=I R_{1} \rightarrow I=\frac{V_{1}}{R_{1}} \quad V_{2}=I R_{2} \rightarrow \\
& R_{2}=\frac{V_{2}}{I}=R_{1} \frac{V_{2}}{V_{1}}=(14.5 \Omega) \frac{3.0 \mathrm{~V}}{9.0 \mathrm{~V}}=4.833 \Omega \approx 4.8 \Omega
\end{aligned}
$$

(b) If the load has a resistance of $7.0 \Omega$, then the parallel combination of $R_{2}$ and the load must be used to analyze the circuit. The equivalent resistance of the circuit can be found and used to calculate the current in the circuit. Then the terminal voltage can be found from Ohm's law, using the parallel combination resistance.

$$
\begin{aligned}
& R_{2 \text { tood }}=\frac{R_{2} R_{\text {load }}}{R_{2}+R_{\text {load }}}=\frac{(4.833 \Omega)(7.0 \Omega)}{11.833 \Omega}=2.859 \Omega \quad R_{\text {eq }}=2.859 \Omega+14.5 \Omega=17.359 \Omega \\
& I=\frac{V}{R_{\text {eq }}}=\frac{12.0 \mathrm{~V}}{17.359 \Omega}=0.6913 \mathrm{~A} \quad V_{\mathrm{T}}=I R_{2+\text { toad }}=(0.6913 \mathrm{~A})(2.859 \Omega)=1.976 \mathrm{~V} \approx 2.0 \mathrm{~V}
\end{aligned}
$$

The presence of the load has affected the terminal voltage significantly.

DC Circuits
77. There are two answers because it is not known which direction the given current is flowing through the $4.0 \mathrm{k} \Omega$ resistor. Assume the current is to the right. The voltage across the $4.0 \mathrm{k} \Omega$ resistor is given by Ohm's law as $V=I R=\left(3.10 \times 10^{-3} \mathrm{~A}\right)(4000 \Omega)=12.4 \mathrm{~V}$. The voltage drop across the $8.0 \mathrm{k} \Omega$ must be the same, and the current through it is $I=\frac{V}{R}=\frac{12.4 \mathrm{~V}}{8000 \Omega}=1.55 \times 10^{-3} \mathrm{~A}$. The total current in the circuit is the sum of the two currents, and so $I_{\text {tot }}=4.65 \times 10^{-3} \mathrm{~A}$. That current can be used to find the terminal voltage of the battery. Write a loop equation, starting at the negative terminal of the unknown battery and going clockwise.

$$
\begin{aligned}
& V_{\mathrm{ab}}-(5000 \Omega) I_{\text {tot }}-12.4 \mathrm{~V}-12.0 \mathrm{~V}-(1.0 \Omega) I_{\text {tot }} \rightarrow \\
& V_{\mathrm{ab}}=24.4 \mathrm{~V}+(5001 \Omega)\left(4.65 \times 10^{-3} \mathrm{~A}\right)=47.65 \mathrm{~V} \approx 48 \mathrm{~V}
\end{aligned}
$$

If the current is to the left, then the voltage drop across the parallel combination of resistors is still 12.4 V , but with the opposite orientation. Again write a loop equation, starting at the negative terminal of the unknown battery and going clockwise. The current is now to the left.

$$
\begin{aligned}
& V_{\mathrm{ab}}+(5000 \Omega) I_{\text {tot }}+12.4 \mathrm{~V}-12.0 \mathrm{~V}+(1.0 \Omega) I_{\text {tot }} \rightarrow \\
& V_{\mathrm{ab}}=-0.4 \mathrm{~V}-(5001 \Omega)\left(4.65 \times 10^{-3} \mathrm{~A}\right)=-23.65 \mathrm{~V} \approx-24 \mathrm{~V}
\end{aligned}
$$

78. The terminal voltage and current are given for two situations. Apply Eq. 26-1 to both of these situations, and solve the resulting two equations for the two unknowns.

$$
\begin{aligned}
& V_{1}=\mathscr{E}-I_{1} r ; V_{2}=\mathscr{E}-I_{2} r \rightarrow \mathscr{E}=V_{1}+I_{1} r=V_{2}+I_{2} r \rightarrow \\
& r=\frac{V_{2}-V_{1}}{I_{1}-I_{2}}=\frac{47.3 \mathrm{~V}-40.8 \mathrm{~V}}{7.40 \mathrm{~A}-2.80 \mathrm{~A}}=1.413 \Omega \approx 1.4 \Omega \\
& \mathscr{E}=V_{1}+I_{1} r=40.8 \mathrm{~V}+(7.40 \mathrm{~A})(1.413 \Omega)=51.3 \mathrm{~V}
\end{aligned}
$$

79. The current in the circuit can be found from the resistance and the power dissipated. Then the product of that current and the equivalent resistance is equal to the battery voltage.

$$
\begin{aligned}
& P=I^{2} R \rightarrow I=\sqrt{\frac{P_{33}}{R_{33}}}=\sqrt{\frac{0.80 \mathrm{~W}}{33 \Omega}}=0.1557 \mathrm{~A} \\
& R_{\mathrm{eq}}=33 \Omega+\left(\frac{1}{68 \Omega}+\frac{1}{75 \Omega}\right)^{-1}=68.66 \Omega \quad V=I R_{\mathrm{eq}}=(0.1557 \mathrm{~A})(68.66 \Omega)=10.69 \mathrm{~V} \approx 11 \mathrm{~V}
\end{aligned}
$$

80. If the switches are both open, then the circuit is a simple series circuit. Use Kirchhoff's loop rule to find the current in that case.

$$
6.0 \mathrm{~V}-I(50 \Omega+20 \Omega+10 \Omega)=0 \rightarrow I=6.0 \mathrm{~V} / 80 \Omega=0.075 \mathrm{~A}
$$

If the switches are both closed, the $20-\Omega$ resistor is in parallel with $R$. Apply Kirchhoff's loop rule to the outer loop of the circuit, with the $20-\Omega$ resistor having the current found previously.

$$
6.0 \mathrm{~V}-I(50 \Omega)-(0.075 \mathrm{~A})(20 \Omega)=0 \rightarrow I=\frac{6.0 \mathrm{~V}-(0.075 \mathrm{~A})(20 \Omega)}{50 \Omega}=0.090 \mathrm{~A}
$$

This is the current in the parallel combination. Since 0.075 A is in the $20-\Omega$ resistor, 0.015 A must be in $R$. The voltage drops across $R$ and the $20-\Omega$ resistor are the same since they are in parallel.

$$
V_{20}=V_{R} \rightarrow I_{20} R_{20}=I_{R} R \rightarrow R=R_{20} \frac{I_{20}}{I_{R}}=(20 \Omega) \frac{0.075 \mathrm{~A}}{0.015 \mathrm{~A}}=100 \Omega
$$

81. (a) We assume that the ammeter is ideal and so has 0 resistance, but that the voltmeter has resistance $R_{\mathrm{v}}$. Then apply Ohm's law, using the equivalent resistance. We also assume the voltmeter is accurate, and so it is reading the voltage across the battery.

$$
V=I R_{\mathrm{eq}}=I \frac{1}{\frac{1}{R}+\frac{1}{R_{\mathrm{v}}}} \rightarrow V\left(\frac{1}{R}+\frac{1}{R_{\mathrm{v}}}\right)=I \rightarrow \frac{1}{R}+\frac{1}{R_{\mathrm{v}}}=\frac{I}{V} \rightarrow \frac{1}{R}=\frac{I}{V}-\frac{1}{R_{\mathrm{v}}}
$$

(b) We now assume the voltmeter is ideal, and so has an infinite resistance, but that the ammeter has resistance $R_{\mathrm{A}}$. We also assume that the voltmeter is accurate and so is reading the voltage across the battery.

$$
V=I R_{\mathrm{eq}}=I\left(R+R_{\mathrm{A}}\right) \rightarrow R+R_{\mathrm{A}}=\frac{V}{I} \rightarrow R=\frac{V}{I}-R_{\mathrm{A}}
$$

82. (a) The $12-\Omega$ and the $25-\Omega$ resistors are in parallel, with a net resistance $R_{1-2}$ as follows.

$$
R_{1-2}=\left(\frac{1}{12 \Omega}+\frac{1}{25 \Omega}\right)^{-1}=8.108 \Omega
$$

$R_{1-2}$ is in series with the $4.5-\Omega$ resistor, for a net resistance $R_{1-2 \cdot 3}$ as follows.

$$
R_{1-2 \cdot 3}=4.5 \Omega+8.108 \Omega=12.608 \Omega
$$

That net resistance is in parallel with the $18-\Omega$ resistor, for a final equivalent resistance as follows.

$$
R_{\mathrm{eq}}=\left(\frac{1}{12.608 \Omega}+\frac{1}{18 \Omega}\right)^{-1}=7.415 \Omega \approx 7.4 \Omega
$$

(b) Find the current in the $18-\Omega$ resistor by using Kirchhoff's loop rule for the loop containing the battery and the $18-\Omega$ resistor.

$$
\mathscr{E}-I_{18} R_{18}=0 \rightarrow I_{18}=\frac{\mathscr{E}}{R_{18}}=\frac{6.0 \mathrm{~V}}{18 \Omega}=0.33 \mathrm{~A}
$$

(c) Find the current in $R_{1-2}$ and the $4.5-\Omega$ resistor by using Kirchhoff's loop rule for the outer loop containing the battery and the resistors $R_{1-2}$ and the $4.5-\Omega$ resistor.

$$
\mathscr{E}-I_{1-2} R_{1-2}-I_{1-2} R_{4.5}=0 \rightarrow I_{1-2}=\frac{\mathcal{E}}{R_{1-2}+R_{4.5}}=\frac{6.0 \mathrm{~V}}{12.608 \Omega}=0.4759 \mathrm{~A}
$$

This current divides to go through the $12-\Omega$ and $25-\Omega$ resistors in such a way that the voltage drop across each of them is the same. Use that to find the current in the $12-\Omega$ resistor.

$$
\begin{aligned}
& I_{1-2}=I_{12}+I_{25} \rightarrow I_{25}=I_{1-2}-I_{12} \\
& V_{R_{12}}=V_{R_{25}} \rightarrow I_{12} R_{12}=I_{25} R_{25}=\left(I_{1-2}-I_{12}\right) R_{25} \rightarrow \\
& I_{12}=I_{1-2} \frac{R_{25}}{\left(R_{12}+R_{25}\right)}=(0.4759 \mathrm{~A}) \frac{25 \Omega}{37 \Omega}=0.32 \mathrm{~A}
\end{aligned}
$$

(d) The current in the $4.5-\Omega$ resistor was found above to be $I_{1-2}=0.4759 \mathrm{~A}$. Find the power accordingly.

$$
P_{4.5}=I_{1-2}^{2} R_{4.5}=(0.4759 \mathrm{~A})^{2}(4.5 \Omega)=1.019 \mathrm{~W} \approx 1.0 \mathrm{~W}
$$

83. Write Kirchhoff's loop rule for the circuit, and substitute for the current and the bulb resistance based on the bulb ratings.

$$
\begin{aligned}
& P_{\text {bulb }}=\frac{V_{\text {bulb }}^{2}}{R_{\text {bulb }}} \rightarrow R_{\text {bulb }}=\frac{V_{\text {bulb }}^{2}}{P_{\text {bulb }}} \quad P_{\text {bulb }}=I_{\text {bulb }} V_{\text {bulb }} \rightarrow I_{\text {bulb }}=\frac{P_{\text {bulb }}}{V_{\text {bulb }}} \\
& \mathscr{E}-I_{\text {bulb }} R-I_{\text {bulb }} R_{\text {bulb }}=0 \rightarrow \\
& R=\frac{\mathscr{E}}{I_{\text {bulb }}}-R_{\text {bulb }}=\frac{\mathscr{E}}{P_{\text {bulb }} / V_{\text {bulb }}}-\frac{V_{\text {bulb }}^{2}}{P_{\text {bulb }}}=\frac{V_{\text {bulb }}}{P_{\text {bulb }}}\left(\mathscr{E}-V_{\text {bulb }}\right)=\frac{3.0 \mathrm{~V}}{2.0 \mathrm{~W}}(9.0 \mathrm{~V}-3.0 \mathrm{~V})=9.0 \Omega
\end{aligned}
$$

84. The equivalent resistance of the circuit is the parallel combination of the bulb and the lower portion of the potentiometer, in series with the upper portion of the potentiometer. With the slide at position $x$, the resistance of the lower portion is $x R_{\mathrm{var}}$, and the resistance of the upper portion is $(1-x) R_{\mathrm{var}}$.
From that equivalent resistance, we find the current in the loop, the voltage across the bulb, and then the power expended in the bulb.

$$
\begin{aligned}
& R_{\text {parallel }}=\left(\frac{1}{R_{\text {lower }}}+\frac{1}{R_{\text {bulb }}}\right)^{-1}=\frac{R_{\text {lower }} R_{\text {bulb }}}{R_{\text {lower }}+R_{\text {bulb }}}=\frac{x R_{\text {var }} R_{\text {bulb }}}{x R_{\text {var }}+R_{\text {bulb }}} \\
& R_{\text {eq }}=(1-x) R_{\text {var }}+R_{\text {parallel }} ; I_{\text {loop }}=\frac{\mathscr{C}}{R_{\text {eq }}} ; V_{\text {bulb }}=I_{\text {loop }} R_{\text {parallel }} ; P_{\text {bulb }}=\frac{V_{\text {bulb }}^{2}}{R_{\text {bulb }}}
\end{aligned}
$$

(a) Consider the case in which $x=1.00$. In this case, the full battery potential is across the bulb, and so it is obvious that $V_{\text {bulb }}=120 \mathrm{~V}$. Thus $P_{\text {bulb }}=\frac{V_{\text {bulb }}^{2}}{R_{\text {bulb }}}=\frac{(120 \mathrm{~V})^{2}}{240 \Omega}=60 \mathrm{~W}$.
(b) Consider the case in which $x=0.65$.

$$
\begin{aligned}
& R_{\text {parallel }}=\frac{x R_{\text {var }} R_{\text {bulb }}}{x R_{\text {var }}+R_{\text {bulb }}}=\frac{(0.65)(150 \Omega)(240 \Omega)}{(0.65)(150 \Omega)+240 \Omega}=69.33 \Omega \\
& R_{\text {eq }}=(1-x) R_{\text {var }}+R_{\text {parallel }}=(0.35)(150 \Omega)+69.33 \Omega=121.83 \Omega \\
& I_{\text {loop }}=\frac{\mathscr{E}}{R_{\text {eq }}}=\frac{120 \mathrm{~V}}{121.83 \Omega}=0.9850 \mathrm{~A} ; V_{\text {bulb }}=(0.9850 \mathrm{~A})(69.33 \Omega)=68.29 \mathrm{~V} \\
& P_{\text {bulb }}=\frac{(68.29 \mathrm{~V})^{2}}{240 \Omega}=19.43 \mathrm{~W} \approx 19 \mathrm{~W}
\end{aligned}
$$

(c) Consider the case in which $x=0.35$.

$$
\begin{aligned}
& R_{\text {parallel }}=\frac{x R_{\text {var }} R_{\text {bulb }}}{x R_{\text {var }}+R_{\text {bulb }}}=\frac{(0.35)(150 \Omega)(240 \Omega)}{(0.35)(150 \Omega)+240 \Omega}=43.08 \Omega \\
& R_{\text {eq }}=(1-x) R_{\text {var }}+R_{\text {parallel }}=(0.65)(150 \Omega)+69.33 \Omega=140.58 \Omega \\
& I_{\text {loop }}=\frac{\mathscr{E}}{R_{\text {eq }}}=\frac{120 \mathrm{~V}}{140.58 \Omega}=0.8536 \mathrm{~A} ; V_{\text {bulb }}=(0.8536 \mathrm{~A})(43.08 \Omega)=36.77 \mathrm{~V} \\
& P_{\text {bulb }}=\frac{(36.77 \mathrm{~V})^{2}}{240 \Omega}=5.63 \mathrm{~W} \approx 5.6 \mathrm{~W}
\end{aligned}
$$

85. (a) When the galvanometer gives a null reading, no current is passing through the galvanometer or the emf that is being measured. All of the current is flowing through the slide wire resistance. Application of the loop rule to the lower loop gives $\mathscr{E}-I R=0$, since there is no current through the emf to cause voltage drop across any internal resistance. The amount of current flowing through the slide wire resistor will be the same no matter what emf is used since no current is flowing through the lower loop. Apply this relationship to the two emf's.

$$
\mathscr{E}_{x}-I R_{x}=0 ; \mathscr{E}_{\mathrm{s}}-I R_{\mathrm{s}}=0 \rightarrow ; I=\frac{\mathscr{C}_{x}}{R_{x}}=\frac{\mathscr{E}_{\mathrm{s}}}{R_{\mathrm{s}}} \rightarrow \mathscr{E}_{x}=\left(\frac{R_{x}}{R_{\mathrm{s}}}\right) \mathscr{E}_{\mathrm{s}}
$$

(b) Use the equation derived above. We use the fact that the resistance is proportional to the length of the wire, by Eq. 25-3, $R=\rho \ell / A$.

$$
\mathscr{E}_{x}=\left(\frac{R_{x}}{R_{\mathrm{s}}}\right) \mathscr{E}_{\mathrm{s}}=\left(\frac{\rho \frac{\ell_{x}}{A}}{\rho \frac{\ell_{\mathrm{s}}}{A}}\right) \mathscr{E}_{\mathrm{s}}=\left(\frac{\ell_{x}}{\ell_{\mathrm{s}}}\right) \mathscr{E}_{\mathrm{s}}=\left(\frac{45.8 \mathrm{~cm}}{33.6 \mathrm{~cm}}\right)(1.0182 \mathrm{~V})=1.39 \mathrm{~V}
$$

(c) If there is current in the galvanometer, then the voltage between points A and C is uncertainty by the voltage drop across the galvanometer, which is $V_{\mathrm{G}}=I_{\mathrm{G}} R_{\mathrm{G}}=\left(0.012 \times 10^{-3} \mathrm{~A}\right)(35 \Omega)$ $=4.2 \times 10^{-4} \mathrm{~V}$. The uncertainty might of course be more than this, due to uncertainties compounding from having to measure distance for both the standard emf and the unknown emf. Measuring the distances also has some uncertainty associated with it.
(d) Using this null method means that the (unknown) internal resistance of the unknown emf does not enter into the calculation. No current passes through the unknown emf, and so there is no voltage drop across that internal resistance.
86. (a) In normal operation, the capacitor is fully charged by the power supply, and so the capacitor voltage is the same as the power supply voltage, and there will be no current through the resistor. If there is an interruption, the capacitor voltage will decrease exponentially - it will discharge. We want the voltage across the capacitor to be at $75 \%$ of the full voltage after 0.20 s . Use Eq. $26-9 \mathrm{~b}$ for the discharging capacitor.

$$
\begin{aligned}
& V=V_{0} e^{-t / R C} ; 0.75 V_{0}=V_{0} e^{-(0.20 \mathrm{~s}) / R C} \rightarrow 0.75=e^{-(0.2 \mathrm{~s}) / R C} \rightarrow \\
& R=\frac{-(0.20 \mathrm{~s})}{C \ln (0.75)}=\frac{-(0.20 \mathrm{~s})}{\left(8.5 \times 10^{-6} \mathrm{~F}\right) \ln (0.75)}=81790 \Omega \approx 82 \mathrm{k} \Omega
\end{aligned}
$$

(b) When the power supply is functioning normally, there is no voltage across the resistor, so the device should NOT be connected between terminals $a$ and $b$. If the power supply is not functioning normally, there will be a larger voltage across the capacitor than across the capacitor-resistor combination, since some current might be present. This current would result in a voltage drop across the resistor. To have the highest voltage in case of a power supply failure, the device should be connected between terminals $b$ and $c$.
87. Note that, based on the significant figures of the resistors, that the $1.0-\Omega$ resistor will not change the equivalent resistance of the circuit as determined by the resistors in the switch bank.

Case 1: $n=0$ switch closed. The effective resistance of the circuit is $16.0 \mathrm{k} \Omega$. The current in the circuit is $I=\frac{16 \mathrm{~V}}{16.0 \mathrm{k} \Omega}=1.0 \mathrm{~mA}$. The voltage across the $1.0-\Omega$ resistor is $V=I R$ $=(1.0 \mathrm{~mA})(1.0 \Omega)=1.0 \mathrm{mV}$.

Case 2: $n=1$ switch closed. The effective resistance of the circuit is $8.0 \mathrm{k} \Omega$. The current in the circuit is $I=\frac{16 \mathrm{~V}}{8.0 \mathrm{k} \Omega}=2.0 \mathrm{~mA}$. The voltage across the $1.0-\Omega$ resistor is $V=I R$ $=(2.0 \mathrm{~mA})(1.0 \Omega)=2.0 \mathrm{mV}$.
Case 3: $n=2$ switch closed. The effective resistance of the circuit is $4.0 \mathrm{k} \Omega$. The current in the circuit is $I=\frac{16 \mathrm{~V}}{4.0 \mathrm{k} \Omega}=4.0 \mathrm{~mA}$. The voltage across the $1.0-\Omega$ resistor is $V=I R$ $=(4.0 \mathrm{~mA})(1.0 \Omega)=4.0 \mathrm{mV}$.
Case 4: $n=3$ and $n=1$ switches closed. The effective resistance of the circuit is found by the parallel combination of the $2.0-\mathrm{k} \Omega$ and $8.0-\mathrm{k} \Omega$ resistors.

$$
R_{\mathrm{eq}}=\left(\frac{1}{2.0 \mathrm{k} \Omega}+\frac{1}{8.0 \mathrm{k} \Omega}\right)^{-1}=1.6 \mathrm{k} \Omega
$$

The current in the circuit is $I=\frac{16 \mathrm{~V}}{1.6 \mathrm{k} \Omega}=10 \mathrm{~mA}$. The voltage across the $1.0-\Omega$ resistor is

$$
V=I R=(10 \mathrm{~mA})(1.0 \Omega)=10 \mathrm{mV} .
$$

So in each case, the voltage across the $1.0-\Omega$ resistor, if taken in mV , is the expected analog value corresponding to the digital number set by the switches.
88. We have labeled the resistors and the currents through the resistors with the value of the specific resistance, and the emf's with the appropriate voltage value. We apply the junction rule to points a and b , and then apply the loop rule to loops 1,2 , and 3. This enables us to solve for all of the currents.

$$
\begin{array}{ll}
I_{5}=I_{6}+I_{\text {top }} ; I_{\text {top }}+I_{6.8}=I_{12} & \rightarrow I_{5}-I_{6}=I_{12}-I_{6.8} \rightarrow \\
I_{5}+I_{6.8}=I_{12}+I_{6} & {[1]}  \tag{1}\\
\mathscr{E}_{6}+\mathscr{G}_{0}-I_{5} R_{5}-I_{6} R_{6}=0 & {[2](\text { loop } 1)} \\
\mathscr{E}_{4}+\mathscr{C}_{8}-I_{12} R_{12}-I_{6.8} R_{6.8}=0 & {[3](\text { loop 2) }} \\
I_{12} R_{12}-I_{6} R_{6}=0 & {[4](\text { loop 3) }}
\end{array}
$$



Use Eq. 4 to substitute $I_{6} R_{6}=I_{12} R_{12}$ and $I_{6}=I_{12} \frac{R_{12}}{R_{6}}=2 I_{12}$. Also combine the emf's by adding the voltages.

$$
\begin{equation*}
I_{5}+I_{6.8}=3 I_{12}[1] ; \mathscr{G}_{5}-I_{5} R_{5}-I_{12} R_{12}=0 \quad[2] ; \mathscr{G}_{12}-I_{12} R_{12}-I_{6.8} R_{6.8}=0 \tag{3}
\end{equation*}
$$

Use Eq. 1 to eliminate $I_{6.8}$ by $I_{6.8}=3 I_{12}-I_{5}$.

$$
\begin{align*}
& \mathscr{G}_{5}-I_{5} R_{5}-I_{12} R_{12}=0 \quad[2] \\
& \mathscr{G}_{2}-I_{12} R_{12}-\left(3 I_{12}-I_{5}\right) R_{6.8}=0 \rightarrow \mathscr{G}_{2}-I_{12}\left(R_{12}+3 R_{6.8}\right)+I_{5} R_{6.8}=0 \tag{3}
\end{align*}
$$

Use Eq. 2 to eliminate $I_{5}$ by $I_{5}=\frac{\mathscr{G}_{5}-I_{12} R_{12}}{R_{5}}$, and then solve for $I_{12}$.

$$
\mathscr{G}_{12}-I_{12}\left(R_{12}+3 R_{6.8}\right)+\left[\frac{\mathscr{G}_{5}-I_{12} R_{12}}{R_{5}}\right] R_{6.8}=0 \rightarrow
$$

$$
\begin{aligned}
I_{12} & =\frac{\mathscr{G}_{2} R_{5}+\mathscr{G}_{5} R_{6.8}}{R_{12} R_{5}+3 R_{6.8} R_{5}+R_{12} R_{6.8}}=\frac{(12.00 \mathrm{~V})(5.00 \Omega)+(15.00 \mathrm{~V})(6.800 \Omega)}{(12.00 \Omega)(5.00 \Omega)+3(6.800 \Omega)(5.00 \Omega)+(12.00 \Omega)(6.800 \Omega)} \\
& =0.66502 \mathrm{~A} \approx 0.665 \mathrm{~A}=I_{12} \\
I_{5} & =\frac{\mathscr{G}_{5}-I_{12} R_{12}}{R_{5}}=\frac{(15.00 \mathrm{~V})-(0.66502 \mathrm{~A})(12.00 \Omega)}{(5.00 \Omega)}=1.40395 \mathrm{~A} \approx 1.40 \mathrm{~A}=I_{5} \\
I_{6.8} & =3 I_{12}-I_{5}=3(0.66502 \mathrm{~A})-1.40395 \mathrm{~A}=0.59111 \mathrm{~A} \approx 0.591 \mathrm{~A}=I_{6.8} \\
I_{6} & =2 I_{12}=2(0.66502 \mathrm{~A}) \approx 1.33 \mathrm{~A}=I_{6}
\end{aligned}
$$

89. (a) After the capacitor is fully charged, there is no current through it, and so it behaves like an "open" in the circuit. In the circuit diagram, this means that $I_{5}=0, I_{1}=I_{3}$, and $I_{2}=I_{4}$. Write loop equations for the leftmost loop and the outer loop in order to solve for the currents.

$$
\begin{aligned}
& \mathscr{E}-I_{2}\left(R_{2}+R_{4}\right)=0 \rightarrow I_{2}=\frac{\mathscr{E}}{R_{2}+R_{4}}=\frac{12.0 \mathrm{~V}}{10.0 \Omega}=1.20 \mathrm{~A} \\
& \mathscr{E}-I_{1}\left(R_{1}+R_{3}\right)=0 \rightarrow I_{1}=\frac{\mathscr{E}}{R_{1}+R_{3}}=\frac{12.0 \mathrm{~V}}{15.0 \Omega}=0.800 \mathrm{~A}
\end{aligned}
$$



Use these currents to find the voltage at points c and d , which will give the voltage across the capacitor.

$$
\begin{aligned}
& V_{\mathrm{c}}=\mathscr{E}-I_{2} R_{2}=12.0 \mathrm{~V}-(1.20 \mathrm{~A})(1.0 \Omega)=10.8 \mathrm{~V} \\
& V_{\mathrm{d}}=\mathscr{E}-I_{1} R_{1}=12.0 \mathrm{~V}-(0.800 \mathrm{~A})(10.0 \Omega)=4.00 \mathrm{~V} \\
& V_{\mathrm{cd}}=10.8 \mathrm{~V}-4.00 \mathrm{~V}=6.8 \mathrm{~V} ; Q=C V=(2.2 \mu \mathrm{~F})(6.8 \mathrm{~V})=14.96 \mu \mathrm{C} \approx 15 \mu \mathrm{C}
\end{aligned}
$$

(b) When the switch is opened, the emf is taken out of the circuit. Then we have the capacitor discharging through an equivalent resistance. That equivalent resistance is the series combination of $R_{1}$ and $R_{2}$, in parallel with the series combination of $R_{3}$ and $R_{4}$. Use the expression for discharging a capacitor, Eq. 26-9a.

$$
\begin{aligned}
& R_{\mathrm{eq}}=\left(\frac{1}{R_{1}+R_{2}}+\frac{1}{R_{3}+R_{4}}\right)^{-1}=\left(\frac{1}{11.0 \Omega}+\frac{1}{14.0 \Omega}\right)^{-1}=6.16 \Omega \\
& Q=Q_{0} e^{-t / R_{\mathrm{ec}} c}=0.030 Q_{0} \rightarrow \\
& t=-R_{\mathrm{eq}} C \ln (0.030)=-(6.16 \Omega)\left(2.2 \times 10^{-6} \mathrm{~F}\right) \ln (0.030)=4.8 \times 10^{-5} \mathrm{~s}
\end{aligned}
$$

90. (a) The time constant of the $R C$ circuit is given by Eq. 26-7.

$$
\tau=R C=(33.0 \mathrm{k} \Omega)(4.00 \mu \mathrm{~F})=132 \mathrm{~ms}
$$

During the charging cycle, the charge and the voltage on the capacitor increases exponentially as in Eq. $26-6 \mathrm{~b}$. We solve this equation for the time it takes the circuit to reach 90.0 V .

$$
V=\mathscr{E}\left(1-e^{-t / \tau}\right) \rightarrow t=-\tau \ln \left(1-\frac{V}{\mathscr{E}}\right)=-(132 \mathrm{~ms}) \ln \left(1-\frac{90.0 \mathrm{~V}}{100.0 \mathrm{~V}}\right)=304 \mathrm{~ms}
$$

(b) When the neon bulb starts conducting, the voltage on the capacitor drops quickly to 65.0 V and then starts charging. We can find the recharging time by first finding the time for the capacitor to reach 65.0 V , and then subtract that time from the time required to reach 90.0 V .

$$
\begin{aligned}
& t=-\tau \ln \left(1-\frac{V}{\mathscr{E}}\right)=-(132 \mathrm{~ms}) \ln \left(1-\frac{65.0 \mathrm{~V}}{100.0 \mathrm{~V}}\right)=139 \mathrm{~ms} \\
& \Delta t=304 \mathrm{~ms}-139 \mathrm{~ms}=165 \mathrm{~ms} ; t_{2}=304 \mathrm{~ms}+165 \mathrm{~ms}=469 \mathrm{~ms}
\end{aligned}
$$

(c) The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH26.XLS," on tab "Problem 26.90c."

91. We represent the $10.00-\mathrm{M} \Omega$ resistor by $R_{10}$, and the resistance of the voltmeter as $R_{\mathrm{V}}$. In the first configuration, we find the equivalent resistance $R_{\mathrm{eq}}$, the current in the circuit $I_{\mathrm{A}}$, and the voltage drop across $R$.

$$
R_{\mathrm{eqA}}=R+\frac{R_{10} R_{\mathrm{V}}}{R_{10}+R_{\mathrm{V}}} ; I_{A}=\frac{\mathscr{E}}{R_{\mathrm{eqA}}} ; V_{\mathrm{R}}=I_{\mathrm{A}} R=\mathscr{E}-V_{\mathrm{A}} \rightarrow \mathscr{E} \frac{R}{R_{\mathrm{eqA}}}=\mathscr{E}-V_{\mathrm{A}}
$$

In the second configuration, we find the equivalent resistance $R_{\text {eq }}$, the current in the circuit $I_{\mathrm{B}}$, and the voltage drop across $R_{10}$.

$$
R_{\mathrm{eqB}}=R_{10}+\frac{R R_{\mathrm{V}}}{R+R_{\mathrm{V}}} ; I_{B}=\frac{\mathscr{E}}{R_{\mathrm{eqB}}} ; V_{\mathrm{R}_{10}}=I_{\mathrm{B}} R_{10}=\mathscr{E}-V_{\mathrm{B}} \rightarrow \mathscr{E} \frac{R_{10}}{R_{\mathrm{eqB}}}=\mathscr{E}-V_{\mathrm{B}}
$$

We now have two equations in the two unknowns of $R$ and $R_{\mathrm{V}}$. We solve the second equation for $R_{\mathrm{V}}$ and substitute that into the first equation. We are leaving out much of the algebra in this solution.

$$
\begin{aligned}
& \mathscr{E} \frac{R}{R_{\mathrm{eqA}}}=\mathscr{E} \frac{R}{R+\frac{R_{10} R_{\mathrm{V}}}{R_{10}+R_{\mathrm{V}}}}=\mathscr{E}-V_{\mathrm{A}} ; \\
& \mathscr{E} \frac{R_{10}}{R_{\mathrm{eqB}}}=\mathscr{E} \frac{R_{10}}{R_{10}+\frac{R R_{\mathrm{V}}}{R+R_{\mathrm{V}}}}=\mathscr{E}-V_{\mathrm{B}} \rightarrow R_{\mathrm{V}}=\frac{V_{\mathrm{B}} R_{10} R}{\left(\mathscr{E} R-V_{\mathrm{B}} R_{10}-V_{\mathrm{B}} R\right)} \\
& \mathscr{E}-V_{\mathrm{A}}=\mathscr{E} \frac{R}{R+\frac{R_{10} R_{\mathrm{V}}}{R_{10}+R_{\mathrm{V}}}}=\mathscr{E} \frac{R}{R+\frac{R_{10}\left[\frac{V_{\mathrm{B}} R_{10} R}{\left(\mathscr{E} R-V_{\mathrm{B}} R_{10}-V_{\mathrm{B}} R\right)}\right]}{R_{10}+\left[\frac{V_{\mathrm{B}} R_{10} R}{\left(\mathscr{E} R-V_{\mathrm{B}} R_{10}-V_{\mathrm{B}} R\right)}\right]}} \rightarrow \\
& R=\frac{V_{\mathrm{B}}}{V_{\mathrm{A}}} R_{10}=\frac{7.317 \mathrm{~V}}{0.366 \mathrm{~V}}(10.00 \mathrm{M} \Omega)=199.92 \mathrm{M} \Omega \approx 200 \mathrm{M} \Omega \\
& \text { (3 sig. fig.) }
\end{aligned}
$$

92. Let the internal resistance of the voltmeter be indicated by $R_{\mathrm{V}}$, and let the $15-\mathrm{M} \Omega$ resistance be indicated by $R_{15}$. We calculate the current through the probe and voltmeter as the voltage across the probe divided by the equivalent resistance of the problem and the voltmeter. We then set the voltage drop across the voltmeter equal to the product of the current and the parallel combination of $R_{\mathrm{V}}$ and $R_{15}$. This can be solved for the unknown resistance.

$$
\begin{aligned}
I & =\frac{V}{R+\frac{R_{15} R_{\mathrm{V}}}{R_{15}+R_{\mathrm{V}}}} ; V_{\mathrm{V}}=I \frac{R_{15} R_{\mathrm{V}}}{R_{15}+R_{\mathrm{V}}}=\frac{V}{R+\frac{R_{15} R_{\mathrm{V}}}{R_{15}+R_{\mathrm{V}}}} \frac{R_{15} R_{\mathrm{V}}}{R_{15}+R_{\mathrm{V}}}=\frac{V R_{15} R_{\mathrm{V}}}{R\left(R_{15}+R_{\mathrm{V}}\right)+R_{15} R_{\mathrm{V}}} \rightarrow \\
R & =\frac{\frac{V}{V_{\mathrm{V}}} R_{15} R_{\mathrm{V}}-R_{15} R_{\mathrm{V}}}{\left(R_{15}+R_{\mathrm{V}}\right)}=\frac{R_{15} R_{\mathrm{V}}}{\left(R_{15}+R_{\mathrm{V}}\right)}\left(\frac{V}{V_{\mathrm{V}}}-1\right)=\frac{(15 \mathrm{M} \Omega)(10 \mathrm{M} \Omega)}{(25 \mathrm{M} \Omega)}\left(\frac{50,000 \mathrm{~V}}{50 \mathrm{~V}}-1\right) \\
& =5994 \mathrm{M} \Omega \approx 6000 \mathrm{M} \Omega=6 \mathrm{G} \Omega
\end{aligned}
$$

93. The charge and current are given by Eq. 26-6a and Eq. 26-8, respectively.

$$
\begin{aligned}
& Q=C \mathscr{E}\left(1-e^{-t / R C}\right) ; I=\frac{\mathscr{E}}{R} e^{-t / R C} ; \tau=R C=\left(1.5 \times 10^{4} \Omega\right)\left(3.0 \times 10^{-7} \mathrm{~F}\right)=4.5 \times 10^{-3} \mathrm{~s} \\
& 0.63 Q_{\text {final }}=0.63 C \mathscr{E}=0.63\left(3.0 \times 10^{-7} \mathrm{~F}\right)(9.0 \mathrm{~V})=1.70 \times 10^{-6} \mathrm{C} \\
& 0.37 I_{\text {initial }}=0.37 \frac{\mathscr{E}}{R}=0.37\left(\frac{9.0 \mathrm{~V}}{1.5 \times 10^{4} \Omega}\right)=2.22 \times 10^{-4} \mathrm{~A}
\end{aligned}
$$

The graphs are shown. The times for the requested values are about 4.4 or 4.5 ms , about one time constant, within the accuracy of estimation on the graphs.

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH26.XLS," on tab "Problem 26.93."



