EconS 425, Spring 2012
Exercise #1: Game Theory (01/17/2012)

Instructor: Ana Espinola, *anaespinola@wsu.edu*

1 Question #1 Mixed Strategies

Consider the Battle of the sexes game described in the following table (Table 2.2, page 17):

	Opera		Football	
Opera	2	1	0	0
Football	0	0	1	2

a. Denote by θ the probability that Jacob goes to the Opera, and β the probability that Rachel goes to the Opera. Formulate the expected payoff of each player.

b. Draw the best-response function for each player $[R^J(\beta)$ and $R^R(\theta)]$.

c. What is the NE in mixed actions for this game?

d. Calculate the expected payoff to each player in this NE

1.1 Solution

a. Expected payoff for Jacob when he goes to the opera and football, respectively:$E\pi_J\left(opera\right) = 2\times\beta + 0\times(1-\beta)$ and $E\pi_J\left(football\right) = 0\times\beta + 1\times(1-\beta)$ .Hence, Jacob goes to the opera if

$$\begin{aligned} E\pi_J\left(opera\right) &> E\pi_J\left(football\right) \\ 2\times\beta + 0\times(1-\beta) &> 0\times\beta + 1\times(1-\beta) \\ 2\times\beta &> 1\times(1-\beta) \\ \beta &> \frac{1}{3} \end{aligned}$$

Expected payoff for Rachel when he goes to the opera and football, respectively:$E\pi_R\left(opera\right) = 1\times\theta + 0\times(1-\theta)$ and $E\pi_R\left(football\right) = 0\times\theta + 2\times(1-\theta)$ .Hence, Rachel goes to the opera if

$$\begin{aligned} E\pi_J\left(opera\right) &> E\pi_J\left(football\right) \\ 1\times\theta + 0\times(1-\theta) &> 0\times\theta + 2\times(1-\theta) \\ 1\times\theta &> 2\times(1-\theta) \\ \theta &> \frac{2}{3} \end{aligned}$$

b. The best-response function for Jacob is

$$R^J(\beta) = \begin{cases} 0 \text{ if } \beta < \frac{1}{3} \\ (0,1) \text{ if } \beta = \frac{1}{3} \\ 1 \text{ if } \beta > \frac{1}{3} \end{cases}$$

The best-response function for Rachel is

$$R^R(\theta) = \begin{cases} 0 \text{ if } \theta < \frac{2}{3} \\ (0,1) \text{ if } \theta = \frac{2}{3} \\ 1 \text{ if } \theta > \frac{2}{3} \end{cases}$$

Graphically:

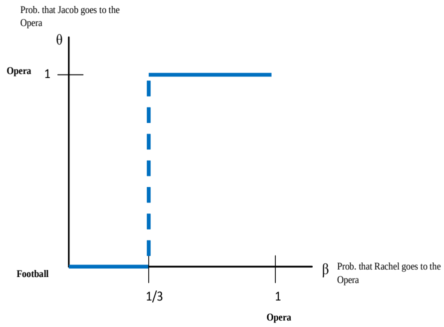


Figure 1: Best-response function for Jacob

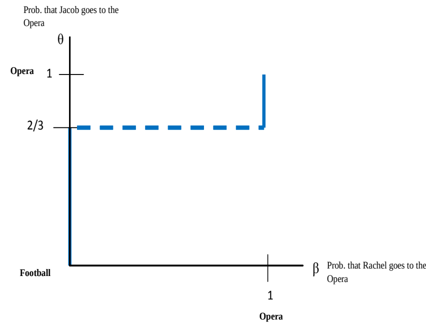
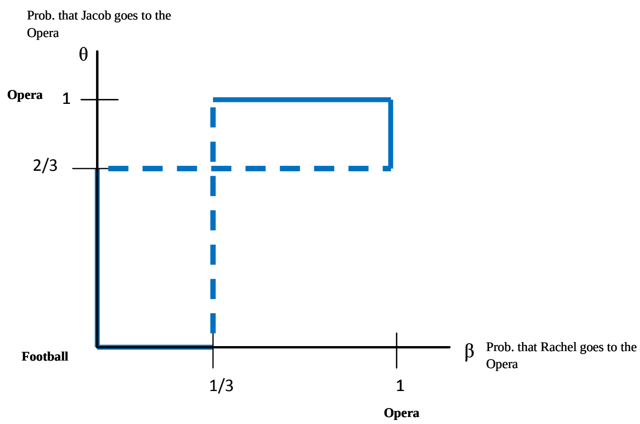


Figure 2: Best-response function for Rachel



2 Question #2 - Centipede Game

Consider a sequential game which is known as the Centipede Game. In this game, each of two players chooses between "Left" and "Right" each time he or she gets a turn. The game does not, however, automatically proceed to the next stage unless players choose to go "Right" rather than "Left".

Player 1 begins, and if he plays Left, the game ends with payoff of $(1, 0)$ (where here, and throughout this exercise, the first payoff refers to player 1 and the second to player 2). If, however, he plays Right, the game continues and it is player 2's turn. If player 2 then plays Left, the game once again ends, this time with payoffs $(0, 2)$, but if she plays Right, the game continues and player 1 gets another turn. Once again, the game ends if player 1 decides to play Left, this time with payoffs of $(3, 1)$, but if he plays Right the game continues and it is once again player 2's turn. Now the game ends regardless of whether player 2 plays Left or Right, but payoffs are $(2, 4)$ if she plays Left and $(3, 3)$ if she plays Right.

a. Draw the game tree for this game. What is the subgame perfect Nash equilibrium of this game?

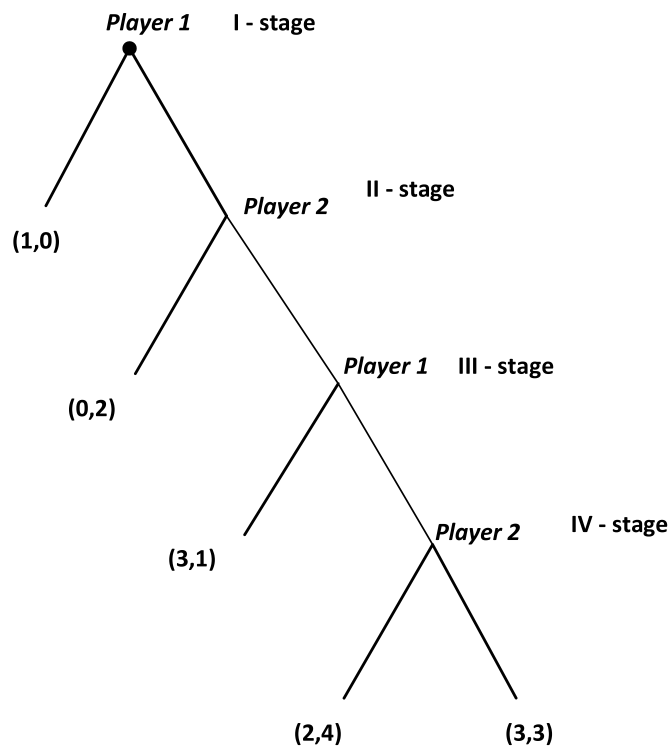
b. Write down the 4 by 4 payoff matrix for this game. What are the pure strategy Nash equilibria in this game? Is the SPNE you derived in (a) among these?

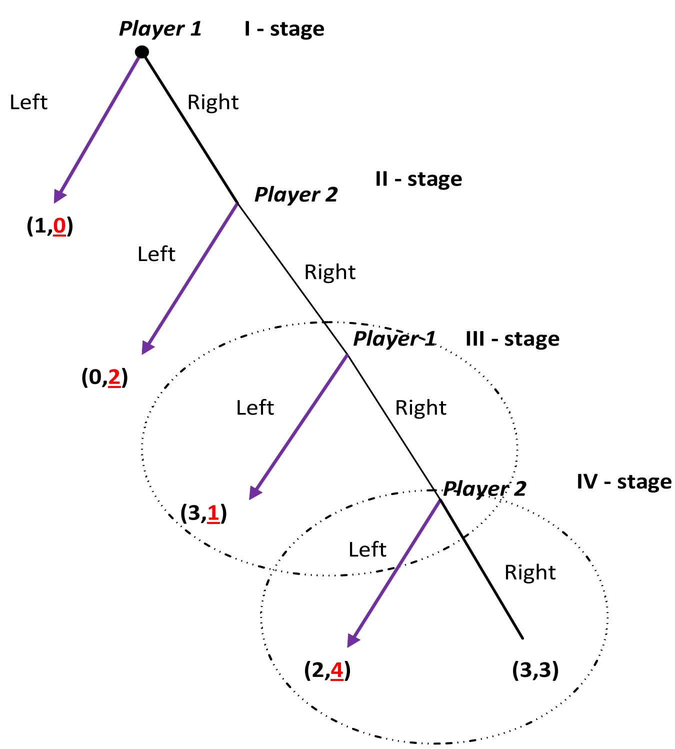
c. Why are the other Nash Equilibria in the game not subgame perfect?

d. Suppose you changed the $(2, 4)$ payoff pair to $(2, 3)$. Do we now have more than one SPNE?

2.1 Solution

a. The extensive-form game is





b.

		Rachel			
		L,L	L,R	R,L	R,R
Jacob	L,L	$(1,0)$	$(1,0)$	$(1,0)$	$(1,0)$
	L,R	$(1,0)$	$(1,0)$	$(1,0)$	$(1,0)$
	R,L	$(0,2)$	$(0,2)$	$(3,1)$	$(3,1)$
	R,R	$(0,2)$	$(0,2)$	$(2,4)$	$(3,3)$

d.