# Sample Questions Trading and Exchanges 

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## Question 1: Sample Theory

- The Stoll model for inventory costs starts from the following assumptions and notation
- 2 period model
- dealer:
* posts bid and ask quotes
* is risk-averse
* wants to be compensated for providing liquidity
* when doing a trade, he moves away from his efficient frontier
* will set his prices such that expected utility of his terminal wealth when the portfolio is on the efficient frontier $W^{E F}$, is equal to expected utility of his terminal wealth, as computed when he takes the opposite side of a trade, $W^{\text {dealer }}$
- CARA utility function with coefficient of risk aversion equal to $A$
* exponential utility framework
* agent's preferences over final wealth $W$ are given by

$$
U(W)=-e^{-A W}
$$

* he chooses strategy to maximize expected utility $E[U(W)]$
* if we assume normality $W \sim N\left(\mu_{W}, \sigma_{W}^{2}\right)$, expected utility becomes

$$
E[U(W)]=E\left[-e^{-A \mu_{W}+\frac{A^{2}}{2} \sigma_{W}^{2}}\right]
$$

* to optimize this, it suffices to optimize $A \mu_{W}-\frac{A^{2}}{2} \sigma_{W}^{2}$ or

$$
\mu_{W}-\frac{A}{2} \sigma_{W}^{2}
$$

- Also, use in the derivations below the following notation
- $p_{h}$ : the price of asset $h, q_{h}^{*}$ : the optimal choice at $t=0$ for the dealer of asset's $h$ holding, $W_{0}$ : initial wealth
- $\widetilde{F}_{h} \sim N\left(F, \sigma_{h}^{2}\right)$ the price of the asset at $t=1$, and $R^{*}$ : the return on the risky asset portfolio that lies on the dealer's efficient frontier
- a trade has size $q_{i}-q_{i}^{*}=-x_{i}$, and price $p_{i}$ equal to the ask $p_{i}^{A}$ for buy order of size $x_{i}>0$ (so dealer sells $-x_{i}<0$ ), and $p_{i}$ equal to the bid price $p_{i}^{B}$ for sell order of size $x_{i}<0$ (so dealer buys $-x_{i}>0$ )

Answer the following $\mathbf{3}$ subquestions:
a) Derive formally that the price function $p_{i}$ is given by:

$$
p_{i}=\bar{F}_{i}-A W_{0} \sigma_{*, i}+\frac{A}{2} \sigma_{i}^{2} x_{i}
$$

(10 p.)
b) Next, give clear and detailed interpretation in words for the price function that you derived in point b). ( 7 p. )
c) Show formally that the spread is equal to

$$
S_{i}=A \sigma_{i}^{2}\left|x_{i}\right|
$$

and provide an interpretation in words for this expression. (8 p.)

## Question 2: Sample Empirical Question

a) When analyzing order choice in order driven markets, i.e. the choice between market orders (MO) and limit order (LO), Ranaldo (2004) tests 7 hypotheses, given below. For each of the hypotheses, provide the economic intuition in words. Why do you expect the hypothesis to hold? A few sentences per hypothesis are enough (14 p.)

1. The thicker the book on the buy (sell) side, the stronger the order aggressiveness of the incoming buyer (seller).
2. The thicker the book on the sell (buy) side, the weaker the order aggressiveness of the incoming buyer (seller).
3. The wider the spread, the weaker the order aggressiveness.
4. The higher the volatility, the weaker the order aggressiveness.
5. The faster the process of order submission, the less aggressive the incoming order.
6. There is symmetry between buyer's and seller's order submissions.
7. Changes in the order book affect the limit and market order trading in opposite ways.
8. To test these hypotheses, Ranaldo estimates the following ordered probit regression for the buy side

$$
\begin{aligned}
y_{t}^{* B} & =\sum_{i=1}^{l} \alpha_{i}^{B} x_{i, t-1}^{B}+\varepsilon_{t}^{* B} \\
y_{n, t}^{B} & =\left\{\begin{array}{lll}
1 & \text { if } & -\infty<y_{t}^{* B} \leq \gamma_{1}^{B} \\
m & \text { if } & \gamma_{m-1}^{B}<y_{t}^{* B} \leq \gamma_{m}^{B} \\
5 & \text { if } & \gamma_{4}^{B}<y_{t}^{* B}<\infty
\end{array}\right.
\end{aligned}
$$

(and a similar model for the sell side)

- with:
- let $y_{t}^{* B}$ be the unobservable continuous variable denoting the order aggressiveness of the buy side in $t$
- orders are classified in 5 categories according to their aggressiveness:
* large buy $($ sell $)=$ most aggressive order $=$ a market order that demands more trading volume than is available at the prevailing quote
* small buy (sell) $=$ market order that demands less volume than the quoted depth
* buy (sell) within $=$ limit order within the prevailing quotes
* buy (sell) at $=$ limit order at the prevailing quotes
* least aggressive category is an order cancellation
- $y_{n, t}^{B}$ is the discrete dependent variable in which $n=1, \ldots, 5$ indicates the order type
- $\alpha_{i}^{B}$ is the regression coefficient
- $x_{i, t}^{B}$ are the regressors $i$ which are defined as follows:
* samevol: Pending volume in number of shares divided by 10,000 at the best quote on the same side of the market as the incoming trader
* oppvol: Pending volume in number of shares divided by 10,000 at the best quote on the opposite side of the market with respect to the incoming trader
* spread: Quoted spread as the difference between the lowest ask and the highest bid quotes
* wait: Average waiting time in seconds between the last 3 subsequent orders, divided by 100
* volat: Transitory return volatility as the standard deviation of the last 20 midquote returns

Explain in words the intuition underlying this equation (you are not required to give econometric details such as estimation method). Why is Ranaldo using this specification of the regression? ( 5 p. )
c) The following table presents the results. Discuss the results that can be derived from this table in words. Are they in line with the hypotheses in point a) of this question? Why or why not? (6 p.)

| ROCHE | BUYER |  | SELLER |  | SAMPLE | BUYER |  | SELLER |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Coeff | $t$-Stat | Coeff | $t$-Stat |  | Coeff | Sig. 1\% | Coeff | Sig. 1\% |
| samevol | -2.760 | -2.587 | -3.170 | -2.566 | samevol | -0.796 | 11 | -0.328 | 8 |
| oppvol | 5.140 | 4.964 | -3.710 | -3.725 | oppvol | 0.868 | 12 | -0.061 | 6 |
| spread | 0.038 | 32.494 | 0.026 | 22.548 | spread | 0.403 | 15 | 0.395 | 15 |
| wait | -0.137 | -3.164 | -0.233 | -4.600 | wait | -0.100 | 11 | -0.146 | 13 |
| $\gamma_{1}$ | -0.789 | -41.341 | -0.802 | -38.283 | $\gamma_{1}$ | -0.905 | 15 | -0.857 | 15 |
| $\gamma_{2}$ | 0.793 | 40.975 | 0.406 | 19.404 | $\gamma_{2}$ | 0.711 | 15 | 0.577 | 15 |
| $\gamma_{3}$ | 1.286 | 64.007 | 0.949 | 44.208 | $\gamma_{3}$ | 1.162 | 15 | 1.061 | 15 |
| $\gamma_{4}$ | 1.920 | 88.655 | 1.634 | 70.923 | $\gamma_{4}$ | 1.861 | 15 | 1.822 | 15 |
| ROCHE | BUYER |  | SELLER |  | SAMPLE | BUYER |  | SELLER |  |
|  | Coeff | $t$-Stat | Coeff | $t$-Stat |  | Coeff | Sig. 1\% | Coeff | Sig. 1\% |
| volat | 250.14 | 4.139 | 361.49 | 5.762 | volat | 150.18 | 12 | 154.623 | 8 |
| $\gamma_{1}$ | -1.067 | -58.059 | -0.889 | -44.993 | $\gamma_{1}$ | -1.147 | 15 | -1.081 | 15 |
| $\gamma_{2}$ | 0.452 | 25.847 | 0.284 | 14.950 | $\gamma_{2}$ | 0.415 | 15 | 0.310 | 15 |
| $\gamma_{3}$ | 0.929 | 51.348 | 0.816 | 41.647 | $\gamma_{3}$ | 0.854 | 15 | 0.786 | 15 |
| $\gamma_{4}$ | 1.571 | 78.244 | 1.506 | 69.206 | $\gamma_{4}$ | 1.568 | 15 | 1.551 | 15 |

## Question 3: Sample Open Question

Assume that you want to investigate empirically the claim that after the introduction of the MiFID directive, and the emergence of a number of alternative trading systems (ATSs), transaction costs on traditional exchanges are lower than before MiFID. Describe how you would undertake such empirical analysis.
Which type of regression would you use to investigate this research question? In your answer, specify clearly the regression specification, including the dependent variable and key independent variables. Moreover, also explain a number of control variables that you would include in the regression and why you would include them. Moreover, for each of the variables, try to think of your expectation about the sign of the coefficient of the variable (positive or negative) and explain why. (15 p.)

