

dia 89

(≈ cannot game)

us 6

Firm 2

		C	N
Firm 1	C	5, 5	1, 7
	N	7, 1	3, 3

NE

Normal form

operation C & N (cooperation & non-cooperation)

→ bc of the benefit
 If both cooperate, 1 starts to deviate, other will have no mercy. They result in N, N = 3, 3.

However, they would be better off if they cooperate

"Repeated play" → infinite horizon

⇒ can grim trigger lead to cooperation?

GRIM TRIGGERS → assume both C

= applied by both companies

=> if 1 company tends to deviate from the cooperation strat (& thus doesn't cooperate)

=> other will have no mercy till the end of days, so he won't cooperate anymore

Want to know the NE

(Stick to GT)

	-n	0	1	2	3	
F1 C	C	C	C	C	...	(Δ)
F2 C	C	C	C	C	...	

only NE if the deviation isn't beneficial

Take perspective of F1

F1 C	(N)	N	N	N	...	(*)
F1 C	(C)	N	N	N	...	

↳ companies act the same way

↳ Sticks to GT

Have no mercy till the end, same for F1

To find the NE

Use present value

$$PV = A + \delta A + \delta^2 A + \dots$$

$$PV = A + \delta PV$$

$$\Leftrightarrow PV = \frac{A}{1-\delta}$$

$$\rightarrow \frac{5}{1-\delta}$$

$$7 + \frac{3\delta}{1-\delta}$$

=> need to discount for all consecutive periods!

↳ die future value → actualiseren

↳ both will receive 5 till the end of days
firm 1 receives 7 at t=0 + 3 in the rest of days

take persp of F1

For which discount factor is GRIM TRIGGER a strat that ensure coop?

$$\frac{5}{1-\delta} > 7 + \frac{3\delta}{1-\delta} \Leftrightarrow 5 > 7 - 7\delta + 3\delta \Leftrightarrow 4\delta > 2$$

$$\Leftrightarrow \delta > 1/2$$

the PV of cooperation from GT must be higher than the PV of deviation from GT

GT is a NE strat for all $\delta > 1/2$

=> IS IT SGP? WANT TO CHECK ALL SCENARIOS

GT is a NE strat
 "can we have a NE for alls?"

GRIM TRIGGER

SITUATION 1

Firm 1 defected (no cooperation in $t=0$)

pijltjes
wijden
gwn
nr de
tydelynen

	$t=-1$	$t=0$	$t=1$	$t=2$	
F1	N	N	N	N ...	$\Rightarrow 3 + \frac{3\delta}{1-\delta}$
F2	C	N	N	N ...	$\frac{1-\delta}{1-\delta}$

stick to GT (1 didn't coop in past, so also doesn't in future)

PVs $\rightarrow$ for F1

$3 > 1 =$
TRUE FOR
ALL δ

Firm 1 deviates from GT

Sticking to
GT would
mean that
if you didn't
cooperate in
the past, you
never coop.

F1	N	C	N	N	N ...	$\Rightarrow 1 + \frac{3\delta}{1-\delta}$
F2	C	N	N	N	N ...	

deviate from GT(1)
 sticks to GT

F1 gets 1 in $t=0$
 + 3 in the rest of days

SITUATION 2

Firm 2 defected (no cooperation in $t=0$)

F1	C	N	N	N ...	$\Rightarrow 3 + \frac{3\delta}{1-\delta}$
F2	N	N	N	N ...	

stick to GT (2 didn't in past, doesn't now)

$3 > 1 =$
TRUE FOR
ALL δ

F1	C	C	N	N	N ...	$\Rightarrow 1 + \frac{3\delta}{1-\delta}$
F2	N	N	N	N	N ...	

deviate from GT(1)
 deviates: didn't coop in past, but does now
 sticks to GT

F1 gets 1 in $t=0$
 + 3 in the rest of days

SITUATION 3 : both do not coop

sticks to GT (both)

F1	N	N	N	N	...
F2	N	N	N	N	...

$$\Rightarrow 3 + \frac{3\delta}{1-\delta}$$

$$3 > 1 =$$

TRUE FOR ALL δ

V

F1	N	C	N	N	...
F2	N	N	N	N	...

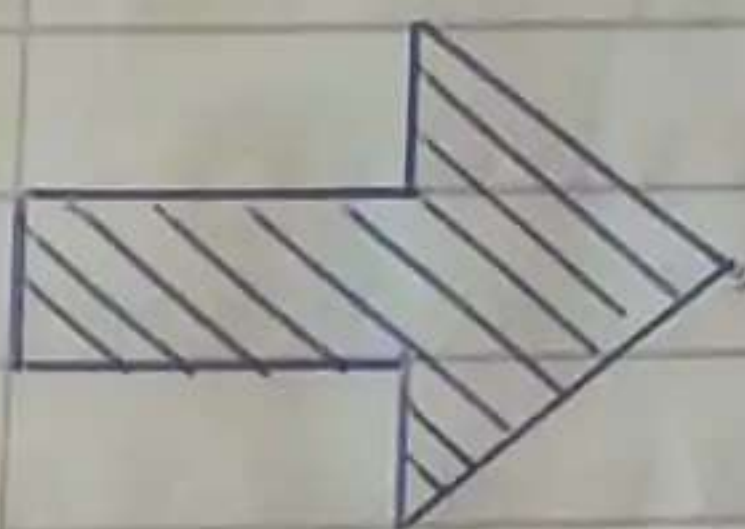
$$\Rightarrow 1 + \frac{3\delta}{1-\delta}$$

deviate from GT (1)

sticks to GT

F1 gets 1 in $t=0$

+3 in the rest of days



After checking scenarios 1-3, we can conclude that GT is a SGP NE strat for all $\delta \geq 1/2$

!!! for all the δ you determine for the NE

see calculations

for GT (both C in the past)

TRIGGER 1

= Grim trigger but with mercy after 1 period

"Is trigger 1 a NE strat?"

NE strategy?

stick to trigger 1

both coop in past, will also coop now & all following periods

	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	
F1	C	C	C	C	C	$\dots$
F2	C	C	C	C	C	$\dots$

$\Rightarrow \frac{5}{1-\delta}$ PV

deviate from T1

	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	
F1	C	N	N	C	C	$\dots$
F2	C	C	N	C	C	$\dots$

$\Rightarrow 7 + 3\delta + \frac{5\delta^2}{1-\delta}$

Annotations:
 - F1 deviates
 - binned 1 period
 - binned 2 periods
 - period in ex
 - PV from F1
 - slides to T1
 - both don't coop for 1 period

When is T1 = NE strat? Sticking to T1 should be larger than the deviation

$$\frac{5}{1-\delta} \geq 7 + 3\delta + \frac{5\delta^2}{1-\delta}$$

$$5 \geq 7 - 7\delta + 3\delta - 3\delta^2 + 5\delta^2$$

$$0 \geq 2 - 4\delta + 2\delta^2$$

$$0 \geq 1 - 2\delta + \delta^2$$

$$0 \geq (1 - \delta)^2$$

$\Rightarrow$ Trigger 1 is only a NE strat with $\delta = 1$, which means no defection over time
 - if there is no defection over time
 None the less, it can be a NE & SGP

look for this:
 The different scenarios

to see if they are SGP!

3 scenarios you always need to check! EXAM!

SITUATION 1

Firm 1 defected (no coop in $t=0$)

Stick to T1

F1	N	N	C	C	C	...	$\Rightarrow$	$3 + \frac{5\delta}{1-\delta}$	$3 > 1$ TRUE FOR ALL δ
F2	C	N	C	C	C	...		$\frac{1-\delta}{1-\delta}$	

PV of F1
 PV of F2
 V

deviate from T1

F1	N	(C)	C	C	C	...	$\Rightarrow$	$1 + \frac{5\delta}{1-\delta}$
F2	C	(N)	C	C	C	...		$\frac{1-\delta}{1-\delta}$

deviates
 sticks to T1
 PV of F1
 PV of F2

SITUATION 2

Firm 2 defected (no coop in $t=0$)

Stick to T1

F1	C	N	C	C	C	...	$\Rightarrow$	$3 + \frac{5\delta}{1-\delta}$	$3 > 1$ TRUE FOR ALL δ
F2	N	N	C	C	C	...		$\frac{1-\delta}{1-\delta}$	

PV of F1
 PV of F2
 V

deviate from T1

F1	C	(C)	C	C	C	...	$\Rightarrow$	$1 + \frac{5\delta}{1-\delta}$
F2	N	(N)	C	C	C	...		$\frac{1-\delta}{1-\delta}$

deviates
 sticks to T1

SITUATION 3

Both firms do not coop
stick to T1

F1	N	C	C	C	C	...	$\Rightarrow \frac{5}{1-\delta}$
F2	N	C	C	C	C	...	

= no depreciation over time

deviate from T1

V

T1 = NE only when $\delta = 1$

F1	N	(N)	N	C	C	C	...	$\Rightarrow 7 + 3\delta + \frac{5\delta^2}{1-\delta}$
F2	N	(C)	N	C	C	C	...	

↖ deviates
↘ sticks to T1

$\Rightarrow$ T1 is a SGP NE strat if $\delta = 1$,
i.e. no depreciation over time

$\Rightarrow 0 > (1-\delta)^2$ } see previously
 $\hookrightarrow \delta = 1!$

Additional manual :

GRIM TRIGGER

* First thing : NE

* $\Rightarrow$ We start off with GRIM TRIGGER

1) we assume that both companies cooperate! (Δ)

2) when they stick to GT they will both cooperate if they cooperated in the last period

$\Rightarrow$ this strat can only be a NE if the deviation is not beneficial

What would be the deviation?

From GT the deviation would be (*)

$\rightarrow$ if both firms in the past period cooperated BUT now I decide to not cooperate

$\Rightarrow$ Firm 2 sticks to GT bc we cooperated in the past. Firm 1 deviates.

$\Rightarrow$ till the end of time I won't have mercy & will not cooperate again

$\Rightarrow$ what do we need to compare now?

how to determine the NE? = PRESENT VALUE

* GT is a NE strat, but is it SGP NE?

$\rightarrow$ what do we need to do in order to determine SGP NE?

$\rightarrow$ we want to check for all potential scenarios:

what could have happened in the past periods?

You not only check this case (both coop & then I deviated), but you also check other cases

$\Rightarrow$ it follows the same reasoning & the same thing you need to do. Need to see how GT affects it & look at the PVs & see for which δ GT can be a NE strat