Introduction

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Stress
- Something that is applied to a material by loading it

Strain
- A change of shape
- Response to stress

Mode of loading
- The way stress is applied

Elastic modulus
- The ability to cope with stress

Stiffness
- The resistance to change of shape that is elastic

Strength
- Resistance to permanent distortion or total failure

Density, stress, strain and moduli

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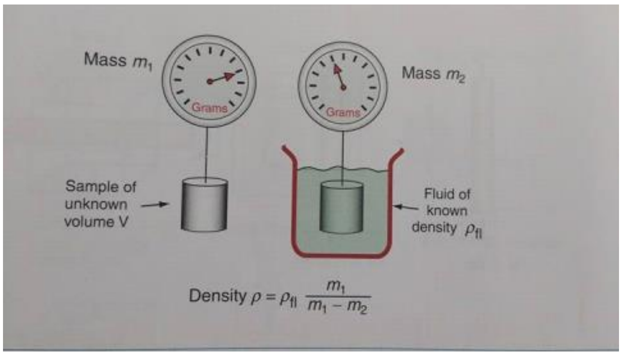
Density

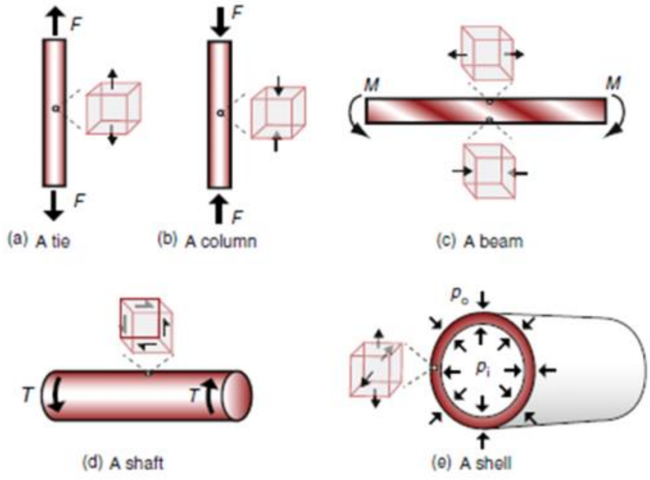
- Mass per unit volume
  - Mg/m^3 (= 1000 kg/m^3)

- Double weighing method
  - First : weighed in air
  - Second : weighted in a liquid of known density
    - = Archimedes' principle
  - Then we can derive density

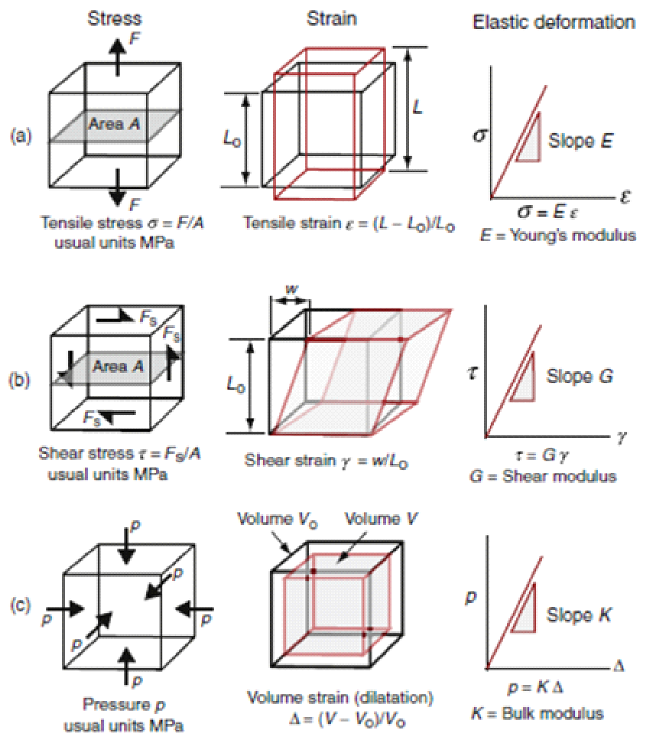


Modes of loading

- The elastic response depends on the way the loads are applied



Stress



- Tensile stress σ
  - $\sigma = \frac{F}{A}$
  - Unit = Mpa

- Shear stress τ
  - $\tau = \frac{F_s}{A}$
  - Unit = Mpa

- Pressure p
  - Positive when they push, negative when they pull
  - Unit = Mpa

Strain

- Tensile strain ε
  - $\varepsilon = \frac{\delta L}{L_O}$
    - L_O = original length
    - L = new length
    - δL = L - L_O

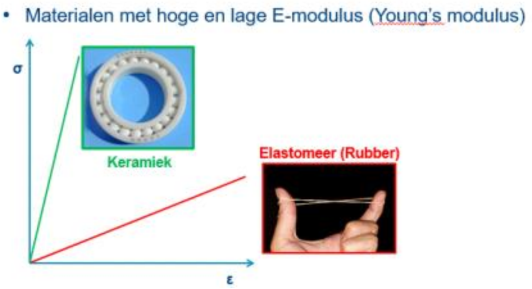
- Shear strain γ
  - $\tan(\gamma) = \frac{w}{L_O} = \gamma$
    - w = distance

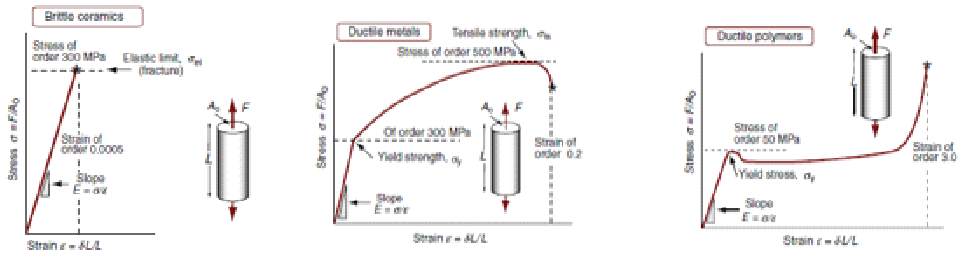
- Dilatation = volumetric strain
  - $\boldsymbol{\Delta} = \frac{\delta V}{V}$

- V = volume

Stress - strain curves and moduli

- Young's modulus E
  - $\sigma = E\varepsilon$





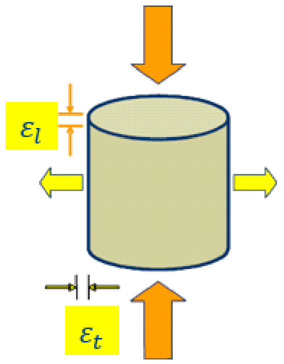
Brosse Keramieken	Ductiele Metalen	Ductiele Polymeren
Elastische rek: 0.05%	Totale rek: 20 %	Totale rek: 300%
Sterkte: 300 MPa	Sterkte: 300 MPa	Sterkte: 50 MPa

- Shear modulus G
  - $\tau = G\gamma$

- Bulk modulus K
  - $p = k\boldsymbol{\Delta}$

- Poisson's ratio

Poisson verhouding ν: $-\frac{\varepsilon_t}{\varepsilon_l}$

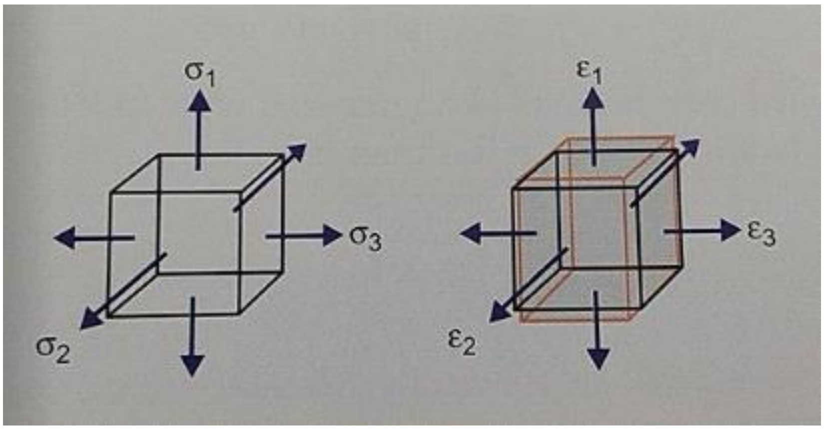


$$G = \frac{E}{2(1+v)}$$

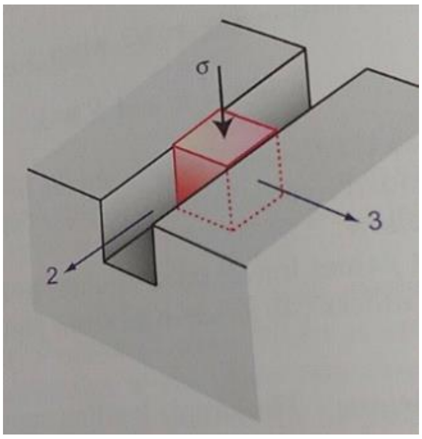
$$K = \frac{E}{3(1-2v)}$$

- Metalen : v ≈ 1/3
  - $G = \frac{3}{8}E$
  - $K = E$
- Elastomeren : v ≈ 1/2
  - $G = \frac{1}{3}E$
  - K >> E

Hooke's Law in 3 dimensions



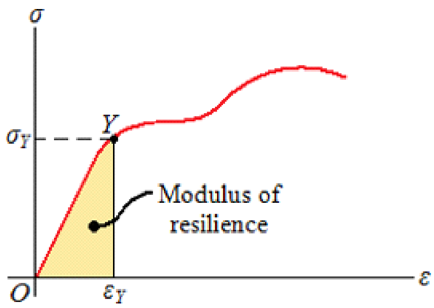
- General cubic element of material with 3 different stresses
  - $\varepsilon_1 = \frac{\sigma_1}{E}$
  - $\varepsilon_2 = \varepsilon_3 = -v\,\varepsilon_1 = -\frac{v\,\sigma_1}{E}$
  - (repeat that for σ_2 and σ_3)
  - Hooke's law in 3 dimensions :
    - $\varepsilon_1 = \frac{1}{E}\,(\sigma_1 - v\,\sigma_2 - v\,\sigma_3)$
    - $\varepsilon_2 = \frac{1}{E}\,(\sigma_2 - v\,\sigma_1 - v\,\sigma_3)$
    - $\varepsilon_3 = \frac{1}{E}\,(\sigma_3 - v\,\sigma_1 - v\,\sigma_2)$
- Example



- $\sigma_2 = 0$
- $\varepsilon_3 = 0$
- $\dfrac{\sigma_1}{\varepsilon_1} = \dfrac{E}{1 - v^2}$
- Result : cube behaves like material with 'effective modulus'
  - Because : $\frac{E}{(1-v^2)} > E$

Elastic energy

- If you stretch an elastic band, energy is stored in it
- $dW = \dfrac{F\ dL}{A\ L} = \sigma\ d\varepsilon$
  - Unit : J/m³
- Area under the stress - strain curve
  - $W = \displaystyle\int_0^{\sigma *} \sigma\, d\varepsilon = \frac{1}{2}\frac{(\sigma *)^2}{E}$



Measurement of Young's modulus

- Moduli are measured dynamically
  - By measuring the frequency of natural vibrations of a beam or wire, or by measuring the velocity of sound waves in the material
  - Both depend on $\sqrt{\frac{E}{\rho}}$

Stress-free strain

- Ex. Magnetic field, electrostatic field, thermal expansion

- Thermal expansion
  - $\varepsilon_T = \alpha \mathbf{\Delta T}$
    - $\varepsilon_T =$ thermal strain
    - α = expansion coefficient
    - $\mathbf{\Delta}$T = temperature change

The big picture : material property charts

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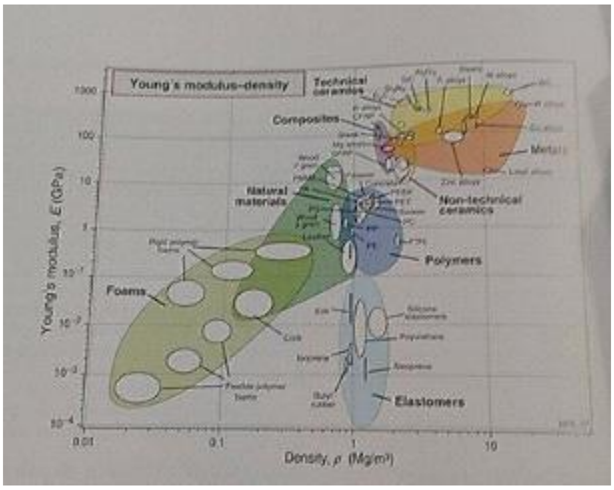
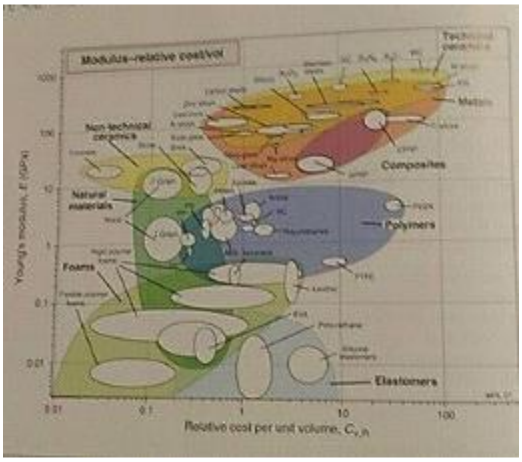
Modulus - density chart



Modulus - relative cost chart



Anistropy

- When their properties depend on the direction in the material they are measured
- Ex. Single crystals, drawn polymers and fibres
- Not ex. Glasses and polymers

What determines stiffness and density

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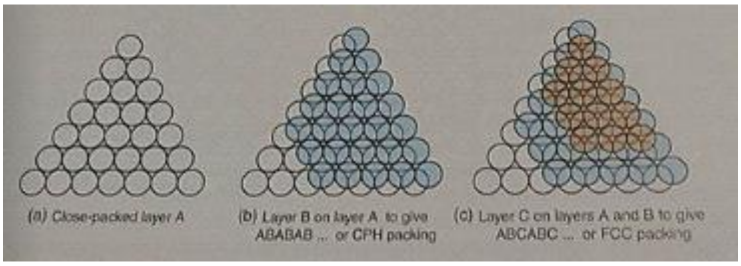
Introduction to GLU 1 : simple ideas of crystallography

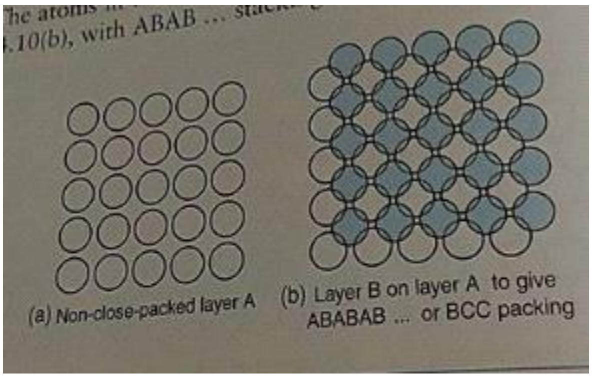
- Many of properties of materials depend directly on the way the atoms or molecules within them are packed

- Glasses : arrangement is disorderd with no regularity
- Polymers : largely made up of tangled long-chain molecules
- Metals and ceramics : crystalline, regularly pattern

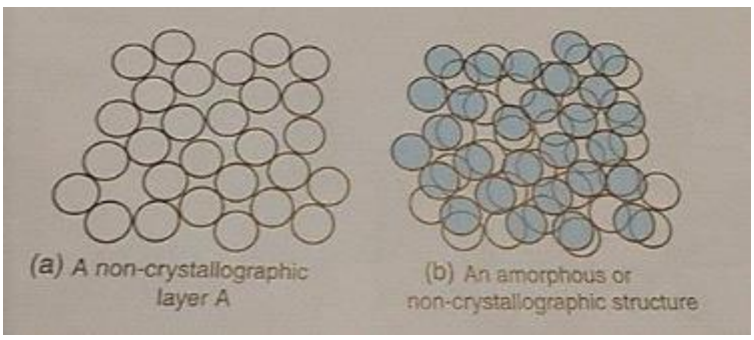
Atom packing in metals and the unit cell



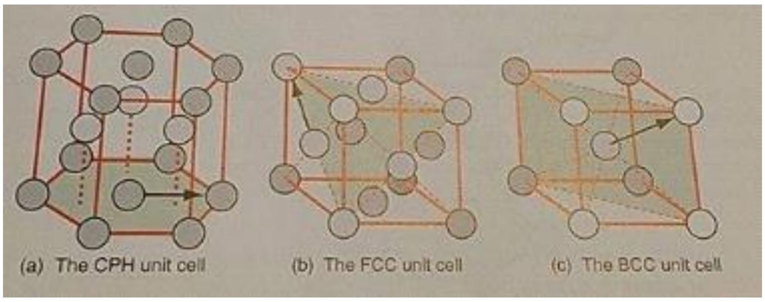
a) Close-packed
b) Close-packed hexagonal = CPH
c) Face-centred cubic = FCC



a) Non-close-packed
b) Body-centred cubic = BCC

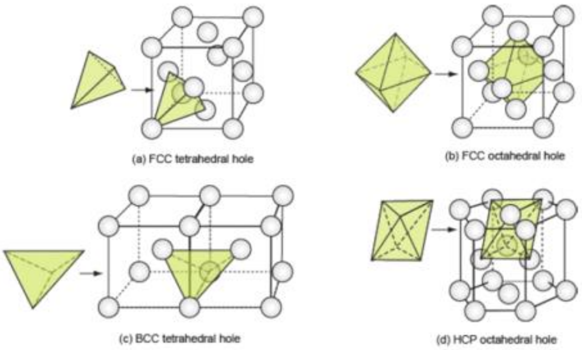


a) Non-crystallographic



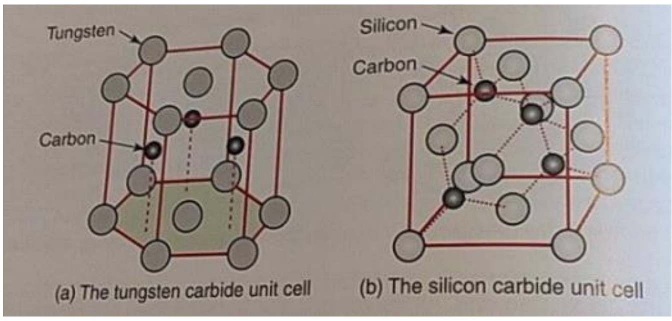
a) CPH unit cell (crystal structure)
b) FCC unit cell
c) BCC unit cell

- Rare holtes



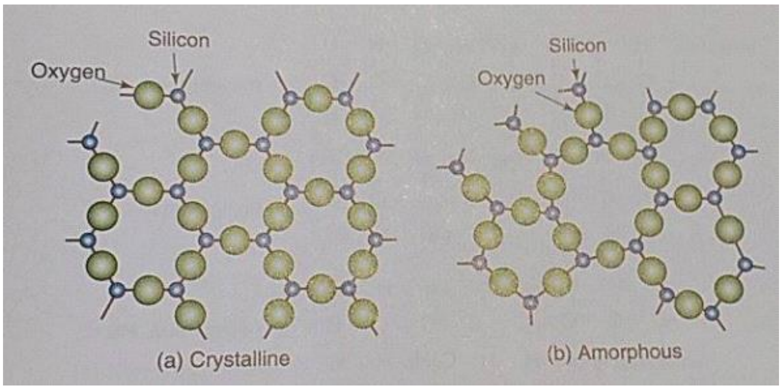
Atom packing in ceramics

- They also have characteristic unit cells



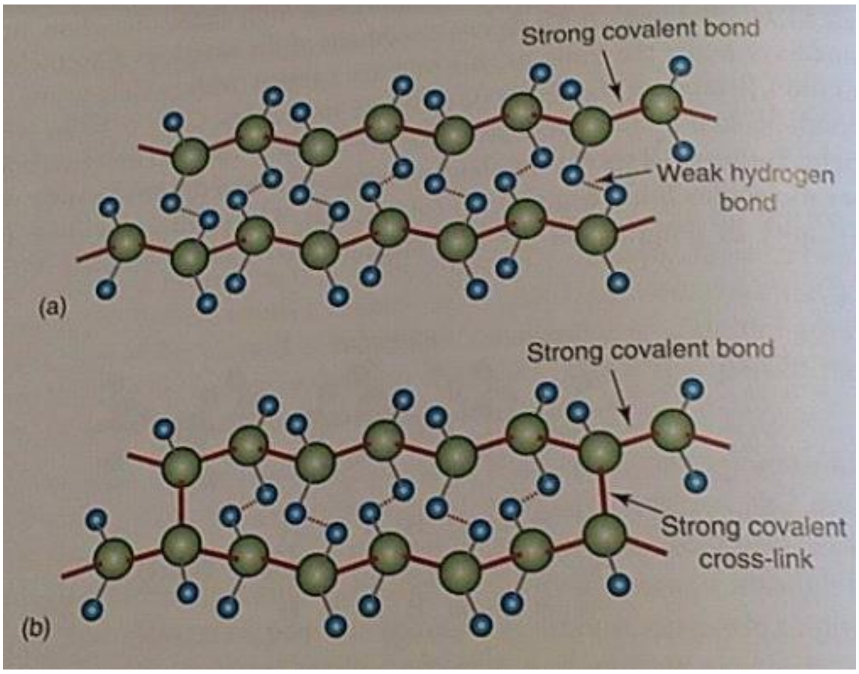
Atom packing in glasses

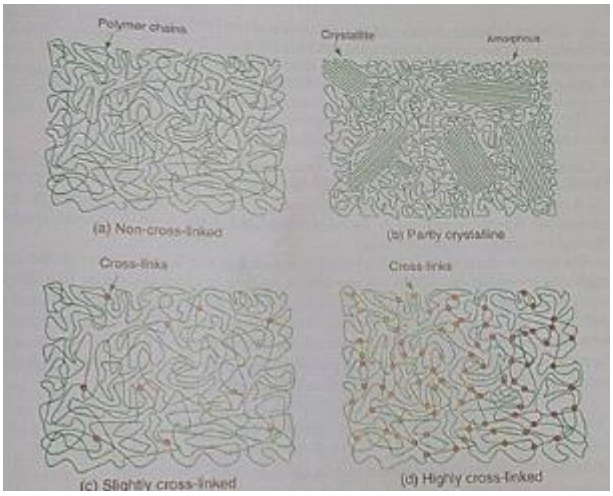
- Crystalline state is lowest energy state
- Glassy structure ≈ amorphous structure



Atom packing in polymers

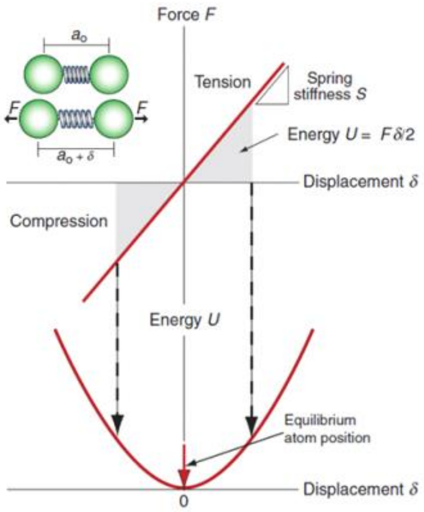
- Long chain of carbon atoms, to whick side groups are attached
- Separated chains attract each other , but weakly
- Cross-links : strong and weak





Cohesive energy and elastic moduli

- Cohesive energy = strenth of the bonding of atoms
    - = energy per mol required to separate the atoms of a solid
- The greater the cohesive energy, the stronger the bonds



- $\sigma = \varepsilon.E$ (Hooke)
- $F/\delta = S$
- $\frac{F}{(a_0)^2} = \sigma$
- $\delta/a_0 = \varepsilon$

$\frac{F}{a_0} = E.\delta$

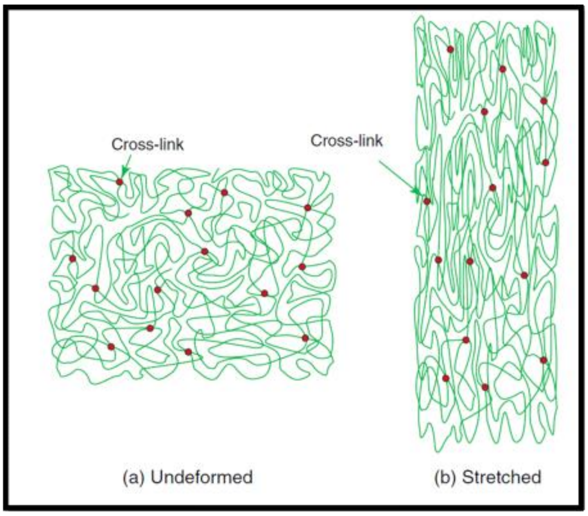
$$E = \frac{S}{a_0}$$

- S = stiffness
- δ = distance
- a_0 = distance between atoms

- Lower limit for E for a true solid = 1GPa

- Verband met thermische expansie coefficiënt :
    - $\alpha \approx \frac{1.6 * 10^{-3}}{E}$

<u>The elastic moduli of elastomers</u>



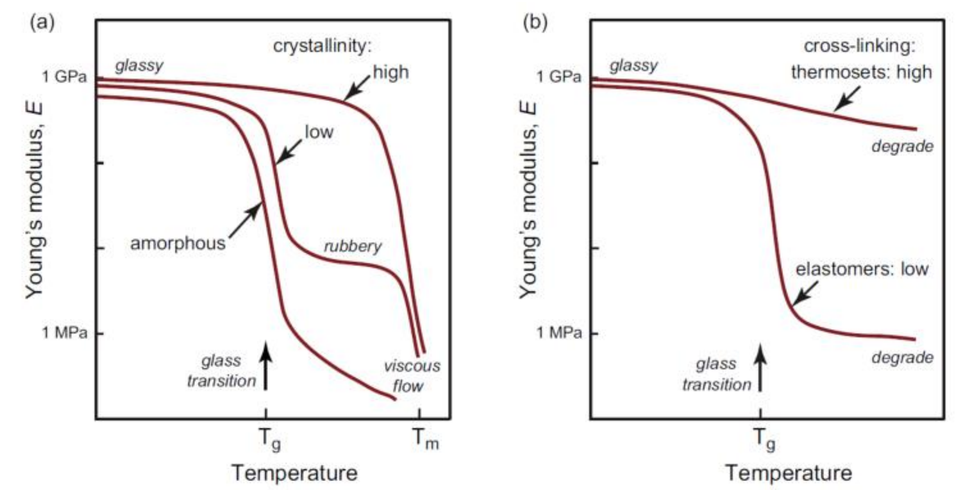
- Structure has high randomness = entropy is high
- When stretched, the molecules get aligned => entropy gets lower => looks more like crystallites

- Moduli of elastomers are low
- Higher temparture favours randomness => when heatening elastomers, they become more random

Temperature - dependence of polymer moduli : the glass transition temperature

- Crystalline solids
  - Melting temperature = T_m
- Amorphous materials
  - Glass transition temperature = T_g
    - = temperature at which the weak inter-chain bonds start to melt

- Typical : $T_m \approx 1.5 * T_g$



Manipulating modulus and density

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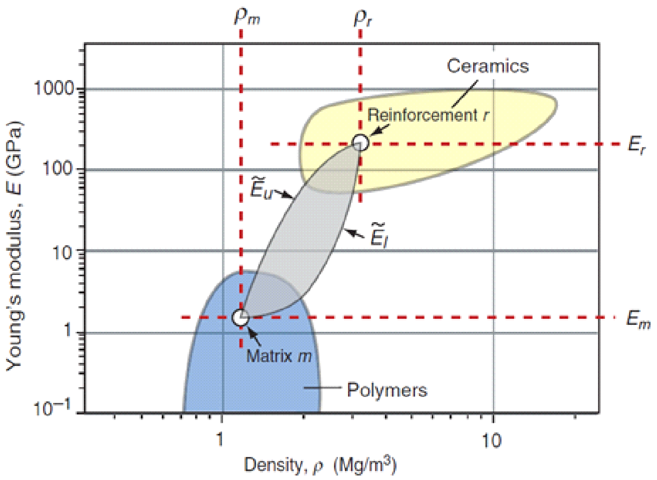
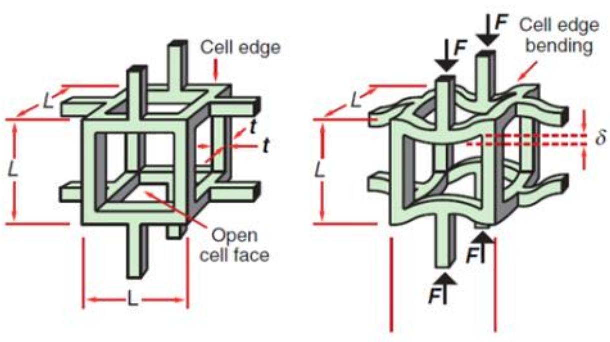
Composites

- Mix 2 materials
- Density
  - $\tilde{\rho} = f\rho_r + (1-f)\rho_m$
    - f = fraction
    - ρ_r = reinforecement density
    - (1-f) = volume fraction
    - ρ_m = matrix density
- The modulus of a composite is bracked between 2 bounds, upper bound $\tilde{E}_U$ and lower bound $\tilde{E}_L$
  - $\tilde{E}_U = fE_r + (1-f)E_m$
    - Parallel
  - $\tilde{E}_L = \frac{E_m E_r}{fE_m + (1-f)E_r}$
    - Serie



Foams

- Mixing a matrix material with a foaming agent



- Density
    - $\frac{\tilde{\boldsymbol{\rho}}}{\boldsymbol{\rho}_s} = \left(\frac{t}{L}\right)^2$
        - $\tilde{\boldsymbol{\rho}}$ = density of foam
        - $\boldsymbol{\rho}_s$ = density of the solid of which it is made
        - L = cell size
        - t = thickness of cell edges

- Modulus E
    - $\frac{\tilde{E}}{E_s} = \left(\frac{\tilde{\boldsymbol{\rho}}}{\boldsymbol{\rho}_s}\right)^2$

Summary

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