Introduction

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Modes of loading

- Tension
- Compression
- Bending
- Torsing

Standard solutions to elastic problems

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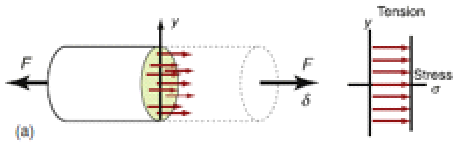
Elastic extension or compression



$\varepsilon = \delta / L_0$

$\sigma = F/A$

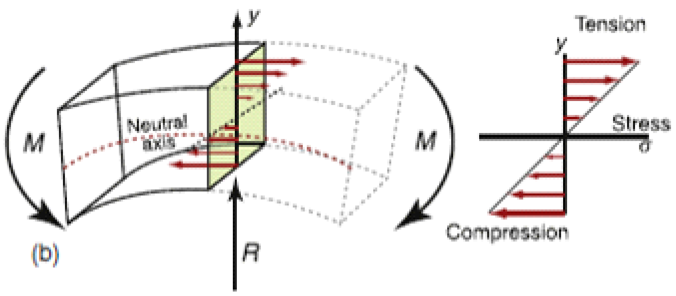
$\sigma = E.\varepsilon$

$$\delta = \frac{L_0 F}{AE}$$

$S = F/\delta = E.A/L_0$

- Shape of cross-section area does not matter because the stress is uniform over the section

Elastic bending of beams



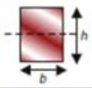
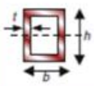
- $$\kappa = \frac{d^2 u}{dx^2} = \frac{1}{R}$$

  - K = curvature (=kromming)
  - u = verplaatsing Y-richting

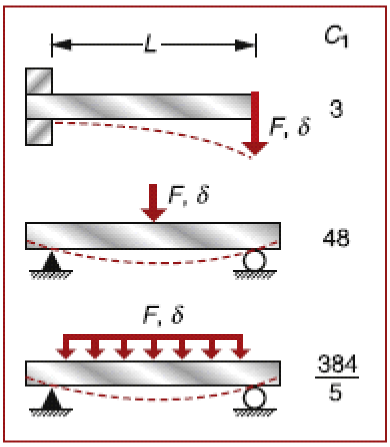
- Elastic beam theory

$$\frac{\sigma}{y} = \frac{M}{I} = E\kappa$$
$$= E\frac{d^2 u}{dx^2}$$

- M = bending moment
- y = distance from neutral axis
- I = second moment of area
  - $$I = \int_{\text{section}} y^2 b(y)\,dy$$
- b(y) = width of the section at y

Section shape	Area A m^2	Moment I m^4	Moment K m^4
	bh	$\frac{bh^3}{12}$	$\frac{bh^3}{3}(1 - 0.58\frac{b}{h})$ $(h > b)$
	πr^2	$\frac{\pi}{4}r^4$	$\frac{\pi}{2}r^4$
	$\pi(r_o^2 - r_i^2)$ $\approx 2\pi rt$	$\frac{\pi}{4}(r_o^4 - r_i^4)$ $\approx \pi r^3 t$	$\frac{\pi}{2}(r_o^4 - r_i^4)$ $\approx 2\pi r^3 t$
	$2t(h + b)$ $(h, b >> t)$	$\frac{1}{6}h^3 t(1 + 3\frac{b}{h})$	$\frac{2tb^2h^2}{(b + h)}(1 - \frac{t}{h})^4$

- Ratio of moment to curvature
  - Flexural rigidity (= buigstijfheid)
    - $\frac{M}{\kappa} = E.I$



- $$S = \frac{F}{\delta} = \frac{C_1 EI}{L^3}$$
    - The only thing that depends on distribution is value of C_1

Torsion of shafts

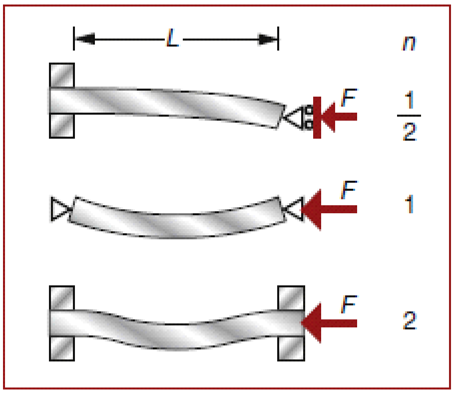
- A torque , T, is applied to the ends of an isotropic bar of a uniform section generates a shear stress τ
- Shear stress varies with radial distance r form the axis of symmetry
- K measures the resistance of the section to twisting
- Twist per unit length = $\frac{\theta}{L}$
- G is shear modulus
    - $\frac{\tau}{r} = \frac{T}{K} = \frac{G\theta}{L}$

- Torsiestijfheid
    - $S_T = \frac{T}{\theta} = \frac{KG}{L}$

Buckling of columns and plates

$$F_{crit} = \frac{n^2 \pi^2 EI}{L^2}$$

- F_{crit} = critical load
- L = length
- EI = flexural rigidity
- n = number of half-wavelengths of the buckled shape

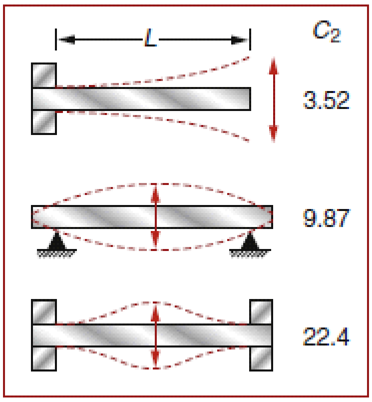


Vibrating beams and plates

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

- m = mass
- k = restoring force per unit displacement
- f = lowest natural frequency of this system

$$f = \frac{C_2}{2\pi}\sqrt{\frac{EI}{m_o L^4}}$$



- m = A***ρ**

- f ~ $\sqrt{\frac{E}{\rho}}$

Material indices for elastic design

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<u>Minimising weight : a light, stiff tie-rod (= staaf in rek)</u>

- Minimising m (**OBJECTIVE**)

$$m = AL_o\rho$$

- At least S* (**CONSTRAINT**)

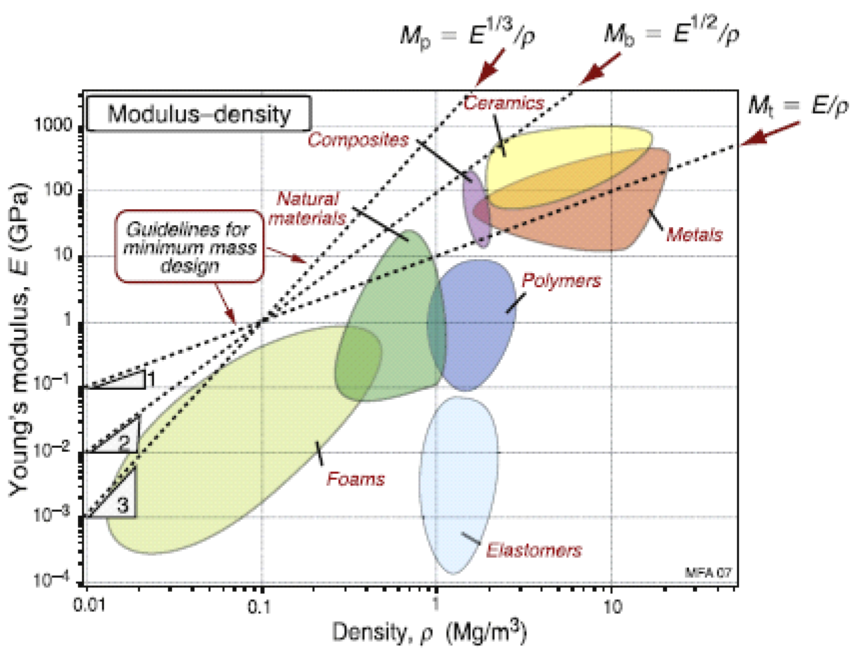
$$S^* = \frac{AE}{L_o}$$

- Area A is free => eliminate it (**FREE VARIABLE**)

$$m = S^* L_o^2 \left(\frac{\rho}{E}\right)$$

- Invert material properties in equation
  - Specific stiffness

$$M_t = \frac{E}{\rho}$$



Minimising weight : a light , stiff panel (= licht, stijf paneel)

- **OBJECTIVE**
  - Minimise m

$$m = AL\rho = bhL\rho$$

- **CONSTRAINT**
  - At least S*

$$S^* = \frac{C_1 EI}{L^3}$$

- **FREE VARIABLE**
  - Thickness h
    - Use

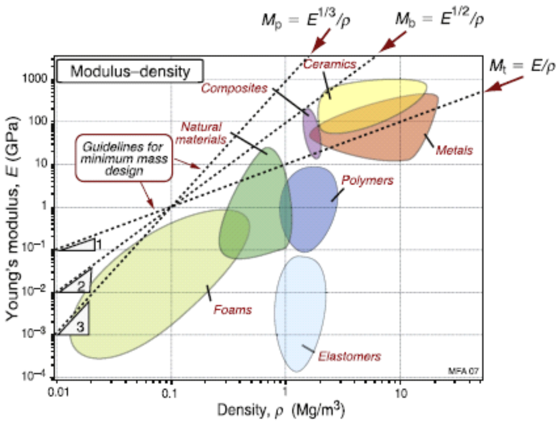
$$I = \frac{bh^3}{12}$$

    - To eliminate h

$$m = \left(\frac{12S^*}{C_1 b}\right)^{1/3} (bL^2)\left(\frac{\rho}{E^{1/3}}\right)$$

- Invert material properties in equation

$$M_p = \frac{E^{1/3}}{\rho}$$



Minimising weight: a light, stiff beam (= lichte, stijve balk)

- **OBJECTIVE**

- Minimise m

$$m = AL\rho = b^2 L\rho$$

- **CONSTRAINT**
  - At least S*

$$S^* = \frac{C_1 EI}{L^3}$$

- **FREE VARIABLE**
  - Area A
    - Use

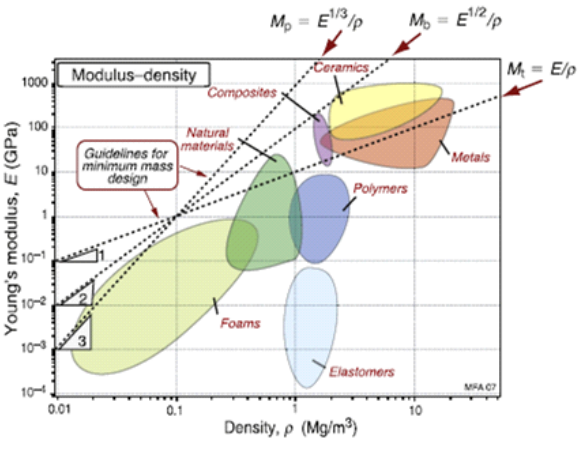
$$I = \frac{b^4}{12} = \frac{A^2}{12}$$

    - To eliminate A

$$m = \left(\frac{12S^* L^3}{C_1}\right)^{1/2} (L) \left(\frac{\rho}{E^{1/2}}\right)$$

- Invert material properties in equation

$$M_b = \frac{E^{1/2}}{\rho}$$



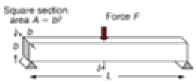
<u>Conclusion</u>

Trek, vrije variabele *A*

$$M_t = \frac{E}{\rho}$$

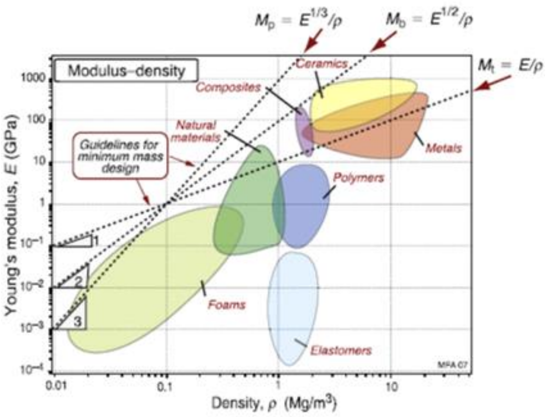
Buiging, vrije variabele *h*

$$M_p = \frac{E^{1/3}}{\rho}$$



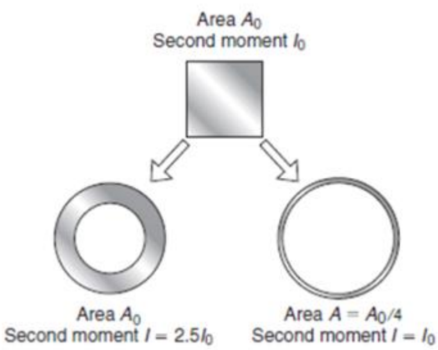
Buiging, vrije variabele *A*

$$M_b = \frac{E^{1/2}}{\rho}$$



Shape factor

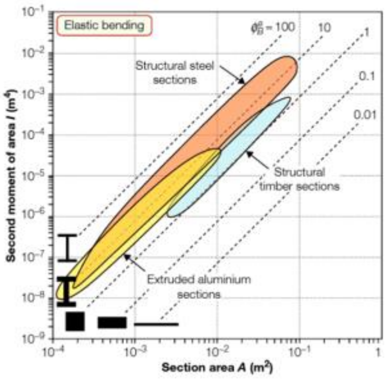
- φ
- It expresses how many times a certain shape is more efficient than a solid square section
  - Ration of I
- I is larger when mass is further away from neutral axis



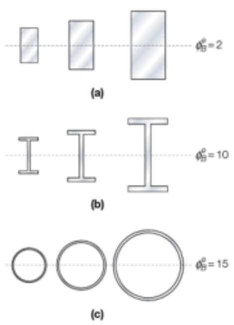
$$S \propto \frac{EI}{L^3}$$

- Square solid section
  - $$I_o = \frac{b_o^4}{12} = \frac{A^2}{12}$$

- Verhouding willekeurig / referentie opp
    - $\phi_B^e = \frac{S}{S_o} = \frac{EI}{EI_o} = \frac{12\,I}{A^2}$

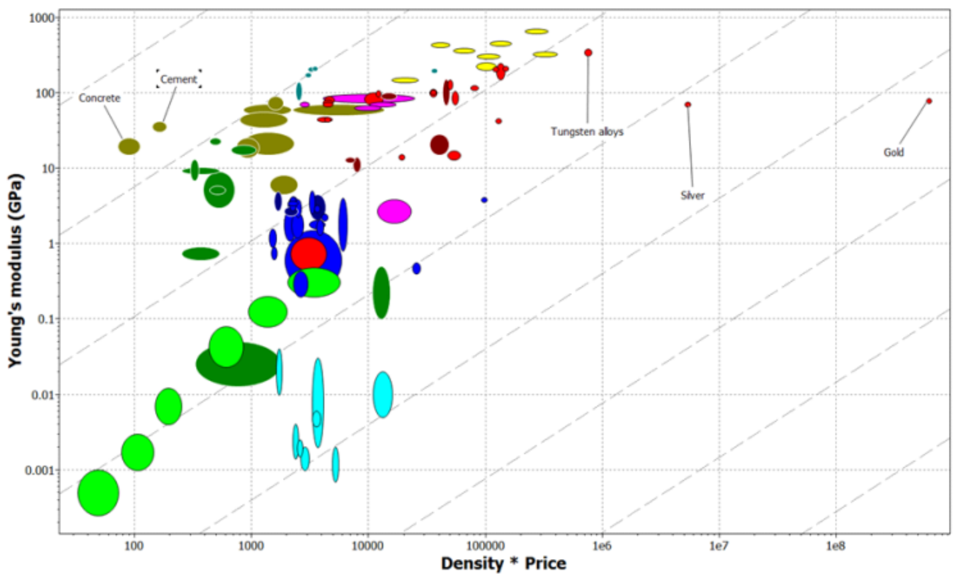


- Characteristics
    - Dimensionless
    - Dependent of shape
    - Independent of scale



Minimising material cost

$$C = mC_m = ALC_m\boldsymbol{\rho}$$



Conclusion

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Case studies

- HB p 115 - 121

Conclusion