Introduction

maandag 5 december 2016 10:25

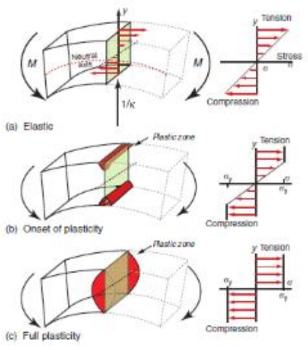
Stiffness - limited design (H5)

- Design to avoid excessive elastic deflection

Strength - limited design

- Design to avoid plastic collapse

- To avoid yield (3 options)
  - Elastic design = totally elastic
    - Not always possible
  - Local yielding
    - Permissible
  - General yield
    - Avoided



Other fuctions of plasticity

- Absorb energy
  - Ex: Car crash
- Give formation
  - Ex: metal shapes

Standard solutions to plastic problems

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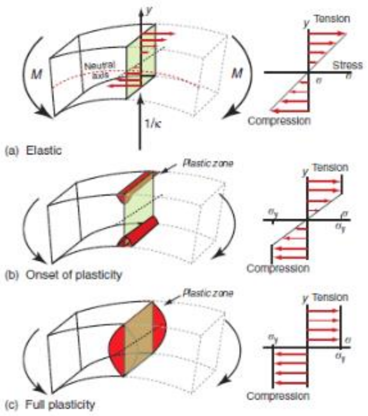
<u>Yielding of ties and columns</u>

- Tie = rod (=staaf) loaded in tension
- Column = rod loaded in compression

- Stress in them is the same, if σ > σ_y then it yields

<u>Yielding of beams and pannels</u>

- A bending moment M generates a linear variation of longitudinal stress σ across the section
    - $$\frac{\sigma}{y} = \frac{M}{I} = E\kappa$$
        - y = distance to neutral axis
        - I = influence of cross-section shape
        - κ = kromtestraal

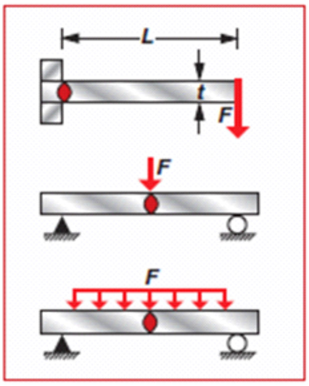


- Elastic section modulus = Z_e
    - $Z_e = \frac{I}{y_m}$
    - $$\sigma_{max} = \frac{M\, y_m}{I} = \frac{M}{Z_e}$$
        - σ_{max} = maximum longitudinal stress
        - y_m = greatest distance from neutral axis

    - If σ_{max} > σ_y , then small zones of elasticity appear at the surface where the stress is highest
        - Beam is no longer elastic
    - If moment is increased further
        - Stress near the surface remains σ_y

- But plastic zones grow inward from surface until completely plastic => forming plastic hinges

- Plastic bending of beams

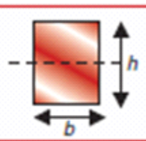
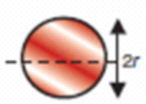
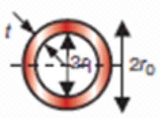


  - Momentum
    - M1 = FL
    - M2 = FL/4
    - M3 = FL/8
  - Plastic hinges form at red areas
    - When maximum moment reaches right before collapse
    - Failure moment = M_f

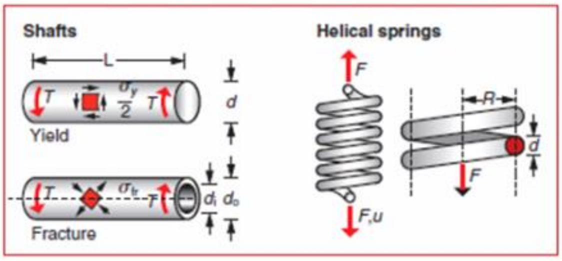
$$M_f = \int_{section} b(y)\, y\, \sigma_y\, d_y = Z_p \sigma_y$$

      - Z_p = plastic section modulus

- 2 new functions have been formed
  - Z_e : for first yielding
  - Z_p : for full plasticity
- Ratio $\frac{Z_p}{Z_e} > 1$
  - A measure for safety margin between initial yield and collapse
  - Form with more efficiency : $Z_p \approx Z_e$

Section shape	Area A m^2	Elastic section modulus Z_e m^3	Plastic section modulus Z_p m^3
	$b\,h$	$\frac{bh^2}{6}$	$\frac{bh^2}{4}$
	πr^2	$\frac{\pi}{4} r^3$	$\frac{\pi}{3} r^3$
	$\pi(r_o^2 - r_i^2)$ $\approx 2\pi r t$	$\frac{\pi}{4 r_o}(r_o^4 - r_i^4)$ $\approx \pi r^2 t$	$\frac{\pi}{3}(r_o^3 - r_i^3)$ $\approx \pi r^2 t$

<u>**Yielding of shafts**</u>



- Torque (= wringingsmoment) T produces shear stress τ
    - $\tau = \frac{T\,r}{K} = \frac{G\,\theta\,r}{K}$
        - r = distance to central axis
        - K = polar second moment of area
        - $\frac{\theta}{L}$ = angle of twist per unit length

- Failure occurs when $\tau_{max} > \sigma_y$
    - $\tau_{\max} = \frac{T\,R}{K}$
        - R = radius of the shaft

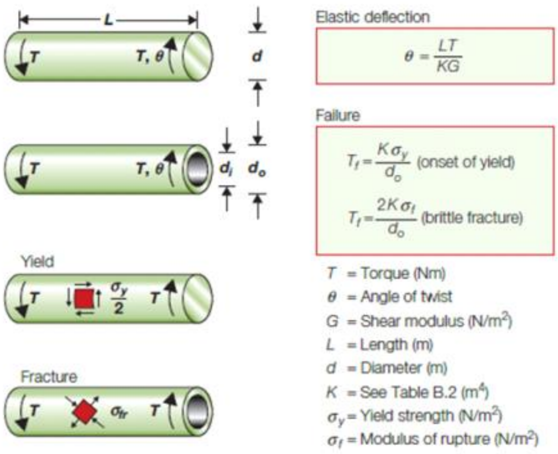
- Yield occurs when $\tau_{max} = \frac{TR}{K}$
    - When torque is increased further, plasticity spreads inward
    - Maximum torque that shaft can carry : τ = k
    - Example : for a solid circular section , the collapse torque =
        - $T = \frac{2}{3}\pi r^3 k$

- Helical springs
    - Special case of torsional loading
    - When the spring is loaded axially, the individual turns twist

$$S = \frac{F}{u} = \frac{G\,d^4}{64\,n\,R^3}$$

- S = spring stiffness
- n = numver of turns of wire
- G = shear modulus
- d = diameter
- R = radius
- F = axial force
- u = extension

- Elastic extension is limited by the onset of plasticity, this occurs at force :
    - $F_{\text{crit}} = \frac{\pi}{32}\frac{d^3\,\sigma_y}{R}$



Equivalentie

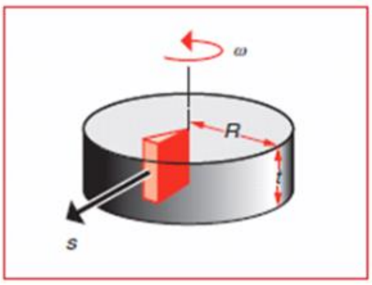
$S = F/\delta = C.E.I/L^3$

$S_T = T/\theta = G.K/L$

$M/I = \sigma/y = E.\kappa$

$T/K = \tau/r$ en $\tau_{max} = \sigma_y/2$
(d = 2r)

Spinning disks



- They store kinetic energy U
- Centrifugal forces generate a radial tensile stress

- Kinetic energy

$$U = \frac{\pi}{4}\rho t\, \omega^2 R^4$$

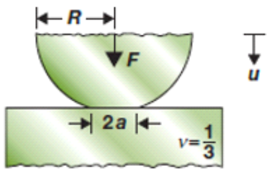
  - ρ = density
  - R = radius
  - t = thickness
  - ω = angular velocity
  - WITH poisson's ratio : ν = 1/3

- Maximum stress

$$\sigma_{max} = 0.42\,\rho\,\omega^2 R^2$$

- The disk yields when $\sigma_{max} > \sigma_y$
    - This defines maximum allowable ω and limits the inertial energy storage

<u>Contact stresses</u>

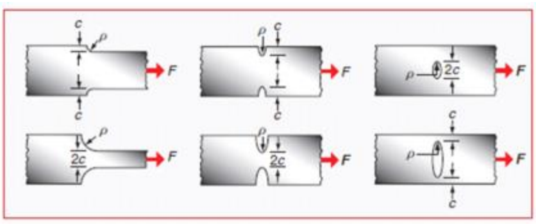


- If the surfaces are loaded, the contacts flatten elastically and the contact areas grow
- Stress state is very complicated

$$\left.\begin{aligned} a &= 0.7\left(\frac{FR}{E}\right)^{\frac{1}{3}} \\ u &= 1.0\left(\frac{F^2}{E^2R}\right)^{\frac{1}{3}} \end{aligned}\right\} \nu = 0.33$$

- Maximum value of shear stress
    - It causes first yield
    - It is beneath the contact at a depth of a/2
    - $\tau_{max} = \dfrac{F}{2\pi a^2}$
        - If this exceeds the shear yield strength $k = \frac{\sigma_y}{2}$
            - Then plastic zones appear beneath the centre of contact

<u>Stress concentrations</u>



Stress concentrations. The change of section concentrates stress most strongly where the curvature of the surface is greatest.

- Yielding will start at places with holes, slots and threads (they concentrate stress)

- Nominal stress : σ_{nom}
    - Load divided by the cross-section, ignoring features that cause the stress concentration
- Maximum local stress : σ_{max}
    - $\sigma_{max} = \sigma_{nom} * K_{sc}$
- Stress concentration factor : K_{sc}

$$K_{sc} = \frac{\sigma_{max}}{\sigma_{nom}} = 1 + \alpha\left(\frac{c}{\rho_{sc}}\right)^{1/2}$$

- $\boldsymbol{\rho}_{sc}$ = minimm radius of curvature of the stress-concentration feature
- c = characteristic dimension
    - Least of :
        - Half-thickness of remaining ligament
        - Half-length of contained notch
        - Length of an edge notch
        - Height of a shoulder
- α
    - For tension ≈ 2
    - For torsion and bending ≈ 1/2

Material indices for yield-limited design

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Minimising weight : a light, strong tie-rod

Function	• Tie-rod
Constraints	• Length *L* specified • Tie must support tensile load *F* without yielding
Objective	• Minimize the mass *m* of the tie
Free variables	• Choice of cross-section area, *A* • Choice of material

- Constraint:
  - $\sigma = FA < \sigma_y$
- Objective:
  - m = **ρ***A*L
- Together :
  - $m \geq F L \left(\frac{\rho}{\sigma_y} \right)$
- Material index
  - $M_t = \frac{\sigma_y}{\rho}$

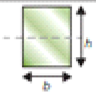
Minimising weigth : light, strong panels

Function	• Panel in bending
Constraints	• Width *b* and span *L* specified • Panel must support bending load *F* without yield
Objective	• Minimize the mass *m* of the panel
Free variables	• Choice of thickness *h* • Choice of material

$$M_p = \frac{\sigma_y^{1/2}}{\rho}$$

Light, strong beams : the effect of shape

Function	• Beam in bending
Constraints	• Span *L* specified, section shape square • Beam must support bending load *F* without yielding
Objective	• Minimize the mass *m* of the beam
Free variables	• Area *A* (or square section dimension *b*) • Choice of material

Section Shape	Area A m^2	Moment I m^4	Moment K m^4	Moment Z m^3	Moment Z_p m^3
	bh	$\frac{bh^3}{12}$	$\frac{bh^3}{3}(1-0.58\frac{b}{h})$ $(h>b)$	$\frac{bh^2}{6}$	$\frac{bh^2}{4}$

$$M_b = \frac{\sigma_y^{2/3}}{\rho}$$

- Vormfactoren

$$Z = I/y_m$$

$$\phi_B^y = \frac{Z_e^{shaped}}{Z_e^{solid}}$$

Case studies

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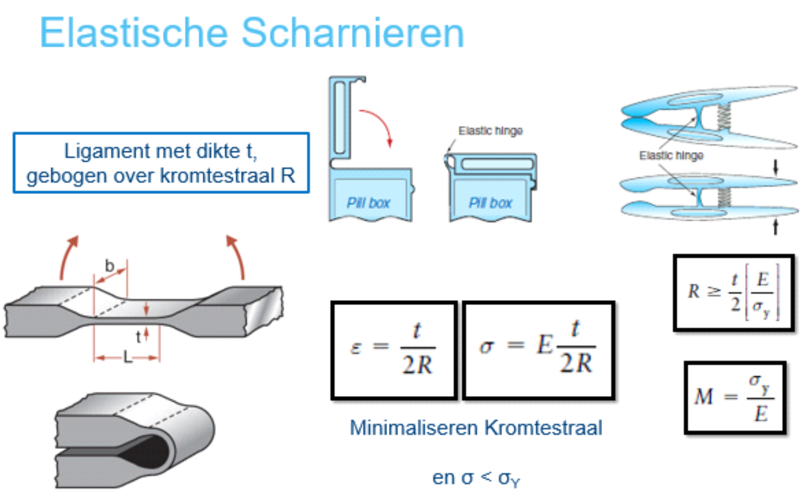
There are two types of plasticity problems
- The condition $\sigma < \sigma_y$ must be met everywhere
    - Compleet elasticity
- Full plasticity is the aim

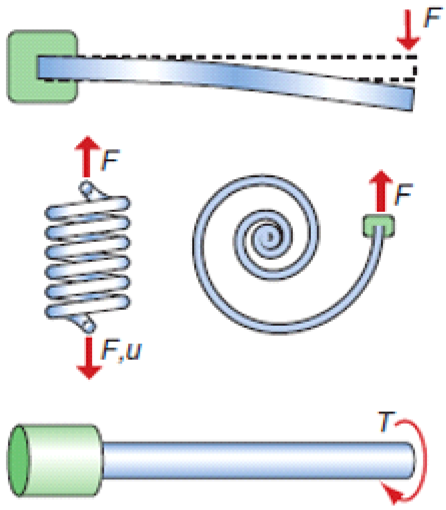
HB p 179 - 184

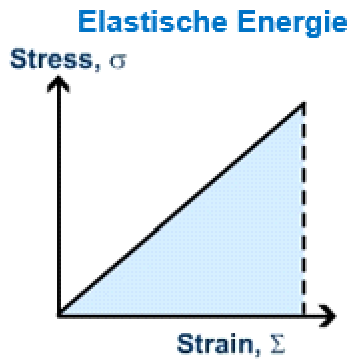
Elastische scharnieren



Veren

- Elastische Energie Opslag

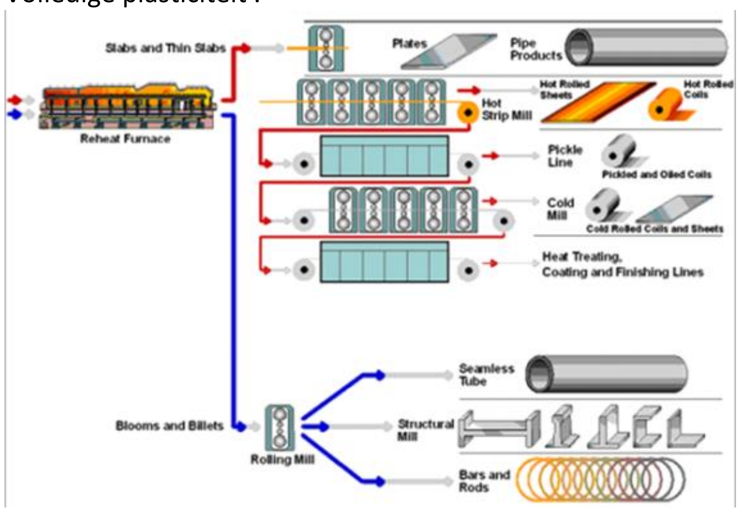




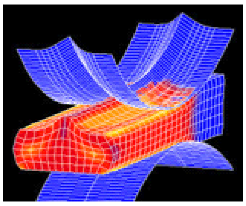
$$M = \frac{\sigma_y^2}{E}$$

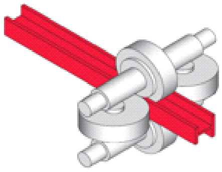
Arcellor Mittal

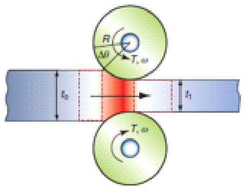
- Volledige plasticiteit :



- Walsen van metalen







Plastische Arbeid (per eenheid volume):
$\sigma_y . \varepsilon_{pl}$ en $\varepsilon_{pl} = \Delta t / t_0$

Volume per eenheid dwarsopp gevoed in matrijs:
$R.\Delta\theta.t_0$

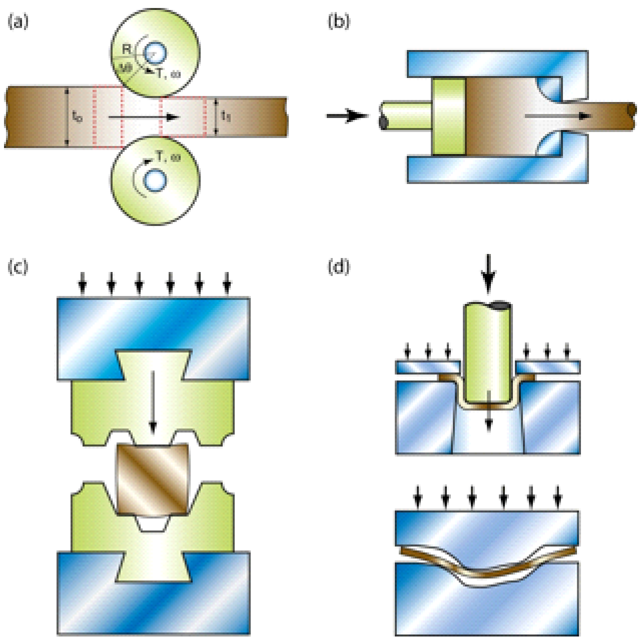
Arbeid geleverd door de twee wals matrijzen:
$2.T.\Delta\theta$

KOPPEL T
(Torque)

$$T = \frac{1}{2} R \sigma_y \Delta t$$

$$P = 2T\omega = R\omega\, \sigma_y\, \Delta t$$

WALSEN



EXTRUSIE

SMEDEN

DIEPDUWEN

STAMPEN