Incentives and Behavior
Final Exam 2020

Closed-ended questions (40 points)

This section contains ten multiple-choice questions, each with four possible answers (A, B, C, or D). For each question, exactly one answer is correct. Write your answers in the answer sheet on page 6. Every correct answer gives you 4 points. Every incorrect answer gives you 0 points. You can opt not to answer a question by leaving the answer box blank.

Game theory

Stag hunt. Two players, $i = 1, 2$, each have to decide whether to hunt stag (S) or hare (H). Hunting hare yields player i a payoff of 1, irrespective of the other player's action. Hunting stag yields player i a payoff that depends on what the other player chooses to hunt. If both hunt stag, each of them gets a payoff of 3. If player i hunts stag while player $j \neq i$ hunts hare, player i gets a payoff of zero. We thus consider the following payoff matrix:

		Player 2	
		S	H
Player 1	S	$3, 3$	$0, 1$
	H	$1, 0$	$1, 1$

Question 1: Which of the following statements is true?

(A) Playing S is a dominant strategy for player 1.

(B) The strategy profile (H, S) survives iterated deletion of strictly dominated strategies.

(C) Playing H is strictly dominated for player 2.

(D) Player 1's best response to player 2 hunting stag is to hunt hare.

Question 2: Which of the following statements is **not** true?

(A) In any Nash equilibrium of the game, each player's strategy is a best response to the strategy of the other player.

(B) There is a pure-strategy Nash equilibrium in which both players hunt stag.

(C) There is a pure-strategy Nash equilibrium in which both players hunt hare.

(D) There is a mixed-strategy Nash equilibrium in which each player hunts stag with probability 1/2 and hunts hare with probability 1/2.

Risk preferences

Choosing between risky alternatives. Consider an individual with initial wealth $x_0 = 10000$ who evaluates risky alternatives according to their expected utility and whose utility function is $u(x) = \sqrt{x}$. Consider a gamble with two possible outcomes: the individual can either lose an amount Δ_1, in which case her wealth becomes $x = x_0 - \Delta_1$, or win an amount Δ_2, in which case her wealth becomes $x = x_0 + \Delta_2$. Let p denote the probability of winning Δ_2, so that $1 - p$ is the probability of losing Δ_1. The individual must decide whether to accept the gamble or to reject it; if she rejects she preserves her initial wealth x_0.

Question 3: Which of the following statements is true?

(A) If $\Delta_1 = \Delta_2 = 1000$ and $p = 0.5$, the individual accepts the gamble.

(B) If $\Delta_1 = 1900$, $\Delta_2 = 2100$, and $p = 0.51$, the individual rejects the gamble.

(C) If $\Delta_1 = 1900$, $\Delta_2 = 4400$, and $p = 0.4$, the individual rejects the gamble.

(D) If $\Delta_1 = 3600$, $\Delta_2 = 2100$, and $p = 0.7$, the individual accepts the gamble.

Prospect theory. Kahneman and Tversky propose an alternative to expected utility theory called prospect theory. In what follows, you are asked to evaluate a few statements about prospect theory.

Question 4: Which of the following statements about prospect theory is **not** true?

(A) The prospect-theory value function is convex in the domain of gains and concave in the domain of losses.

(B) Prospect theory assumes that individuals evaluate outcomes in relation to a reference point.

(C) Prospect theory assumes that individuals are loss averse (i.e., losses weigh heavier than same-sized gains).

(D) Prospect theory assumes that individuals overweight small probabilities.

Time preferences

Consumption-saving decisions. Consider a consumption-saving model with three periods, $t = 0, 1, 2$. A decision maker with income stream $y = (y_0, y_1, y_2) = (0, w, 0)$ chooses a consumption plan $c = (c_0, c_1, c_2)$ and a savings plan $s = (s_0, s_1, s_2)$ to maximize her sum of discounted payoffs,

$$u(c_0) + \delta u(c_1) + \delta^2 u(c_2),$$

subject to the per-period budget constraint

$$c_t + s_t \leq y_t + s_{t-1}$$

for $t = 0, 1, 2$ (using the convention that $s_{-1} = 0$), where $\delta \in (0, 1)$ is the discount factor. Let $u(c_t) = \ln(c_t)$. Borrowing is not possible, so $c_0 = s_0 = 0$. Compute the decision maker's optimal consumption and savings for $t = 1, 2$.

Question 5: Which of the following statements is **not** true?

(A) $s_1 = \delta w/(1 + \delta)$

(B) $s_2 = 0$

(C) $c_1 = w(1 - \delta)/(1 + \delta)$

(D) $c_2 = \delta w/(1 + \delta)$

Now suppose the decision maker has $\beta - \delta$ preferences: in period t, she values a stream of consumption $(c_t, c_{t+1}, \ldots, c_T)$ at

$$u(c_t) + \beta \sum_{\tau=t+1}^{T} \delta^{\tau-t} u(c_\tau).$$

Compute the decision maker's consumption and savings for $t = 1, 2$.

Question 6: Which of the following statements is true?

(A) $c_1 = w/(1 + \beta\delta)$

(B) $s_1 = \beta w/(1 + \beta\delta)$

(C) The decision maker would like to commit herself to save more at $t = 2$.

(D) Due to her self-control problem, the decision maker procrastinates and only starts saving at $t = 2$.

Exploiting consumers

Add-on pricing. A hotel room costs hotel chain Tonhil 100 Euros to supply. Consumers are naive: they do not think about add-ons when they plan a hotel visit. Once they are at the hotel, they discover the need for an add-on (e.g., water from the mini-bar), which they can only get from the hotel. All consumers have a willingness-to-pay of 20 Euros for the add-on, and Tonhil's cost of providing it is zero. The market for hotel rooms is competitive, so that Tonhil's profit is driven down to zero.

Question 7: Which of the following statements about the equilibrium prices is true?

(A) Tonhil charges 100 Euros for the hotel room and 0 Euros for the add-on.

(B) Tonhil charges 80 Euros for the hotel room and 20 Euros for the add-on.

(C) Tonhil charges 90 Euros for the hotel room and 10 Euros for the add-on.

(D) Tonhil charges 100 Euros for the hotel room and 20 Euros for the add-on.

A competitor of Tonhil, called Transparent, is choosing its business strategy. A consultant argues that Transparent should charge 100 Euros for the hotel room and 0 Euros for the add-on. The consultant also proposes that Transparent should educate consumers by advertising the fact that Transparent's hotel rooms come with a free add-on. All consumers would then become aware of the add-on. Even though they could substitute away from the add-on (and the substitute is costless), the consultant argues that they have no incentive to do so, given that the add-on is free.

Question 8: Which of the following statements about ~~about~~ what would happen if Transparent implemented the business strategy proposed by the consultant is true?

(A) Consumers would switch from Tonhil to Transparent.

(B) Consumers would stay with Tonhil and substitute away from the add-on.

(C) Transparent would make a loss.

(D) Tonhil would benefit.

Performance pay

Moral hazard. In the principal-agent model with moral hazard seen in class, a risk-neutral principal contracts with a risk-averse agent. The agent carries out a task on the principal's behalf and must decide whether to exert high or low effort.

Question 9: Which of the following statements about the optimal incentive contract is **not** true?

(A) If the agent's effort is observable, the principal pays the agent a fixed wage.

(B) If the agent's effort is unobservable and the principal pays a fixed wage, the agent exerts low effort.

(C) If the agent's effort is unobservable, the agent's wage increases as the principal's profit increases.

(D) If the agent's effort is unobservable, the optimal contract minimizes variability in the agent's wage subject to providing incentives for effort provision.

Multitasking. In the multitasking model seen in class, the agent performs two tasks, one of which can be precisely measured (task 1) while the other can only be measured with noise (task 2). The cost of effort is $C(a_1, a_2) = \frac{1}{2}(a_1^2 + a_2^2) + \delta a_1 a_2$, where a_1 and a_2 are the efforts on tasks 1 and 2, respectively, and $\delta \in [0, 1)$ is a parameter.

Question 10: Which of the following statements about the optimal contract is **not** true?

(A) If $\delta = 0$, the optimal contract provides first-best incentives on task 1.

(B) If $\delta = 0$, the incentives on task 2 are set below the first-best level.

(C) If $\delta > 0$, the incentives on task 1 are set above the first-best level.

(D) If $\delta > 0$, the incentives on both tasks decrease with the amount of noise in the measurement of task 2.

ANSWERS

Indicate your answer (A, B, C, or D) to each of the 10 closed-ended questions above in the table below. If you do not want to answer a question, leave the corresponding row empty.

Your name: ....

Question	Answer
1	B
2	D
3	D
4	A
5	C
6	A
7	B
8	B
9	C
10	C

Open-ended questions (40 points)

This section contains two problems with open-ended questions. Write your answers in the boxes provided below the questions. Write your name on the pages on which you provide answers.

Performance pay for a loss-averse agent (20 points)

Consider a principal who wants to delegate a task to a loss-averse agent. The task can result in two possible levels of profit, $x \in \{\underline{x}, \overline{x}\}$ with $\overline{x} > \underline{x}$. The probability that a given level of profit is realized depends on the agent's effort, $a \in \{a^L, a^H\}$. Denote $p^H \equiv \Pr(x = \overline{x} | a = a^H)$ the probability that profit is high given a^H, and similarly $p^L \equiv \Pr(x = \overline{x} | a = a^L)$. Assume $0 < p^L < p^H < 1$. The cost of effort is $C(a^L) = 0$ and $C(a^H) = c > 0$. The principal's payoff is $\pi = x - w$, where w is the wage paid to the agent. The agent's payoff is $u = v(w - r) - C(a)$, where

$$v(w - r) = \begin{cases} w - r & \text{for } w \geq r \\ \lambda(w - r) & \text{for } w < r. \end{cases}$$

The parameter $\lambda \geq 1$ measures the agent's loss aversion, and r denotes his reference point. The value of the agent's outside option is $\bar{V} = 0$. Assume that the agent's reference point is his outside option, i.e., $r = \bar{V} = 0$. The principal offers a contract $(\underline{w}, \overline{w})$ to the agent, where $\underline{w}$ corresponds to the wage when profit is $\underline{x}$ and $\overline{w}$ corresponds to the wage when profit is $\overline{x}$.

(a) Write down the principal's maximization problem assuming she wants to implement high effort ($a = a^H$). (5 points)

$$\max_{(\underline{w},\bar{w})} \; p^H(\bar{x}-\bar{w}) + (1-p^H)(\underline{x}-\underline{w}) \qquad 1$$

$$\text{s.t.} \begin{cases} p^H v(\bar{w}) + (1-p^H)v(\underline{w}) - c \geq 0 & (PC) \quad 2 \\ p^H v(\bar{w}) + (1-p^H)v(\underline{w}) - c \geq p^L v(\bar{w}) + (1-p^L)v(\underline{w}) & (IC) \quad 2 \end{cases}$$

(b) Provide an informal argument suggesting that the agent's participation constraint must be binding at the solution to the principal's problem. (2 points)

If (PC) isn't binding, P can reduce $\underline{w}$ without affecting (IC), as $1-p^H < 1-p^L$.

(c) Assume that the incentive-compatibility constraint also binds. Solve for the optimal contract $(\underline{w}, \overline{w})$. (5 points)

$$(IC) \text{ binding} \Rightarrow v(\bar{w}) - v(\underline{w}) = \frac{c}{p^H - p^L}$$

$$(PC) \Rightarrow v(\underline{w}) + p^H \frac{c}{p^H - p^L} = c \iff v(\underline{w}) = -\frac{p^L c}{p^H - p^L} < 0$$

$$v(\underline{w}) = \lambda \underline{w} \Rightarrow \underline{w} = -\frac{p^L c}{\lambda(p^H - p^L)} \qquad 2.5$$

$$v(\bar{w}) = \bar{w} = v(\underline{w}) + \frac{c}{p^H - p^L} = \frac{1-p^L}{p^H - p^L} c \qquad 2.5$$

(d) Suppose the principal wants to implement low effort ($a = a^L$). Argue informally that the contract $(\underline{w}, \overline{w}) = (0, 0)$ is optimal for the principal. (2 points)

Constant wage → no risk of a loss for the agent, which would need to be compensated.

(e) Derive a necessary and sufficient condition under which the principal prefers to implement high effort. Show that this condition becomes more difficult to satisfy as λ increases. (4 points)

$p^H(\bar{x}-\bar{w}) + (1-p^H)(\underline{x}-\underline{w}) \geq p^L(\bar{x}-0) + (1-p^L)(\underline{x}-0)$

$\iff (p^H-p^L)(\bar{x}-\underline{x}) \geq p^H \frac{(1-p^L)c}{p^H-p^L} - (1-p^H)\frac{p^L c}{\lambda(p^H-p^L)}$ 3

The RHS is increasing in λ. 1

(f) Provide a brief intuition for the last result. (2 points)

The more loss-averse the agent, the greater the compensation the principal needs to give to achieve (IC).

Selling to time-inconsistent consumers (20 points)

Consider a consumer who has to make a consumption decision concerning a good with immediate costs and delayed benefits. If she consumes a quantity x of the good, the consumer incurs costs of $x^2/2$ in period $t = 1$ and receives benefits of bx in period $t = 2$, where $b > 0$. At time t, the consumer's overall utility from a payoff stream $(u_t, u_{t+1}, \ldots, u_T)$ is

$$u_t + \beta \sum_{\tau=t+1}^{T} u_\tau,$$

where $0 < \beta \leq 1$. The good is sold by a monopolist charging a fixed fee F and a price per unit p. The monopolist's cost of providing the good is κx, where $\kappa \geq 0$. There are three periods, $t = 0, 1, 2$, and the timing is as follows. At $t = 0$, the monopolist offers a contract (F, p) to the consumer, and the consumer accepts or rejects. If she rejects, her payoff is zero and the game ends. If she accepts, then at $t = 1$ the consumer chooses $x \geq 0$, pays $F + px$, and incurs costs $x^2/2$; at $t = 2$, the consumer obtains benefits bx.

(a) Give an example of a good that fits this model and briefly explain your choice. (2 points)

Health club/gym: exercising is costly in the short run but has long-run health benefits.

(b) Suppose $\beta = 1$. Derive the quantity the consumer chooses at $t = 1$ (conditional on having accepted the contract). State the condition for her to accept the contract at $t = 0$. (3 points)

$$\max_x \; bx - \frac{x^2}{2} - px - F$$ 1

FOC: $b - p = x$ 1

Accept if $F \leq (b-p)x - \frac{x^2}{2} = \frac{(b-p)^2}{2}$ 1

(c) Still for the case $\beta = 1$, state the monopolist's profit-maximization problem and determine the optimal fixed fee and per-unit price. (5 points)

$$\max_{p,F} \; (p-k)(b-p) + F \quad \text{s.t. } F \leq \frac{(b-p)^2}{2}$$ 3

$$\Rightarrow \max_p \; \left(p - k + \frac{b-p}{2}\right)(b-p)$$

FOC: $\frac{b-p}{2} - \left(p - k + \frac{b-p}{2}\right) = 0 \iff p = k$ 1

$F = \frac{(b-k)^2}{2}$ 1

(d) Now suppose $\beta < 1$ and the consumer is *naive*, i.e., at $t = 0$, she mistakenly thinks that at $t = 1$ she will choose as if $\beta = 1$. Derive the quantity the consumer actually chooses at $t = 1$ and state the condition for her to accept the contract at $t = 0$. (2 points)

$$\max_x \; (\beta b - p)x - \frac{x^2}{2} - F$$ 1

FOC: $\beta b - p = x$

Accept if $F \leq \frac{(b-p)^2}{2}$ as consumer thinks she will choose $x = b - p$ 1

(e) For the case of a naive consumer with $\beta < 1$, state the monopolist's profit-maximization problem and determine the optimal fixed fee and per-unit price. (5 points)

$$\max_{p,F} (p-\kappa)(\beta b - p) + F \quad \text{s.t.} \quad F \leq \frac{(b-p)^2}{2}$$ 3

$$\Rightarrow \max_{p} (p-\kappa)(\beta b - p) + \frac{(b-p)^2}{2}$$

$$\text{FOC: } \beta b - p - p + \kappa - (b-p) = 0 \Leftrightarrow p = \kappa - (1-\beta) b$$ 1

$$F = \frac{(b-\kappa+(1-\beta)b)^2}{2}$$ 1

(f) Explain briefly why a monopolist would want to set the unit price below marginal cost when selling to a naive time-inconsistent consumer. (3 points)

Slightly lowering the unit price below MC increases the consumer's anticipated consumption by more than her actual consumption. The monopolist can exploit this by raising the fixed fee