Exam Risk Management in Financial Institutions

June 2020

1 True/False Questions

10 true/false questions with justification. Points are only given if the justification is correct and detailled enough. A blank page was given at the beginning in order to write down any assumptions or possible ambiguities and the assumptions that needed to be made in order to give an answer to the questions.

1. A bank should only take into account unexpected risk and not expected risk.

2. Given that risk appetite is individual specific, banks should not take risk appetite into account.

3. Let us suppose we have a portfolio with an annual volatility of 20%. If the daily return of the portfolio is negligible, then the 1-day normal expected shortfall associated with a 99.9% VaR is equal to 4.26%.

4. A reverse stress test can be used in order to give subjective probabilities to given stress scenarios.

5. The conditional Value at Risk is a risk measure that reflects diversification benefits.

6. Due to the inherent skewness of credit losses, the relative credit Value at Risk is more appropriate to use than the absolute credit Value at Risk.

7. Consider a company with the value of assets equal to €125 mio. The assets are funded by a zero-coupon bond with face value €80 mio (T = 1 year) and an equity tranche. The risk-free rate is at 3% p.a. and asset price volatility is 25% p.a. Then the implied probability of default in Merton's model is equal to 3.75%.

8. Let us suppose we have a portfolio of two bonds X and Y. The 1-year probability of default of bond X is 0.6% and the 1-year probability of default of Y is 4.8%. We know that the joint probability of default is equal to 0.194%. Then the 1-year probability of no-default in the portfolio is equal to 94.79%.

9. If we have a portfolio with non-linear financial instruments, the use of the normal Value at Risk will underestimate the risk of the portfolio.

10. Let us suppose we have a portfolio of n options on a stock S with delta relative to that stock equal to δ. Then the delta-normal 1-day 95% relative Value at Risk of that portfolio is equal to:

$$VaR^r_{0.95,1} = -(n \cdot \delta) \cdot \sigma_{\Delta S} \cdot (-1.645)$$

2 Credit VaR of a portfolio of bonds

We suppose that we have a portfolio of 3 bonds: a bond rated AA, a bond rated B and a bond rated CCC. The 1-year probability of default of bonds in function of the rating is the following:

Rating	**No Default(%)**	**Default(%)**
AAA	0.01	99.99
AA	0.03	99.97
A	0.5	99.5
BBB	0.85	99.15
BB	1.06	98.94
B	1.62	98.38
CCC	15.56	84.44

The exposures for the different bonds in the portfolio are the following:

Rating	**Exposure**
AA	23
B	8
CCC	7

The probability of default of the different bonds are supposed to be independent. The recovery rates of the different bonds are supposed to be known and equal to 0.

a) Compute the 1-year credit VaR of the portfolio at level 99.9% of the portfolio.

b) How can we improve the estimation of the risk in this portfolio?

3 Solutions

3.1 True/False questions

1. A bank should only take into account unexpected risk and not expected risk.

Solution: False. Both expected and unexpected risk should be taken into account when doing risk management.

2. Given that risk appetite is individual specific, banks should not take risk appetite into account.

Solution: False. Financial institutions can also have a given risk appetite: a pension institution will not have the same risk appetite as a highly speculative hedge fund.

3. For an annual volatility of 20%, the 1-day normal expected shortfall associated with a 99.9% VaR is equal to 4.26%.

Solution: False. Using the assumption of 250 trading days a year, the 1-day expected shortfall associated with a 99.9% VaR is equal to:

$$ES_{0.999,1} = \frac{1}{0.001}\phi(\Phi^{-1}(0.001))\frac{0.2}{\sqrt{250}} = 4.6957\%$$

4. A reverse stress test can be used in order to give subjective probabilities to given stress scenarios.

Solution: True (?). Reverse stress testing allows us to start from a failure, the identifying the possible events that could trigger such a failure. By allocating subjective probabilities for each possible events that can trigger a failure, subjective probabilities for a given stress scenario could be obtained.

5. The conditional Value at Risk is a risk measure that reflects diversification benefits.

Solution: True. The conditional Value at Risk or Expected Shortfall is a coherent risk measure. All coherent risk measure are subadditive, which means that they reflect diversification benefits.

6. Due to the inherent skewness of credit losses, the relative credit Value at Risk is more appropriate to use than the absolute credit Value at Risk.

Solution: True (?). Due to the fact that credit loss distribution is likely to be highly skewed, with low probabilities of default and a possibly high premium for holding the bonds, the absolute credit Value at Risk is likely to underestimate the real amount of money at risk in the portfolio. The relative credit Value at Risk is therefore more appropriate to be used than the absolute credit Value at Risk.

7. Consider a company with the value of assets equal to €125 mio. The assets are funded by a zero-coupon bond with face value €80 mio (T = 1 year) and an equity tranche. The risk-free rate is at 3% p.a. and asset price volatility is 25% p.a. Then the implied probability of default in Merton's model is equal to 3.75%.

Solution: True. The implied probability of default in Merton model is given by:

$$PD = \Phi(-d_2)$$

with:

$$d_2 = \frac{\log\left(\frac{A_0}{B}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

Using the data given in the statement, we obtain $d_2 = 1.7801$ and $PD = 3.25\%$.

8. Let us suppose we have a portfolio of two bonds X and Y. The 1-year probability of default of bond X is 0.6% and the 1-year probability of default of Y is 4.8%. We know that the joint probability of default is equal to 0.194%. Then the 1-year probability of no-default in the portfolio is equal to 94.794%.

Solution: True. If we denote by X and Y the random variables representing the default of the bond and we write $PD_x = P[X=1]$, $PD_y = P[Y=1]$ and $x = \rho\sqrt{PD_x(1-PD_x)PD_y(1-PD_y)}$, then the joint probability of default of the two bonds is given by:

$$\begin{aligned} P[X=1,Y=1] &= P[X=1]P[Y=1] + x \\ x &= P[X=1,Y=1] - P[X=1]P[Y=1] \\ &= 0.006 \cdot 0.048 - 0.00194 \\ &= 0.001652 \end{aligned}$$

The probability of no default in the portfolio is then equal to:

$$\begin{aligned} P[X=0,Y=0] &= P[X=0]P[Y=0] + x \\ &= (1-0.006)\cdot(1-0.048) + 0.001652 \\ &= 0.94794 \end{aligned}$$

9. If we have a portfolio with non-linear financial instruments, the use of the normal Value at Risk will underestimate the risk of the portfolio.

Solution: True/False (depending on justification). Using a normal VaR for non-linear financial instruments can lead to an underestimation or overestimation of the risk in the portfolio but whether the Value at Risk is over- or underestimated depends on the financial instruments used and on the exact portfolio.

10. Let us suppose we have a portfolio of n options on a stock S with delta relative to that stock equal to δ. Then the delta-normal 1-day 95% relative Value at Risk of that portfolio is equal to:

$$VaR^r_{0.95,1} = -(n\cdot\delta)\cdot\sigma_{\Delta S}\cdot(-1.645)$$

Solution: True. The value of the portfolio is given by:

$$V = nV_c$$

with V_c: the value of a single option. Using a delta-approximation, the change of value of the portfolio is given by:

$$\begin{aligned} \Delta V &= n\cdot\Delta V_C \\ &\approx n\cdot\frac{\partial V_C}{\partial S}\Delta S \\ &\approx n\cdot\delta\cdot\Delta S \end{aligned}$$

The volatility of the portfolio is then given by $\sigma_{\Delta V} = n\cdot\delta\sigma_{\Delta S}$. The delta-normal 1-day 95% relative Value at Risk of that portfolio is then equal to:

$$\begin{aligned} VaR^r_{0.95,1} &= -\sigma_{\Delta V}\cdot\Phi^{-1}(0.05) \\ VaR^r_{0.95,1} &= -n\cdot\delta\cdot\sigma_{\Delta S}\cdot(-1.645) \end{aligned}$$

3.2 Credit VaR of a portfolio of bonds

3.2.1 Credit Var of the portfolio

Note: the probability of default for the bonds in the portfolio are the ones that were given in the exam, the other probabilities of default are made up.

The random variables representing the losses associated with each bond are the following:

$$L_A = \begin{cases} 23 & p = 0.0003 \\ 0 & p = 0.9997 \end{cases} \qquad L_B = \begin{cases} 8 & p = 0.0162 \\ 0 & p = 0.9838 \end{cases} \qquad L_C = \begin{cases} 7 & p = 0.1556 \\ 0 & p = 0.8444 \end{cases}$$

Given that the probabilities of default are independent from each other, the joint probabilities of default are simply equal to the product of the marginal probabilities of default. The full distribution of the losses is then equal to:

$$L = L_A + L_B + L_C = \begin{cases} 38 & p = 7.56216 \cdot 10^{-7} \\ 31 & p = 4.103784 \cdot 10^{-6} \\ 30 & p = 4.5923784 \cdot 10^{-5} \\ 23 & p = 2.49216216 \cdot 10^{-4} \\ 15 & p = 2.519963784 \cdot 10^{-3} \\ 8 & p = 1.367517622 \cdot 10^{-2} \\ 7 & p = 0.1530333562 \\ 0 & p = 0.8304715038 \end{cases}$$

Using the credit loss distribution, we have that $\mathbb{E}[L] = 1.2257$.

The 1-year 99.9% Credit VaR of the portfolio is then equal to:

$$\begin{aligned} VaR^r_{0.999,1} &= L^* - \mathbb{E}[L] \\ &= 15 - 1.2257 \\ &= 13.7743 \end{aligned}$$

3.2.2 Possible ways to improve

Some ideas:

- Zero recovery rates: unrealistic → use recovery rates in function of seniority
- Bonds that can only stay with their current rating: unrealistic → use transition matrix and forward yields in order to allow for more general family of credit losses
- Independence between different bonds: unrealistic → use of historical data in order to estimate correlations
- VaR doesn't always reflect diversification benefits → computes credit ES in addition to VaR