SAMENVATTING III

WISKUNDIGE MODELLEN

WISKUNDIGE ANALYSE: MACRO-ECONOMISCHE PROBLEMEN

ACADEMIEJAAR ’22 - ‘23

INHOUD

CONSUMPTION

Consumption is 2/3 of the GDP and the whole point in economics: the increase in welfare

2 faces:

- AD short run: IS $Y = C(Y_d) + I + G + NX$
- AS: complement consuming is saving → national investments and money growth

4 major approaches:

1. Keynes approach
2. Fisher's approach
3. Life-cycle or permanent income approach
4. Rational expectations approach

KEYNESIAN CONSUMPTION FUNCTION

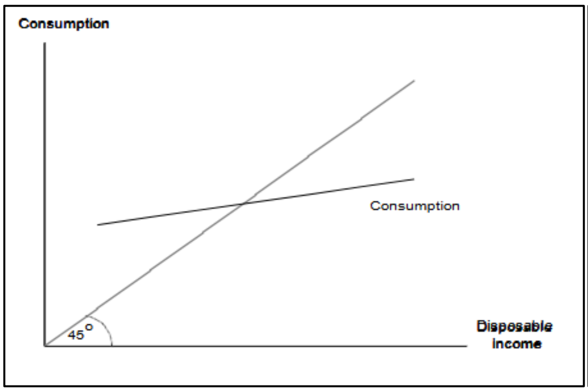
Consumption depends fundamentally on personal income

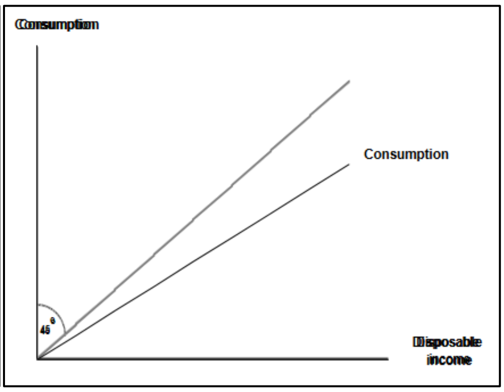
$$C = a + bY_d$$

a = autonomous spending
$b = dC/dY_d$ = marginal tendency to consume $0 < b < 1$
$\frac{C}{Y_d} = b + \frac{a}{Y_d}$ = average tendency to consume
Y_d = current income

Micro-economics: $a > 0, b < \varepsilon <$ SHORT RUN

Macro-economic: $a \approx 0, b < 1$ LONG RUN





Problems:

- Theoretical: consumption does not depend on income only
- Empirical:
  - Micro-economic data: average tendency falls when income rises
  - Macro-economic data: average tendency does not systemically fall, roughly constant

FISHER'S APPROACH

Basic idea: smooth consumption over lifetime because of lifetime income expectations

Assumptions: 2 periods, income Y_1, Y_2, consumption C_1, C_2

→ goal = maximalize lifetime utility

Lifetime utility: $U = u(C_1) + \frac{u(C_2)}{1+\phi}$

u two times differentiable → concave function

ϕ = time preference. The consumer is impatient so the utility today is bigger than tomorrow

The budget constraint: savings in period 1, no savings in period 2

- Period 1: $V_2 = (1+r)(V_1 + Y_1 - T_1 - C_1$
- Period 2: $C_2 = V_2 + Y_2 - T_2$

V = real financial wealth
r = real interest rate
Y = real labour income
T = real net tax payement
C = real consumption

ALGEBRAICALLY:

$$C_1 + \frac{C_2}{1+r} = V_1 + Y_1 - T_1 + \frac{Y_2 - T_2}{1+r}$$

→ present value total consumption can not exceed present value disposable income plus initial stock of wealth

→ concept of **human wealth** $H_1 \equiv Y_1 - T_1 + \frac{Y_2 - T_2}{1+r}$

Simplified budget constraint:

$$C_1 + \frac{C_2}{1+r} = V_1 + H_1$$

Maximalize U, taking budget constraint into account.

$$u'(C_1) = \left(\frac{1+r}{1+\phi}\right) u'(C_2)$$

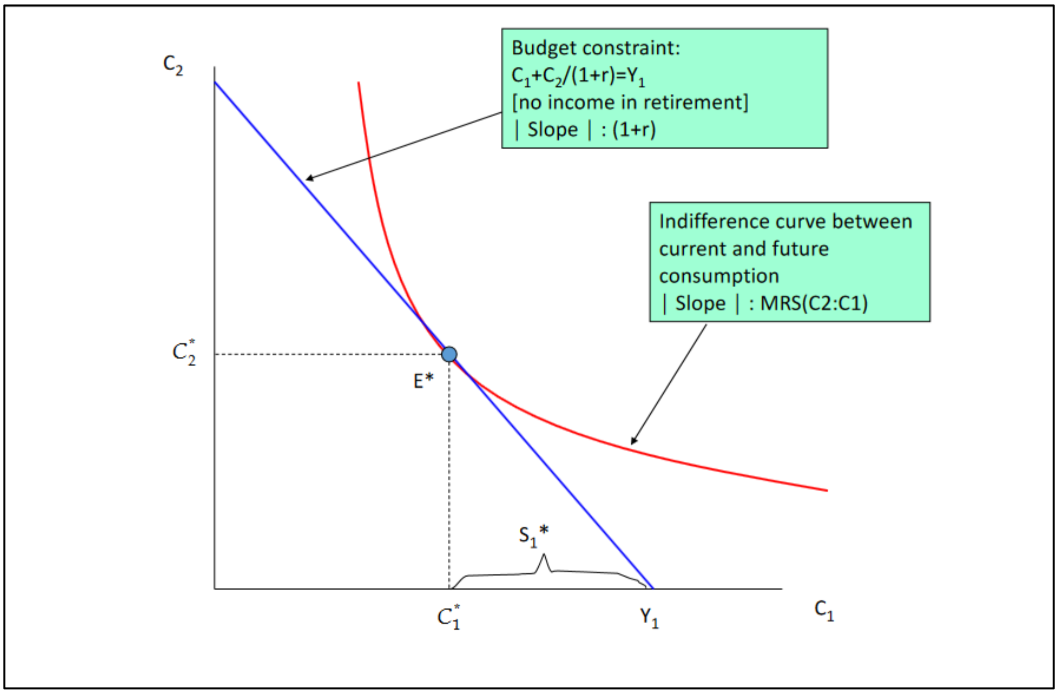
→ the consumer is indifferent between consuming one more unit or saving one more unit today

Keynes-Ramsey rule:

$$\frac{u'(C_1)}{u'(C_2)/(1+\phi)} = 1 + r$$

- Utility cost reducing C_2 by 1 = $u'(C_2)/(1+\phi)$ → period 2
- Utility gain reducing C_2 by 1 = $u'(C_1)/(1+r)$ → period 1

VISUALLY:



Consume C_1^*: save $Y_1 - C_1^* = S_1^*$ → interest on savings $(1 + r)S_1^*$

Optimum is where curves cross and the Keynes-Ramsey rule applies
→ consumption smoothing

ANALOGY:

Appelen en peren in micro-economie

LIFE CYCLE MODEL

= extension Fisher model to multi period: planning from age 1 to D

Utility function:

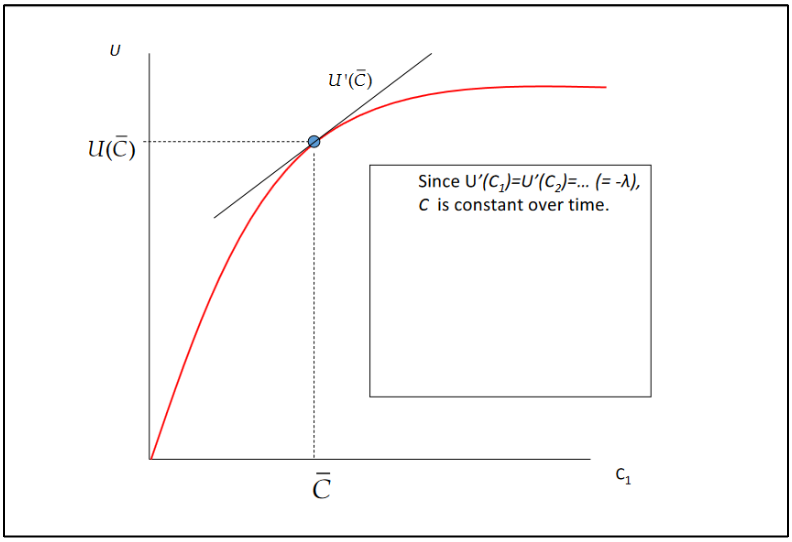
$$\max V(C_1, C_2, \dots, C_D) = \sum_{z=1}^{D} (1+\delta)^{-z} U(C_z)$$

Budget constraint:

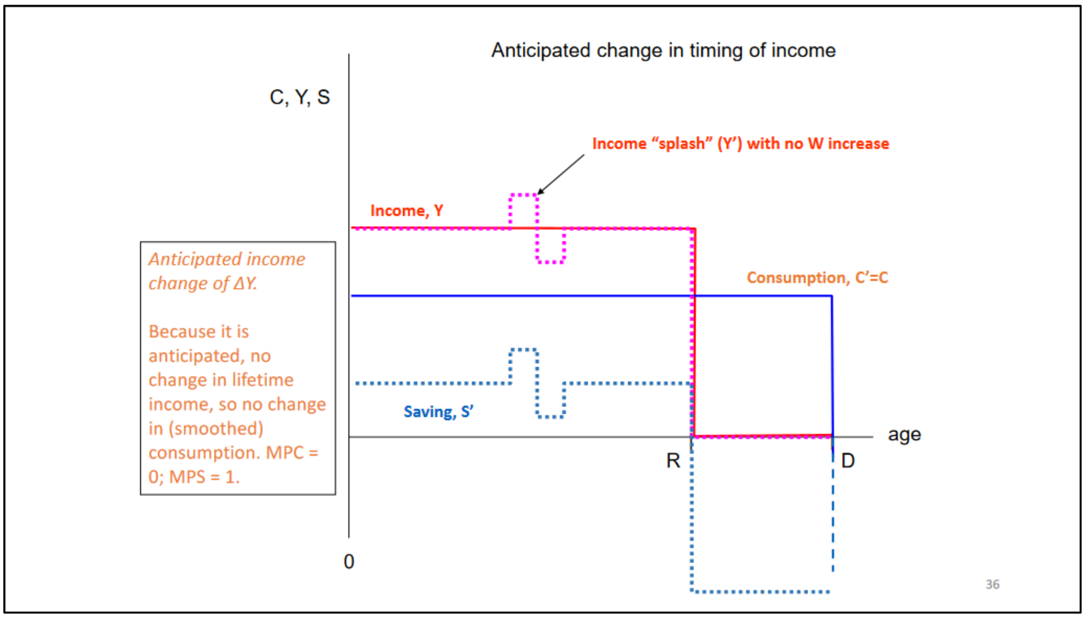
$$\sum_{z=1}^{D} (1+r)^{-z} C_z = \sum_{z=1}^{D} (1+r)^{-z} Y_z$$

ALGEBRAICALLY: Lagrange maximization

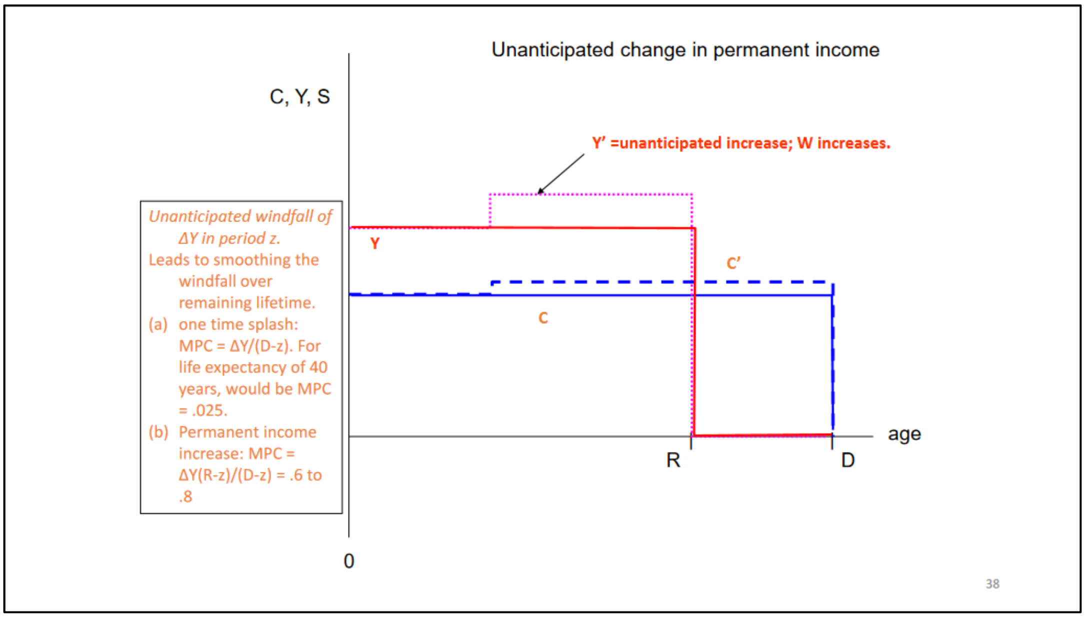
VISUALLY:



Consumption smoothing and anticipated income change:



Unanticipated increase in income:



→ redistribution consumption

EXAMPLE LIFE-CYCLE MODEL

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RATIONAL EXPECTATIONS

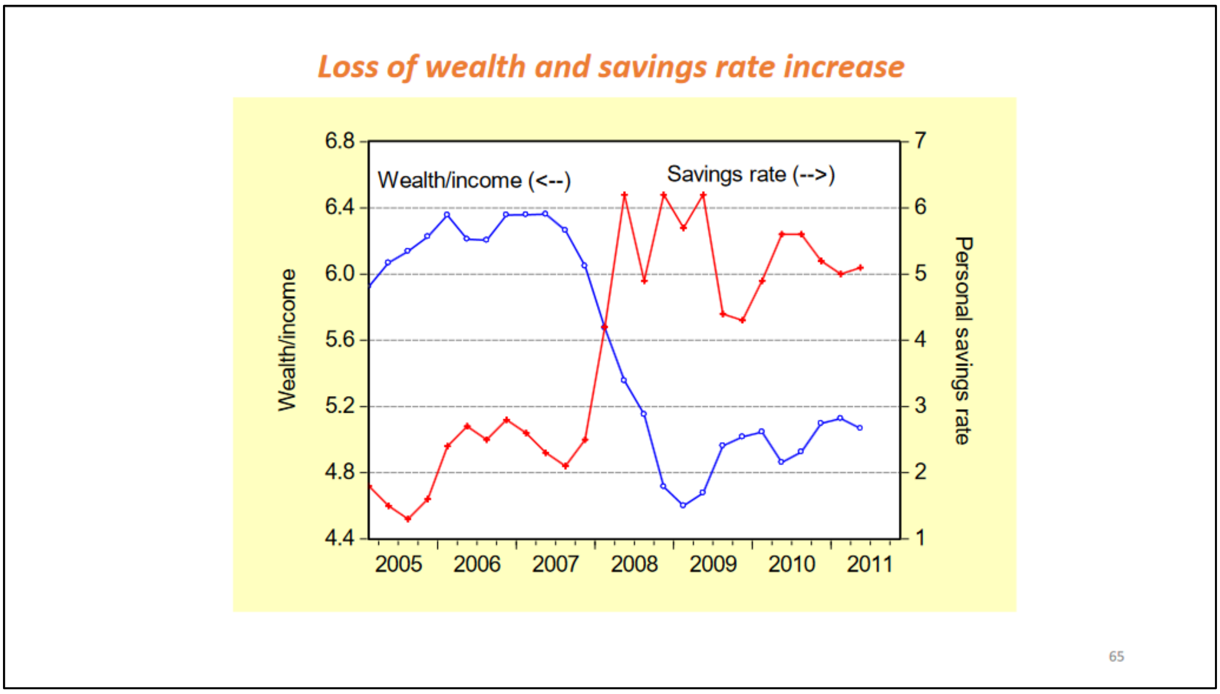
Consumption and Wealth

Initial wealth spread over life cycle
Augmented model:

$$C_t = \beta_0 + \beta_1 Y_t + \beta_2 W_t$$

$0 < \beta_2 < 1$

GFC: wealth effect



Consumption and Interest Rates

Empirical and theoretical very hard

Human wealth effect: rise in interest rate reduces present value of income, because $H \equiv Y_1 - T_1 + \frac{Y_2 - T_2}{1+r}$

Financial wealth effect: rise in state of interest reduces market value of financial wealth, because stock and housing pricing vary negatively with real interest rate.

CONSUMPTION AND TAXES

How consumption responds to taxes: stimulating economic activity

Partial view: responding to timing taxes

Complete view: consumer understands that government has also budget constraint

Tax cuts:

- Temporary

$$\frac{\partial C_1}{\partial T_1} = -\theta$$

- Permanent

$$\frac{\partial C_1}{\partial T_1} + \frac{\partial C_2}{\partial T_1} = -\theta\left(1 + \frac{1}{1+r}\right)$$

RICARDIAN EQUIVALENCE

- Government budget constraint

$$D_1 + G_1 + \frac{G_2}{1+r} = T_1 + \frac{T_2}{1+r}$$

  D_1 = real debt at beginning period 1

- Equivalence tax and debt finance

$$dC_1 = -\theta\left(dT_1 + \frac{dT_2}{1+r}\right) = 0$$

  Remarkable contrast $\frac{\partial C_1}{\partial T_1} = -\theta$

  → prediction that fiscal policy = temporary tax cut has no effect!
  Interesting theoretically frame of mind

- Critic: intergeneration distribution effects, effect on human behaviour, credit constraints (a fraction of the population is poor)
  - Aggregate consumption function

$$\frac{\partial C_1}{\partial(Y_1 - T_1)} = \mu + (1-\mu)\theta > \theta$$

  - Debt-finance tax cut

$$\frac{dC_1}{dT_1} = -\mu < 0$$

  - Temporary tax cut

$$\frac{dC_1}{dT_1} = -(\mu + (1-\mu)\theta) \qquad \frac{\partial}{\partial\mu}(|dC_1/dT_1|) > 0$$

Note: behavioural economics, people are not optimizers → inconsistent decisions

FINANCIAL CRISES

Different views on financial crises:

1. Traditional
2. Externalities
3. Financial cycle & non-rational beliefs
4. Efficient: crisi maait financiële sector weg en dat is gezond om slechte firms weg te snijden

DIAMOND-DYBVIG: BANK RUNS

Assumption: one bank, metaphor for all banks, the dynamic stays the same

Bank model:

Assets	Liabilities
Loans	Deposits

Essence of a bank: maturity transformation, short term deposits to long term investments

Consumer model:

	T = 0	T = 1	T = 2
TYPE 1	1	1	/
TYPE 2	1	/	R

Type 1 = vroege consument, vragen zonder rente bedrag al terug
Type 2 = late consumenten, hebben geld niet nodig in tijdstip T = 1, worden beloond met R>1

Doel consument: nut maximaliseren (concave functie!)

UNOBSERVABLE TYPES

Banks don't know if you are a type 1 or type 2 consumer, so they cannot treat you differently

- T = 0: investing as good as not investing
- T = 1: type 1 consume 1, type 2 waits
- T = 2: type 2 consumes R

No strategic behoviour:
Type 1 cannot profit from acting as type 2
Type 2 cannot profit from acting as type 1

Lottery: propability t get 1, propability $1 - t$ get R

Observable Types

Risk sharing at T = 0: insurance against being unlucky type 1

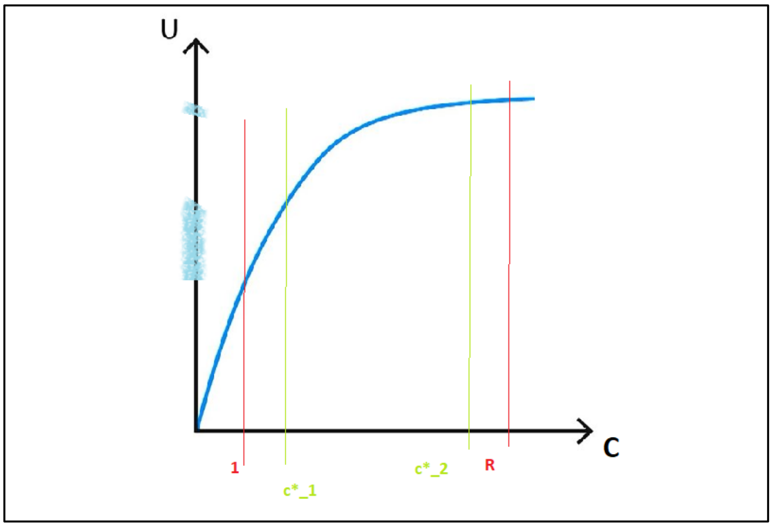
We proberen de type 1 en 2 consumenten even gelukkig te maken:

$$\begin{cases} tc_1 + \frac{(1-t)c_2}{R} = 1 \\ U_1' = RU_2' \end{cases}$$

De budgetvergelijking en de nutsvergelijking, hieruit volgt

$$1 < c_1^* < c_2^* < R$$

Everybody wants to avoid $1 = c_1^* < c_2^* = R$, because the expected utility is the lowest



Lottery with smaller variance: the unlucky ones don't lose as much, the lucky ones don't gain as much.

→ More preferable, but the lottery is more realistic

Demand Deposit Contract

Fraction type 1 consumers is f and so fraction type 2 is $(1-f)$
T = 1: fraction f withdraws r_1, total fr_1
T = 2: fractions $(1-f)$ withdraws R
→ BUT: bank runs out of money if $fr_1 = 1$, all the money of type 2 consumers is paid to type 1: only first in line get money, rest nothing → BANK RUN

How to fix this sequential service:
set $r_1 = c_1^*$ → but what about type 2 agents?

- Optimal risk sharing: all type 2 wait, get c_2^*
- Actual action depends on what type 2 thinks about type 1 actions: GAME THEORY

Type 2	Withdraw	Wait
Withdraw	c_1^*p, c_1^*p	$c_1^*p, 0$
Wait	$0, c_1^*p$	c_2^*, c_2^*

p = kans dat consument aan de bank is voordat ze geen geld meer heft

Equilibrium 1: (c_2^*, c_2^*) → everybody waits, bank stays in business and everyone gets c_2^* at T = 2
Equilibrium 2: $(c_1^* p, c_1^* p)$ → everybody withdraws: BANK RUN

= example of the classic <u>prisoner's dilemma</u>

POLICY

Other reasons that bank go bankrupt: bad investments → actions of the bank itself instead of irrational behaviour of the consumers.

Regulation:

- Bank: control mechanism of the government
- Consumer: prevent from choosing the bad equilibrium by reducing p: suspension of convertibility (bank op slot), deposit insurance

Suspension of Convertibility
= bank closes until T = 2 when an amount of deposits are withdrawn
→ type 2 is guaranteed that its paid-off, no incentive to run

Does not work if t is stochastic: more withdrawn before shutdown, still incentive to run

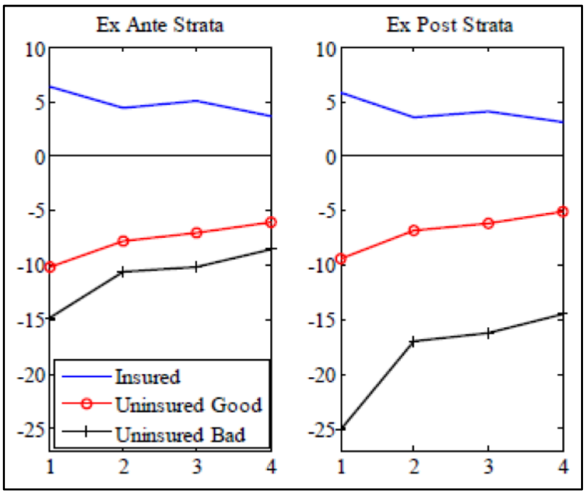
Deposit insurance
= afstraffen van risicovol gedrag

→ Clou blijft het wegnemen van de incentive to run: belofte van geld terug krijgen doet dit

Why do depositors run? They suspect others will run

- Sunspots = panic: ensuring big enough aggregate pool of liquidity
- Fundamentals: ensuring big enough own pool of liquidity

Widespread bank run: (good and bad aims to the investment policy of the bank)



Other runs: oil and grain due war (fundamental), toilet paper due pandemic (sunspot)

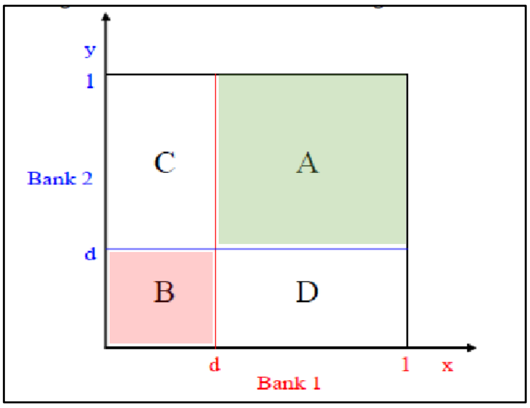
WAGNER: SYSTEMATIC RISK

= interest in overall health of the financial system more than one bank

SYSTEMATIC RISK

Externalities: the government as bank-rescuer

Model:
Bank 1: investment X, return x
Bank 2: investment Y, return y
each asset return μ, variance σ^2, no correlation (by transactions, they do have similar exposures)
default when x < d or y < d



A: alles in orde, genoeg return voor beide banken
B: beide hebben tekorten SYSTEMIC CRISIS
C, D: 1 return schiet tekort INDIVIDUAL CRISIS

Default → sell of assets at discount

- INDIVIDUAL CRISIS: sell to other bank, discount c_I
- SYSTEMIC CRISIS: sell to outsider, discount $c_S > c_I$
  alle banken zijn failliet, iemand buiten de sector is minder bekwaam om de assts te gebruiken of waarderen, daarom hogere discount

Expected return bank 2, symmetrisch voor bank 1:

$$V_2 = \mu - \pi_2 c_I - \pi_S c_S$$

π_2 = kans dat bank 2 failliet gaat
π_S = kans op een systeemcrisis

DIVERSIFICATION

Banken proberen het risico op (individueel en systematisch) faillisement te verkleinden door te diversifiëren in hun investeringen.

Bank 2 invests fraction $0 \leq r_2 \leq 1/2$ in investment X

Total return:

$$return = r_2 x + (1 - r_2) y$$

Variance on this return:

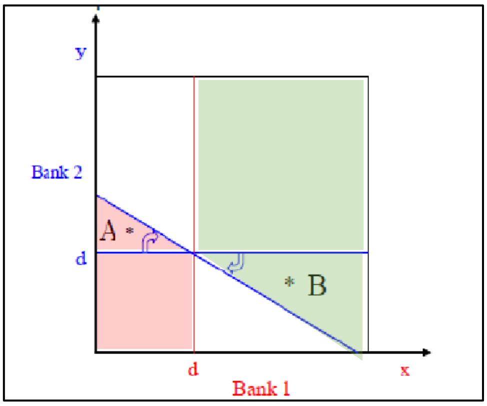
$$Var(r_2 x + (1 - r_2) y) = r_2^2 \sigma^2 + (1 - r_2)^2 \sigma^2 = (r_2^2 + (1 - r_2)^2) \sigma^2$$

no diversification $r_2 = 0$: $Var = \sigma^2$

full diversification $r_2 = 1/2$: $Var = (r_2^2 + (1 - r_2)^2)\sigma^2 < \sigma^2$

seems a good idea!

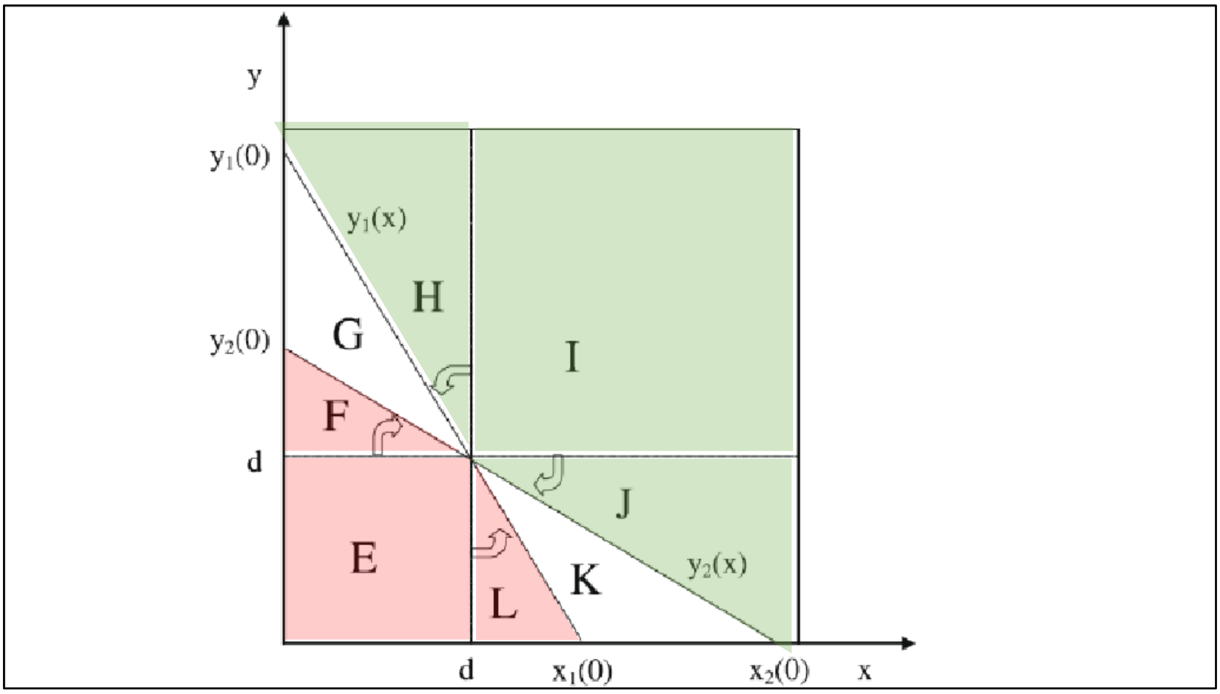
if y < d, then bank 2 can still have a good return from its X investment



A* = no crisis for bank 2, individual crisis for bank 1 → systemic crisis

B* = individual crisis for bank 2 → no crisis

Now bank 1 will also diversify:



Both banks have now a lower probability of failure, BUT there is a higher risk at systemic crises.

→ joint liquidity is more costly! Remember the high discount when a systemic crisis occurs

The micro-incentive to diversify is natural and has micro-rewards, but on a macro scale is diversifying a bad idea. Bank I optimizes $V_i = \mu - \pi_i c_I - \pi_S c_S$, but does not see the effect on π_S. The efficient outcome is welfare optimalisation where you look at joint profits.

POLICY

Align incentives banks and the system

- Pigou-taxing: tax the bad guys
- Measuring systemic risk contributions

Should be valued by bank sector: don't correlate to survive by buying others at a discount

> → waarom spelen banken niet in op dicverificatie en crisissen? Economisten kennen speltheorie! Één bank alleen zal niet gered worden, dus liever samen meer risico lopen en gered worden door de overheid dan alleen voort te bestaan met een groter risico op individuele crisis.

Model can be applied more generally: different assets or liabilities
→ remember GFC, alle banken waren zeer ster gediversifieerd dus zakten ze ook samen snel weg in een systeemcrisis.