Module 6

oef. 1 a) $\begin{cases} x+y-2z=10 \\ 4x+2y+2z=1 \\ 6x+2y+3z=4 \end{cases} \Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & -2 & 10 \\ 4 & 2 & 2 & 1 \\ 6 & 2 & 3 & 4 \end{array}\right)$

$\xrightarrow[R_3-6R_1]{R_2-4R_1} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 10 \\ 0 & -2 & 10 & -39 \\ 0 & -4 & 15 & -56 \end{array}\right) \xrightarrow{R_3-2R_2} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 10 \\ 0 & -2 & 10 & -39 \\ 0 & 0 & -5 & 22 \end{array}\right)$

$\Rightarrow \begin{cases} x+y-2z=10 \\ -2y+10z=-39 \\ -5z=22 \end{cases} \Rightarrow \begin{cases} x=\frac{37}{10} \\ y=\frac{-1}{2} \\ z=-\frac{22}{5} \end{cases}$

b) $\begin{cases} x+2y-3z=-1 \\ 3x-y+2z=7 \\ 5x+3y-4z=2 \end{cases} \Rightarrow \left(\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 3 & -1 & 2 & 7 \\ 5 & 3 & -4 & 2 \end{array}\right)$

$\xrightarrow[R_3-5R_1]{R_2-3R_1} \left(\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 0 & -7 & 11 & 10 \\ 0 & -7 & 11 & 7 \end{array}\right) \xrightarrow{R_3-R_2} \left(\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 0 & -7 & 11 & 10 \\ 0 & 0 & 0 & -3 \end{array}\right)$

$\Rightarrow$ geen oplossingen

c) $\begin{cases} x+2y-3z+2t=2 \\ 2x+5y-8z+6t=5 \\ 3x+4y-5z+2t=4 \end{cases} \Rightarrow \left(\begin{array}{cccc|c} 1 & 2 & -3 & 2 & 2 \\ 2 & 5 & -8 & 6 & 5 \\ 3 & 4 & -5 & 2 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$

$\xrightarrow[R_3-3\cdot R_1]{R_2-2\cdot R_1} \left(\begin{array}{cccc|c} 1 & 2 & -3 & 2 & 2 \\ 0 & 1 & -2 & 2 & 1 \\ 0 & -2 & 4 & -4 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{R_3+2R_2} \left(\begin{array}{cccc|c} 1 & 2 & -3 & 2 & 2 \\ 0 & 1 & -2 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$

$$\Rightarrow \begin{cases} x + 2y - 3z + 2t = 2 \\ \quad y - 2z + 2t = 1 \\ \quad z = \mu \\ \quad t = \lambda \end{cases} \Rightarrow \begin{cases} x = -\mu + 2\lambda \\ y = 1 + 2\mu - 2\lambda \\ z = \mu \\ t = \lambda \end{cases}$$

$$\{(-\mu + 2\lambda,\ 1 + 2\mu - 2\lambda,\ \mu,\ \lambda) \text{ met } \mu, \lambda \in \mathbb{R}\}$$

d) $$\begin{cases} 3x + y + z = 0 \\ 3y + x + z = 0 \\ 3z + x + y = 0 \end{cases} \Rightarrow \left(\begin{array}{ccc|c} 3 & 1 & 1 & 0 \\ 1 & 3 & 1 & 0 \\ 1 & 1 & 3 & 0 \end{array}\right)$$

$$\xrightarrow{R_3 \leftrightarrow R_1} \left(\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 1 & 3 & 1 & 0 \\ 3 & 1 & 1 & 0 \end{array}\right) \xrightarrow[R_2 - R_1]{R_3 - 3R_1} \left(\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & 2 & -8 & 0 \end{array}\right)$$

$$\xrightarrow{R_3 + R_2} \left(\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & 0 & -10 & 0 \end{array}\right) \Rightarrow \begin{cases} x + y + 3z = 0 \\ \quad 2y - 2z = 0 \\ \quad -10z = 0 \end{cases}$$

$$\Rightarrow \begin{cases} x = 0 \\ y = 0 \\ z = 0 \end{cases}$$

e) $$\begin{cases} 3x = 29 + y - z \\ 3y = 6 - 30z - x \\ 3z = 51 - 3x + 3y \end{cases} \Rightarrow \begin{cases} 3x - y + z = 29 \\ x + 3y + 30z = 6 \\ 3x - 3y + 3z = 51 \end{cases}$$

$$\Rightarrow \left(\begin{array}{ccc|c} 3 & -1 & 1 & 29 \\ 1 & 3 & 30 & 6 \\ 3 & -3 & 3 & 51 \end{array}\right) \xrightarrow{R_3/3} \left(\begin{array}{ccc|c} 3 & -1 & 1 & 29 \\ 1 & 3 & 30 & 6 \\ 1 & -1 & 1 & 17 \end{array}\right)$$

$$\xrightarrow{R_1 \leftrightarrow R_3} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 17 \\ 1 & 3 & 30 & 6 \\ 3 & -1 & 1 & 29 \end{array}\right) \xrightarrow[R_3 - 3R_1]{R_2 - R_1} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 17 \\ 0 & 4 & 29 & -11 \\ 0 & 2 & -2 & -22 \end{array}\right)$$

$$\xrightarrow{R_2 - 2R_3} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 17 \\ 0 & 0 & 33 & 33 \\ 0 & 2 & -2 & -22 \end{array}\right) \xrightarrow[\frac{R_2}{33}]{\frac{R_3}{3}} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 17 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & -1 & -11 \end{array}\right) \Rightarrow \begin{cases} x = 6 \\ y = -10 \\ z = 1 \end{cases}$$

f) $\begin{cases} x + y + z = 1 \\ y + z + u = 0 \\ x + u = 0 \end{cases} \Rightarrow \left(\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$

$\xrightarrow{R_3 \leftrightarrow R_1} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{R_3 - R_1} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$

$\xrightarrow{R_3 - R_2} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{cases} x + u = 0 \\ y + z + u = 0 \\ z = \lambda \\ -2u = 1 \end{cases} \Rightarrow \begin{cases} x = \frac{1}{2} \\ y = \frac{1}{2} - \lambda \\ z = \lambda \\ u = -\frac{1}{2} \end{cases}$

$\left\{ \left(\frac{1}{2}, \frac{1}{2} - \lambda, \lambda, -\frac{1}{2} \right) \text{ met } \lambda \in \mathbb{R} \right\}$

g) $\begin{cases} 2s + 3t + u = 11 \\ 3s + t + 2u = 11 \\ s + 2t + 3u = 14 \end{cases} \Rightarrow \left(\begin{array}{ccc|c} 2 & 3 & 1 & 11 \\ 3 & 1 & 2 & 11 \\ 1 & 2 & 3 & 14 \end{array}\right)$

$\xrightarrow{R_3 \leftrightarrow R_1} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 14 \\ 3 & 1 & 2 & 11 \\ 2 & 3 & 1 & 11 \end{array}\right) \xrightarrow[R_3 - 2R_1]{R_2 - 3R_1} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 14 \\ 0 & -5 & -7 & -31 \\ 0 & -1 & -5 & -17 \end{array}\right)$

$\xrightarrow{R_2 - 5R_3} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 14 \\ 0 & 0 & 18 & 54 \\ 0 & -1 & -5 & -17 \end{array}\right) \xrightarrow[\frac{R_3}{-1}]{\frac{R_2}{18}} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 14 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & 5 & 17 \end{array}\right)$

$\Rightarrow \begin{cases} x + 2y + 3z = 14 \\ z = 3 \\ y + 5z = 17 \end{cases} \Rightarrow \begin{cases} x = 1 \\ y = 2 \\ z = 3 \end{cases}$

1) $\begin{cases} 2x - 3z + y = 5 \\ -2y + 3x + 2z = 5 \\ -z + 5x - 3y = 16 \end{cases} \Rightarrow \left(\begin{array}{ccc|c} 2 & 1 & -3 & 5 \\ 3 & -2 & 2 & 5 \\ 5 & -3 & -1 & 16 \end{array}\right)$

$\xrightarrow{R_2 - R_1} \left(\begin{array}{ccc|c} 2 & 1 & -3 & 5 \\ 1 & -3 & 5 & 0 \\ 5 & -3 & -1 & 16 \end{array}\right) \xrightarrow{R_2 \leftrightarrow R_1} \left(\begin{array}{ccc|c} 1 & -3 & 5 & 0 \\ 2 & 1 & -3 & 5 \\ 5 & -3 & -1 & 16 \end{array}\right)$

$\xrightarrow[R_3 - 5R_1]{R_2 - 2R_1} \left(\begin{array}{ccc|c} 1 & -3 & 5 & 0 \\ 0 & 7 & -13 & 5 \\ 0 & 12 & -26 & 16 \end{array}\right) \xrightarrow{\frac{R_3}{2}} \left(\begin{array}{ccc|c} 1 & -3 & 5 & 0 \\ 0 & 7 & -13 & 5 \\ 0 & 6 & -13 & 8 \end{array}\right)$

$\xrightarrow{R_2 - R_3} \left(\begin{array}{ccc|c} 1 & -3 & 5 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 6 & -13 & 8 \end{array}\right) \Rightarrow \begin{cases} x - 3y + 5z = 0 \\ y = -3 \\ 6y - 13z = 8 \end{cases} \Rightarrow \begin{cases} x = 1 \\ y = -3 \\ z = -2 \end{cases}$

c) $\begin{cases} r + 2s + 3t + 4u = 5 \\ 2r - s + t + 2u = 2 \\ 3r + s + 4t + 6u = 7 \end{cases} \Rightarrow \left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 5 \\ 2 & -1 & 1 & 2 & 2 \\ 3 & 1 & 4 & 6 & 7 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$

$\xrightarrow[R_3 - 3R_1]{R_2 - 2R_1} \left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 5 \\ 0 & -5 & -5 & -6 & -8 \\ 0 & -5 & -5 & -6 & -8 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{R_3 - R_2} \left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 5 \\ 0 & -5 & -5 & -6 & -8 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$

$\xrightarrow{\frac{R_2}{-1}} \left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 5 \\ 0 & 5 & 5 & 6 & 8 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{cases} r + 2s + 3t + 4u = 5 \\ 5s + 5t + 6u = 8 \\ t = \lambda \\ u = \mu \end{cases}$

$\Rightarrow \begin{cases} r = \frac{9}{5} - \lambda - \frac{8}{5}\mu \\ s = \frac{8}{5} - \lambda - \frac{6}{5}\mu \\ t = \lambda \\ u = \mu \end{cases}$

j) $\begin{cases} x_1 + 2x_2 - x_3 = 4 \\ x_1 + 3x_2 + x_3 = 7 \\ 2x_1 + 3x_2 - 4x_3 = 5 \\ 2x_1 + x_2 - 8x_3 = -1 \end{cases} \Rightarrow \left(\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 1 & 3 & 1 & 7 \\ 2 & 3 & -4 & 5 \\ 2 & 1 & -8 & -1 \end{array}\right)$

$\xrightarrow{\substack{R_2 - R_1 \\ R_3 - 2R_1 \\ R_4 - 2R_1}} \left(\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & -1 & -2 & -3 \\ 0 & -3 & -6 & -9 \end{array}\right) \xrightarrow{\substack{R_2 + R_3 \\ R_4 + 3R_3}} \left(\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & -2 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$\xrightarrow{\frac{R_3}{-1}} \left(\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{cases} x_1 + 2x_2 - x_3 = 4 \\ x_2 + 2x_3 = 3 \\ x_3 = \lambda \end{cases} \Rightarrow \begin{cases} x_1 = -2 + 5\lambda \\ x_2 = 3 - 2\lambda \\ x_3 = \lambda \end{cases}$

$\{(5\lambda - 2, 3 - 2\lambda, \lambda) \text{ met } \lambda \in \mathbb{R}\}$

k) $\begin{cases} x + 2y + 2z = 2 \\ 3x - 2y - z = 5 \\ 2x - 5y + 3z = -4 \\ x + 4y + 6z = 0 \end{cases} \Rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 2 & 2 \\ 3 & -2 & -1 & 5 \\ 2 & -5 & 3 & -4 \\ 1 & 4 & 6 & 0 \end{array}\right)$

$\xrightarrow{\substack{R_2 - 3R_1 \\ R_3 - 2R_1 \\ R_4 - R_1}} \left(\begin{array}{ccc|c} 1 & 2 & 2 & 2 \\ 0 & -8 & -7 & -1 \\ 0 & -9 & -1 & -8 \\ 0 & 2 & 4 & -2 \end{array}\right) \xrightarrow{R_4 \leftrightarrow R_2} \left(\begin{array}{ccc|c} 1 & 2 & 2 & 2 \\ 0 & 2 & 4 & -2 \\ 0 & -9 & -1 & -8 \\ 0 & -8 & -7 & -1 \end{array}\right)$

$\xrightarrow{\frac{R_2}{2}} \left(\begin{array}{ccc|c} 1 & 2 & 2 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & -9 & -1 & -8 \\ 0 & -8 & -7 & -1 \end{array}\right) \xrightarrow{\substack{R_3 + 9R_2 \\ R_4 + 8R_2}} \left(\begin{array}{ccc|c} 1 & 2 & 2 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 9 & -9 \\ 0 & 0 & 17 & -17 \end{array}\right)$

$\Rightarrow \begin{cases} x + 2y + 2z = 2 \\ y + 2z = -1 \\ z = -1 \end{cases} \Leftrightarrow \begin{cases} x = 2 \\ y = 1 \\ z = -1 \end{cases}$

l) $\begin{cases} 5x + 2y - 3z = 1 \\ 2x + y + 4z = 5 \end{cases} \Rightarrow \left(\begin{array}{ccc|c} 5 & 2 & -3 & 1 \\ 2 & 1 & 4 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$\xrightarrow{R_1 - 2R_2} \left(\begin{array}{ccc|c} 1 & 0 & -11 & -9 \\ 2 & 1 & 4 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{R_2 - 2R_1} \left(\begin{array}{ccc|c} 1 & 0 & -11 & -9 \\ 0 & 1 & 26 & 23 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$\Rightarrow \begin{cases} x - 11z = -9 \\ y + 26z = 23 \\ z = \lambda \end{cases} \Rightarrow \begin{cases} x = -9 + 11\lambda \\ y = 23 - 26\lambda \\ z = \lambda \end{cases}$

$\{(-9 + 11\lambda, 23 - 26\lambda, \lambda) \text{ met } \lambda \in \mathbb{R}\}$

m) $\begin{cases} 3x + y + z = 5 \\ x + 3y + z = 5 \\ x + y + 3z = 5 \end{cases} \Rightarrow \left(\begin{array}{ccc|c} 3 & 1 & 1 & 5 \\ 1 & 3 & 1 & 5 \\ 1 & 1 & 3 & 5 \end{array}\right)$

$\xrightarrow{R_3 \leftrightarrow R_1} \left(\begin{array}{ccc|c} 1 & 1 & 3 & 5 \\ 1 & 3 & 1 & 5 \\ 3 & 1 & 1 & 5 \end{array}\right) \xrightarrow{\substack{R_2 - R_1 \\ R_3 - 3R_1}} \left(\begin{array}{ccc|c} 1 & 1 & 3 & 5 \\ 0 & 2 & -2 & 0 \\ 0 & 2 & -8 & -10 \end{array}\right)$

$\xrightarrow{R_3 + R_2} \left(\begin{array}{ccc|c} 1 & 1 & 3 & 5 \\ 0 & 2 & -2 & 0 \\ 0 & 0 & -10 & -10 \end{array}\right) \Rightarrow \begin{cases} x + y + 3z = 5 \\ 2y - 2z = 0 \\ z = 1 \end{cases} \Rightarrow \begin{cases} x = 1 \\ y = 1 \\ z = 1 \end{cases}$

n) $3x - 2y - 3z = 1 \Rightarrow \left(\begin{array}{ccc|c} 3 & -2 & -3 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$\Rightarrow \begin{cases} 3x - 2y - 3z = 1 \\ y = \lambda \\ z = \mu \end{cases} \Rightarrow 3x - 2\lambda - 3\mu = 1$

$x = \dfrac{1 + 2\lambda + 3\mu}{3}$

$\left\{\left(\dfrac{1 + 2\lambda + 3\mu}{3}, \lambda, \mu\right) \text{ met } \mu, \lambda \in \mathbb{R}\right\}$

af.2 a) $3x+2a=1 \Rightarrow x = \frac{1-2a}{3}$

b) $ax+3=6 \Rightarrow x = \frac{6-3}{a} = \frac{3}{a}$

$\frac{3}{a}$ Als $a \neq 0$ en $\emptyset$ als $a=0$

c) $3(x-1)+a(b+x)=x \Rightarrow 3x-3+ab+ax=x$

$3x+ax-x=3-ab$

$2x+ax=3-ab$

$(2+a)x=3-ab$

$x = \frac{3-ab}{(2+a)}$

$\frac{3-ab}{2+a}$ als $a \neq -2$

als $a=-2$ en $b \neq -\frac{3}{2}$ dan $\emptyset$

Als $a=-2$ en $b=-\frac{3}{2}$ dan $\mathbb{R}$

d) $ax=bx$

als $a=b$: $\mathbb{R}$

als $a \neq b$: $\emptyset$

af.3 a) $\begin{cases} x+ay=a \\ ax+y=1 \end{cases} \Rightarrow \left(\begin{array}{cc|c} 1 & a & a \\ a & 1 & 1 \end{array}\right)$

$\xrightarrow{R_2-a.R_1} \left(\begin{array}{cc|c} 1 & a & a \\ 0 & 1-a^2 & 1-a^2 \end{array}\right) \xrightarrow{\frac{R_2}{(1-a^2)} \text{ als } a \neq \pm 1} \left(\begin{array}{cc|c} 1 & a & a \\ 0 & 1 & 1 \end{array}\right)$

$\xrightarrow{R_1-aR_2} \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 1 \end{array}\right) \Rightarrow \begin{cases} x=0 \\ y=1 \end{cases}$

Stel $a=1$ dan is $\left(\begin{array}{cc|c} 1 & a & a \\ 0 & 1-a^2 & 1-a^2 \end{array}\right) = \left(\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right)$

$\Rightarrow \begin{cases} x=1-\lambda \\ y=\lambda \end{cases}$

Stel $\alpha = -1$ dan is: $\left(\begin{array}{cc|c} 1 & \alpha & \alpha \\ 0 & 1-\alpha^2 & 1-\alpha^2 \end{array}\right) = \left(\begin{array}{cc|c} 1 & -1 & -1 \\ 0 & 0 & 0 \end{array}\right)$

$\Rightarrow \begin{cases} x = \lambda - 1 \\ y = \lambda \end{cases}$

b) $\begin{cases} x + 2y - 3z = 6 \\ 2x - y + 4z = 2 \\ 4x + 3y - 2z = \alpha \end{cases} \Rightarrow \left(\begin{array}{ccc|c} 1 & 2 & -3 & 6 \\ 2 & -1 & 4 & 2 \\ 4 & 3 & -2 & \alpha \end{array}\right)$

$\xrightarrow[R_3 - 4R_1]{R_2 - 2R_1} \left(\begin{array}{ccc|c} 1 & 2 & -3 & 6 \\ 0 & -5 & 10 & -10 \\ 0 & -5 & 10 & \alpha - 24 \end{array}\right)$

Stel $\alpha = 14$ dan:

$\left(\begin{array}{ccc|c} 1 & 2 & -3 & 6 \\ 0 & -5 & 10 & -10 \\ 0 & -5 & 10 & -10 \end{array}\right) \xrightarrow{R_3 - R_2} \left(\begin{array}{ccc|c} 1 & 2 & -3 & 6 \\ 0 & -5 & 10 & -10 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$\xrightarrow{-\frac{R_2}{5}} \left(\begin{array}{ccc|c} 1 & 2 & -3 & 6 \\ 0 & +1 & -2 & +2 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{cases} x + 2y - 3z = 6 \\ y - 2z = 2 \\ z = \lambda \end{cases} \Rightarrow \begin{cases} x = 2 - \lambda \\ y = 2 + 2\lambda \\ z = \lambda \end{cases}$

Stel $\alpha \neq 14$ dan:

$\xrightarrow{R_3 - R_2} \left(\begin{array}{ccc|c} 1 & 2 & -3 & 6 \\ 0 & -5 & 10 & -10 \\ 0 & 0 & 0 & \alpha - 14 \end{array}\right) \Rightarrow$ geen opl.

c) $\begin{cases} x + (\alpha+1)y + z = 0 \\ x + y + (\alpha+1)z = 0 \\ 2x + y + z = \alpha + 1 \end{cases} \Rightarrow \left(\begin{array}{ccc|c} 1 & \alpha+1 & 1 & 0 \\ 1 & 1 & \alpha+1 & 0 \\ 2 & 1 & 1 & \alpha+1 \end{array}\right)$

stel $\alpha = -1$ dan: $\left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 \end{array}\right) \xrightarrow[R_3 - 2R_1]{R_2 - R_1} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right)$

$\xrightarrow{R_3-R_2} \begin{pmatrix} 1 & 0 & 1 & | & 0 \\ 0 & 1 & -1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix} \Leftrightarrow \begin{cases} x+y=0 \\ y-z=0 \\ z=\lambda \end{cases} \Rightarrow \begin{cases} x=-\lambda \\ y=\lambda \\ z=\lambda \end{cases}$

Si tel $\alpha=0$:

$\begin{pmatrix} 1 & 1 & 1 & | & 0 \\ 1 & 1 & 1 & | & 0 \\ 2 & 1 & 1 & | & 1 \end{pmatrix} \xrightarrow[R_3-2R_1]{R_2-R_1} \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \\ 0 & -1 & -1 & | & 1 \end{pmatrix}$

$\Rightarrow \begin{cases} x+y+z=0 \\ -y-z=1 \\ z=\lambda \end{cases} \Rightarrow \begin{cases} x=1 \\ y=-1-\lambda \\ z=\lambda \end{cases}$

Si tel $\alpha \in \mathbb{R}\setminus\{0,-1\}$:

$\begin{pmatrix} 1 & \alpha+1 & 1 & | & 0 \\ 1 & 1 & \alpha+1 & | & 0 \\ 2 & 1 & 1 & | & \alpha+1 \end{pmatrix} \xrightarrow[R_3-2R_1]{R_2-R_1} \begin{pmatrix} 1 & \alpha+1 & 1 & | & 0 \\ 0 & -\alpha & \alpha & | & 0 \\ 0 & -2\alpha-1 & -1 & | & \alpha+1 \end{pmatrix}$

$\xrightarrow{R_3-2R_2} \begin{pmatrix} 1 & \alpha+1 & 1 & | & 0 \\ 0 & -\alpha & \alpha & | & 0 \\ 0 & -1 & -1-2\alpha & | & \alpha+1 \end{pmatrix} \xrightarrow{\frac{R_2}{\alpha}} \begin{pmatrix} 1 & \alpha+1 & 1 & | & 0 \\ 0 & -1 & 1 & | & 0 \\ 0 & -1 & -1-2\alpha & | & \alpha+1 \end{pmatrix}$

$\xrightarrow{R_3-R_2} \begin{pmatrix} 1 & \alpha+1 & 1 & | & 0 \\ 0 & -1 & 1 & | & 0 \\ 0 & 0 & -2\alpha-2 & | & \alpha+1 \end{pmatrix} \Rightarrow \begin{cases} x+(\alpha+1)y+z=0 \\ -y+z=0 \\ (-2\alpha-2)z=\alpha+1 \end{cases}$

$\Rightarrow \begin{cases} x=1+\frac{1}{2}\alpha \\ y=-\frac{1}{2} \\ z=-\frac{1}{2} \end{cases}$

d) $\begin{cases} ax+by+z=1 \\ 5ax+by+3z=0 \\ 4ax+by+z=1 \end{cases} \Leftrightarrow \begin{pmatrix} a & b & 1 & | & 1 \\ 5a & 3b & 3 & | & 0 \\ 4a & b & 1 & | & 1 \end{pmatrix}$

$\xrightarrow[R_3-4R_1]{R_2-5R_1} \begin{pmatrix} a & b & 1 & | & 1 \\ 0 & -2b & -2 & | & -5 \\ 0 & -3b & -3 & | & -3 \end{pmatrix} \xrightarrow[\frac{R_2}{-1}]{\frac{R_3}{-3}} \begin{pmatrix} a & b & 1 & | & 1 \\ 0 & 2b & 2 & | & 5 \\ 0 & 1b & 1 & | & 1 \end{pmatrix}$

$\xrightarrow{R_2-2R_3}$ $\left(\begin{array}{ccc|c} a & b & 1 & 1 \\ 0 & 0 & 0 & 3 \\ 0 & b & 1 & 1 \end{array}\right)$ $\Rightarrow$ geen oplossingen $\forall a, b \in \mathbb{R}$

e) $\begin{cases} ax + y + z = 0 \\ x + ay + z = 0 \\ x + y + az = 0 \end{cases} \Rightarrow \left(\begin{array}{ccc|c} a & 1 & 1 & 0 \\ 1 & a & 1 & 0 \\ 1 & 1 & a & 0 \end{array}\right)$

Stel $a = 1$: $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{array}\right) \xrightarrow[R_3-R_1]{R_2-R_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$\Rightarrow \begin{cases} x + y + z = 0 \\ y = \lambda \\ z = \mu \end{cases} \Rightarrow \begin{cases} x = \lambda - \mu \\ y = \lambda \\ z = \mu \end{cases}$

$\left(\begin{array}{ccc|c} a & 1 & 1 & 0 \\ 1 & a & 1 & 0 \\ 1 & 1 & a & 0 \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_3} \left(\begin{array}{ccc|c} 1 & 1 & a & 0 \\ 1 & a & 1 & 0 \\ a & 1 & 1 & 0 \end{array}\right)$

$\xrightarrow[R_3-aR_1]{R_2-R_1} \left(\begin{array}{ccc|c} 1 & 1 & a & 0 \\ 0 & a-1 & 1-a & 0 \\ 0 & 1-a & 1-a^2 & 0 \end{array}\right) \xrightarrow{R_3+R_2} \left(\begin{array}{ccc|c} 1 & 1 & a & 0 \\ 0 & a-1 & 1-a & 0 \\ 0 & 0 & -a^2-a+2 & 0 \end{array}\right)$

$(a-1)y + (1-a)z = 0$ als $a = 1$ of als $y = z = 0$

$\Rightarrow \begin{cases} x + y + az = 0 \\ (a-1) \cdot y + (1-a) \cdot z = 0 \\ (a^2 + a + 2)z = 0 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = z = 0 \\ z = 0 \end{cases}$

Aangezien we werken in $\mathbb{R} \setminus \{1\}$ is $y = 0 = z$

Stel $a = -2$: $\left(\begin{array}{ccc|c} 1 & 1 & a & 0 \\ 0 & a-1 & 1-a & 0 \\ 0 & 0 & -a^2-a+2 & 0 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$\xrightarrow{\frac{R_2}{3}} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{cases} x + y - 2z = 0 \\ y + z = 0 \\ z = \lambda \end{cases} \Rightarrow \begin{cases} x = \lambda \\ y = \lambda \\ z = \lambda \end{cases}$

f) $\begin{cases} ax + (a+1)y + z = 0 \\ ax + y + (a+1)\cdot z = 0 \\ 2ax + y + z = a+1 \end{cases} \Rightarrow \left(\begin{array}{ccc|c} a & a+1 & 1 & 0 \\ a & 1 & a+1 & 0 \\ 2a & 1 & 1 & a+1 \end{array}\right)$

$$\xrightarrow[R_3-2R_1]{R_2-R_1} \left(\begin{array}{ccc|c} a & a+1 & 1 & 0 \\ 0 & -a & a & 0 \\ 0 & -2a-1 & -1 & a+1 \end{array}\right) \xrightarrow{\frac{R_2}{a}} \left(\begin{array}{ccc|c} a & a+1 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -2a-1 & -1 & a+1 \end{array}\right)$$

$$\xrightarrow{R_3-(2a+1)R_2} \left(\begin{array}{ccc|c} a & a+1 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -2a-2 & a+1 \end{array}\right)$$

$$\xrightarrow{\frac{R_3}{a+1}} \left(\begin{array}{ccc|c} a & a+1 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -2 & 1 \end{array}\right) \Rightarrow \begin{cases} ax+(a+1)y+z=0 \\ -y+z=0 \\ -2z=1 \end{cases} \Rightarrow \begin{cases} x = \frac{2+a}{2a} \\ y = -\frac{1}{2} \\ z = -\frac{1}{2} \end{cases}$$

Stel $a=0$ dan is:

$$\left(\begin{array}{ccc|c} a & a+1 & 1 & 0 \\ a & 1 & a+1 & 0 \\ 2a & 1 & 1 & a+1 \end{array}\right) = \left(\begin{array}{ccc|c} 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right)$$

$$\xrightarrow[R_3-R_1]{R_2-R_1} \left(\begin{array}{ccc|c} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right) \Rightarrow \text{heeft geen oplossingen}$$

Stel $a=-1$ dan is:

$$\left(\begin{array}{ccc|c} a & a+1 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -2(a+1) & a+1 \end{array}\right) = \left(\begin{array}{ccc|c} -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\Rightarrow \begin{cases} -x+z=0 \\ -y+z=0 \\ z=\lambda \end{cases} \Rightarrow \begin{cases} x=\lambda \\ y=\lambda \\ z=\lambda \end{cases}$$

g) $\begin{cases} x + y - az = 0 \\ x + 4y + 2az = 3 \\ ax + (a+1)y + (a-1)z = a \end{cases} \Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & -a & 0 \\ 1 & 4 & 2a & 3 \\ a & a+1 & a-1 & a \end{array}\right)$

$\xrightarrow[R_3 - a \cdot R_1]{R_2 - R_1} \left(\begin{array}{ccc|c} 1 & 1 & -a & 0 \\ 0 & 3 & 3a & 3 \\ 0 & 1 & a^2+a-1 & a \end{array}\right) \xrightarrow{\frac{R_2}{3}} \left(\begin{array}{ccc|c} 1 & 1 & -a & 0 \\ 0 & 1 & a & 1 \\ 0 & 1 & a^2+a-1 & a \end{array}\right)$

$\xrightarrow{R_3 - R_2} \left(\begin{array}{ccc|c} 1 & 1 & -a & 0 \\ 0 & 1 & a & 1 \\ 0 & 0 & a^2-1 & a-1 \end{array}\right)$

Stel $a = 1$:

$\left(\begin{array}{ccc|c} 1 & 1 & -a & 0 \\ 0 & 1 & a & 1 \\ 0 & 0 & a^2-1 & a-1 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$\Rightarrow \begin{cases} x + y - z = 0 \\ y + z = 1 \\ z = \lambda \end{cases} \Rightarrow \begin{cases} x = 2\lambda - 1 \\ y = 1 - \lambda \\ z = \lambda \end{cases}$

Stel $a = -1$:

$\left(\begin{array}{ccc|c} 1 & 1 & -a & 0 \\ 0 & 1 & a & 1 \\ 0 & 0 & a^2-1 & a-1 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & -2 \end{array}\right)$

$\Rightarrow$ heeft geen opl.

Stel $a \in \mathbb{R} \setminus \{-1, 1\}$:

$\left(\begin{array}{ccc|c} 1 & 1 & -a & 0 \\ 0 & 1 & a & 1 \\ 0 & 0 & (a+1)(a-1) & (a-1) \end{array}\right) \xrightarrow{\frac{R_3}{a-1}} \left(\begin{array}{ccc|c} 1 & 1 & -a & 0 \\ 0 & 1 & a & 1 \\ 0 & 0 & a+1 & 1 \end{array}\right)$

$\Rightarrow \begin{cases} x + y - az = 0 \\ y + az = 1 \\ (a+1)z = 1 \end{cases} \Rightarrow \begin{cases} x = az - \frac{1}{a+1} \\ y = 1 - a \cdot \frac{1}{a+1} = \frac{a+1-a}{a+1} \\ z = \frac{1}{a+1} \end{cases}$

$\begin{cases} x = \frac{a}{a+1} - \frac{1}{a+1} = \frac{a-1}{a+1} \\ y = \frac{1}{a+1} \\ z = \frac{1}{a+1} \end{cases}$

2) $\begin{cases} ax + y + z = b \\ x + ay + z = b \\ x + y + az = b \end{cases} \Rightarrow \left(\begin{array}{ccc|c} a & 1 & 1 & b \\ 1 & a & 1 & b \\ 1 & 1 & a & b \end{array}\right)$

Stel $a = 1$: $\left(\begin{array}{ccc|c} 1 & 1 & 1 & b \\ 1 & 1 & 1 & b \\ 1 & 1 & 1 & b \end{array}\right) \xrightarrow[R_3 - R_1]{R_2 - R_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & b \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$\Rightarrow \begin{cases} x + y + z = b \\ y = \lambda \\ z = \mu \end{cases} \Rightarrow \begin{array}{l} x = b - \lambda - \mu \\ y = \lambda \\ z = \mu \end{array}$

$\left(\begin{array}{ccc|c} a & 1 & 1 & b \\ 1 & a & 1 & b \\ 1 & 1 & a & b \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_3} \left(\begin{array}{ccc|c} 1 & 1 & a & b \\ 1 & a & 1 & b \\ a & 1 & 1 & b \end{array}\right)$

$\xrightarrow[R_3 - a \cdot R_1]{R_2 - R_1} \left(\begin{array}{ccc|c} 1 & 1 & a & b \\ 0 & a-1 & 1-a & 0 \\ 0 & 1-a & 1-a^2 & b-ab \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 1 & a & b \\ 0 & a-1 & 1-a & 0 \\ 0 & 1-a & 1-a^2 & (1-a)b \end{array}\right)$

$\xrightarrow{R_3 + R_2} \left(\begin{array}{ccc|c} 1 & 1 & a & b \\ 0 & a-1 & 1-a & 0 \\ 0 & 0 & -a^2-a+1 & (1-a)b \end{array}\right)$

Stel $a = -2$ en $b \in \mathbb{R}$:

$\left(\begin{array}{ccc|c} 1 & 1 & -2 & b \\ 0 & -3 & 3 & 0 \\ 0 & 0 & 0 & 3b \end{array}\right) \Rightarrow$ geen oplossingen

Stel $a = -2$ en $b = 0$:

$\left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{cases} x + y - 2z = 0 \\ y = z \\ z = \lambda \end{cases}$

$\Rightarrow \begin{cases} x = \lambda \\ y = \lambda \\ z = \lambda \end{cases}$

Stel $\alpha \in \mathbb{R} \setminus \{-2, 1\}$:

$$\left(\begin{array}{ccc|c} 1 & 1 & \alpha & b \\ 0 & \alpha-1 & 1-\alpha & 0 \\ 0 & 1-\alpha & 1-\alpha^2 & (1-\alpha)b \end{array}\right) \begin{array}{c} \frac{R_2}{\alpha-1} \\ \to \\ \frac{R_3}{1-\alpha} \end{array} \left(\begin{array}{ccc|c} 1 & 1 & \alpha & b \\ 0 & 1 & \frac{-(\alpha-1)}{\alpha-1} & 0 \\ 0 & 1 & \frac{1-\alpha^2}{1-\alpha} & b \end{array}\right)$$

$$\to \left(\begin{array}{ccc|c} 1 & 1 & \alpha & b \\ 0 & 1 & -1 & 0 \\ 0 & 1 & \frac{1-\alpha^2}{1-\alpha} & b \end{array}\right) \longrightarrow \begin{cases} x + y + \alpha \cdot z = b \\ y = z \\ y + \left(\frac{1-\alpha^2}{1-\alpha}\right) z = b \end{cases}$$

$$\Rightarrow \begin{cases} x + y + \alpha \cdot z = b \\ y = z \\ z + \frac{1-\alpha^2}{1-\alpha} z = b \end{cases} \Rightarrow \begin{cases} x + y + \alpha z = b \\ y = z \\ \left(\frac{1-\alpha+1-\alpha^2}{1-\alpha}\right) z = b \end{cases}$$

$$\Rightarrow \begin{cases} x + y + \alpha \cdot z = b \\ y = z \\ z = \frac{b \cdot (1-\alpha)}{-\alpha^2-\alpha+2} = \frac{b(\cancel{1-\alpha})}{(\cancel{1-\alpha})(\alpha+2)} \end{cases}$$

$$-\alpha^2 - \alpha + 2 = (1-\alpha)(\alpha+2)$$

$$\Rightarrow \begin{cases} x = b - y - \alpha \cdot z \\ y = z \\ z = \frac{b}{\alpha+2} \end{cases} \Rightarrow \begin{cases} x = b - \frac{b}{\alpha+2} - \frac{\alpha b}{\alpha+2} \\ y = z \\ z = \frac{b}{\alpha+2} \end{cases}$$

$$\Rightarrow \begin{cases} x = \frac{(\alpha+2)b - b - \alpha b}{\alpha+2} = \frac{(\alpha+2-1-\alpha)b}{\alpha+2} \\ y = z \\ z = \frac{b}{\alpha+2} \end{cases}$$

$$\Rightarrow \begin{cases} x = \frac{b}{\alpha+2} \\ y = \frac{b}{\alpha+2} \\ z = \frac{b}{\alpha+2} \end{cases}$$

i) $\begin{cases} x+y+z=2 \\ x-2y+3z=10 \\ 2x+3y+z=1 \\ x+y-z=\alpha \end{cases} \Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & -2 & 3 & 10 \\ 2 & 3 & 1 & 1 \\ 1 & 1 & -1 & \alpha \end{array}\right)$

R_2-R_1, R_3-2R_1, R_4-R_1 $\rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -3 & 2 & 8 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & -2 & \alpha-2 \end{array}\right)$ $R_2+3R_3 \rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 0 & -1 & -1 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & -2 & \alpha-2 \end{array}\right)$

$R_4-2.R_2 \rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 0 & -1 & -1 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & \alpha \end{array}\right)$

stel $\alpha \neq 0$; dan heeft het geen oplossingen.

Stel $\alpha = 0$;

$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 0 & -1 & -1 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{cases} x+y+z=2 \\ -z=-1 \\ y-z=-3 \end{cases}$

$\Rightarrow \begin{cases} x+y+z=2 \\ z=1 \\ y-z=-3 \end{cases} \Rightarrow \begin{cases} x=3 \\ y=-2 \\ z=1 \end{cases}$

oef. 4 $\begin{cases} x+2y+3z+2s+4t=11 \\ 3x+3y+s+4t=9 \\ 4x+5y+3z+2s+4t=20 \end{cases} \Rightarrow \left(\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 4 & 11 \\ 3 & 3 & 6 & 1 & 4 & 9 \\ 4 & 5 & 3 & 2 & 1 & 20 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$

R_2-3R_1, R_3-4R_1 $\rightarrow \left(\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 4 & 11 \\ 0 & -3 & -9 & -5 & -8 & -24 \\ 0 & -3 & -9 & -6 & -15 & -24 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$

$$\xrightarrow{R_3-R_2} \left(\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 4 & 11 \\ 0 & -3 & -9 & -5 & -8 & -24 \\ 0 & 0 & 0 & -1 & -7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow[R_3/-1]{R_2/-3} \left(\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 4 & 11 \\ 0 & 3 & 9 & 5 & 8 & 24 \\ 0 & 0 & 0 & 1 & 7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\Rightarrow \begin{cases} x + 2y + 3z + 2s + 4t = 11 \\ 3y + 9z + 5s + 8t = 24 \\ s + 7t = 0 \\ z = \lambda \\ t = \mu \end{cases} \Rightarrow \begin{cases} x + 2y + 3\lambda + 2(-7\mu) + 4\mu = 11 \\ 3y + 9\lambda + 5(-7\mu) + 8\mu = 24 \\ s = -7\mu \\ z = \lambda \\ t = \mu \end{cases}$$

$$\Rightarrow \begin{cases} x = -5 + 3\lambda - 8\mu \\ y = 8 - 3\lambda + 9\mu \\ s = -7\mu \\ z = \lambda \\ t = \mu \end{cases}$$

oef. 5 R

oef. 6

A	B
20	25
0,3	0,20
0,10	0,05

$32 - 20 = 12$

$32 - 25 = 7$

$$\begin{cases} 0,3x + 0,10y = 12 \\ 0,20x + 0,05y = 7 \end{cases} \Rightarrow \begin{cases} y = \frac{12}{0,1} - \frac{0,3x}{0,1} \\ y = \frac{7}{0,05} - \frac{0,2}{0,05}x \end{cases}$$

$$y = \frac{12}{0,1} - \frac{0,3x}{0,1} = \frac{7}{0,05} - \frac{0,2}{0,05}x$$

$$120 - 3x = 140 - 4x$$

$$x = 140 - 120$$

$$x = 20$$

$$y = 120 - 3 \cdot 20$$

$$= 120 - 60 = 60$$

$$\begin{cases} 0,3x + 0,10y = 12 \\ 0,20x + 0,05y = 7 \end{cases} \Rightarrow \begin{cases} x = 20 \\ y = 60 \end{cases}$$

oef. 7

$$\begin{cases} 4x + y + 2z + t = 420 \\ 2x + 2y + 2z + t = 460 \\ x + 3y + z + 3t = 540 \\ 3x + 4y + 3z + 2t = 800 \end{cases} \longrightarrow \left(\begin{array}{cccc|c} 4 & 1 & 2 & 1 & 420 \\ 2 & 2 & 2 & 1 & 460 \\ 1 & 3 & 1 & 3 & 540 \\ 3 & 4 & 3 & 2 & 800 \end{array}\right)$$

$$\xrightarrow{R_3 \leftrightarrow R_1} \left(\begin{array}{cccc|c} 1 & 3 & 1 & 3 & 540 \\ 2 & 2 & 2 & 1 & 460 \\ 4 & 1 & 2 & 1 & 420 \\ 3 & 4 & 3 & 2 & 800 \end{array}\right) \xrightarrow{\substack{R_2 - 2R_1 \\ R_3 - 4R_1 \\ R_4 - 3R_1}} \left(\begin{array}{cccc|c} 1 & 3 & 1 & 3 & 540 \\ 0 & -4 & 0 & -5 & -620 \\ 0 & -11 & -2 & -11 & -1740 \\ 0 & -5 & 0 & -7 & -820 \end{array}\right)$$

$$\xrightarrow{\substack{\frac{R_2}{-1} \\ \frac{R_3}{-1}\ \frac{R_4}{-1}}} \left(\begin{array}{cccc|c} 1 & 2 & 1 & 3 & 540 \\ 0 & 4 & 0 & 5 & 620 \\ 0 & 11 & 2 & 11 & 1740 \\ 0 & 5 & 0 & 7 & 820 \end{array}\right) \xrightarrow{R_3 - 2.R_4} \left(\begin{array}{cccc|c} 1 & 2 & 1 & 3 & 540 \\ 0 & 4 & 0 & 5 & 620 \\ 0 & 1 & 2 & -3 & 100 \\ 0 & 5 & 0 & 7 & 820 \end{array}\right)$$

$$\xrightarrow{R_3 \leftrightarrow R_2} \left(\begin{array}{cccc|c} 1 & 2 & 1 & 3 & 540 \\ 0 & 1 & 2 & -3 & 100 \\ 0 & 4 & 0 & 5 & 620 \\ 0 & 5 & 0 & 7 & 820 \end{array}\right) \xrightarrow{\substack{R_3 - 4R_2 \\ R_4 - 5R_2}} \left(\begin{array}{cccc|c} 1 & 2 & 1 & 3 & 540 \\ 0 & 1 & 2 & -3 & 100 \\ 0 & 0 & -8 & 17 & 220 \\ 0 & 0 & -10 & 22 & 320 \end{array}\right)$$

$$\xrightarrow{\frac{R_4}{10}} \left(\begin{array}{cccc|c} 1 & 2 & 1 & 3 & 540 \\ 0 & 1 & 2 & -3 & 100 \\ 0 & 0 & -8 & 17 & 220 \\ 0 & 0 & -1 & 2,2 & 32 \end{array}\right) \xrightarrow{R_3 - 8R_4} \left(\begin{array}{cccc|c} 1 & 2 & 1 & 3 & 540 \\ 0 & 1 & 2 & -3 & 100 \\ 0 & 0 & 0 & -0,6 & -36 \\ 0 & 0 & -1 & 2,2 & 32 \end{array}\right)$$

$$\Rightarrow \begin{cases} x + 2y + z + 3t = 540 \\ y + 2z - 3t = 100 \\ -z + 2,2\,t = 32 \\ -0,6t = -36 \end{cases} \Rightarrow \begin{cases} x = 540 - 3y - z - 3t \\ y = 100 - 2z + 3t \\ z = 2,2t - 32 \\ t = 60 \end{cases}$$

$$\Rightarrow \begin{cases} x = 20 \\ y = 80 \\ z = 100 \\ t = 60 \end{cases}$$

Ant: $x + 3y + 2z + 2t = 20.1 + 3.80 + 2.100 + 2.60 = 580$

oef. 8 a) $10x_1 - 10x_2 + 5x_3 - 12x_4 + 15x_5 - 8x_6$

$$\begin{cases} 10 = x_1 - x_2 & \Rightarrow x_1 = 10 + x_2 \\ -5 = x_2 - x_3 & \Rightarrow x_2 = -5 + x_3 \\ 12 = x_3 - x_4 & \Rightarrow x_3 = 12 + x_4 \\ -15 = x_4 - x_5 & \Rightarrow x_4 = -15 + x_5 \\ 8 = x_5 - x_6 & \Rightarrow x_5 = 8 + x_6 \\ -10 = x_6 - x_1 & \Rightarrow x_6 = -10 + x_1 \end{cases}$$

$$\Rightarrow \left(\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & 0 & 0 & 10 \\ 0 & 1 & -1 & 0 & 0 & 0 & -5 \\ 0 & 0 & 1 & -1 & 0 & 0 & 12 \\ 0 & 0 & 0 & 1 & -1 & 0 & -15 \\ 0 & 0 & 0 & 0 & 1 & -1 & 8 \\ -1 & 0 & 0 & 0 & 0 & 1 & -10 \end{array}\right)$$

b)

$R_6 + R_1 + R_2 + R_3 + R_4 + R_5$

$$\longrightarrow \left(\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & 0 & 0 & 10 \\ 0 & 1 & -1 & 0 & 0 & 0 & -5 \\ 0 & 0 & 1 & -1 & 0 & 0 & 12 \\ 0 & 0 & 0 & 1 & -1 & 0 & -15 \\ 0 & 0 & 0 & 0 & 1 & -1 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\Rightarrow \begin{cases} x_1 = 10 + \lambda \\ x_2 = \lambda \\ x_3 = 5 + \lambda \\ x_4 = \lambda - 7 \\ x_5 = 8 + \lambda \\ x_6 = \lambda \end{cases}$$

c) $x_5 \geqslant 15$

oef. 9 $f: \mathbb{R}^3 \to \mathbb{R}$ $f(1,2,3) = 1$, $f(4,5,6) = 0$ en $f(5,7,9) = 1$

$$\begin{cases} x + 2y + 3z = 1 \\ 4x + 5y + 6z = 0 \\ 5x + 7y + 9z = 1 \end{cases} \Rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 4 & 5 & 6 & 0 \\ 5 & 7 & 9 & 1 \end{array}\right)$$

$$\xrightarrow{\substack{R_2 - 4R_1 \\ R_3 - 5R_1}} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -3 & -6 & -4 \\ 0 & -3 & -6 & -4 \end{array}\right) \xrightarrow{R_3 - R_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -3 & -6 & -4 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\Rightarrow \begin{cases} x + 2y + 3z = 1 \\ 3y + 6z = 4 \\ z = \lambda \end{cases} \Rightarrow \begin{cases} x = \lambda - \frac{5}{3} \\ y = \frac{4}{3} - 2\lambda \\ z = \lambda \end{cases}$$

oef. 10

$$A = \begin{pmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{pmatrix}$$

$$\xrightarrow{R_2 \leftrightarrow R_1} \begin{pmatrix} 1 & 3 & -5 & 1 & 5 \\ -2 & -5 & 8 & 0 & -17 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{pmatrix} \xrightarrow{\substack{R_2 + 2R_1 \\ R_3 - 3R_1 \\ R_4 - R_1}} \begin{pmatrix} 1 & 3 & -5 & 1 & 5 \\ 0 & 1 & -2 & 2 & -7 \\ 0 & 2 & -4 & 4 & -14 \\ 0 & 4 & 8 & 4 & -8 \end{pmatrix}$$

$$\xrightarrow{\substack{R_3 - 2R_2 \\ R_4 + 4R_2}} \begin{pmatrix} 1 & 3 & -5 & 1 & 5 \\ 0 & 1 & -2 & 2 & -2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & -4 & 20 \end{pmatrix} \xrightarrow{R_3 \leftrightarrow R_4} \begin{pmatrix} 1 & 3 & -5 & 1 & 5 \\ 0 & 1 & -2 & 2 & -2 \\ 0 & 0 & 16 & -4 & 20 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$\Rightarrow$ de rang is 3

oef. 11
a) rang = 3 vrijheidsgraden: 0
b) rang = 2 (niet oplosbaar)
c) rang = 2 vrijheidsgraden: 2
d) rang = 3 vrijheidsgraden: 0
e) rang = 3 vrijheidsgraden: 0
f) rang = 3 vrijheidsgraden: 1
g) rang = 3 vrijheidsgraden: 0
h) rang = 3 vrijheidsgraden: 0

i) rang = 2 Vrijheidsgraden : 2
j) rang = 2 Vrijheidsgraden : 1
k) rang = 3 Vrijheidsgraden : 0
l) rang = 2 Vrijheidsgraden : 1
m) rang = 3 Vrijheidsgraden : 0
n) rang = 1 Vrijheidsgraden : 2

oef. 12 $AX = B$

$\text{rang}(A) \leq \text{rang}(A|B) \leq \text{rang}(A)+1$

Stel het stelsel is oplosbaar, dan is de rang(A) = rang(A|B)

Dus: $\text{rang}(A) = \text{rang}(A|B) \leq \text{rang}(A)+1$

Stel het stelsel is niet oplosbaar (dus strijdig), dan is
$\text{rang}(A) \leq \text{rang}(A|B)$.

Als A een rang heeft die 2 of meer kleiner is dan rang(A|B) dan is dit een verschil van 1 door het naar echelonvorm om te vormen

Bv:
$$\underbrace{\left(\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 3 \end{array}\right)}_{\text{rang}(A)=2,\ \text{rang}(B)=4} \xrightarrow{R_4+3R_3} \underbrace{\left(\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right)}_{\text{rang}(A)=2,\ \text{rang}(B)=3}$$

oef. 13 a) Vals, Bv: $\begin{cases} 7x+2y+z=0 \\ 0x+0y+0z=7 \end{cases} \Rightarrow \left(\begin{array}{ccc|c} 7 & 2 & 1 & 0 \\ 0 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \end{array}\right)$

b) Vals, Bv: $\begin{cases} x+2y+3z=0 \\ 4y+3z=0 \\ 4y+4z=0 \\ -x+2y+3z=0 \end{cases} \Rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 4 & 3 & 0 \\ 0 & 4 & 4 & 0 \\ -1 & 2 & 3 & 0 \end{array}\right)$

c) Waar

d) Vals, Bv: $\begin{cases} 7x + 3y + z = 1 \\ 8x + 7y + 2z = 4 \\ 0x + 0y + 0z = 2 \end{cases} \Rightarrow \left(\begin{array}{ccc|c} 7 & 3 & 1 & 1 \\ 8 & 7 & 2 & 4 \\ 0 & 0 & 0 & 2 \end{array}\right)$

oef. 14

$q_{v1} = 10 - 2 \cdot p_1 + p_2$

$q_{v2} = 5 + 2p_1 - 2 \cdot p_2$

$q_{a1} = -3 + 2p_1$

$q_{a2} = -2 + 3p_2$

a) Concurrerende, als de prijs van het andere goed stijgt zal er meer gevraagd zijn van het ene, het omgekeerd als de prijs daalt

b) $\begin{cases} \begin{cases} q_{v1} = 10 - 2 \cdot p_1 + p_2 \\ q_{v2} = 5 + p_1 - 2 \cdot p_2 \end{cases} \\ \begin{cases} q_{a1} = -3 + 2p_1 \\ q_{a2} = -2 + 3p_2 \end{cases} \\ \begin{cases} q_{v1} = q_{a1} \\ q_{v2} = q_{a2} \end{cases} \end{cases} \Rightarrow \begin{cases} 10 - 2 \cdot p_1 + p_2 = -3 + 2p_1 \\ 5 + p_1 - 2 \cdot p_2 = -2 + 3 \cdot p_1 \end{cases}$

$\Rightarrow \begin{cases} 4p_1 - p_2 = 13 \\ 5p_2 - 2p_1 = 7 \end{cases} \Rightarrow \left(\begin{array}{cc|c} -2 & 5 & 7 \\ 4 & -1 & 13 \end{array}\right)$

$\xrightarrow{R_2 + 2R_1} \left(\begin{array}{cc|c} -2 & 5 & 7 \\ 0 & 9 & 27 \end{array}\right) \xrightarrow{\frac{R_2}{9}} \left(\begin{array}{cc|c} -2 & 5 & 7 \\ 0 & 1 & 3 \end{array}\right) \Rightarrow \begin{cases} -2p_1 + 5p_2 = 7 \\ p_2 = 3 \end{cases}$

$\Rightarrow \begin{cases} p_1 = 4 \\ p_2 = 3 \end{cases}$

$\begin{cases} q_{v1} = 10 - 2p_1 + p_2 \\ q_{v2} = 5 + 2p_1 - 2p_2 \\ q_{a1} = -3 + 2p_1 \\ q_{a2} = -2 + 3p_2 \\ p_2 = 3 \\ p_1 = 4 \end{cases} \Rightarrow \begin{cases} q_{v1} = 5 \\ q_{v2} = 7 \\ q_{a1} = 5 \\ q_{a2} = 7 \\ p_2 = 3 \\ p_1 = 4 \end{cases}$

oef. 15 $q_v = 25 - \frac{1}{2}p$
$q_a = -50 + 2p$

a) $25 - \frac{1}{2}p = -50 + 2p$ $\quad q_a = q_v = 25 - \frac{1}{2} \cdot 30 = 10$
$75 = 2p + \frac{1}{2}p$
$75 = \frac{5}{2}p$
$p = 30$ $\quad q = 10$

b) $-50 + 2(p-5) = 25 - \frac{1}{2}p$ $\quad q = q_a = q_v = 25 - \frac{1}{2} \cdot 34$
$-50 + 2p - 10 = 25 - \frac{1}{2}p$ $\quad q = 8$
$85 = \frac{5}{2}p$
$p = 34$

oef. 16

(p_1, p_2)	6,8	7,7	9,7
q_1	3200	3100	2900

a) $\begin{cases} 3200 = -6a + 8b + c \\ 3100 = -7a + 7b + c \\ 2900 = -9a + 7b + c \end{cases} \Rightarrow \left(\begin{array}{ccc|c} -6 & 8 & 1 & 3200 \\ -7 & 7 & 1 & 3100 \\ -9 & 7 & 1 & 2900 \end{array}\right)$

$\xrightarrow{R_3 - R_2} \left(\begin{array}{ccc|c} -6 & 8 & 1 & 3200 \\ -7 & 7 & 1 & 3100 \\ -2 & 0 & 0 & -200 \end{array}\right) \xrightarrow{R_3 / -2} \left(\begin{array}{ccc|c} -6 & 8 & 1 & 3200 \\ -7 & 7 & 1 & 3100 \\ 1 & 0 & 0 & 100 \end{array}\right)$

$\xrightarrow{R_3 \leftrightarrow R_1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 100 \\ -7 & 7 & 1 & 3100 \\ -6 & 8 & 1 & 3200 \end{array}\right) \xrightarrow{R_2 + 7R_1,\ R_3 + 6R_1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 100 \\ 0 & 7 & 1 & 3800 \\ 0 & 8 & 1 & 3800 \end{array}\right)$

$\xrightarrow{R_3 - R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 100 \\ 0 & 7 & 1 & 3800 \\ 0 & 1 & 0 & 0 \end{array}\right) \Rightarrow \begin{cases} a = 100 \\ b = 0 \\ c = 3800 \end{cases}$

$q_v = -100 + p_1 + 3800$

b) Het zijn onafhankelijke goederen, want de prijs van product 2 heeft geen invloed op de q_v van product 1