Module 9

oef. 1 $A = \begin{pmatrix} 5 & 4 & -1 \\ -6 & 0 & 3 \\ 2 & -3 & 2 \end{pmatrix}$

a) $\begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$

$A\begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 & 4 & -1 \\ -6 & 0 & 3 \\ 2 & -3 & 2 \end{pmatrix}\begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 8 \\ -6 \\ 8 \end{pmatrix} \Rightarrow$ geen EV

b) $A\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 & 4 & -1 \\ -6 & 0 & 3 \\ 2 & -3 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 6 \end{pmatrix} \Rightarrow$ EV met EW = 3

c) $A\begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 & 4 & -1 \\ -6 & 0 & 3 \\ 2 & -3 & 2 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ -8 \end{pmatrix} \Rightarrow$ geen EV

oef. 2 $A = \begin{pmatrix} 1 & -9 \\ -1 & 1 \end{pmatrix}$

a) $\begin{pmatrix} 1 & -9 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} -15 \\ -1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & -9 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 2 \end{pmatrix} \neq \lambda\begin{pmatrix} -15 \\ -1 \end{pmatrix}$

$\Rightarrow$ geen EV

b) $\begin{pmatrix} 1 & -9 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -6 \\ -2 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & -9 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \end{pmatrix} = -2\begin{pmatrix} 3 \\ 1 \end{pmatrix}$

c) $A^5\begin{pmatrix} -3 \\ -1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & -9 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} -3 \\ -1 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & -9 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} -3 \\ -1 \end{pmatrix} = -2\begin{pmatrix} -3 \\ -1 \end{pmatrix}$

$A^5 = (-2)^5 \cdot \begin{pmatrix} -3 \\ -1 \end{pmatrix} = -32\begin{pmatrix} -3 \\ -1 \end{pmatrix} = \begin{pmatrix} 96 \\ 32 \end{pmatrix}$

d) $\det\begin{pmatrix}1-\lambda & -9\\ -1 & 1-\lambda\end{pmatrix} = (1-\lambda)(1-\lambda)-9 = \lambda^2-2\lambda+1-9$
$= \lambda^2-2\lambda-8=0$

$\Delta = (-2)^2-4.1.(-8) = 4+32 = 36$

$\frac{2\pm 6}{2} \begin{cases} 4 \\ -2 \end{cases}$

$\lambda_1 = 4$; $\left(\begin{array}{cc|c}1-4 & -9 & 0\\ -1 & 1-4 & 0\end{array}\right) \to \left(\begin{array}{cc|c}-3 & -9 & 0\\ -1 & -3 & 0\end{array}\right)$

$\xrightarrow{R_1-3R_2} \left(\begin{array}{cc|c}0 & 0 & 0\\ -1 & -3 & 0\end{array}\right) \Rightarrow \begin{cases}x_1=\alpha\\ -x_1-3y=0\end{cases} \Rightarrow \begin{cases}x_1=\alpha\\ y=-\frac{x_1}{3}\end{cases}$

$\Rightarrow \begin{cases}x_1=\alpha\\ y=-\frac{\alpha}{3}\end{cases} \quad (\alpha, -\frac{1}{3}\alpha) \to (3\alpha, -\alpha) \to (-3\alpha, \alpha)$

$\lambda_2 = -2$: $\left(\begin{array}{cc|c}1-(-2) & -9 & 0\\ -1 & 1-(-2) & 0\end{array}\right) \to \left(\begin{array}{cc|c}3 & -9 & 0\\ -1 & 3 & 0\end{array}\right)$

$\xrightarrow{R_1+3R_2} \left(\begin{array}{cc|c}0 & 0 & 0\\ -1 & 3 & 0\end{array}\right) \Rightarrow \begin{cases}x_1=\alpha\\ -x_1+3x_2=0\end{cases} \Rightarrow \begin{cases}x_1=\alpha\\ x_2=\frac{\alpha}{3}\end{cases}$

$\Rightarrow (\alpha, \frac{1}{3}\alpha) \to (3\alpha, \alpha)$

e) $Q = \begin{pmatrix}-3 & 3\\ 1 & 1\end{pmatrix} \quad \Delta = \begin{pmatrix}4 & 0\\ 0 & -2\end{pmatrix}$

$\left(\begin{array}{cc|cc}-3 & 3 & 1 & 0\\ 1 & 1 & 0 & 1\end{array}\right) \xrightarrow{R_1+2R_2} \left(\begin{array}{cc|cc}-1 & 5 & 1 & 2\\ 1 & 1 & 0 & 1\end{array}\right)$

$\xrightarrow{R_2+R_1} \left(\begin{array}{cc|cc}-1 & 5 & 1 & 2\\ 0 & 6 & 1 & 3\end{array}\right) \xrightarrow{\frac{R_2}{6}} \left(\begin{array}{cc|cc}-1 & 5 & 1 & 2\\ 0 & 1 & \frac{1}{6} & \frac{3}{6}\end{array}\right)$

$\xrightarrow{R_1-5.R_2} \left(\begin{array}{cc|cc}-1 & 0 & \frac{1}{6} & -\frac{1}{2}\\ 0 & 1 & \frac{1}{6} & \frac{3}{6}\end{array}\right) \xrightarrow{-1R_1} \left(\begin{array}{cc|cc}1 & 0 & -\frac{1}{6} & \frac{1}{2}\\ 0 & 1 & \frac{1}{6} & \frac{1}{2}\end{array}\right)$

$Q^{-1} = \begin{pmatrix}-\frac{1}{6} & \frac{1}{2}\\ \frac{1}{6} & \frac{1}{2}\end{pmatrix}$

$A = \begin{pmatrix}-3 & 3\\ 1 & 1\end{pmatrix}\begin{pmatrix}4 & 0\\ 0 & -2\end{pmatrix}\begin{pmatrix}-\frac{1}{6} & \frac{1}{2}\\ \frac{1}{6} & \frac{1}{2}\end{pmatrix}$

1) $A^7 = \begin{pmatrix} -3 & 3 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 4 & 0 \\ 0 & -2 \end{pmatrix}^7 \begin{pmatrix} -\frac{1}{6} & \frac{1}{2} \\ \frac{1}{6} & \frac{1}{2} \end{pmatrix}$

$= \begin{pmatrix} -3 & 3 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 16\,384 & 0 \\ 0 & -128 \end{pmatrix} \begin{pmatrix} -\frac{1}{6} & \frac{1}{2} \\ \frac{1}{6} & \frac{1}{2} \end{pmatrix}$

$= \begin{pmatrix} -49\,152 & -384 \\ 16\,384 & -128 \end{pmatrix} \begin{pmatrix} -\frac{1}{6} & \frac{1}{2} \\ \frac{1}{6} & \frac{1}{2} \end{pmatrix}$

$= \begin{pmatrix} 8128 & -24\,768 \\ -2752 & 8128 \end{pmatrix}$

oef. 3 $A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \Rightarrow \det \begin{pmatrix} 0-\lambda & 0 & 0 \\ 0 & 1-\lambda & 0 \\ 1 & 0 & 1-\lambda \end{pmatrix} = -\lambda . \det \begin{pmatrix} 1-\lambda & 0 \\ 0 & 1-\lambda \end{pmatrix}$

$= -\lambda((1-\lambda)(1-\lambda)) = -\lambda(\lambda^2 - 2\lambda + 1) = -\lambda(\lambda-1)^2 = 0$

$\lambda = 0 \quad \vee \quad \underbrace{\lambda = 1}_{\hookrightarrow \ MP = 2}$

$\lambda = 0$: $\left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{array}\right) \Rightarrow \begin{cases} x = a \\ y = 0 \\ z = -a \end{cases}$

$\lambda = 1$: $\left(\begin{array}{ccc|c} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{array}\right) \xrightarrow{R_1 + R_3} \left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{array}\right)$

$\Rightarrow \begin{cases} x = 0 \\ y = a \\ z = b \end{cases}$

$Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \quad D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_3 + R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right)$

$\Rightarrow Q^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \qquad A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

$$B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \Rightarrow \det \begin{pmatrix} 1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 0 \\ 1 & 0 & 1-\lambda \end{pmatrix} = (1-\lambda)(1-\lambda)(1-\lambda)$$

$$= (1-\lambda)(1-\lambda)^2$$

$$= (1-\lambda)^3 = 0$$

$$\lambda = 1 \quad \hookrightarrow MP = 3$$

$$\left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{cases} x = 0 \\ y = a \\ z = b \end{cases}$$

$$Q = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & 0 \end{pmatrix} \Leftarrow$$

$\det(Q) = 0 \Rightarrow Q$ is niet inverteerbaar

Dus B is niet diagonaliseerbaar

$$C = \begin{pmatrix} -3 & 1 & -1 \\ -7 & 5 & -1 \\ -6 & 6 & -2 \end{pmatrix} \Rightarrow \det \begin{pmatrix} -3-\lambda & 1 & -1 \\ -7 & 5-\lambda & -1 \\ -6 & 6 & -2-\lambda \end{pmatrix}$$

$$= (-3-\lambda) \cdot \det \begin{pmatrix} 5-\lambda & -1 \\ 6 & -2-\lambda \end{pmatrix} - \det \begin{pmatrix} -7 & -1 \\ -6 & -2-\lambda \end{pmatrix}$$

$$+ (-1) \cdot \det \begin{pmatrix} -7 & 5-\lambda \\ -6 & 6 \end{pmatrix}$$

$$= (-3-\lambda)\big((5-\lambda)(-2-\lambda) - (-1)\cdot 6\big) - \big(-7\cdot(-2-\lambda) - 6\big) - \big(-7\cdot 6 - (-6(5-\lambda)\big)$$

$$= (-3-\lambda)\big((5-\lambda)(-2-\lambda) + 6\big) - (7\lambda + 14 - 6) - \big(-42 + 6(5-\lambda)\big)$$

$$= (-3-\lambda)(-10 - 5\lambda + 2\lambda + \lambda^2 + 6) - 7\lambda - 8 - (-42 + 30 - 6\lambda)$$

$$= (-3-\lambda)(\lambda^2 - 3\lambda - 4) - 7\lambda - 8 + 12 + 6\lambda$$

$$= \cancel{-3\lambda^2} + 9\lambda + 12 - \lambda^3 + \cancel{3\lambda^2} + 4\lambda - \lambda + 4$$

$$= -\lambda^3 + 12\lambda + 16 = 0$$

$$\lambda = -2 \vee \lambda = 4$$

$$\hookrightarrow MP = 2$$

$$\lambda = 4: \left(\begin{array}{ccc|c} -3-4 & 1 & -1 & 0 \\ -7 & 5-4 & -1 & 0 \\ -6 & 6 & -2-4 & 0 \end{array}\right) = \left(\begin{array}{ccc|c} -7 & 1 & -1 & 0 \\ -7 & 1 & -1 & 0 \\ -6 & 6 & -6 & 0 \end{array}\right)$$

$$\xrightarrow[\frac{R_3}{6}]{R_2 - R_1} \left(\begin{array}{ccc|c} -7 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 1 & -1 & 0 \end{array}\right) \xrightarrow{R_1 - 7 \cdot R_3} \left(\begin{array}{ccc|c} 0 & -6 & 6 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 1 & -1 & 0 \end{array}\right)$$

$$\xrightarrow[R_2 \leftrightarrow R_3]{\frac{R_1}{6}} \left(\begin{array}{ccc|c} 0 & -1 & 1 & 0 \\ -1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{R_2 + R_1} \left(\begin{array}{ccc|c} 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{cases} x = 0 \\ y = a \\ z = a \end{cases}$$

$\lambda=-2$; $\left(\begin{array}{ccc|c} -3-(-2) & 1 & -1 & 0 \\ -7 & 5-(-2) & -1 & 0 \\ -6 & 6 & -2-(-2) & 0 \end{array}\right)$

$= \left(\begin{array}{ccc|c} -1 & 1 & -1 & 0 \\ -7 & 7 & -1 & 0 \\ -6 & 6 & 0 & 0 \end{array}\right) \xrightarrow[\frac{R_3}{6}]{R_2-7R_1} \left(\begin{array}{ccc|c} -1 & 1 & -1 & 0 \\ 0 & 0 & 6 & 0 \\ -1 & 1 & 0 & 0 \end{array}\right)$

$\xrightarrow[R_1-R_3]{\frac{R_2}{6}} \left(\begin{array}{ccc|c} 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 \end{array}\right) \xrightarrow{R_1+R_2} \left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 \end{array}\right)$

$\Rightarrow \begin{cases} x = \alpha \\ y = \alpha \\ z = 0 \end{cases}$

$Q = \begin{pmatrix} 1 & 0 & 2 \\ 1 & 1 & 2 \\ 0 & 1 & 0 \end{pmatrix}$ $\Rightarrow$ det$(Q) = 0 \Rightarrow Q$ is niet inverteerbaar, Dus is C niet diagonaliseerbaar

oef.4 $\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \rightarrow \det \begin{pmatrix} a-\lambda & b \\ 0 & c-\lambda \end{pmatrix} = (a-\lambda)(c-\lambda) = 0$

$\lambda = a \vee \lambda = c$

de EW van een 2×2-diagonaalmatrix zijn de elementen op de diagonaal

$\begin{pmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{pmatrix} \rightarrow \det \begin{pmatrix} a-\lambda & b & c \\ 0 & d-\lambda & e \\ 0 & 0 & f-\lambda \end{pmatrix}$

$= (a-\lambda).\det \begin{pmatrix} d-\lambda & e \\ 0 & f-\lambda \end{pmatrix} - b.\det \begin{pmatrix} 0 & e \\ 0 & f-\lambda \end{pmatrix} + c.\det \begin{pmatrix} 0 & d-\lambda \\ 0 & 0 \end{pmatrix}$

$= (a-\lambda)(d-\lambda)(f-\lambda) - b.0 + c.0$

$= (a-\lambda)(d-\lambda)(f-\lambda)$

$\lambda = a \vee \lambda = d \vee \lambda = f$

$\Rightarrow$ De Eigenwaardes van een 2×2 en en 3×3 diagonaalmatrix zijn de elementen op de diagonaal

Aef. 5 $A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$ $\lambda_1 = 1$ en $\lambda_2 = \lambda_3 = -2$

$e_1 = (1, -1, 1)$

$e_{-2} = (-a-b, a, b)$

$$A\begin{pmatrix} -4 \\ -3 \\ 9 \end{pmatrix} = ?$$

$$A^k \cdot \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} = \alpha \cdot \overset{EW}{\overset{\shortparallel}{A}}{}^k \cdot \overset{EV}{\begin{pmatrix} x \\ y \\ z \end{pmatrix}} + \beta \cdot \overset{EW}{\overset{\shortparallel}{A}}{}^k \cdot \overset{EV}{\begin{pmatrix} x \\ y \\ z \end{pmatrix}} + \delta \cdot \overset{EW}{\overset{\shortparallel}{A}}{}^k \overset{EV}{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}$$

$$A^1\begin{pmatrix} -4 \\ -3 \\ 9 \end{pmatrix} = \alpha \cdot 1^1 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + \beta \cdot (-2)^1 \begin{pmatrix} -a-b \\ a \\ b \end{pmatrix} + \delta \cdot (-2)^1 \begin{pmatrix} -a-b \\ a \\ b \end{pmatrix}$$

$$\begin{pmatrix} -4 \\ -3 \\ 9 \end{pmatrix} = \alpha \cdot \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + \beta \underset{a=0\ b=1}{\begin{pmatrix} -a-b \\ a \\ b \end{pmatrix}} + \delta \underset{a=-2\ b=1}{\begin{pmatrix} -a-b \\ a \\ b \end{pmatrix}}$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & -4 \\ -1 & 0 & -2 & -3 \\ 1 & 1 & 1 & 9 \end{array}\right) \begin{array}{c} R_2 + R_1 \\ \to \\ R_3 - R_1 \end{array} \left(\begin{array}{ccc|c} 1 & -1 & 1 & -4 \\ 0 & -1 & -1 & -7 \\ 0 & 2 & 0 & 13 \end{array}\right)$$

$$\Rightarrow \begin{cases} \alpha - \beta + \delta = -4 \\ -\beta - \delta = -7 \\ 2 \cdot \beta = 13 \end{cases} \Rightarrow \begin{cases} \alpha = 2 \\ \beta = \frac{13}{2} \\ \delta = \frac{1}{2} \end{cases}$$

$$A\begin{pmatrix} -4 \\ -3 \\ 9 \end{pmatrix} = 2 \cdot 1 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + \frac{13}{\cancel{2}} \cdot (-\cancel{2}) \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + \frac{1}{\cancel{2}} (-\cancel{2}) \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ -2 \\ 2 \end{pmatrix} - 13 \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 2 \end{pmatrix} + \begin{pmatrix} 13 \\ 0 \\ -13 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} 14 \\ 0 \\ -12 \end{pmatrix}$$

oef. 6 $\begin{pmatrix} X_{n+1} \\ Y_{n+1} \end{pmatrix} = M \begin{pmatrix} x_n \\ y_n \end{pmatrix}$ met $M = \begin{pmatrix} \frac{15}{16} & \frac{1}{8} \\ \frac{1}{16} & \frac{7}{8} \end{pmatrix}$

a) X_n = werklozen en y_n = werkenden

$x_n = 0,5$ $\quad y_n = 4$

$$\begin{pmatrix} x_n \\ y_n \end{pmatrix} = M^k \begin{pmatrix} 4 \\ 0,5 \end{pmatrix}$$

b) $\det \begin{pmatrix} \frac{15}{16} - \lambda & \frac{1}{8} \\ \frac{1}{16} & \frac{7}{8} - \lambda \end{pmatrix} = \left(\frac{15}{16} - \lambda\right)\left(\frac{7}{8} - \lambda\right) - \frac{1}{8} \cdot \frac{1}{16}$

$= \frac{105}{128} - \frac{15}{16}\lambda - \frac{7}{8}\lambda + \lambda^2 - \frac{1}{128} = \lambda^2 - \frac{29}{16}\lambda + \frac{105}{128} - \frac{1}{128}$

$= \lambda^2 - \frac{29}{16}\lambda + \frac{13}{16} = 0$

$\Delta = \left(\frac{-29}{16}\right)^2 - 4 \cdot 1 \cdot \frac{13}{16} = \frac{9}{256}$

$$\frac{\frac{29}{16} \pm \frac{3}{16}}{2} \begin{cases} 1 \\ \frac{13}{16} \end{cases}$$

$\lambda_1 = 1$: $\begin{pmatrix} \frac{15}{16} - 1 & \frac{1}{8} \\ \frac{1}{16} & \frac{7}{8} - 1 \end{pmatrix} \Rightarrow \left(\begin{array}{cc|c} -\frac{1}{16} & \frac{1}{8} & 0 \\ \frac{1}{16} & -\frac{1}{8} & 0 \end{array}\right)$

$\xrightarrow{R_2 + R_1} \left(\begin{array}{cc|c} -\frac{1}{16} & \frac{1}{8} & 0 \\ 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{cases} -\frac{1}{16}x + \frac{1}{8}y = 0 \\ y = \alpha \end{cases} \Rightarrow \begin{cases} x = 2\alpha \\ y = \alpha \end{cases}$

$(2\alpha, \alpha)$

$\lambda_2 = \frac{13}{16}$: $\begin{pmatrix} \frac{15}{16} - \frac{13}{16} & \frac{1}{8} \\ \frac{1}{16} & \frac{7}{8} - \frac{13}{16} \end{pmatrix} \Rightarrow \left(\begin{array}{cc|c} \frac{1}{8} & \frac{1}{8} & 0 \\ \frac{1}{16} & \frac{1}{16} & 0 \end{array}\right)$

$\xrightarrow{R_2 - \frac{1}{2}R_1} \left(\begin{array}{cc|c} \frac{1}{8} & \frac{1}{8} & 0 \\ 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{cases} \frac{1}{8}x + \frac{1}{8}y = 0 \\ y = \alpha \end{cases} \Rightarrow \begin{cases} x = -\alpha \\ y = \alpha \end{cases}$

$(-\alpha, \alpha)$

c) $\begin{pmatrix} 0,5 \\ 4 \end{pmatrix} = a \begin{pmatrix} 1 \\ -1 \end{pmatrix} + b \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\left(\begin{array}{cc|c} 1 & 2 & 0,5 \\ -1 & 1 & 4 \end{array}\right) \xrightarrow{R_1+R_2} \left(\begin{array}{cc|c} 0 & 3 & 4,5 \\ -1 & 1 & 4 \end{array}\right) \Rightarrow \begin{cases} 3b = 4,5 \\ -a+b = 4 \end{cases}$

$\Rightarrow \begin{cases} b = 1,5 \\ a = -2,5 \end{cases}$

d) $M^k \begin{pmatrix} 0,5 \\ 4 \end{pmatrix} = \left(\frac{13}{16}\right)^k \cdot (-2,5) \begin{pmatrix} 1 \\ -1 \end{pmatrix} + (1)^k \cdot (1,5) \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

e) $\lim_{k \to \infty} \underbrace{\left(\frac{13}{16}\right)^k}_{\text{gaat nr } 0} \cdot (-2,5) \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \underbrace{(1)^k}_{\text{gaat nr } 1} \cdot (1,5) \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$= 1 \cdot (1,5) \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1,5 \end{pmatrix}$

oef. 7 $\begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = \begin{pmatrix} 0,6 & 0,3 \\ 0,4 & 0,7 \end{pmatrix} \begin{pmatrix} 500\,000 \\ 900\,000 \end{pmatrix}$

$\det \begin{pmatrix} 0,6-\lambda & 0,3 \\ 0,4 & 0,7-\lambda \end{pmatrix} = (0,6-\lambda)(0,7-\lambda) - 0,3 \cdot 0,4$
$= 0,42 - 1,3\lambda + \lambda^2 - 0,12$
$= \lambda^2 - 1,3\lambda + 0,3 = 0$
$\Delta = (-1,3)^2 - 4 \cdot 1 \cdot 0,3 = 0,49$
$\frac{1,3 \pm 0,07}{2} \begin{cases} 1 \\ 0,3 = \frac{3}{10} \end{cases}$

$\lambda_1 = 1: \begin{pmatrix} 0,6-1 & 0,3 \\ 0,4 & 0,7-1 \end{pmatrix} = \left(\begin{array}{cc|c} -0,4 & 0,3 & 0 \\ 0,4 & -0,3 & 0 \end{array}\right)$

$\xrightarrow{R_1+R_2} \left(\begin{array}{cc|c} 0 & 0 & 0 \\ 0,4 & -0,3 & 0 \end{array}\right) \Rightarrow \begin{cases} x = \frac{3}{4} a \\ y = a \end{cases} \Rightarrow \left(\frac{3}{4}a, a\right) \to (3a, 4a)$

$\lambda_2 = \frac{3}{10}: \begin{pmatrix} 0,6-0,3 & 0,3 \\ 0,4 & 0,7-0,3 \end{pmatrix} = \left(\begin{array}{cc|c} 0,3 & 0,3 & 0 \\ 0,4 & 0,4 & 0 \end{array}\right)$

$\xrightarrow[\frac{R_2}{0,4}]{\frac{R_1}{0,3}} \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 1 & 1 & 0 \end{array}\right) \xrightarrow{R_2 - R_1} \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{cases} x = -a \\ y = a \end{cases} \Rightarrow (-a, a)$

$$\begin{pmatrix} 500\,000 \\ 900\,000 \end{pmatrix} = \alpha \cdot \begin{pmatrix} 3 \\ 4 \end{pmatrix} + b \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\Rightarrow \left(\begin{array}{cc|c} 3 & -1 & 500\,000 \\ 4 & 1 & 900\,000 \end{array}\right) \xrightarrow{R_2 + R_1} \left(\begin{array}{cc|c} 3 & -1 & 500\,000 \\ 7 & 0 & 1\,400\,000 \end{array}\right)$$

$$\Rightarrow \begin{cases} 7\alpha = 1\,400\,000 \\ -b = 500\,000 - 3\alpha \end{cases} \Rightarrow \begin{cases} \alpha = 200\,000 \\ b = 100\,000 \end{cases}$$

$$M^k \begin{pmatrix} 500\,000 \\ 900\,000 \end{pmatrix} = 1^k \cdot 200\,000 \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \left(\frac{3}{10}\right)^k \cdot 100\,000 \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\lim_{k \to \infty} 1^k \cdot 200\,000 \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \left(\frac{3}{10}\right)^k \cdot 100\,000 \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$= 1 \cdot 200\,000 \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 600\,000 \\ 800\,000 \end{pmatrix}$$ $\Rightarrow$ 600 000 in de stad
800 000 op het platteland

oef. 8 $|\lambda| < 1$ $\lim_{k \to \infty} A^k = 0$ A is een diagonaliseerbare 3x3-matrix

$$A = Q \cdot D \cdot Q^{-1} \Rightarrow A^k = Q \cdot D^k \cdot Q^{-1}$$

$$D^k = \begin{pmatrix} \lambda^k & 0 & 0 & \cdots & 0 \\ 0 & \lambda^k & 0 & \cdots & 0 \\ 0 & 0 & \lambda^k & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \lambda^k \end{pmatrix}$$

en aangezien elke $|\lambda| < 1$ is $\lim_{k \to \infty} \lambda^k = 0$, voor elke λ

dus als k naar oneindig gaat is D^k de nulmatrix

$$\lim_{k \to \infty} D^k = 0$$

Dan is $\lim_{k \to \infty} A^k = \lim_{k \to \infty} Q \cdot D^k \cdot Q^{-1} = \lim_{k \to \infty} Q \cdot 0 \cdot Q^{-1} = 0$

Module 10

oef.1 $(3,-4)$ $\quad 6x - 2.y = 26$

$6.3 - 2.(-4) = 18 + 8 = 26$

oef.2 $a = (2,1)$ $\quad b = (3,-2)$ $\quad c = (7,0)$ $\quad d = (1,4)$

Parametervergelijking:

$ab \leftrightarrow \begin{cases} x = 2 + \lambda(3-2) \\ y = 1 + \lambda(-2-1) \end{cases} \Rightarrow \begin{cases} x = 2 + \lambda \\ y = 1 - 3\lambda \end{cases}$

$cd \leftrightarrow \begin{cases} x = 7 + \lambda(1-7) \\ y = 0 + \lambda(4-0) \end{cases} \Rightarrow \begin{cases} x = 7 - 6\lambda \\ y = 4\lambda \end{cases}$

Cartesiaanse vergelijking:

$ab \leftrightarrow \begin{cases} x = 2+\lambda \\ y = 1-3\lambda \end{cases} \Rightarrow \begin{cases} \lambda = x-2 \\ \lambda = -\frac{y}{3} + \frac{1}{3} \end{cases} \Rightarrow \lambda = x-2 = -\frac{1}{3}y + \frac{1}{3}$

$x - 2 = -\frac{1}{3}y + \frac{1}{3} \Rightarrow 3x - 6 = -y + 1 \Rightarrow 3x + y = 7$

OF: $y - 1 = \frac{-2-1}{3-2}(x-2) \Rightarrow y = -3x + 7 \Rightarrow 3x + y = 7$

$cd \leftrightarrow \begin{cases} x = 7 - 6\lambda \\ y = 4\lambda \end{cases} \Rightarrow \begin{cases} \lambda = \frac{x-7}{-6} \\ \lambda = \frac{y}{4} \end{cases} \Rightarrow \lambda = \frac{x-7}{-6} = \frac{1}{4}y$

$\frac{x-7}{-6} = \frac{1}{4}y \Rightarrow 4x - 28 = -6y \Rightarrow 4x + 6y = 28$

$\Rightarrow 2x + 3y = 14$

OF: $y - 0 = \frac{4-0}{1-7}(x-7) \Rightarrow y = \frac{4}{-6}(x-7) \Rightarrow y = -\frac{2}{3}x + \frac{14}{3}$

$3y = -2x + 14 \Rightarrow 2x + 3y = 14$

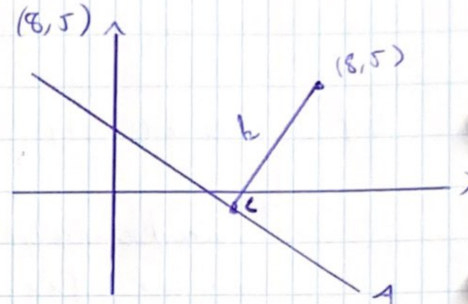
Snijden?: $\begin{cases} 3x + y = 7 \\ 2x + 3y = 14 \end{cases} \Rightarrow \begin{cases} y = 7 - 3x \\ 2x + 3(7-3x) = 14 \end{cases} \Rightarrow \begin{cases} y = 7 - 3x \\ 2x - 9x + 21 = 14 \end{cases}$

$\begin{cases} -7x = 14 - 21 \\ y = 7 - 3x \end{cases} \Rightarrow \begin{cases} x = 1 \\ y = 4 \end{cases}$

Ze snijden in het punt $(1,4)$

oef.3 Je zoekt dan een punt waar alle 3 de rechten doorgaan

oef.4 $\mathbb{R}^2$ $\quad A \leftrightarrow 3x + 2y = 8$



$b \leftrightarrow (8,5) + \lambda(3,2)$

$$\begin{cases} x = 8 + 3\lambda \\ y = 5 + 2\lambda \end{cases} \Rightarrow \begin{cases} \lambda = \frac{x-8}{3} \\ \lambda = \frac{y-5}{2} \end{cases} \Rightarrow \lambda = \frac{x}{3} - \frac{8}{3} = \frac{y}{2} - \frac{5}{2}$$

$$\frac{x-8}{3} = \frac{y-5}{2} \Rightarrow 2x - 16 = 3y - 15$$
$$2x - 3y = 1$$

$b \leftrightarrow 2x - 3y = 1$

$$\begin{cases} 2x - 3y = 1 \\ 3x + 2y = 8 \end{cases} \Rightarrow \begin{cases} 2x - 3y = 1 \\ y = \frac{8}{2} - \frac{3}{2} \cdot x \end{cases} \Rightarrow \begin{cases} 2x - 3y = 1 \\ y = 4 - \frac{3}{2}x \end{cases}$$

$$\begin{cases} 2x - 3\left(-\frac{3}{2}x + 4\right) = 1 \\ y = 4 - \frac{3}{2}x \end{cases} \Leftrightarrow \begin{cases} 2x + \frac{9x}{2} - 12 = 1 \\ y = 4 - \frac{3}{2}x \end{cases} \Rightarrow \begin{cases} \frac{13}{2}x - 12 = 1 \\ y = 4 - \frac{3}{2}x \end{cases}$$

$$\Rightarrow \begin{cases} \frac{13}{2}x = 13 \\ y = 4 - \frac{3}{2}x \end{cases} \Rightarrow \begin{cases} x = 2 \\ y = 4 - \frac{3 \cdot \cancel{2}}{\cancel{2}} \end{cases} \Rightarrow \begin{cases} x = 2 \\ y = 1 \end{cases}$$

de afstand:

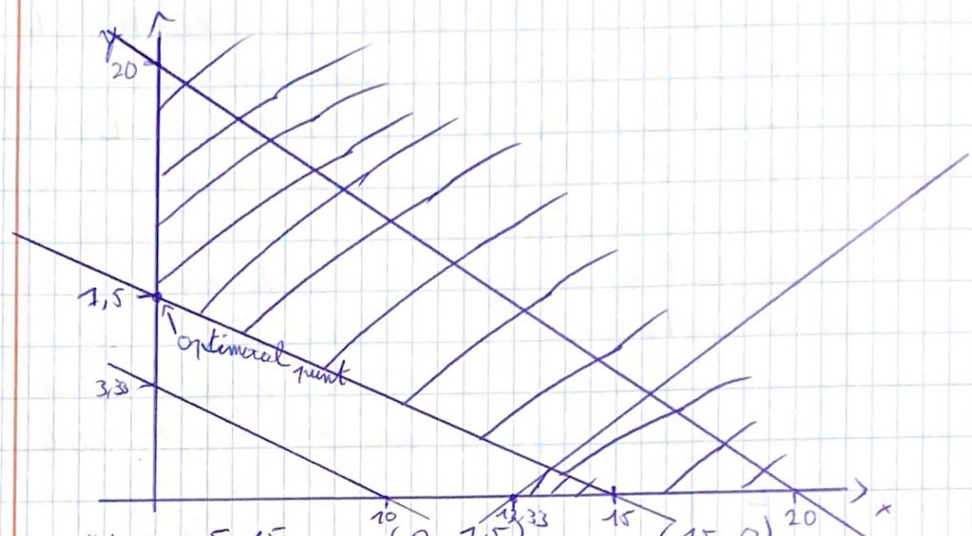
$$\sqrt{(2-8)^2 + (1-5)^2} = \sqrt{(-6)^2 + (-4)^2} = \sqrt{36 + 16} = \sqrt{52}$$

oef.5 Maxi $x + 3y$

$$\begin{cases} x + 2y \leq 15 \\ x + y \leq 20 \\ 3x - 4y \leq 40 \end{cases}$$

$x \geq 0$ en $y \geq 0$

doel: maximaliseer: $x + 3y$



$x + 2y \leq 15$ $(0, 7,5)$ $(15, 0)$

$x + y \leq 20$ $(0, 20)$ $(20, 0)$

$3x - y \leq 40$ $(0, -10)$ $(13,33, 0)$

$x + 3y = 10 \Rightarrow (10, 0)$ $(0, 3,33)$

$0 + 7,5 \cdot 3 = 22,5 \Rightarrow$ optimaal punt

oef. 6 400 Vit., 500 Min. en 1400 kcal

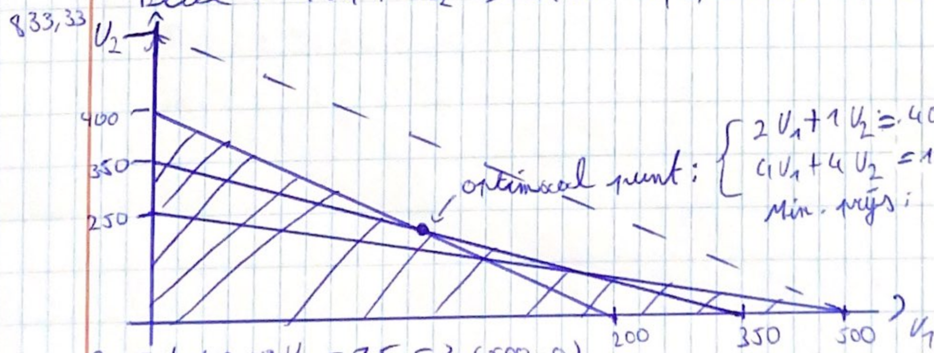
Min.: $0,15\, V_1 + 0,09\, V_2$

	Vit.	Min.	kcal
V_1	2	1	4
V_2	1	2	4

Vit: $2V_1 + 1V_2 \geq 400 \Rightarrow (0, 400)$ $(200, 0)$

Min.: $1V_1 + 2V_2 \geq 500 \Rightarrow (0, 250)$ $(500, 0)$

kcal: $4V_1 + 4V_2 \geq 1400 \Rightarrow (0, 350)$ $(350, 0)$



optimaal punt: $\begin{cases} 2V_1 + 1V_2 = 400 \\ 4V_1 + 4V_2 = 1400 \end{cases} \Rightarrow \begin{cases} V_1 = 50 \\ V_2 = 300 \end{cases}$

Min. prijs: $0,15 \cdot 50 + 0,09 \cdot 300 = 34,5$

$0,15\, V_1 + 0,09\, V_2 = 75 \Rightarrow (500, 0)$ $(0, 833,33)$

oef. 7 Winstfunctie:

$400H + 240L - 336H - 160L = 64H + 80L$

H: 20 W + 40 n — max: 1200 Wol en 1000 nylon

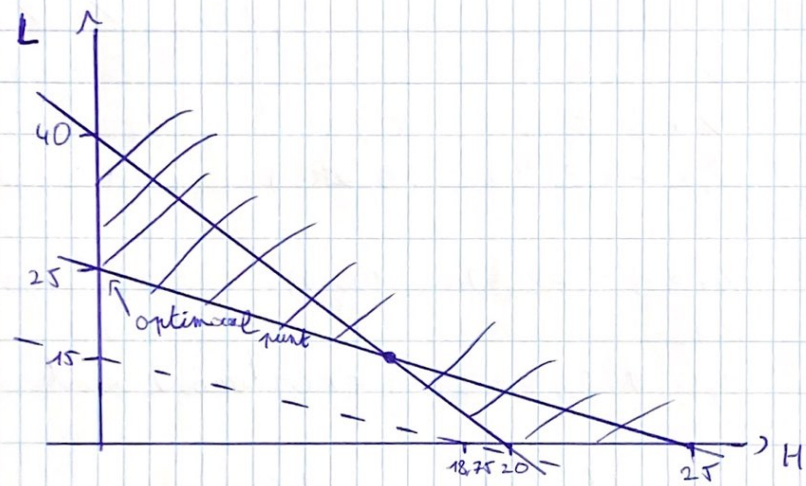
L: 0 W + 40 n — $\Rightarrow 40H + 40L \leq 1000$

H: 40 u . 4,8 — max. 800 u $\Rightarrow 40H + 20L \leq 800$

L: 20 u . 4,8

Wol kost 4 en nylon 1,6, arbeid 4,8.

Kosten voor 1:	Wol		Nylon		Veen	
H:	80	+	64	+	192	= 336 H
L:	0	+	64	+	96	= 160 L



$40H + 40L \leq 1000 \Rightarrow$ (25,0) (0,25)

$40H + 20L \leq 800 \Rightarrow$ (20,0) (0,40)

$64H + 80L = 1200 \Rightarrow$ (18,75,0) (0,15)

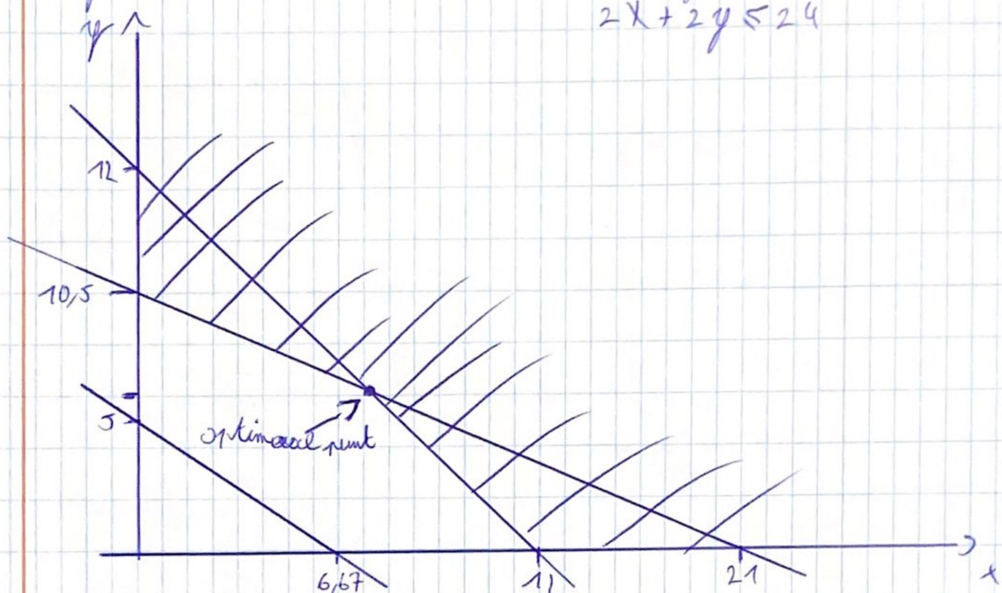
Optimaal punt: H = 0 en L = 25

$64 \cdot 0 + 80 \cdot 25 = 2000$

opg. 8 Winstfunctie: $15x + 20y$

$x = 4W + 2k$
$y = 8W + 2k$

Max: 24 k en 84 W
$\Rightarrow 4x + 8y \leq 84$
$2x + 2y \leq 24$



$4x + 8y \leq 84 \Rightarrow (21, 0) \quad (0, 10{,}5)$
$2x + 2y \leq 24 \Rightarrow (12, 0) \quad (0, 12)$

$15x + 20y = 100 \Rightarrow (6{,}67, 0) \quad (0, 5)$

$$\begin{cases} 4x + 8y = 84 \\ 2x + 2y = 24 \end{cases} \Rightarrow \left(\begin{array}{cc|c} 4 & 8 & 84 \\ 2 & 2 & 24 \end{array}\right) \xrightarrow{R_1 - 2 \cdot R_2} \left(\begin{array}{cc|c} 0 & 4 & 36 \\ 2 & 2 & 24 \end{array}\right)$$

$$\xrightarrow[\frac{R_2}{2}]{\frac{R_1}{4}} \left(\begin{array}{cc|c} 0 & 1 & 9 \\ 1 & 1 & 12 \end{array}\right) \Rightarrow \begin{cases} x = 12 - y \\ y = 9 \end{cases} \Rightarrow \begin{cases} x = 3 \\ y = 9 \end{cases}$$

$15 \cdot 3 + 9 \cdot 20 = 225$