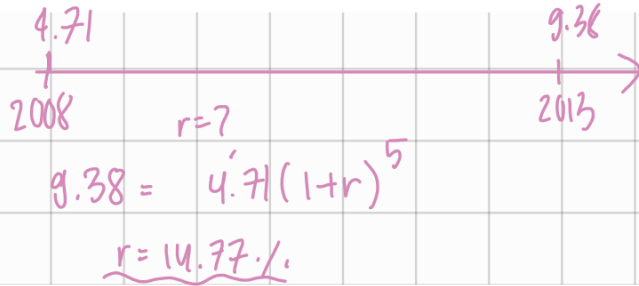


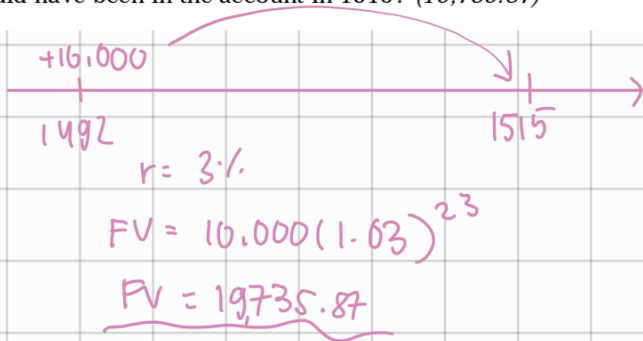
# Chapter 1

## TIME VALUE OF MONEY

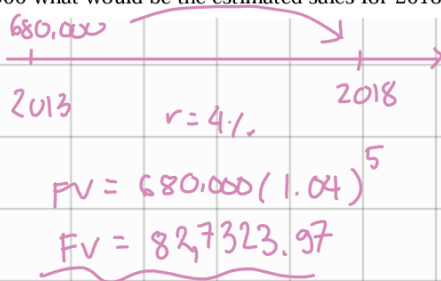
1. In 2008 the earnings per share of a company were \$4.71. By 2013 they were \$9.38. What is the annual rate of increase in the earnings per share in this five-year period? (annual rate of increase =  $r$ ). (14.77%)



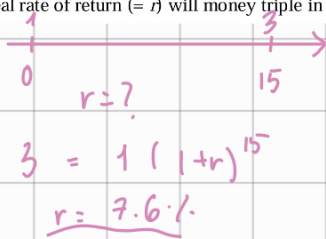
2. In 1492, Queen Isabella sponsored Christopher Columbus' journey by giving him \$10,000. If she had placed this money in a bank account at an annual rate  $r = 3\%$ , how much money would have been in the account in 1515? (19,735.87)



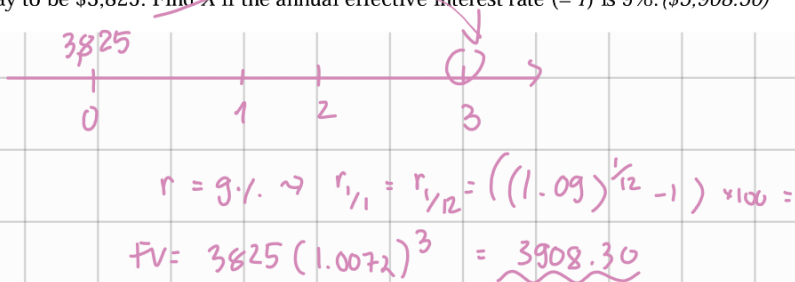
3. The IXYZET Company has had an increase in sales of 4% per annum. If sales in 2013 are \$680,000 what would be the estimated sales for 2018? (\$827,323.97)



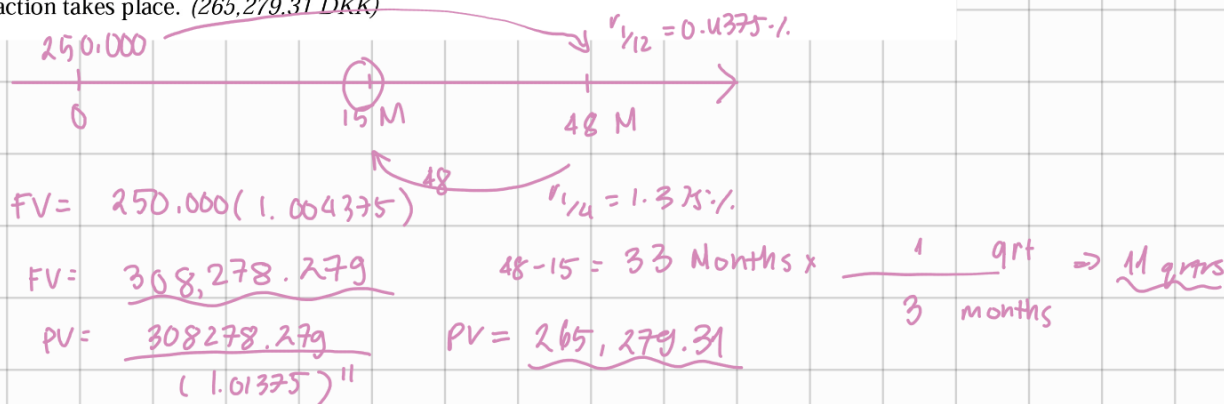
4. At what real rate of return ( $= r$ ) will money triple in 15 years? (7.6%)



5. An amount \$X is due in 3 months. A finance company calculates the value of this amount today to be \$3,825. Find X if the annual effective interest rate ( $= r$ ) is 9%. (\$3,908.30)



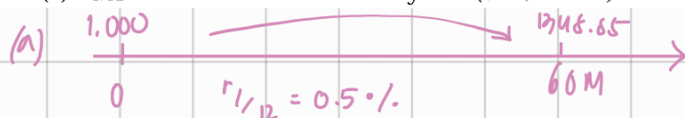
6. A financial product of 250,000 DKK at a monthly rate  $r_{1/12} = 0.4375\%$  will be repaid four years from now. Suppose the owner of this product is selling it 15 months from now to another person who wants to earn a quarterly rate  $r_{1/4} = 1.375\%$ . Compute the price at which this transaction takes place. (265,279.31 DKK)



7. Find the total interest earned on \$1,000 at

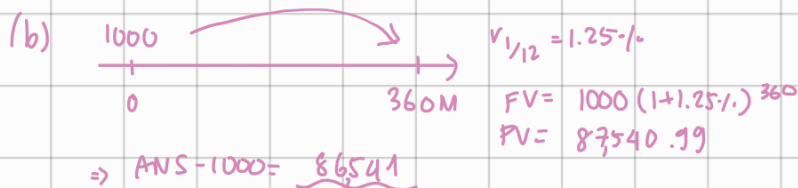
(a)  $r_{1/12} = 0.5\%$  invested for 5 years (\$348.85)

(b)  $r_{1/12} = 1.25\%$  invested for 30 years (\$86,541.00)



$$FV = 1000 (1 + 0.5\%)^{60} = 1348.85$$

$$\text{ANS} - 1000 = 348.85$$

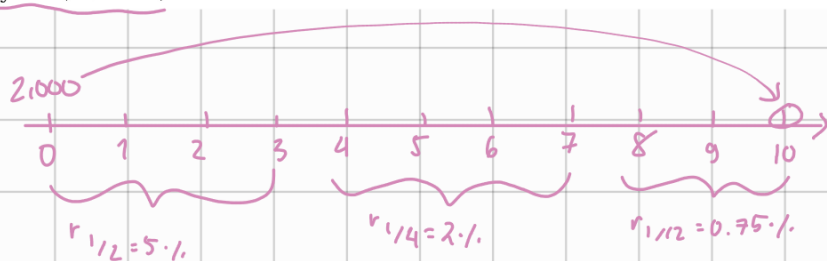


$$FV = 1000 (1 + 1.25\%)^{360}$$

$$FV = 87,540.99$$

$$\Rightarrow \text{ANS} - 1000 = 86,541$$

8. \$2,000 are invested for 10 years at  $r_{1/2} = 5\%$  for the first 3 years, at  $r_{1/4} = 2\%$  for the next 4 years and at  $r_{1/12} = 0.75\%$  for the last 3 years. Find the accumulated value of the investment after 10 years. (\$4,814.94)



$$FV = 2000 (1 + 5\%)^6 \cdot (1 + 2\%)^{16} \cdot (1 + 0.75\%)^{36}$$

$$FV = 4,814.96$$

9. Calculate

- ① the annual equivalent to  $r_{1/12} = 0.84\%$  (10.56%)  

$$r = \left( (1.0084)^{12} - 1 \right) \times 100 = 10.56\%$$
- ② the  $r_{0.5}$  equivalent to  $r_{1/4} = 3\%$  (6.09%)  

$$r_{1/2} = \left( (1.03)^2 - 1 \right) \times 100 = 6.09\%$$
- ③ the  $r_{0.25}$  equivalent to a daily rate of 0.025% (2.28%)  

$$r_{1/4} = \left( (1.00025)^{\frac{360}{4}} - 1 \right) \times 100 = 2.28\%$$
- the per month equivalent of 5% per half year (0.82%)  

$$r_{1/12} = \left( (1.05)^{\frac{2}{12}} - 1 \right) \times 100 = 0.82\%$$

10. A savings and loan association offers guaranteed investment certificates paying interest at  $r_{1/12} = 0.9375\%$ ,  $r_{0.5} = 6.75\%$  and  $r = 11.75\%$ . Which option performs best? (second option)

↳ Performs best: Highest interest rate

①  $r_{1/12} \Rightarrow r = ((1.009375)^{12} - 1) \times 100 = 11.85\%$

②  $r_{1/2} \Rightarrow r = ((1.0675)^2 - 1) \times 100 = 13.96\% \rightarrow \checkmark$

③  $r = 11.75\%$

11. You have 2 options:

PV<sub>1</sub>

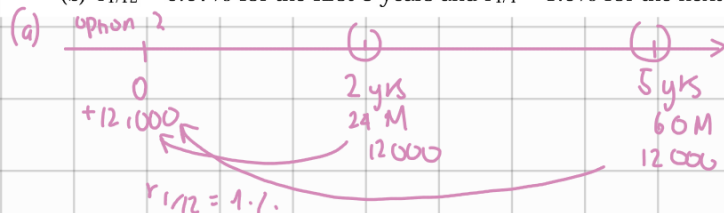
Option 1: Receive \$30,000.00 now;

Option 2: Receive \$12,000.00 now, \$12,000.00 in 2 years and \$12,000.00 in 5 years.

Which option is better if money can be invested at

(a)  $r_{1/12} = 1\%$ ? (Option 1) ✓

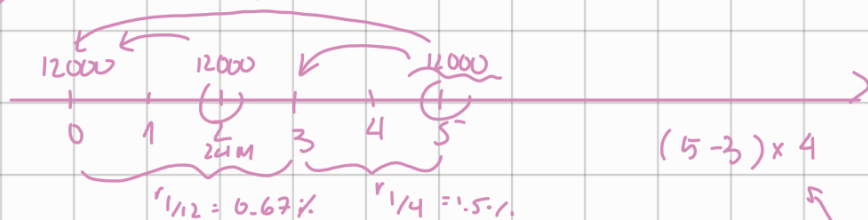
(b)  $r_{1/12} = 0.67\%$  for the first 3 years and  $r_{1/4} = 1.5\%$  for the next 2 years? (Option 2)



$$PV_2 = 12000 + 12000(1+1\%)^{-24} + 12000(1+1\%)^{-60}$$

$$PV_2 = 28,056$$

(b)  $PV_1 = 30,000$



$$PV_2 = 12000 + 12000(1.0067)^{-24} + 12000(1.015)^{-8} \times (1.0067)^{-36}$$

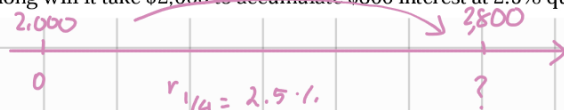
$$PV_2 = 30,599 \rightarrow \text{option 2} \checkmark$$

12. An investment fund advertises that it will triple your money in 10 years. What monthly rate of interest is implied? (0.92%)



$$3 = 1(1+r)^{120} \Rightarrow r = 0.92\%$$

13. How long will it take \$2,000 to accumulate \$800 interest at 2.5% quarterly? (13.63 quarters)



$$2800 = 2000(1.025)^n$$

$$n = 13.63 \text{ quarters}$$

14. If an investment doubles in value in 6 years at a certain monthly rate of interest how many months will it take for the same investment to triple in value at the same monthly rate? (113.81 months)



$$2 = 1(1+r_{1/12})^{72} \Rightarrow r_{1/12} = 0.967$$

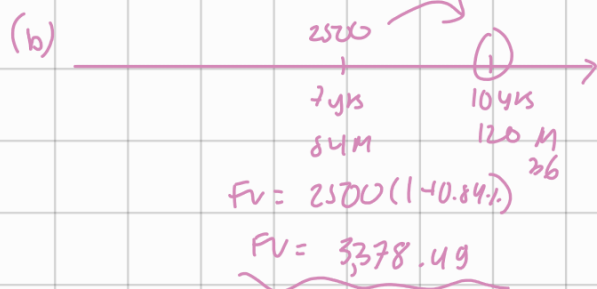
$$\Rightarrow 3 = 1(1+0.967\%)^n$$

$$n = 113.81$$

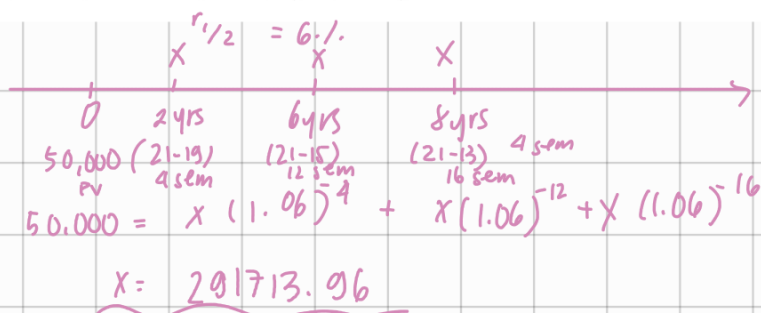
15. You need to pay \$2,500 at the end of 7 years. If money is valued at  $r_{1/12} = 0.84\%$  find the equivalent debt at the end of

(a) 3 years, (\$1,673.26)

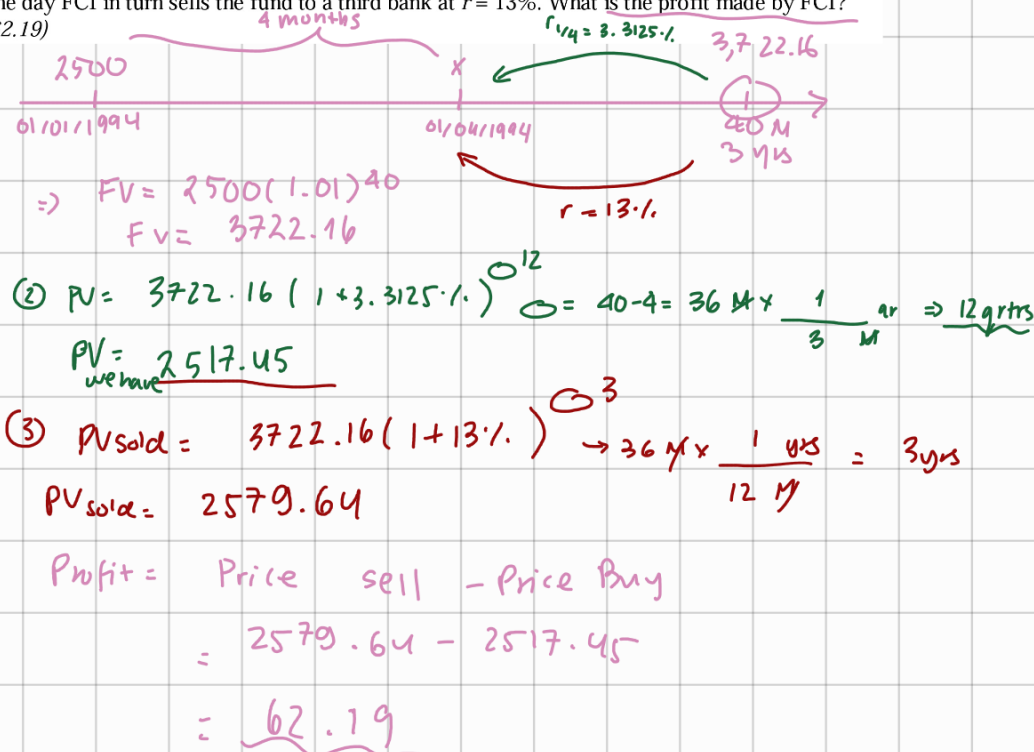
(b) 10 years. (\$3,378.49)



16. A man stipulates in his will that \$50,000.00 from his estate is to be placed in a fund from which his three daughters are each to receive the same amount when aged 21. When the man dies, the girls are aged 19, 15 and 13. How much will each daughter receive if the fund earns interest at a semestrial rate of 6%? (\$29,713.99)

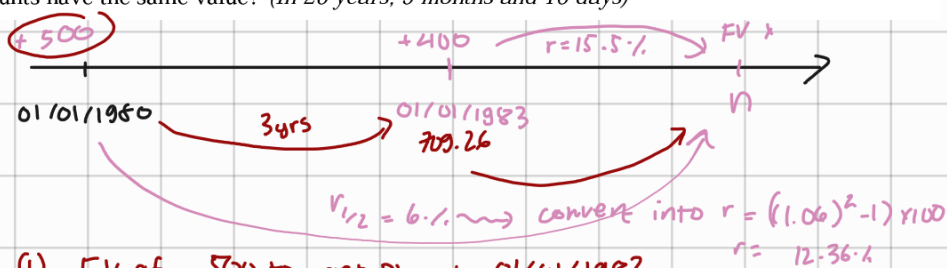


17. A fund of \$2,500 dated January 1, 1994, is due with monthly interest of 1% forty months later. On May 1, 1994, the holder of the fund sells it to FCI at a quarterly rate of 3.3125%. The same day FCI in turn sells the fund to a third bank at  $r = 13\%$ . What is the profit made by FCI? (\$62.19)





18. \$500 was deposited on January 1, 1980 in an account paying 6% per half year. On January 1, 1983, \$400 was deposited in another account paying 15.5% annually. When will the two accounts have the same value? (In 20 years, 9 months and 10 days)



(1) FV of 500 to get PV at 01/01/1983

$$FV = 500(1.06)^6$$

$$FV = 709.26$$

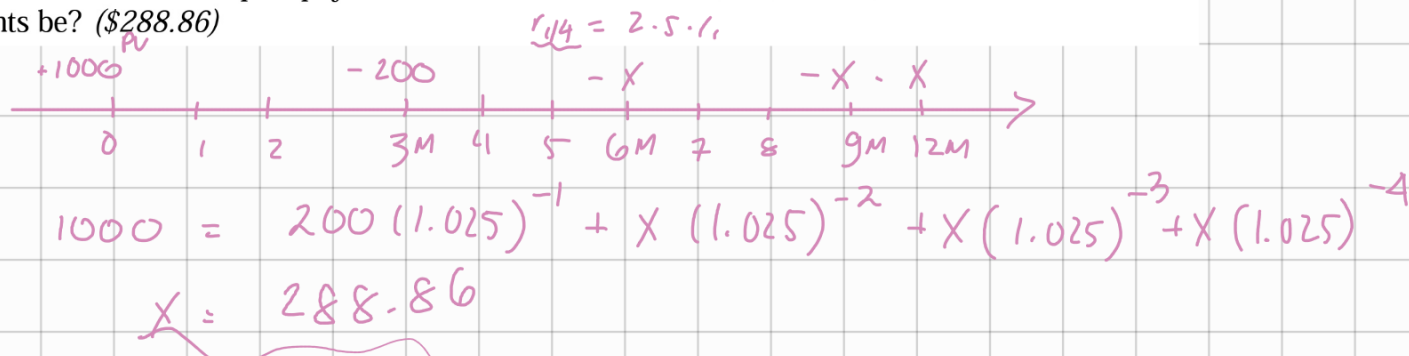
$$FV \Rightarrow 400(1.155)^n = 709.26(1.1236)^n$$

$$n = 20.78 \text{ years}$$

OR

$$\Rightarrow 400(1.155)^n = 500(1.1236)^{(n+3)}$$

19. A debt of \$1,000 with interest at  $r_{1/4} = 2.5\%$ , will be repaid by a payment of \$200 at the end of 3 months and three equal payments at the ends of 6, 9 and 12 months. What will these payments be? (\$288.86)

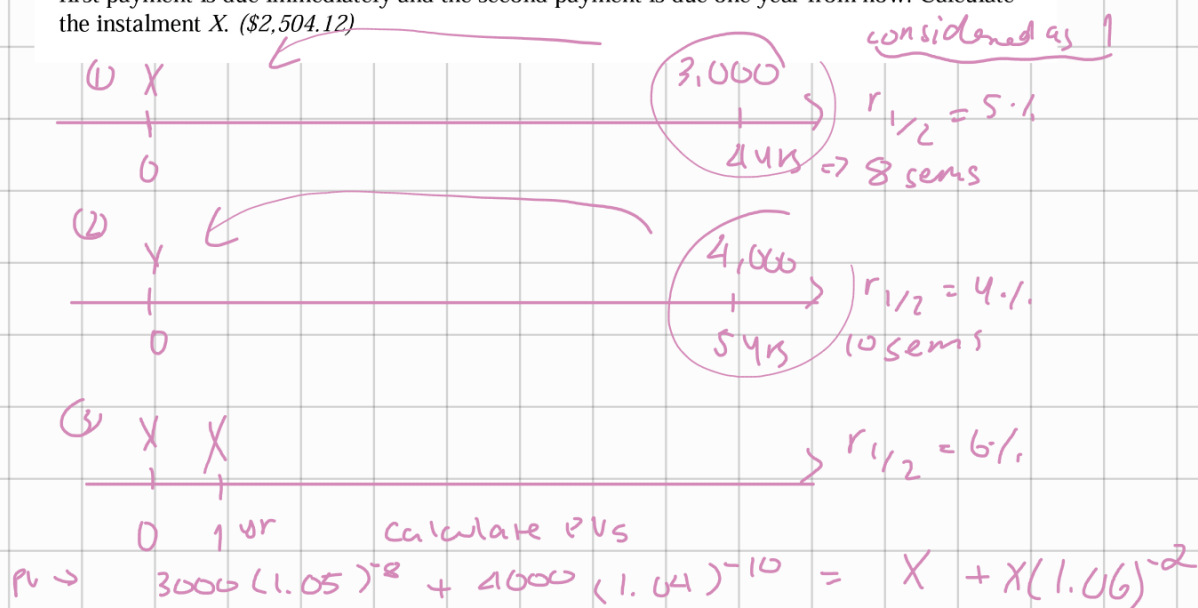


20. You are given two loans, with each loan to be repaid by a single payment in the future.

The first loan is to be repaid by a single payment of \$3,000 at the end of four years. The interest on this loan is accrued at a semi-annual rate of 5%.

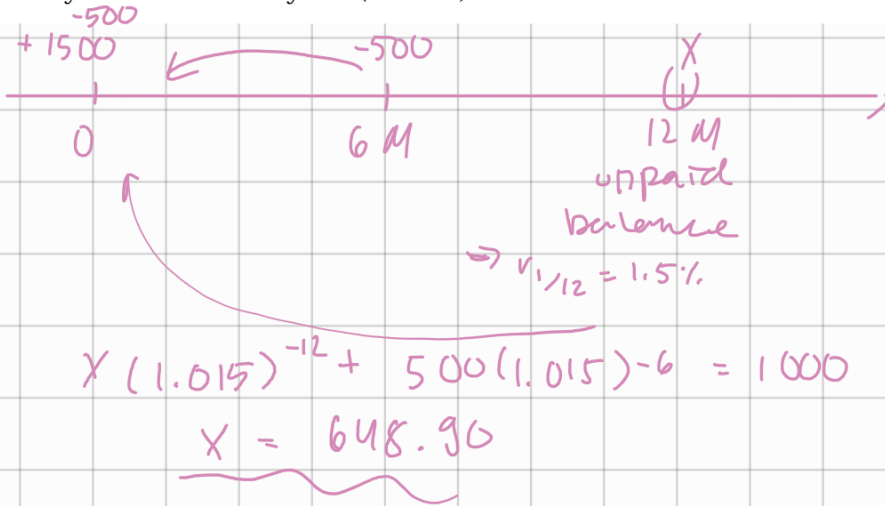
The second loan is to be repaid by a single payment of \$4,000 at the end of five years. The interest on this loan is accrued at a semi-annual rate of 4%.

Now suppose that the two loans are to be consolidated (i.e. considered as one). The consolidated loan is to be repaid by two equal instalments of size  $X$ , with interest at 6% per half year. The first payment is due immediately and the second payment is due one year from now. Calculate the instalment  $X$ . (\$2,504.12)



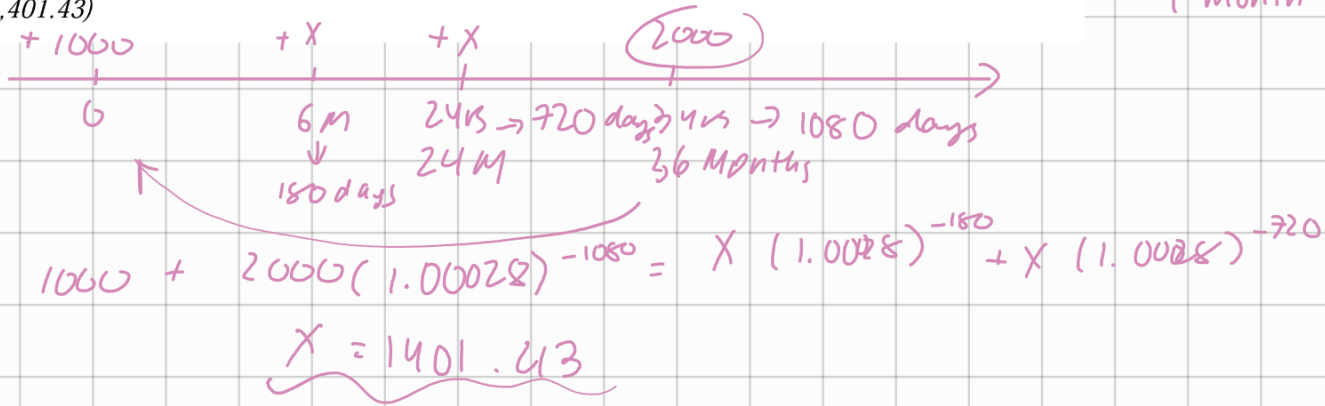
$$X = 2504$$

21. A consumer buys goods worth \$1,500, paying \$500 down and \$500 at the end of six months. If the store charges interest at  $r_{1/12} = 1.5\%$  on the unpaid balance, what final payment will be necessary at the end of one year? (\$648.90)

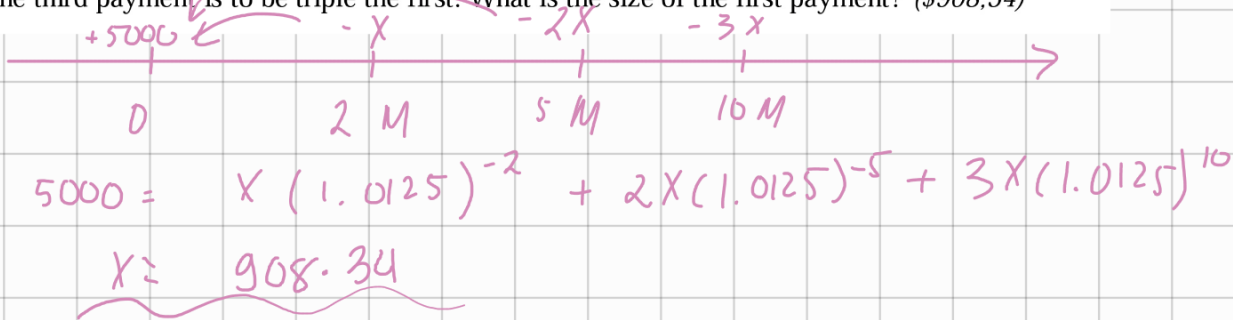


1 year = 360 days  
1 month = 30 days

22. If money is worth 0.028% daily, what equal payments  $X$  at the end of six months and two years will equitably replace the obligations: \$1,000 due now and \$2,000 due in three years? (\$1,401.43)



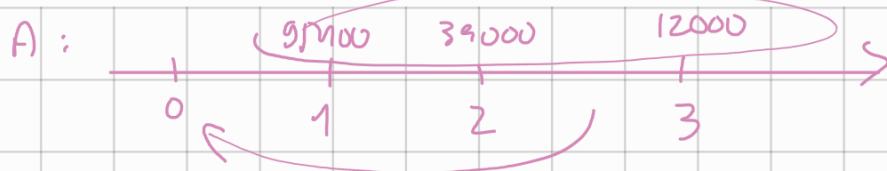
23. To pay off a loan of \$5,000 at a monthly rate of 1.25%, Mrs. Jones agrees to make three payments in two, five and ten months respectively. The second payment is to be double the first and the third payment is to be triple the first. What is the size of the first payment? (\$908.34)



24. The management of a company must decide between two proposals, on the basis of the following information:

Proposal	Investment Now	Net Cash Inflow at the End of		
		Year 1	Year 2	Year 3
A	80 000	95 400	39 000	12 000
B	100 000	35 000	58 000	80 000

Advise management regarding the proposal that should be selected, assuming that on projects of this type the company can earn  $r = 14\%$ . (Proposal A is the best. Hint: Calculate the Net Present Value ([Link to Investopedia](#)) of each proposal and choose the proposal with the highest Net Present Value)



$$\Rightarrow PV = 95400(1.14)^{-1} + 39000(1.14)^{-2} + 12000(1.14)^{-3} - 85400$$

$$PV = 41,793$$

B: something

$$PV = 29,328.59$$

Choose A because it creates more money

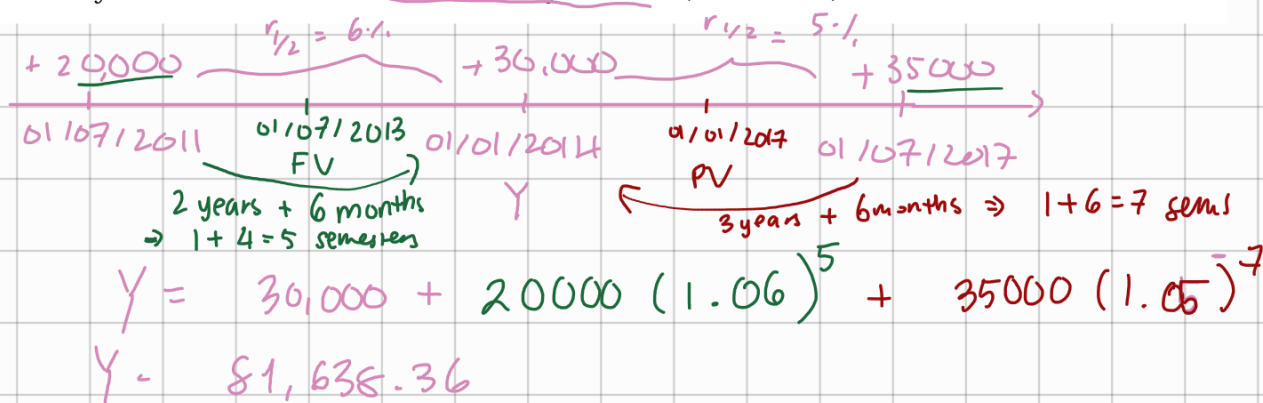
25. A company wishes to replace the following three debts:

\$20,000 due on July 1, 2011

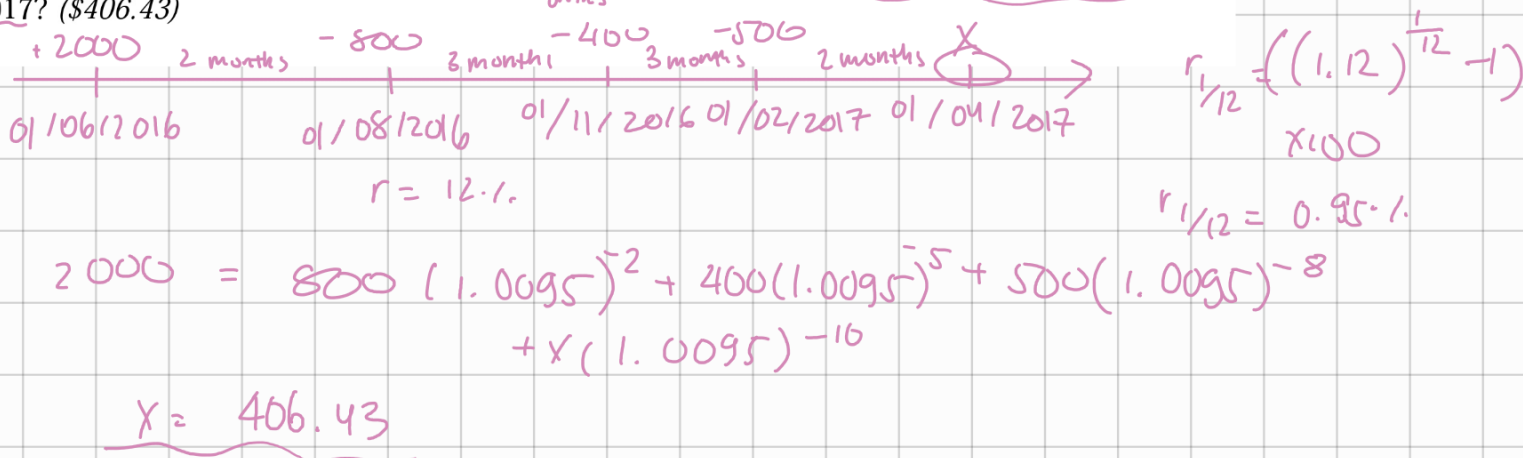
\$30,000 due on January 1, 2014

\$35,000 due on July 1, 2017

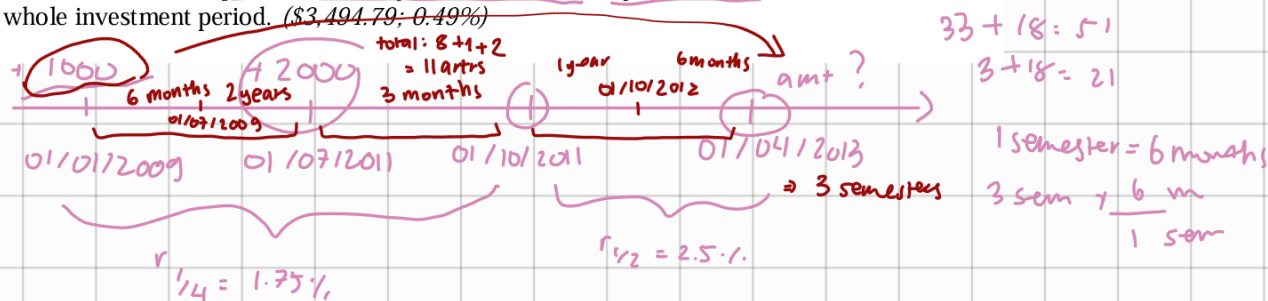
with a single debt of \$Y payable on January 1, 2014. Calculate the value of Y if  $r_{0.5} = 6\%$  prior to January 1, 2014 and  $r_{0.5} = 5\%$  after January 1, 2014. (\$81,638.36)



26. On June 1, 2016, Sheila borrows \$2,000 at 12% annually. She pays \$800 on August 1, 2016, \$400 on November 1, 2016 and \$500 on February 1, 2017. What is the balance due on April 1, 2017? (\$406.43)



27. \$1,000 was deposited on January 1, 2009 and \$2,000 was deposited in an account on July 1, 2011. Interest was paid on the account at 1.75% quarterly from January 1, 2009 to October 1, 2011 and at 2.5% semi-annually from that day onwards. Find the amount in the account on April 1, 2013 and the equivalent monthly interest rate actually earned on the investment over the whole investment period. (\$3,494.79; 0.49%)



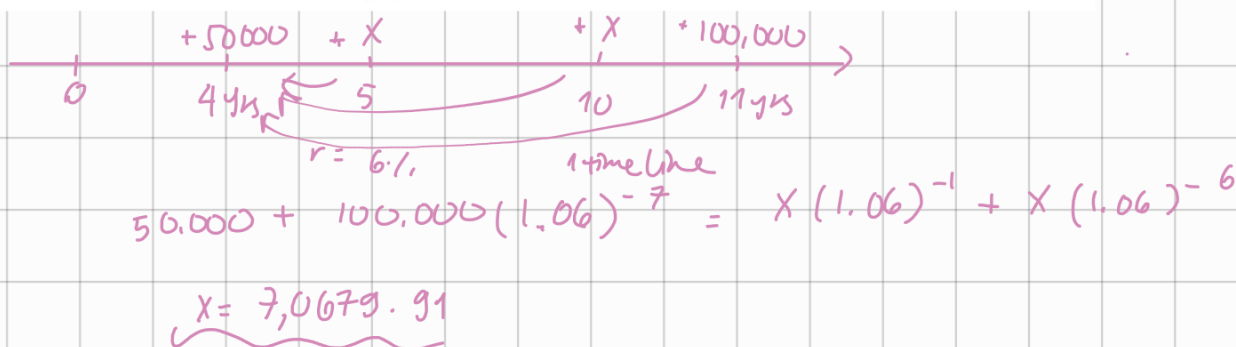
$$FV = 1000(1.0175)^{11} \cdot (1.025)^3 + 2000(1.0175)^1 \cdot (1.025)^3$$

$$FV = 3494.79$$

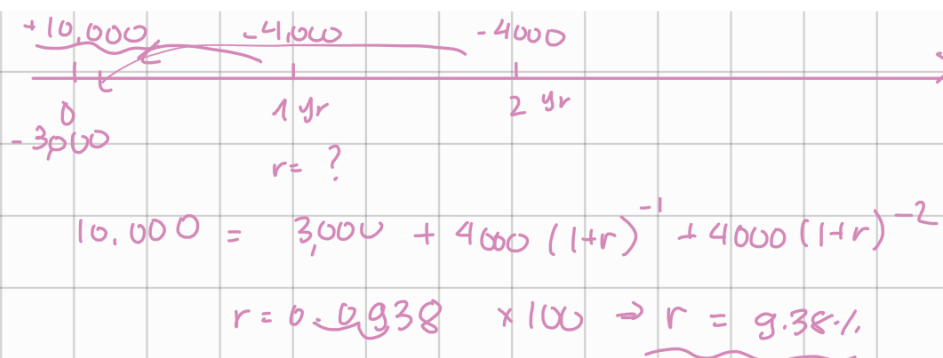
$$\Rightarrow 3494.79 = 1000(1 + r_{1/12})^{51} + 2000(1 + r_{1/12})^{21}$$

$$r_{1/12} = 0.0049 \times 100 = 0.49\%$$

28. A debt of €50,000 due in four years and a debt of €100,000 due in 11 years will be replaced by two equal payments in five and in ten years respectively. If money is worth 6% per annum (real rate of return), what are these payments? (€70,679.91)

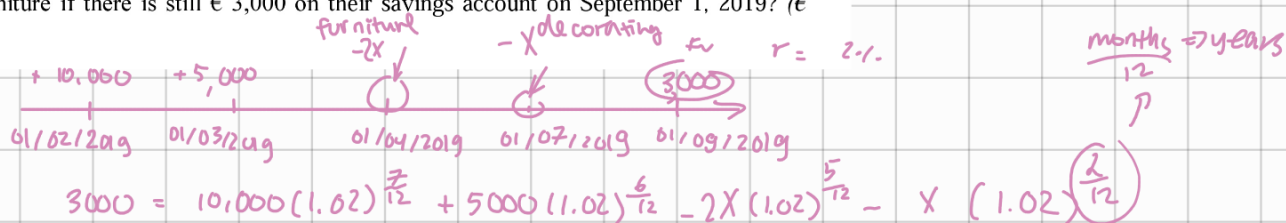


29. You don't have the €10,000 you are due today. Your banker proposes to replace your debt by payments of €3,000 today and payments of €4,000 in one year and in two years respectively. Compute the real rate of return your banker charges. (9.38%)



#### Previous exam question

Elisabeth and Philip, a young couple, decide to rent a house and see how living together works out. As a starting budget, Elisabeth deposits € 10,000 on February 1 2019 and Philip deposits € 5,000 on March 1, 2019 on a savings account with an annual rate of return of 2%. On April 1, 2019, they buy some new furniture and on July 1, 2019 they buy some other decorating stuff. If the cost of the decoration is half the cost of the furniture, how much can they spend maximum on the furniture if there is still € 3,000 on their savings account on September 1, 2019? (€ 8,057.24)



$$X = 4028.62 \leftarrow$$

$$\text{But } \rightarrow \text{ on furniture} = 2 \cdot X = \underline{8,057.24}$$