

Chapter 4: BONDS

1. A 20-year €5,000 bond maturing at 103% on October 1, 2039, has annual coupons of 5.25%. Find the purchase price on October 1, 2024, if the yield to maturity is 4.75%. (106.77% or €5,338.71)

$$n = 15 \text{ years}$$

$$r = 4.75\%$$

$$c = 5.25\%$$

$$P = c \times a_{n|r} + \frac{D}{(1+r)^n}$$

$$D = 103\%$$

$$P = ?$$

$$P = 5.25 \cdot a_{15|4.75} + 103 (1.0475)^{-15} \quad \text{!!} = 106.77\%$$

October 1
2039

October 1
2024

$$\rightarrow P = 5000 \times 106.77\% = 5338.5$$

2. A €1,000 bond, redeemable at par on December 1, 2028, pays annual coupons of 4.5%. The bond was issued on December 1, 2023. Find the issue price if the required yield is 4%. (102.23% or €1,022.26)

December 1,
2023

December 1,
2028

$$n = 5 \text{ years}$$

$$D = 100\% \quad c = 4.5\%$$

$$P = ?$$

$$r = 4\%$$

$$P = 4.5 a_{5|4} + 100 (1.04)^{-5}$$

$$P = 102.23\%$$

$$\text{or } \underline{1022.3}$$

3. A 2-year €1,000 bond with a purchase price of 95.68% is redeemable at par. The bond pays half-yearly coupons of 3%. Find the real annual return when the investor holds the bond for 2 years? (8.57%)

$$P = 95.68\%$$

$$D = 100\%$$

$$C_{1/2} = 3\% \rightarrow C_{1/1} = 3$$

$$r = ?$$

$$n = 2 \text{ yrs}$$

$$95.68 = 3 a_{\overline{4}|r_{1/2}} + 100 (1+r)^{-4}$$

$$\frac{1 - (1+r)^{-n}}{r}$$

$$r_{1/2} = 4.196\%$$

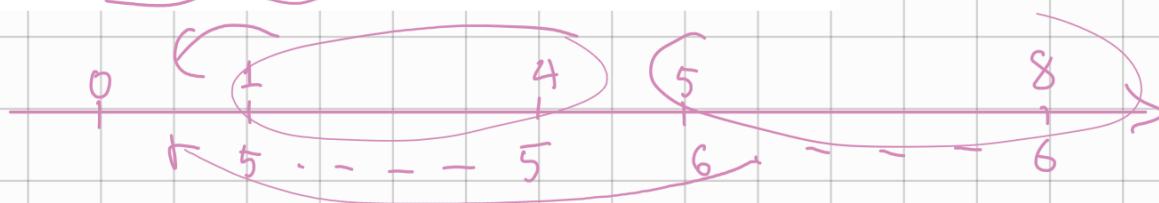
$$r \rightarrow ((1.04196)^2 - 1) \times 100$$

$$r = 8.57\%$$

4. You buy an 8-year bond with the following features:

- Annual coupons of 5% the first 4 years
- Annual coupons of 6% the last 4 years
- Redemption value is 102%
- Annual return is 4.53%

Find the price of this bond. (107.5%)



$$D = 102\%$$

$$r = 4.53\%$$

$$P = ?$$

$$P = 5 a_{\overline{4}|4.53} + 6 \times a_{\overline{4}|4.53} \cdot (1.0453)^{-4} + 102 (1.0453)^{-8}$$

$$P = 17.93 + 18.02 +$$

$$P = 107.5\%$$

5. A bond with face value of €1,000 was issued on February 4th, 2016. Its maturity was 8 years, the coupon rate 3% in nominal terms and the bonds were repaid at par. Investors claimed a net yield of 1.85% at the time. Assume a withholding tax of 30%.

- a) At what price was the bond issued? (€1,018.43)
b) What was the gross return to the investor? (2.74%)

$$C = 3\% \quad n = 8$$

$$1 - 0.30$$

$$r = 1.85\%$$

$$D = 100\%$$

$$P = ?$$

$$(a) P = C \cdot a_{\overline{n}|r} + D \cdot (1+r)^{-n}$$

$$P = 30 \cdot (1-0.30) a_{\overline{8}|1.85\%} + 1000 \cdot (1.0185)^{-8}$$

$$P = 1018.43$$

$$(b) \text{ gross return} = ?$$

$$1018.43 = 30 \cdot a_{\overline{8}|R} + 1000(1+R)^{-8}$$

$$R = 2.74\%$$

6. Today you buy a bond that is redeemable at 101% in 4 years and 2 months. Annual coupons are 4% and the return is 3.8%.

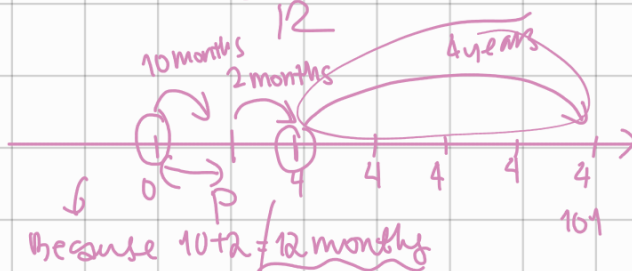
- a) What is the accrued interest today? (3.33%)
b) What is the quoted price today for this bond? (101.61%)

$$D = 101\%$$

$$n = 4 + \frac{2}{12}$$

$$r = 3.8\%$$

$$P = ?$$



Should be
1 year before payment
because of PV of annuities

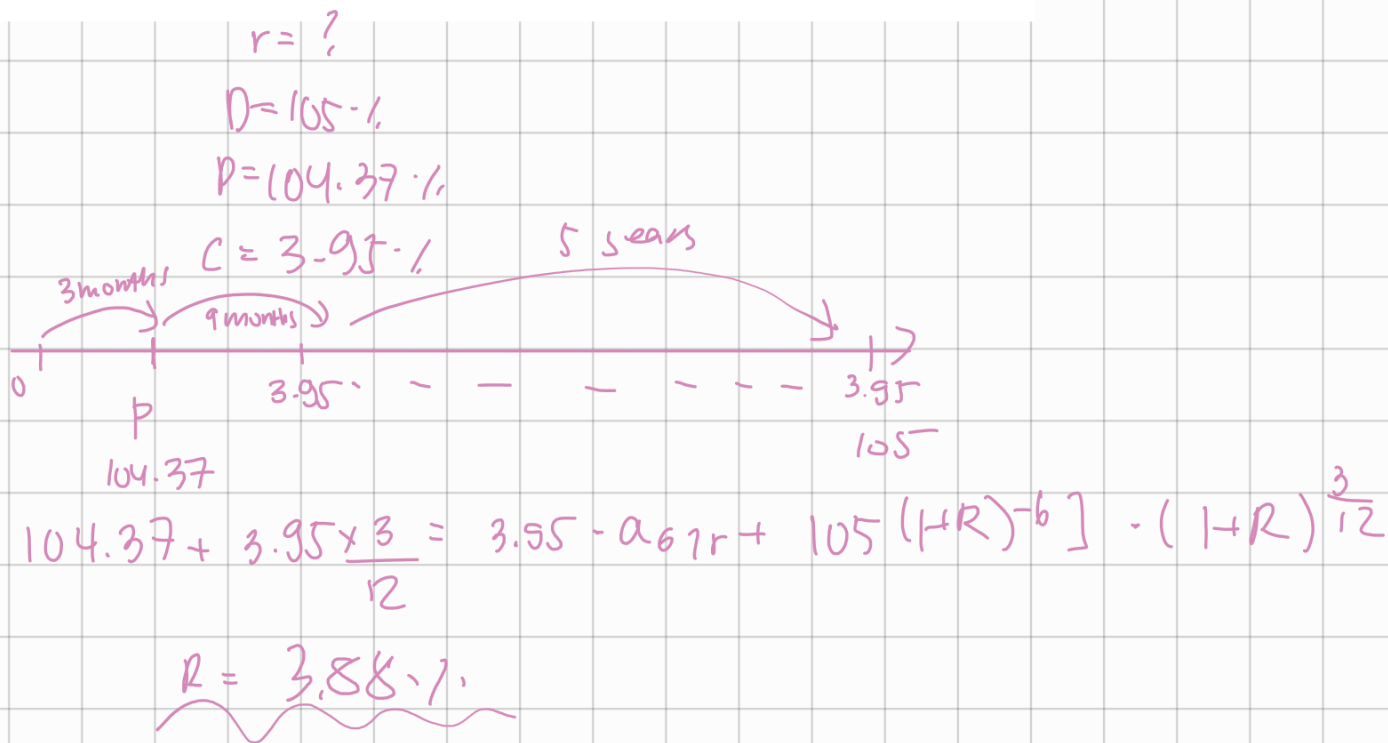
$$(a) \text{ Face value} \times \text{coupon rate} \times \text{Accrual period} \\ \Rightarrow 4 \times \frac{10}{12} = 3.33\%$$

$$(b) P + C \cdot (1-t) = (C \cdot a_{\overline{n}|r} + D)(1+R)^{-n} \times (1+r)^{1-t}$$

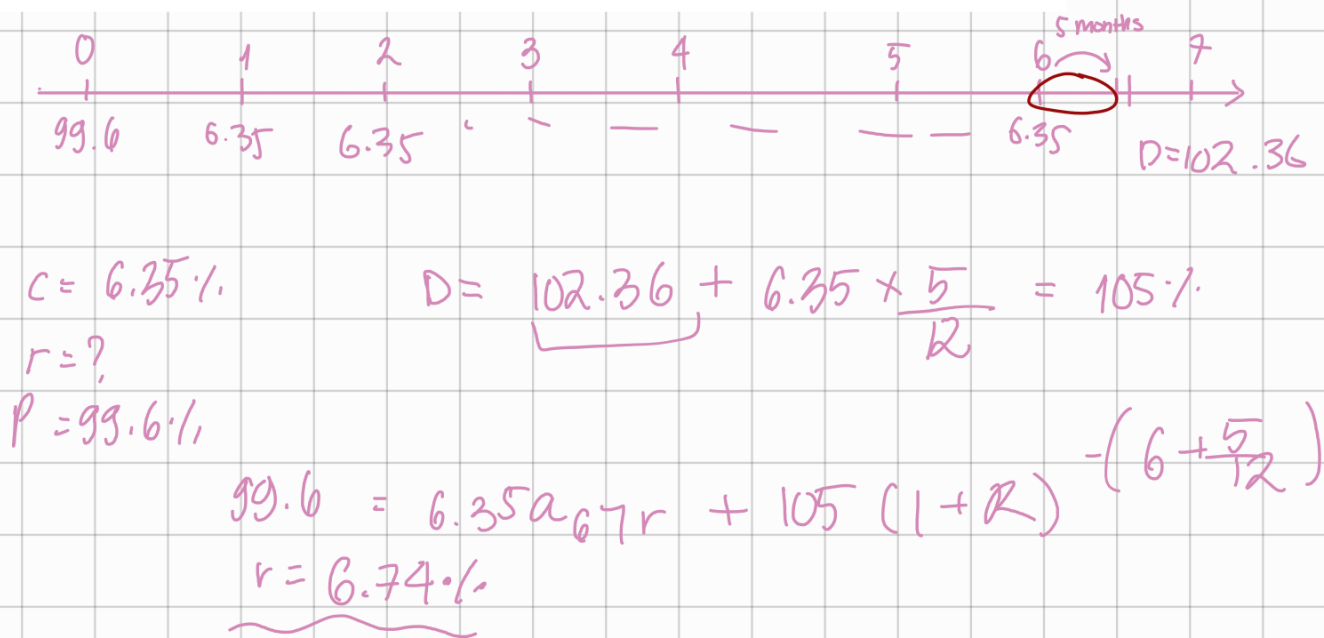
$$P + 3.33 = (4 \cdot a_{\overline{5}|3.8} + 101(1.038)^{-5}) \times (1.038)^{\frac{10}{12}}$$

$$P = 101.61\%$$

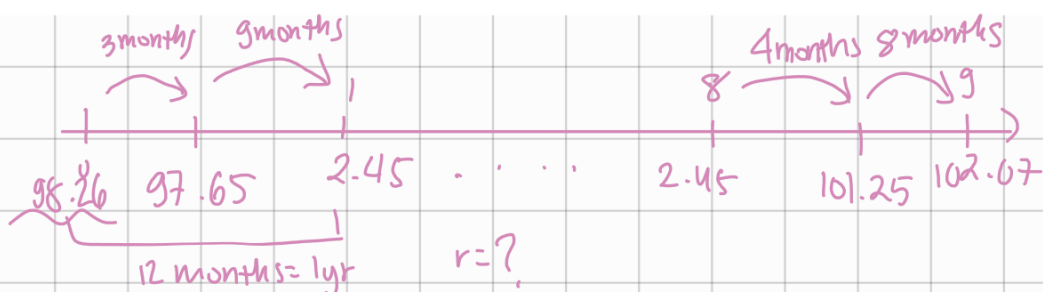
7. Today you buy a bond from ABC Company at a quoted price of 104.37%. The bond is 5 years and 9 months before the final maturity date, has annual coupons of 3.95% and is redeemed at 105%. What yield to maturity do you get? (3.88%)



8. Assume you buy a bond from DEF Company at issue at a price of 99.6% and sell it at a quoted price of 102.36% five months after the sixth coupon date but before its maturity date. The bond has annual coupons of 6.35%. What is the yield you realize on this investment? (6.74%)



9. GHI Company decides to issue a bond at an annual coupon rate of 2.45%. Assume you buy this bond at a quoted price of 97.65% three months after the issue date and sell it eight months before the ninth coupon at a quoted price of 101.25%. What yield do you realize on your investment? (2.92%)



$$D = 101.25 + 2.45 \times \frac{4}{12} = 102.07\%$$

because 1 month +
it 3 months
after issued
4 - 3 months

$$P = 97.65 + 2.45 \times \frac{3}{12} = 98.26\%$$

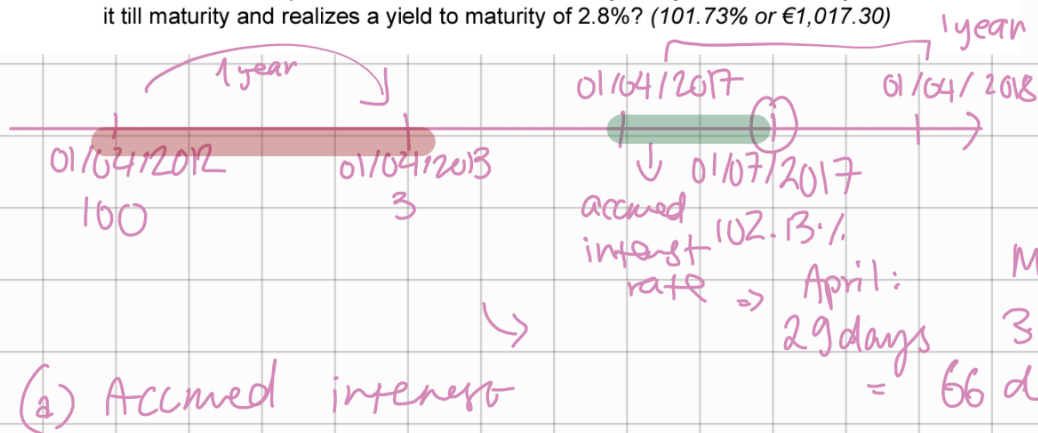
$r = ?$

$$98.26 = 2.45 a_{\overline{8}|r} \cdot (1+r)^{\frac{3}{12}} + 102.07 (1+r)^{-\left(8 + \frac{1}{12}\right)}$$

$$r = 2.92\%$$

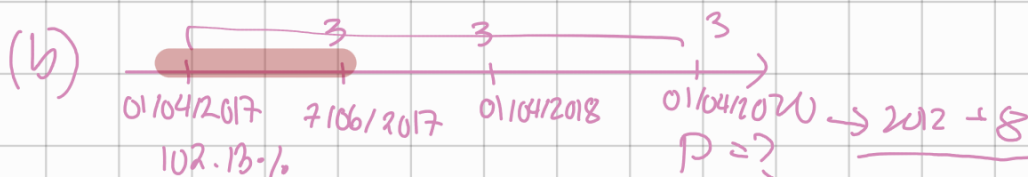
Previous exam questions

- A divorce forces Tim and Carla to sell their bonds. The one with a face value of €1,000 was bought at par on April 1, 2012 with a maturity of 8 years and a coupon rate of 3%. Today, June 7, 2017, the bond is sold at 102.13% of face value.
 - What is the accrued interest on this bond today? Assume a year has 360 days. (0.55%)
 - What is the redemption value of the bond if the buyer buys the bond today and holds it till maturity and realizes a yield to maturity of 2.8%? (101.73% or €1,017.30)



(a) Accrued interest

$$= 3 \times \frac{66}{360} = 0.55\%$$



$$102.13 + 0.55 = \left(3 \times a_{\overline{8}|2.8\%} + D \cdot (1.028)^{-8} \right) \times (1.028)$$

$$D = 101.73\%$$

- Britney thinks it is interesting to issue 'Britney bonds'. These bonds are issued at 101.5%, have a coupon rate of 3%, a maturity of 15 years, and are redeemable at par.
 - What is the yield of this bond? (2.88%, rounded)
 - What will be the price of a similar, same maturity, zero bond? (65.36%)

$r = ?$

$C = 3\%$

(a) $D = 100$

$n = 15$

$P = 101.5$

$$101.5 = 3 a_{\overline{15}|R} + 100(1+R)^{-15}$$

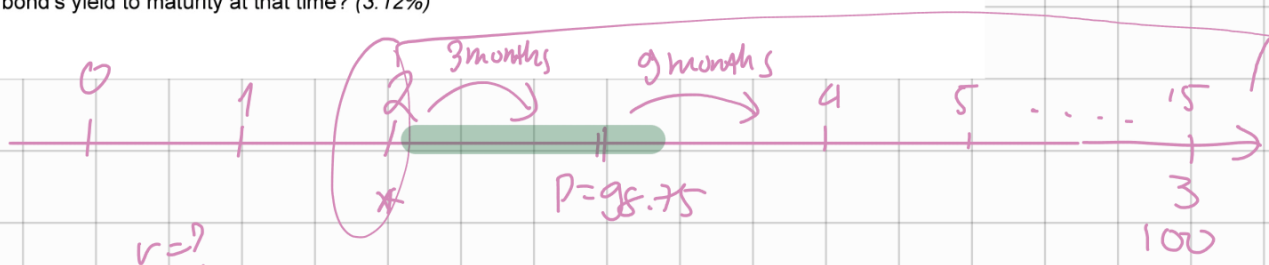
$$R = 2.88\%$$

(b) zero bond:

$$P = \frac{D}{(1+r)^n} = \frac{100}{(1.0288)^{15}} = 65.32\%$$

3. 2 years and 3 months later, the 'Britney bond' has a quoted price of 98.75%. What is the bond's yield to maturity at that time? (3.12%)

$$15 - 2 = 13$$



$$98.75 + 3 \times \frac{3}{100} = (3 \cdot 0.137R + 100(1+R)^{-13}) \times (1+R)^{\frac{3}{12}}$$

$$R = 3.12\%$$