

1. Today, January 1, 2022, Cindy is worried about her ability to pay the bills. As a single mom with 2 kids, she faces increasing costs, but her income is rather limited. She receives a monthly allowance of €800 in the beginning of each month from her ex-husband. She also earns €1500 a month as a waitress, which is always paid at the end of month. How much can she spend at the end of every month in 2022 if she does not save and she expects that her monthly costs increase by 5% every 4 months? The real annual interest rate is 1.2%. (1.5p)

$$r = 1.2\% \quad r_{\frac{1}{12}} = 0.099\%$$

$$\begin{aligned} PV &= 1500 \cdot a_{\overline{12}|0.099\%} = 17\,884.70 \\ PV &= (800 \cdot a_{\overline{12}|0.099\%}) \cdot 1.00099 = 9547.95 \end{aligned} \quad \left. \vphantom{\begin{aligned} PV &= 1500 \cdot a_{\overline{12}|0.099\%} \\ PV &= (800 \cdot a_{\overline{12}|0.099\%}) \cdot 1.00099 \end{aligned}} \right\} 27,432.65$$

let S be monthly outflow. Let a^n be discount factor $(1+r)^n$

$$\begin{aligned} PV(\text{month } 1-4) &= S \cdot \bar{a}^1 + S \cdot \bar{a}^2 + S \cdot \bar{a}^3 + S \cdot \bar{a}^4 \\ PV(\text{month } 5-8) &= 1.05 \cdot S \cdot \bar{a}^5 + 1.05 \cdot S \cdot \bar{a}^6 \dots + \\ PV(\text{month } 9-12) &= (1.05)^2 \cdot S \cdot \bar{a}^9 + (1.05)^2 \cdot S \cdot \bar{a}^{10} \dots \end{aligned} \quad \left. \vphantom{\begin{aligned} PV(\text{month } 1-4) &= S \cdot \bar{a}^1 + S \cdot \bar{a}^2 + S \cdot \bar{a}^3 + S \cdot \bar{a}^4 \\ PV(\text{month } 5-8) &= 1.05 \cdot S \cdot \bar{a}^5 + 1.05 \cdot S \cdot \bar{a}^6 \dots + \\ PV(\text{month } 9-12) &= (1.05)^2 \cdot S \cdot \bar{a}^9 + (1.05)^2 \cdot S \cdot \bar{a}^{10} \dots \end{aligned}} \right\} \begin{array}{l} \text{find } S, \text{ so sum} \\ \text{of all equals} \\ 27,432.65 \end{array}$$

$$\begin{aligned} S &= 2,189.84 \quad (\text{month } 1-4) \\ S(1.05) &= 2299.33 \quad (\text{month } 5-8) \\ S(1.05^2) &= 2414.30 \quad (\text{month } 9-12) \end{aligned}$$

2. Unfortunately, her ex-husband Jim refuses to pay the allowance of €800 a month from January till April. A judge therefore decides that he should pay them anyway. He should pay €1,632.24 on June 1, 2022 and the same amount on September 1, 2022. What is the annual interest rate the judge has taken into account in this case? (1p)

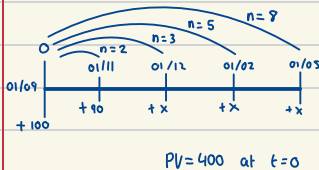
Amount should have been paid = 3200, actual = 3264.48

Find PV of each on Jan 1st and set payment equal

$$(800 \cdot a_{\overline{4}|r}) \cdot (1+r) = 1632.24 \cdot (1+r)^6 + 1632.24 \cdot (1+r)^9 \quad \text{solve } (r)$$

$$r = 0.004 \quad \therefore (1.004^{12} - 1) \cdot 100 = 4.91\% \quad \text{annual I.R.}$$

3. September is a very expensive month for Cindy. Her 14-year-old daughter has to buy a laptop which costs her €400 on September 1, 2022. An alternative is that she pays €100 on September 1 and €90 on November 1, 2022. How much will she have to pay on December 1, 2022, on February 1, 2023 and on May 1, 2023 if the annual interest rate is 3% and the amount in those 3 payments is constant? (1p)



$$r_{\frac{1}{12}} = 0.247\% \quad \text{or } 0.00247$$

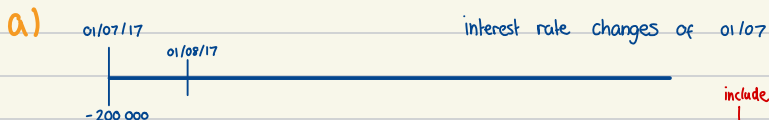
$$\text{let } r = 1.00247$$

$$400 (PV) = 100 + \frac{90}{r^2} + \frac{x}{r^3} + \frac{x}{r^5} + \frac{x}{r^9} \quad \text{Solve for } x$$

$$x = 71.07 \quad \text{each payment}$$

4. Given her limited financial resources Cindy was given the ability to buy a house on July 1, 2017. To finance it she obtained a mortgage of €200,000 with a maturity of 25 years that she pays off with a constant annuity, paid for the first time on August 1, 2017. The mortgage has a variable interest rate that will be revised every 5 years. Her initial real annual interest rate was 1% but five years later it increased to 2%. (1.5p)

- a) If she keeps the maturity of the loan constant, how much will she pay from August 1, 2022 on?
b) How much interest does she pay in the first 10 years?



$$200\,000 = X \cdot a_{\overline{300}|0.083\%} = PMT = 753.33$$

$$\text{balance after 60 payments} = 163,877.99 \quad (\text{balance on } 01/07/22)$$

$$163,877.99 = X \cdot a_{\overline{240}|0.17\%} \quad x = 832.15$$

- b) first 5 years (using $753.33 \cdot a_{\overline{300}|0.083\%} \leq \ln(1,60)$)
 $= 9083.06$
 Second 5 years (using $832.15 \cdot a_{\overline{240}|0.17\%} \leq \ln(1,60)$)
 $= 14,994.03$

$$\text{total interest paid} = 24\,077.93$$

5. Suppose the interest of Cindy's mortgaged doesn't change and remains 1% annually the entire 25 years. In which month does she repay more principal as compared to the situation where she would have repaid the mortgage with constant principal repayments? Also show clearly the formula that allows you to calculate the exact month using the "equation solver" of your calculator? (1p)

find when $\leq Pm(n, n) = 666.67$

$$\frac{200000}{300} = 666.67 \text{ monthly principal payments}$$

$$bal(n) = \text{Principal} \cdot \frac{(r+1)^N - (r+1)^{n-1}}{(r+1)^N - 1}$$

the -1 is to reflect that interest for next payment is calculated off compounded amount just before payment

$$\leq Int(n) = \left(\text{Principal} \cdot \frac{(r+1)^N - (r+1)^{n-1}}{(r+1)^N - 1} \right) \cdot r$$

therefore $\leq Pm(n, n) = PMT - \text{Interest}$ let $r = 0.0083$

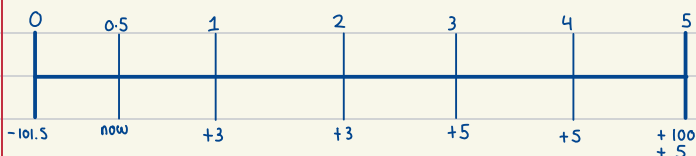
$$\leq Pm(n, n) = 753.33 - \text{Principal} \cdot \frac{(1+r)^{300} - (1+r)^{n-1}}{(1+r)^{300}} \cdot r$$

(666.67) solve for n

$n = 153.71 \Rightarrow$ the 154th payment is first with more

cannot enter $\leq Pm(n)$ into numerical solver!

6. Despite her financial worries, Cindy has found love with Wouter, a young wealthy bachelor. Since he is not a very romantic person, he decides to give her a bond instead of flowers. He bought the bond when it was issued, 6 months ago, at an issue price of 101.5 and is redeemable at par. The bond had at that time a maturity of 5 years and a coupon rate of 3% in the first 2 years and 5% afterwards. What was the yield of the bond? (1p)



$$101.5 = \frac{3}{(r+1)} + \frac{3}{(r+1)^2} + \frac{5}{(r+1)^3} + \frac{5}{(r+1)^4} + \frac{105}{(r+1)^5} \quad \text{solve for } r$$

$$r = 0.0382 \Rightarrow YTM = 3.82\%$$

7. When Cindy receives the bond, 6 months after issue, she decides to sell it immediately since she needs the money. How much money will she receive from the sale if the yield is now 2%? (1p)

$$PV = \frac{3}{(r+1)} + \frac{3}{(r+1)^2} + \frac{5}{(r+1)^3} + \frac{5}{(r+1)^4} + \frac{105}{(r+1)^5} \quad r = 0.02$$

↓

at $t=0$ $PV = 110.257$ (clean price)

$$+ 1.5 = 111.76 \cdot 1.02^{0.5} = 112.87$$

Question 1 (3 points)

- a) Dimi wants to start up his own business. Unfortunately, he doesn't have a lot of money. That's why his bank is offering him a credit cards on December 1, 2015. This allows him to spend money and only pay it back when it suits him. On January 1, 2016 Dimi spends 3,000 and then on April 1, 2017 another 5,000 euros. On January 1, 2018, he spends 150 euros every month during one year. Off course a credit card is not cheap. Dimi will need to pay an annual effective interest rate of 3.2% for the first 12 months and an annual effective interest rate of 16.8% for the period after. What does Dimi need to repay the credit card company on December 1, 2018?

CC offer : 01/12/15

$r = 3.2\%$ (01/12/15 - 01/12/16)

$r = 16.8\%$ (01/12/16 - ...)

3000 : 01/01/16

5000 : 01/04/16

$r_{1/12} = 0.26\%$

150 : 01/01/17 - 01/12/17

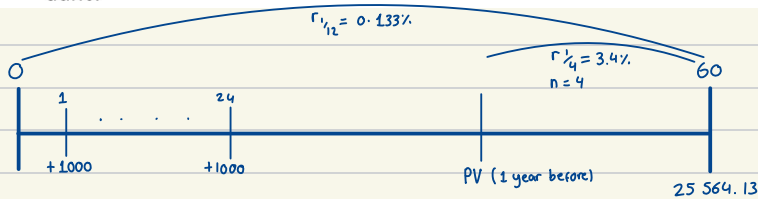
Value on 01/12/17

$$\left. \begin{array}{l} 3000 \cdot (1.00263)^{11} \cdot (1.01303)^{12} \\ 5000 \cdot (1.00263)^8 \cdot (1.01303)^{12} \end{array} \right\} 9570.58$$

$$150 \cdot S_{\overline{12}|1.303\%} = 1934.77$$

$$\text{amount owed} = \$11,505.35$$

- b) Dimi's favorite aunt wants to help him to start his business. She is going to sell a financial product that she once bought and she is going to give him this money. The financial product she invested in, was a fund that would pay her back after five years at $r_{1/2} = 0.8\%$. During the first 2 years, she had to invest 1,000 euros at the beginning of every month. Then the money would stay in the fund for another three years. Today, after four years, she is going to sell the product to someone who wants to earn a quarterly rate of $r_{1/4} = 3.4\%$. What sum of money will Dimi receive from his favorite aunt?



$$FV \text{ (at end of 5 years)} = (1000 \cdot S_{\overline{24}|0.133\%}) \cdot 1.008^6 = 25,564.13$$

$$25,564.13 \cdot (1.034)^{-4} = 22,363.96$$

- c) Another option is to set up an endowment mortgage. He would also borrow 50,000 euros, but pay off the full amount after 10 years. In order to be able to pay this amount after 10 years, he will deposit money each month in a fund which will earn him 0.7% monthly. What is the interest rate charged by the bank for this loan if you know that the total monthly installment is 300 euros? (1 point)

$$\text{Payment} = \text{Interest} + \text{Saving}$$

$$50\,000 = x \cdot S_{\overline{120}|0.7\%} \quad x = 267.26$$

$$\Rightarrow 300 - 267.26 = 32.74 \cdot 120 = 3274.25 \quad (\text{total interest})$$

$$50000 \cdot r \text{ (monthly)} = 32.74 \quad r_{1/2} = 0.065\%$$

Question 2 (3 points)

- a) A few years ago Dimi bought himself a house. He received a loan from his bank of 250,000 euros that he needed to pay back with constant monthly payments during 20 years at $r_{1/12} = 0.12\%$. After five years of paying back the loan, Dimi wants to borrow an additional amount to buy a warehouse for his business. The amount is equal to the principal that has been repaid in that period. How much can he borrow? (0.5 points)

$$250\,000 = x \cdot a_{\overline{240}|0.12\%} \quad x = 1199.48$$

$$\leq \text{Prn}(1, 60) = 55,924.20 \text{ principal paid}$$

- b) He finally decides to borrow 50,000 euros on top of his existing loan. It is also agreed that the interest rate will not change and that the remaining maturity remains 15 years for both loans. They agree that he will pay 1,400 euros as a monthly installment which is always paid at the end of the month in those 15 years. How much will he have pay to the bank to pay off both loans (1.5 points)

$$\text{Principal outstanding} = 250000 - 55\,924.20 = 194,075.80$$

$$\text{PV (principal)} = 244,075.80 \Rightarrow \text{FV} = 302,884$$

$$\text{FV} = 1400 \cdot S_{\overline{180}|0.12\%} = 281,098.62 \quad (\text{fv amount he paid})$$

$$= 21,785.21$$

Method 2 - (In finance solver)

$$N = 180$$

$$I(\%) = 0.12$$

$$\text{PV} = 244,075.80$$

$$\text{PMT} = -1400$$

$$\text{FV} = 21,785.21$$

- c) Dimi will put 20,000 euros from the money he receives from his aunt on an account. During the first year he is going to withdraw an amount every month. The second year he will withdraw the double of that amount every month. And then in the third year the triple of that amount. What amount will Dimi be able to withdraw when the monthly rate is $r_{1/12} = 0.15\%$

$$r = 1.815\%$$

$$20000 = \frac{x}{1.01815} + \frac{2x}{1.01815^2} + \frac{3x}{1.01815^3}$$

$$x = 3475.90 \Rightarrow \text{in first year can spend } 289.66 / \text{month}$$

$$\text{second year} = 579.32 / \text{month}$$

$$\text{third year} = 868.98 / \text{month}$$

Question 3 (1.5 points)

- a) When Dimi was a student, he was inspired by Professor Paepen to buy a bond. He bought a bond which is redeemable at par after 3 years with coupons of 5% at a yield of maturity of 5.5%. What is the duration of this bond? (0.5 points)

$$\text{Maturity} = 100 \quad \text{Coupon Payments} = 5 \quad \text{YTM} = 5.5\%$$

$$\text{Macaulay Duration} = D = \frac{\left(\sum_{t=1}^n t \cdot \frac{C}{(1+YTM)^t} + n \cdot \frac{\text{Par}}{(1+YTM)^n} \right)}{\text{PV of Bond}}$$

time weighted PV see (2)
PV see (1)

$$1 \quad PV = \frac{5}{1.055} + \frac{5}{1.055^2} + \frac{105}{1.055^3} = 98.65$$

PV (time weighted)

$$\frac{281.98}{98.65} = 2.86 \text{ yrs}$$

$$2 \quad \left(\frac{5}{1.055} \cdot 1 \right) + \left(\frac{5}{1.055^2} \cdot 2 \right) + \left(\frac{105}{1.055^3} \cdot 3 \right) = 281.98$$

a) this means time expected on average to recover investment

therefore larger duration = price changes more with int Δ

b) this is direct correlation between interest & price
eg) int ↑ 1% ⇒ Price ↓ 2.86%